Spurious regression and econometric trends

Antonio E. Noriega[1]
Daniel Ventosa-Santaulària[2]

School of Economics
University of Guanajuato

JEL Classi…cation: C12 ,C13, C22
Keywords: Spurious regression, trends, unit roots, trend stationarity, structural breaks

Abstract

This paper analyses the asymptotic and …nite sample implications of di¤erent types of nonstationary behavior among the dependent and explanatory variables in a linear spurious regression model. We study cases when the nonstationarity in the dependent and explanatory variables is deterministic as well as stochastic. In particular, we derive the order in probability of the t¡statistic in a spurious regression equation under a variety of empirically relevant data generation processes, and show that the spurious regression phenomenon is present in all cases when at least one of the variables behaves in a nonstationary way. Simulation experiments con…rm our asymptotic results.

[1] Escuela de Economia, Universidad de Guanajuato., e-mail: noriegam@quijote.ugto.mx.
[2] Corresponding author. Escuela de Economia, Universidad de Guanajuato. UCEA, Campus Mar…l, Fracc. I, El Establo, Guanajuato, Gto. 36250, México. Phone and Fax +52-(473)7352925, e-mail: daniel@ventosa-santaularia.com

1 Introduction

It has been documented in recent studies that the phenomenon of spurious regression is present under different forms of nonstationarity in the data generating process (DGP). In particular, when the variables y_t and x_t are nonstationary, independent of each other, ordinary least squares applied to the regression model

$$y_t = \alpha + \delta x_t + u_t$$

have the following implications: 1) the estimator of δ ($\widehat{\delta}$) does not converge to its true value of zero, and 2) the t-statistic for testing the null hypothesis $H_0 : \delta = 0$ ($t_{\widehat{\delta}}$) diverges, thus indicating the presence of an asymptotic spurious relationship between y_t and x_t.

The rate at which $t_{\widehat{\delta}}$ diverges depends on the type of nonstationarity present in the process generating y_t and x_t. In Phillips (1986), where a driftless random walk is assumed for both variables, the t-statistic is $O_p(T^{1/2})$. For the case of a random walk with drift, Entorf (1997) shows that $t_{\widehat{\delta}}$ diverges at rate T. More recently, Kim, Lee and Newbold (2004) (KLN henceforth) show that the phenomenon of spurious regression is still present even when the nonstationarity in individual series is of a deterministic nature: they find that, under a linear trend stationary assumption for both variables, the t-statistic is $O_p(T^{3/2})$. Extending KLN's results, Noriega and Ventosa-Santaulària (2005) (NVS hereafter), show that adding breaks in the DGP still generates the phenomenon of spurious regression, but at a reduced divergence rate; i.e. $t_{\widehat{\delta}}$ is $O(T^{1/2})$ under either single or multiple breaks in each variable. In all these works, the implicit assumption is that both variables share the same type of nonstationarity, either stochastic (Phillips, Entorf), or deterministic (KLN, NVS).[3]

Although the literature on this issue has grown considerably, there are still gaps, particularly when regressions involve variables with different types of trending mechanisms. The purpose of the present paper is to fill these gaps. Our results uncover the presence of spurious regressions under a wide variety of combinations of empirically relevant DGPs, not explored before in the literature. For instance, we consider regressions of a random walk with drift on a trend (with and without breaks)-stationary process (and viceversa). We find that the rate at which the phenomenon occurs is generally $T^{1/2}$, as predicted by Phillips (1998). However, for a few combinations of trending mechanisms the divergence rate is higher. We also show that the spurious regression vanishes when one of the variables is stationary.

The rest of the paper is organized as follows. Section 2 discusses the DGPs considered. Section 3 and 4 present the main asymptotic results, and Monte Carlo evidence for finite samples, respectively. Section 5 concludes.

[3] Some related papers share this same feature: Marmol (1995, 1996, 1998), Cappuccio and Lubian (1997), Granger et. al. (1998) and Tsay and Chung (1999).

2 Trending mechanisms in the data generating process

In a simple regression equation, the nature of the trending mechanism in the dependent and explanatory variables is unknown a priory. This is mainly due to a lack of economic knowledge on trending mechanisms. We study the spurious regression phenomenon under eight different DGPs, widely used in applied work in economics.

We consider the following spurious ordinary least squares regression model:

$$y_t = \hat{\alpha} + \hat{\delta} x_t + \hat{u}_t \tag{1}$$

used as a vehicle for testing the null hypothesis $H_0 : \delta = 0$. The following assumption summarizes the DGPs considered below for both the dependent and the explanatory variables in model (1).

Assumption. The DGPs for $z_t = y_t, x_t$ are as follows.

Case	Name*	Model
1.	I(0)	$z_t = \mu_z + u_{zt}$
2.	I(0)+br	$z_t = \mu_z + \sum_{i=1}^{N_z} \theta_{iz} DU_{izt} + u_{zt}$
3.	TS	$z_t = \mu_z + \beta_z t + u_{zt}$
4.	TS+br	$z_t = \mu_z + \sum_{i=1}^{N_z} \theta_{iz} DU_{izt} + \beta_z t + \sum_{i=1}^{M_z} \gamma_{iz} DT_{izt} + u_{zt}$
5.	I(1)	$\Delta z_t = u_{zt}$
6.	I(1)+dr	$\Delta z_t = \mu_z + u_{zt}$
7.	I(1)+dr+br	$\Delta z_t = \mu_z + \sum_{i=1}^{N_z} \theta_{iz} DU_{izt} + u_{zt}$
8.	I(2)	$\Delta^2 z_t = u_{zt}$

* TS, br, and dr stand for Trend-Stationary, breaks, and drift, respectively.

Note that cases 5, 6 and 7 can be written as

$$\begin{aligned} z_t &= z_0 + S_{zt} \\ z_t &= z_0 + \mu_z t + S_{zt} \\ z_t &= z_0 + \mu_z t + \sum_{i=1}^{M_z} \theta_{iz} DT_{izt} + S_{zt} \end{aligned}$$

where $S_{zt} = \sum_{i=1}^{t} u_{zi}$, $DT_{izt} = \sum_{i=1}^{t} DU_{izt}$, z_0 is an initial condition, $u_{zt} = \phi_z u_{zt-1} + \varepsilon_{zt}$, $|\phi_z| < 1$, ε_{zt} are $iid(0, \sigma_z^2)$ independent of each other, and DU_{izt}, DT_{izt} are dummy variables allowing changes in the trend's level and slope respectively, that is, $DU_{izt} = 1(t > T_{b_{iz}})$ and $DT_{izt} = (t - T_{b_{iz}})1(t > T_{b_{iz}})$, where $1(\cdot)$ is the indicator function, and $T_{b_{iz}}$ is the unknown date of the i^{th} break in z. We denote the break fraction as $\lambda_{iz} = (T_{b_{iz}}/T) \in (0, 1)$, where T is the sample size. We maintain the same structure for the innovations u_{zt} as in KLN, although it can also be assumed that innovations obey the (general-level) conditions stated in Phillips (1986, p. 313).

Cases 1 and 2 are used to model the behaviour of (theoretically) mean stationary variables, such as real exchange rates, unemployment rates, great ratios (i.e. output-capital ratio), and the current account. Examples of $I(0)$ and $I(0)$ with breaks variables have been presented in Perron and Vogelsang (1992), Wu (2000), and D'Adda and Scorcu (2003). Cases 3 to 8 are widely used to model growing variables, real and nominal, such as output, consumption, money, prices, etc. Macro variables have been described as $I(0)$ around a linear trend, $I(0)$ around a linear trend with structural breaks, and $I(1)$ in Perron (1992, 1997), Lumsdaine and Papell (1997), Mehl (2000), and Noriega and de Alba (2001). Combinations of case 8 with other cases are often behind the empirical modelling of nominal speci…cations expressed in terms of $I(2)$ (nominal) and $I(1)$ or $I(0)$+breaks (real) variables. Economic models involving $I(2)$ variables include models of money demand relations, purchasing power parity, and in‡ation and the markup. Examples of variables described as $I(2)$ can be found in Juselius (1996, 1999), Haldrup (1998), Muscatelli and Spinelli (2000), Coenen and Vega (2001), and Nielsen (2002).

The DGPs include both deterministic and stochastic trending mechanisms, with 49 possible nonstationary combinations of them among the dependent and the explanatory variables, where case 1 is included as a benchmark.[4]

The spurious regression phenomenon has already been analyzed for a few combinations of DGPs in the assumption. For instance, the case of both variables following a unit root (case 5) was studied by Granger and Newbold (1974) and Phillips (1986), and case 6 by Entorf (1997). The case (3) of a trend-stationary model for both variables was studied by KLN, while its extension to multiple breaks (case 4) by NVS. Mixtures of integrated processes were studied by Marmol (1995), who considers cases 5 and 8 (y follows a unit root, while x follows two unit roots, and viceversa). Many other combinations, however, have not been analyzed. Among them, combinations 3-6 and 4-6, which have practical importance, given the empirical relevance of structural breaks in the time series properties of many macro variables.

3 Asymptotics for spurious regressions

In this section we present the asymptotic behaviour of the t-statistic for testing the null hypothesis $H_0 : \delta = 0$ ($t_{\widehat{\delta}}$) in model (1) when the dependent and explanatory variables are generated according to combinations of DGPs in the Assumption. Table 1 summarizes the main results.

According to our …ndings, Table 1 is symmetric, implying that the order in probability does not depend on the type of nonstationarity among dependent and explanatory variables. As expected, when both variables are stationary (case 1-1) the spurious regression phenomenon is not present, since the t-statistic collapses to zero (at rate $T^{1/2}$). When one of the variables is stationary the t-

[4] We do not consider the cases of I(1) processes with long memory errors, and fractionally integrated processes, studied in Cappuccio and Lubian (1997), and Marmol (1998), respectively.

statistic converges (to a constant, or to a random variable, depending on the DGP), as indicated in row 1, columns 2-8: the t-statistic is $O_p(1)$. For the rest of combinations, divergence at rate $T^{1/2}$ is prominent, indicating the presence of spurious regression.

Table 1. Orders in probability of $t_{\hat{\delta}}$ [1]

DGP	1	2	3	4	5	6	7	8
1	$T^{-1/2}$	1	1	1	1	1	1	1
2		$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$
3			$T^{3/2}$	$T^{1/2}$	$T^{1/2}$	T	$T^{1/2}$	$T^{1/2}$
4				$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$
5					$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$
6						T	$T^{1/2}$	$T^{1/2}$
7							$T^{1/2}$	$T^{1/2}$
8								$T^{1/2}$

1. Results for combinations 3-3 and 1-3 come from KLN; for 5-5, 6-6, 8-8, and 5-8 results come from Phillips (1986), Entorf (1997), Marmol (1995), and Marmol (1996), respectively. For the rest see the appendix at the end of the paper.

Therefore, when two independent random variables follow any of the nonstationary combinations considered in the Assumption, OLS inference will indicate, asymptotically, a significant (spurious) relationship among them.

The representation theory developed by Phillips (1998) shows that a trending stochastic (or deterministic) process can be represented in various ways. In particular, it can be written as an infinite linear combination of trending deterministic (stochastic) functions with random coefficients. In such an asymptotic environment, he shows that the regression t-ratios of the fitted coefficients diverge at rate $O_p(T^{1/2})$. Results from the Table indicate that relatively simple nonstationary time series models correctly indicate the presence of the limiting representation.[5]

4 Experimental results

We computed rejection rates of the t-statistic for testing the null hypotheses $H_0 : \delta = 0$, in model (1), using a 1.96 critical value (5% level) for a standard normal distribution. In order to assess the usefulness for a finite sample of the asymptotic results presented in Table 1, rejection rates were based on simulated data, for samples of size $T = 25, 50, 100, 250, 500, 1,000$, and $10,000$, under various combinations of the DGPs in the Assumption.[6] In all experiments, the

[5] A few cases do not conform with the general level results of Phillips (1998) in terms of divergence rates. These are the cases 3-3, 3-6, and 6-6 (the first one studied in KLN and the last one in Entorf (1997)). In all three cases the divergence rates are higher.

[6] Note that the experimental results in this section do not pretend to analyze every possible combination of DGPs in the assumption. They serve as a guide on the finite sample behaviour of some particular cases.

number of replications is 10,000.

Table 2. Rejection Rates for $t_{\hat{\delta}}$; the case of two breaks

T	Combinations of cases (DGPs) in the Assumption											
	1-7	2-2	2-4	2-6	3-4	3-6	4-4	4-6	4-7	4-8	6-8	7-8
25	.06	.06	.06	.07	.14	.25	.32	.61	.66	.65	.93	.94
50	.06	.06	.06	.06	.89	.95	.99	.99	.99	.94	.95	.96
100	.05	.05	.06	.06	1.0	.99	1.0	1.0	1.0	.97	.97	.97
250	.05	.05	.07	.07	1.0	1.0	1.0	1.0	1.0	.98	.98	.98
500	.06	.05	.10	.10	1.0	1.0	1.0	1.0	1.0	.99	.99	.99
1000	.05	.05	.14	.15	1.0	1.0	1.0	1.0	1.0	.99	.99	.99
10000	.05	.05	.80	.79	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

The values of the parameters in the DGPs are as follows: $\sigma_z = 1$, $\phi_z = 0$, $\mu_x = 0.4$, $\mu_y = 0.7$, $\beta_x = 0.07$, $\beta_y = 0.04$, $\theta_{xi} = 0.07$, $\theta_{yi} = 0.04$, $\gamma_{xi} = 0.02$, $\gamma_{yi} = 0.04$, for $i = 1, ...M_z$, $M_z = 2$, for $z = x, y$. Breaks in x (y) occur at 20% (40%) and 70% (80%) of total data length.

Table 3. Rejection Rates for $t_{\hat{\delta}}$; the case of four breaks

T	Combinations of cases (DGPs) in the Assumption											
	1-7	2-2	2-4	2-6	3-4	3-6	4-4	4-6	4-7	4-8	6-8	7-8
25	.06	.06	.06	.07	.18	.25	.81	.89	.97	.90	.93	.94
50	.06	.06	.07	.07	.94	.94	1.0	.99	.99	.96	.96	.96
100	.05	.05	.09	.09	1.0	1.0	1.0	1.0	1.0	.97	.97	.97
250	.05	.05	.14	.14	1.0	1.0	1.0	1.0	1.0	.98	.98	.98
500	.05	.05	.23	.23	1.0	1.0	1.0	1.0	1.0	.99	.99	.99
1000	.05	.05	.41	.41	1.0	1.0	1.0	1.0	1.0	.99	.99	.99
10000	.05	.08	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

The values of the parameters in the DGPs are as follows: $\sigma_z = 1$, $\phi_z = 0$, $\mu_x = 0.4$, $\mu_y = 0.7$, $\beta_x = 0.07$, $\beta_y = 0.04$, $\theta_{xi} = 0.07$, $\theta_{yi} = 0.04$, $\gamma_{xi} = 0.02$, $\gamma_{yi} = 0.04$, for $i = 1, ...M_z$, $M_z = 4$, for $z = x, y$. Breaks in x (y) occur at 15% (20%), 30% (35%), 50% (55%) and 70% (80%) of total data length.

Tables 2 and 3 present rejection rates under 12 di¤erent combinations of DGPs in the Assumption. In Table 2, the cases where breaks are considered (all but 3-6 and 6-8) include 2 breaks, while Table 3 presents results when 4 breaks are allowed. The column labeled 1-7 in both tables presents the …nite sample counterpart of the $O_p(1)$ result in Table 1: the t-statistic does not diverge, revealing that the spurious regression phenomenon is not a problem in …nite samples either. The tables also show that a nonsense regression is not likely in small samples ($25 \cdot T \cdot 250$) when one of the variables is generated by a stationary process with structural breaks (combinations of cases 2-2, 2-4, and 2.6), as long as there are not too many breaks.[7] For the rest of cases,

[7] For case 2-2 the simulation results seem not to detect a spurious regression even with 10,000 observations. For this case the …nite sample results converge very slowly to the asymp-

the asymptotic results presented in Table 1 are supported by our simulation experiments: the spurious regression phenomenon is present even for samples as small as 25. In comparing results from Tables 2 and 3, it can be noted that a nonsense regression is more likely when the number of structural breaks increases in the DGP.

5 Conclusions

This paper has presented an asymptotic and experimental analysis of the spurious regression phenomenon under a wide variety of empirically relevant data generating processes in a linear regression model. It has shown that the t-statistic for testing a linear relationship among independent variables diverges if both variables are driven by a trending mechanism, whether deterministic or stochastic. Our results particularize Phillips' (1998) general results to empirically useful models, by showing that the phenomenon of spurious regression is present for time series with relatively simple trending mechanisms. This phenomenon depends on the commonality of trends in both dependent and explanatory variables. If a (stochastic or deterministic) trend is present in only one of the variables, however, the spurious regression vanishes. Our simulation experiments reveal that a spurious regression will also be present in finite samples, except for the case of one of the variables following a stationary process around a mean with a few structural breaks in level.

6 Appendix

We present a guide on how to obtain the order in probability of one combination of DGPs, namely, the combination 1-7, for which

$$y_t = \mu_y + u_{yt}$$
$$x_t = x_0 + \mu_x t + \sum_{i=1}^{M_x} \theta_{ix} DT_{ixt} + S_{xt}$$

The orders in probability for the rest of cases follow the same steps. The proofs were assisted by the software Mathematica 4.1. The corresponding codes for all combinations of DGPs are available at www.ventosa-santaularia.com/NVS_06a.zip. Below, we describe the steps involved in the computerized calculations.

Write regression model $y_t = \alpha + \delta x_t + u_t$ in matrix form: $y = X\beta + u$. The vector of OLS estimators is $\widehat{\beta} = (\widehat{\alpha}\ \ \widehat{\delta})' = (X'X)^{-1}X'y$, and the t-statistic of interest $t_{\widehat{\delta}} = \widehat{\delta}\left[\widehat{\sigma}_u^2 (X'X)^{-1}_{22}\right]^{-1/2}$, where $(X'X)^{-1}_{22}$ is the 2^{nd} diagonal element of $(X'X)^{-1}$ and $\widehat{\sigma}_u^2 = T^{-1}\sum_{t=1}^{T} \widehat{u}_t^2 = T^{-1}\sum_{t=1}^{T}\left(y_t - \widehat{\alpha} - \widehat{\delta} x_t\right)^2$. $t_{\widehat{\delta}}$ is a function of the following objects:

$$\sum_{t=1} y_t = \mu_y T + \Sigma_{uy} T^{1/2}$$

totic one. When the sample size is increased to 100,000 (400,000), we obtained rejection rates of 7.2% (11.8%) for $M_z = 2$, and 39.9% (91.9%) for $M_z = 4$.

$$\sum_{t=1} y_t^2 = \left[\mu_y^2 + \Sigma_{u2y}\right] T + 2\mu_y \Sigma_{uy} T^{1/2}$$

$$\sum_{t=1} x_t = \frac{1}{2}\left[\mu_x + \sum_{i=1}^{M_x}\theta_i(1-\lambda_i)^2\right] T^2 + \Sigma_{sx}T^{3/2} + \left(x_0 + \frac{1}{2}\left[\mu_x + \sum_{i=1}^{M_x}\theta_i(1-\lambda_i)\right]\right) T$$

$$\sum_{t=1} x_t^2 = \left[\frac{1}{3}\mu_x^2 + \lambda^+ + \frac{1}{3}\mu_x \sum_{i=1}^{M_x}\theta_i(1-\lambda_i)^2(\lambda_i+2)\right] T^3 + 2\left(\mu_x\Sigma_{tsx} + \Sigma_{ts1xi}\right) T^{5/2} + O_p(T^2)$$

$$\sum_{t=1} y_t x_t = \left[\frac{1}{2}\mu_y\left(\mu_x + \sum_{i=1}^{M_x}\theta_i(1-\lambda_i)^2\right)\right] T^2 + O_p(T^{3/2})$$

with

$$\Sigma_{uy} = T^{-1/2}\sum_{t=1}^{T} u_{yt}$$
$$\Sigma_{u2y} = T^{-1}\sum_{t=1}^{T} u_{yt}^2$$
$$\Sigma_{sx} = T^{-3/2}\sum_{t=1}^{T} S_{xt}$$
$$\Sigma_{tsx} = T^{-5/2}\sum_{t=1}^{T} tS_{xt}$$
$$\Sigma_{ts1xi} = T^{-5/2}\sum_{i=1}^{M_x}\theta_i\left[\sum_{t=T_{b_i}+1}^{T} tS_{xt} - \lambda_i\sum_{t=T_{b_i}+1}^{T} S_{xt}\right]$$

$$\lambda^+ = \frac{1}{3}\sum_{i=1}^{M_x}\theta_i^2(1-\lambda_i)^2 + \sum_{i=1}^{M_x-1}\sum_{j=i+1}^{M_x}\theta_i\theta_j\left[\frac{2}{3}(1-\lambda_{u(i,j)})^3 + \lambda_{d(i,j)}(1-\lambda_{u(i,j)})^2\right]$$

$$\lambda_{u(i,j)} = \max(\lambda_i,\lambda_j),\ i,j = 1,2,...,M_x$$
$$\lambda_{l(i,j)} = \min(\lambda_i,\lambda_j)$$
$$\lambda_{d(i,j)} = \lambda_{u(i,j)} - \lambda_{l(i,j)}$$

where (see for instance Phillips (1986)),

$$\Sigma_{uy} \Rightarrow \sigma_y W_y(1)$$
$$\Sigma_{u2y} \Rightarrow \sigma_y^2$$
$$\Sigma_{sx} \Rightarrow \sigma_x \int_0^1 W_x(r)dr$$
$$\Sigma_{tsx} \Rightarrow \sigma_x \int_0^1 rW_x(r)dr$$
$$\Sigma_{ts1xi} \Rightarrow \sigma_x \sum_{i=1}^{M_x}\theta_i\int_{\lambda_i}^1 (r-\lambda_i)W_x(r)dr$$

where $\Rightarrow$ signifies convergence in distribution, and $W_z(r), z = y, x$ is the standard Wiener process on $r \in [0,1]$.

Using these expressions, Mathematica computes the limiting distribution of the parameter vector and the rest of the elements of $t_{\hat{\delta}}$ by factoring out the relevant expressions in powers of the sample size. In this way, the orders in probability can be determined, and the limiting expression obtained, by retaining only the asymptotically relevant terms, upon a suitable normalization. From Mathematica's output it can be deducted that, for the case at hand

$$T^{3/2}\hat{\theta}\left[\hat{\sigma}_u^2 T^3 (X'X)^{-1}_{22}\right]^{-1/2} = \hat{\theta}\left[\hat{\sigma}_u^2 (X'X)^{-1}_{22}\right]^{-1/2} = O_p(1),$$

as reported in Table 1.

7 References

Cappuccio, N. and D. Lubian (1997), "Spurious Regressions Between $I(1)$ Processes with Long Memory Errors", Journal of Time Series Analysis, 18, 341-354.

Coenen, G. and J.-L. Vega (2001), "The Demand for M3 in the Euro Area", Journal of Applied Econometrics, 16, 727-48.

D'Adda, C. and A. E. Scorcu (2003), "On the Time Stability of the Output-Capital Ratio", Economic Modelling, 20, 1175-1189.

Entorf, H. (1997), "Random Walks with Drifts: Nonsense Regression and Spurious Fixed-E¤ect Estimation", Journal of Econometrics, 80, 287-296.

Granger, C.W.J. and P. Newbold (1974), "Spurious Regression in Econometrics", Journal of Econometrics, 2, 111-120.

Granger, C.W.J., N. Hyung, and Y. Jeon (1998), "Spurious Regressions with Stationary Series", Mimeo.

Haldrup, N. (1998), "An Econometric Analysis of $I(2)$ Variables", Journal of Economic Surveys, 12(5), 595-650.

Juselius, K. (1996), "A Structured VAR under Changing Monetary Policy", Journal of Business and Economic Statistics, 16, 400-12.

Juselius, K. (1999), "Price Convergence in the Medium and Long Run: An $I(2)$ Analysis of Six Price Indices", in Engle, R. F. and H. White (eds.): Cointegration, Causality and Forecasting. A Festschrift in Honour of Clive W. J. Granger, Oxford University Press.

Kim, T.-H., Y.-S. Lee and P. Newbold (2004), "Spurious Regressions with Stationary Processes Around Linear Trends", Economics Letters, 83, 257-262.

Lumsdaine, R.L. and D.H. Papell (1997), "Multiple Trend Breaks and the Unit Root Hypothesis", The Review of Economics and Statistics, 79, 212-218.

Marmol (1995), "Spurious Regressions for $I(d)$ Processes", Journal of Time Series Analysis, 16, 313-321.

Marmol (1996), "Nonsense Regressions Between Integrated Processes of Different Orders", Oxford Bulletin of Economics and Statistics, 58(3), 525-36.

Marmol (1998), "Spurious Regression Theory with Nonstationary Fractionally Integrated Processes", Journal of Econometrics, 84, 233-50.

MATHEMATICA 4.1 (2000), Wolfram Research Inc.

Mehl, A. (2000), "Unit Root Tests with Double Trend Breaks and the 1990s Recession in Japan", Japan and the World Economy, 12, 363-379.

Muscatelli, V. A. and F. Spinelli (2000), "The Long-Run Stability of the Demand for Money: Italy 1861-1996", Journal of Monetary Economics, 45, 717-39.

Nielsen, H. B. (2002), "An I(2) Cointegration Analysis of Price and Quantity Formation in Danish Manufactured Exports", Oxford Bulletin of Economics and Statistics, 64(5), 449-72.

Noriega, A. and E. de Alba (2001), "Stationarity and Structural Breaks: Evidence from Classical and Bayesian Approaches", Economic Modelling, 18, 503-524.

Noriega, A. and D. Ventosa-Santaulària (2005), "Spurious Regression Under Broken Trend Stationarity", Working Paper EM-0501, School of Economics, University of Guanajuato.

Park, J. Y. and P. C. B. Phillips (1989), "Statistical Inference in Regressions with Integrated Processess: Part 2", Econometric Theory, 5, 95-131.

Perron, P. (1992), "Trend, Unit Root and Structural Change: A Multi-Country Study with Historical Data", in Proceedings of the Business and Economic Statistics Section, American Statistical Association, 144-149.

Perron, P. (1997), "Further Evidence on Breaking Trend Functions in Macroeconomic Variables", Journal of Econometrics, 80, 355-385.

Perron, P. and T. J. Vogelsang (1992), "Nonstationarity and Level Shifts with and Application to Purchasing Power Parity", Journal of Business and Economic Statistics, 10(3), 301-20.

Phillips, P.C.B. (1986), "Understanding Spurious Regressions in Econometrics", Journal of Econometrics, 33, 311-340.

Phillips, P.C.B. (1998), "New Tools for Understanding Spurious Regressions", Econometrica, 66(6), 1299-1325.

Tsay, W.-J. and C.-F. Chung (1999), "The Spurious Regression of Fractionally Integrated Processes", Mimeo.

Wu, J.-L. (2000), "Mean Reversion of the Current Account: Evidence from the Panel Data Unit-Root Test", Economics Letters, 66, 215-22.