Impact of International Technological Diffusion on Southern Convergence

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Abstract

In the standard models of North-South technological-knowledge diffusion, the larger the initial technological-knowledge gap between countries is, the higher the Southern catching up. However, this result does not adjust well to Southern reality as a whole. The purpose of this paper is to demonstrate that the disparity between the theoretical outcome and the empirical findings can be reduced by considering that: (i) the South can only imitate Northern technological knowledge when it is sufficiently close to the Northern frontier; (ii) the advantage of the South's moderate backwardness, together with its imitation capacity, is a mechanism of catching up with the North; and (iii) the Southern catching-up specification can be country specific. In particular, we show that the behavior of the South's relative level of employed human capital affects Southern imitation capacity and depends on the catching-up specifications.

Key words: North-South; R&D; Human Capital; Convergence; Numerical Computations.

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1 Introduction

The world productivity data reveals that the benefits of innovative R&D are much more evenly distributed in the world than are expenditures on innovative R&D, indicating that technological knowledge is diffused internationally (*e.g.*, Coe and Helpman, 1995; Coe *et al.*, 1997). Thus, focusing exclusively on Northern innovative technological-knowledge progress, as considered by the standard R&D growth models, is inappropriate, especially when the analysis also includes a Southern imitator country.

Therefore, standard models of North-South technological diffusion (*e.g.*, Grossman and Helpman, 1991a, ch. 12; Barro and Sala-i-Martin, 1997), extend the standard R&D growth models (*e.g.*, Romer, 1990; Aghion and Howitt, 1992) so that they can be applicable to all countries. Basically, the analysis in these models is accomplished taking into account a dynamic setting where the North innovates whereas the South imitates Northern innovations and imitation is a vehicle for international technological-knowledge transfer. More specifically, as a result of the proposed standard Southern catching-up specification, the initial relatively low cost of imitation tends to rise as the pool of imitable goods that embody technological knowledge decreases. That is, in line with, *e.g.*, Mansfield *et al.* (1981), during the transitional dynamics towards the steady state, the cost of imitation is lower than the cost of innovation and, thereby, the initial high Southern growth rate tends to fall towards the Northern one.

In particular, in standard models of North-South technological-knowledge diffusion, the larger the initial technological-knowledge gap between countries, the higher the catching up. However, this result does not adjust well to Southern reality as a whole (*e.g.*, Bernard and Jones, 1996a, b; Quah, 1997; Hall and Jones, 1999; Acemoglu and Zilibotti, 2001), which suggests that the standard catching-up specification needs to be re-evaluated, if we intend to reduce the disparity between the theoretical outcome and the empirical findings. This is the main purpose of this paper.

In our model, we consider that: (i) growth is driven by advances in the quality of intermediate goods (Schumpeterian R&D, as formalized by, *e.g.*, Aghion and Howitt, 1992) and by human-capital accumulation *à la* Lucas (1988); (ii) scale effects are removed (as suggested by, *e.g.*, Jones, 1995a, b); (iii) different catching-up specifications are possible (in this paper we are going to consider three specifications). The main specificities that we introduce in the South's probability of successful R&D, through the catching-up specifications, are imitation capacity and an advantage of moderate backwardness (*i.e.*, the South can only imitate Northern technological knowledge when it is sufficiently close to the Northern frontier), which, together, are capable of inducing catching up with the North.

Imitation capacity, *i.e.*, capacity to implement advanced technological knowledge, is enhanced by domestic policies promoting R&D, as mentioned by Aghion *et al.* (2001), and decreases with the human-capital gap in relation to the North, in the lines of Nelson and Phelps (1966). With this specification of imitation capacity, we are able to add a new channel through which different levels of human capital have a bearing on the effects of technological-knowledge diffusion.

As in standard models, the advantage of backwardness considers that the rate of technological-knowledge progress in the South is an increasing function of the gap between its own technological-knowledge level and that of the North. However, we require the gap to be below a threshold distance, beyond which the cost of imitation becomes prohibitive. This is because low-income countries, which are far from the technological-knowledge frontier, are stagnant and show no potential for rapid growth (*e.g.*, Quah, 1997). Thus, a suitable classification for our North and South countries would be developed versus developing, rather than developed versus underdeveloped. Additionally, we further explore the pattern of the Southern middle-income country convergence, by considering that convergence occurs as long as the South has the imitation capacity to take advantage of Northern technological knowledge and that convergence depends on the catching up specification. Moreover, we analyze not only the Southern path of technological knowledge, but also the Southern path of human capital.

With regards to the literature on international technological-knowledge diffusion, our main contributions is the interrelated analysis of: (i) the effects of interactions between endogenous R&D and human-capital accumulation; (ii) the mechanism of international technological-knowledge diffusion as a vehicle for an eventual partial convergence between countries, in a context without scale effects; and (iii) the impact of the catching-up specifications on the results. In particular, by considering endogenous human-capital accumulation, we aspire to analyze to what extent the behavior of the South's relative level of employed human capital depends on the catching-up specification.

We model a standard economic structure for the North and South as stipulated in endogenous R&D growth theory. Production of perfectly competitive final goods uses quality-adjusted intermediate goods as inputs, together with human capital. Intermediate goods, in turn, use innovative or imitative designs as inputs, under monopolistic competition. As to the human capital, we assume an accumulation function *à la* Lucas (1988).

As stated by Song (2000), transitional dynamics is often avoided in endogenous growth models. Consequently, the same occurs in international technological-knowledge diffusion models. For example, Grossman and Helpman (1991a, b) analyze only the steady state. Like Barro and Sala-i-Martin (1997), we derive the Southern transitional dynamics. However, as our model is complex due to the interaction between endogenous R&D and human-capital accumulation, we solve numerically the differential system of equations using a Runge-Kutta method.

The paper is organized as follows. In section 2, we characterize the domestic North and South economies. In section 3, we derive the equilibrium R&D, the world steady state defined by the North and Southern transitional dynamics. Finally, in section 4, we present some conclusions, including several paths for future research.

2 Modeling North and South economies

2.1 Product markets

Final goods, continuously indexed by $n \in [0,1]$, are produced in perfect competition. Following the Schumpeterian set-up, each final good is produced by human capital, H, and by a continuum of intermediate goods indexed by $j \in [0,J]$. The output of n, Y_n, at time t is,

$$Y_n(t) = A \left[\int_0^J \left(q^{k(j,t)}\ x_n(k,j,t) \right)^{1-\alpha} dj \right] \ [H_{w,n}(t)]^{\alpha} . \tag{1}$$

The production function is the same in both countries, except for term A, which is a positive exogenous variable representing the level of productivity, dependent on the country's institutions (where $A_S < A_N$, indexing the South by S and the North by N). The integral sums up the contributions of intermediate goods to production. In the Schumpeterian tradition, the quantity of each j, x, is quality-adjusted. The constant quality upgrade is $q > 1$, and k is the highest quality rung at time t. The expression with exponent $\alpha \in]0,1[$ represent the role of the H input, and index w in H_n identifies the quantity of H employed in the production of n, *i.e.*, that works and earns a wage (as opposed to H in accumulating human capital).

Plugging the demand for the highest quality of each j by the producer of n into (1), the supply of final good n is

$$Y_n(t) = A^{\frac{1}{\alpha}} \left[\frac{p_n(t)\ (1-\alpha)}{p(k,j,t)} \right]^{\frac{1-\alpha}{\alpha}} H_{w,n}(t)\ Q(t), \text{ where } Q(t) \equiv \int_0^J q^{k(j,t)\left[\frac{1-\alpha}{\alpha}\right]}\ dj \tag{2}$$

is the aggregate domestic quality index, measuring domestic technological knowledge, where $p_n(t)$ and $p(k,j,t)$ are, respectively, the prices of n and j at time t.

We define the aggregate output, Y, *i.e.*, the composite final good, as

$$Y(t) = \int_0^1 p_n(t)\ Y_n(t)\ dn = \exp\left[\int_0^1 \ln Y_n(t)\ dn \right], \tag{3}$$

and normalize its price at each time t to one (numeraire). Resources in the economy measured in terms of Y can be used in the production of the intermediate goods, X, in the R&D sector, R, or consumed, C,

$$Y = X + R + C. \tag{4}$$

Since Y is the input in the production of each j and final goods are produced in perfect competition, the marginal cost of producing j is one. Its production requires a start-up cost of R&D, which can only be recovered if profits at each time are positive for a certain period in the future. This is assured by a system of intellectual property rights, while at the same time, almost without costs, disseminating acquired technological knowledge to other firms.

The profit-maximization price of the monopolistic intermediate good firms yields $p(k,j,t) = p = \frac{1}{1-\alpha} > 1$. This mark-up is constant over time, across all intermediate goods and for all quality grades. Since the leader firm is the only one legally allowed to produce the highest quality, it will use pricing to wipe out sales of lower quality. Depending on whether $q(1-\alpha)$ is greater or lesser than the marginal cost it will use, respectively, either the monopoly pricing $p = \frac{1}{1-\alpha}$ or the limit pricing $p = q$ to capture the entire market. Like, for example, Grossman and Helpman (1991a, ch. 4), we assume that the limit pricing strategy is binding. Thus, since the lowest price that the closest follower can charge without negative profits is one, the leader can successfully capture the entire market by selling at a price slightly below q, since q is the quality advantage over the closest follower.

The aggregate output at each time t, from (3), is expressible as a function of the currently aggregate domestic quality index, Q, in (2) and employed human capital, H_w, which is assumed to be fully employed at each t :[1]

$$Y(t) = A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{q}\right)^{\frac{1-\alpha}{\alpha}} Q(t)\ H_w(t). \tag{5}$$

Since Q and H_w grow over time due to R&D activities and human-capital accumulation, output Y also grows over time.

2.2 R&D sector

The value of the leading-edge patent depends on the profit-yields accrued by the monopolist at each t, and on the duration of the monopoly. The duration, in turn, depends on the probability of successful R&D, which creatively destroys the current leading-edge design (*e.g.*, Aghion and Howitt, 1992). The determinants of the probabilities of success, innovation and imitation, are thus at the heart of the Schumpeterian R&D models.

Let Z index the country, and $pb_Z(k, j, t)$ denote the instantaneous probability of successful innovation $Z = N$ or imitation $Z = S$ in the next higher quality $[k(j,t)+1]$ of intermediate good j. Formally,

$$pb_Z(k,j,t) = rs_Z(k,j,t) \cdot DC_Z(k,j,t) \cdot [CU_i(t)]^{\Gamma_Z}, \tag{6}$$

where:

(i) $rs_Z(k,j,t)$ is the flow of domestic Y in (3) devoted to R&D in j.

(ii) $DC_Z = \beta_Z \; q^{k_Z(j,t)} \cdot \zeta_Z^{-1} q^{-\alpha^{-1} k_Z(j,t)} \cdot H_{w,Z}^{-1}$, $\beta_Z > 0$, $\zeta_Z > 0$, represents the domestic causes promoting domestic R&D; $\beta_Z \; q^{k_Z(j,t)}$ is the positive learning effect of accumulated public technological knowledge from past successful research (*e.g.*, Grossman and Helpman, 1991a, ch. 12; Connolly, 2003); ζ_Z^{-1} $q^{-\alpha^{-1} k_Z(j,t)}$ is the adverse effect due to the increasing complexity of quality improvements (*e.g.*, Kortum, 1997);[2] $H_{w,Z}^{-1}$ is the adverse effect of the market size, capturing the idea that the difficulty of replacing old quality-adjusted intermediate goods is proportional to the size of the market as measured by the human-capital employed, in line with Dinopoulos and Segerstrom (1999) and Dinopoulos and Thompson (1999).

(iii) $[CU_i(t)]^{\Gamma_Z}$ (where $\Gamma_N = 0$ and $\Gamma_S = 1$) is a catching-up term, specific to the South, which sums up the positive effects of imitation capacity and backwardness on the probability of successful imitation. By considering $i = 1, 2, 3$, we take the following three different specifications:

$$CU_1(t) = \exp(s_{R\&D}) \cdot \left\{ f\left[\widetilde{H}_w(t)\right]\right\}^{\overline{\sigma}_1} \cdot \left\{ g\left[\widetilde{Q}(t), d\right]\right\}^{-\overline{\sigma}_2 + \widetilde{Q}(t)}; \tag{7a}$$

$$CU_2(t) = \exp(s_{R\&D}) \cdot \exp\left\{ \overline{\sigma}_3 \; h\left[\widetilde{H}_w(t), \widetilde{Q}(t), d\right]\right\}; \tag{7b}$$

$$CU_3(t) = \exp(s_{R\&D}) \cdot \left\{ f\left[\widetilde{H}_w(t)\right]\right\}^{\overline{\sigma}_1} \cdot \exp\left\{ g\left[\widetilde{Q}(t), d\right]\right\}^{\overline{\sigma}_4 + \frac{\overline{\sigma}_5}{\widetilde{Q}(t)}}; \tag{7c}$$

where: $\overline{\sigma}_1,\ \overline{\sigma}_2,\ \overline{\sigma}_4,\ \overline{\sigma}_5,\ s_{R\&D} > 0$; $\overline{\sigma}_2 > \widetilde{Q}$; $\overline{\sigma}_3 > 1$; $0 < \widetilde{H}_w \equiv \frac{H_{w,S}}{H_{w,N}}, \widetilde{Q} \equiv \frac{Q_S}{Q_N} < 1$; $\widetilde{H}_w$ is the South's relative level of employed human capital and $\widetilde{Q}$ is the relative technological-knowledge level of the South's intermediate good; the exponential $\exp(s_{R\&D})$ captures one important determinant of imitation capacity, which are the policies promoting R&D (*e.g.*, Aghion *et al.*, 2001); and functions f, g and h are formally given at each t, by:

$$f\left(\widetilde{H}_w\right) = 1 + \frac{\exp\left(\widetilde{H}_w\right)}{1+\exp\left(\widetilde{H}_w\right)}; \tag{8a}$$

$$g\left(\widetilde{Q}, d\right) = \begin{cases} 0 & \text{, if } 0 < \widetilde{Q} \leq d \\ -\widetilde{Q}^2 + (1+d)\widetilde{Q} - d & \text{, if } d < \widetilde{Q} < 1 \end{cases}; \tag{8b}$$

$$h\left(\widetilde{H}_w, \widetilde{Q}, d\right) = g\left(\widetilde{Q}, d\right)\ \widetilde{H}_w. \tag{8c}$$

That is, from f, human capital at work enhances the imitation capacity and so speeds up convergence with the North (in line with, *e.g.*, Nelson and Phelps, 1966; Benhabib and Spiegel, 1994), and parameter $\overline{\sigma}_1$ indicates how quickly $pb_S(k,j,t)$ rises as $\widetilde{H}_w$ also rises. Function g attempts to capture the benefits obtained from relative backwardness, *i.e.*, provided that the technological-knowledge gap is not very wide, the South can benefit from an advantage of backwardness (*e.g.*, Barro and Sala-i-Martin, 1997).[3] Function h brings together the role of the gap in human-capital employed and the role of the technological-knowledge gap. Equations (8b) and (8c) guarantee that functions g and h are non-negative and so economically feasible. Moreover, they are quadratic over the range of main interest, and, once affected by the exponents, yield an increasing (in the technological-knowledge gap) advantage of backwardness.

2.3 Consumption and human-capital accumulation

A time-invariant number of heterogeneous individuals decides on the allocation of time and income. Time is divided between accumulation of human capital and working to earn a share of Y in (3), proportional to the individual's human capital. Income is partly spent directly on the consumption of Y, and partly lent in return for future interest. Assuming a CIES instantaneous utility function, a homogeneous discount rate $\rho > 0$ and a constant elasticity of marginal utility with respect to consumption $\theta > 0$, the infinite horizon lifetime utility is

$$U(t) \equiv \int_0^\infty \left[\frac{c(t)^{1-\theta} - 1}{1-\theta} \right] \exp(-\rho t) \; dt \tag{9}$$

where $c(t)$ is individual consumption at time t.

Savings consists of accumulation of financial assets (K, with return r) in the form of ownership of the firms that produce intermediate goods. The value of these firms, in turn, corresponds to the value of patents in use. The individual budget constraint equalizes savings to income earned minus consumption,[4]

$$\dot{K}(t) = r(t)K(t) + \left[1 - u_T\,(t)\right] w(t)H(t) - c(t), \tag{10}$$

where $u_T\,(t)$ is the fraction of time t that is spent accumulating human capital and $w(t)$ is the wage per unit of human capital.

As in Lucas (1988), we consider that the productivity of the time spent in accumulation increases with the amount of human capital at each time t, *i.e*,

$$\dot{H} = \left(\chi_T \; u_T - \delta\right) H, \tag{11}$$

where $\chi_T \; u_T > \delta$, δ is the depreciation rate of human capital and χ_T is an efficiency parameter, which measures the productivity of formation.

The maximization of the lifetime utility (9), subject to the budget constraint (10) and to human-capital accumulation (11) yields the solution for the consumption path, which is the standard Euler equation

$$\widehat{c}(t) = \widehat{C}(t) = \frac{1}{\theta}\left[r(t) - \rho\right], \tag{12}$$

where $\widehat{c}$ is the growth rate of c.

An interior solution to the maximization problem requires positive amounts of both assets, K and H, which is not sustainable unless their returns are equalized at all times, and the following resulting condition ensures this:

$$\widehat{w}(t) = r(t) + \delta - \chi_T. \tag{13}$$

This completes the description of the model.

3 Equilibrium

3.1 Equilibrium R&D and steady state

In order to analyze the equilibrium R&D, we start comparing the incremental profits of follower firms taking over the leader position, which is given by

$$\Pi(k, j, t) = H_w(t)\ \ (q-1)\ \ \left[\frac{A\ (1-\alpha)}{q}\right]^{\frac{1}{\alpha}}\ \ q^{k(j,t)\left[\frac{1-\alpha}{\alpha}\right]}, \tag{14}$$

with the incremental profits of leaders replacing themselves. It becomes easier to show that the gain achieved by the followers is greater since $q^{\frac{1}{\alpha}} > 1$. Thus, considering the same R&D technology for all firms, all R&D will by supported by entrants. Moreover, due to the technological complementarity in (1), the size of the market for intermediate goods is H_w, as shown in (14).

The expected current value of the flow of profits to the monopolist producer of intermediate good j (the market value of the patent or still the value of the monopolist firm, owned by domestic consumers), $V(k,j,t)$, depends on the amount in each period in (14), on r, and on the expected duration of the flow, which is the expected duration of the technological-knowledge leadership. Such duration, in turn, depends on $pb\,(k,j,t)$. Considering that over time between each successful R&D in j, $rs\,(k,j,t)$ grows at the same rate as $H_w(t)$ and, thereby, $pb\,(k,j,t)$ does not vary, we have that (*e.g.*, Barro and Sala-i-Martin, 2004, ch. 7):

$$V(k,j,t) = \frac{\Pi(k,j,t)}{r(t) + pb(k,j,t)}. \tag{15}$$

Assuming free-entry equilibrium into R&D, which is defined by the equality between expected revenue and resources spent, *i.e.*, $pb(k,j,t)\ V(k+1,j,t) = rs(k,j,t)$, and then plugging (15) and (6), and bearing in mind (14), the term $rs(k,j,t)$ cancels out. As a result, the equilibrium probability of a successful R&D, pb, given r and H_w is

$$pb_Z(t) = \frac{\beta_Z}{\zeta_Z}\ \left[CU_i(t)\right]^{\Gamma_Z}\ \left(\frac{q-1}{q}\right)\left[A_Z\ (1-\alpha)\right]^{\frac{1}{\alpha}} - r_Z(t). \tag{16}$$

The equilibrium pb turns out to be independent of j and k, because the positive influence of the quality rung on profits and on the learning effect is exactly offset by its effect on the complexity cost. As has been suggested by the R&D endogenous growth literature since Jones' (1995a, b) critique, pb is also independent of the market-size effect. This is because the adverse effect of market size due to the scale-proportional difficulty of replacing old quality intermediate goods is designed to offset the scale effect on profits. Considering (6) and (16), it can be shown that equilibrium aggregate resources devoted to R&D by all follower firms at each t, $R(t)$, depend positively on Q and H_w

$$R_Z = \frac{\zeta_Z \; pb_Z}{\beta_Z \;\; [CU_i(t)]^{\Gamma_Z}} \; Q_Z \; H_{w,Z}. \tag{17}$$

Equation (17) shows that the increased resources devoted to R&D as Q_Z and/or $H_{w,Z}$ rise(s) are needed to offset the greater difficulty of R&D as Q_Z and/or $H_{w,Z}$ increase(s).

Since the probability of successful R&D determines the speed of technological-knowledge progress, equilibrium can be translated into the path of technological knowledge. The relationship comes to yield the following expression for the equilibrium growth rate (*e.g.*, Barro and Sala-i-Martin, 2004, ch. 7)

$$\widehat{Q}(t) = pb(t) \;\; \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right]. \tag{18}$$

As Y, X, R and C are all multiples of $Q \cdot H_w$,[5] the constant and unique steady-state endogenous growth rate, which, through the Euler equation (12), also implies a constant steady-state interest rate, r^*, denoted by g^* is

$$g^* = \widehat{Q}^* + \widehat{H}_w^* = \widehat{Y}^* = \widehat{X}^* = \widehat{R}^* = \widehat{C}^* = \frac{r^* - \rho}{\theta}. \tag{19}$$

Thus, r^* is obtained by first plugging (18) into the human capital demand path obtained from (5), resulting in $\widehat{w}(t) = \widehat{Q}(t)$, and then by equating this latter expression to the condition for optimization by individuals (13). In particular, the constant r_N^* (and g_N^*) is a direct result. However, the constant r_S^* (and g_S^*) requires that $\widetilde{Q}^*$ and $\widetilde{H}_w^*$ must be constant as well. Thus, it is due to the North-South technological-knowledge diffusion that both countries grow at the same rate, $g^* = g_N^* = g_S^*$.

3.2 Southern transitional dynamics

Having established that in steady state there is a world growth rate common to both countries, we now need to find out whether the Southern country converges towards that steady state or not, by assuming that the North is and remains always in steady state. Thus, first, we build the system of differential equations governing the transitional dynamics,[6] and then we solve the system through numerical integration.

Due to the exploitation of technological-knowledge backwardness, the Southern growth rate can be higher than the Northern one during the transitional phase, *i.e.*, from $t = 0$, when imitation starts, until t^*, when the South reaches the Northern steady state. In this phase, the probability of successful imitation changes with variations in both $\widetilde{Q}$ and $\widetilde{H}_w$ towards $\widetilde{Q}^*$ and $\widetilde{H}_w^*$, respectively. The Southern transitional path is fully described by a system of differential equations in $\widetilde{Q}$, $\widetilde{H}_w$, $\vartheta \equiv C/Q_S H_{w,S}$, $u_{T,S}$ and $u_{w,S} \equiv 1 - u_{T,S}$, which must be constants in steady state.

By equating the human capital demand path to the 'supply' in (13) and after that subtracting the resulting expression for the North and South we have

$$\widehat{\widetilde{Q}}(t) = r_S(t) - r_N^*(t), \tag{20}$$

where $\widehat{\widetilde{Q}}(t) = \widehat{Q}_S(t) - \widehat{Q}_N^*(t)$. To find $r_S(t)$, we start by considering the expression for the expected present value of profits for the respective leading imitator at time v

$$V\left(k, av, v\right) = \Pi(k, av, v) \int_t^{\infty} \exp\left\{ - \int_t^s \left[r\left(v\right) + pb\left(k, av, v\right) \right] dv \right\} ds, \tag{21}$$

where av represents a representative domestic intermediate good.[7] Since there is free entry into R&D, then at all times $V(k+1, av, v)\ pb(k, av, v)$ equals the cost of R&D, $rs(k, av, v)$. Differentiating both sides of this expression, bearing in mind Leibniz's rule for the left-hand side, and considering (20) and (14), the expression for r_S at each time is:[8]

$$\begin{array}{r} r_S = -\overline{\sigma}_1\ \widehat{f} - \dot{\widetilde{Q}}\ \ln g - (-\overline{\sigma}_2 + \widetilde{Q})\ \widehat{g} + \\ + \frac{pb_S(k,av)\ \Pi(k+1,av)}{rs(k,av)} + pb_S(k+1, av) \end{array}, \tag{22}$$

where the first three terms on the right-hand side represent a capital gains term, the fourth is the dividend term, and the fifth reflects market losses due to creative destruction when the next successful imitation occurs.

Plugging (22) into (20), we have an expression that relates the paths of $\widetilde{Q}$ and $\widetilde{H}_w$. Moreover, from (4), and bearing additionally in mind the expressions for Y, X and R, as well as (18), we obtain an expression which relates the paths of $\widetilde{Q}$, $\widetilde{H}_w$ and ϑ. In addition, since $\widehat{\vartheta} = \widehat{C} - \widehat{Q}_S - \widehat{H}_{w,S}$, and given that $\widehat{Q}_S = \widehat{\widetilde{Q}} + \widehat{Q}_N^*$, $\widehat{H}_{w,S} = \widehat{\widetilde{H}}_{w,S} + \widehat{H}_{w,N}^*$ and (12), we achieve another expression for the relationship through the paths of $\widetilde{Q}$, $\widetilde{H}_w$ and ϑ. In other words, we reach a system of three differential equations in $\widetilde{Q}$, $\widetilde{H}_w$, ϑ.

Finally, we obtain the path for $u_{w,S}$, which must be constant in steady state. To this end, we take into account that:

$$\begin{array}{r} H_S(t) = \underbrace{u_{w,S}(t) H_S(t)}_{\equiv H_{w,S}(t)} + \underbrace{u_{T,S}(t) H_T(t)}_{\equiv H_{T,S}(t)} \Rightarrow \\ \Longrightarrow \widehat{H}_{w,S}(t) = \widehat{u}_{w,S}(t) + \widehat{H}_S(t) \end{array}. \tag{23}$$

Thus, bearing in mind (11) and the dynamics of $\widetilde{H}_w$, we obtain:

$$\widehat{u}_{w,S}(t) = \widehat{\widetilde{H}}_{w,S}(t) + \widehat{H}_{w,N}^*(t) - \chi_T \left[1 - u_{w,S}(t)\right] + \delta. \tag{24}$$

As the stability of $u_{w,S}$ and $u_{T,S}$ is analyzed after the stability of $\widetilde{Q}$, $\widetilde{H}_w$, ϑ, the properties of the South's dynamics are block recursive.

It should be borne in mind that the three catching-up specifications originate three cases: cases 1, 2 and 3 related respectively, with catching-up specifications (7a), (7b) and (7c). In all cases, however, our idea underlying technological-knowledge imitation differs from that found in the literature. The later assumes the notion of pure relative backwardness and so the rate of imitation (and growth) is a positive and monotonic function of the relative technological-knowledge gap between countries. Thus, very poor countries are the ones with the most potential to imitate, while in our case very poor countries will remain stagnant due to their inability to imitate and only middle income countries will benefit from existing Northern technological-knowledge progress.

For the numerical solution of the ordinary differential equations system describing Southern transitional dynamics, we use a Runge-Kutta method. More specifically, we use the explicit Runge-Kutta (4,5) pair of Dormand and Price (*e.g.*, Dormand and Price, 1980; Shampine and Reichelt, 1997). We solve the above system for a set of baseline parameter values and initial conditions in the Appendix. In particular, the values for $\overline{\sigma}_3$, $\overline{\sigma}_4$ and $\overline{\sigma}_5$ in the two alternative specifications for the catching-up term are chosen in order to get the same steady-state technological-knowledge gap between countries under all specifications. In all computations we require a relative error tolerance of $1.0e-05$ and an absolute error tolerance of $1.0e-07$.

Table 1 condenses the main results, by comparing initial and steady-state values of the relevant variables under the three catching-up specifications.

First, from Table 1, we can immediately observe that the adjustment processes are globally stable. Moreover, the speed of convergence towards the

	t^*	$\widetilde{Q}^*$	$\widetilde{H}_w^*$	$u_{w,S}^*$
case 1	56	0.55	0.40	0.72
case 2	200	0.55	0.26	0.72
case 3	360	0.55	0.14	0.72

Table 1: Steady-state values of the relative technological-knowledge level of the South $\widetilde{Q}$, of the South's relative level of employed human capital $\widetilde{H}_w$ and of the Southern fraction of time spent at work $u_{w,S}$ for the three cases considered.

steady state is different according to the case. The fastest speed of convergence is observed in case 1, where the steady state is reached after 56 years. In cases 2 and 3, the steady state is reached after, respectively, 200 and 360 years. The prolonged time scale towards the steady state is not however surprising. For example, in the model without human-capital accumulation and with scale effects proposed by Papageorgiou (2002), the steady state emerges at the end of 160 years. The long time towards the steady state suggests that transitional dynamics is important and should not be neglected.[9]

Regardless of the case, the South improves its relative technological knowledge (from 0.34 to 0.55) and, in case 1, its relative level of human capital at work (from 0.30 to 0.40). However, under cases 2 and 3, its relative human capital at work decreases (from 0.30 to 0.26 and 0.14, respectively), which partly offsets the benefit obtained in terms of technological knowledge. This question is not analyzed in the standard technological-knowledge literature, since human-capital accumulation is not taken into account.

Table 1 also shows, as expected, that the fraction of time spent working in steady state is the same in all cases, which, in turn, corresponds to the Northern value.

Figure 1 completes the set of results, by showing the path of variables $\widetilde{Q}$, $\widetilde{H}_w$ and $u_{w,S}$ under the catching-up specification ($7a$). For the other two cases, the behavior of these variables is similar.

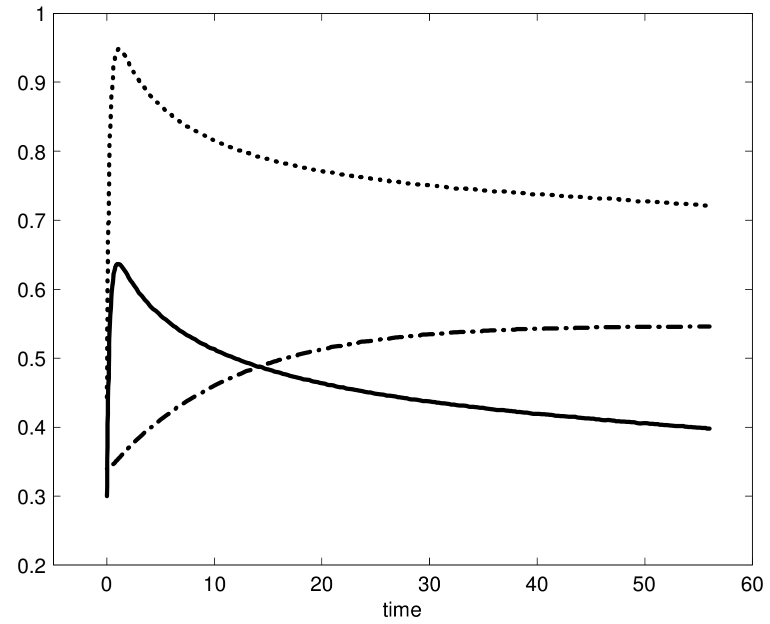

Figure 1: Transitional dynamics towards the steady state of $\widetilde{Q}$ (dashdot line), $\widetilde{H}_w$ (solid line) and $u_{w,S}$ (dotted line).

We observe that $\widetilde{Q}$ increases from its initial value, 0.34, towards its steady-state value, 0.55. This is because, initially the probability of successful imitation rises. However, as the rung of quality left to be copied decreases, the cost of imitation increases and the probability of successful imitation falls towards the steady-state value. Thus, Southern technological knowledge grows more quickly than the Northern one during the transitional dynamics, but slows down until steady state is reached.

The paths of $\widetilde{H}_w$ and $u_{w,S}$ indicate that there is an initial abrupt surge in the South's relative level of employed human capital owing to the immediate increase in the share of time dedicated at work. After that, $\widetilde{H}_w$ drops due to the greater share of time devoted to work in the South in comparison to the North.

Thus, the drop is a result of the (relatively) smaller Southern human-capital accumulation during the transition phase. As shown in Table 1, the biggest drop occurs under case 3, which generates the smallest Southern human-capital accumulation. In light of the initial values, at the end of the adjustment process, a new higher steady-state level of $\widetilde{H}_w$ is only reached in case 1.

The joint behavior of $\widetilde{Q}$ and $\widetilde{H}_w$ implies that initially the South grows at a higher rate than the North owing to the immediate jump in the Southern share of time devoted to work. Afterwards, it is the magnitude of the probability of successful imitation when compared with the probability of successful innovation that accounts for the higher Southern growth rate. Finally, both countries grow at the same rate due to the successive increases in the cost of imitation, which represents a form of diminishing returns. In the end, both the technological-knowledge gap and the gap in human capital at work remain constant. That is, the interest rate and the growth rate of the South fall steadily towards their (or Northern) steady-state values.

The initial conditional value of $u_{w,S}$ indicates that the South starts with a relative scarcity of human capital, due to the smaller value of $u_{w,S}$ at $t = 0$. The differential is greater in case 1 (see the Appendix). As reported above, at the beginning of the transitional phase, the growth rate of human capital at work drives economic growth, due to both the previous increase in the amount of human capital and the reallocation of human capital to production. After that, when the economy moves towards the steady state, R&D becomes the main engine of growth.

We also checked the robustness of the results of the transitional dynamics to shocks. The results were obtained from numerical simulations in which one parameter or an initial condition at a certain time is allowed to deviate from its baseline value. The general conclusion is that the model's qualitative behavior is similar for the ranges of parameter values tested. In fact, similar stable saddle paths to steady state were obtained, differing only slightly in the specific levels of the steady state of the variables which they approach.

4 Concluding remarks

This paper presents a non-scale endogenous growth model through R&D and human-capital accumulation, which are considered the two driving forces of endogenous growth by empirical growth literature. By considering two countries, the Northern innovator and the Southern imitator, it highlights the effects of North-South technological-knowledge diffusion through imitation for the less developed country (the South).

In our model the South can only benefit from Northern R&D when its technological-knowledge backwardness is moderate. Moreover, we propose three catching-up specifications in which the mechanism of catching-up results from the advantage of moderate backwardness and imitation capacity. Thus, we show that the convergence of the Southern middle-income country differs under each specification. Indeed, regardless of the case, the South improves its relative technological knowledge equally, but not the relative level of human capital at work, which can partly offset the benefit obtained in terms of technological knowledge. Thus, the results depend on the connection between the North-South technological-knowledge gap and the South's relative level of employed human capital.

We can alternatively state that our approach extends standard North-South technological-knowledge diffusion models, by considering human-capital accumulation, which, in turn, affects the imitation capacity, and that the South can only imitate Northern technological knowledge when it is sufficiently close to the Northern frontier. Furthermore, our approach can accommodate the behavior of all Southern countries and it is in line with the existence of 'club convergence'. That is, we can provide a theoretical explanation for the divergence that has occurred between certain countries, as well as for the convergence that has taken place between others.

Our argument is based on the premise that the process of Northern technological-knowledge progress can only favor some Southern countries. In this case, technological-knowledge imitation is a window of opportunity for the South; during the transition towards the steady state it achieves higher growth rates. In our case, convergence is at first driven by the behavior of the Southern human capital and then by Southern imitation. This makes our framework similar to the neoclassical model, in the sense that the decreasing probability of imitation towards the steady state is analogous to the diminishing returns on capital in the neoclassical model.

However, our assumptions with regard to exogenous levels of productivity related to institutions, and to the probabilities of successful R&D tend to keep the North as the technological-knowledge leader country. In fact, in steady state there is convergence in growth rates but not in levels. Thus, our framework also agrees with the evidence in that it suggests that absolute convergence only takes place within groups of homogeneous countries, which operate in similar institutional, legal and economic environments with the same technologies and tastes.

As a future step of this research, we intend to extend the framework to international trade of intermediate goods, where R&D is directly applied, and international trade of intermediate and final goods. In this latter case, the effects are limited to the somewhat more traditional ones, in international trade theory, such as the pattern of final-goods specialization of the economies. In both cases, the discussion of international intellectual properties rights, in addition to the domestic ones, can be done.

Appendix: Baseline parameter values and initial conditions

Baseline parameter values

Parameter	Value
A_N	1.56
A_S, β_S	1.00
α	0.60
q	2.50
β_N	1.60
ζ_N	2.00

Parameter	Value
ζ_S	4.00
$s_{R\&D}$	0.25
d	0.10
$\overline{\sigma}_1$	0.25
$\overline{\sigma}_2$	0.60
$\overline{\sigma}_3$	3.92

Parameter	Value
$\overline{\sigma}_4$	0.05
$\overline{\sigma}_5$	0.23
θ	1.05
ρ	0.03
δ	0.02
χ_T	0.09

Parameters are chosen to calibrate the steady-state world growth rate around 2%, which approximately matches the average per capita growth rate of the U.S. in the post-war period (*e.g.*, Jones, 1995b). For some parameters the choice is guided by empirical findings, while other parameter values are based on theoretical specifications. When the range of choice is large we have opted for a value close to some critical value.

Initial values of the variables

Variable	Value
$\widetilde{Q}$	0.34
$\widetilde{H}_w$	0.30
ϑ	0.20

Variable	Value
$u_{w,S}\|_{\text{case 1}}$	0.44
$u_{w,S}\|_{\text{case 2}}$	0.51
$u_{w,S}\|_{\text{case 3}}$	0.50

Notes

[1]It can be shown that X in (4) is also expressible as a function of Q and H_w.

[2]The positive learning effect is modeled in such a way that, together with the adverse effect, it exactly offsets the positive influence of the quality rung on the profits of each intermediate good leader firm, as we will see below. This is the technical reason for the presence of the production function parameter α in (6).

[3]However, when the gap is wider such that $\widetilde{Q}$ is below threshold d, backwardness is no longer an advantage. Hence, the rule that the wider the initial technological-knowledge gap, the higher the catching up, does not apply unconditionally, as it does in the standard backwardness hypothesis.

[4]We will use the dot above a variable to denote a time change in that variable.

[5]Indeed, bearing in mind (4), (5), (17) and the first footnote, C is expressible as a function of Q and H_w.

[6]More specifically, by considering the human capital market equilibrium, the free-entry condition into R&D, individual utility maximization with individual optimal time allocation and that the North is in steady state, we will be able to characterize the Southern transitional dynamics.

[7]That is, to characterize the transition path for the South, we need to consider the representative domestic intermediate good (defined as the average domestic intermediate good).

[8]By taking for example into consideration the catching-up specification $CU_1(t)$.

[9]For example, the capital gains (see (22)) are only present during the transitional phase.

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