Discrete-Time Implementation of Continuous-Time Portfolio Strategies

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Abstract

Since trading cannot take place continuously, the optimal portfolio calculated in a continuous-time model cannot be held, but the investor has to implement the continuous-time strategy in discrete time. This leads to the question how severe the resulting discretization error is. We analyze this question in a simulation study for a variety of models. First, we show that discrete trading can be neglected if only the stock and the money market account are traded, even in models with additional risk factors like stochastic volatility and jump risk in the stock and in volatility. Second, we show that the opposite is true if derivatives are traded. In this case, the utility loss due to discrete trading may be much larger than the utility gain from having access to derivatives. To profit from trading derivatives, the investor has to rebalance his portfolio at least every day.

Keywords: Asset Allocation, Discrete Trading, Use of Derivatives

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1 Introduction

Over the last 35 years, beginning with the seminal article by Merton (1971), the theory on optimal dynamic portfolio strategies in continuous time has made considerable progress. Optimal portfolio strategies are known today for a variety of market models with stochastic volatility (SV), jumps in returns and even jumps in volatility, and for a variety of tradable securities. Models with both SV and jumps have, e.g., been studied in Liu (2006) and Liu, Longstaff, and Pan (2003). Liu and Pan (2003) pushed the general theory a large step forward by adding derivatives to the set of tradable assets, thereby completing the market.

The optimal portfolio in these models is given by a dynamic trading strategy, and investors have to trade continuously in order to hold this portfolio. This is obvious when the optimal portfolio depends on time and on state variables like the stock price, interest rates, or stochastic volatility. However, continuous rebalancing is necessary even if the optimal share of wealth invested in the stock and the money market account remains constant over time as it is the case in a Black-Scholes economy. Without intervention, different returns for different assets cause the real portfolio to drift away from the original portfolio. Theory thus demands that the investor rebalances his portfolio at every instant. In practice, however, this can never be achieved, since trading even on fully electronic systems is only possible at discrete points in time. Furthermore, investors may not want to rebalance their portfolio too frequently because of trading costs, liquidity constraints or their opportunity costs of trading.

The impact of discrete trading on hedging strategies for derivatives is studied in many papers. Examples for this type of analysis can be found, among many others, in Dudenhausen (2003) or Branger and Schlag (2004). For the case of dynamic portfolio strategies however, until now very few attempts are documented that study the impact of rebalancing frequency and different rebalancing strategies on the welfare of the investor. Laue (2002) investigates the welfare impact of different rebalancing strategies based on calendar time and on threshold values on the deviation of the actual portfolio from the optimal portfolio. She considers a BS model with investors having constant absolute or relative risk aversion, and she finds that utility losses are small even for wide no-intervention intervals and low rebalancing frequencies. Sun, Fan, Chen, Schouwenaars, and Albota (2004) study a model where returns follow a multivariate normal distribution. They derive the optimal portfolio strategies when trading is only possible at discrete points in time and investors face transaction costs. Again, the differences between the performance of their optimal strategy and classical rebalancing methods like 'calendar rebalancing' or 'tolerance band rebalancing' are surprisingly small. With five assets, they find annualized suboptimality costs for 'no trading' of about 30 basis points, for 'annual rebalancing' of about 1.5 basis points, for 'monthly rebalancing' of basically zero, and for 5% tolerance band rebalancing of about 0.7 basis points. The performance of different rebalancing strategies in case of transaction costs is also analyzed in Donohue and Yip (2003). Using the quasi-optimal value function approach of Leland (1999), they derive optimal rebalancing strategies and compare these strategies to heuristic rebalancing strategies in a simulation study. Again, the loss due to the use of heuristic strategies is quite small as long as transaction costs are not taken into account. In summary, these results suggest that the problem of discrete trading is not economically significant for portfolio planning. Naive discretization, i.e. using the continuous-time strategy in discrete time without any modification, performs rather well. The gains from using more sophisticated rebalancing schemes or even finding the optimal strategy in discrete time are rather small.

Our paper contributes to the literature by extending these studies in two directions. First, we analyze whether these results still hold in markets with a stochastic investment opportunity set or even with jumps in stock prices and volatility. Second, we add derivatives to the set of traded assets. On the one hand, derivatives help to complete the market in case of stochastic volatility or jumps, but on the other hand, they are more sensitive to changes in time to maturity and state variables than stocks or index funds. Thus, discrete trading might be a more severe problem in case of derivatives.

First, we extend the analysis to markets with a stochastic investment opportunity set or even with jumps in stock prices and volatility. If only the stock and the money market account are available to the investor, the market is incomplete. To explore the economic significance of discrete trading, we perform a simulation study. We find that, although welfare losses due to infrequent rebalancing become somewhat larger when introducing additional risk factors, they are still surprisingly small. For example, for a model including stochastic volatility and downward jumps in the stock price and for a rebalancing frequency as low as once per year over a 10 year horizon, 1.00115 times the initial wealth W_0 is enough to achieve the same expected utility as in the case of continuous trading with initial wealth W_0. In all cases considered here, the loss due to discrete trading is of the order of only one or two basis-points. Even not rebalancing at all over the 10 year investment period does not result in economically significant utility losses, except when the investor has an extremely low degree of risk aversion.

In a second step we add derivatives to the set of traded securities in order to complete the market. This completely changes the picture. While discrete trading is an economically insignificant problem when only the stock and the money market account are traded, it becomes highly relevant when the investor includes derivatives in his portfolio. In a model including stochastic volatility and downward jumps in the stock price, e.g., the initial wealth would have to be multiplied by a factor of more than 6 – and in some cases of more than 90 – to achieve the same expected utility with monthly rebalancing as in the case of continuous trading. As shown in our examples, the losses from discrete trading may be so high that they more than offset the utility gains from having access to derivatives. In this case, the investor is better off if he does not include derivatives in his portfolio. In our numerical examples, he or she has to rebalance the portfolio monthly in order to profit from trading derivatives in a jump-diffusion model with deterministic jumps in the stock price. In all other models with stochastic volatility, the portfolio has to be rebalanced at least daily to profit from derivatives.

Our findings have several implications. First, investors and fund managers who want to rely on the results from continuous time asset allocation models have to implement the strategies in discrete time. Our results show that the induced utility loss can be ignored if only stocks and bonds are used. With derivatives, however, the portfolio has to be adjusted at least once a day to avoid prohibitively high utility losses due to discrete trading. This will only be possible if trading costs for the investor are quite low, which provides a strong argument for investing into (hedge) funds as they can exploit economies of scale in trading. Second, for a lot of models and trading strategies, closed form solutions for the indirect utility function are not available. When analyzing these models and trading strategies, one has to resort to numerical techniques like for example Monte Carlo simulations. The strategy is then necessarily discretized, and an important question is how fine the time grid must be chosen to obtain reliable results. Our analysis shows that the grid might be quite coarse if only the stock and the money market account are traded. On the other hand, with derivatives, time steps of at least one day have to be used. Third, as an alternative to naive discretization strategies, one can use trading strategies that are optimal in a discrete time setup. Our results suggest that these strategies can offer a great improvement when derivatives are to be used, and that this holds even for moderate trading frequencies. The additional effort for deriving a more robust strategy would certainly pay off, but is well beyond the scope of this paper and left for future research.

The structure of the remainder of the paper is as follows. In Section 2 we introduce the model and briefly review optimal dynamic portfolio strategies for the incomplete and complete market cases. Section 3 describes the rebalancing strategies in discrete time and gives a brief exposition of the simulation technique. In Section 4 we present the results for the case without derivatives, and in Section 5 for the case when derivatives are also available to the investor. Section 6 concludes.

2 The Model

2.1 Stochastic Setup

The model we consider in this paper is one with stochastic volatility, jumps in the stock price, and jumps in volatility. In the context of option pricing, this SVCJ model is studied in, e.g., Duffie, Pan, and Singleton (2000) or Broadie, Chernov, and Johannes (2005), while Liu, Longstaff, and Pan (2003) and Branger, Schlag, and Schneider (2005) analyze its implications for portfolio planning. The dynamics of the stock price S_t and the local variance V_t are given by the following stochastic differential equations

$$dS_t = \left(r + \eta_1 V_t + \mu_X(\lambda^P - \lambda^Q)V_t\right) S_t dt + \sqrt{V_t} S_t dB_t^{(1)} + S_{t-}\mu_X(dN_t - \lambda^P V_t dt) \quad (1)$$
$$dV_t = \kappa^P(\bar{v}^P - V_t)dt + \sigma_V \sqrt{V_t}\left(\rho dB_t^{(1)} + \sqrt{1-\rho^2} dB_t^{(2)}\right) + \mu_Y(dN_t - \lambda^P V_t dt). \quad (2)$$

The Poisson process N_t has a jump intensity of $\lambda^P V_t$. μ_X and μ_Y are the return and the variance jump size, respectively. $\eta_1\sqrt{V_t}$ and $\eta_2\sqrt{V_t}$ are the risk premia for the diffusions $B_t^{(1)}$ and $B_t^{(2)}$, respectively. This means that the investor receives on average a compensation of $\eta_1 V_t$ and $\eta_2 V_t$ for taking on one unit of $\sqrt{V_t}dB_t^{(1)}$ and $\sqrt{V_t}dB_t^{(2)}$, respectively. The difference $\mu_X(\lambda^P - \lambda^Q)V_t$ represents the instantaneous compensation for jump risk.

Given the market prices of risk for the two diffusion processes and for the jump risk factor, the dynamics of the stock price and the volatility process under the risk-neutral measure are

$$dS_t = rS_t dt + \sqrt{V_t} S_t d\widetilde{B}_t^{(1)} + S_{t-}\mu_X(dN_t - \lambda^P V_t dt) \quad (3)$$
$$dV_t = \kappa^Q(\bar{v}^Q - V_t)dt + \sigma_V \sqrt{V_t}\left(\rho d\widetilde{B}_t^{(1)} + \sqrt{1-\rho^2} d\widetilde{B}_t^{(2)}\right) + \mu_Y(dN_t - \lambda^Q V_t dt), \quad (4)$$

where

$$\kappa^Q = \kappa^P + \sigma_V\left(\rho\eta_1 + \sqrt{1-\rho^2}\eta_2\right) + \left(\lambda^P - \lambda^Q\right)\mu_Y$$
$$\kappa^Q \bar{v}^Q = \kappa^P \bar{v}^P.$$

$\widetilde{B}_t^{(1)}$ and $\widetilde{B}_t^{(2)}$ are standard Brownian motions under the risk-neutral measure Q, and N_t is a Poisson process under the risk-neutral measure with intensity $\lambda^Q V_t$.

The model with jumps in returns only (SVJ model) is obtained as a special case of the model presented above by setting $\mu_Y = 0$. By additionally setting $\mu_X = 0$ and $\lambda^P = \lambda^Q = 0$ we get the SV model or Heston model without jumps. The SJ model or Merton model with jumps in prices, but constant volatility results from the SVJ model if we discard Equations (2) and (4) and replace $\sqrt{V_t}$ in Equations (1) and (3) by some constant σ.

If derivatives are traded, the dynamics of the price of a derivative $O_t^{(i)} = g^{(i)}(t, S_t, V_t)$ are given by

$$\begin{aligned} dO_t^{(i)} &= rO_t^{(i)}dt + \left(g_s^{(i)} S_t + \sigma_V \rho g_v^{(i)}\right)\left(\eta_1 V_t dt + \sqrt{V_t} dB_t^{(1)}\right) \\ &\quad + \sigma_V \sqrt{1-\rho^2} g_v^{(i)}\left(\eta_2 V_t dt + \sqrt{V_t} dB_t^{(2)}\right) \\ &\quad + \Delta g^{(i)}\left((\lambda^P - \lambda^Q)V_t dt + dN_t - \lambda^P V_t dt\right), \end{aligned}$$

where

$$
\begin{aligned}
g_s^{(i)} &= \left.\frac{\partial g^{(i)}(s,v)}{\partial s}\right|_{(S_t,V_t)} \\
g_v^{(i)} &= \left.\frac{\partial g^{(i)}(s,v)}{\partial v}\right|_{(S_t,V_t)} \\
\Delta g^{(i)} &= g^{(i)}\left((1+\mu_X)S_{t-}, V_{t-}+\mu_Y\right) - g^{(i)}(S_{t-},V_{t-}).
\end{aligned}
$$

For the SJ model, these so-called 'sensitivities' $g_s^{(i)}$ and $\Delta g^{(i)}$ and the prices of put and call options are calculated following Merton (1976). For the other models, the methodology presented in Duffie, Pan, and Singleton (2000) is used, and the resulting differential equations are solved numerically. Note that $g_s^{(i)}$, $g_v^{(i)}$ and $\Delta g^{(i)}$ depend on the maturity of the option, the current stock price and the current volatility. When one of these variables changes, the sensitivities change as well.

2.2 Optimal Strategies with Stock and Money Market Account

For the case of an SVCJ model where only the stock and the money market account are traded, the portfolio planning problem is solved by Liu, Longstaff, and Pan (2003). The results are repeated here only for reasons of completeness.

We assume that the investor has a standard CRRA utility function with risk aversion γ, i.e. $U(w) = w^{1-\gamma}/1-\gamma$. The investor maximizes expected utility derived from terminal wealth W_T only, i.e. we do not consider intermediate consumption. The indirect utility function J is defined via the equation

$$
J(w,v,t) = \max_{\{\phi_s, t\le s\le T\}} E_t\left[U(W_T)|W_t = w, V_t = v\right],
$$

where ϕ_t denotes the share of wealth invested in the stock at time t. To find the optimal portfolio strategy $\{\phi_t^\star\}_{\{0\le t\le T\}}$, one has to apply the principle of optimal stochastic control. This leads to the following Hamilton-Jacobi-Bellman equation for J

$$
\begin{aligned}
\max_{\phi_t}\Bigg\{ & \frac{\phi_t^2 w^2 v}{2} J_{ww} + \phi_t \rho \sigma_V w v J_{wv} + \frac{\sigma_V^2 v}{2} J_{vv} \\
& + \big(r + \phi_t(\eta_1 - \mu_X \lambda^Q) v\big) w J_w + (\kappa^P \bar{v}^P - \kappa^P v - \mu_Y \lambda^P v) J_v \\
& + \lambda^P V_t \big(J(w(1+\phi_t\mu_X), v+\mu_Y, t) - J(w,v,t)\big) + J_t \Bigg\} = 0, \qquad (5)
\end{aligned}
$$

where subscripts of J denote partial derivatives. The equation for the case of no jumps in volatility is obtained by setting μ_Y to zero, and the case without any jumps is obtained by setting λ^P and λ^Q to zero. The HJB equation for a model with constant volatility, but possibly with return jumps, is obtained by omitting all partial derivatives with respect to v and replacing v by σ^2 in (5).

The functional form of the indirect utility function is

$$J(w, v, t) = \frac{1}{1-\gamma} w^{1-\gamma} \exp(a(t) + b(t)v)$$

for the general case, and

$$J(w, t) = \frac{1}{1-\gamma} w^{1-\gamma} \exp(a(t))$$

for the case of constant volatility with boundary condition $a(T) = b(T) = 0$. Calculating the necessary partial derivatives and plugging them back into the HJB equation gives the first-order condition for $\phi_t^\star$:

$$(\eta_1 - \mu_X \lambda^Q) + \rho \sigma_V b(t) - \gamma \phi_t^\star + \lambda^P (1 + \phi_t^\star \mu_X)^{-\gamma} \mu_X \exp(b(t)\mu_Y) = 0. \tag{6}$$

Again, the equations for the special cases without jumps are obtained by setting the related jump sizes to zero. For the case with constant volatility, the first-order condition is obtained by setting $\mu_Y = \sigma_V = 0$ and replacing V_t by a constant. In this case, the optimal portfolio strategy will not be time-dependent.

Collecting the v terms in the HJB equation yields the following ordinary differential equation for $b(t)$:

$$\begin{aligned} b'(t) + \frac{\sigma_V^2 b(t)^2}{2} + \left(\phi_t^\star \rho \sigma_V (1-\gamma) - \kappa^P - \mu_Y \lambda^P\right) b(t) \\ + \Bigg[\frac{\gamma(\gamma-1)(\phi_t^\star)^2}{2} + \left(\eta_1 - \mu_X \lambda^Q\right)(1-\gamma)\phi_t^\star \\ + \lambda^P \left(1 + \phi_t^\star \mu_X\right)^{1-\gamma} \exp\left(b(t)\mu_Y\right) - \lambda^P \Bigg] &= 0, \end{aligned} \tag{7}$$

and collecting the remaining terms in the HJB equation yields the following ordinary differential equation for $a(t)$:

$$a'(t) + \kappa^P \bar{v}^P b(t) + (1-\gamma) r = 0.$$

In general, equations (6) and (7) have to be solved jointly by numerical methods. For the SV model, there are closed-form solutions which can for example be derived following Liu (2006). They are given in Appendix A.

2.3 Optimal Strategies in a Complete Market

With stochastic volatility or jumps, the market is incomplete so long as only the stock and the money market account are traded. The introduction of options enlarges the set of traded payoffs the investor can choose from. The market is complete if the number of traded risky assets equals the number of risk factors, where every possible combination of jump sizes in the stock price and the variance counts as one additional risk factor. In this case, the investor can achieve every payoff profile he would like to by a dynamic trading strategy in the basic assets.

Here, we only allow for simultaneous jumps, and we assume that the jump size both for the stock return and the variance is constant. Thus, we need only two additional derivatives to complete the market, one for volatility risk and one for jump risk. If either jump risk or volatility risk is absent, one derivative is enough to complete the market.

In a complete market, the investor can either decide on his optimal portfolio holdings or on his optimal exposure with respect to the risk factors. Given the desired exposure, it is always possible to find a portfolio of traded assets with exactly this risk exposure. As argued by Liu, Longstaff, and Pan (2003), working in terms of the optimal exposure is both economically more intuitive and mathematically more convenient than working with the optimal portfolio. In terms of factor exposures the dynamics of wealth are

$$\begin{aligned} dW_t &= rW_t dt + \theta_t^{B1} W_t \left(\eta_1 V_t dt + \sqrt{V_t} dB_t^{(1)}\right) + \theta_t^{B2} W_t \left(\eta_2 V_t dt + \sqrt{V_t} dB_t^{(2)}\right) \\ &\quad + \theta_t^N W_{t-} \left(dN_t - \lambda^Q V_t dt\right), \end{aligned} \tag{8}$$

where θ_t^{B1} denotes the fraction of wealth invested into $\sqrt{V_t} B_t^{(1)}$. Equivalently, θ_t^{B2} and θ_t^N are the fractions of wealth invested into diffusive volatility risk and jump risk, respectively. The corresponding positions in the assets implementing the exposure given by θ_t^{B1}, θ_t^{B2}, and θ_t^N then follow from the sensitivities of the derivatives and the stock with respect to these risk factors.

The optimal factor exposures and the indirect utility functions for the case of complete markets were first derived by Liu and Pan (2003) in an SVJ model and by Branger, Schlag, and Schneider (2005) in an SVCJ model. The optimal factor exposures in the latter model are given by

$$\begin{aligned} \theta_t^{\star B1} &= \frac{\eta_1}{\gamma} + \rho \sigma_V H(\tau) \\ \theta_t^{\star B2} &= \frac{\eta_2}{\gamma} + \sqrt{1-\rho^2}\sigma_V H(\tau) \\ \theta_t^{\star N} &= \left[\left(\frac{\lambda^P}{\lambda^Q}\right)^{1/\gamma} - 1\right] + \left(\frac{\lambda^P}{\lambda^Q}\right)^{1/\gamma} \left(\exp\left(H(\tau)\mu_Y\right) - 1\right), \end{aligned}$$

where the function $H(\tau)$ solves the ordinary differential equation

$$H'(\tau) = a + bH(\tau) + cH^2(\tau) + \lambda^Q \left(\frac{\lambda^P}{\lambda^Q}\right)^{1/\gamma} \exp\left(\mu_Y H(\tau)\right)$$

with boundary conditions $H(0) = 0$, $\tau = T - t$ and

$$\begin{aligned} a &= \frac{1-\gamma}{2\gamma^2}\left[(\eta_1)^2 + (\eta_2)^2\right] + \frac{1-\gamma}{\gamma}\lambda^Q - \frac{1}{\gamma}\lambda^P \\ b &= -\left(\kappa^P + \mu_Y \lambda^P\right) + \frac{1-\gamma}{\gamma}\sigma_V \left(\rho\eta_1 + \sqrt{1-\rho^2}\eta_2\right) \\ c &= \frac{1}{2}\sigma_V^2. \end{aligned}$$

The optimal factor demands can be decomposed into a myopic demand (first summand) and a hedging demand (second summand). By the myopic demand, the investor tries to exploit the risk premium, while the hedging demand arises due to future uncertainty about the investment opportunity set. It converges to zero for $\tau \to 0$. A more detailed discussion is provided in Liu and Pan (2003) and Branger, Schlag, and Schneider (2005).

3 Discrete Rebalancing Strategies

3.1 Discretization Error, Calendar Rebalancing, and Threshold Rebalancing

The optimal portfolio is calculated under the assumption that investors can trade continuously. If trading is possible at discrete points in time only, these optimal strategies can no longer be implemented. Rather, the actual portfolio held by the investor will deviate from the optimal portfolio between two rebalancing dates. In the incomplete market where the stock is the only traded risky asset, deviations from the optimal portfolio composition may arise for two reasons. First, the stock return may be greater or less than the risk-free rate. In this case the value of the stock holdings after some holding period is greater or less than the target equity fraction. Second, the intertemporal hedging demand due to a stochastic investment opportunity set is time-dependent. Thus, the target equity share changes over time, and the portfolio would have to be adjusted continuously. In the complete market where the investor can trade derivatives as well, there is yet a third reason for deviations between the realized and the optimal portfolio. As argued above, the investor first determines the optimal exposure with respect to the risk factors, and then calculates the portfolio that achieves this exposure, based on the sensitivities of the assets with respect to the risk factors. For options, these sensitivities change significantly over time, as the stock price, volatility, and time to maturity change, causing the realized portfolio exposure to deviate from the original one.

In the following, we consider what could be called a *naive discretization strategy:* when the investor buys the initial portfolio or rebalances his portfolio, he uses the portfolio composition determined in the continuous-time model at this point in time. This portfolio is optimal over the next instant, but not over the discrete holding period to follow, as argued above.

To implement the discrete trading strategy, investors do not only have to decide on how to discretize the strategy, but also on when to rebalance their portfolio. We consider two methods, calender rebalancing and threshold rebalancing, which are also used in studies concerned with transaction costs. In case of *calender rebalancing*, the investors adjust their portfolio at pre-specified points in time, e.g. once a year. Over the interval between two successive trading dates, they keep the composition of the portfolio constant. In case of *threshold rebalancing*, investors monitor the deviation of the actual from the optimal portfolio composition. Whenever the deviation exceeds some prespecified boundary, they adjust their portfolio holdings. For computational reasons, we apply this rebalancing method in the incomplete market only.

3.2 Losses Due to Discrete Trading

The utility of an investor decreases if he can only trade at discrete points in time. In the incomplete market where only the stock and the money market account are traded, we measure his utility loss by the multiple $\widetilde{W}_{disc\to cont,w/o}$ of initial capital that is necessary to compensate him for not being able to trade continuously. This multiple is defined by

$$J_{disc,w/o}(0, W_0 \cdot \widetilde{W}_{disc\to cont,w/o}, V_0) = J_{cont,w/o}(0, W_0, V_0),$$

where $J_{disc,w/o}$ and $J_{cont,w/o}$ denote the indirect utility function in case of discrete and continuous trading, respectively. Since the relative risk aversion is constant, the optimal portfolio weights do not depend on initial capital, and the indirect utility function can be factorized into $J_{\cdot,\cdot}(0, W_0, V_0) = W_0^{1-\gamma} J_{\cdot,\cdot}(0, 1, V_0)$. Thus, $\widetilde{W}_{disc\to cont,w/o}$ can be calculated as

$$\widetilde{W}_{disc\to cont,w/o} = \left(\frac{J_{cont,w/o}(0, 1, V_0)}{J_{disc,w/o}(0, 1, V_0)} \right)^{\frac{1}{1-\gamma}}.$$

The larger $\widetilde{W}_{disc\to cont,w/o}$, the larger the utility gain from continuous trading as compared to discrete trading, and the larger the utility loss from discrete trading. It is intuitively clear that $\widetilde{W}_{disc\to cont,w/o} \geq 1$. In a complete market, $\widetilde{W}_{disc\to cont,w}$ is defined in an analogous way as

$$\widetilde{W}_{disc\to cont,w} = \left(\frac{J_{cont,w}(0, 1, V_0)}{J_{disc,w}(0, 1, V_0)} \right)^{\frac{1}{1-\gamma}}.$$

Finally, we consider the benefits from having access to derivatives when trading is possible only at discrete points in time and when the investor uses a naive discretization strategy. We thus compare the utility with and without derivatives. $\widetilde{W}_{disc,w\to w/o}$ is defined as

$$\widetilde{W}_{disc,w\to w/o} = \left(\frac{J_{disc,w/o}(0, 1, V_0)}{J_{disc,w}(0, 1, V_0)} \right)^{\frac{1}{1-\gamma}}.$$

Note that, different from the cases above, $\widetilde{W}_{disc,w\to w/o}$ might be smaller or larger than one. With discrete trading, the investor profits from including derivatives in his portfolio if and only if $\widetilde{W}_{disc,w\to w/o} < 1$.

Given that derivatives increase the set of attainable payoffs, it seems surprising that the expected utility of the investor may actually decrease due to trading derivatives. The reason is that the investor chooses his portfolio in a suboptimal way. He does not decide on his portfolio in the discrete-time model he is actually facing, but rather uses a continuous-time model, and then applies the portfolio weights in discrete time. The utility loss from using this suboptimal strategy may be quite large. In particular, it may be larger in the complete market than in the incomplete market, and it may even more than offset the utility gain from having access to derivatives. If this is true, $\widetilde{W}_{disc,w\to w/o} > 1$, and the investor is better off if he does not use derivatives (or determines an optimal trading strategy in discrete time).

Finally, note that

$$\widetilde{W}^{-1}_{disc,w\to w/o}\widetilde{W}_{disc\to cont,w} \quad = \quad \widetilde{W}_{disc\to cont,w/o}\widetilde{W}_{cont,w/o\to w}$$

with the obvious definition of $\widetilde{W}_{cont,w/o\to w}$. Both products provide a measure for the utility gain when going from an incomplete market with discrete trading to a complete market with continuous trading.

3.3 Simulation Setup

For the case of continuous trading, the indirect utility can be calculated either in closed form or by numerically solving partial differential equations. This is no longer true for the discrete trading strategies. In order to evaluate their performance, we use Monte Carlo simulations.

The stochastic processes for the stock and the local variance are simulated using an Euler scheme with one time step per hour. To obtain accurate results, we use 2,000,000 replications in the case without options, and 100,000 in the case with options. Since the results remain stable when increasing the number of simulations from 10,000 to 100,000, we think that this number of paths is sufficient. We also checked that the indirect utility in the discrete case converges towards the indirect utility with continuous trading if the trading interval becomes smaller and smaller.

3.4 Calibration

The basic parametrization for the SV model with jumps in prices and volatilities is taken from Branger, Schlag, and Schneider (2005), whose parameter values are in turn based on Liu, Longstaff, and Pan (2003). The parameters of the other models are chosen in such a way that the levels of overall risk and of risk premia for the stock and for options are as similar as possible.

First, we match several moments for the stock and for local volatility perfectly. This are the expected instantaneous stock return, given by $(\eta_1 + (\lambda^P - \lambda^Q)\mu_X)\bar{v}^P$ in models with jumps in returns and by $\eta_1 \bar{v}^P$ in models without, and the expected instantaneous stock return variance ($\bar{v}^P(1+\mu_X^2\lambda^P)$ in models with jumps in returns, $\bar{v}^P$ in models without). Furthermore, we equalize the expected instantaneous variance of variance ($\bar{v}^P(\sigma_V^2+\mu_Y^2\lambda^P)$ in the model with jumps in volatility, $\bar{v}^P\sigma_V^2$ in models without) and the expected time between jumps (equal to $\frac{1}{\lambda^P \bar{v}^P}$ in all models containing jumps). The remaining free parameters ρ, $\bar{v}^Q$, λ^Q, and η_2 are calibrated by minimizing the sum of squared differences in expected instantaneous returns (weight 60%) and expected instantaneous variances (weight 40%) for four standard options. These options are at-the-money calls with maturities equal to one and three months, respectively, and out-of-the-money puts with the same maturities and a strike price equal to 90% of the current stock price. The resulting parameters for the models are shown in Table 1.

4 Discrete Trading With Stock and Money Market Account Only

In this section we analyze the effect of discrete portfolio rebalancing in an incomplete market, where only the stock and the money market account are traded. Besides the model of Black-Scholes, we consider models with stochastic volatility (SV model or Heston model), with jumps (SJ model or Merton model), with both stochastic volatility and jumps in the stock price (SVJ model) and with jumps in volatility, too (SVCJ model). This allows us to analyze whether discrete trading is a more severe problem the more sophisticated the model is, and it allows us to compare the impact of discrete trading for stochastic volatility and jump risk. The different models have been calibrated as described in Section 3.4. The planning horizon T is 10 years, the risk aversion of the investor is set equal to 3.

The results are shown in Tables 2 to 4. The most striking result in Table 2 is that the utility loss of even a buy-and-hold strategy (over 10 years) is very small. As a consequence, stretching the period between successive portfolio changes has only very little impact on the performance measure (or 'implied initial wealth') $\widetilde{W}_{disc \to cont, w/o}$. This holds true irrespective of the model. The utility losses seem to become somewhat larger when jumps in returns or in volatilities are added to the model, but neither the absolute level nor the differences between models are anywhere near economic significance.

The same is true for the threshold strategy, as can be seen from Table 3. Nevertheless, one might argue that the threshold strategy is slightly better than calendar rebalancing for two reasons. First, to achieve a given $\widetilde{W}_{disc \to cont, w/o}$, the average number of portfolio adjustments shown in Table 4 is smaller than the number of adjustments with calendar rebalancing. Second, the investor can not only monitor the deviation between current and optimal portfolio holdings, but he can also check whether some other conditions on the portfolio composition still hold. This is in particular important in case of jump risk. Here, a limit on stock holdings is needed to ensure that wealth cannot become negative after a jump event, which would result in an expected utility of minus infinity.

The interesting question in this context is, of course, why the utility losses from discrete rebalancing are so surprisingly small. One key to the answer can be found in Table 5, which shows the optimal stock position ϕ_0 at the beginning of a 10 year investment horizon, and ϕ_{10}, the optimal stock holdings immediately before the end of the investment period. Note that these stock holdings only depend on the time, but neither on the stock price nor on the current volatility. The numbers show that the impact of the time horizon on the optimal portfolio is quite small. If we were to see large deviations of the actual from the optimal portfolio composition, then these deviations would have to arise from very different rates of returns on the stock and the bond which cause the actual portfolio weights to drift away from the (nearly) constant optimal weights. To get the intuition, assume $\phi_0 < 1$. If the stock grows at a rate larger than the risk free rate, then the share of wealth invested in the stock increases over time and will thus become too large. A similar argument holds for $\phi_0 > 1$, while there is no need to rebalance if the optimal weight of the stock is constantly equal to one. The more $\phi^\star$ differs from one, the larger the impact of the stock return, and the more severe the utility loss due to discrete trading will be. However, our results show that for sensible parameter values which result in a moderate $\phi^\star$, the problem caused by different rates of return is still quite small. This is true even in the case of jumps where large stock price changes may occur.

Our results are robust with respect to relative risk aversion. We have redone the analysis for $\gamma = 1$ and $\gamma = 8$, and the results for calendar rebalancing in the SVCJ model with $\gamma = 8$ are given in Table 2. For these two risk aversions, the optimal stock position deviates more from one, and we expect the benefits from more frequent rebalancing to be larger, which is indeed the case. Nevertheless, the differences between discrete and continuous trading are still very small. However, note that for $\gamma = 1$, the investor puts more than 200% of his wealth into the stock, which implies that he faces the risk of ending up with a negative wealth if he trades at discrete points in time only.

The results in this section show that discrete trading is not a severe problem if only the stock and the money market account are used. They also suggest that we can replace the average utility in case of discrete trading by the expected utility in case of continuous trading, which can usually be calculated much easier and faster. This will indeed be done in some calculations in the next section.

5 Discrete Trading With Derivatives

We now turn to the case of a complete market. In addition to the stock and the money market account, we need one derivative in case of the SV and the SJ model, and two derivatives for the SVJ and the SVCJ model to achieve market completeness. We use an at-the-money call and, in case a second derivative is needed, a call option with strike price equal to 90%, 95% or 105% of the current stock price. We perform the analysis also for varying times to maturity. This allows us to assess the impact of these characteristics of the option on the performance of the discrete strategy (which is quite pronounced in some cases, as shown below). The investment horizon is 10 years, and the relative risk aversion is 3. The results are based on 100,000 runs. In order to limit the influence of extreme outcomes and to avoid negative terminal wealth, we bound the terminal wealth from below by 1% of the initial wealth.

Table 6 shows the results for the analysis in a market with stochastic volatility and jumps according to the SVCJ model in Equations (1)–(4), and Tables 8, 9 and 10 show the results for the SVJ model, the SV model and the SJ model, respectively. Table 7 gives the results for an SVCJ model with a zero volatility risk premium ($\eta_2 = 0$) in order to assess the impact of volatility risk premia on the results. To describe the impact of discrete trading on the well-being of the investor, we consider the two multiples $\widetilde{W}_{disc \to cont,w}$ and $\widetilde{W}_{disc,w \to w/o}$ defined in Section 3.2. The higher $\widetilde{W}_{disc \to cont,w}$, the higher the utility loss due to discrete trading. $\widetilde{W}_{disc,w \to w/o}$ caputures the benefits from derivatives in case of discrete trading. If $\widetilde{W}_{disc,w \to w/o} > 1$, the investor is better off if he does not trade derivatives.

The overall result is that the utility loss due to discrete trading is economically highly significant for all models where volatility is stochastic, while it is quite moderate with stock jump risk only. This stands in sharp contrast to the case of an incomplete market where only the stock and the money market account are traded and where the utility losses are barely recognizable. In models with stochastic volatility, the investor has to rebalance his portfolio daily or even hourly to profit from derivatives. Furthermore, the choice of the option(s) used in the portfolio can be quite important both in terms of the strike price and in terms of time to maturity.

In the SVCJ model, the multiple $\widetilde{W}_{disc \to cont,w}$ is between 40 and 60 for a holding period of one month. The discrete trading strategy becomes better if rebalancing takes place more often. For weekly rebalancing, however, $\widetilde{W}_{disc \to cont,w}$ is still above 5.9, and even with daily rebalancing, the multiples are at least 1.06, so that the utility losses due to discrete trading are still significant. Even hourly rebalancing is, with multiples of at least 1.02, not perfect. However, with daily or hourly rebalancing, the losses from discrete trading are smaller than the gains from market completeness, and the investor profits from having access to derivatives. With a $\widetilde{W}_{disc,w \to w/o}$ of 0.79 or below, he can indeed realize a big chunk of the utility gain due to the availability of derivatives.

These results are even more pronounced for the SVJ model shown in Table 8. Here, $\widetilde{W}_{disc \to cont,w}$ is between 6 and 120 for a holding period of one month. To profit from trading derivatives, the investor has to adjust his portfolio at least daily or hourly, depending on the time to maturity and the strikes of the options he decides to invest in. Furthermore, the utility loss depends on the time to maturity of the options and, in case of the SVJ model, also on the strike price of the second option.

The numerical results for the SV model and the SJ model given in Tables 9 and 10 allow us to assess whether there are fundamental differences between stochastic volatility and jumps. This is indeed the case. With SV, daily rebalancing is again necessary in order to make holding derivatives worthwhile. And even if utility losses for monthly rebalancing are smaller than in the SVJ and the SVCJ model, they are still prohibitively high and will prevent the investor from trading in derivatives. With constant volatility but stock jump risk, the picture changes completely. Discrete trading is, different from the other models, not a problem. Even in case of monthly rebalancing, $\widetilde{W}_{disc,w \to w/o}$ is at most 0.95 and thus well below 1 if the investor uses options with at most six months to maturity.

In the SVCJ model with a zero volatility risk premium, the results are somewhat different than with a negative volatility risk premium. With a $\widetilde{W}_{disc \to cont,w}$ between 5.7 and 13, the losses due to monthly trading are not that large anymore. The speed of convergence toward the optimal continuous strategy however is slower. Different from the other cases, even daily trading is with most options not sufficient to make holding derivatives worthwhile. Hourly rebalancing however is practically perfect.

To explain our results, first note that very high utility losses can to a large extent be attributed to very low levels of terminal wealth. Figure 1 shows the distribution of terminal wealth for 10,000 simulation runs in the SVCJ model for monthly, weekly, daily and hourly rebalancing. We have – mainly for numerical reasons – imposed a lower bound on the terminal wealth equal to 1% of initial wealth, and the more often this lower bound is attained, the higher the utility loss due to discrete trading. This explains the very high losses due to monthly trading, and also the losses in case of weekly rebalancing, where the lower boundary is still attained a few times. Note that without the lower boundary, we would see negative wealth levels, which result in an indirect utility of minus infinity. With this significant default risk, the corresponding strategies are basically not acceptable. With daily and hourly rebalancing, the lower boundary is no longer attained in our simulation (even if the probability of ending up with a negative wealth is not identically equal to zero). Now, the utility losses are mainly driven by the increased variance due to discrete trading and the higher risk of ending up with a quite low (but positive) terminal wealth.

In a next step, we take a further look at the impact of the portfolio composition on the utility loss. In Tables 6 to 10, the two or three rightmost columns show the average portfolio composition at the points in time when the investor buys derivatives. ϕ is the multiple of wealth invested into the stock, ψ_1 respectively ψ in the case with one derivative only is the multiple of wealth invested into the ATM call and ψ_2 is the multiple of wealth invested in the second derivative. Negative numbers denote short sales. The money market account is used to balance the budget. The numbers show that the optimal portfolio of the investor may be quite extrem. In the SVJ model, e.g., an investor who uses an ATM call and an ITM call with strike price equal to 90% of the stock price and a time to maturity of one year, is about 95 times his wealth short in the stock, 22 times his wealth short in the ATM call and 50 times his wealth long in the ITM call. Such a high leverage makes the position extremely risky, since even small changes in the sensitivities of the options will cause large changes in the sensitivities of the portfolio. The realized portfolio will then deviate significantly from the optimal one, which induces quite high utility losses.

To confirm this intuition, we analyze the actual exposure of the portfolio and its deviation from the optimal exposure. First, the deviations due to discrete trading might actually be quite high in particular for longer holding periods, and the realized exposure is highly volatile. Figures 2 to 4 show 10,000 simulated paths of the evolution of the actual factor exposures over time when the portfolio is not adjusted. They are based on the SVCJ model, the options used are the at-the-money call and a call with strike equal to 90% of the stock price and a maturity of 1 year. One can see that there is already a large deviation between the actual and the original exposure after one day which then increases over time. After one month, there is barely any relation between the actual and the original risk exposure of the portfolio. To assess which of these deviations is most important in explaining the utility loss, we regress $\widetilde{W}_{disc \to cont,w}$ on the absolute value of the average relative deviation

$$\left|\Delta\theta^i\right| = \left|\overline{\left(\frac{\theta_t^{i,\star} - \theta_t^{i,actual}}{\theta_t^{i,\star}}\right)}\right|, \qquad i \in \{B^{(1)}, B^{(2)}, N\}$$

at the time of rebalancing the portfolio. The results from this regression are shown in Table 11. In all models a large part of the variation in $\widetilde{W}_{disc \to cont,w}$ can be attributed to variation in the difference between optimal and actual factor exposures, especially in all models containing jumps. The deviation of the actual from the optimal stock price diffusion exposure plays the most important role.

Overall, the results show that the utility losses are driven by the risk of large losses and even negative wealth levels, by a high leverage, and by quite unstable realized portfolio exposures. In the next step, we want to link these findings to the differences between the models, and we want to explain why the choice of both the maturity of the options and the strike price of the second option can be important. To do so, we analyze the optimal exposure and the sensitivities of the options (which together determine the portfolio composition) and their dependence on time and on the state variables.

Remember that our analysis shows distinctive differences between stochastic volatility and jumps. In particular, it is the combination of stochastic volatility and discrete trading which causes large utility losses. To get the intuition, note that two things are special about stochastic volatility. First, it induces a hedging demand which changes over time, so that the current portfolio will deviate from the optimal one simply because time passes. Figure 6 shows the evolution of the optimal exposure over time. The main changes are seen towards the end of the planning horizon, and they are quite moderate. Second, the sensitivities of the options depend on volatility. Changes in volatility are thus yet another reason for deviations of the realized from the optimal exposure of the portfolio held by the investor, beyond the deviations already induced by the passage of time and changes in the stock price.

For the different models, Figure 5 shows the option price, delta, vega and the price change in case of a jump as a function of volatility for options with different moneyness. Note that in models without jumps in volatility, only volatilities of up to 0.2 are attained. In the SJ model where volatility is constant, the corresponding numbers are 0.04032 for the price of an ATM call, 0.61520 for the delta, and -0.04036 for the price change in case of a jump. One sees that the price and especially the vega are considerably higher in the SVCJ than in the SVJ and SV model. Therefore, the investor has to take less extreme positions in the assets in the SVCJ than in the SVJ model in order to obtain his optimal factor exposure.

We now consider the SVCJ model with a zero volatility risk premium again. The optimal demand for volatility risk is significantly lower than in the benchmark case with $\eta_2 = -2$, since the myopic demand is equal to zero. As can be seen from a comparison of Tables 6 and 7, the positions in the options are reduced significantly, and the utility losses in case of hourly trading are lower, too. These findings suggests that it is mainly the exposure to volatility risk that causes the high leverage in the option positions. Furthermore, note that the investor now takes a long position in stocks instead of a short position.

Finally, the maturity of the option turns out to be important. There are two opposite effects. First, the sensitivities of the options change as time to maturity decreases. This dependence on time to maturity is the more pronounced the shorter the time to maturity, which gives long-term options an advantage over short-term options. Second, the investor has to achieve a certain exposure to the risk factors. The smaller the sensitivities of the options with respect to the risk factors and the more similar the options, the higher the leverage in the optimal portfolio will be. As argued above, a high leverage makes the position extremely risky, and it is mainly the exposure to volatility risk that causes this high leverage. The sensitivity with respect to volatility is highest for options with intermediate times to maturity which in all our numerical examples are options with a time to maturity of three months, which gives these short-term options an advantage. Put together, it depends on the trade-off between the stability of the sensitivities over time and the size of the sensitivities in particular to volatility risk which time to maturity will ultimately be optimal. As it can be seen in our numerical examples, long-term options are superior in the SVCJ model, whereas the investor would certainly prefer short-term options in the SVJ model. Short-term options which allow for lower leverage also tend to be better in the other examples.

While the choice of the second option is not important in the SVCJ model with a negative volatility risk premium, it turn out to be important in the SVCJ model with zero volatility risk premium and in the SVJ model. In these cases one can see clearly that those combinations of assets which yield the least extreme, i.e. the least leveraged portfolio positions perform best. A low leverage results if the sensitivities of the assets used in the portfolio differ significantly, so one should pick the option which is the most different from the stock and the ATM call. This explains why an OTM call with a strike of 105% of the current stock price is better than the ITM call with a strike of 90% of the stock price. The latter suffers in particular from the problem that its delta is quite high, so that almost offsetting positions in the stock and this option must be taken to exploit the differences between these two assets.

6 Conclusion

Optimal dynamic asset allocation strategies derived in continuous-time models pose a problem for the investor. Since he cannot trade continuously, he has to apply the continuous strategy in discrete time, which necessarily creates a discretization error and results in a utility loss. However, the question is how large this error actually is. Previous studies have shown that the utility loss due to discrete trading can be neglected in a Black-Scholes model. In this paper, we have extended these studies in two directions. First, we consider models with further risk factors like stochastic volatility and jumps in the stock price and in volatility. Second, we analyze the impact of discretization when derivatives are traded in addition to the stock and the money market account.

In the incomplete market where only the stock and the money market account are traded, our results confirm that discrete trading is not much of a problem. While the utility loss seems to increase in the number of risk factors, it is far from being economically significant even in the SVCJ model with stochastic volatility and simultaneous jumps in the stock price and in volatility. When derivatives are introduced, discrete trading is still not a problem in the SJ model. In models with stochastic volatility, however, the picture changes completely. The loss due to discrete trading is very large, and the investor would need 5, 10 or even 100 times his initial wealth to compensate him if he would be able to trade only weekly or monthly. These utility losses due to discrete trading are offset by the benefits from having access to derivatives only if the investor rebalances his portfolio every day. Otherwise, the investor is better off if he holds just the stock and the money market account.

Our results show that discrete trading is a significant problem when derivatives are involved. Since daily trading is needed to profit from derivatives, transaction costs might also be a significant problem when it comes to implementing optimal trading strategies. Naive discretization strategies, where the portfolio composition derived in a continuous time setup is just used in discrete time, are at least problematic. It would thus certainly be worthwhile to search for strategies which explicitly take discrete trading into account. Even if the overall optimal strategy might be too hard to find in realistic models, one could try to come up with locally optimal strategies or ad-hoc adjustments. While this paper has shown that there is a need for such strategies, it is left to future research to come up with these strategies. A second strand for future research is the question which derivatives are superior in case of discrete trading. It is clear that a claim to the future optimal wealth as determined in the continuous-time model will certainly solve the problem of discrete trading since it will result in a buy-and-hold strategy. However, such a claim will in general not be traded. This gives rise to the question which liquidly traded standard derivatives are best, and it may also be a motivation for financial innovation.

A Optimal Portfolio Strategy and Indirect Utility for the SV Model

In the case of the SV model the system of Equations (6)–(7) can be solved analytically. The solutions given in the next paragraph are derived from the results in Liu (2006), who calculates the optimal stock weight as:

$$\phi_t^* = \frac{\eta_1}{\gamma} + \frac{\rho\sigma_V b(t)}{\gamma},$$

where

$$b(t) = \begin{cases} -\frac{2(\exp(\xi\tau)-1)}{(\tilde{\kappa}+\xi)(\exp(\xi\tau)-1)+2\xi}\gamma\delta, & \text{if } \xi^2 \geq 0 \\ -\frac{2}{\tilde{\kappa}+\zeta\cos(\zeta\tau/2)/\sin(\zeta\tau/2)}\gamma\delta, & \text{if } \zeta^2 > 0, \end{cases}$$

and

$$\begin{aligned} \zeta &= -i\xi \\ \tau &= T-t \\ \delta &= -\frac{1-\gamma}{2\gamma^2}\eta_1^2 \\ \tilde{\kappa} &= \kappa^P - \frac{1-\gamma}{\gamma}\eta_1\sigma_V\rho \\ \xi &= \sqrt{\tilde{\kappa}^2 + 2\delta(\rho^2+\gamma(1-\rho^2))\sigma_V^2}. \end{aligned}$$

In order to calculate the expected utility we need $a(t)$ as well. Following Liu (2006), $a(t)$ is given by

$$a(t) = -\gamma\kappa^P\bar{v}^P B(t) + (1-\gamma)r\tau + \gamma\bar{c},$$

where $B(t)$ is the primitive of $b(t)$ given by

$$B(t) = \begin{cases} \frac{(-4)\ln\left[\tilde{\kappa}\exp(\xi\tau)-\tilde{\kappa}+\xi\exp(\xi\tau)+\xi\right]}{(\tilde{\kappa}^2-\xi^2)}\gamma\delta & \text{if } \xi^2 \geq 0 \\ \frac{(-4)\gamma\delta\ln(2\tilde{\kappa}\tan(-\frac{1}{4}\zeta\tau)+\zeta\tan(-\frac{1}{4}\zeta\tau)^2-\zeta)+4\gamma\delta\ln(1-\tan(-\frac{1}{4}\zeta\tau)^2)}{\zeta^2+\tilde{\kappa}^2} - \frac{8\gamma\delta\tilde{\kappa}\arctan(\tan(-\frac{1}{4}\zeta\tau))}{\zeta(\zeta^2+\tilde{\kappa}^2)} & \text{if } \zeta^2 > 0 \end{cases}$$

and

$$\bar{c} = -\frac{4\kappa^P\bar{v}^P\delta\ln(2\xi)}{\tilde{\kappa}^2-\xi^2}.$$

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	SVCJ	SVCJ ($\eta_2 = 0$)	SVJ	SV	SJ
κ^P	5.3	5.3	8.81561	5.94614	
$\bar{v}^P$	0.02172	0.0272	0.02172	0.02422	0.02172
λ^P	1.84156	1.84156	1.84156		1.84156
σ_V	0.22478	0.22478	0.38000	0.35986	
ρ	− 0.57	− 0.57	− 0.79715	− 0.1	
μ_X	− 0.25	− 0.57	− 0.25		− 0.25
μ_Y	0.22578	0.22578			
η_1	1.38117	1.38117	1.38073	2.47721	1.38073
η_2	− 2.0	0.0	− 1.99752	− 0.46769	
λ^Q	7.36623	7.36623	7.36799		7.36799
κ^Q	3.50630	3.87568	7.93905	5.68953	
$\bar{v}^Q$	0.03283	0.02970	0.02412	0.02531	0.02172

Table 1: Calibrated Parameters

The table shows the calibrated parameters for the SV, SJ, SVJ and SVCJ model. The models are calibrated such that the overall level of risk and of risk compensation is as similar as possible.

Frequency	BS	SV	SJ	SVJ	SVCJ	SVCJ($\gamma = 8$)
buy and hold	1.00082	1.00085	1.00142	1.00155	1.00145	1.00600
1/year	1.00009	1.00010	1.00092	1.00115	1.00089	1.00134
2/year	1.00004	1.00005	1.00046	1.00059	1.00050	1.00086
4/year	1.00002	1.00002	1.00023	1.00030	1.00026	1.00050
1/month	1.00001	1.00001	1.00009	1.00011	1.00010	1.00022
1/week	1.00000	1.00000	1.00002	1.00002	1.00002	1.00006
1/day	1.00000	1.00000	1.00001	1.00001	1.00000	1.00001

Table 2: $\widetilde{W}_{disc \to cont,w/o}$ for Calendar Rebalancing

The table shows the performance measure $\widetilde{W}_{disc \to cont,w/o}$ for calendar rebalancing strategies when only the stock and the money market account are traded. The investor has a relative risk aversion of 3, his planning horizon is 10 years.

Threshold	BS	SV	SJ	SVJ	SVCJ
buy and hold	1.00082	1.00085	1.00142	1.00155	1.00145
10%	1.00049	1.00048	1.00077	1.00063	1.00060
5%	1.00015	1.00014	1.00024	1.00016	1.00017
2%	1.00002	1.00002	1.00005	1.00003	1.00003
1%	1.00001	1.00000	1.00001	1.00001	1.00000
0.5%	1.00000	1.00000	1.00000	1.00000	1.00000
0.25%	1.00000	1.00000	1.00000	1.00000	1.00000

Table 3: $\widetilde{W}_{disc\rightarrow cont,w/o}$ for Threshold Rebalancing

The table shows the performance measure $\widetilde{W}_{disc\rightarrow cont,w/o}$ for threshold rebalancing strategies when only the stock and the money market account are traded. The investor has a relative risk aversion of 3, his planning horizon is 10 years.

Threshold	BS	SV	SJ	SVJ	SVCJ
buy and hold	0	0	0	0	0
10%	0.30	0.28	1.60	1.57	1.64
5%	1.91	1.84	3.81	3.62	3.59
2%	12.16	11.71	17.01	15.49	14.44
1%	47.32	45.28	61.39	54.37	48.93
0.5%	178.77	169.65	226.03	196.65	174.59
0.25%	638.95	598.97	792.77	682.22	606.56

Table 4: Average Number of Portfolio Changes for Threshold Rebalancing

This table shows the average number of portfolio changes for threshold rebalancing over the 10 year investment period for an investor with risk aversion $\gamma = 3$ when only the stock and the money market account are traded.

Model	ϕ_0	ϕ_{10}
SVCJ	0.8180	0.7801
SVJ	0.8009	0.7801
SJ	0.7800	0.7800
SV	0.8299	0.8257
BS	0.8257	0.8257

Table 5: Optimal Share of Wealth in the Stock at the Beginning and an the End of a 10 Year Investment Period

The table shows ϕ_0, the optimal share of wealth invested in the stock at the beginning of the 10 year investment period, and ϕ_{10}, the optimal share immediately before the end of the 10 year period.

Maturity	Hold time	Moneyness	$\widetilde{W}_{disc\to cont,w}$	$\widetilde{W}_{disc,w\to w/o}$	$\overline{W}_T$	(se)	ϕ	ψ_1	ψ_2
1 year	1 month	0.90	40.834	28.757	4.194	(0.0099)	− 5.16	− 4.18	6.88
		0.95	40.675	28.645	4.193	(0.0097)	− 3.38	− 7.37	9.30
		1.05	40.029	28.190	4.186	(0.0096)	− 1.23	5.78	− 4.73
	1 week	0.90	5.914	4.165	4.038	(0.0061)	− 5.17	− 4.18	6.88
		0.95	6.023	4.242	4.027	(0.0061)	− 3.38	− 7.37	9.30
		1.05	6.519	4.591	4.010	(0.0061)	− 1.23	5.78	− 4.73
	1 day	0.90	1.067	0.752	4.111	(0.0052)	− 5.17	− 4.18	6.88
		0.95	1.066	0.751	4.112	(0.0052)	− 3.38	− 7.37	9.30
		1.05	1.065	0.750	4.112	(0.0052)	− 1.23	5.78	− 4.73
	1 hour	0.90	1.023	0.720	4.066	(0.0049)	− 5.17	− 4.18	6.88
		0.95	1.023	0.720	4.066	(0.0049)	− 3.38	− 7.37	9.30
		1.05	1.024	0.721	4.042	(0.0049)	− 1.23	5.78	− 4.73
6 months	1 month	0.90	42.879	30.197	4.209	(0.0102)	− 1.23	− 1.27	2.27
		0.95	42.404	29.863	4.199	(0.0101)	− 0.22	− 2.09	2.71
		1.05	42.385	29.849	4.171	(0.0100)	0.82	1.37	− 1.13
	1 week	0.90	6.526	4.596	4.017	(0.0062)	− 1.23	− 1.27	2.27
		0.95	6.848	4.823	4.008	(0.0062)	− 0.22	− 2.09	2.71
		1.05	8.321	5.860	3.990	(0.0062)	0.82	1.37	− 1.13
	1 day	0.90	1.071	0.754	4.117	(0.0053)	− 1.23	− 1.27	2.27
		0.95	1.070	0.754	4.117	(0.0053)	− 0.22	− 2.09	2.71
		1.05	1.071	0.754	4.115	(0.0053)	0.82	1.37	− 1.13
	1 hour	0.90	1.023	0.720	4.069	(0.0049)	− 1.23	− 1.27	2.27
		0.95	1.023	0.721	4.067	(0.0049)	− 0.22	− 2.09	2.71
		1.05	1.025	0.722	4.059	(0.0049)	0.82	1.37	− 1.13
3 months	1 month	0.90	58.032	40.869	4.180	(0.0130)	−0.21	−0.56	1.14
		0.95	57.475	40.477	4.156	(0.0127)	0.64	−0.85	1.15
		1.05	58.336	41.083	4.106	(0.0126)	1.37	0.44	−0.38
	1 week	0.90	12.923	9.101	3.976	(0.0069)	−0.21	−0.56	1.14
		0.95	14.083	9.918	3.964	(0.0069)	0.64	− 0.85	1.15
		1.05	16.864	11.876	3.944	(0.0068)	1.37	0.44	− 0.38
	1 day	0.90	1.121	0.789	4.112	(0.0054)	−0.21	−0.56	1.14
		0.95	1.120	0.789	4.111	(0.0054)	0.64	−0.85	1.15
		1.05	1.119	0.788	4.110	(0.0054)	1.37	0.44	−0.38
	1 hour	0.90	1.032	0.727	4.054	(0.0049)	−0.21	−0.56	1.14
		0.95	1.033	0.727	4.052	(0.0049)	0.64	−0.85	1.15
		1.05	1.035	0.729	4.042	(0.0049)	1.37	0.44	−0.38

Table 6: Utility Loss due to Discrete Rebalancing in SVCJ Model with Derivatives

The table shows the performance measures $\widetilde{W}_{disc\to cont,w}$ (comparing the performance of continuous and discrete strategy with derivatives) and $\widetilde{W}_{disc,w\to w/o}$ (comparing the performance of the discrete strategy with and without derivatives) and the average terminal wealth $\overline{W}_T$ for the SVCJ model for a 10 year investment horizon and an investor with risk aversion $\gamma = 3$. ϕ is the investment in the stock, ψ_1 the investment in the ATM call and ψ_2 the investment in the second option. One ATM option and one with moneyness given in Column 3 were used.

Maturity	Hold time	Moneyness	$\widetilde{W}_{disc\to cont,w}$	$\widetilde{W}_{disc,w\to w/o}$	$\overline{W}_T$	(se)	ϕ	ψ_1	ψ_2
1 year	1 month	0.90	13.001	11.148	3.062	(0.0028)	4.27	0.60	− 1.59
		0.95	11.979	10.272	3.061	(0.0028)	3.82	1.31	− 2.12
		1.05	10.358	8.882	3.061	(0.0028)	3.29	− 1.65	1.05
	1 week	0.90	6.732	5.772	3.001	(0.0027)	4.27	0.60	− 1.59
		0.95	6.020	5.163	3.003	(0.0027)	3.82	1.31	− 2.12
		1.05	4.583	3.930	3.006	(0.0027)	3.29	− 1.65	1.05
	1 day	0.90	3.227	2.767	3.030	(0.0027)	4.27	0.60	− 1.59
		0.95	2.442	2.094	3.030	(0.0027)	3.82	1.31	− 2.12
		1.05	1.523	1.306	3.030	(0.0027)	3.29	− 1.65	1.05
	1 hour	0.90	1.000	0.858	3.029	(0.0027)	4.27	0.60	− 1.59
		0.95	1.000	0.858	3.029	(0.0027)	3.82	1.31	− 2.12
		1.05	1.000	0.857	3.030	(0.0027)	3.29	− 1.65	1.05
6 months	1 month	0.90	10.142	8.697	3.051	(0.0028)	3.12	0.16	− 0.67
		0.95	8.931	7.658	3.052	(0.0028)	2.80	0.39	− 0.79
		1.05	7.036	6.034	3.058	(0.0029)	2.47	− 0.60	0.32
	1 week	0.90	4.345	3.726	3.003	(0.0027)	3.12	0.16	− 0.67
		0.95	3.634	3.116	3.005	(0.0027)	2.80	0.39	− 0.79
		1.05	2.480	2.127	3.009	(0.0027)	2.47	− 0.60	0.32
	1 day	0.90	1.328	1.138	3.029	(0.0027)	3.12	0.16	− 0.67
		0.95	1.283	1.100	3.029	(0.0027)	2.80	0.39	− 0.79
		1.05	1.278	1.096	3.030	(0.0027)	2.47	− 0.60	0.32
	1 hour	0.90	1.000	0.858	3.028	(0.0027)	3.12	0.16	− 0.67
		0.95	1.000	0.858	3.028	(0.0027)	2.80	0.39	− 0.79
		1.05	1.000	0.857	3.030	(0.0027)	2.47	− 0.60	0.32
3 months	1 month	0.90	8.714	7.472	3.037	(0.0028)	2.65	0.04	− 0.36
		0.95	6.963	5.970	3.042	(0.0028)	2.36	0.13	− 0.35
		1.05	5.691	4.880	3.056	(0.0031)	2.11	− 0.25	0.11
	1 week	0.90	3.444	2.953	3.001	(0.0027)	2.65	0.04	− 0.36
		0.95	2.453	2.103	3.004	(0.0027)	2.36	0.13	− 0.35
		1.05	1.831	1.570	3.007	(0.0028)	2.11	− 0.25	0.11
	1 day	0.90	1.282	1.099	3.028	(0.0027)	2.65	0.04	− 0.36
		0.95	1.278	1.096	3.028	(0.0027)	2.36	0.13	− 0.35
		1.05	1.015	0.871	3.028	(0.0027)	2.11	− 0.25	0.11
	1 hour	0.90	1.000	0.858	3.028	(0.0027)	2.65	0.04	− 0.36
		0.95	1.000	0.858	3.028	(0.0027)	2.36	0.13	− 0.35
		1.05	1.000	0.857	3.030	(0.0027)	2.11	− 0.25	0.11

Table 7: Utility Loss due to Discrete Rebalancing in SVCJ Model with Derivatives if the Volatility Risk Premium is 0

The table shows the performance measures $\widetilde{W}_{disc\to cont,w}$ (comparing the performance of continuous and discrete strategy with derivatives) and $\widetilde{W}_{disc,w\to w/o}$ (comparing the performance of the discrete strategy with and without derivatives) and the average terminal wealth $\overline{W}_T$ for the SVCJ model for a 10 year investment horizon and an investor with risk aversion $\gamma = 3$. ϕ is the investment in the stock, ψ_1 the investment in the ATM call and ψ_2 the investment in the second option. One ATM option and one with moneyness given in Column 3 were used.

Maturity	Hold Time	Moneyness	$\widetilde{W}_{disc\to cont,w}$	$\widetilde{W}_{disc,w\to w/o}$	$\overline{W}_T$	(se)	ϕ	ψ_1	ψ_2
1 year	1 month	0.90	119.637	90.056	3.769	(0.0308)	− 95.10	− 22.24	50.16
		0.95	95.140	71.616	3.794	(0.0188)	− 47.05	− 29.17	42.66
		1.05	33.830	25.465	3.950	(0.0131)	− 15.39	16.19	− 11.70
	1 week	0.90	75.600	56.907	4.445	(0.0118)	− 95.10	− 22.24	50.16
		0.95	45.178	34.007	4.098	(0.0089)	− 47.05	− 29.17	42.66
		1.05	6.431	4.841	3.927	(0.0073)	− 15.39	16.19	− 11.70
	1 day	0.90	29.355	22.097	3.850	(0.0063)	− 95.10	− 22.24	50.16
		0.95	9.258	6.969	3.907	(0.0059)	− 47.05	− 29.17	42.66
		1.05	1.110	0.836	3.951	(0.0056)	− 15.39	16.19	− 11.70
	1 hour	0.90	1.180	0.888	3.810	(0.0051)	− 95.10	− 22.24	50.16
		0.95	1.070	0.806	3.861	(0.0051)	− 47.05	− 29.17	42.66
		1.05	1.035	0.779	3.891	(0.0051)	− 15.39	16.19	− 11.70
6 months	1 month	0.90	42.667	32.117	4.025	(0.0135)	− 12.22	− 2.38	5.88
		0.95	16.112	12.128	4.113	(0.0124)	− 5.46	− 3.57	5.32
		1.05	7.862	5.918	4.152	(0.0115)	− 0.77	2.13	− 1.56
	1 week	0.90	12.383	9.321	3.932	(0.0073)	− 12.22	− 2.38	5.88
		0.95	2.381	1.792	3.906	(0.0070)	− 5.46	− 3.57	5.32
		1.05	1.312	0.987	3.878	(0.0068)	− 0.77	2.13	− 1.56
	1 day	0.90	1.160	0.873	3.967	(0.0063)	− 12.22	− 2.38	5.88
		0.95	1.068	0.804	3.978	(0.0056)	− 5.46	− 3.57	5.32
		1.05	1.048	0.789	3.989	(0.0057)	− 0.77	2.13	− 1.56
	1 hour	0.90	1.031	0.776	3.914	(0.0052)	− 12.22	− 2.38	5.88
		0.95	1.025	0.771	3.918	(0.0052)	− 5.46	− 3.57	5.32
		1.05	1.024	0.771	3.911	(0.0051)	-0.77	2.13	-1.56
3 months	1 month	0.90	9.727	7.322	4.176	(0.0135)	− 2.54	− 0.67	1.67
		0.95	6.677	5.026	4.191	(0.0130)	− 0.46	− 0.99	1.47
		1.05	11.880	8.942	4.111	(0.0118)	1.09	0.44	− 0.34
	1 week	0.90	1.432	1.078	3.897	(0.0072)	− 2.54	− 0.67	1.67
		0.95	1.368	1.030	3.886	(0.0071)	− 0.46	− 0.99	1.47
		1.05	1.340	1.009	3.850	(0.0069)	1.09	0.44	− 0.34
	1 day	0.90	1.062	0.799	3.986	(0.0057)	− 2.54	− 0.67	1.67
		0.95	1.054	0.793	3.988	(0.0057)	− 0.46	− 0.99	1.47
		1.05	1.052	0.792	3.981	(0.0056)	1.09	0.44	− 0.34
	1 hour	0.90	1.022	0.770	3.924	(0.0052)	− 2.54	− 0.67	1.67
		0.95	1.022	0.769	3.921	(0.0052)	− 0.46	− 0.99	1.47
		1.05	1.027	0.773	3.901	(0.0051)	1.09	0.44	− 0.34

Table 8: Utility Loss due to Discrete Rebalancing in SVJ Model with Derivatives

The table shows the performance measures $\widetilde{W}_{disc\to cont,w}$ (comparing the performance of continuous and discrete strategy with derivatives) and $\widetilde{W}_{disc,w\to w/o}$ (comparing the performance of the discrete strategy with and without derivatives) and the average terminal wealth $\overline{W}_T$ for the SVJ model for a 10 year investment horizon and an investor with risk aversion $\gamma = 3$. Furthermore the average multiples of wealth invested into the different assets are given. ϕ is the investment in the stock, ψ_1 the investment in the ATM call and ψ_2 the investment in the second option. One ATM option and one with moneyness given in Column 3 were used.

Maturity	Hold Time	$\widetilde{W}_{disc \to cont,w}$	$\widetilde{W}_{disc,w \to w/o}$	$\overline{W}_T$	(se)	ϕ	ψ
1 year	1 month	10.335	10.144	2.818	(0.0043)	2.62	− 0.24
	1 week	1.839	1.805	2.741	(0.0038)	2.63	− 0.24
	1 day	1.001	0.983	2.764	(0.0037)	2.63	− 0.24
	1 hour	1.000	0.980	2.762	(0.0037)	2.63	− 0.24
6 months	1 month	5.305	5.207	2.794	(0.0040)	2.03	− 0.11
	1 week	1.020	1.001	2.740	(0.0037)	2.03	− 0.11
	1 day	1.000	0.981	2.763	(0.0037)	2.03	− 0.11
	1 hour	1.000	0.979	2.761	(0.0037)	2.03	− 0.11
3 months	1 month	3.661	3.593	2.776	(0.0039)	1.79	− 0.06
	1 week	1.021	1.002	2.735	(0.0037)	1.79	− 0.06
	1 day	1.000	0.981	2.762	(0.0037)	1.79	− 0.06
	1 hour	1.000	0.980	2.761	(0.0037)	1.79	− 0.06

Table 9: Utility Loss due to Discrete Rebalancing in SV(Heston) Model with Derivatives

The table shows the performance measures $\widetilde{W}_{disc \to cont,w}$ (comparing the performance of continuous and discrete strategy with derivatives) and $\widetilde{W}_{disc,w \to w/o}$ (comparing the performance of the discrete strategy with and without derivatives) and the average terminal wealth $\overline{W}_T$ for the SV model for a 10 year investment horizon and an investor with risk aversion $\gamma = 3$. ϕ is the investment in the stock and ψ the investment in the ATM call. One ATM option was used.

Maturity	Hold Time	$\widetilde{W}_{disc\to cont,w}$	$\widetilde{W}_{disc,w\to w/o}$	$\overline{W}_T$	(se)	ϕ	ψ
1 year	1 month	1.516	1.347	2.852	(0.0028)	2.83	− 0.35
	1 week	1.037	0.922	2.829	(0.0027)	2.83	− 0.35
	1 day	1.008	0.895	2.877	(0.0027)	2.83	− 0.35
	1 hour	1.006	0.894	2.873	(0.0027)	2.83	− 0.35
6 months	1 month	1.040	0.924	2.899	(0.0030)	2.14	− 0.17
	1 week	1.025	0.911	2.875	(0.0029)	2.14	− 0.17
	1 day	1.004	0.892	2.921	(0.0029)	2.14	− 0.17
	1 hour	1.005	0.893	2.916	(0.0029)	2.14	− 0.17
3 months	1 month	1.051	0.934	2.916	(0.0032)	1.85	− 0.09
	1 week	1.028	0.913	2.889	(0.0030)	1.85	− 0.09
	1 day	1.004	0.892	2.937	(0.0030)	1.85	− 0.09
	1 hour	1.005	0.893	2.931	(0.0030)	1.85	− 0.09

Table 10: Utility Loss due to Discrete Rebalancing in SJ(Merton) Model with Derivatives

The table shows the performance measures $\widetilde{W}_{disc\to cont,w}$ (comparing the performance of continuous and discrete strategy with derivatives) and $\widetilde{W}_{disc,w\to w/o}$ (comparing the performance of the discrete strategy with and without derivatives) and the average terminal wealth $\overline{W}_T$ for the SJ model for a 10 year investment horizon and an investor with risk aversion $\gamma = 3$. ϕ is the investment in the stock and ψ the investment in the ATM call. One ATM option was used.

Multivariate Regression

	SVCJ	SVJ	SJ	SV
dev. B1	236.0 (0.003)	226.8 (0.002)	0.40 (0.130)	1199.7 (0.255)
dev. B2	− 175.0 (0.185)	− 698.1 (0.014)		−105.7 (0.316)
dev. jump	164.9 (0.786)	− 48.2 (0.566)	1.53 (0.273)	
R^2	85.6%	71.9%	84.1%	52.3%

Univariate Regression

	SVCJ	SVJ	SJ	SV
dev. B1	176.7 (0.000)	58.9 (0.123)	0.69 (0.000)	217.5 (0.558)
R^2	82.3%	53.3%	82.5%	14.3%
dev. B2	322.0 (0.000)	252.9 (0.135)		8.6 (0.477)
R^2	54.6%	37.5%		2.2%
dev. jump	1834.8 (0.000)	70.5 (0.320)	3.5 (0.000)	
R^2	59.9%	43.5%	81.2%	

Table 11: Determinants of Utility Loss

This table shows the results of regressions of $\widetilde{W}_{disc\to cont,w}$ on the absolute value of the average relative deviation of the actual factor exposures from optimal factor exposures. Numbers in parenthesis are p-values calculated using heteroscedasticity-consistent standard errors.

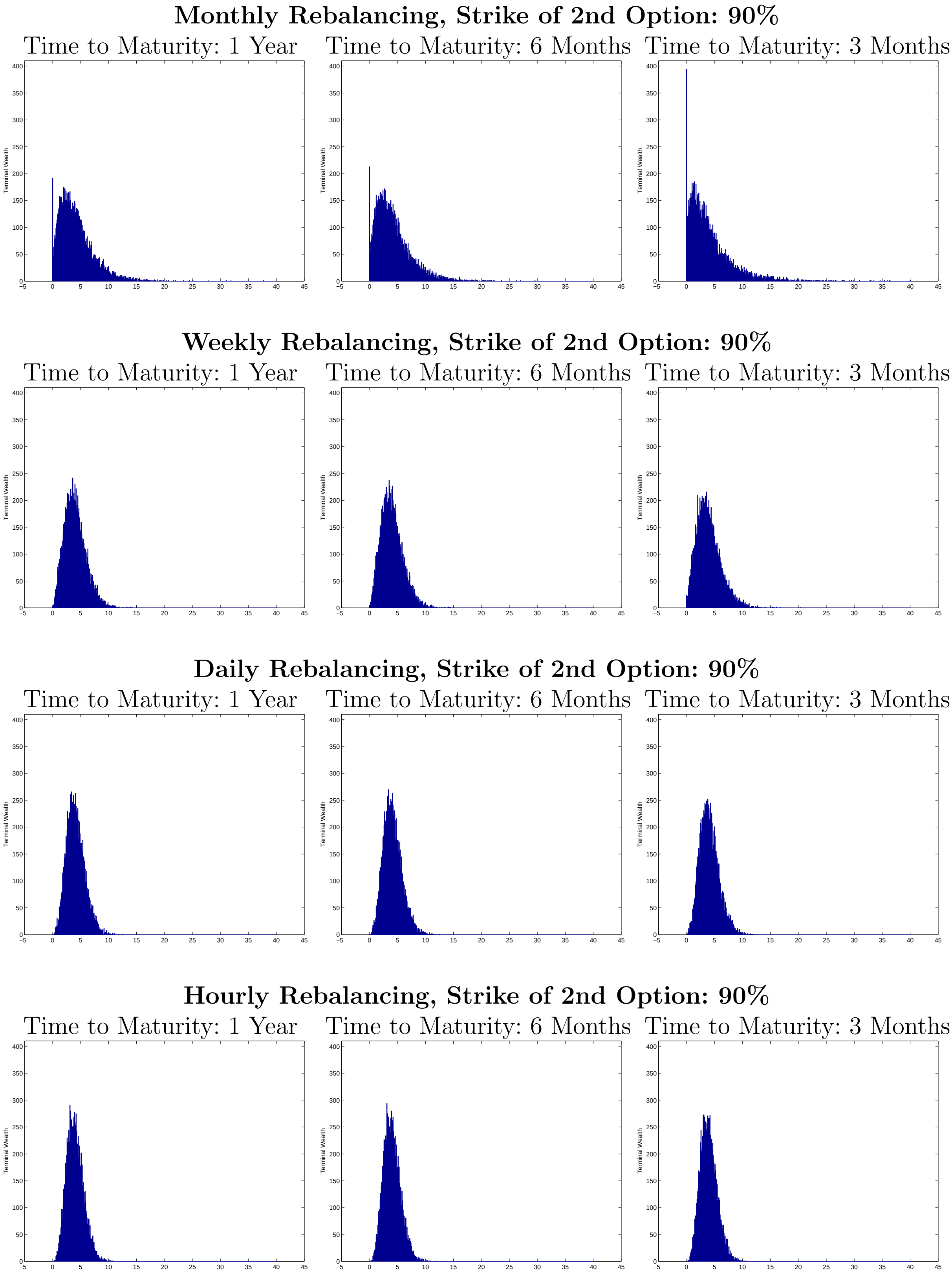


Figure 1: Distribution of terminal wealth in SVCJ model for different rebalancing frequencies

The figure shows the histogram of terminal using 10,000 simulation runs for different rebalancing frequencies and different times to maturity. The option employed are one ATM call and an ITM call with strike price equal to 90 % of the stock price at time of purchase.

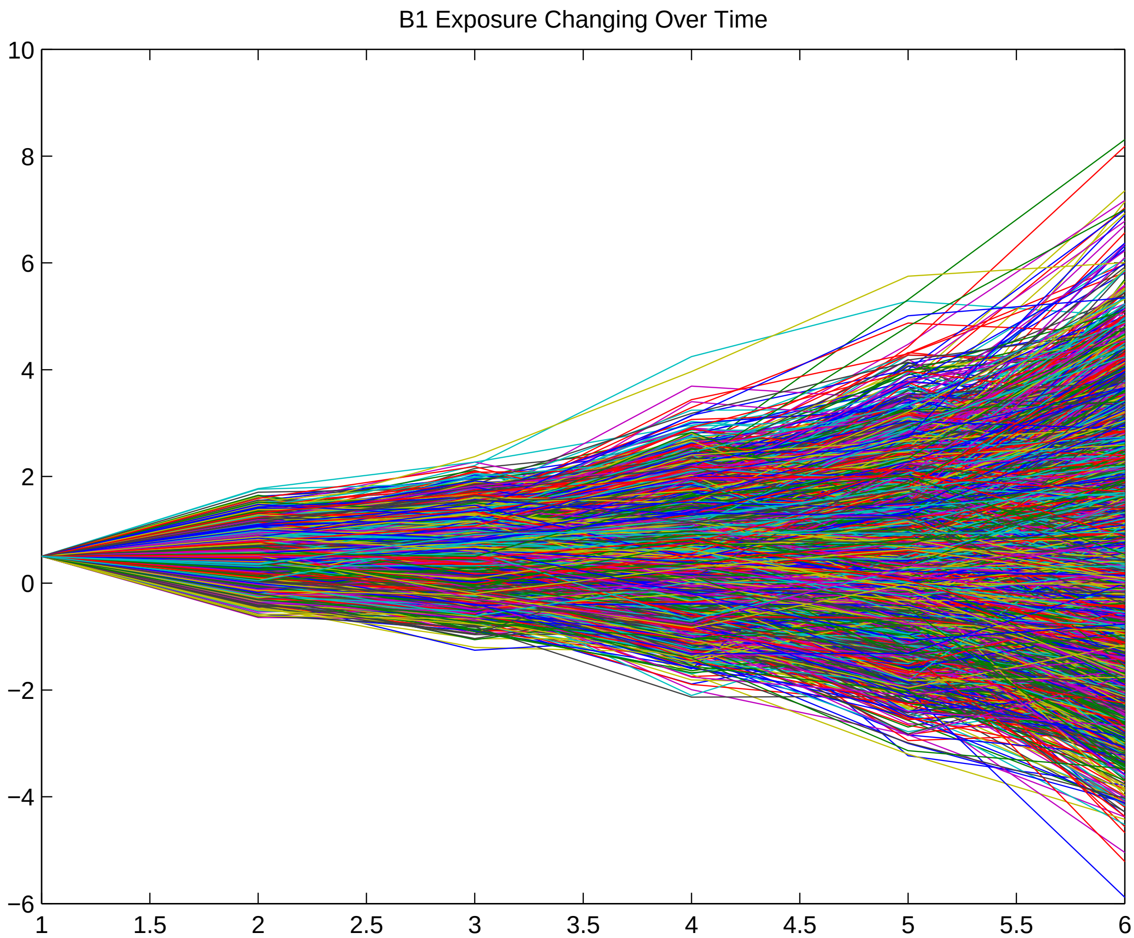


Figure 2: Evolution of $B1$ Exposure over 1 Month
This figure shows 10.000 paths of the evolution of the actual B1 exposure over 1 month, 1 on the time scale corresponds to the re-balancing time, 2 to 1 day after re-balancing, 3 to 2 days, 4 to 5 days, 5 to 10 days and 6 to 1 month.

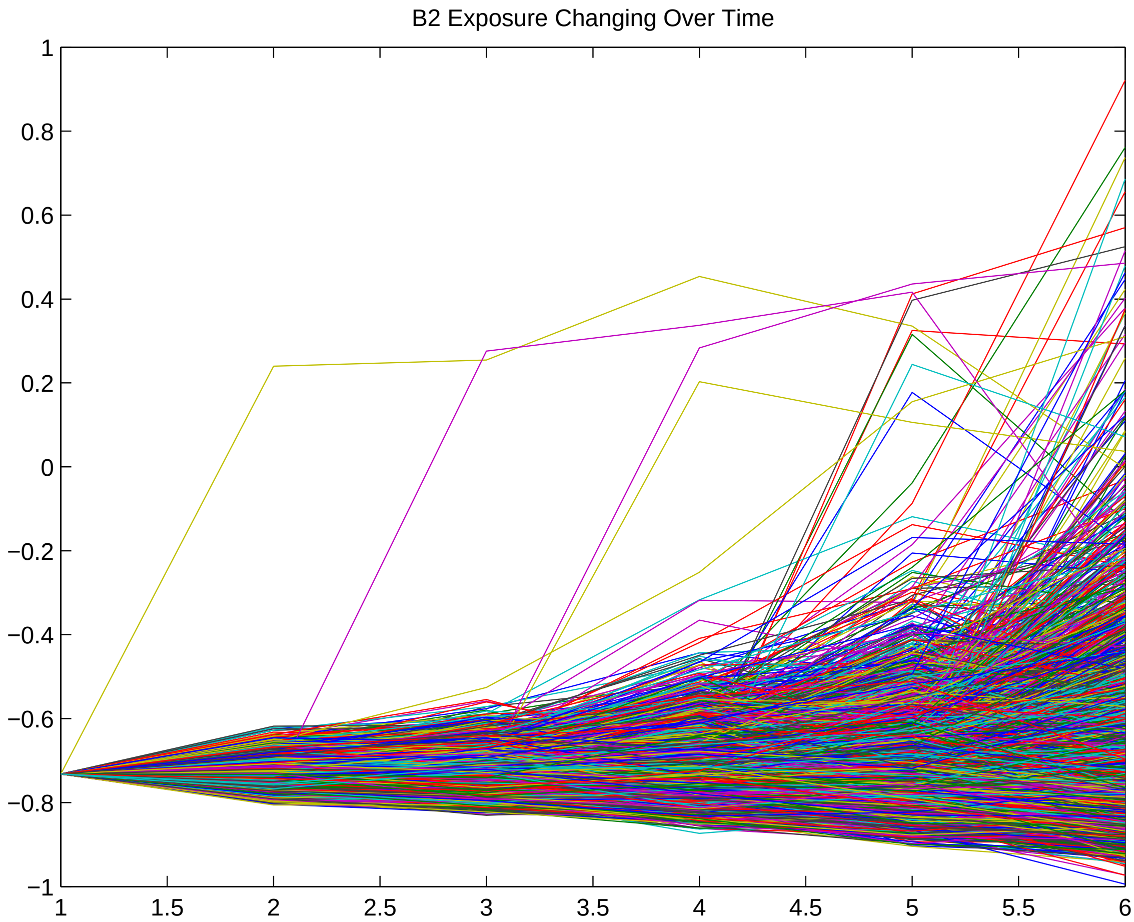


Figure 3: Evolution of $B2$ Exposure over 1 Month
This figure shows 10.000 paths of the evolution of the actual B2 exposure over 1 month, 1 on the time scale corresponds to the re-balancing time, 2 to 1 day after re-balancing, 3 to 2 days, 4 to 5 days, 5 to 10 days and 6 to 1 month.

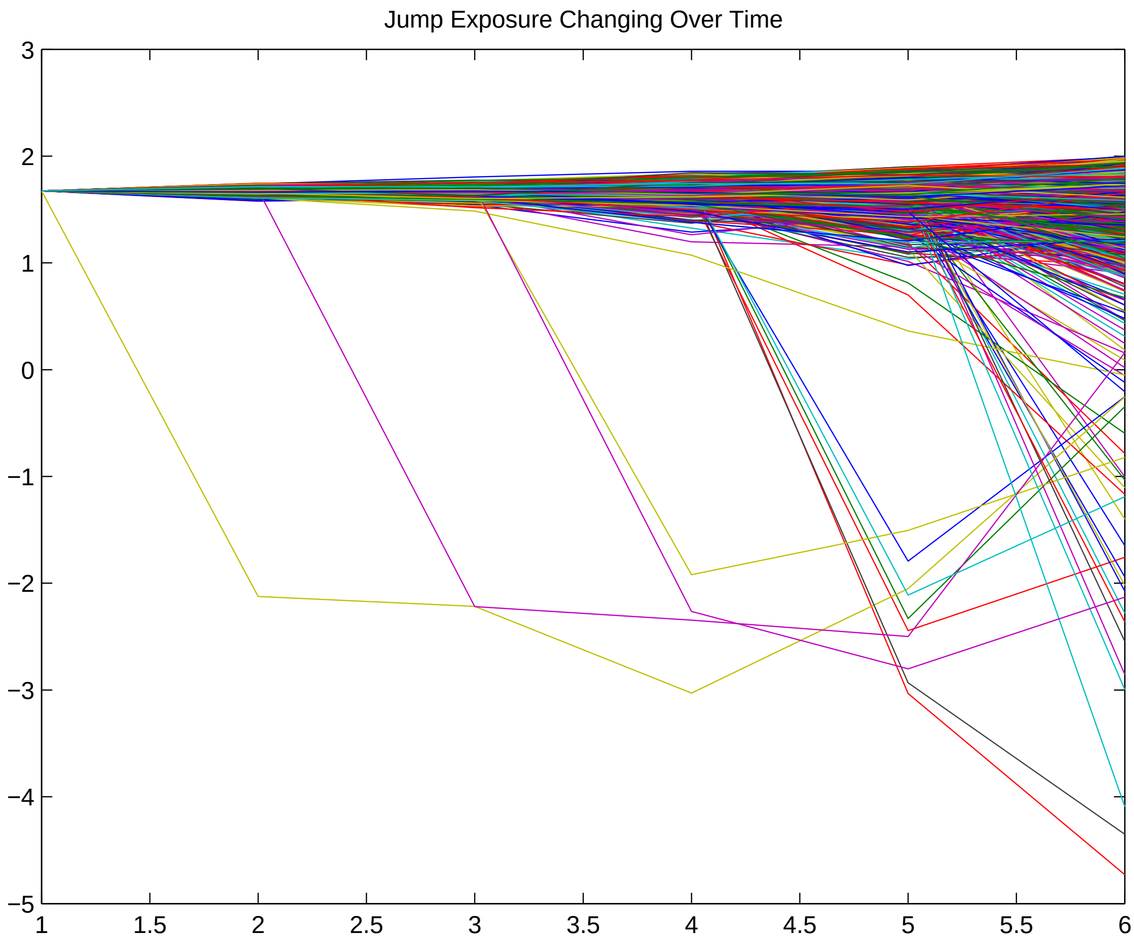


Figure 4: Evolution of Jump Exposure over 1 Month
This figure shows 10.000 paths of the evolution of the actual jump exposure over 1 month, 1 on the time scale corresponds to the re-balancing time, 2 to 1 day after re-balancing, 3 to 2 days, 4 to 5 days, 5 to 10 days and 6 to 1 month.

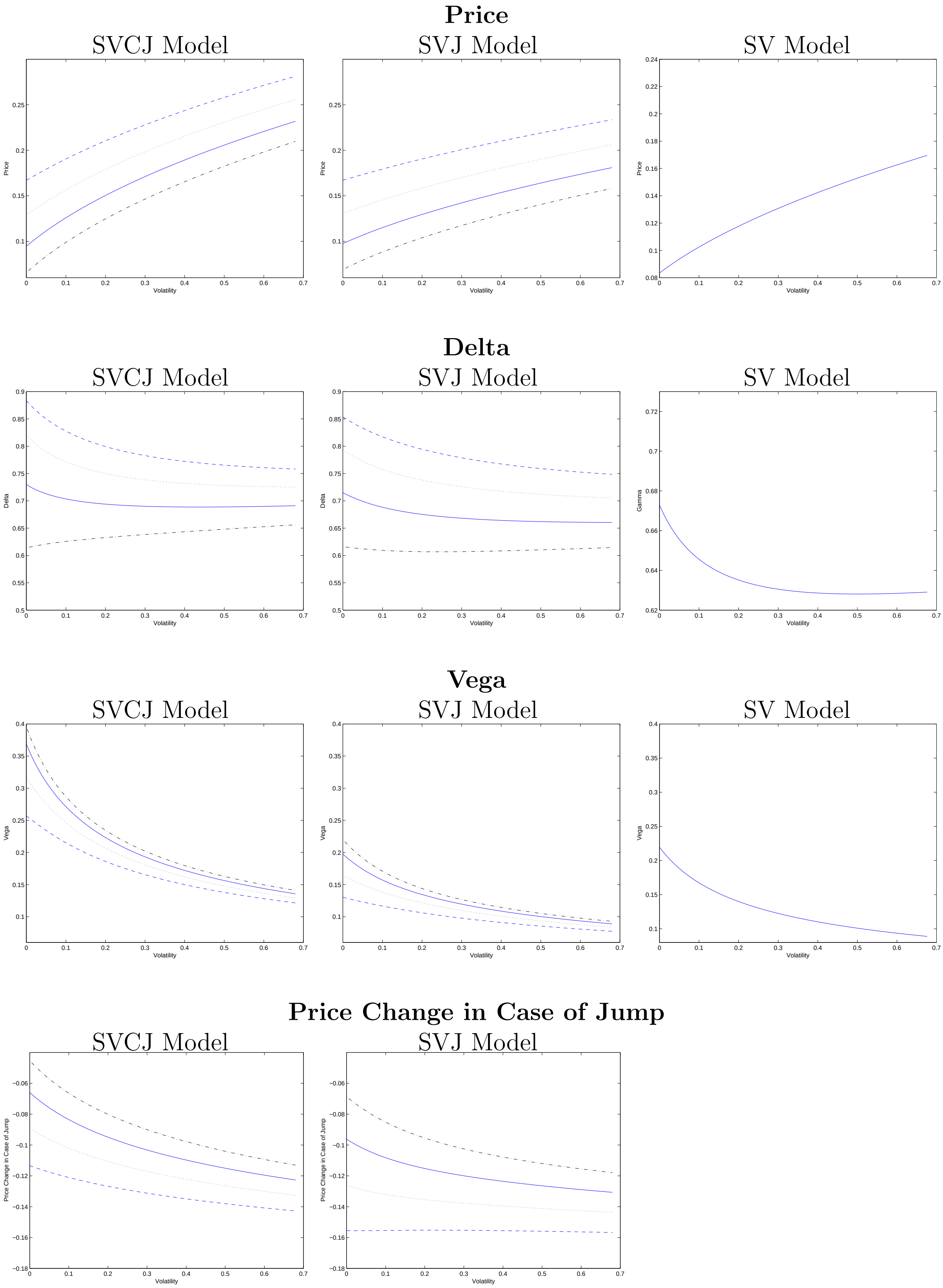
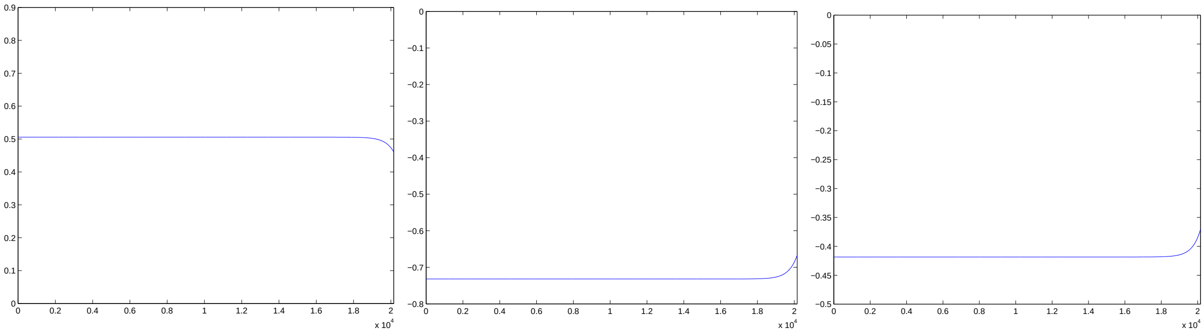


Figure 5: Sensitivities for Different Volatility Levels

This figure shows price, delta, vega and and the price change in case of a jump for an ATM calls with 1 year time-to-maturity and strikes from 90% (blue dashed line), 95% (black dotted line), 100% (blue solid line) and 105 % (black dash-dotted line) for different volatility levels and different models.

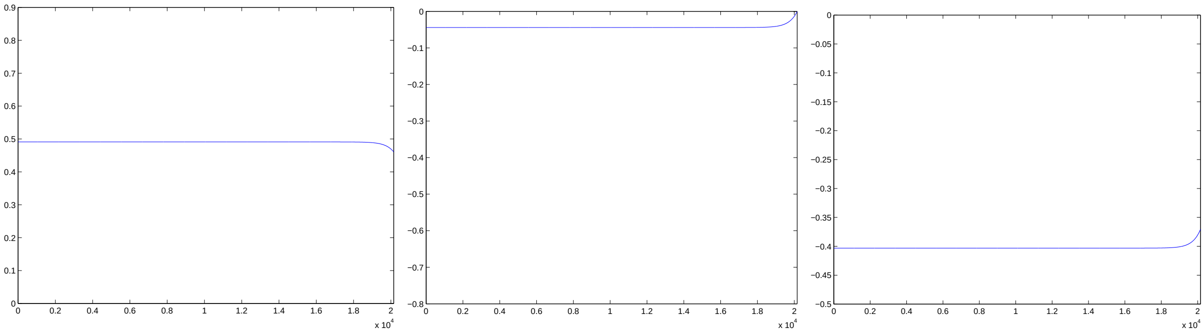
Optimal Factor Exposures in SVCJ Model

B1 Exposure | B2 Exposure | Jump Exposure



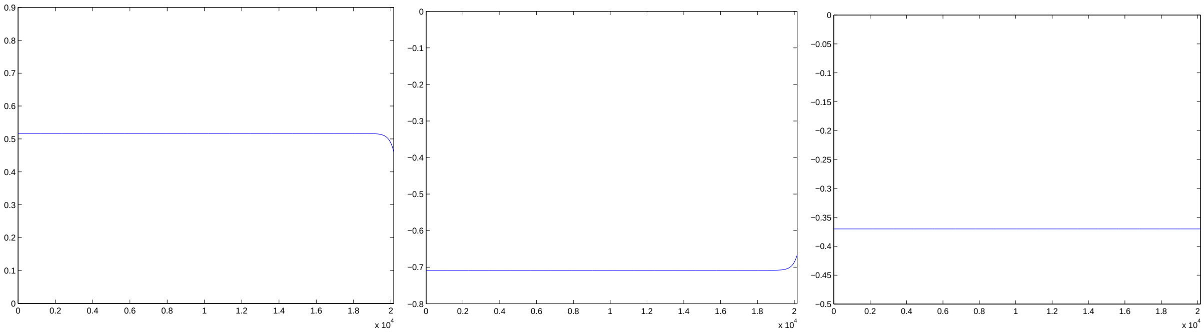
Optimal Factor Exposures in SVCJ Model with Zero Volatility Risk Premium

B1 Exposure | B2 Exposure | Jump Exposure



Optimal Factor Exposures in SVJ Model

B1 Exposure | B2 Exposure | Jump Exposure



Optimal Factor Exposures in SV Model

B1 Exposure | B2 Exposure

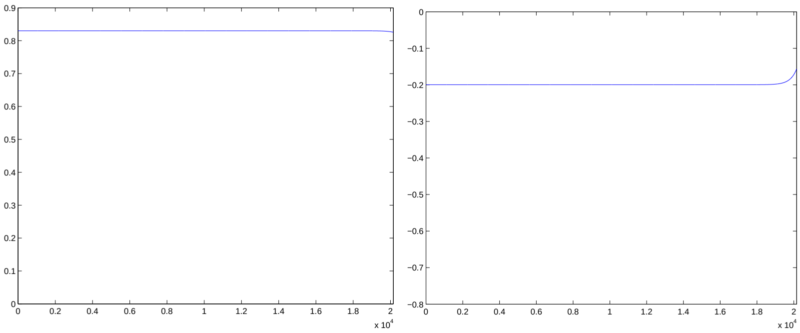


Figure 6: Optimal Factor Exposures

These figures show the development of the optimal factor exposures for the different models over time.