“Credit Cycles” in a OLG Economy with Money and Bequest (Preliminary draft)

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Abstract

In this paper we develop an overlapping generation version of Kiyotaki and Moore’ s (hereafter KM) model of the "credit cycle". In each period the population consists of two classes of agents: a group of financially constrained agents ("farmers") and one of unconstrained agents ("gatherers"). Each class in turn consists of young and old agents.Each class of agents uses different technologies to produce the same perishable good ("fruit") by means of labour and "land". Land is a durable asset which plays the role not only of an input for production processes but also of collateralizable wealth to secure lenders from the risk of borrowers’ default. In a context of intergenerational altruism, old agents leave a bequest to their offspring. Money enters the picture as a means of payment and a reserve of value because it enables to access consumption in old age. In the original paper in which people are infinitely lived (an money as such is absent), KM study the fluctuations following a technological shock. In the OLG version of the model, self susteained oscillations arise naturally. We study the complex dynamics of the allocation of land to farmers and gatherers – which determines aggregate output – and of the price of the durable asset. The next step is the analysis of the conditions under which money is or is not superneutral.

1. Introduction

In the early '90s two modelling strategies have been followed to develop *Financial Accelerator* models, i.e. theoretical frameworks in which financial factors – such as the degree of firms'financial fragility – affect investment and production: the Bernanke-Gertler (BG) framework based on agency costs (Bernanke and Gertler, 1989, 1990, 1996) and the Greenwald-Stiglitz (GS) one centered on bankruptcy costs (Greenwald and Stiglitz, 1988, 1990, 1993). [1]In the late '90s Kiyotaki and Moore (KM) have put forward a new framework (Kiyotaki and Moore,1997, 2002) in which financial constraints play a crucial role. The novel and appealing feature of their model is a "dynamic feedback process between asset prices and borrowing constraints." (Kasa, 1998, p. 17): booming asset prices relax borrowing constraints and boost economic activity, driving the expansion; the upswing, in turn, affects asset prices. Thanks to this feature, the KM framework has gained the reputation of being particularly suitable to explore the intertwined dynamics of asset prices and aggregate output. [2] Open economy variants of such a model have been adopted to study the *twin crises*, i.e. the currency and financial crises which hit the Far East (Edison et al. 1998, Kasa, 1998). More recently, the same framework has been applied to an empirical study of the US and Europe (Jacoviello, 2005a,b).

Kiyotaki and Moore (1997) consider an economy populated by infinitely lived agents, which will referred hereafter as a ILA-KM economy. The reduced form of the model boils down to the law of motion of the borrowers' landholding (which determines aggregate output and asset prices). KM use a linearized version of the law of motion to assess the dynamic impact of productivity shocks according to the impulse-propagation approach. In a sense, therefore, they are exploring *credit fluctuations.* The evocative term "credit cycles" showing up in the title of the 1997 paper sounds inappropriate because the model does not yield self-sustained oscillations. In fact the the law of motion is non-linear but yields trajectories not significantly different from those obtained by similar linear law.

In an appendix, KM sketch the building blocks of an overlapping generations variant of their model along the lines of Blanchard's "finite horizon" framework. This suggestion has been followed by Kasa (1998). The dynamics, however, is not

[1]Bernanke et al. (1999) provides a recapitulation and extended new version of the BG framework. As to GS, an in depth reflection on the main issues and suggestions for further developments can be found in Greenwald and Stiglitz (2001).

[2]For an extended refelction on this see Kiyotaki...Clarendon Lectures)

significantly different from that of the original KM framework.

To the best of our knowledge no other attempt has been made to develop an OLG framework. In this paper we model an OLG-KM economy along Samuelsonian lines. The novel and intriguing feature of the present model is the complexity of the dynamics. Not only cyclical patterns are routinely generated – so that in this context the expression *credit cycles* deserves a mention in the title[3] – but the periodicity and amplitude are irregular. A route to chaotic dynamics is open.

In our OLG-KM economy, in each period the population consists of two classes of heterogeneous interacting agents. Young and old agents, infact, can be either lenders ("gatherers") or financially constrained borrowers ("farmers"). By assumption each young agent is endowed with one unit of labour. Heterogeneity is introduced in the model by assuming that each class of agents use different technologies to produce the same non durable good. As in KM we develop a dynamic model in which the durable asset is not only an input for production processes but also collateralizable wealth to secure lenders from the risk of borrowers'default.

The paper is organized as follows. In section 2 we present a quick refresher course on ILA-KM economies. Section 3 is devoted to the background assumptions concerning our OLG-KM economy. The optimization problems of the farmer and the gatherer are discussed in sections 4 and 5 respectively. Section 6 is devoted to the discussion of constraints and their consolidation. Section 7 describe the trickling down process by which money spreads in the economy and is carried out from one period to the next by means of exchanges and bequests. The welfare criterion and the first best solution are discussed in section 8. Section 9 is devoted to the dynamics.

2. A quick refresher course on ILA-KM economies

KM assume that in a principal-agent relationship between borrowers and lenders, characterized by asymmetric information and moral hazard, borrowers face a financing constraint: the loans they get is smaller or equal to the *value of their collateralizable assets*, which plays, in this framework a role analogous to that of net worth or the equity base in GS and entrepreneurs' savings (internal finance) in BG.

[3] Cordoba e Ripoll (....) obtain credit cycles in a ILA-KM economy with a cash in advance constraint.

KM assume that infinitely lived agents can be either financially constrained borrowers ("farmers") or lenders ("gatherers"). A farmer is an agent endowed with inalienable human capital. Therefore, he can get from lenders no more than the value of his collateralizable assets. This is the reason of the financing constraints.[4] A gatherer, on the contrary, is not endowed with inalienable human capital and does not face financing constraints.

There are two types of goods, output ("fruit") and a collateralizable, durable, non-reproducible asset ("land") whose total supply is fixed ($\bar{K}$). Output can be consumed or lent. If lent, each unit of output yields a constant return $R = 1 + r$.Output is produced by means of a technology which uses land and labour.

By assumption farmers and gatherers have access to different technologies.

The production function of each financially constrained agent (farmer) is: $y_t^F = (\alpha + \bar{c})K_{t-1}^F$ where y_t^F is output of the farmer in t, $\alpha, \bar{c}$ are positive technological parameters and K_{t-1}^F is land of the farmer in t-1. $\bar{c}\,K_{t-1}^F$ is the output which deteriorates ("bruised fruit") and is therefore non-tradable.

Each farmer's technology is idiosyncratic in the sense that once production has started only the farmer has the skills to successfully complete the production process, i.e. to make land bear fruit. If the farmer withdrew her labour, production would not be carried out, i.e. land would bear no fruit. In the words of Hart and Moore (1994), the farmer's human capital is inalienable. As a consequence, if the farmer is indebted, he may have an incentive to threaten her creditors to withdraw her labour and repudiate her debt. Creditors protect themselves against this threat by collateralizing the farmer's land. This is the reason why farmers face a *financing constraint:*

$$b_t = \frac{q_{t+1}K_t^F}{R} \tag{1}$$

According to (1), the maximum amount of debt a farmer succeeds to get "today" b_t is such that the sum of principal and interest Rb_t is equal to the value of the farmer's land when the debt is due, i.e. $q_{t+1}K_t^F$ where q_{t+1} is the price of land at time t+1.

Farmers face also a *flow-of-funds constraint*:

$$y_t^F + b_t^F = q_t(K_t^F - K_{t-1}^F) + Rb_{t-1}^F + c_t^F \tag{2}$$

where c_t^F is the farmer's consumption. Substituting (1) into (2) we get the budget constraint:

$$c_t^F = (\alpha + \bar{c})K_{t-1}^F - \mu_t K_t^F \tag{3}$$

[4] On this issue see Hart and Moore (1994, 1998).

where $\mu_t = q_t - \frac{q_{t+1}}{R}$.

Preferences are such that farmers consume only non-tradable output, i.e. $c_t^F = \bar{c}\,\mathrm{K}_{t-1}^F$. The farmer's demand for land, therefore, is:

$$K_t^F = \frac{1}{\mu_t}\left[(\alpha + q_t)K_{t-1}^F - Rb_{t-1}^F\right] = \frac{\alpha}{\mu_t}K_{t-1}^F \tag{4}$$

Substituting (4) into (1), we obtain:

$$b_t^F = \frac{q_{t+1}}{R}\frac{1}{\mu_t}\left[(\alpha + q_t)K_{t-1}^F - Rb_{t-1}^F\right] \tag{5}$$

The production function of each gatherer is: $y_t^G = f(K_{t-1}^G)$ where y_t^G is output of the gatherer in t $f(.)$ is a well behaved production function and K_{t-1}^G is land of the gatherer in t-1. The gatherers' human capital is not inalienable. Therefore, gatherers face only a *flow-of-funds constraint*:

$$y_t^G + Rb_{t-1}^G = q_t(K_t^G - K_{t-1}^G) + b_t^G + c_t^G \tag{6}$$

Substituting the production function of gatherers and the financing constraint of farmers into (6) and assuming, for the sake of simplicity and without loss of generality, that population consists only of one farmer and one gatherer so that $K_t^F = \bar{K} - K_t^G$ we get the budget constraint:

$$c_t^G = f(K_{t-1}^G) + \mu_t\left(\bar{K} - K_t^G\right) \tag{7}$$

Preferences of the gatherer are such that $R\mu_t = f'(K_t^G)$. The demand for land, therefore, is:

$$K_t^G = {f'}^{-1}(R\mu_t) \tag{8}$$

From (8) we know that the following must be true:

$$\bar{K} - K_t^F = {f'}^{-1}(R\mu_t) \tag{9}$$

Substituting this expression into (4) and rearranging we end up with the following:

$$K_t^F = \frac{R\alpha}{f'(\bar{K} - K_t^F)}K_{t-1}^F \tag{10}$$

(10) is a non-linear difference equation in the state variable K_t^F.

In the steady state $K_t^F = K_{t-1}^F = K^*$ Therefore $\mu^* = a$. Moreover $q_t = q_{t+1} = q^*$ so that $q^* = \alpha\frac{R}{R-1}$. Finally $b_t^F = b_{t-1}^F = b^*$so that $b^* = \alpha\frac{K^*}{R-1}$ .From the steady state condition $K_t^G = K_{t-1}^G = \bar{K} - K^*$ follows $\bar{K} - K^* = f'^{-1}(R\alpha)$. As a consequence: $K^* = \bar{K} - f'^{-1}(R\alpha)$.

In this setting KM show that small shocks – for instance to technology – can produce large and persistent fluctuations in output and asset prices. In their model, in fact, the durable, non reproducible asset (land) plays the dual role of a factor of production for both constrained and unconstrained agents and of collateralizable wealth for financially constrained agents. Therefore the price of assets affects the borrowers' financing constraint. At the same time, the size of the borrowers' credit limits feeds back on asset prices. "The dynamic interaction between credit limits and asset prices turns out to be a powerful transmission mechanism by which the effects of shocks persist, amplify and spread out" (Kiyotaki and Moore, 1997:212).

3. The structure of an OLG-KM economy

In each period there are four classes of agents. In order to simplify matters, we assume for the moment that there is only one (representative) agent per class. Therefore in t population consists of

- a financially constrained young agent (young farmer, YF)
- a financially constrained old agent (old farmer, OF)
- an unconstrained young agent (young gatherer, YG)
- an unconstrained old agent (old gatherer, OG).

A YF borrows from a YG. Being endowed with *inalienable human capital,* the former can get from the latter no more than the value of the collateralizable assets, i.e. the future value of the land he is currently owning: $b_t = \dfrac{q_{t+1}}{R}K_t^F$.

There are two types of goods, output ("fruit") and a non-reproducible asset ("land") whose total supply is fixed ($\bar{K}$). Output is produced by means of a technology which uses land and labour. By assumption each young agent is endowed with one unit of labour. Assuming that there is no disutility of labour, this endowment is supplied inelastically. By assumption farmers and gatherers have access to different technologies. The production function of the YF is: $y_t^F = \alpha K_{t-1}^F$

where y_t^F is output of the farmer in t, α is a positive technological parameter and K_{t-1}^F is land of the farmer in t-1. The production function of the YG is: $y_t^G = G\left(K_{t-1}^G\right) = G\left(\bar{K} - K_{t-1}^G\right)$ and $G(.)$ is increasing, concave and satisfies the Inada conditions.Both farmers and gatherers work when young and consume when old.

The paper is organized as follows. OLG-KM economy with money and bequest. Money is a reserve of value and a way of leaving a bequest. Heterogeneity...

4. The farmer/borrower

For simplicity we assume that the agent does not consume when young. Hence preferences are defined over consumption and bequest of the agent when old. Adopting a Cobb-Douglas specification of the utility function, preferences of the farmer are represented by

$$U^F = \gamma \ln c_{t,t+1}^F + (1-\gamma) \ln a_{t+1}^F \tag{11}$$

where $0 < \gamma < 1$; $c_{t,t+1}^F$ is consumption of the agent of generation t in t+1, a_{t+1}^F is bequest left by agent of generation t in t+1 to his child. [5]

The farmer maximizes utility subject to three constraints: the flow-of-funds (FF) constraint of the YF, the FF constraint of the OF and the financing constraint (see appendix A for the derivation).

The FF constraint of the YF in t (in real terms) is:

$$q_t\left(K_t^F - K_{t-1}^F\right) + m_{t,t}^F \leq b_t + a_t^F \tag{12}$$

where $q_t := \dfrac{Q_t}{P_t}$ is the real price of land,[6] $m_{t,t}^F := \dfrac{M_{t,t}^F}{P_t}$ are real money balances of the YF; b_t is credit and a_t^F is bequest, i.e. "wealth" inherited by the YF. According to 12, the "resources" of the YF, of internal or external origin (a_t^F and b_t respectively), can be employed to "invest", $q_t\left(K_t^F - K_{t-1}^F\right)$ – i.e. to change the landholding – and hold money balances.

[5] In the case of bequest, the notation is unambiguous. The bequest left by the agent of generation t in t+1 (i.e. when old) to his child can be denoted by $a_{t,t+1}^F$. The bequest received by agent of generation t+1 in t+1 (i.e. when young) is $a_{t+1,t+1}^F$. Of course the two notions amount to the same magnitude, i.e. $a_{t,t+1}^F = a_{t+1,t+1}^F = a_{t+1}^F$.

[6] Following KM, we purposedly adopt a notation reminiscent of Tobin's q.

The YF may be financially coinstrained. The financing constraint can be expressed as

$$b_t \leq \frac{q_{t+1}}{R} K_t^F \tag{13}$$

where $q_{t+1} := \dfrac{Q_{t+1}}{P_{t+1}}$ is the real price of land in the future, known in advance, and R is the real (gross) interest rate.

In t, the YF uses labour and land K_t^F to produce output which will become available in $t+1$: $y_{t+1}^F = \alpha K_t^F$.When old, the farmer's resources consist of output (produced when young) and money balances.These resources can be employed to repay the loan (if the YF were a borrower), consume and leave a bequest. Therefore the FF constraint of the OF in $t+1$ in real terms is:

$$c_{t,t+1}^F + a_{t+1}^F + b_t R \leq \alpha K_t^F + m_{t,t+1}^F \tag{14}$$

where $b_t R$ is the repayment of the loan; $m_{t,t+1}^F = \dfrac{M_{t,t+1}^F}{P_{t+1}}$are real money balances of the OF of generation t in t+1.

The farmer maximizes 11 subject to 12 14 and 13. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} &= \gamma \ln c_{t,t+1}^F + (1-\gamma)\ln a_{t+1}^F + \lambda_t^F \left[b_t + a_t^F - q_t\left(K_t^F - K_{t-1}^F\right) - m_{t,t}^F\right] + \\
&\quad + \lambda_{t+1}^F \left[\alpha K_t^F + m_{t,t+1}^F - c_{t,t+1}^F - a_{t+1}^F - b_t R\right] + \phi_t \left[b_t - \frac{q_{t+1}}{R} K_t^F\right]
\end{aligned}$$

from which one gets the $FOCS$:

$$\begin{aligned}
(iF) \quad \frac{\partial \mathcal{L}}{\partial c_{t,t+1}^F} &= 0 \Rightarrow \frac{\gamma}{c_{t,t+1}^F} = \lambda_{t+1}^F \\
(iiF) \quad \frac{\partial \mathcal{L}}{\partial a_{t+1}^F} &= 0 \Rightarrow \frac{1-\gamma}{a_{t+1}^F} = \lambda_{t+1}^F \\
(iiiF) \quad \frac{\partial \mathcal{L}}{\partial K_t^F} &= 0 \Rightarrow -\lambda_t^F q_t + \lambda_{t+1}^F \alpha = \phi_t \frac{q_{t+1}}{R} \\
(ivF) \quad \frac{\partial \mathcal{L}}{\partial b_t} &= 0 \Rightarrow \phi_t = \lambda_{t+1}^F R - \lambda_t^F
\end{aligned}$$

From *(iF)* and *(iiF)* follows that $\lambda_{t+1}^F \neq 0$. Therefore the FF constraint of the OF is binding.

Substituting *(ivF)* into *(iiiF)* and rearranging:

$$\frac{\mu_t}{\alpha - q_{t+1}} = \frac{\lambda_{t+1}^F}{\lambda_t^F} \tag{15}$$

where $\mu_t := q_t - \dfrac{q_{t+1}}{R}$ is the downpayment, i.e. the amount of internal finance the borrower has to accumulate in order to get a loan equal to $\dfrac{q_{t+1}}{R}$ (per unit of land he wants to purchase). The downpayment is always positive.

We assume that

(A1) $q_{t+1} \neq \alpha$. In this case, from 15 follows that $\lambda_t^F \neq 0$. Therefore the FF constraint of the YF is also binding.

Substituting 15 into the (ivF) one gets:

$$\phi_t = \left(R \frac{\mu_t}{\alpha - q_{t+1}} - 1 \right) \lambda_t^F$$

We assume that

(A2) $q_{t+1} \neq \alpha / R$ or $R\mu_t \neq \alpha - q_{t+1}$. Therefore $\phi_t \neq 0$. Hence the financing constraint is binding:

$$b_t = \frac{q_{t+1}}{R} K_t^F \tag{16}$$

The farmer is the borrower. Therefore the gatherer is the lender.

Thanks to assumptions (A1) and (A2) all the constraints are binding. Substituting 13 into 12 and 14 respectively, in the case of binding constraints, one gets:

$$\mu_t K_t^F + m_{t,t}^F = a_t^F + q_t K_{t-1}^F \tag{17}$$

$$c_{t,t+1}^F + a_{t+1}^F = (\alpha - q_{t+1}) K_t^F + m_{t,t+1}^F \tag{18}$$

Equation 17 provides a different interpretation of the FF constraint of the young: the YF employs bequests and *collateralizable wealth* $q_t K_{t-1}^F$ to put aside internal finance and hold money balances.

From *(iF)* and *(iiF)* and the FF constraint 18 we derive the optimal consumption and the optimal bequest of the OF:

$$c_{t,t+1}^F = \gamma \left[(\alpha - q_{t+1}) K_t^F + m_{t,t+1}^F \right] \tag{19}$$

$$a_{t+1}^F = (1 - \gamma) \left[(\alpha - q_{t+1}) K_t^F + m_{t,t+1}^F \right] \tag{20}$$

Thanks to the Cobb-Douglas specification of prefences, consumption and bequest are a fraction γ and $1-\gamma$ respectively of the *resources* available in t+1 to the OF, e^F_{t+1},where

$$e^F_{t+1} = (\alpha - q_{t+1})\, K^F_t + m^F_{t,t+1}$$

From the YF constraint in t 12one gets:

$$K^F_t = \frac{a^F_t + q_t K^F_{t-1} - m^F_{t,t}}{\mu_t} \tag{21}$$

Notice now that from 20 follows that the optimal bequest of the OF of generation t-1 in t is

$$a^F_t = (1-\gamma)\left[(\alpha - q_t)\, K^F_{t-1} + m^F_{t-1,t}\right]$$

where $m^F_{t-1,t} = \dfrac{M^F_{t-1,t}}{P_t}$are real money balances of the OF of generation t-1 in t. Substituting this expression into 21 and rearranging one gets:

$$K^F_t = \frac{\left[(1-\gamma)\,\alpha + \gamma q_t\right] K^F_{t-1} + (1-\gamma)\, m^F_{t-1,t} - m^F_{t,t}}{\mu_t} \tag{22}$$

which is the law of motion of the land of the farmer.

Money plays two different and contrasting roles with respect to landholding.On the one hand, *given the bequest,* the higher money of the young $m^F_{t,t}$, the lower landholding: In fact resources of the young (bequest and credit) can be devoted either to money or landholding; On the other hand, the higher money of the old $m^F_{t-1,t}$, the higher resources available to him and the higher bequest and landholding.

In the special and convenient case in which $m^F_{t-1,t} = m^F_{t,t} = m^F_t$,i.e. when the old farmer is exchanging money only with the young farmer (more on this in section...), $(1-\gamma)\, m^F_{t-1,t} - m^F_{t,t} = (1-\gamma)\, m^F_t - m^F_t = -\gamma m^F_t$:The second effect is offset by the first so that in the end the accumulation of money affects negatively land holding. Hence, recalling that $\mu_t = q_t - \dfrac{q_{t+1}}{R}$ 22 boils down to

$$K^F_t = \frac{\left[(1-\gamma)\,\alpha + \gamma q_t\right] K^F_{t-1} - \gamma m^F_t}{q_t - \dfrac{q_{t+1}}{R}} \tag{23}$$

5. The gatherer/lender

Following the same modelling route of the previous section, we assume that preferences of the gatherer are represented as follows

$$U^G = \gamma \ln c^G_{t,t+1} + (1-\gamma) \ln a^G_{t+1} \tag{24}$$

where $c^G_{t,t+1}$ and a^G_{t+1} are consumption and bequest of the OG.Being unconstrained from the financial point of view, the gatherer maximizes utility subject to the FF constraints of the YG and of the OG (see appendix A for the derivation).

The FF constraint of the YG in t is

$$m^G_{t,t} + b_t + q_t \left(K^G_t - K^G_{t-1}\right) \leq a^G_t \tag{25}$$

According to 25, the *resources* of the YG, of internal origin (a^G_t), can be employed to "invest", $q_t \left(K^G_t - K^G_{t-1}\right)$,i.e. to change the landholding, extend credit and hold money balances.

In t, the YG uses labour and land K^G_t to produce output which will become available in $t+1$: $y^G_{t+1} = G\left(K^G_t\right)$.When old, the gatherer's resources consist of output (produced when young), the reimbursement of debt and money balances.These resources can be employed to consume and leave a bequest. Therefore the FF constraint of the OG in $t+1$ in real terms is:

$$c^G_{t,t+1} + a^G_{t+1} \leq G\left(K^G_t\right) + Rb_t + m^G_{t,t+1} \tag{26}$$

The gatherer maximizes 24 subject to 25 26. The Lagrangian is:

$$\begin{aligned}
\mathcal{L} &= \gamma \ln c^G_{t,t+1} + (1-\gamma) \ln a^G_{t+1} + \lambda^G_t \left[a^G_t - q_t \left(K^G_t - K^G_{t-1}\right) - m^G_{t,t} - b_t\right] \\
&\quad + \lambda^G_{t+1} \left[G\left(K^G_t\right) + Rb_t + m^G_{t,t+1} - c^G_{t,t+1} - a^G_{t+1}\right]
\end{aligned}$$

$$\begin{aligned}
(iG) \quad \frac{\partial \mathcal{L}}{\partial c^G_{t,t+1}} &= 0 \Rightarrow \frac{\gamma}{c^G_{t,t+1}} = \lambda^G_{t+1} \\
(iiG) \quad \frac{\partial \mathcal{L}}{\partial a^G_{t+1}} &= 0 \Rightarrow \frac{1-\gamma}{a^G_{t+1}} = \lambda^G_{t+1} \\
(iiiG) \quad \frac{\partial \mathcal{L}}{\partial K^G_t} &= 0 \Rightarrow \lambda^G_{t+1} G'\left(K^G_t\right) = \lambda^G_t q_t \\
(ivG) \quad \frac{\partial \mathcal{L}}{\partial b_t} &= 0 \Rightarrow \lambda^G_t = \lambda^G_{t+1} R
\end{aligned}$$

From the FOCS it is clear that all the constraints are binding. Moreover, from (iiiG) and (ivG) follows $q_t = \dfrac{G'\left(K_t^G\right)}{R}$.Since the total amount of "land" is fixed by assumption, $K_t^F = \bar{K} - K_t^G$, $G'\left(K_t^G\right) = G'\left(\bar{K} - K_t^F\right)$. In nthe following, in order to save on notation, we will write $G'\left(\bar{K} - K_t^F\right) = g\left(K_t^F\right), g' = -G'' > 0$.Therefore we can write

$$q_t = \frac{g\left(K_t^F\right)}{R} \tag{27}$$

Since the financing constraint is binding, the amount of credit extended by the YG in t is equal to the present value of land in t+1: $b_t = \dfrac{q_{t+1}}{R} K_t^F$. Taking into account 16, from 26, *(iG)* and *(iiG)* we derive the optimal consumption and the optimal bequest of the OG:

$$c_{t,t+1}^G = \gamma \left[G\left(K_t^G\right) + q_{t+1} K_t^F + m_{t,t+1}^G\right] \tag{28}$$

$$a_{t+1}^G = (1-\gamma)\left[G\left(K_t^G\right) + q_{t+1} K_t^F + m_{t,t+1}^G\right] \tag{29}$$

Thanks to the Cobb-Douglas specification of prefences, consumption and bequest are a fraction γ and $1-\gamma$ respectively of the *resources* available in t+1 to the OG, e_{t+1}^G,where

$$e_{t+1}^G = G\left(K_t^G\right) + q_{t+1} K_t^F + m_{t,t+1}^G$$

6. Playing with constraints

Since the total amount of "land" is fixed by assumption, $K_t^F = \bar{K} - K_t^G$. Hence an increase of landholding for the farmer can occur only if there is a corresponding decrease of landholding for the gatherer: $K_t^F - K_{t-1}^F = -\left(K_t^G - K_{t-1}^G\right)$. Taking this fact into account, summing side by side the (binding) FF constraints of the young agents 12 and 25, i.e.

$$q_t\left(K_t^F - K_{t-1}^F\right) + m_{t,t}^F = b_t + a_t^F$$

$$m_{t,t}^G + b_t + q_t\left(K_t^G - K_{t-1}^G\right) = a_t^G$$

one gets

$$m_{t,t}^F + m_{t,t}^G = a_t^F + a_t^G \tag{30}$$

In words: the total amount of bequest obtained by the young agents is equal to the total amount of *money of the young agents.* In the special case in which bequest is left exclusively in terms of money, i.e. $m^F_{t,t} = a^F_t$ and $m^G_{t,t} = a^G_t$, investment of the farmer is financed exclusively by means of credit, i.e. $q_t \left(K^F_t - K^F_{t-1}\right) = b_t$

Updating 30 we obtain

$$m^F_{t+1,t+1} + m^G_{t+1,t+1} = a^F_{t+1} + a^G_{t+1} \tag{31}$$

Summing side by side the (binding) FF constraints of the old agents 14 and 26, i.e.

$$c^F_{t,t+1} + a^F_{t+1} + b_t R = y^F_{t+1} + m^F_{t,t+1}$$
$$c^G_{t,t+1} + a^G_{t+1} = y^G_{t+1} + Rb_t + m^G_{t,t+1}$$

yields

$$c^F_{t,t+1} + c^G_{t,t+1} + a^G_{t+1} + a^F_{t+1} = y^G_{t+1} + y^F_{t+1} + m^G_{t,t+1} + m^F_{t,t+1} \tag{32}$$

In words: aggregate output and real *money balances of the old agents* is equal to the sum of aggregate consumption and aggregate bequest.

We assume equilibrium on the goods market, i.e.

$$c^F_{t,t+1} + c^G_{t,t+1} = y^G_{t+1} + y^F_{t+1} \tag{33}$$

Taking 33 into account, 32 boils down to

$$m^F_{t,t+1} + m^G_{t,t+1} = a^F_{t+1} + a^G_{t+1} \tag{34}$$

i.e. the total amount of bequest left by the old agents is equal to the total amount of *money of the old agents*. From 31 and 34 we get

$$m^F_{t+1,t+1} + m^G_{t+1,t+1} = m^G_{t,t+1} + m^F_{t,t+1} \tag{35}$$

i.e the total amount of money of the young agents in t+1 must be equal to the total amount of money of the old agents of generation t in t+1.

7. Money trickles down

In our economy money "trickles down" from one period to the next and from one agent to the other. In fact a network of money transfers is taking place from the pool of monetary resources of one agent to the pool of another agent. In principle we distinguish three types of transfers:

- "within generations" or *horizontal transfers*, i.e. transfers between agents of the same generation but of different types (farmers and gatherers). Horizontal transfers are the monetary counterpart of transactions between agents of different types concerning goods (fruit) or land. Therefore they are motivated by agents' decisions to consume and invest, i.e. modify landholdings;

- "between generations" or *vertical transfers*, i.e. transfers between agents of different generations but of the same type (old and young agents).Vertical transfers coincides with bequests, which are motivated by intergenerational altruism.

- Government transfers, i.e. monetized subsidies to the old.

In order to describe the way in which money spreads in the economy, let's take a look at table 1. In each row we report the inflows and outflows which show up in the FF constraints of the agents in period t+1. The amount in the inflow cell is equal to the amount in the outflow cell. For instance, the first row represents the FF constraint of the YF in t+1 : $a^F_{t+1} = q_{t+1}\left(K^F_{t+1} - K^F_t\right) + m^F_{t+1,t+1} - b_{t+1}$.[7] In other words, we have rewritten in a suitable form equation 12. The third row is the sum of rows 1 and 2 (i.e. of the young agents), the sixth row is the sum of rows 4 and 5 (i.e. of the old agents). Therefore, the table contains adapted equations 12 25 31 14 26 32.

	inf lows	outflows
YF	a^F_{t+1}	$q_{t+1}\left(K^F_{t+1} - K^F_t\right) + m^F_{t+1,t+1} - b_{t+1}$
YG	a^G_{t+1}	$-q_{t+1}\left(K^F_{t+1} - K^F_t\right) + m^G_{t+1,t+1} + b_{t+1}$
$\sum$	$a^G_{t+1} + a^F_{t+1}$	$m^G_{t+1,t+1} + m^F_{t+1,t+1}$
OF	$y^F_{t+1} + m^F_{t,t+1}$	$c^F_{t,t+1} + a^F_{t+1} + b_t R$
OG	$y^G_{t+1} + m^G_{t,t+1}$	$c^G_{t,t+1} + a^G_{t+1} - Rb_t$
$\sum$	$y^F_{t+1} + m^F_{t,t+1} + y^G_{t+1} + m^G_{t,t+1}$	$c^F_{t,t+1} + a^F_{t+1} + c^G_{t,t+1} + a^G_{t+1}$

Let's assume that $y^F_{t+1} - c^F_{t,t+1} = s^F_{t,t+1} > 0$, i.e. the OF consumes less than the output he has produced. In a sense he is "saving" the amount $s^F_{t,t+1}$. Market clearing on the goods market implies $s^G_{t,t+1} = -\left(c^G_{t,t+1} - y^G_{t+1}\right) = -s^F_{t,t+1} < 0$ i.e. the OG consumes more than the output he has produced. He is "dissaving"

[7] Matter of factly b_{t+1} represent an inflow for the YF in t+1. It shows up as a negative component in the outflow cell for convenience.

the amount $-\left(c^G_{t,t+1}-y^G_{t+1}\right)$. In other words, the OG has excess consumption $c^G_{t,t+1}-y^G_{t+1}$.

The OF sells $s^F_{t,t+1}$ units of output to the OG in order to let him consume in excess of his output. The OG pays this output by means of money. Therefore, after the transaction, the OF has money balances equal to $m^F_{t,t+1}+\left(c^G_{t,t+1}-y^G_{t+1}\right)$. This money is used to reimburse debt b_tR and leave the bequest a^F_{t+1}. Accounts are consistent: In fact $c^G_{t,t+1}-y^G_{t+1}=y^F_{t+1}-c^F_{t,t+1}$ so that $m^F_{t,t+1}+y^F_{t+1}-c^F_{t,t+1}=a^F_{t+1}+b_tR$ which is the FF of the OF.

The YF receives a^F_{t+1} from his parents and b_{t+1} from the YG and employs these resources to invest, i.e. $q_{t+1}\left(K^F_{t+1}-K^F_t\right)$. The difference between bequest and credit on the one hand and investment on the other consists of money balances $m^F_{t+1,t+1}=a^F_{t+1}+b_{t+1}-q_{t+1}\left(K^F_{t+1}-K^F_t\right)$ that the farmer holds idle when young (since he does not consume) in order employ them when old to access consumption and leave a bequest. Notice that, since $a^F_{t+1}=(1-\gamma)\left[(\alpha-q_{t+1})K^F_t+m^F_{t,t+1}\right]$,

$$m^F_{t+1,t+1}=\left[(1-\gamma)\alpha+\gamma q_{t+1}\right]K^F_t+(1-\gamma)m^F_{t,t+1}-\mu_{t+1}K^F_{t+1}$$

This equation links the money of the young farmer to the money of the old farmer in period t+1. It is 22 rewritten and updated.

Thanks to the Cobb-Douglas specification of the utility function,from the FOCs (iF)(iiF) and (iG)(iiG) one gets

$$c^i_{t,t+1}=\frac{\gamma}{1-\gamma}a^i_{t+1}\qquad i=F,G \tag{36}$$

Substituting 36 and the market clearing condition 33 into 32 we obtain

$$m^G_{t,t+1}+m^F_{t,t+1}=\frac{1-\gamma}{\gamma}\left(y^G_{t+1}+y^F_{t+1}\right) \tag{37}$$

Total real money balances are proportional to aggregate output. Equation 37 is a sort of quantity theory of money in this context.

In the following we will write:

$$m^F_{t,t+1}=\frac{1-\gamma}{\gamma\left(1+\sigma_{t+1}\right)}\left(y^G_{t+1}+y^F_{t+1}\right) \tag{38}$$

where $\sigma_{t+1}:=\dfrac{m^G_{t,t+1}}{m^F_{t,t+1}}$.

In principle there is no reason to assume that the money the young agent has must be equal to the money of the old agent of the same class. This equality holds in the aggregate (see equation 35) but not for each class of agent. We assume however exactly this: $m^F_{t+1,t+1} = m^F_{t,t+1} = m^F_{t+1}$ and $m^G_{t+1,t+1} = m^G_{t,t+1} = m^G_{t+1}$ in order to simplify the analysis. Therefore we can write

$$m^F_t = \frac{1-\gamma}{\gamma\left(1+\sigma_t\right)}\left[\alpha K^F_{t-1} + G\left(\bar{K} - K^F_{t-1}\right)\right] \tag{39}$$

Moreover we assume the following $M^i_{t+1} = M^i_t\left(1+\eta^i_{t+1}\right)$, $i = F, G$. Therefore

$$m^i_{t+1} = m^i_t \frac{1+\eta^i_{t+1}}{1+\pi_{t+1}} \qquad i = F, G$$

The ratio of money of the gatherer to money of the farmer will be denoted by

$$\sigma_t = \frac{M^G_t}{M^F_t} = \frac{m^G_t}{m^F_t}$$

Hence

$$\sigma_{t+1} = \frac{M^G_{t+1}}{M^F_{t+1}} = \frac{m^G_{t+1}}{m^F_{t+1}} = \frac{m^G_t}{m^F_t}\frac{1+\eta^G_{t+1}}{1+\eta^F_{t+1}} = \sigma_t \frac{1+\eta^G_{t+1}}{1+\eta^F_{t+1}}$$

Equation ... is the law of motion of the composition of money. This ratio is constant iff $\eta^G_{t+1} = \eta^F_{t+1}$.

8. A simple welfare criterion

The utility function 11 is a logarithmic transformation of $U^F = \left(c^F_{t,t+1}\right)^{\gamma}\left(a^F_{t+1}\right)^{1-\gamma}$ and therefore represents the same preferences. In order to compute indierct utility we plug opimal consumption and bequests into the function, obtaining $U^F = \gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma} e^F_{t+1}$ i.e. indirect utility is increasing linearly with resources e^F_{t+1} of the old farmer where $e^F_{t+1} = \left(\alpha - q_{t+1}\right) K^F_t + m^F_{t,t+1}$.

Following the same reasoning, we can draw the conclusion that $U^G = \gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma} e^G_{t+1}$ i.e. indirect utility is increasing linearly with resources e^G_{t+1} of the old gatherer where $e^G_{t+1} = G\left(K^G_t\right) + q_{t+1}K^F_t + m^G_{t,t+1}$.

A measure of society's welfare can be roughly be

$$U^S = U^F + U^G = \gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}\left(e^F_{t+1} + e^G_{t+1}\right)$$

i.e. society's well being is increasing with the sum of resources of the farmer and the gatherer.

Notice now that $e_{t+1}^{F} + e_{t+1}^{G} = \alpha K_t^F + G\left(K_t^G\right) + m_{t,t+1}^{F} + m_{t,t+1}^{G}$. The term $q_{t+1}K_t^F = Rb_t$, i.e. debt service, is a positive component of the gatherer's resources and a negative component of the farmer's resources. It cancels out in the aggregate.

Notice, moreover, that $m_{t,t+1}^{G} + m_{t,t+1}^{F} = \frac{1-\gamma}{\gamma}\left(y_{t+1}^{G} + y_{t+1}^{F}\right)$. Substituting this expression into the expression above and rearranging we get: $e_{t+1}^{F} + e_{t+1}^{G} = \frac{1}{\gamma}\left[\alpha K_t^F + G\left(K_t^G\right)\right]$. Hence

$$U^S = U^F + U^G = \left(\frac{1-\gamma}{\gamma}\right)^{1-\gamma}\left[\alpha K_t^F + G\left(K_t^G\right)\right]$$

i.e. society's well being is increasing with aggregate output.

Maximization of society's welfare therefore occurs when the marginal productivity of the farmer equals that of the gatherer's, i.e.

$$\alpha = G'\left(K_t^G\right) \tag{40}$$

i.e. when $K_t^G = G'^{-1}(\alpha) = \bar{K} - K_t^F$. Hence $K_f^F = \bar{K} - G'^{-1}(\alpha)$. In this case

$$y_f = y_f^G + y_f^F = G'\left(\bar{K} - K_f^F\right) + \alpha K_f^F = y\left(K_f^F\right)$$

is the maximum aggregate output society can obtain.

The same conclusion in KM.

Notice that from equation... follows that the only level of q_t such that the first best would be obtained is

$$q_f = \frac{\alpha}{R}$$

which we have ruled out (see assumption A2 above) because it would imply that the financing constraint is not binding. In other words, the first best could be attained only if the financing constraint were not binding. From the FF constraints it turns out that in the steady state $b = a^F - m^F$ and $a^F = (1-\gamma)\left(y_f^F - Rb + m^F\right)$. Finally, from the quantity theory $m^F = \frac{1-\gamma}{\gamma(1+\sigma)}y_f$. Substituting we get:

$$b_f = \frac{1-\gamma}{1+\sigma}\left(\sigma y_f^F - y_f^G\right)$$

9. Dynamics

The dynamics of the macroeconomy are described by equation 23, i.e the law of motion of the farmer's land, equation 27, which links the asset price to the farmer's land, and equation 39, i.e. the quantity theory of money. The state variables are K_t^F, q_t and m_t^F.We list the equations below for the reader's convenience.

$$K_t^F = \frac{\left[(1-\gamma)\alpha + \gamma q_t\right] K_{t-1}^F - \gamma m_t^F}{q_t - \frac{q_{t+1}}{R}}$$
$$q_t = \frac{g\left(K_t^F\right)}{R}$$
$$(1+\sigma) m_t^F = \frac{1-\gamma}{\gamma}\left[\alpha K_{t-1}^F + G\left(\bar{K} - K_{t-1}^F\right)\right]$$

Plugging the third equation into the first one, the system boils down to

$$K_t^F = \frac{\left[\frac{(1-\gamma)\alpha\sigma}{1+\sigma} + \gamma q_t\right] K_{t-1}^F - \frac{1-\gamma}{1+\sigma} G\left(\bar{K} - K_{t-1}^F\right)}{q_t - \frac{q_{t+1}}{R}}$$
$$q = \frac{g\left(K^F\right)}{R}$$

Substituting the second equation into the first one and noting that $q_{t+1} = \frac{g\left(K_{t+1}^F\right)}{R}$ the system boils down to

$$\left[\frac{g\left(K_t^F\right)}{R} - \frac{g\left(K_{t+1}^F\right)}{R^2}\right] K_t^F - \left[\frac{(1-\gamma)\alpha\sigma}{1+\sigma} + \frac{\gamma g\left(K_t^F\right)}{R}\right] K_{t-1}^F + \frac{1-\gamma}{1+\sigma} G\left(\bar{K} - K_{t-1}^F\right) = 0 \tag{41}$$

which is a non linear second order difference equation in the state variable K_t^F in implicit form.

9.1. Steady states

An interesting way of computing the steady state is the following. Rewriting the system ignoring time indices and recalling that, in the steady state $\mu = q\delta$ with $\delta = 1 - \frac{1}{R}$ we get

$$
\begin{aligned}
K^F &= \frac{\left[(1-\gamma)\,\alpha+\gamma q\right]K^F-\gamma m^F}{q\delta} \\
q &= \frac{g\left(K^F\right)}{R} \\
(1+\sigma)\,m^F &= \frac{1-\gamma}{\gamma}\left[\alpha K^F+G\left(\bar{K}-K^F\right)\right]
\end{aligned}
$$

Plugging the third equation into the first one, the system boils down to

$$
\begin{aligned}
q &= \theta\left[h\left(K^F\right)-\sigma\alpha\right] \\
q &= \frac{g\left(K^F\right)}{R}
\end{aligned}
$$

where

$$
\begin{aligned}
\theta &= \frac{1-\gamma}{(\gamma-\delta)\,(1+\sigma)} \\
h\left(K^F\right) &= \frac{G\left(\bar{K}-K^F\right)}{K^F}
\end{aligned}
$$

i.e. a system of two equations in $K^F$and q. Notice that $h\left(K^F\right)$ is clearly decreasing with K^F.

The first equation yields two different curves on the $\left(K^F,q\right)$ plane depending upon the relative value of γ and δ.

In the case $\gamma>\delta$, the curve is downward sloping. In the opposite case, $\gamma<\delta$ the curve is upward sloping. In both cases, the curve and crosses the x-axis when $K^F=h^{-1}(\sigma\alpha)$.

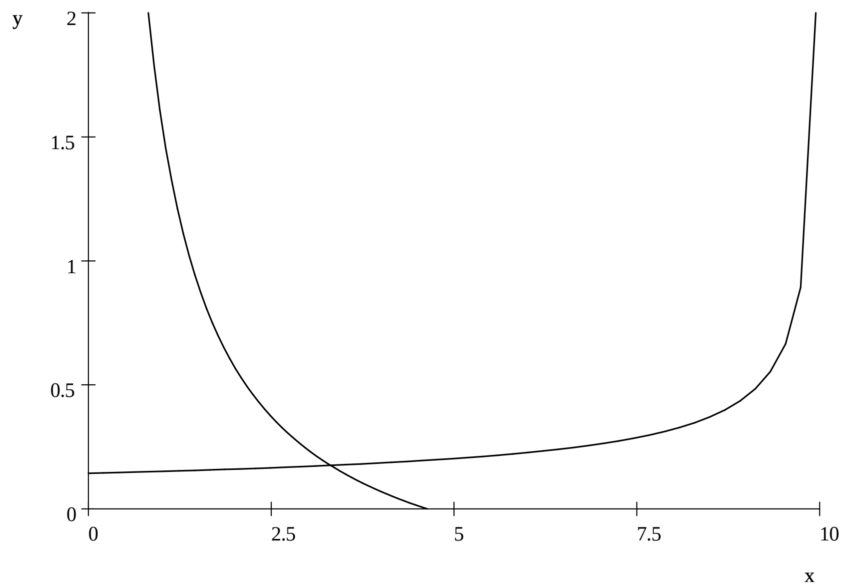
For instance, in the case of a Cobb-Douglas production function $G\left(\bar{K}-K^F\right)=\sqrt{\bar{K}-K^F}$ we can have two cases.

In fig.9.1 gamma=0.5, R=1.1 (delta=0.1), sigma 1, alfa=0.5, Kbar=10

In fig.9.1 gamma=0.5, R=2.5 (delta=0.6), sigma 1, alfa=0.5, Kbar=10

9.2. A convenient special case

In the open economy case, the market clearing condition is $c_{t,t+1}^F+c_{t,t+1}^G+x_{t+1}=y_{t+1}^G+y_{t+1}^F$, where x_{t+1} represent net exports. Assuming that net exports accomodate discrepancies between output and domestic demand (because of changes in the real exchange rate, for instance) we can allow for $c_{t,t+1}^F+c_{t,t+1}^G\neq y_{t+1}^G+y_{t+1}^F$ so that the quantity theory should not necessarily hold any longer. This is a simple


y
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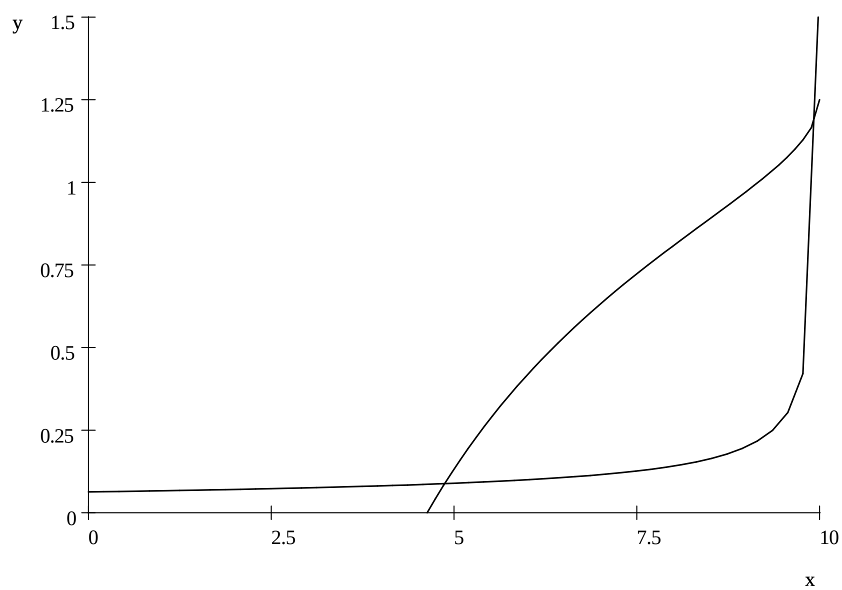
1.5
1
0.5
0
0
2.5
5
7.5
10
x


y
1.5
1.25
1
0.75
0.5
0.25
0
0
2.5
5
7.5
10
x

way of simplifying the framework in order to get a convenient special case. The dynamic system in this case is

$$K_t^F = \frac{\left[(1-\gamma)\,\alpha + \gamma q_t\right] K_{t-1}^F - \gamma m_t^F}{q_t - \dfrac{q_{t+1}}{R}}$$

$$q_t = \frac{g\left(K_t^F\right)}{R}$$

In order to assess the properties of the trajectories generated by this system, we can assume that the gatherer's production function is $G = \sqrt{\bar{K} - K_t^F}$. Hence the system becomes

$$K_t^F = \frac{\left[(1-\gamma)\,\alpha + \gamma q_t\right] K_{t-1}^F - \gamma m_t^F}{q_t - \dfrac{q_{t+1}}{R}}$$

$$q_t = \frac{1}{2R\sqrt{\bar{K} - K_t^F}}$$

The state variables are K_t^F (on the x-axis of the following diagrams) and q_t (on the y-axis). The phase diagrams are depicted for following parameter values: $\gamma = 0.5; m_t^F = 12; \bar{K} = 10; R = 1.02$. When the productivity of the farmer is relatively low ($\alpha = 1.1$) (see fig.1) the trajectories are generally converging to a steady state (black point). Points belonging to the pink (grey) region, in fact, generate trajectories which converge to (diverge from) the steady state.

When the productivity of the farmer goes up ($\alpha = 1.15$) (see fig.2) the steady state becomes unstable (white circle) via Neimark-Hopf bifurcation and an invariant attracting curve emerges. Also attracting cycles emerge (not reported) as it is usually the case in the case of a Neimark-Sacker bifurcation (the so-called Arnold tongues). Our educated guess is that this is a route to chaos .

In the end a "strange attractor" appears (see below, fig.3). Fluctuations are irregular. Chaotic dynamics is a possible occurrence. (to be continued)

A. Constraints at current and constant prices

In the following we denote magnitudes at current (constant) prices with capital (small) letters.

We assume that each young farmer is endowed at birth with *bequest* A_t^F. The YF employs the bequest he got and credit B_t (since it turns out that he is a borrower) to invest in land – $Q_t\left(K_t^F - K_{t-1}^F\right)$ – *and hold money balances* $M_{t,t}^F$.

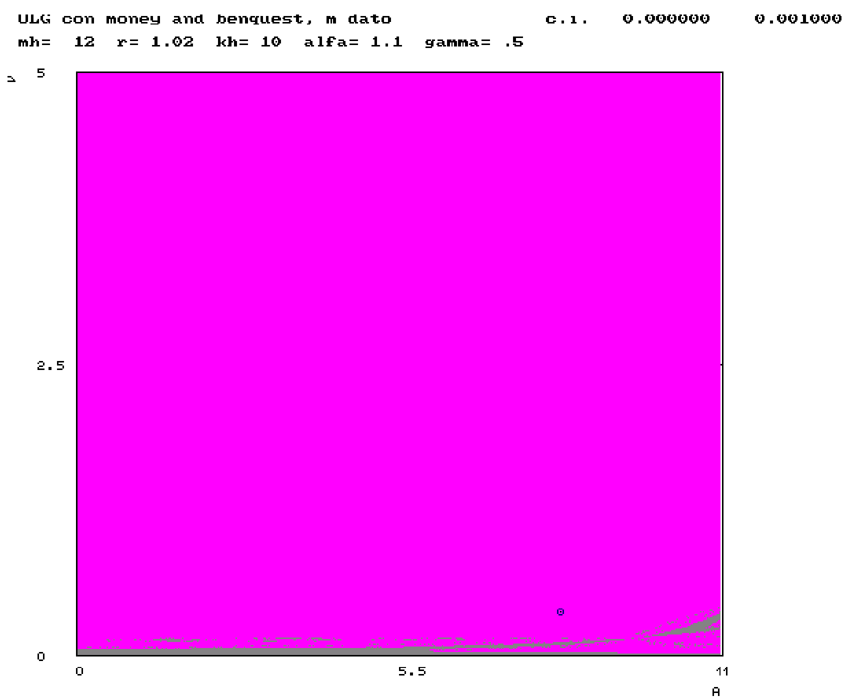
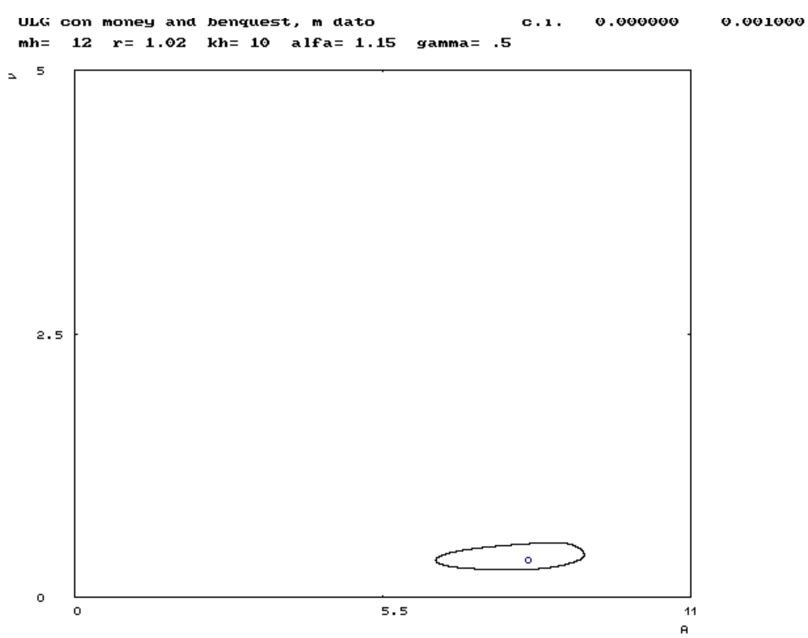

ULG con money and benquest, m dato c.i. 0.000000 0.001000
mh= 12 r= 1.02 kh= 10 alfa= 1.1 gamma= .5
5
2.5
0
0
5.5
11
A

Figure 1: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c.i. 0.000000 0.001000
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5
2.5
0
0
5.5
11
A

Figure 2:

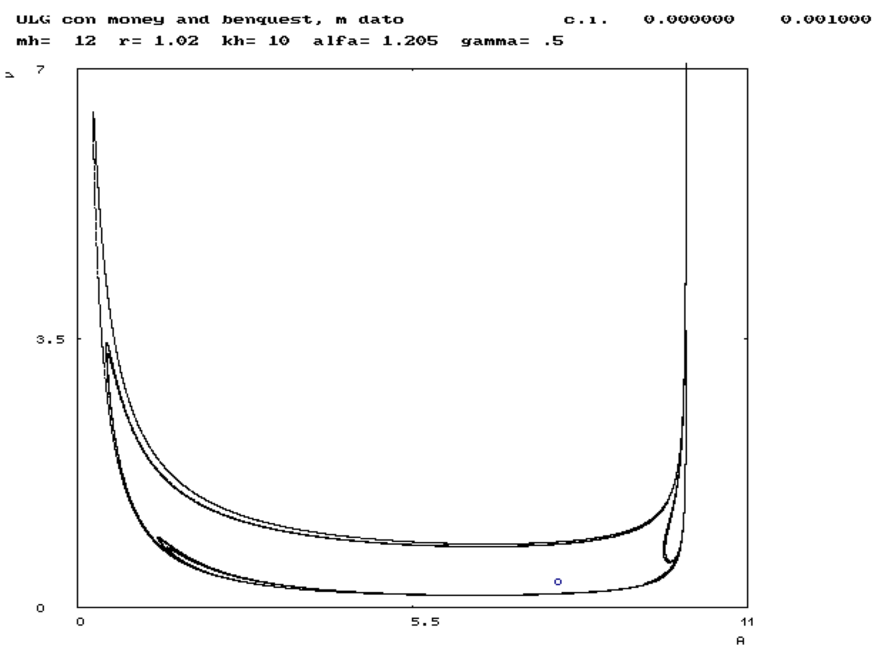
ULG con money and benquest, m dato c.i. 0.000000 0.001000
mh= 12 r= 1.02 kh= 10 alfa= 1.205 gamma= .5
7
3.5
0
0
5.5
11
A

Figure 3:

Since the young does not derive utility from consumption, *money is a reserve of value for the YF: He carries money over from youth to old age in order to use it as a means of payment in the latter stage of his life, i.e. to access consumption when old.*

The flow-of-funds (*fof*) constraint of the YF in t is:

$$Q_t\left(K_t^F - K_{t-1}^F\right) + M_{t,t}^F \leq B_t + A_t^F \tag{42}$$

Dividing by P_t and rearranging we get:

$$q_t\left(K_t^F - K_{t-1}^F\right) + m_{t,t}^F \leq b_t + a_t^F \tag{43}$$

The YF may be financially coinstrained. The financing constraint can be expressed as follows:

$$B_t \geq \frac{Q_{t+1}}{1+i_t} K_t^F$$

where i_t is the nominal interest rate. In words: The YF gets a loan in t greater or equal to the present value of *collateralizable wealth, i.e. of the market value in t+1 of land owned in t.* If the constraint is binding, $B_t = \dfrac{Q_{t+1}}{1+i_t} K_t^F$. In real terms: $b_t = \dfrac{Q_{t+1}}{P_t\left(1+i\right)} K_t^F$. Multiplying and dividing the expression above by P_{t+1}

$$b_t = \frac{q_{t+1}}{R} K_t^F$$

where $R := \left(1+i\right)/\left(1+\pi_{t+1}\right)$ is the real (gross) interest rate and $1+\pi_{t+1} := P_{t+1}/P_t$ is the (gross) rate of inflation.

In t, the YF uses labour and land K_t^F to produce output which will become available in $t+1$, $y_{t+1}^F = \alpha K_t^F$.When old, the farmer has an "inflow" equal to the revenues from sale of output (produced when young) and money balances. Part of the money balances are carried over from youth, part are conferred to the old by the Government as a (monetized) transfer payment. [8]The "outflow" consists of

[8] Let's assume

$$M_{t+1}^F = M_t^F + T_{t+1}$$

where T_{t+1} is a monetized transfer payment to the old agent. The transfer is proportional to the individual money holdings, i.e.

$$T_{t+1} = \eta^F M_t^F$$

Hence

$$m_{t+1}^F = \frac{M_{t+1}^F}{P_{t+1}} = \frac{\left(1+\eta^F\right) M_t^F}{P_{t+1}} = \frac{1+\eta^F}{1+\pi_{t+1}} m_t^F$$

the repayment of the loan (if the YF were a borrower), consumption and bequest. Therefore the fof constraint of the OF in $t+1$ in nominal terms is:

$$P_{t+1}c^F_{t,t+1} + A^F_{t+1} + B_t(1+i_t) \leq P_{t+1}y^F_{t+1} + M^F_{t,t+1}$$

Dividing by P_{t+1} and recalling that $y^F_{t+1} = \alpha K^F_t$

$$c^F_{t,t+1} + a^F_{t+1} + b_t R \leq \alpha K^F_t + m^F_{t,t+1}$$

Thanks to assumptions A1 and A2, all the constraints are binding, i.e.

$$q_t\left(K^F_t - K^F_{t-1}\right) + m^F_{t,t} = b_t + a^F_t$$
$$b_t = \frac{q_{t+1}}{R}K^F_t$$
$$c^F_{t,t+1} + a^F_{t+1} + b_t R = \alpha K^F_t + m^F_{t,t+1}$$

Substituting the second constraint into the first and the third one gets

$$\mu_t K^F_t + m^F_{t,t} = a^F_t + q_t K^F_{t-1}$$
$$c^F_{t,t+1} + a^F_{t+1} = (\alpha - q_{t+1})K^F_t + m^F_{t,t+1}$$

where $\mu_t := q_t - \dfrac{q_{t+1}}{(1+i_t)}(1+\pi_{t+1}) = q_t - \dfrac{q_{t+1}}{R}$ is the downpayment, i.e. the amount of internal finance the borrower has to accumulate in order to get a loan equal to $\dfrac{q_{t+1}}{R}$ (per unit of land he wants to purchase). The fof constraint of the YF $\mu_t K^F_t + m^F_{t,t} = a^F_t + q_t K^F_{t-1}$provides a different interpretation of the fof constraint: the YF employs bequests and *collateralizable wealth* $q_t K^F_{t-1}$ to put aside internal finance and hold money balances.

The flow-of-funds constraint of the YG in t is

$$Q_t\left(K^G_t - K^G_{t-1}\right) + B_t + M^G_{t,t} \leq A^G_t$$

In real terms

$$q_t\left(K^G_t - K^G_{t-1}\right) + b_t + m^G_{t,t} \leq a^G_t$$

Since $b_t = \dfrac{q_{t+1}}{R}K^F_t$ and $K^G_t - K^G_{t-1} = -\left(K^F_t - K^F_{t-1}\right)$

$$-\mu_t K^F_t + m^G_t = a^G_t - q_t K^F_{t-1} \tag{44}$$

When old, the gatherer employs "income", the repayment of the loan, money carried out from youth and transfers to consume and leave a bequest. Therefore the flow-of-funds constraint of the OG in $t+1$ is:

$$P_{t+1}c^G_{t,t+1} + A^G_{t+1} = P_{t+1}y^G_{t+1} + B_t(1+i_t) + M^G_{t,t+1}$$

Dividing by P_{t+1} and recalling that $y^G_{t+1} = G\left(K^G_t\right)$ we get

$$c^G_{t,t+1} + a^G_{t+1} = G\left(K^G_t\right) + Rb_t + m^G_{t,t+1}$$

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