Multiple Equilibria in a Modified Solow-Swan Model*

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Abstract

In this paper, we study the effect of education on economic growth. In particular, we want to show that education can cause some nonlinearities in the human capital accumulation process. These nonlinearities may affect the economic growth path. In the first part of this work, we will provide a model of human capital accumulation. According to this model, a non constant human capital obsolescence rate can cause non constant returns to scale of education to produce human capital. Subsequently - to explain why, under some conditions, multiple equilibria can appear - we will modify a traditional Solow-Swan model by introducing our theoretical contribution. Furthermore, we will calibrate our model to see if given reasonable values of parameters, it is possible to generate multiple steady states. In the second part of this work, we will conduct some econometric analyses to prove that the returns to scale in producing human capital are non constant. Finally, in the last part, we will discuss the main implications of our paper.

1cm1cm Economic growth, education, human capital accumulation, multiple equilibria E13, I2, O4

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1 Introduction

Since the 1960s, the subject of education and economic growth has interested an increasing number of economists. Nevertheless, it is mainly in the last ten years that the flow of statistical analyses on this topic has improved. This is due to the relatively recent availability of aggregate data on country-level education.

In the literature, education is considered one of the most important inputs to produce human capital. Stiglitz and Boadway (1994) defined human capital as, "...the stock of accumulated skills and experience that make workers more productive." Human capital plays a secondary role in the neoclassical growth theory¹, on the contrary, it is central in the endogenous growth models.²

Mankiw, Romer D. and Weil (1992), using a cross-country analysis, show that data are fairly consistent with a Solow-Swan model (or Solow's model) augmented to take account of human capital.³ By taking a linear specification for the human capital contribution to the economic growth, they obtain a rather satisfactory estimate of the aggregate production function. Therefore, education does not produce externalities at the aggregate level. That is, education seems to be a private input, which is remunerated according to its marginal product. Barro and Sala-i-Martin (1995), through an extensive test on cross-country data, prove that the neoclassical model could explains the empiric facts as well as a models of technological diffusion across countries.

Lucas (1988) constructs an augmented Solow-Swan model, starting from a different point of view. Considering a firm's production function, Lucas models a scenario where externalities from human capital can affect the individual production functions. In a context of imperfect competition, Lucas provides a model where R&D activity may sustain the GDP growth in the long-term. Romer P. (1990) shows that international data are consistent with an endogenous growth model, in which human capital is an important input for the R&D sector.

Studying the relationship between education and growth, we can divide the models into two branches. The first branch considers the stock of human

¹The neoclassical growth models, which consider technological change an external factor for growth, are named also *exogenous growth models*. In this sense, Solow's model is the first example of exogenous growth model.

²See Paul Romer (1990), and Aghion and Howitt (1998).

³See Solow (1956), and Swan (1956).

capital a determinant of growth. This approach is based on the contribution of Nelson and Phelps (1966), who argue that education plays a fundamental role in explaining economic growth when an economic system has a relatively high rate of technological change. They proved that high levels of human capital are necessary to sustain high rates of technological progress. Nelson and Phelps use only the initial stock of human capital to achieve their result and not the human capital accumulation process. The second branch uses the process of human capital accumulation over time to explain long-run growth. Unfortunately, the empirical literature cannot help us to understand which approach is more appropriate. Some studies suggest that both the variation and the initial level of education can help us explain GDP growth, while other works show that only the initial stock of education has a significant effect on economic growth.

A traditional assumption in the economic growth theory is that the human capital is produced under constant returns to scale, using education as single input. Yet, there is no evidence to support this assumption. On the contrary, an increasing number of analyses shows that human capital exhibits increasing returns to scale for low educational levels and decreasing returns to scale for high educational levels. Therefore, since human capital is an important input to produce outcome, these nonlinearities in the process of human capital accumulation could also affect the economic growth path of a country.

In 1995, Donal O'Neill found that for the period between 1967-1985, the returns to education rose by 58 percent in industrialized countries and by 64 percent in less developed countries. Analyzing the effect of human capital in an open economy, Isaksson (2002) found evidence of a non-linear relationship between education and its productivity. Also, Trostel (2004) obtained the same results, using micro-data to estimate an adapted Mincerian equation.

This paper can be included in this new generation of studies. Here, we provide a theoretical framework in which education can generate a non-linear human capital accumulation process. The fundamental assumption of our model will be the existence of an increasing human capital obsolescence rate.⁷ We will then use this result to obtain a Solow-Swan model in which

⁴See Uzawa (1965), and Lucas (1988).

⁵See Gemmell (1996), Krueger and Lindhal (2001).

⁶See Benhabib and Spiegel (1994), Barro and Sala-i-Martin (1995).

⁷This assumption is supported by several studies. For a rather detailed review of these studies we suggest to see Andries de Grip (2004).

three different Equilibria may appear. In particular, the lowest equilibrium, in terms of physical capital per capita, is also known in literature as the poverty trap case. Subsequently, we will calibrate our model to see if given reasonable values of parameters, it is possible to generate multiple steady states.

In the second part of this paper, we use a cross-section of 78 countries to conduct some econometric analyses. To show how the returns to scale in producing human capital are not constant, we will compare a traditional regression method with a semiparametric technique used also in Liu and Stengos (1999).

Section 2, contains our modified Solow-Swan model, and Section 3 provides some numerical examples to prove the aforementioned main result and its consistency with the real values of our parameters. In Section 4, using a cross-section of countries, we show how the level of schooling has a non-linear effect on the economic growth rate. Thus, we will conclude that a non-linear relationship emerges between education and the human capital stock. Finally, in the last section, we illustrate the main conclusions of this work.

⁸A situation of *povety trap* is a permanent condition of low income per capita, in which some countries remain trapped even when they show positive GDP growth rates. See Durlauf and Johnson (1995), Quah (1997), Fiascchi and Lavezzi (2003).