

# A Dynamic Heterogeneous Beliefs CAPM

Carl Chiarella

School of Finance and Economics  
University of Technology, Sydney  
PO Box 123 Broadway, NSW 2007, Australia

Roberto Dieci

Universita' degli Studi di Bologna  
Facolta' di Economia del Polo di Rimini  
Via D. Anghera' 22, I-47900 Rimini, Italy

Xue-Zhong He

School of Finance and Economics  
University of Technology, Sydney  
PO Box 123 Broadway, NSW 2007, Australia

We reconsider the derivation of the traditional capital asset pricing model (CAPM) in the discrete time setting for a portfolio of one risk-free asset and many risky assets. In contrast to the standard setting we consider heterogeneous agents whose expectations of future returns based on statistical properties of past returns induce expectations feedback. We assume that agents are heterogeneous in their conditional means and covariances of the risky asset returns. A Walrasian auctioneer scenario is used for the determination of the market clearing price.

In this framework we first construct a consensus belief (with respect to the means and covariances of the risky asset returns) to represent the aggregate market belief and derive a heterogeneous CAPM which relates aggregate excess return on risky assets with aggregate excess return on the market portfolio via an aggregate beta coefficient for risky assets. The aggregate expectations involve the expectations of the various heterogeneous agents, which in turn involve the expectations feedback referred to earlier.

We then use the result in the first part to establish a market fraction model in which agents are grouped according to their beliefs. The impact of different beliefs on the market equilibrium returns and the  $\beta$  coefficient is analysed. In particular, we focus on the three "classical" heterogeneous

agents types - fundamentalists, trend chasers and noise traders - and investigate how some of the key agent characteristics such as strength of fundamental demand, strength of trend chasing, intensity of noise trading and risk aversion affect the time varying behaviour of the aggregate beta coefficient. We contrast this behaviour with that observed in empirical data.

The paper builds on and extends earlier contributions to the literature on portfolio theory with boundedly rational heterogeneous agents, in particular Böhm and Chiarella (2005), Chiarella, Dieci and Gardini (2005) and Chiarella, Dieci and He (2006).

### **References**

Böhm, V., Chiarella, C., 2005. Mean variance preferences, expectations formation and the dynamics of random asset prices. *Mathematical Finance* 15(1), 61-97.

Chiarella, C., Dieci, R. and Gardini, L., 2005. The dynamic interaction of speculation and diversification. *Applied Mathematical Finance* 12(1), 17-52.

Chiarella, C., Dieci, R. and He, X.-Z., 2006. Heterogeneous expectations and speculative behaviour in a dynamic multi-asset framework. *Journal of Economic Behavior and Organization*, in press.