The Role of Consumer's Risk Aversion on Price Rigidity*

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Abstract

This paper aims at contributing to the research agenda on the sources of price stickiness, showing that the adoption of nominal price rigidity may be an optimal firms' reaction to the consumers' behavior, even if firms have no adjustment costs. With regular broadly accepted assumptions on economic agents behavior, we show that firms' competition can lead to the adoption of sticky prices as an (sub-game perfect) equilibrium strategy. We introduce the concept of a consumption centers model economy in which there are several complete markets. Moreover, we weaken some traditional assumptions used in standard monetary policy models, by assuming that households have imperfect information about the inefficient time-varying cost shocks faced by the firms, e.g. the ones regarding to inefficient equilibrium output levels under flexible prices. Moreover, the timing of events are assumed in such a way that, at every period, consumers have access to the actual prices prevailing in the market only after choosing a particular consumption center. Since such choices under uncertainty may decrease the expected utilities of risk averse consumers, competitive firms adopt some degree of price stickiness in order to minimize the price uncertainty and "attract more customers".

Keywords: Inflation dynamics, price rigidity, risk aversion, choice under uncertainty, Calvo type model, monetary policy, welfare analysis, DSGE models.

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1 Introduction

This paper aims at contributing to the research agenda on the sources of price stickiness, showing that the adoption of nominal price rigidity may be an optimal firms' reaction to the consumers behavior, even if firms have no adjustment costs. With regular broadly accepted assumptions on economic agents behavior, we show that firms' competition can lead to the adoption of sticky prices as an (sub-game perfect) equilibrium strategy in order to attract more customers. The intuition behind the model formal conclusions are explained as follows.

We introduce the concept of a *consumption centers* model economy in which there are several complete markets that also compete with each other. Moreover, we weaken some traditional assumptions used in standard monetary policy models, by assuming that households have imperfect information about the inefficient time-varying cost shocks faced by the firms, e.g. the ones regarding to inefficient equilibrium output levels under flexible prices. Moreover, the timing of events are assumed in such a way that, at every period, consumers have access to the actual prices prevailing in the market only after choosing a particular consumption center. Indeed in a real world economy with several consumption centers as supermarkets or shopping malls, for instance, high frequent decisions on which one to choose are made before knowing the actual prices. Since such choices under uncertainty may decrease the expected utilities of risk averse consumers, competitive firms adopt some degree of price stickiness in order to minimize price uncertainty and "attract more customers". On the other hand, increasing such a degree reduces the unconditional expected discounted flow of firms' profit, so there is a trade off between attracting more costumers and increasing profits..

In such a context, we proof two theorems stating that: (a) there is no equilibrium in which households always choose the same consumption center; and (b) the equilibrium degree of price stickiness is the highest, provided that firms have non-negative unconditional expected discounted profit flows, e.g. the unconditional expected discounted profit flows will be zero in non-trivial cases. Such a result follows from the two types of competition inputted in the model. The first one is the traditional monopolistic competition that allows each firm to choose an optimal price that maximizes its expected discounted profit flow. The second one is the Bertrand flavor competition played by the consumption centers using the degree of price stickiness in order to be more "attractive" for the households.

1.1 Background literature

Considerable empirical evidence suggests that prices are sticky in the short run. Some pricing behavior studies, carried out among representative firm samples, indicate that several prices remain fixed on average for more than one quarter[1]. This price setting behavior suggests that prices do not change as frequently as the observed alterations in the state of the economy, which occur more often.

Those facts motivated the development of a broad theoretical research agenda about price stickiness modeling. In order to filter the spectrum of possible theories concerning price stickiness, indicating correct ways to be followed by future researches, Blinder et al. (1998) surveyed

[1] Blinder et al. (1998) findings, inferred from a sample of 200 firms in United States, suggest that the median price adjustment frequency are 1.4 times per year, meaning that prices remain stick for almost 9 months, on average. And about 50% of firms adjust their prices once a year, at most. In a similar work, run by the Bank of England, Hall et al. (1997) infer that more than 60% of the surveyed firms, on a sample of 654 United Kingdom firms, adjusted their prices once or twice during the studied year. These results are consistent with the ones found by Blinder et al., even though coming from a different country. In the same line, Chakrabarti and Scholnick (2005) focused at two internet-leader bookstore (Amazon and Barnes & Noble), on a sample of 3124 books. Regardless the fact that those firms have no physical adjusting costs, the authors found out that prices change only 2.4 times a year, on average.

the reasons why firms do not adopt flexible prices among a significative sample of 200 firms in the United States, from several industries. They asked business people about their price-setting practices and their opinions about which academic theories, expressed in laymen's terms, matched the actual price-setting procedures in United States[2].

It is interesting to note that, although it was never asked whether their costumers were averse to price variation, most of the surveyed firms voluntarily mentioned that changing prices would "antagonize" or "cause difficulties" with their customers, and such a fact would be a strong reason why firms fear to adjust their prices. Indeed, the authors stressed that this issue "came up so often that figuring out precisely what it means should be a high-priority item on any future research agenda." Surprisingly, Hall et al. (1997) also stressed that fact that their surveyed firms "stated that physical menu costs of changing prices were a less important source of price rigidity than the need to preserve customer relationships". And at the same direction, Zbaracki et al. (2004) stated: "Changes in prices harmed the customer perceptions of the firm's reputation, integrity, and reliability."

Nowadays the mainstream in price stickiness macroeconomic modeling, whose main reference relays in Woodford (2003), incorporate adjusting costs, strategic complementarities measures and the presence of differentiated goods, on a monopolistic competition environment. Taking the real business cycle (RBC) analysis structure[3], those models focus on the agents optimization problems with intertemporal budget constrains and are so general that the neoclassical or new-Keynesian features are just a result of a particular relationship assumed between the basic preference and technology parameters. Those parameters define the degree of price setting strategic complementarity, among the suppliers of different goods, whose magnitude defines how sticky prices are.

Moreover, the majority of the existing analysis on price stickiness directly assumes a Calvo's (1983) type source of nominal rigidity or its extensions[4]. Briefly speaking, the simplest model state in *ad hoc* way that firms maintain unchanged their prices for two consecutive periods probability α, independently on the other firms' behavior. Such a modelling approach has been often used in monetary policy analysis for allowing a straightforward derivation of the central bank's loss function, as a second order approach of the welfare function, besides a good empirical adherence as well as an easy analytical treatment. As a matter of fact, Calvo's type models may be interpreted as stylized simplifications of the more plausible state dependent adjustment cost models[5], generating similar results with less analytical effort.

In spite of such appealing features, Calvo's type models have been subjected to some criticism due to the fact that the stochastic process is imposed into the model economy in a rather ad hoc way. Furthermore if firms have no adjustment costs, a Calvo's type economy with time-invariant inefficient shocks is not efficient, for under usual assumptions the adoption of flexible prices will be the optimal choice from both the firms and consumers point of view. Therefore, there are no reason why firms would rationally submit themselves to a Calvo's lottery. The case of time-varying inefficient shocks is still inconclusive, depending on whether consumers prefer a flexible price environment or not.

[2] They confirmed the empirical relevance of the theory in which "firms hold back on price changes, waiting for others to go first", e.g., in which multiple equilibria may arise from the interaction of menu costs and strategic complementarities in price setting. In second place, there were the theory referring to "delaying price increases until cost rise", pointing to some markup procedure on price setting.

[3] A good reference on several RBC models can be found in Barro and Sala-I-Martin (1995).

[4] Good references are Rotemberg and Woodford (1997), Rotemberg and Woodford (1998), Galí and Gertler (1999), Amato e Laubach (2000), Galí et al. (2001), Clarida et al. (2002), Woodford (2003), Giannoni e Woodford (2003), Woodford (2004), Galí and Monacelli (2004), Loyo e Vereda (2004), and Alves and Areosa (2005), among others.

[5] Good references are Caplin e Leahy (1991), Caballero and Engel (1993), Dotsey et al. (1999), Bonomo and Garcia (2001) and Bonomo and Almeida (2002), among others.

Rotemberg (2002), on the other hand, presented a model in which the probability α of not adjusting the prices for two periods is determined on an endogenous way. If consumers' utility functions have a psychological component, regarding the expected degree of firms' altruism, they strongly react to unfair price increases. Hence if consumers have imperfect information about the actual costs, firms will be unwilling to adjust prices so frequently due to the possibility of being interpreted as an unfair pricing setter by the consumers. The key point of this study is to regard the consumers' behavior as the source of price stickiness, as suggested by Blinder et al. (1998). However, his results apply only to unfair price increases, so consumers' aversion to price variations still remains to be carefully understood and analytically treated.

1.2 The paper approach

Within the Calvo's basic framework, the present study aims at build a model of pricing behavior in which the degree of the price rigidity is strategically chosen by profit maximizing firms, as an optimal decision to face consumer's risk aversion. Thus, the probability of not adjusting prices is endogenously determined. Moreover, some of the assumptions adopted in the basic Calvo's type model considered in Woodford (2003) are relaxed, allowing for the presence of several (unit mass) complete markets in the model economy. These markets are herein called *consumption centers*. We state that households do not assess the information about the actual state of the economy, in terms of the real firms' costs and prices, prior to each period consumption center choice. Once such a choice is made the actual state of the economy is revealed, but then consumers optimal shopping decisions are restricted to the elected consumption center[6]. In each of the following periods, new choices on consumption center are made in similar conditions.

In equilibrium, it will be shown that firms adopt a randomization strategy to decide when to adjust prices. Such an equilibrium will be found with traditional assumptions about consumers' preferences. As presented further on, price stickiness will be a consequence of the broadly accept assumption of consumers risk aversion, formalizing the research lacuna mentioned by Blinder et al. (1998). In such an environment, the price uncertainty of a flexible price economy decreases the consumers' expected discounted utility flow, so competitive firms adopt a price stickiness strategy as a best response in order to attract more clients. On the other hand, increasing price stickiness reduces the present value of firms expected profit flow. So equilibrium implies that firms increase the price nominal rigidity until the point in which the present value of firms expected profit flow is zero[7]. Consequently, our results represent a plausible solution to the unsolved problem of the case of firms facing time-varying inefficient cost shocks.

This paper is organized as follows. Section 2 presents the necessary modeling extensions to the Woodford's (2003) standard model, formally deriving the main result of this study, namely the implicitly defined degree of price stickiness in the model economy. Moreover, this section also presents original contributions to the theoretical analysis nominal rigidity sources. Section 3 presents Taylor approximations to the structural results derived in section 2 and introduces some related conclusions. Simulations on the endogenous degree of price stickiness and the volatility of the aggregate variables are also shown in this section. Finally, Section 4 concludes.

2 The Model

In this section, we introduce an extension to Calvo's type basic model. But now competitive firms strategically choose the degree of price stickiness, which in equilibrium depends on the

[6] Several shopping decisions can be modeled in such a fashion, as in supermarkets and malls, for instance.

[7] Such a result follows from the Bertrand flavor competition of the consumption centers.

economy deep parameters, namely the consumers' risk aversion and the ones related to the production function and the stochastic cost shocks distribution.

Furthermore, it depends on the way monetary policy is conducted. Thus, the Lucas's critique applies in this latter sense. But a similar critique also comes up. Adapting Woodford (2003) words: since price stickiness depends on the exogenous cost shock distribution, traditional monetary evaluation exercises using macroeconometric models are flawed by a failure to recognize that the relations typically estimated, even with quasi-structural equations containing future expectations derived with an ad hoc imposed nominal rigidity source, are reduced-form rather than truly structural relations, for structural changes in the stochastic cost shocks generating process may change the optimal degree of price stickiness chosen by the firms.

As in standard recent literature (see Woodford (2003) for more details), we model a cashless economy, in which there is a monetary unit of account in terms of which prices are quoted. This unit of account is defined in terms of a claim to a certain quantity of a liability of the central bank, which may or may not have any physical existence[8].

2.1 Households

In real world, purchasing decisions of great part of goods, as durables, are sufficiently sparse to allow enough time to gather price information before purchases are actually concluded. Thus traditional assumptions stating that consumers know all the prices before consuming is quite a good description of reality. Such an economic decision is exhaustively modeled and its consequences are well understood.

But there are situations in which such a premise does not work so well. Consumers frequently face the following recurrent questions: which shopping mall should I choose? Or which supermarket? People's habitual behavior is to choose a supermarket before knowing the actual prices, only effectively known when walking through its rows. And doing so, empirical evidence points that after choosing a place to buy, consumers restrict their purchasing decisions only to the goods found in the elected market.

Therefore, the following question arises: how to incorporate such a decision pattern in formal analytical models? And what are the consequent optimal agents decisions?

In an effort to answer such a question, we assume the existence of several complete markets, or *consumption centers* ($\mathbb{C}_j$, henceforth), indexed by j. In each one, monopolist firms i hire specialized labor force $h_{j,t}(i)$ at nominal wage $w_{j,t}(i)$ and produce differentiated goods i. As usual, we assume that $i \in (0,1)$ in a unit mass continuum and that individual firm's decisions have no influence on wages. Each market is then characterized by monopolistic competition. We also assume that firms are subjected to exogenous cost shocks, formalized further on, but none of them are subjected to price adjustment costs.

[8] As analytically shown in Woodford (2003), such an approach is justified by two facts:

(a) In an economy in which the central bank uses a short-term nominal interest rate as their instrument, often empirically characterized by central bank reaction functions as Taylor type rules, the old theoretical models considering money growth targets are not convenient since it is not necessary to first determine the endogenous evolution of money supply in order to understand the consequences, in terms of product, inflation and welfare, of such interest rate rules. Money, prices and interest rates are rather simultaneously determined given a central bank reaction function;

(b) In an economy in which households optimally choose to hold money balances in order to reduce transaction frictions, frequently modeled including real balances in the utility function or assuming cash-in-advance constraint, equilibrium relations are direct generalizations of those for the cashless economy. However, since its quantitative results are not too different if monetary frictions are parameterized in an empirically plausible way, a cashless analysis is a useful simplification.

Here, we consider markets transacting non-durable goods, so that the purchase decisions happen with high frequency[9]. Given the great number of goods and given the decision frequency, it is not reasonable to consider that consumers are informed of all the prices prior to each period market choices. Not because of information cost, but due to the fact that the period length between consecutive consumption decisions is lower than the necessary to memorize make optimal decisions based on the huge information set[10].

Therefore, the above consideration leads us to assume that the consumers' buying decision is based on historical data on prices. In other words, we assume that the consumers know the historical average pricing strategy adopted by each firm. In general, this information can be summarized by indexes such as price averages, price volatility and so on.

Even though it seems to be a strong assumption, it captures the observed consumers behavioral pattern previously exemplified. For illustrative purposes, we can take the traditional grocery shops as examples of our model's consumption centers. Each item[11] is sufficiently differentiated and they all are diversified. Another example would be the set of large shopping malls, which gather several differentiated firms.

For simplification purposes, we build a model in an environment with only two consumption centers[12]. Due to such considerations, we make the model's primary assumption:

Assumption 1 *In every period, preceding the choice of a consumption center, only historical price patterns are households common knowledge. Hence, they choose a consumption center before they have the information about the prevailing prices of the chosen center. Once this choice is made, their consumption decisions are restricted to the chosen market.*

Furthermore, it is important to make use of some tools from game theory, in particular some concepts and their rationale, for they explicitly handle the agents' rationality. Indeed, the microfounded macroeconomic rational expectations equilibrium concept have their peer in game theory sub-game perfect equilibrium concept, for embedding the same rationale of backward inductions methodological algorithm: rational expectations optimal decisions in period t are agents' best responses, given the best responses to be made in the future.

Under certain assumptions, described in Woodford (2003), we may use the concept of a representative household. In order to characterize its preferences, we define $u(\cdot)$ and $\nu(\cdot)$ denoting the consumption utility and labor disutility respectively[13]. It is convenient to make some regularity assumptions:

Assumption 2 *The domains of $u(\cdot)$ and $\nu(\cdot)$ are strictly positive*[14]*, in other words $u, v : (0, +\infty) \to \mathbb{R}$.*

[9] Hence, the model is not proposed to explain the whole economy, but only specific sectors.

[10] Even with computer assistance to find which firms are cheaper, time would still be an issue, due to the length of time required by price researches to catalog and release price information.

[11] Since the model assumes an infinite number of agents, one may argue that the real world finite number of agents may flaw the model results. Nevertheless, a known result of Debreu (1975) states that a Walras equilibrium convergence rate to the core, in regular economies, is of order O(1/n). Since Walras equilibria have a finite number of agents in spite of the infinite number of agents of the core, one may conjecture that the problem concerning the number agents may be minimized at least as fast as the actual number of agents.

[12] However, the analytical treatment and results can be easily expanded for the case of several consumption centers.

[13] Note that they are not subject to preference shocks, as in traditional literature. Also, we assume further on the absence of technology shocks in the production function. Such assumptions aim only to simplify the analysis allowing us to better understand the consequences of inefficient time-varying exogenous shocks hitting firms' marginal costs, formally introduced in subsection 2.2. And due to this last disturbance source, the model distinguishes the concepts of natural and steady state products, formally defined in subsection 2.2.1.

[14] Economic modeling usually assumes that equilibrium consumption and labor force are not zero. Therefore, such an assumption does not restrict the model results.

Assumption 3 *The function consumption utility* $u(\cdot)$ *is increasing in consumption, strictly concave, and its third derivative satisfies* $u_{CCC}(\cdot) > 0$ *in its domain. Furthermore, the function labor disutility* $\nu(\cdot)$ *is increasing in labor and strictly convex in its domain.*

According to Assumption 1, consumption decisions are restricted to the chosen $\mathbb{C}_j$. Therefore, we may aggregate consumption in such a consumption center considering the Dixit and Stiglitz (1977) standard way, which assumes a constant elasticity of substitution $\theta > 1$ among the differentiated transitioned goods, as shown in equation (1) below, where $c_{j,t}(i)$ indicates the consumption of good i from $\mathbb{C}_j$, in period t.

$$C_{j,t} \equiv \left[\int_0^1 c_{j,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad , \quad \forall j \in \{1,2\} \tag{1}$$

In each period t, after choosing a particular $\mathbb{C}_j$, households gets the instantaneous consumption utility $u(C_{j,t})$, e.g. no matter how $C_{j,t}$ is distributed among each good i from $\mathbb{C}_j$, only the aggregate consumption in the consumption center is important in terms of preference issues[15].

One can easily derive[16] the hicksian demand function of good i from $\mathbb{C}_j$ and an expression for the aggregate price $P_{j,t}$ in each consumption center, as shown in (2) and (3), below.

$$c_{j,t}(i) = C_{j,t} \left(\frac{p_{j,t}(i)}{P_{j,t}} \right)^{-\theta} \tag{2}$$

$$P_{j,t} = \left[\int_0^1 p_{j,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{3}$$

Where $P_{j,t}$ satisfies $P_{j,t}C_{j,t} = \int_0^1 p_{j,t}(i)\, c_{j,t}(i)\, di$.

In order to capture either deterministic or stochastic (randomizations) choices among each consumption center, define γ as the probability[17] of choosing the consumption centar $\mathbb{C}_1$. This modeling procedure allows, in turn, to capture deterministic choices by $\gamma = 0$ or $\gamma = 1$, as the events in which the household always chooses $\mathbb{C}_1$ or $\mathbb{C}_2$, respectively. Moreover, this probabilistic treatment allows for possible randomizations, without the need of modifying the corresponding expressions.

As a consequence of Assumption 1, the choice of the consumption center can be interpreted as a choice among lotteries, with the corresponding pay off's depending on the prevailing prices found at the chosen center. Hence, the aggregate consumption C_t from both consumption centers must satisfy the equality (4), e.g. C_t is the equivalent consumption, under absence of uncertainty on the lottery choice, which generates the same utility level as the one measured by the expected utility. However, C_t is not a certainty equivalent aggregate consumption, for it is still a random variable due either to the uncertainty regarding the prices found in each consumption center and to the other random variables present in the model economy.

$$u(C_t) = E_\gamma u(C_{j,t}) = \gamma \cdot u(C_{1,t}) + (1-\gamma) \cdot u(C_{2,t}) \tag{4}$$

For simplification sake, we assume as well that the representative household supply labor only at the chosen consumption center $\mathbb{C}_j$. Thus we aggregate the labor force $h_{j,t}(i)$ supplied in each $\mathbb{C}_j$ to produce good i by the same way we did with the aggregate consumption, for there are similar uncertainties as the previous considered ones. Therefore we aggregate the amount of labor force as indicated below in equation (5).

[15] Such an assumption is very in line with the one adopted in recent literature.

[16] Similar results are standard in recent literature, so the analytical derivation is not shown here.

[17] In future extensions, one may consider a time-varying parameter γ_t instead.

$$v\left(h_t(i)\right)=\gamma\cdot v\left(h_{1,t}(i)\right)+(1-\gamma)\cdot v\left(h_{2,t}(i)\right) \tag{5}$$

We may now define P_t and $w_t(i)$ denoting the aggregate price and the aggregate wage of labor force of type i among all consumption centers in period t, respectively, satisfying the following relations:

$$P_tC_t=\gamma\cdot P_{1,t}C_{1,t}+(1-\gamma)\cdot P_{2,t}C_{2,t} \tag{6}$$

$$w_t(i)\,h_t(i)=\gamma\cdot w_{1,t}(i)\,h_{1,t}(i)+(1-\gamma)\cdot w_{2,t}(i)\,h_{2,t}(i) \tag{7}$$

As standard, we assume that financial assets are evenly shared among all households in period zero, so complete markets imply in identical budget restrictions for every household. Moreover, define W_t as the nominal financial wealth held by the household in the beginning of period t, $Q_{t,t+1}$ as the stochastic discounting factor that must exist under absence of arbitrage, β as the preference intertemporal discounting factor, i_t is the nominal interest rate satisfying $(1+i_t)^{-1}=E_tQ_{t,t+1}$ and $\Pi_t(i)$ as the nominal profit from selling each good i. Also define $Q_{t,\tau}=\prod_{s=t+1}^{\tau}Q_{s-1,s}$.

Regarding the representative household, note that after choosing a $\mathbb{C}_j$ in each period t, the prevailing prices of the chosen center are known. However, expectations regarding future consumption and prices are summarized by C_τ and P_τ, for $\forall\tau>t$, for the future choices on consumption centers are still lottery choices. Hence, the household problem can be formally represented by (8) and its solution depends on the non-ponzi constraint (9).

$$\begin{array}{ll}\max\limits_{\substack{\{C_{j,t}\,,\,h_{j,t}(i)\}\\ \{C_\tau\,,\,h_\tau(i)\}}} & \left(u\left(C_{j,t}\right)-\int_0^1\nu\left(h_{j,t}(i)\right)di\right)+E_t\sum\limits_{\tau=t+1}^{\infty}\beta^{\tau-t}\left(u\left(C_\tau\right)-\int_0^1\nu\left(h_\tau(i)\right)di\right)\\ s.t. & \begin{array}{l}P_{j,t}C_{j,t}+E_\tau\left[Q_{t,t+1}W_{t+1}\right]\leq W_t+\int_0^1\left(w_{j,t}(i)\,h_{j,t}(i)+\Pi_t(i)\right)di\\ P_\tau C_\tau+E_\tau\left[Q_{\tau,\tau+1}W_{\tau+1}\right]\leq W_\tau+\int_0^1\left(w_\tau(i)\,h_\tau(i)+\Pi_\tau(i)\right)di\ ,\ \forall\tau>t\end{array}\end{array} \tag{8}$$

$$W_t+\int_0^1\left(w_{j,t}(i)\,h_{j,t}(i)+\Pi_t(i)\right)di+E_t\sum_{\tau=t+1}^{\infty}Q_{t,\tau}\left[\int_0^1\left(w_\tau(i)\,h_\tau(i)+\Pi_\tau(i)\right)di\right]\geq 0 \tag{9}$$

Denote by $Y_{j,t}$ and Y_t aggregate production levels to be further discussed. Considering that all production must be consumed in equilibrium for every period, e.g. $C_{j,t}=Y_{j,t}$ and $C_t=Y_t$, it is straightforward to solve the problem (8) and obtain the standard shaped Euler equations (10) and (11).

$$\frac{u_C\left(Y_{j,t}\right)}{P_{j,t}}=\beta E_t\left[\left(Q_{t,t+1}\right)^{-1}\frac{u_C\left(Y_{t+1}\right)}{P_{t+1}}\right] \tag{10}$$

$$\frac{u_C\left(Y_\tau\right)}{P_\tau}=\beta E_\tau\left[\left(Q_{\tau,\tau+1}\right)^{-1}\frac{u_C\left(Y_{\tau+1}\right)}{P_{\tau+1}}\right]\quad,\ \forall\tau>t \tag{11}$$

Note that in period $t-1$, the following equation would hold as a consequence of (11).

$$\frac{u_C\left(Y_t\right)}{P_t}=\beta E_t\left[\left(Q_{t,t+1}\right)^{-1}\frac{u_C\left(Y_{t+1}\right)}{P_{t+1}}\right] \tag{12}$$

Purging equations (10) and (12), we present the new generalized Euler equation (13) associated to our consumption center economy:

$$\frac{u_C(Y_t)}{P_t} = \frac{u_C(Y_{1,t})}{P_{1,t}} = \frac{u_C(Y_{2,t})}{P_{2,t}} = \Lambda_t \tag{13}$$

Where

$$\Lambda_t = \beta E_t \left[(Q_{t,t+1})^{-1} \frac{u_C(Y_{t+1})}{P_{t+1}} \right] \tag{14}$$

The last results lead us to an interesting interpretation. Even after the choice of a consumption center, the expectations about future aggregate consumption and prices do not change and are the same as the ones prevailing just before that choice. Furthermore the rationale of optimal current consumption planning, as a function of current aggregate price, remain unchanged even after the choice of a center.

Considering the rationale of backward induction, since households actually know they will optimally behave in period $t+1$, optimal decisions in period t are made assuming that the expectation term at the right hand side of (13) is given. It means that such a term do not depend on contemporaneous decisions[18]. Therefore, we state the following remark:

Remark 1 *The expectation term at the right hand side of the previously depicted consumption generalized Euler equation is not a function of contemporaneous decisions.*

Another important first order condition that solves the problem (8) is the expression (15) for the real wage $w_t(i)/P_t$ of type i.

$$\frac{w_t(i)}{P_t} = \frac{v_h(h_t(i))}{u_C(Y_t)} \tag{15}$$

Moreover, it is not difficult to verify that the next relation must hold:

$$\frac{v_h(h_t(i))}{w_t(i)} = \frac{v_h(h_{1,t}(i))}{w_{1,t}(i)} = \frac{v_h(h_{2,t}(i))}{w_{2,t}(i)} \tag{16}$$

2.2 Firms

As usual in this type of modeling, we assume that each firm is specialized in the production of a unique good i, holding monopoly of its production, in an environment of monopolistic competition. Furthermore, the only input of each firm is the specialized labor force. In addition, some other simplifying assumptions are made[19].

Assumption 4 *Firm are price takers, regarding the nominal wage $w_{j,t}(i)$, in the labor market.*

Assumption 5 *Even though there is a committed price in each period, if there is no demand for a good i, the firm i will make no expenses.*

We assume a "just in time" process of inputs supplying, so that the elapsed time between producing and supplying is negligible. Therefore, firms do not need to anticipate the production decision, e.g. $y_{j,t}(i) = c_{j,t}(i)$, $\forall j \in \{1,2\}$ and $\forall t \geq 0$.

[18] Such a conclusion is standard and is simply a consequence of the Euler equation, for it implies that contemporaneous decisions depend only on the expectation concerning the future, not the past. Therefore, expectations on future optimal choices are not affected by contemporaneous decisions. If instead households had habit persistence as in some modeling approaches, contemporaneus consumption decisions would depend on the past, implying that expectations on future optimal choices would be affected by contemporaneous decisions.

[19] Such simplifying assumptions can be weakened in future extensions.

Assumption 7 *There is a steady state*[20] *level for prices*[21]*, namely* $\overline{P}$*. However, the distribution of the stochastic process generating the exogenous shocks can vary.*

Thus, if such a shock term follows an autoregressive process it is possible that prices remain above, or below, its stationary level for an arbitrarily long length of time, allowing for persistent inflation. However, the assumption on $\overline{P}$ implies that inflationary periods will be followed by deflationary ones.

Assumption 8 *The unconditional distribution of all the random variables considered in this model economy is time stationary.*

Before presenting our equilibrium analysis, we formally characterize the outcomes of three types of possible environments: (a) the standard flexible price environment, for the outcomes under most price equilibria are better understood when compared with the former; (b) the efficient producing and the steady state producing environments; and (c) the standard sticky price environment. In the latter environment, we show some useful results in order to conduct our equilibrium analysis. In particular, we show that the unconditionally expected profits of competing firms under a standard sticky price environment decreases with the degree of price rigidity.

2.2.1 Flexible prices

Let $Cost_{j,t}(i)$ be the total cost of firm i from $\mathbb{C}_j$ in the period t. Since the produced good is differentiated, and given the assumption that, once a consumption center is chosen, households cannot go to another one until the next period, the firm i is subject to monopolistic competition.

Let then γ_j be the consumer's probability of choosing $\mathbb{C}_j$, e.g. $\gamma_1 = \gamma$ and $\gamma_2 = 1 - \gamma$. With such a notation, we formalize the problem of the firm i from $\mathbb{C}_j$ as to maximize its expected profit in each period subject to the demand curve (2), e.g.

$$\begin{array}{cl} \max\limits_{\{p_{j,t}(i)\}} & \Pi_{j,t}(i) = \gamma_j \left[p_{j,t}(i)\, y_{j,t}(i) - Cost_{j,t}(i) \right] \\ s.t. & y_{j,t}(i) = Y_{j,t} \left(\frac{p_{j,t}(i)}{P_{j,t}} \right)^{-\theta} \end{array} \tag{17}$$

Optimal solution implies that $p^*_{j,t}(i)$ is determined with a markup μ over the nominal marginal cost $S^{\varepsilon}_{j,t}(i)$, as expected due to the monopolistic competition environment, e.g.

$$p^*_{j,t}(i) = \mu\, S^{\varepsilon}_{j,t}(i) \tag{18}$$

Where $S^{\varepsilon}_{j,t}(i) = \frac{\partial Cost_{j,t}(i)}{\partial y_{j,t}(i)}$ and $\mu = \frac{\theta}{\theta - 1} > 1$.

We assume that each firm i from $\mathbb{C}_j$ have that same production function as shown in (19), so that its only input is the labor force $h_{j,t}(i)$.

$$y_{j,t}(i) = A\, f\left(h_{j,t}(i) \right) \tag{19}$$

Where the parameter A denotes the average production technology used by firms from all consumption centers, and $f(\cdot)$ satisfies the following assumptions:

[20] We define the steady state as the equilibrium environment that would occur if all exogenous random variables remain fixed in their expected values.

[21] Such an assumption is not too strong, since several works in the literature concludes that the optimal monetary policy rule is to target a fixed price level. Moreover, our model conclusions are consistent with such an assumption.

Assumption 9 *The domain of $f(\cdot)$ is strictly positive.*

Assumption 10 *The function $f(\cdot)$ is strictly increasing in labor force and strictly concave.*

We assume that each firm faces a total cost function represented in (20) below.

$$Cost_{j,t}(i) = w_{j,t}(i)\, h_{j,t}(i) + y_{j,t}(i)\, P_{j,t} \cdot \varepsilon_t \tag{20}$$

Where ε_t is a time-varying exogenous shock such that:

$$\varepsilon_t = \overline{\varepsilon} + \xi_t^{\varepsilon} \tag{21}$$

Where ξ_t^{ε} are *i.i.d.* for all t with $E(\xi_t^{\varepsilon}) = 0$.

We define $s_{j,t}^{\varepsilon}(i)$ and $s_{j,t}(i)$ as the real marginal cost and the real labor marginal cost, respectively, e.g. $s_{j,t}^{\varepsilon}(i) = \frac{S_{j,t}^{\varepsilon}(i)}{P_{j,t}}$ and $s_{j,t}(i) = \frac{1}{P_{j,t}} \frac{\partial(w_{j,t}(i) h_{j,t}(i))}{\partial y_{j,t}(i)}$.

Since firms are price takers in labor market, we may derive the following expression for $s_{j,t}(i)$ considering the equations for real wages (15) and for the production function (19):

$$s_{j,t}(i) = s(y_{j,t}(i), Y_{j,t}) = \frac{v_h\left[f^{-1}\left(A^{-1} y_{j,t}(i)\right)\right]}{u_C(Y_{j,t})} \frac{\Psi\left(A^{-1} y_{j,t}(i)\right)}{A} \tag{22}$$

Where $\Psi(y) = \frac{\partial f^{-1}(y)}{\partial y}$.

Note that we may represent the real marginal cost $s_{j,t}^{\varepsilon}(i)$ as follows:

$$s^{\varepsilon}(y_{j,t}(i), Y_{j,t}; \varepsilon_t) = s(y_{j,t}(i), Y_{j,t}) + \varepsilon_t \tag{23}$$

Now, given the demand equation (2), we may rewrite (18) as follows:

$$\left(\frac{y_{j,t}(i)}{Y_{j,t}}\right)^{-\frac{1}{\theta}} = \mu\, s^{\varepsilon}(y_{j,t}(i), Y_{j,t}; \varepsilon_t)$$

Note that given the regularity properties of preferences and production function, all firms from every consumption center optimally choose the same production level in a flexible price equilibrium, which must equal the aggregate production, e.g. $y_{j,t}(i) = Y_t$. Moreover, given the demand equation (2), they all choose the same optimal price, which must equal the aggregate price in $\mathbb{C}_j$, e.g. $p_{j,t}^{*}(i) = P_{j,t}^{*}$. Moreover, due to the symmetry among the consumption centers, it is not difficult to verify that $P_{1,t}^{*} = P_{2,t}^{*} = P_t^{*}$. So we state the following definitions:

Definition 1 *The (time-varying) natural product Y_t^n, the equilibrium aggregate production level prevailing in all consumption centers under a fully flexible prices environment, is implicitly defined by relation* (24).

$$\mu\, s^{\varepsilon}(Y_t^n, Y_t^n; \varepsilon_t) = 1 \tag{24}$$

Definition 1 implies a money neutrality regarding the natural product, for it is independent on monetary policy. Furthermore, considering the above equation (23) we may rewrite (24) as follows and conclude that the exogenous shock is a very relevant variable, for it determines a time-varying markup μ_t applied over the nominal labor marginal cost under a flexible price equilibrium price setting, e.g.

$$\left(\mu^{-1} - \varepsilon_t\right)^{-1} s(Y_t^n, Y_t^n) = 1 \tag{25}$$

or equivalently,

$$P_t^{*} = \mu_t S(Y_t^n, Y_t^n)$$

Where $\mu_t = \left(\mu^{-1} - \varepsilon_t\right)^{-1}$.

2.2.2 Efficient and steady state productions

Consider now the efficient output level Y^e that maximizes the representative household's instantaneous utility $\left[u\left(C_t\right)-\int_0^1 \nu\left(h_t\left(i\right)\right)di\right]$ in a given period t. It is easy to conclude that the first order condition satisfies the following equation. Due to the absence of any time-varying term, the efficient production shall be time-invariant.

$$\frac{v_h\left[f^{-1}\left(A^{-1}Y^e\right)\right]}{u_C\left(Y^e\right)}\frac{\Psi\left(A^{-1}Y^e\right)}{A}=1$$

Note that the left hand side of the previous result is an equivalent representation of the real labor marginal cost $s_{j,t}\left(i\right)$ evaluated in the efficient level of production. Therefore, we state the following definition.

Definition 2 *The (time-invariant) efficient product Y^e, the equilibrium aggregate production level that maximizes the representative household's instantaneous utility, is implicitly defined by relation (26).*

$$s\left(Y^e,Y^e\right)=1 \tag{26}$$

Comparing equations (25) and (26), we conclude that Y_t^n equals Y^e only if $\left(\mu^{-1}-\varepsilon_t\right)=1$, e.g. in an event of measure zero for practical purposes. Thus, it is expected the natural product to be inefficient in general[22].

Regarding the steady state production level, one easily shows that it must satisfy the following definition:

Definition 3 *The steady state product $\overline{Y}$, the equilibrium aggregate production level prevailing in all consumption centers if the shock term ε_t remains fixed in its mean $\overline{\varepsilon}$ in all periods, is implicitly defined by relation (27).*

$$\mu\, s^{\varepsilon}\left(\overline{Y},\overline{Y};\overline{\varepsilon}\right)=1 \tag{27}$$

Since (27) may be rewritten as $s\left(\overline{Y},\overline{Y}\right)=\left(\mu^{-1}-\overline{\varepsilon}\right)$, the efficiency of the steady state product depends on the value of the parameter $\overline{\varepsilon}$. As standard, we assess its inefficiency degree considering the parameter $\phi_y\leq 1$, implicitly defined as follows, so that the steady state product is efficient if and only if $\phi_y=0$.

$$s\left(\overline{Y},\overline{Y}\right)=1-\phi_y$$

Considering our model features, the inefficiency degree parameter ϕ_y may be defined as in (28) below.

$$\phi_y=1-\left(\mu^{-1}-\overline{\varepsilon}\right) \tag{28}$$

Note that our model has possibly two sources of inefficiency: (a) the monopolistic power of firms, captured by the price markup μ; and (b) the cost shock, captured by its average $\overline{\varepsilon}$. Therefore, in order to correct the inefficiency sources and make $\overline{Y}$ efficient the model economy needs a time-varying subsidy, for $\phi_y=0$ if and only if $\overline{\varepsilon}_{ef}=-\theta^{-1}<0$, e.g. the cost shock must actually be a subsidy averaging the inverse of the elasticity of substitution among the differentiated goods.

[22] Thus, our time varying exogenous shock generates inefficiencies in the same way the standard time varying nominal income tax does.

2.2.3 Sticky prices

Consider the standard assumption in which a particular firm adjusts its price in period t with the timeless probability $(1-\alpha)$, within the staggered pricesetting framework of Calvo's (1983) nominal rigidity structure. Denote by $\bar{p}_{j,t}(i)$ the new price if the firm adjusts in period t. Thus the probability α of not readjusting is the firm's measure of price stickiness. Note that the situation in which the firm always chooses flexible prices is modeled by $\alpha = 0$.

Therefore, considering that some properties of uniform convergence apply, the firm's expected sum of profit flow $\Pi^d_{j,0}(i)$ discounted at period $t=0$ may be represented as in (29) for $\alpha < 1$. Details are in Appendix A.1.

$$\begin{aligned}\Pi^d_{j,0}(i) &= E_0 \sum_{t=0}^{\infty} \gamma_j \Bigg[\alpha^{t+1} Q_{0,t} \Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi^{\varepsilon}_t\right) + \\ &\qquad + (1-\alpha) \sum_{T=t}^{\infty} \alpha^{T-t} Q_{0,T} \Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi^{\varepsilon}_T\right)\Bigg] \qquad (29)\end{aligned}$$

Where

$$\Pi\left(p_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi^{\varepsilon}_T\right) = p_{j,t}(i)\, y_{j,T}(i) - \left[w_{j,T}(i)\, h_{j,T}(i) + y_{j,T}(i)\, P_{j,T} \cdot \varepsilon_T\right]$$

Due to its monopolistic power, the profit-maximizing firm i must choose an optimal price sequence $\left\{p^*_{j,t}(i)\right\}_{t=0}^{\infty}$ that maximize (29) given its demand equation (2). Note that such a problem is separable into several independent simpler problems like (30), one for each branch on the possibility tree, depicted in Figure 4 of Appendix A.1, regarding the event "the firm adjust its price in period t, once for good".

$$\begin{array}{ll} \max\limits_{\{\bar{p}_{j,t}(i)\}} & E_t \sum\limits_{T=t}^{\infty} \gamma_j \alpha^{T-t} Q_{t,T} \Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi^{\varepsilon}_T\right) \\ s.t. & y_{j,t}(i) = Y_{j,t} \left(\frac{p_{j,t}(i)}{P_{j,t}}\right)^{-\theta} \end{array} \qquad (30)$$

Optimal solution implies:

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left.\frac{\partial \Pi_{j,t,T}(i)}{\partial \bar{p}_{j,t}(i)}\right|_{\bar{p}_{j,t}(i) = p^*_{j,t}(i)} = 0 \qquad (31)$$

Where $\Pi_{j,t,T}(i) = \Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi^{\varepsilon}_T\right)$ and:

$$\left.\frac{\partial \Pi_{j,t,T}(i)}{\partial \bar{p}_{j,t}(i)}\right|_{\bar{p}_{j,t}(i) = p^*_{j,t}(i)} = (1-\theta)\, Y_{j,T} \left[\left(\frac{p^*_{j,t}(i)}{P_{j,T}}\right)^{-\theta} - \mu \left(\frac{p^*_{j,t}(i)}{P_{j,T}}\right)^{-(1+\theta)} \cdot s^{\varepsilon}_{j,T}(i)\right]$$

Note that the optimal price $p^*_{j,t}(i)$ is a continuous function[23] of α, e.g. $p^*_{j,t}(i) = p^*_{j,t}(\alpha)$. From now on we assess some properties of the firm's profit flow value-function $\Pi^*_j(\alpha)$, the profit flow function discounted at period $t=0$ evaluated at the optimal prices $\left\{p^*_{j,t}(\alpha)\right\}_{t=0}^{\infty}$.

[23]Given the assumed regularity hypothesis regarding the functions at hand, the first order condition (31) implicitly defines an unique solution $p^*_{j,t}(i)$.

In the equilibrium analysis conducted in next section, we search for an equilibrium in which firms optimally decide a timeless[24] price rigidity degree to be maintained in all periods, e.g. firms decide on the stickiness degree before knowing the future realizations of random variables. Hence, such decisions must be based on unconditional expectations.

Note that the cost shock may lead to negative profits in some periods. Therefore, we state the following assumption on the necessary condition of market existence, where $E_{\xi^\varepsilon}\Pi_j^*(\alpha)$ denotes the unconditional expected value, in ξ^ε, of $\Pi_j^*(\alpha)$.

Assumption 11 *If the firm i from $\mathbb{C}_j$ is present in the market then there exists a non-zero probability measure for the households to choose this market, e.g. $\gamma_j > 0$, and its unconditional expected profit flow $E_{\xi^\varepsilon}\Pi_j^*(\cdot)$ is a continuous and non-negative function of α.*

Moreover, one easily conjectures that a particular firm maximizes $\Pi_j^*(\alpha)$ when choosing a flexible price strategy, e.g. $\alpha = 0$, for its expected profit $\Pi_{j,t}(i)$ is maximized period-to-period as previously shown in (17). Thus $E_{\xi^\varepsilon}\Pi_j^*(\cdot)$ is also optimized when $\alpha = 0$. Since higher values of α imply in stronger restrictions[25] to the firm's optimization problem (30), one expects that $E_{\xi^\varepsilon}\Pi_j^*(\alpha)$ is a decreasing function of its argument. The following statement formalizes such a conjecture:

Proposition 1 *Under a timeless perspective, if a particular firm i from $\mathbb{C}_j$ is present in the market then its expected profit $E_{\xi^\varepsilon}\Pi_j^*(\alpha)$ is a decreasing function of the degree of nominal price rigidity summarized by the probability α.*

The proof is given in Appendix A.2.

Corollary 1 *If a particular firm i is present in the market then $E_{\xi^\varepsilon}\Pi_j^*(0) \geq 0$.*

The proof is straightforward once $E_{\xi^\varepsilon}\Pi_j^*(0) \geq E_{\xi^\varepsilon}\Pi_j^*(\alpha)$, $\forall\alpha \in [0,1]$.

We next assess the case in which the firm adopts the probability α of price stickiness, but when adjusting the firm decides instead for the sub-optimal price[26] $p_{j,t}^*(\bar{\alpha})$, where $\bar{\alpha} \leq \alpha$. Such a case is relevant, for we consider it when testing best responses in the further discussed equilibrium analysis.

In such a context, we define $\Pi_j^d(\alpha, p^*(\bar{\alpha}))$ as the profit flow function of a firm that readjusts with probability $(1-\alpha)$, discounted at period $t=0$ and evaluated at the sub-optimal prices $\left\{p_{j,t}^*(\bar{\alpha})\right\}_{t=0}^\infty$, where $\bar{\alpha} \leq \alpha$. Formally, $\Pi_j^d(\alpha, p^*(\bar{\alpha}))$ satisfies:

$$\begin{aligned}\Pi_j^d(\alpha, p^*(\bar{\alpha})) &= E_0\sum_{t=0}^{\infty}\gamma_j\Bigg[\alpha^{t+1}Q_{0,t}\Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^\varepsilon\right) + \\ &\quad + (1-\alpha)\sum_{T=t}^{\infty}\alpha^{T-t}Q_{0,T}\Pi\left(p_{j,t}^*(\bar{\alpha}), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^\varepsilon\right)\Bigg] \qquad (32)\end{aligned}$$

Now, we make the following statement:

[24] Under such a timeless perspective, we mean that a particular firm has always chosen the same price stickiness degree α, and has always behaved the same way, even before the initial period $t=0$. Such a time consistent approach is standard and "tastes" like the Woodford's (2003) strategy in deriving the time consistent optimal monetary rules.

[25] Since prices remain fixed for about $\alpha/(1-\alpha)$ periods, on average, the restriction works almost as if higher values of α "increased" the number of restrictions of the type "$p_{j,t}(i) = p_{j,t-1}(i)$".

[26] Note that $p_{j,t}^*(\bar{\alpha})$ would be the optimal price if the firm adjusted with probability $(1-\bar{\alpha})$ instead.

Proposition 2 *Under a timeless perspective, suppose that the i-th firm from $\mathbb{C}_j$ is present in the market and adopts the probability α of price stickiness, but when adjusting it decides instead for the sub-optimal price $p^*_{j,t}(\bar{\alpha})$, where $\bar{\alpha} \leq \alpha$. In such a context the unconditional expectance of $\Pi^d_j(\alpha, p^*(\bar{\alpha}))$, previously defined in* (32)*, satisfies the following inequality:*

$$E_{\xi^\varepsilon}\Pi^d_j(\alpha, p^*(\bar{\alpha})) \leq E_{\xi^\varepsilon}\Pi^*_j(\bar{\alpha}) \quad \text{for} \quad \bar{\alpha} \leq \alpha \tag{33}$$

The proof is given in Appendix A.3.

Therefore, a profit-maximizing firm that optimally readjust its price with probability $(1-\bar{\alpha})$ have its expected profit decreased when increasing its price stickiness degree to $\alpha \geq \bar{\alpha}$ even when readjusted prices are $\left\{p^*_{j,t}(\alpha)\right\}_{t=0}^{\infty}$ instead of $\left\{p^*_{j,t}(\bar{\alpha})\right\}_{t=0}^{\infty}$.

We next search for a particular equilibrium in which all firms of a particular consumption center endogenously choose the same time-invariant price stickiness degree. Such an equilibrium is interesting for the standard literature assumes that all firms are identical regarding their exogenously given nominal price rigidity degree, even if no adjusting costs apply. Our approach leads to an endogenous price stickiness degree as an optimal strategy of competing firms. We stress the fact that such a result follows from the traditionally assumed consumer risk aversion, so it applies even in an economy where firms have no adjusting costs.

2.3 The equilibrium

In line with the former assumption on distributions stationarity, we focus our analysis in searching for equilibrium outcomes in which agents' decisions are also time stationary regarding the firms' choices on the degree of price stickiness. Although there should be other equilibria with idiosyncratic time-varying parameter $\alpha_{j,t}(i)$, our choice for such a specific equilibrium simplifies our analysis while still allowing for a broadening of the understanding of the sources of nominal rigidities. Moreover, such an equilibrium is in line with the basic standard approach in which the degree of nominal rigidity is time-invariant.

In order to simplify the argument, we may consider that a coalition is formed among all the firms from each $\mathbb{C}_j$, so that they all decide to adopt the same degree α_j of price stickiness. Such coalition is formed for long-run reputation purposes, and its plausibility depends on a mechanism that penalizes each firm that refuses to adopt the group strategy. Indeed, consider the case in which no penalizing mechanism is created. Since every firm knows that its strategy has no influence over aggregate variables, they will choose the "free rider" flexible price strategy, for it maximizes their individual profits. Thus the existence of such a coalition depends on the penalizing mechanism. However such a coalition strategy permits a competitive advantage over the strategies adopted by the firms from the other consumption center, as will be shown further on. Therefore such a coalition assumption is not so strong. We abstract from issues concerning the coalition, such as the coalition central planner, and focus on its consequences.

Formalizing our arguments we define the equilibrium we search for:

Definition 4 *The equilibrium of the above model economy consists on a set of dynamic equations characterizing the agents optimal behavior and a set of endogenously determined probability measures, such as the timeless degrees of nominal rigidity α_j adopted by each $\mathbb{C}_j$, and also the timeless probabilities γ_j of choosing $\mathbb{C}_j$, both consistent with the agents solutions to their inter-temporal maximization problem.*

Note that the previous definition implies that all the firms, from the same $\mathbb{C}_j$, that readjust prices in period t choose the same optimal price $p^*_t(\alpha_j)$. Note that the optimal price ultimately depends only on the chosen price stickiness degree, so if both consumption centers choose

$\alpha_1 = \alpha_2 = \alpha$, every firm that optimize in period t choose the same optimal price $p_t^*(\alpha)$, regardless the consumption center they belong to. However, if the consumption centers adopt different degrees of price stickiness, e.g. in case of $\alpha_1 \neq \alpha_2$, the readjusted prices will differ from center to center.

Thus the aggregate price $P_{j,t}$ of each $\mathbb{C}_j$, defined in (3), may now be represented by the following expression.

$$P_{j,t} = \left[\alpha_j \, P_{j,t-1}^{1-\theta} + (1-\alpha_j) \; p_t^* (\alpha_j)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{34}$$

An interesting feature of our modelling assumptions is that once chosen the consumption center, everything tends to mimic the standard models in the literature, at least until the following period.

Before presenting the next proposition, which states that the representative household's instantaneous utility function[27] is concave in prices, we need some lemmas[28] regarding the following functions:

$$\begin{aligned} Þ_u &: \mathbb{R}_{++} \to \mathbb{R} \\ Þ_u(\varkappa) &= u\left[u_C^{-1}\left(\varkappa^{1/(1-\theta)}\right)\right] \end{aligned} \tag{35}$$

$$\begin{aligned} Þ_\nu &: \mathbb{R}_{++} \to \mathbb{R} \\ Þ_\nu(\varkappa) &= \nu\left[f^{-1}\left(k\,\varkappa^{\theta/(1-\theta)} \cdot u_C^{-1}\left(K\,\varkappa^{1/(1-\theta)}\right)\right)\right] \end{aligned} \tag{36}$$

Where $k > 0$ and $K > 0$.

Lemma 1 *The previously defined function $Þ_u(\cdot)$ is strictly concave.*

Lemma 2 *If the consumption utility function satisfies the following restriction, e.g. if the households are sufficiently prudent and risk averse, then the previously defined function $Þ_\nu(\cdot)$ is strictly convex.*

$$\frac{\eta(\imath)}{\sigma_a^{-1}(\imath)} + \theta(2\theta-1)\,\sigma_r^{-1}(\imath) > 3\theta \tag{37}$$

Where $\eta(\imath) = -\frac{u_{CCC}(\imath)}{u_{CC}(\imath)} > 0$ is the *Absolute Prudence Index*[29], $\sigma_a^{-1}(\imath) = -\frac{u_{CC}(\imath)}{u_C(\imath)} > 0$ is the *Absolute Risk Aversion Index* and $\sigma_r^{-1}(\imath) = -\frac{u_{CC}(\imath)\cdot\imath}{u_C(\imath)} > 0$ is the *Relative Risk Aversion Index* for $\imath > 0$.

At a first glance the restriction (37) seems to be very strong. However in the case of the widely used Constant Relative Risk Aversion (CRRA) utility functions[30], the parameters locus satisfying such a restriction is quite wide. Note that the narrow gray area of Figure 1 indicates the region where (37) is not satisfied.

[27] Remember that the instantaneous utility function considers both the consumption utility and the labor disutility, e.g. $\left[u(C_t) - \int_0^1 \nu(h_t(i))\,di\right]$.

[28] Since their proofs are easy and purely algebraic, we omit them.

[29] Its usual to consider the Absolute Prudence Index in economic analysis in which agents make optimal choices in an inter-temporal decision environment with uncertainties. A good reference is Kimball (1990).

[30] For CRRA utility functions as $u(c) = \frac{c^{1-\sigma^{-1}}-1}{1-\sigma^{-1}}$, with a constant relative risk aversion index σ^{-1}, the absolute index of risk aversion and prudence are $\sigma_a^{-1}(c) = \sigma^{-1}/c$ and $\eta(c) = \left(1+\sigma^{-1}\right)/c$, respectively. So, the inequality (37) can be simplified to $\theta(2\theta-1)\,\sigma^{-2} + (1-3\theta)\,\sigma^{-1} + 1 > 0$, whose solution set is: $\left\{\sigma^{-1} \in \mathbb{R} : \sigma^{-1} < \frac{1}{2\theta-1} \text{ or } \sigma^{-1} > \frac{1}{\theta}\right\}$.

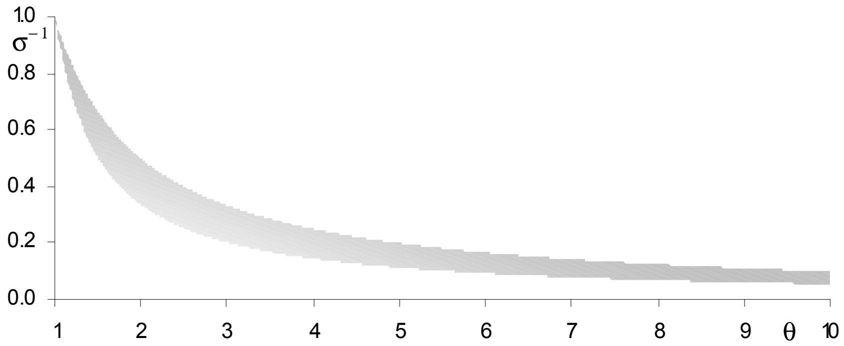

Figure 1: Convexity of $Þ_{\nu}(\cdot)$.

It is important to stress that such a restriction is just a sufficient condition for the Lemma 2 to hold. The necessary and sufficient condition also includes labor disutility and production function parameters. Since its interpretation is less intuitive we did not present it. However such a new restriction narrows the gray area of Figure 1, making even wider the acceptance area where $Þ_{\nu}(\cdot)$ is strictly convex in the case of the CRRA utility functions.

Now we are able to announce and prove an important proposition stating that a best response of the firms from one consumption center is to increase the degree of price stickiness relatively to the one adopted by the firms from the other consumption center.

Proposition 3 *Provided that restriction* (37) *is satisfied and that* $E_{\xi^{\varepsilon}}\Pi_1^*(0) > 0$, *suppose that the households always choose the* $\mathbb{C}_1$, *e.g.* $\gamma_1 = 1$ *and* $\gamma_2 = 0$. *Therefore, there is a small enough probability* $\alpha_2 > 0$ *such that if the firms from* $\mathbb{C}_2$ *announce the following price setting mechanism from a given period* t *onwards*

$$p_{2,t}(i) = \begin{cases} p_t^*(0) & , \text{ with probability } (1-\alpha_2) \\ p_{2,t-1}(i) & , \text{ with probability } \alpha_2 \end{cases} \tag{38}$$

then all households realize that they have better changing their strategies to $\bar{\gamma}_1 = 0$ *and* $\bar{\gamma}_2 = 1$, *benefiting the firms from* $\mathbb{C}_2$.

The proof is given in Appendix A.4.

Based on the above arguments, we are able to formalize the characterization of the equilibrium concept from the households' behavior standpoint, as the next theorem assesses.

Theorem 1 *Provided that restriction* (37) *is satisfied and that* $E_{\xi^{\varepsilon}}\Pi_j^*(0) > 0$, *there is no equilibrium in which the representative household always chooses the same consumption center. Therefore, under such assumptions, households are indifferent between consumption centers in equilibrium.*

The proof is given in Appendix A.5.

Now, turning our attention to the firms behavior, the next theorem assess that firms choose the equilibrium degree of nominal rigidity α_{eq} as the highest degree of price stickiness consistent with a non-negative expected profit. Therefore such a theorem constitutes the key result of the present study.

Theorem 2 *Provided that households are sufficiently prudent and risk averse, according to the inequality* (37) *of above Lemma 2 and that* $E_{\xi^{\varepsilon}}\Pi_j^*(0) \geq 0$, *equilibrium requires that all firms* *from both consumption centers adopt the same highest degree of price stickiness $\alpha_1 = \alpha_2 = \alpha_{eq}$, for which the expected profit is non-negative, e.g. $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) \geq 0$, $\forall j$. Non-trivial solutions implies $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, $\forall j$. Otherwise, if $E_{\xi^\varepsilon}\Pi_j^*(1) \geq 0$ then $\alpha_{eq} = 1$ represents the trivial solution*[31].

The proof is given in Appendix A.6.

Therefore, the above theorem implicitly defines the equilibrium stating that $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, $\forall j$ in the non-trivial case.

Note that it has a Bertrand equilibrium flavor. However, instead of competition on prices *per se* as in the Bertrand case, the equilibrium at hand considers a competition on the parameter capturing the degree of price rigidity.

It is interesting to note that such a result follows from the two types of competition inputted in the model. The first one is the traditional monopolistic competition that allows each firm to choose an optimal price that maximizes its expected discounted profit flow. The second one is the contribution of our modelling assumption on consumption centers. Indeed in the first decision moment of each period t, households must decide from two "identical goods", namely the homogeneous consumption centers. Therefore, a oligopoly game must apply to model competition among both consumption centers. Since such a competition is conducted in terms of the degree of price stickiness, the Bertrand game captured the strategic behavior. As a consequence, the expected discounted profit flows turned to be zero in the non-trivial equilibrium, despite the fact they are the best firms can make optimally choosing their individual prices.

In order to close this section, three comments are in order. The first one concern the number of consumption centers in the model economy. In spite of adopting only two consumption centers in the above economy, the obtained results can be easily extended to a larger number of consumption centers.

The second one refers to the fact that the above theorem generalizes the perfect competitive equilibrium result of zero profits. Theorem 2 states that such a profit is zero on average or in expected terms.

The third one is based on Proposition 3. The uncertainty regarding the exogenous shock ε_t, which does not affect the households' preferences neither the firms' productivity, make households postpone the flexible prices environment.

Such a result is achieved from the fact that the expected utility decreases with the uncertainty regarding the flexible prices environment. We showed that a sticky price environment, at least a Calvo's type one, is preferred to the one with flexible prices. However it is important to point out that we did not proved that households prefer the Calvo's type nominal rigidity the most. It is possible that other price filtering procedures may also reduce the implied uncertainty, and we suggest that approach as future extensions.

Note that $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq})$ depends on the distribution of $\Pi\left(p_t^*(\alpha_{eq}), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^\varepsilon\right)$, so the non-trivial equilibrium condition $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$ implies that he endogenous degree of price stickiness α_{eq} depends on the distributions of aggregate price and production. But such distributions surely depend on the way monetary policy is conducted, for it determines the expected path of aggregate variables. Therefore the Lucas' critique may be applied, for changes on the way monetary policy is conducted may lead to changes in the endogenous degree of price stickiness and in the coefficients of structural equations.

Moreover, we expect that the equilibrium price rigidity would depend on the distribution of the exogenous shock, so structural breaks in the stochastic process of ε_t affect α_{eq}.

Therefore, we could extend the concept behind the Lucas' critique. The dependency of the degree of price rigidity on the distribution of the exogenous cost shock strongly suggests that

[31] Note that if $E_{\xi^\varepsilon}\Pi_j^*(1) < 0$, $\forall j$ then there is no equilibrium, for there are no firms in the market.

the traditional monetary policy evaluation exercises using macroeconometric models could be flawed. Typically estimated relations, even with quasi-structural equations, containing future expectations derived with an ad hoc imposed nominal rigidity source, are reduced-form rather than truly structural relations, for structural changes in the stochastic process generating the cost shocks can change the optimal degree of price stickiness chosen by the firms.

Therefore, as a policy-oriented implication of the present study we recommend the utilization of econometric models with time-varying parameters in order to assess possible parameters structural breaks even if the implemented policy remains unchanged.

In the following section, we introduce the model's first and second order approximations for the corresponding structural equations. Among other results, we show that: (a) the degree of inefficiency ϕ_y constitutes a source of nominal price rigidity; and (b) the equilibrium (optimal) degree of price rigidity α_{eq} depends on the coefficient of variation of the random shock ε_t, for a given monetary policy rule.

3 Log-approximated structural equations

Initially, it is convenient to derive log-approximations for aggregate product and prices through the consumption centers, adopting the following notation as the percentage deviation of each variable from its steady state value. For any variable $\varkappa_t$, always positive or negative, with a steady state value $\bar{\varkappa}$, we define $\hat{\varkappa}_t \equiv \log\left(\varkappa_t/\bar{\varkappa}\right)$.

It is easy to verify that the expressions (4) and (6) imply the following first order Taylor approximations:

$$\begin{aligned} \widehat{Y}_t &= \gamma \widehat{Y}_{1,t} + (1-\gamma)\,\widehat{Y}_{2,t} && (39) \\ \widehat{P}_t &= \gamma \widehat{P}_{1,t} + (1-\gamma)\,\widehat{P}_{2,t} && (40) \end{aligned}$$

Moreover, from (24) and (26), we log-linearize the natural and the efficient[32] products as follows:

$$\begin{aligned} \widehat{Y}^n_t &= -\left(1-\phi_y\right)^{-1}\left(\omega+\sigma^{-1}\right)^{-1} \bar{\varepsilon}\,\widehat{\varepsilon}_t && (41) \\ \widehat{Y}^e &= \phi_y\left(1-\phi_y\right)^{-1}\left(\omega+\sigma^{-1}\right)^{-1} && (42) \end{aligned}$$

Where ϕ_y is the previously defined inefficiency degree parameter and γ denotes the time-invariant probability of choosing the $\mathbb{C}_1$, e.g. $\gamma = \gamma_1$.

Assuming that the distribution support of ε_t is completely inside $\mathbb{R}_+$ or $\mathbb{R}_-$, we obtain the following log-linearizations for the real marginal cost:

$$\begin{aligned} \widehat{s}^{\,\varepsilon}_{j,t} &= \mu\left(1-\phi_y\right)\left(\omega+\sigma^{-1}\right)\widehat{Y}_{j,t} + \mu\bar{\varepsilon}\,\widehat{\varepsilon}_t \\ \widehat{s}^{\,\varepsilon}_t &= \mu\left(1-\phi_y\right)\left(\omega+\sigma^{-1}\right)\widehat{Y}_t + \mu\bar{\varepsilon}\,\widehat{\varepsilon}_t && (43) \end{aligned}$$

Where $\widehat{s}^{\,\varepsilon}_{j,t} \equiv \log\left(\mu s^{\varepsilon}_{j,t}\right)$, $\omega \equiv \frac{s_y\left(\overline{Y},\overline{Y}\right)}{s\left(\overline{Y},\overline{Y}\right)}\overline{Y}$, $\sigma^{-1} \equiv -\frac{u_{CC}\left(\overline{Y}\right)}{u_C\left(\overline{Y}\right)}\overline{Y}$ is the steady state relative risk aversion index, and $\widehat{s}^{\,\varepsilon}_t \equiv \gamma\widehat{s}^{\,\varepsilon}_{1,t} + (1-\gamma)\,\widehat{s}^{\,\varepsilon}_{2,t}$ aggregates the real marginal costs from each consumption center.

[32]If ϕ_y is close enough to zero, the approximated log-deviation of the efficient production from the steady state product turns into $\widehat{Y}^e = \phi_y\left(\omega+\sigma^{-1}\right)^{-1}$, as traditionally presented in the literature.

In turn, we may relate the aggregate real marginal cost to the output gap $x_t \equiv \left(\widehat{Y}_t - \widehat{Y}_t^n\right)$ as follows:

$$\widehat{s}_t^{\varepsilon} = \mu\left(1-\phi_y\right)\left(\omega+\sigma^{-1}\right)x_t \tag{44}$$

Finally, the time-varying markup can be log-linearized as shown below. Note that $\widehat{\mu}_t$ is proportional to $\widehat{Y}_t^n$, but oscillates in opposite directions.

$$\widehat{\mu}_t = \left(1-\phi_y\right)^{-1}\bar{\varepsilon}\,\widehat{\varepsilon}_t \tag{45}$$

3.1 The structural aggregate supply curve

Log-linearizing the first order condition (31) from the firms' problem under price stickiness, we obtain the following New-Keynesian Phillips Curve (NKPC):

$$\pi_t = \kappa\ x_t + \beta E_t\pi_{t+1} \tag{46}$$

Where $\kappa \equiv \zeta_{\phi_y}\cdot\frac{(1-\alpha_{eq})(1-\alpha_{eq}\beta)}{\alpha_{eq}}\cdot\zeta$, $\zeta_{\phi_y} \equiv \frac{\mu\left(1-\phi_y\right)(1+\omega\theta)}{1+\mu\left(1-\phi_y\right)\omega\theta}$ and $\zeta \equiv \frac{\omega+\sigma^{-1}}{1+\omega\theta}$.

The term ζ is well known in the literature and is related to the degree of strategic complementarity between firms' price setting decisions. If ζ is low enough, the aggregate price tends to be more sticky even when a great fraction of firms adjust their prices more often.

Note now that the friction captured by the inefficiency degree also affects the nominal price rigidity through the parameter ζ_{ϕ_y}. Indeed, such a parameter increases with the degree of efficiency $\left(1-\phi_y\right)$. Hence we argue that, in our set up, such a friction works also as a source for price rigidity.

3.1.1 Welfare and cost push shocks

Under certain conditions[33] it can be shown that in order to maximize the welfare of the representative household the monetary authority should minimize[34]:

$$W_0 = -\Omega E_0\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + \lambda\left(\widehat{Y}_t - \widehat{Y}^e\right)^2\right] \tag{47}$$

Where the parameters $\Omega > 0$ and $\lambda \geq 0$ are based on the deep parameters of the economy. More specifically $\lambda \equiv \frac{\kappa}{\theta}$, where κ is the coefficient associated with output in the NKPC and θ represents the elasticity of substitution between goods in the economy.

Note that the monetary authority must also concern about dispersions of the aggregate production from its efficient level rather than from the steady state level, e.g. $\widetilde{x}_t \equiv \widehat{Y}_t$ is the relevant concept of output gap for monetary policy issues.

Thus we may rewrite the NKPC in terms of $\widetilde{x}_t$ rather than x_t as follows, implying a cost push shock term related to the exogenous cost shock term $\widehat{\varepsilon}_t$:

$$\pi_t = \kappa\ \widetilde{x}_t + \beta E_t\pi_{t+1} + u_t \tag{48}$$

[33] In particular, such an approximation is possible if the steady state product gets close to the efficient product faster than the inefficiency degree gets close to zero. Since such an approximation is always valid in the case of $\phi_y = 0$, several researches assume a government subsidy amounting the necessary value to offset the remaining inefficiency sources in their model.

[34] For an extensive explanation on deriving microfounded welfare based central bank loss functions, see Woodford (2003), chapter 6. Basically, it is a second order Taylor approximation of the welfare function around an efficient steady state. Its functional form is crucially influenced by the assumed source of price stickiness.

Where $\widetilde{x}_t \equiv \widehat{Y}_t$, $u_t \equiv \frac{\kappa}{(\omega+\sigma^{-1})}\widehat{\mu}_t$ and $\widehat{\mu}_t = \left(1-\phi_y\right)^{-1}\bar{\varepsilon}\,\widehat{\varepsilon}_t$.

Thus the unconditional expectance of u_t satisfies

$$E_{\xi^\varepsilon} u_t = \frac{\kappa}{(\omega+\sigma^{-1})\left(1-\phi_y\right)}\bar{\varepsilon}\; E_{\xi^\varepsilon}\widehat{\varepsilon}_t \tag{49}$$

3.2 The endogenous degree of price stickiness

We obtain a log-linearized expression for the unconditional expected profit flow $E_{\xi^\varepsilon}\Pi_j^*(\alpha)$ as follows:

$$E_{\xi^\varepsilon}\Pi_j^*(\alpha) = \frac{\overline{P}\;\overline{Y}}{(1-\beta)}\left[\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)+\left(1-\mu^{-1}-\frac{1-\phi_y}{\omega\sigma_f\sigma}\right)E_{\xi^\varepsilon}\widehat{Y}_t+\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)E_{\xi^\varepsilon}\widehat{P}_t\right]$$

Where $\sigma_f^{-1} \equiv -\frac{f''(\bar{h})}{f'(\bar{h})}\bar{h}$ is an additional parameter that represents the steady state concavity of firms' production function. So, the assumed regularity properties[35] about the function $f(\cdot)$ implies that $\sigma_f^{-1} > 0$.

Therefore, as a consequence of log-linearizing the necessary condition of a non-trivial equilibrium, e.g. $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, $\forall j$, the following equality must hold:

$$\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)+\left(1-\mu^{-1}-\frac{1-\phi_y}{\omega\sigma_f\sigma}\right)E_{\xi^\varepsilon}\widehat{Y}_t+\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)E_{\xi^\varepsilon}\widehat{P}_t = 0 \tag{50}$$

Note that $E_{\xi^\varepsilon}\widehat{Y}_t$ and $E_{\xi^\varepsilon}\widehat{P}_t$ clearly depend on the degree of price stickiness, for it influences the equilibrium expected path of aggregate product and price, and on the way monetary policy is conducted. Since monetary policy conduction varies, there is no unique solution of α_{eq} determined by (50). So the Lucas' critique may be applied as previously commented.

In the general case, the unconditional expectations of the aggregate variables depend on the distribution of the exogenous shock ε_t, for this is the only exogenous random variable. Indeed we show further on that an optimal monetary policy under a timeless perspective imply that $E_{\xi^\varepsilon}\widehat{Y}_t$ and $E_{\xi^\varepsilon}\widehat{P}_t$ are functions of $E_{\xi^\varepsilon}\widehat{\varepsilon}_t$. So we expect that the equilibrium price rigidity must depend on the distribution of the exogenous shock. In other words, structural breaks in the stochastic process governing the exogenous shock induce changes the (endogenous) degree of price stickiness chosen by the firms.

Hence, we approximate the first[36] and second[37] moments of $\widehat{\varepsilon}_t$:

$$E_{\xi^\varepsilon}\widehat{\varepsilon}_t \approx -\frac{1}{2}\frac{Var_{\xi^\varepsilon}\varepsilon_t}{\bar{\varepsilon}^2} \tag{51}$$

$$Var_{\xi^\varepsilon}\widehat{\varepsilon}_t \approx \frac{Var_{\xi^\varepsilon}\varepsilon_t}{\bar{\varepsilon}^2}\left(1+\frac{1}{4}\frac{Var_{\xi^\varepsilon}\varepsilon_t}{\bar{\varepsilon}^2}\right) \tag{52}$$

Note that both the unconditional expectance and variance of $\widehat{\varepsilon}_t$ can be approximated as functions of the variation coefficient of ε_t. For simplicity, let $V\varepsilon$ denote the variation coefficient of ε_t, e.g. $V\varepsilon = \frac{Var_{\xi^\varepsilon}\varepsilon_t}{\bar{\varepsilon}^2}$.

Note that, given the approximation of $E_{\xi^\varepsilon}\widehat{\varepsilon}_t$, the unconditional expectance of the cost push shock u_t, shown in (49), is not zero and depends on the volatility of ε_t. In our approach, $E_{\xi^\varepsilon}u_t = 0$ only if the variation coefficient of ε_t is zero.

[35] See Assumptions 9 and 10.

[36] Since the function $log(\cdot)$ is concave, the negative signal was expected.

[37] The approximation of $Var_{\xi^\varepsilon}\widehat{\varepsilon}_t$ is also convenient, since it allows for a volatility analysis of each aggregated variable, as made further on.

Following the analytical analysis, we turn next to numerically simulate the effects of a given monetary policy rule on the endogenous determination of the degree of price stickiness. To this end, a particular specification for a monetary policy instance is chosen.

3.2.1 Simulations under a timeless perspective optimal monetary policy rule

For simulation purposes, we consider a particular solution[38] of the time consistent optimal monetary policy approaches shown in Woodford (2003), chap. 7. Adapting for our model at hand, a particular possible solution satisfies the following expression:

$$\widehat{P}_t = \mu_1 \widehat{P}_{t-1} + \beta^{-1} \sum_{j=0}^{\infty} \mu_2^{-(j+1)} E_t u_{t+j} \tag{53}$$

Where u_t is the cost push shock term in (48), and μ_1 and μ_2 are the roots of the characteristic equation $\beta\mu_j^2 - \left(1+\beta+\frac{\kappa^2}{\lambda}\right)\mu_j + 1 = 0$, satisfying $0 < \mu_1 < 1 < \mu_2$.

In order to simplify our analysis, we assume that the cost push shock term evolves according to an AR(1) process, as follows:

$$u_t = (1-\rho_u) E_{\xi^\varepsilon} u_t + \rho_u u_{t-1} + \nu_t \tag{54}$$

Where ν_t is i.i.d. with zero mean. Therefore the expected realization of u_{t+j} can be derived as described below.

$$E_t u_{t+j} = E_{\xi^\varepsilon} u_t + \rho_u^j \left(u_t - E_{\xi^\varepsilon} u_t\right)$$

Substituting this result back in (53), we derive a more simplified expression for $E_{\xi^\varepsilon} \widehat{P}_t$ below. Note that it does not depend on the persistence parameter ρ_u of the stochastic process describing the cost push shocks.

$$E_{\xi^\varepsilon} \widehat{P}_t = \frac{\lambda}{\kappa^2} E_{\xi^\varepsilon} u_t$$

It is easy to verify that the particular monetary policy rule implies that

$$E_{\xi^\varepsilon} \widehat{Y}_t = -\frac{1}{\kappa} E_{\xi^\varepsilon} u_t$$

Thus, from the expression of the unconditional expectance of the cost push shock u_t, shown in (49), we obtain the unconditional expectations on aggregate price and output (percentage deviations from their respective steady state values) as follows:

$$E_{\xi^\varepsilon} \widehat{P}_t = \frac{\lambda}{\kappa} \frac{1}{\left(\omega+\sigma^{-1}\right)\left(1-\phi_y\right)} \bar{\varepsilon} \; E_{\xi^\varepsilon} \widehat{\varepsilon}_t \tag{55}$$

$$E_{\xi^\varepsilon} \widehat{Y}_t = -\frac{1}{\left(\omega+\sigma^{-1}\right)\left(1-\phi_y\right)} \bar{\varepsilon} \; E_{\xi^\varepsilon} \widehat{\varepsilon}_t \tag{56}$$

Since the microfounded endogenous parameter λ is defined as $\lambda \equiv \frac{\kappa}{\theta}$ in the previously shown central bank loss function, we may simplify the expression for $E_{\xi^\varepsilon} \widehat{P}_t$ as follows:

$$E_{\xi^\varepsilon} \widehat{P}_t = \frac{1}{\theta} \frac{1}{\left(\omega+\sigma^{-1}\right)\left(1-\phi_y\right)} \bar{\varepsilon} \; E_{\xi^\varepsilon} \widehat{\varepsilon}_t \tag{57}$$

[38] For a more realist simulation, regarding to the empirical features of a particular economy, one must conveniently model the way monetary policy is actually conducted by a particular central bank.

Nevertheless, it is also convenient to use the above result (55) due to the fact that some central banks chooses a discretionary $\bar{\lambda}$ value that penalizes the dispersion of $\widehat{Y}_t$ from $\widehat{Y}^e$ as shown in the central bank loss function.

Note that if the monetary authority considers the microfounded parameter λ, output, price and consequently firms' profit expectations will not depend on the price rigidity parameter. Hence, provided a non negative expected profit, firms will choose a total price stickiness, e.g. $\alpha_{eq} = 1$. But such a decision implies that $\kappa = 0$. Since $\lambda \equiv \frac{\kappa}{\theta}$, the only equilibrium occurs with $\lambda = 0$. Thus, under such a timeless perspective optimal monetary policy rule, the central bank will not penalize the aggregate product volatility when choosing a welfare maximizing criterion. As a result, prices will be completely stabilized while the product dispersion will have its maximum volatility.

It is important to stress that such a strong conclusion relies: (a) on the Assumption 7 on the existence of a steady state price level $\bar{P}$; and (b) on the assumption on absence of any preference disturbing shocks. For instance, consider the traditional literature case, with just one complete market of unit mass, in which households are homogeneous regarding their preferences, which are subjected to common knowledge disturbing shocks. A traditional result states that the aggregate price is also a function of such shocks[39]. Although not formally proved in our analysis, we can make the conjecture that not filtering the implicit volatility on optimal prices implied only by such preference shocks would increase the economy welfare, for there is no household uncertainty concerning this volatility source. As a result, firms would not choose the maximum price stickiness and α_{eq} is likely to be lower. Again, it is a conjecture to be tested in future extensions.

Returning to our formal analysis, we assess now the usual practice adopted by central banks to consider a discretionary weight for the aggregate output gap into the loss function, namely $\bar{\lambda}$. Therefore $E_{\xi^\varepsilon}\widehat{P}_t$ depends on the price rigidity parameter α, as indicated in (46) and (55), for κ is a function of α. In such a general case, the parameter α_{eq} depends on the volatility of the exogenous shock $E_{\xi^\varepsilon}\widehat{\varepsilon}_t$. Following, we show the expected discounted profits flow in the case of a discretionary weight $\bar{\lambda}$.

$$\begin{aligned} E_{\xi^\varepsilon}\Pi_j^*(\alpha) &= \frac{\overline{P}\,\overline{Y}}{(1-\beta)}\left[\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)-\left(1-\mu^{-1}-\frac{1-\phi_y}{\omega\sigma_f\sigma}\right)\frac{1}{(\omega+\sigma^{-1})\left(1-\phi_y\right)}\bar{\varepsilon}\; E_{\xi^\varepsilon}\widehat{\varepsilon}_t+\right. \\ &\quad \left.+\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)\frac{\bar{\lambda}}{\kappa}\frac{1}{(\omega+\sigma^{-1})\left(1-\phi_y\right)}\bar{\varepsilon}\; E_{\xi^\varepsilon}\widehat{\varepsilon}_t\right] \end{aligned} \tag{58}$$

Since Corollary 1 states that $E_{\xi^\varepsilon}\Pi_j^*(0) \geq 0$ must hold in order to firms be present in the market then the following inequality must also hold[40].

$$\left(1-\bar{\varepsilon}-\frac{1-\phi_y}{\omega\sigma_f}\right)-\left(1-\mu^{-1}-\frac{1-\phi_y}{\omega\sigma_f\sigma}\right)\frac{1}{(\omega+\sigma^{-1})\left(1-\phi_y\right)}\bar{\varepsilon}\; E_{\xi^\varepsilon}\widehat{\varepsilon}_t \geq 0 \tag{59}$$

Therefore, provided the assumptions of Theorem 2, the non-trivial equilibrium condition $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, $\forall j$, implies the following expression for the endogenous degree of nominal rigidity:

$$\alpha_{eq} = \frac{(1+\beta+\Theta)-\sqrt{(1+\beta+\Theta)^2-4\beta}}{2\beta} \tag{60}$$

Where

[39] See Woodford (2003).

[40] Since $\lim_{\alpha\to 0}(1/\kappa) = 0$, the result is straightforward.

$$\Theta = \frac{\bar{\lambda}}{\zeta_{\phi_y} \cdot \zeta} \cdot \frac{\left(1 - \bar{\varepsilon} - \frac{1-\phi_y}{\omega \sigma_f}\right) \bar{\varepsilon}\ E_{\xi^\varepsilon} \widehat{\varepsilon}_t}{-\left(1 - \bar{\varepsilon} - \frac{1-\phi_y}{\omega \sigma_f}\right)\left(\omega + \sigma^{-1}\right)\left(1 - \phi_y\right) + \left(1 - \mu^{-1} - \frac{1-\phi_y}{\omega \sigma_f \sigma}\right) \bar{\varepsilon}\ E_{\xi^\varepsilon} \widehat{\varepsilon}_t} \tag{61}$$

Consider now the most likely case in which $\bar{\varepsilon} \geq 0$, e.g. ε_t is actually a cost. Thus one can verify[41] that α_{eq} is a decreasing function of the variation coefficient of ε_t, e.g $V\varepsilon$. In other words, if positive cost shocks are expected to happen then prices will be more frequently adjusted in environments where the volatility of ε_t is high. Moreover, if $V\varepsilon = 0$ then the expected discounted profit flow is is non-negative and do not depend on α_{eq}.

Furthermore, the equilibrium degree of price stickiness α_{eq} is a decreasing function of the discretionary value $\bar{\lambda}$. Hence, if the central bank aversion to aggregate product volatility is lower than the aversion to inflation volatility, as it is the case of many central banks, then the degree of price stickiness in the economy, as a sub-game perfect equilibrium strategy, must be high.

In order to numerically simulate the degree of price stickiness, we assume a positive $\bar{\varepsilon}$ value and adopt a particular Cobb-Douglas type production function specification:

$$y_{j,t}(i) = A\, h_{j,t}(i)^n \tag{62}$$

Where $n < 1$.

With such assumptions, it is easy to verify that the steady state concavity of $f(\cdot)$ and the elasticity of the real labor marginal cost for the firms are $\sigma_f^{-1} = (1-n)$ and $\omega = (1-n)/n$, respectively. Moreover, $\bar{\varepsilon}$ must satisfy $0 < \bar{\varepsilon} < \mu^{-1}$.

Additionally, it is possible to verify that the previous restriction (59) is always satisfied if $\sigma^{-1} \leq \sigma_{crit}^{-1}$, where:

$$\sigma_{crit}^{-1} = \frac{\left(1 - \mu^{-1}\right)}{n\left(\mu^{-1} - \bar{\varepsilon}\right)} \tag{63}$$

On the other hand, if $\sigma^{-1} > \sigma_{crit}^{-1}$ then the previous restriction requires that the volatility measure $V\varepsilon$ be lower than a critical value $V\varepsilon_{crit}$, where:

$$V\varepsilon_{crit} = \sqrt{2\frac{\left(\omega + \sigma^{-1}\right)}{\left(\sigma^{-1} - \sigma_{crit}^{-1}\right)} \frac{\left[(1 - \bar{\varepsilon}) - n\left(\mu^{-1} - \bar{\varepsilon}\right)\right]}{n\bar{\varepsilon}}} \tag{64}$$

Note that $V\varepsilon_{crit}$ is a strictly decreasing function of σ^{-1} and is such that its lower bound $V\varepsilon_{crit}^{LB} = \lim_{\sigma^{-1}\to\infty} V\varepsilon_{crit}$ is determined as follows:

$$V\varepsilon_{crit}^{LB} = \sqrt{2\frac{\left[(1 - \bar{\varepsilon}) - n\left(\mu^{-1} - \bar{\varepsilon}\right)\right]}{n\bar{\varepsilon}}} \tag{65}$$

Since our approximations hold for small enough volatility values, the constrain $V\varepsilon \leq V\varepsilon_{crit}$ shall not be very restrictive.

Base on the above results, we simulate α_{eq} as a function of the volatility measure $V\varepsilon$, and of the remaining parameters of the consumption centers economy. These computations generate the graphs[42] depicted in Figures 2 and 3 below. Then, we can graphically infer the following fundamental relations in our model economy, as a sub-game perfect equilibrium outcome.

[41] Such properties rely on $E_{\xi^\varepsilon} \Pi_j^*(\alpha_{eq}) = 0$, $E_{\xi^\varepsilon} \Pi_j^*(0) \geq 0$, $\forall j$, $\left(1 - \phi_y\right) \geq 0$ and $\left(\omega + \sigma^{-1}\right) \geq 0$.

[42] In such graphs, the baseline is the following parameters values: $\beta = 0.986$ (6% per year); $\theta = 9.5$; $\bar{\lambda} = 5$; $\bar{\varepsilon} = 0.15$; $\omega = 0.25$; $\sigma^{-1} = 0.15$; $\sigma_{crit}^{-1} = 0.18$; $\sigma_f^{-1} = 0.20$; $n = 0.80$; $V\varepsilon_{crit}^{LB} = 2.1$; $\zeta = 0.12$ and $\zeta_{\phi_y} = 0.94$. The parameters relate to a quarterly frequency model.

First of all, the parameter of price rigidity α_{eq} is a decreasing function of the volatility of exogenous shocks, as shown in Figure 2 below. Furthermore, the more risk averse are the consumers, the higher is the degree of price stickiness in the economy.

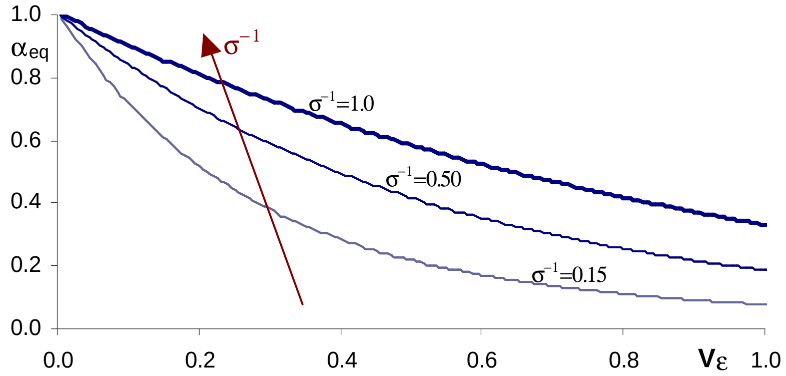

Figure 2: Equilibrium Price Stickiness α_{eq} as a Function of $V\varepsilon$ and σ^{-1}

Second, in Figure 3 one verifies that the parameter $\bar{\lambda}$, which measures the discretionary weight for the aggregate output gap in the central bank loss function, induces a reduction of the degree of price rigidity in the model economy.

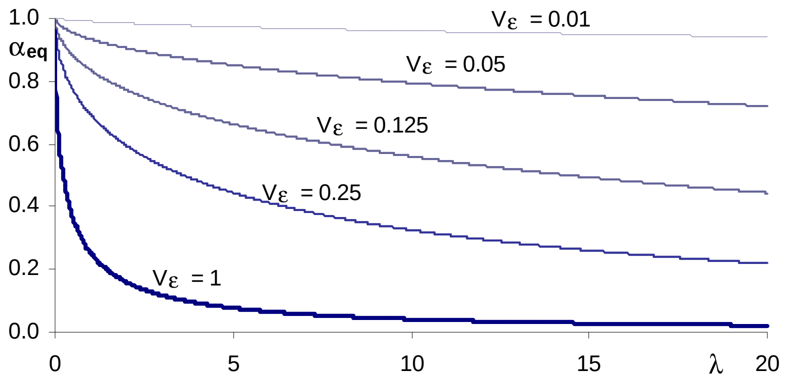

Figure 3: Equilibrium Price Stickiness α_{eq} as a Function of $\bar{\lambda}$

4 Conclusions

The present study aims to contribute suggesting a possible way to fill the research lacuna first stressed by Blinder et al. (1998). To this end, a model economy is built in which firms could choose prices according to the Calvo's approach. Nevertheless, the degree of price rigidity α_{eq} is endogenously determined as a sub-game perfect strategy profile adopted by the firms as an optimum response to consumers' risk aversion in an economy in which firms are not subject to adjustment costs. This main result is formalized in Theorem 2.

In other words, our main results imply that firms monopolistically compete setting optimal prices *a la* Calvo, and choosing the degree of price stickiness in a Bertrand game flavor. In equilibrium, such a behavior leads to a zero expected profit flow, generalizing the traditional result of zero per period profit. However, we need to stress that this particular result depends on the existence of inefficient stochastic shocks.

It is interesting to note that such a result follows from the two types of competition inputted in the model. The first one is the traditional monopolistic competition that allows each firm to choose an optimal price that maximizes its expected discounted profit flow. The second one is the contribution of our modelling assumption on consumption centers. Indeed in the first decision moment of each period t, households must decide from two "identical goods", namely the homogeneous consumption centers. Therefore, a oligopoly game must apply to model competition among both consumption centers. Since such a competition is conducted in terms of the degree of price stickiness, the Bertrand game captured the strategic behavior. As a consequence, the expected discounted profit flows turned to be zero in the non-trivial equilibrium, despite the fact they are the best firms can make optimally choosing their individual prices.

The results also show that the Lucas' critique may apply. Moreover, a relevant extension of the Lucas' critique is presented on the analysis. Since the degree of price rigidity depends on the distribution of the stochastic process governing the cost shocks, traditional monetary evaluation exercises using macro-econometric models are flawed by a failure to recognize that the relations typically estimated, even with quasi-structural equations containing future expectations derived with an ad hoc imposed nominal rigidity source, are reduced-form relations rather than truly structural relations. This is due to the fact that structural changes in the stochastic process generating the cost shocks may change the optimal degree of price stickiness chosen by the firms. Hence, as a policy oriented recommendation we may suggest the use of time-varying parameters econometric models for monetary policy purposes, for it is difficult to accurately asses the occurrence of such structural breaks.

Furthermore, it is shown that the degree of inefficiency captured by the parameter ϕ_y also affects the nominal price rigidity by means of the parameter ζ_{ϕ_y}. Thus, this degree of inefficiency can also be accounted as a source of price stickiness. In short, our analysis shows that inefficiency and uncertainty are both key sources of price rigidities in the economy.

Finally, our numerical simulations indicate that if the monetary authority chooses the microfounded parameter λ, which captures the penalty of aggregate output gap in the central bank's loss function, and conduct a time consistent optimal monetary policy rule, firms will optimally choose a full price stickiness, e.g. $\alpha_{eq} = 1$. In such an instance, the only equilibrium occurs with $\lambda = 0$. Thus, under such a timeless perspective optimal monetary policy rule, the central bank will not penalize the aggregate product volatility when choosing a welfare maximizing criterion. As a result, prices will be completely stabilized while the product dispersion will have its maximum volatility.

It is important to stress that such a strong conclusion relies: (a) on the Assumption 7 on the existence of a steady state price level $\bar{P}$; and (b) on the assumption on absence of any preference disturbing shocks.

On the other hand, if the central bank chooses a discretionary weight $\bar{\lambda}$, under the assumption of positive ε_t cost shocks, then the frequency of (optimal) price adjustments will be an increasing function of the cost shock volatility. Furthermore, it is shown that the degree of price stickiness is a decreasing function of the discretionary weight $\bar{\lambda}$.

Regarding possible extensions, the present analysis should also consider economies in which there are also preferences and production technology shocks, and efficient shocks. In such a case, consumers would not like that the price volatility generated by such shocks were completely filtered. So, the technique applied in this study must be adapted, mainly regarding the fact that preference shocks are consumers' common knowledge and not a source of uncertainty. Probably, the equilibrium degree of nominal price rigidity will be lower than the one derived in this exercise.

Another important extension should consider model economies in which there is a persistent level of inflation that affects the agents' optimal behavior. One can even extend the model for the case of an open small economy. In that case, the inclusion into the model economy of an import sector and exporter firms strategically deciding how much to pass the exchange rate volatility through their prices should be key elements.

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A Appendix

A.1 Firms' profit flow

Since the firm i from $\mathbb{C}_j$ adjust its price with probability α, its possibility tree is the one depicted in Figure 4 below, in which $\bar{p}_{j,t}(i)$ represents a new price adjusted in period t and the dotted lines mean that the structure pattern repeats indefinitely.

As a consequence of the price setting process depicted in Figure 4, the probability distribution of each price $\bar{p}_{j,t}(i)$ is the one shown in the following table:

Therefore, given the previous distribution and considering that some properties of uniform convergence apply, the firm's expected sum of profit flow $\Pi^d_{j,0}(i)$ discounted at period $t = 0$ may be represented as in (66) if $\alpha < 1$.

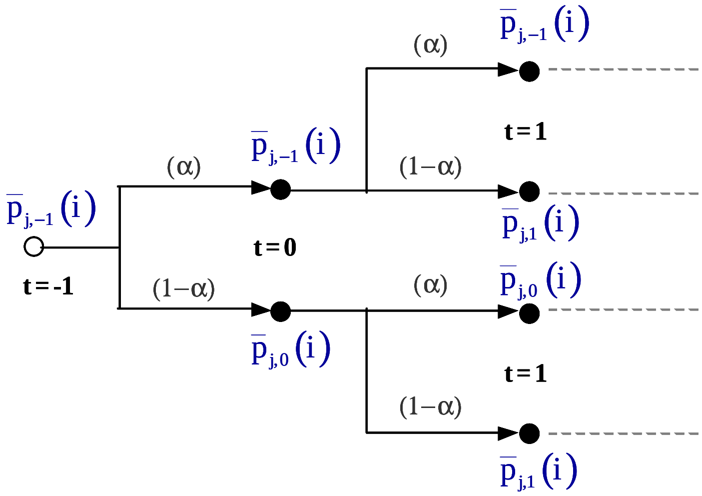

Figure 4: Price Setting

Table 1: Prices Distribution

Period	$p_{j,-1}(i)$	$\bar{p}_{j,0}(i)$	$\bar{p}_{j,1}(i)$	$\bar{p}_{j,2}(i)$	$\bar{p}_{j,3}(i)$	$\cdots$	$\bar{p}_{j,\tau}(i)$
0	α	$(1-\alpha)$	0	0	0	$\cdots$	0
1	α^2	$(1-\alpha)\alpha$	$(1-\alpha)$	0	0	$\cdots$	0
2	α^3	$(1-\alpha)\alpha^2$	$(1-\alpha)\alpha$	$(1-\alpha)$	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	
t	α^{t+1}	$(1-\alpha)\alpha^t$	$(1-\alpha)\alpha^{t-1}$	$(1-\alpha)\alpha^{t-2}$	$(1-\alpha)\alpha^{t-3}$		$(1-\alpha)\alpha^{t-\tau}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		

$$\begin{aligned}\Pi_{j,0}^{d}(i) &= E_0 \sum_{t=0}^{\infty} \gamma_j \left[\alpha^{t+1} Q_{0,t} \Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^{\varepsilon}\right) + \right. \\ & \left. + (1-\alpha) \sum_{T=t}^{\infty} \alpha^{T-t} Q_{0,T} \Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^{\varepsilon}\right) \right] \end{aligned} \tag{66}$$

A.2 Proof of Proposition 1

Proposition 1 *Under a timeless perspective, if a particular firm i from $\mathbb{C}_j$ is present in the market then its expected profit $E_{\xi^\varepsilon}\Pi_j^*(\alpha)$ is a decreasing function of the degree of nominal price rigidity summarized by the probability α.*

Proof. Assuming uniform convergence and considering all regularity assumptions previously made, we obtain the result:

$$\frac{\partial E_{\xi^\varepsilon}\Pi_j^*(\alpha)}{\partial\alpha} = E_{\xi^\varepsilon}\frac{\partial\Pi_j^*(\alpha)}{\partial\alpha} \tag{67}$$

However, in order to determine the first derivative $\partial\Pi_j^*(\alpha)/\partial\alpha$, it is easier to consider the envelope theorem applied on $\Pi_{j,0}^d(i)$, previously defined in (29).

First of all, we determine $\partial\Pi_{j,0}^d(i)/\partial\alpha$ considering some properties of uniform convergence applied over infinite series differentiating:

$$\begin{aligned}
\frac{\partial\Pi_{j,0}^d(i)}{\partial\alpha} = \; & E_0\sum_{t=0}^{\infty}\gamma_j\Bigg[\frac{\partial\alpha^{t+1}}{\partial\alpha}Q_{0,t}\Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i);\xi_t^\varepsilon\right) - \\
& -\sum_{T=t}^{\infty}\alpha^{T-t}Q_{0,T}\Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i);\xi_T^\varepsilon\right) + \\
& +(1-\alpha)\sum_{T=t}^{\infty}\frac{\partial\alpha^{T-t}}{\partial\alpha}Q_{0,T}\Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i);\xi_T^\varepsilon\right)\Bigg]
\end{aligned}$$

$$\begin{aligned}
= \; & E_0\sum_{t=0}^{\infty}\gamma_j(t+1)\alpha^tQ_{0,t}\Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i);\xi_t^\varepsilon\right) + \\
& +E_0\sum_{t=0}^{\infty}\gamma_j\Bigg[-\sum_{T=t}^{\infty}\alpha^{T-t}Q_{0,T}\Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i);\xi_T^\varepsilon\right) + \\
& +(1-\alpha)\sum_{T=t}^{\infty}\frac{\partial\alpha^{T-t}}{\partial\alpha}Q_{0,T}\Pi\left(\bar{p}_{j,t}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i);\xi_T^\varepsilon\right)\Bigg]
\end{aligned}$$

Applying the envelope theorem, we obtain:

$$\begin{aligned}
\frac{\partial\Pi_j^*(\alpha)}{\partial\alpha} = \; & \alpha E_0\sum_{t=0}^{\infty}\gamma_j\frac{\partial\alpha^t}{\partial\alpha}Q_{0,t}\Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i);\xi_t^\varepsilon\right) + \\
& +E_0\sum_{t=0}^{\infty}\gamma_j\alpha^tQ_{0,t}\Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i);\xi_t^\varepsilon\right) + \\
& +E_0\sum_{t=0}^{\infty}\gamma_j\Bigg[-\sum_{T=t}^{\infty}\alpha^{T-t}Q_{0,T}\Pi\left(p_{j,t}^*(\alpha), P_{j,T}, Y_{j,T}, w_{j,T}(i);\xi_T^\varepsilon\right) + \\
& +(1-\alpha)\sum_{T=t}^{\infty}\frac{\partial\alpha^{T-t}}{\partial\alpha}Q_{0,T}\Pi\left(p_{j,t}^*(\alpha), P_{j,T}, Y_{j,T}, w_{j,T}(i);\xi_T^\varepsilon\right)\Bigg]
\end{aligned}$$

Note that $p_{j,0}^{*}(\alpha) = \arg\max E_0 \sum\limits_{T=0}^{\infty} \gamma_j \alpha^T Q_{0,T} \Pi(\ \cdot\ , P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^{\varepsilon})$, as a consequence of the firm's problem (30) under sticky prices. Since $p_{j,-1}(i)$ is likely to be different from $p_{j,0}^{*}(\alpha)$, the following result is straightforward:

$$
\begin{aligned}
\frac{\partial \Pi_j^*(\alpha)}{\partial \alpha} \leq\ & \alpha E_0 \sum_{t=0}^{\infty} \gamma_j \frac{\partial \alpha^t}{\partial \alpha} Q_{0,t} \Pi\left(p_{j,-1}(i), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^{\varepsilon}\right) + \\
& + E_0 \sum_{t=0}^{\infty} \gamma_j \alpha^t Q_{0,t} \Pi\left(p_{j,0}^{*}(\alpha), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^{\varepsilon}\right) + \\
& + E_0 \sum_{t=0}^{\infty} \gamma_j \left[-\sum_{T=t}^{\infty} \alpha^{T-t} Q_{0,T} \Pi\left(p_{j,t}^{*}(\alpha), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^{\varepsilon}\right) + \right. \\
& \left. + (1-\alpha) \sum_{T=t}^{\infty} \frac{\partial \alpha^{T-t}}{\partial \alpha} Q_{0,T} \Pi\left(p_{j,t}^{*}(\alpha), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^{\varepsilon}\right) \right]
\end{aligned}
$$

Before applying the unconditional expectance operator, we need the following results:

(a) The timeless perspective implies that $p_{j,-1}(i) = p_{j,-\tau}^{*}(\alpha)$ for some $\tau > 0$, e.g. $p_{j,-1}(i)$ is the most recent optimal price adjusted before the initial period $t = 0$.

(b) aggregate variables are independent on individual firms' decisions, e.g. $P_{j,t}$ independs on $p_{j,t}^{*}(\alpha)$ for instance.

(c) The last results combined with the Assumption 8 on stationary distributions imply the following equalities:

$$
E_{\xi^{\varepsilon}} \Pi\left(p_{j,t}^{*}(\alpha), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^{\varepsilon}\right) = E_{\xi^{\varepsilon}} \Pi\left(p_{j,t}^{*}(\alpha), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^{\varepsilon}\right)
$$

$$
E_{\xi^{\varepsilon}} \Pi\left(p_{j,-1}(i), P_{j,T}, Y_{j,T}, w_{j,T}(i); \xi_T^{\varepsilon}\right) = E_{\xi^{\varepsilon}} \Pi\left(p_{j,t}^{*}(\alpha), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^{\varepsilon}\right)
$$

We such results in mind, we once more apply some properties of uniform convergence and obtain:

$$
\begin{aligned}
E_{\xi^{\varepsilon}} \frac{\partial \Pi_j^*(\alpha)}{\partial \alpha} \leq\ & \gamma_j \left[\alpha \frac{\partial}{\partial \alpha} \left(\frac{1}{1-\alpha\beta} \right) + \frac{1}{1-\alpha\beta} - \frac{1}{(1-\alpha\beta)(1-\alpha\beta)} + \right. \\
& \left. + \frac{(1-\alpha)}{(1-\beta)} \frac{\partial}{\partial \alpha} \left(\frac{1}{1-\alpha\beta} \right) \right] \cdot E_{\xi^{\varepsilon}} \Pi\left(p_{j,t}^{*}(\alpha), P_{j,t}, Y_{j,t}, w_{j,t}(i); \xi_t^{\varepsilon}\right)
\end{aligned}
$$

It is easy to verify that the expression inside the brackets sum zero. Therefore we obtain the following final result when considering (67):

$$
\frac{\partial E_{\xi^{\varepsilon}} \Pi_j^*(\alpha)}{\partial \alpha} \leq 0
$$

Therefore, the expected profit $E_{\xi^{\varepsilon}} \Pi_j^*(\alpha)$ is a decreasing function of the degree of nominal price stickiness α. ■

A.3 Proof of Proposition 2

Proposition 2 *Under a timeless perspective, suppose that the i-th firm from* $\mathbb{C}_j$ *is present in the market and adopts the probability* α *of price stickiness, but when adjusting it decides instead for the sub-optimal price* $p_{j,t}^{*}(\bar{\alpha})$, *where* $\bar{\alpha} \leq \alpha$. *In such a context the* unconditional expectance of $\Pi_j^d(\alpha, p^*(\bar{\alpha}))$, *previously defined in* (32), *satisfies the following inequality:*

$$E_{\xi^\varepsilon} \Pi_j^d(\alpha, p^*(\bar{\alpha})) \leq E_{\xi^\varepsilon} \Pi_j^*(\bar{\alpha}) \quad \text{for } \bar{\alpha} \leq \alpha$$

Proof. Note that $\Pi_j^d(\alpha, p^*(\bar{\alpha}))$ equals $\Pi_{j,0}^d(i)$ when the latter, defined in (29), is evaluated in $\bar{p}_{j,t}(i) = p_{j,t}^*(\bar{\alpha})$, $\forall t \geq 0$.

Since $\left\{p_{j,t}^*(\alpha)\right\}_{t=0}^{\infty} \in \arg\max \Pi_{j,0}^d(i)$, its value-function $\Pi_j^*(\alpha) = \Pi_j^d(\alpha, p^*(\alpha))$ satisfies the following inequality:

$$\Pi_j^*(\alpha) \geq \Pi_j^d(\alpha, p^*(\bar{\alpha})) \quad , \quad \forall \bar{\alpha} \in [0,1] \tag{68}$$

Moreover, since $\bar{\alpha} \leq \alpha$, the Proposition 1 implies that $E_{\xi^\varepsilon} \Pi_j^*(\bar{\alpha}) \geq E_{\xi^\varepsilon} \Pi_j^*(\alpha)$. Thus, applying the unconditional expectance operator in both sides of (68), the following inequality must hold:

$$E_{\xi^\varepsilon} \Pi_j^d(\alpha, p^*(\bar{\alpha})) \leq E_{\xi^\varepsilon} \Pi_j^*(\bar{\alpha}) \quad \text{for } \bar{\alpha} \leq \alpha$$

Therefore, a profit-maximizing firm that optimally readjust its price with probability $(1-\bar{\alpha})$ have its expected profit decreased when increasing its price stickiness degree to $\alpha \geq \bar{\alpha}$ even when readjusted prices are $\left\{p_{j,t}^*(\alpha)\right\}_{t=0}^{\infty}$ instead of $\left\{p_{j,t}^*(\bar{\alpha})\right\}_{t=0}^{\infty}$. ■

A.4 Proof of Proposition 3

Proposition 3 *Provided that restriction* (37) *is satisfied and that* $E_{\xi^\varepsilon} \Pi_1^*(0) > 0$, *suppose that the households always choose the* $\mathbb{C}_1$, *e.g.* $\gamma_1 = 1$ *and* $\gamma_2 = 0$. *Therefore, there is a small enough probability* $\alpha_2 > 0$ such that *if the firms from* $\mathbb{C}_2$ *announce the following price setting mechanism from a given period* t *onwards*

$$p_{2,t}(i) = \begin{cases} p_t^*(0) & \text{, with probability } (1-\alpha_2) \\ p_{2,t-1}(i) & \text{, with probability } \alpha_2 \end{cases}$$

then all households realize that they have better changing their strategies to $\bar{\gamma}_1 = 0$ *and* $\bar{\gamma}_2 = 1$, *benefiting the firms from* $\mathbb{C}_2$.

Proof. Since households always choose the $\mathbb{C}_1$, the best response of all firms from such a consumption center is to adopt flexible prices in all periods, e.g. $\alpha_1 = 0$, for either $E_{\xi^\varepsilon} \Pi_1^*(0) > 0$ and such a strategy maximizes its expected discounted profit flow as a consequence of Proposition (1). Therefore, the aggregate price and product from $\mathbb{C}_1$ satisfy the following relations:

$$P_{1,t} = p_t^*(0) \tag{69}$$

$$y_{1,t}(i) = Y_{1,t}^n \quad , \quad \forall i \text{ from } \mathbb{C}_1 \tag{70}$$

From (13) we replicate the generalized Euler equation as follows:

$$\frac{u_C(Y_t)}{P_t} = \frac{u_C(Y_{1,t})}{P_{1,t}} = \frac{u_C(Y_{2,t})}{P_{2,t}} = \Lambda_t$$

Note that Assumption (3) on regularity conditions implies that $u_C(\cdot)$ is invertible, so we may obtain the instantaneous consumption utility for consuming only in center 1, e.g.

$$u\left(Y_{1,t}\right)=u\left[u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right] \tag{71}$$

Moreover, from the production function (19), and relations (70) and (71) we obtain the implied instantaneous labor disutility for consuming only in $\mathbb{C}_1$, as follows:

$$\nu\left(h_{1,t}\left(i\right)\right)=\nu\left[f^{-1}\left(A^{-1}u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right)\right]\ ,\ \forall i \text{ from } \mathbb{C}_1 \tag{72}$$

So far note that the firms from $\mathbb{C}_2$ make a zero profit, once their goods will not be demanded.

Suppose now that firms from $\mathbb{C}_2$ decide to adopt the strategy (38) of readjusting to $p_t^*\left(0\right)$ with probability $\left(1-\alpha_2\right)$, where $\alpha_2>0$ is sufficiently close to zero. Hence in the event in which households change their strategy to $\bar{\gamma}_2>0$, the firms from $\mathbb{C}_2$ would make profits such that $E_{\xi^\varepsilon}\Pi_2^d\left(\alpha_2,p^*\left(0\right)\right)\leq E_{\xi^\varepsilon}\Pi_2^*\left(0\right)$ as predicted by Proposition 2.

Note that if $E_{\xi^\varepsilon}\Pi_1^*\left(0\right)>0$ implies $E_{\xi^\varepsilon}\Pi_2^*\left(0\right)>0$ if $\bar{\gamma}_2>0$. Moreover, since the regularity assumptions on preferences and production implies that $E_{\xi^\varepsilon}\Pi_2^d\left(\alpha_2,p^*\left(0\right)\right)$ is a continuos function in α_2, Proposition 2 implies that there is a neighborhood close to zero in which $E_{\xi^\varepsilon}\Pi_2^d\left(\ \cdot\ ,p^*\left(0\right)\right)$ is positive and strictly decreasing. Therefore, if α_2 is in such a neighborhood then $E_{\xi^\varepsilon}\Pi_2^d\left(\alpha_2,p^*\left(0\right)\right)>0$ in the event in which households change their strategy to $\bar{\gamma}_2>0$.

Now we test if the adoption of the previous strategy would induce the households to change their strategy to $\bar{\gamma}_1=0$ and $\bar{\gamma}_2=1$. Adopting the previously described price setting strategy[43], the aggregate price in $\mathbb{C}_2$ can be represented as follows.

$$P_{2,t}=\left[\alpha_2\, P_{2,t-1}^{1-\theta}+\left(1-\alpha_2\right)\, p_t^*\left(0\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

From now on, we evaluate the representative household's utility in case of opting to change the choice strategy to $\bar{\gamma}_1=0$ and $\bar{\gamma}_2=1$. Similarly to the determination of (71) and (72), we obtain the (potential) instantaneous utility and the implied labor disutility derived for consuming only in $\mathbb{C}_2$, respectively, as follows.

$$\begin{aligned} u\left(Y_{2,t}\right) &= u\left[u_C^{-1}\left({\varkappa_{u,t}}^{1/(1-\theta)}\right)\right] \\ \nu\left(h_{2,t}\left(i\right)\right) &= \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\,{\varkappa_{\nu,t}}^{\theta/(1-\theta)}\ u_C^{-1}\left(\Lambda_t\ {\varkappa_{\nu,t}}^{1/(1-\theta)}\right)\right)\right], \forall i \text{ from } \mathbb{C}_2 \end{aligned}$$

Where

$$\begin{aligned} \varkappa_{u,t} &= \alpha_2\left(\Lambda_t\, P_{2,t-1}\right)^{1-\theta}+\left(1-\alpha_2\right)\left(\Lambda_t\, p_t^*\left(0\right)\right)^{1-\theta} \\ \varkappa_{\nu,t} &= \alpha_2\, P_{2,t-1}^{1-\theta}+\left(1-\alpha_2\right)\ p_t^*\left(0\right)^{1-\theta} \end{aligned}$$

Considering Lemma 1 and Lemma 2, we apply the Jensen's inequality twice and obtain the following results:

[43] With such a strategy, the firms from $\mathbb{C}_2$ smooth the flexible prices adopted by the firms from $\mathbb{C}_1$.

$$
\begin{aligned}
u\left(Y_{2,t}\right) &> \alpha_2\, u\left(Y_{2,t-1}\right) + \left(1-\alpha_2\right)\, u\left[u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right] \\
\nu\left(h_{2,t}\left(i\right)\right) &< \alpha_2\, \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, P_{2,t-1}^{\theta}\, u_C^{-1}\left(\Lambda_t\, P_{2,t-1}\right)\right)\right] + \\
&\quad + \left(1-\alpha_2\right)\, \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, \left(p_t^*\left(0\right)\right)^{\theta}\, u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right)\right]
\end{aligned}
$$

Since each decision on consumption centers can be thought as a lottery decision, we must consider the expected utility to evaluate which consumption center is the most preferred one. Therefore we apply the unconditional expectance operator into the previous inequalities:

$$
E_{\xi^\varepsilon} u\left(Y_{2,t}\right) > \alpha_2\, E_{\xi^\varepsilon} u\left(Y_{2,t-1}\right) + \left(1-\alpha_2\right)\, E_{\xi^\varepsilon} u\left[u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right] \tag{73}
$$

$$
\begin{aligned}
E_{\xi^\varepsilon} \nu\left(h_{2,t}\left(i\right)\right) &< \alpha_2\, E_{\xi^\varepsilon} \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, P_{2,t-1}^{\theta}\, u_C^{-1}\left(\Lambda_t\, P_{2,t-1}\right)\right)\right] + \\
&\quad + \left(1-\alpha_2\right)\, E_{\xi^\varepsilon} \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, \left(p_t^*\left(0\right)\right)^{\theta}\, u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right)\right]
\end{aligned} \tag{74}
$$

Remember that Λ_t independs on past aggregate variables[44] and that aggregate prices independ on individual firm price setting, for each single firm decision cannot affect the aggregate variables. Hence, considering the Assumption 8 on stationary distributions, we obtain the following equalities:

$$
\begin{aligned}
E_{\xi^\varepsilon} u\left[u_C^{-1}\left(\Lambda_t\, P_{2,t-1}\right)\right] &= E_{\xi^\varepsilon} u\left[u_C^{-1}\left(\Lambda_t\, P_{2,t}\right)\right] \\
E_{\xi^\varepsilon} \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, P_{2,t-1}^{\theta}\, u_C^{-1}\left(\Lambda_t\, P_{2,t-1}\right)\right)\right] &= E_{\xi^\varepsilon} \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, P_{2,t}^{\theta}\, u_C^{-1}\left(\Lambda_t\, P_{2,t}\right)\right)\right]
\end{aligned}
$$

Hence from (71) and noting that $u\left(Y_{2,t}\right) = u\left[u_C^{-1}\left(\Lambda_t\, P_{2,t}\right)\right]$, it is straightforward to proof that the instantaneous consuming expected utility obtained from consuming only in $\mathbb{C}_2$ is strictly greater than the one obtained from consuming only in $\mathbb{C}_1$, as follows:

$$
E_{\xi^\varepsilon} u\left(Y_{2,t}\right) > E_{\xi^\varepsilon} u\left(Y_{1,t}\right) \tag{75}
$$

Note now that $\nu\left(h_{2,t}\left(i\right)\right) = \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, P_{2,t}^{\theta}\, u_C^{-1}\left(\Lambda_t\, P_{2,t}\right)\right)\right]$. Hence it is also straightforward to derive the following result:

$$
E_{\xi^\varepsilon} \nu\left(h_{2,t}\left(i\right)\right) < E_{\xi^\varepsilon} \nu\left[f^{-1}\left(\frac{\left(p_{2,t}\left(i\right)\right)^{-\theta}}{A}\, \left(p_t^*\left(0\right)\right)^{\theta}\, u_C^{-1}\left(\Lambda_t\, p_t^*\left(0\right)\right)\right)\right]
$$

Aggregating the previous expression over the support $(0,1)$ and considering the uniform convergence theorem, we derive the following inequality:

[44] See Remark 1.

$$E_{\xi^{\varepsilon}} \int_0^1 \nu\left(h_{2,t}(i)\right) di < E_{\xi^{\varepsilon}} \int_0^1 \nu\left[f^{-1}\left(\frac{\left(p_{2,t}(i)\right)^{-\theta}}{A} \left(p_t^*(0)\right)^{\theta} u_C^{-1}\left(\Lambda_t\, p_t^*(0)\right)\right)\right] di$$

We now use the Assumption 8 on stationary distributions, the iterated expectation property and the $\mathbb{C}_2$ price setting strategy defined in (38) to derive the following steps on the right hand side term of the previous expression:

$$E_{\xi^{\varepsilon}} \int_0^1 \nu\left[f^{-1}\left(\frac{\left(p_{2,t}(i)\right)^{-\theta}}{A} p_t^*(0)^{\theta} u_C^{-1}\left(\Lambda_t\, p_t^*(0)\right)\right)\right] di =$$

$$= \alpha_2\, E_{\xi^{\varepsilon}} \int_0^1 \nu\left[f^{-1}\left(\frac{\left(p_{2,t-1}(i)\right)^{-\theta}}{A} p_t^*(0)^{\theta} u_C^{-1}\left(\Lambda_t\, p_t^*(0)\right)\right)\right] di +$$
$$+ (1-\alpha_2) \int_0^1 E_{\xi^{\varepsilon}} \nu\left[f^{-1}\left(A^{-1} u_C^{-1}\left(\Lambda_t\, p_t^*(0)\right)\right)\right] di$$

Thus, considering the previous result on unconditional expectance equalities, we obtain:

$$E_{\xi^{\varepsilon}} \int_0^1 \nu\left[f^{-1}\left(\frac{\left(p_{2,t}(i)\right)^{-\theta}}{A} p_t^*(0)^{\theta} u_C^{-1}\left(\Lambda_t\, p_t^*(0)\right)\right)\right] di = \int_0^1 E_{\xi^{\varepsilon}} \nu\left[f^{-1}\left(A^{-1} u_C^{-1}\left(\Lambda_t\, p_t^*(0)\right)\right)\right] di$$

Therefore, considering the result (72), it is straightforward to proof that the instantaneous implied labor expected disutility obtained from consuming only in $\mathbb{C}_2$ is strictly lower than the one obtained from consuming only in $\mathbb{C}_1$, as follows:

$$E_{\xi^{\varepsilon}} \int_0^1 \nu\left(h_{2,t}(i)\right) di < E_{\xi^{\varepsilon}} \int_0^1 \nu\left(h_{1,t}(i)\right) di \tag{76}$$

The result (75) was not surprising since the price setting strategy adopted by the firms from $\mathbb{C}_2$ reduced the price volatility generated by the $\mathbb{C}_1$ price setting strategy. However, assessing the implied labor expected disutility was not that trivial due to the wage channel. But, provided that households are sufficiently prudent and risk averse, according to the inequality (37), we could prove that the implied labor expected disutility obtained from consuming only in $\mathbb{C}_2$ was strictly lower than the one obtained from consuming only in $\mathbb{C}_1$.

From (75) and (76), the following result is satisfied in every period t:

$$E_{\xi^{\varepsilon}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(u\left(Y_{2,\tau}\right) - \int_0^1 \nu\left(h_{2,\tau}(i)\right) di \right) > E_{\xi^{\varepsilon}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(u\left(Y_{1,\tau}\right) - \int_0^1 \nu\left(h_{1,\tau}(i)\right) di \right)$$

Therefore, in each period t, the expected discounted utility flow obtained from changing the household's strategy to always choosing $\mathbb{C}_2$ (left hand side of the above inequality) is strictly greater than the one obtained from always choosing $\mathbb{C}_1$ (right hand side of the above inequality). Hence all households realize that they have better changing their strategies to $\bar{\gamma}_1 = 0$ and $\bar{\gamma}_2 = 1$, benefiting the firms from $\mathbb{C}_2$, which then make positive profits. ■

A.5 Proof of Theorem 1

Theorem 1 *Provided that restriction* (37) *is satisfied and that* $E_{\xi^\varepsilon}\Pi_j^*(0) > 0$, *there is no equilibrium in which the representative household always chooses the same consumption center. Therefore, under such assumptions, households are indifferent between consumption centers in equilibrium.*

Proof. Suppose, by contradiction and without loss of generality, that the representative household always chooses the $\mathbb{C}_1$, e.g. $\gamma_1 = 1$ and $\gamma_2 = 0$. Therefore, all firms from $\mathbb{C}_2$ make zero profit since their goods have no demand.

However, provided the conditions of Theorem 1 above, if all firms from $\mathbb{C}_2$ adopt the price setting strategy defined in (38) for a small enough probability $\alpha_2 > 0$, households realize that they have better changing their strategies to $\bar{\gamma}_1 = 0$ and $\bar{\gamma}_2 = 1$, as implied by Proposition 3. As a consequence, all firms from $\mathbb{C}_2$ would make positive profits. Therefore, adopting such a price setting strategy is a best response, in a context of sub-game perfect equilibria[45], and so firms have the incentives to adopt it.

Hence $\gamma_1 = 1$ and $\gamma_2 = 0$ is not an equilibrium strategy, for contradicting the statement of Definition 4. Similarly, there is no equilibrium in which $\gamma_1 = 0$ and $\gamma_2 = 1$. Thus, households randomize choosing $\gamma_1 \in (0,1)$ and $\gamma_2 \in (0,1)$.

It is straightforward to conclude that households are indifferent between consumption centers in equilibrium, otherwise they would always choose the favorite one. ■

A.6 Proof of Theorem 2

Theorem 2 *Provided that households are sufficiently prudent and risk averse, according to the inequality* (37) *of above Lemma 2 and that* $E_{\xi^\varepsilon}\Pi_j^*(0) \geq 0$, *equilibrium requires that all firms from both consumption centers adopt the same highest degree of price stickiness* $\alpha_1 = \alpha_2 = \alpha_{eq}$, *for which the expected profit is non-negative, e.g.* $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) \geq 0$, $\forall j$. *Non-trivial solutions implies* $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, $\forall j$. *Otherwise, if* $E_{\xi^\varepsilon}\Pi_j^*(1) \geq 0$ *then* $\alpha_{eq} = 1$ *represents the trivial solution.*

Proof. This theorem is proven for the non-trivial cases in which $E_{\xi^\varepsilon}\Pi_j^*(0) > 0$ and $E_{\xi^\varepsilon}\Pi_j^*(1) < 0$, $\forall j$.

As predicted by Theorem 1, equilibrium requires that $\gamma_1 \in (0,1)$ and $\gamma_2 \in (0,1)$. Moreover, households are indifferent between consumption centers in equilibrium, e.g. $E_{\xi^\varepsilon}U_1 = E_{\xi^\varepsilon}U_2$, where[46]

$$\begin{aligned} E_{\xi^\varepsilon}U_1 &= E_{\xi^\varepsilon}\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(u\left(Y_{1,\tau}\right) - \int_0^1 \nu\left(h_{1,\tau}(i)\right)di\right) \\ E_{\xi^\varepsilon}U_2 &= E_{\xi^\varepsilon}\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(u\left(Y_{2,\tau}\right) - \int_0^1 \nu\left(h_{2,\tau}(i)\right)di\right) \end{aligned}$$

Let α_1 and α_2 be the strategies adopted by firms from $\mathbb{C}_1$ and $\mathbb{C}_2$, respectively, who are present in the market in the sense of Assumption 11. Hence Corollary 1 implies that they make non-negative expected profits for adopting such strategies, e.g.

$$\begin{aligned} E_{\xi^\varepsilon}\Pi_1^*(\alpha_1) &\geq 0 \\ E_{\xi^\varepsilon}\Pi_2^*(\alpha_2) &\geq 0 \end{aligned}$$

[45] Note that there is a Nash equilibrium in which the households always choose a specific consumption center and all firms from both consumption centers always adopt flexible prices. Houever, such an equilibrium is not a subgame perfect equilibrium.

[46] Note that Assumption 8 on stationary distributions implies that $E_{\xi^\varepsilon}U_1$ and $E_{\xi^\varepsilon}U_2$ independs on time.

Suppose, by contradiction and without loss of generality, that $\alpha_1 > \alpha_2$ in equilibrium. Thus Proposition 1 implies that $E_{\xi^\varepsilon}\Pi_1^*(\alpha_1) \leq E_{\xi^\varepsilon}\Pi_2^*(\alpha_2)$. From now on, we consider the non-trivial case where the expected profits are strictly decreasing on the degree of nominal rigidity, e.g. $E_{\xi^\varepsilon}\Pi_1^*(\alpha_1) < E_{\xi^\varepsilon}\Pi_2^*(\alpha_2)$.

Since $E_{\xi^\varepsilon}U_1 = E_{\xi^\varepsilon}U_2$, if all firms from $\mathbb{C}_1$ adopted $\bar{\alpha}_1 = \alpha_2 < \alpha_1$ then they would make a larger expected profit, for consumers would remain indifferent among the consumption centers[47]. Therefore, adopting $\bar{\alpha}_1 = \alpha_2 < \alpha_1$ is a best response for firms in $\mathbb{C}_1$. Thus, $\alpha_1 > \alpha_2$ is not an equilibrium outcome, for contradicting the statement of Definition 4. Similarly, $\alpha_1 < \alpha_2$ cannot occur in equilibrium as well. Therefore, equilibrium requires that $\alpha_1 = \alpha_2 = \alpha_{eq}$, and such a fact is a common knowledge to all firms.

Suppose now, by contradiction, that $E_{\xi^\varepsilon}\Pi_1^*(\alpha_{eq}) > 0$. Consider the non-trivial case in which $\alpha_{eq} < 1$. Therefore, using a similar reasoning made to proof Proposition 3 above, there is a probability $\bar{\alpha} > \alpha_{eq}$ in a neighborhood of α_{eq} such that if the firms from $\mathbb{C}_2$ adopted instead the following price setting mechanism[48]

$$p_{2,t}(i) = \begin{cases} p_t^*(\alpha_{eq}) & \text{, with probability } (1-\bar{\alpha}) \\ p_{2,t-1}(i) & \text{, with probability } \bar{\alpha} \end{cases}$$

then all households would realize a best response of changing their strategies to $\bar{\gamma}_1 = 0$ and $\bar{\gamma}_2 = 1$.

Note that Proposition 2 implies that $E_{\xi^\varepsilon}\Pi_2^d(\bar{\alpha}, p^*(\alpha_{eq})) \leq E_{\xi^\varepsilon}\Pi_2^*(\alpha_{eq})$ if the households did not change their choices on γ_j. However, their best response would be to change them to $\bar{\gamma}_1 = 0$ and $\bar{\gamma}_2 = 1$, benefiting the firms from $\mathbb{C}_2$, for the firms' expected profits would increase to $(\gamma_2)^{-1} E_{\xi^\varepsilon}\Pi_2^d(\bar{\alpha}, p^*(\alpha_{eq}))$. Again, with a similar line of argument used to proof Proposition 3 one easily shows that if $\bar{\alpha}$ is sufficiently close to α_{eq} then $(\gamma_2)^{-1} E_{\xi^\varepsilon}\Pi_2^d(\bar{\alpha}, p^*(\alpha_{eq})) > E_{\xi^\varepsilon}\Pi_2^*(\alpha_{eq})$, e.g. the firms from $\mathbb{C}_2$ realize that adopting the previous price setting strategy is a best response in the case of $E_{\xi^\varepsilon}\Pi_1^*(\alpha_{eq}) > 0$. Hence, $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) > 0$ is not an equilibrium outcome for $\alpha_{eq} < 1$.

In the case in which $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, firms would have no incentive to decrease even more the degree of price stickiness, for such an action would make their expected profits to be negative. Therefore equilibrium requires that $E_{\xi^\varepsilon}\Pi_j^*(\alpha_{eq}) = 0$, if $\alpha_{eq} < 1$.

If $E_{\xi^\varepsilon}\Pi_j^*(1) \geq 0$, one easily verifies that $\alpha_{eq} = 1$. ■

[47] Since $\bar{\alpha}_1 = \alpha_2$, the unconditional expectance on aggregate variables would be the same among the consumption centers, thus there would be no difference in terms of consumption utility and labor disutility among them.

[48] Under such a strategy, the aggregate price of $\mathbb{C}_2$ would be $P_{2,t} = \left[\bar{\alpha}\, P_{2,t-1}^{1-\theta} + (1-\bar{\alpha})\, p_t^*(\alpha_{eq})^{1-\theta}\right]^{\frac{1}{1-\theta}}$.