Ramsey Meets Hosios: The Optimal Capital Tax and Labor Market Efficiency

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Abstract

Heterogeneity between unemployed and employed individuals matters for optimal fiscal policy. This paper considers the consequences of such heterogeneity for the determination of optimal capital and income taxes in a model with matching frictions in the labor market. In line with a recent finding in the literature, we find that the optimal capital tax is typically non-zero because it is used to indirectly mitigate an externality that arises from search and matching frictions, one that cannot be corrected by the labor tax. However, the consideration of heterogeneity makes our result differ in an important way: even for a well-known parameter configuration that typically eliminates this externality, we continue to find a non-zero optimal capital tax. This difference stems from heterogeneity in welfare between the employed and the unemployed that gives rise to an insider/outsider problem in wage bargaining. An employed individual does not internalize how the outcome of wage negotiations affects the welfare of the unemployed, while the Ramsey planner does internalize this effect.

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1 Introduction

Heterogeneity between unemployed and employed individuals matters for optimal fiscal policy. This paper considers the consequences of such heterogeneity for the determination of optimal capital and income taxes in a model with matching frictions in the labor market. In line with a recent finding by Domeij (2005), we find that the optimal capital tax is typically non-zero because it is used to indirectly mitigate an externality that arises from search and matching frictions, one that cannot be corrected by the labor tax. However, the consideration of heterogeneity makes our result differ in an important way: even for a well-known parameter configuration that typically eliminates this externality, we continue to find a non-zero optimal capital tax. This difference stems from the heterogeneity in welfare between the employed and the unemployed.

Models featuring labor search and matching have two central features. First, because matches are costly to form, existing matches generate a surplus to be split between the worker and the firm. Second, matching models exhibit an externality due to the relative number of agents on each side of the labor market. This "market-tightness" externality arises from the fact that one additional job-seeker in the market increases the probability that a firm will match with a worker but decreases the probability that job-seekers already in the market will match with a firm.[1] Hosios (1990) shows these externalities are balanced when a worker's share of the match surplus equals his contribution to the formation of the match, yielding the socially-optimal market tightness. In a model with no taxes, the Hosios condition typically boils down to a simple parameter restriction, specifically that the bargaining power of workers equals the elasticity of workers' input into the matching technology.

In a model in which government spending must be financed by a combination of labor and capital income taxes, Domeij (2005) finds that when this parameter restriction is not satisfied, the optimal capital tax is non-zero because it can be used to correct the market-tightness externality. This result is an instance of using the capital tax to substitute for a missing tax instrument.[2] However, at the parameter restriction typically associated with the Hosios condition, he finds the optimal capital tax is zero. This latter result is quite surprising because it is known that in the presence of distortionary labor taxes this simple parameter restriction generally does not eliminate the matching externality.[3] Domeij's (2005) result seems to depend on the lack of any welfare

[1] Equivalently, the externality can be thought of as arising from the fact that one additional firm with a vacancy increases the probability that a job-seeker will match with a firm but decreases the probability that firms already in the market will match with a job-seeker.

[2] In this case, the missing instrument is one that directly affects market tightness. See Ljungqvist and Sargent (2004, p. 478) for more discussion of using the capital tax as part of an incomplete tax system.

[3] See Pissarides (2000), p. 210-211.

heterogeneity between employed and unemployed individuals.[4]

In contrast, we show that allowing for this heterogeneity makes the coincidence of this parameter restriction and the zero capital tax disappear. In our model, once a match is formed, bargaining occurs between a firm and an individual worker who cares only about his own welfare. These workers do not internalize how the outcome of the bargaining process influences the welfare of the unemployed. In particular, they fail to take into account the fact that bargaining for a higher wage lowers firms' profitability, which reduces firms' incentives to post vacancies. When fewer vacancies are posted, it is more difficult for unemployed individuals to find jobs, lengthening their expected duration of unemployment. This insider/outsider problem does not arise when bargaining occurs at the household level, as in Domeij (2005). The welfare loss of not taking into account this welfare heterogeneity when setting capital taxes is between 0.1 and 0.3 percent of consumption.

On the surface, this point might seem quite technical, but it is important for at least two reasons. First, as an ever-growing list of otherwise-standard dynamic macro models has begun incorporating labor search frictions, it seems inevitable that such models will be used to answer policy questions.[5] One advantage of labor search models is that one can think about unemployed *versus* employed individuals Our results show that whether or not one considers heterogeneity between the unemployed and employed matters for optimal policy. The second point is more conceptual, but is related to the first: it is quite common in this class of models to assume (as we also do) perfect consumption insurance, but for the consideration of optimal policy an outstanding issue is whether or not to assume "perfect leisure insurance" as well. We interpret Domeij's (2005) model as making such an assumption. To the extent that leisure matters for optimal policy, this is an open issue to deal with.

The rest of our paper is organized as follows. In Section 2, we lay out our model. Section 3 presents the Ramsey problem. Section 4 describes how we parameterize our model. Results are presented in Section 5, along with a detailed analysis. Section 6 concludes.

2 Model

The model embeds the Pissarides (2000) textbook search model into a general equilibrium framework. The crucial feature of our model is that there are some individuals who are employed and other individuals who are unemployed, and each individual cares about his own labor market status. Bargaining occurs between an individual worker and the firm. The structure we employ ensures perfect consumption insurance between employed and unemployed individuals. While this simplifi-

[4] Domeij's (2005) model builds on that of Shi and Wen (1999).

[5] A few recent examples of dynamic models incorporating labor search frictions are Gertler and Trigari (2005), Krause and Lubik (2004), Trigari (2003), and Walsh (2005).

cation shuts down an important source of heterogeneity between the unemployed and the employed, it allows us to focus on the implications for fiscal policy stemming from welfare heterogeneity due solely to the utility of work versus non-work.

2.1 Households

There is a continuum of households in the economy. Each household consists of a continuum of measure one of family members. Each member of the household either works during a given time period or is unemployed and searching for a job. There is a measure n_t of workers in the household and a measure $1-n_t$ of unemployed individuals. We assume that total household income is divided evenly amongst all individuals, so each individual has the same consumption.[6]

The household's discounted lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \int_0^{n_t} v(h_t^i) di + \int_{n_t}^1 A^i di \right], \tag{1}$$

where $u(c)$ is each family member's utility from consumption, $v(h^i)$ is the disutility suffered by individual i from working h^i hours, and A^i is the (constant) utility enjoyed by individual i from leisure. The functions u and v satisfy $u'(c) > 0$, $u''(c) < 0$, $v'(h) < 0$, and $v''(h) < 0$. We assume symmetry in hours worked amongst the employed, so that $h^i = h$, as well as symmetry in the utility of leisure amongst the unemployed, so that $A^i = A$. Thus, household lifetime utility can be expressed as

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + n_t v(h_t) + (1 - n_t) A \right]. \tag{2}$$

The household does not choose how many family members work, nor does it choose the number of hours of work for those who do work. As described below, the number of people who work is determined by a random matching process, and hours worked are determined in bargaining between an individual worker and a firm. The household chooses sequences of consumption, real government bond holdings, and capital $\{c_t, b_t, K_{t+1}\}$, to maximize lifetime utility subject to the flow budget constraint

$$c_t + K_{t+1} + b_t = (1 - \tau_t^n) w_t h_t n_t + R_{b,t} b_{t-1} + \left[1 + \left(1 - \tau_t^k \right) (r_t - \delta) \right] K_t + d_t, \tag{3}$$

where b_t denotes bond holdings at the end of period t, each unit of which pays a gross real return $R_{b,t+1}$ at the beginning of $t+1$, K_t denotes the household's capital holdings at the start of period t, and δ is the depreciation rate of capital. The hourly wage is w_t, and the capital rental rate is r_t. Household labor income is taxed at the rate τ_t^n, and capital income net of depreciation os taxed

[6] Thus, we follow much of the literature in this regard by assuming full consumption insurance between employed and unemployed individuals.

at the rate τ_t^k. Finally, d_t denotes profit income received from firms, which the household takes as given.

Denote by ϕ_t the time-t Lagrange multiplier on the flow budget constraint. The first order conditions with respect to c_t, b_t, and K_{t+1} are

$$u'(c_t) - \phi_t = 0, \tag{4}$$

$$-\phi_t + \beta R_{b,t+1}\phi_{t+1} = 0, \tag{5}$$

$$-\phi_t + \beta\phi_{t+1}\left[1 + \left(1 - \tau_{t+1}^k\right)(r_{t+1} - \delta)\right] = 0. \tag{6}$$

These first-order conditions can be combined to yield a standard Euler equation

$$u'(c_t) = \beta R_{b,t+1} u'(c_{t+1}) \tag{7}$$

and the no-arbitrage condition between government bonds and capital

$$R_{b,t+1} = \left[1 + \left(1 - \tau_{t+1}^k\right)(r_{t+1} - \delta)\right]. \tag{8}$$

We again point out that although all individuals are perfectly insured against consumption risk, there is welfare heterogeneity between unemployed and employed because of differences in their work status.

2.2 Firms

There is a representative firm that produces and sells a homogenous final good in a perfectly competitive product market. The firm must engage in costly search for a worker to fill each of its job openings. In each job j that will produce output, the worker and firm bargain simultaneously over the hours h_{jt} that will be worked in that job and the hourly pre-tax real wage w_{jt} paid in that position. Output of job j is given by $y_{jt} = g(k_{jt}, h_{jt})$, where $g(k, h)$ will be assumed to have constant returns to scale. The capital k_{jt} used in production is specific to a particular job and is rented by the firm in a spot capital market. Any two jobs i and j at the firm are identical, so from here on we suppress the first subscript and denote by h_t hours worked in any job, by w_t the wage in any job, and so on. Total output thus depends on the production technology and the measure of matches n_t that produce,

$$y_t = n_t g(k_t, h_t). \tag{9}$$

The total wages paid by the firm in any given job are $w_t h_t$, and capital rental payments for any given job are $r_t k_t$. The total wage bill of the firm is the sum of wages paid at all of its positions, $n_t w_t h_t$, and the total capital rental bill is $n_t r_t k_t$.

The firm begins period t with employment stock n_t. Its future employment stock depends on its current choices as well as the random matching process. With probability $q(\theta)$, taken as given by the firm, a vacancy will be filled by a worker. Labor-market tightness is $\theta \equiv v/u$, and the matching probability depends only on tightness. The firm rents capital k_t for use in each job and chooses vacancies to post v_t and future employment n_{t+1} to maximize

$$\Pi_t = \sum_{t=0}^{\infty} \frac{\beta^t \phi_t}{\phi_0} \left[n_t g(k_t, h_t) - n_t w_t h_t - n_t r_t k_t - \gamma v_t \right] \tag{10}$$

subject to the law of motion for employment

$$n_{t+1} = (1 - \rho^x)(n_t + v_t q(\theta_t)). \tag{11}$$

Firms incur the cost γ for each vacancy created. Job separation occurs with exogenous fixed probability ρ^x. Firms discount profits using the household's pricing kernel, $\beta\phi_{t+1}/\phi_0$, derived in the household problem above.

Associate the multiplier μ_t with the employment constraint. The first-order conditions with respect to n_{t+1}, v_t, and k_t are, respectively,

$$\mu_t = \left[\left(\frac{\beta \phi_{t+1}}{\phi_t} \right) \left(g(k_{t+1}, h_{t+1}) - w_{t+1} h_{t+1} - r_{t+1} k_{t+1} + (1 - \rho^x) \mu_{t+1} \right) \right], \tag{12}$$

$$\frac{\gamma}{q(\theta_t)} = (1 - \rho^x) \mu_t, \tag{13}$$

$$r_t = g_k(k_t, h_t). \tag{14}$$

Combining the optimality conditions (12) and (13) yields the job-creation condition,

$$\frac{\gamma}{q(\theta_t)} = \left[\left(\frac{\beta \phi_{t+1}}{\phi_t} \right) (1 - \rho^x) \left(g(k_{t+1}, h_{t+1}) - w_{t+1} h_{t+1} - r_{t+1} k_{t+1} + \frac{\gamma}{q(\theta_{t+1})} \right) \right], \tag{15}$$

which states that at the optimal choice, the vacancy-creation cost incurred by the firm is equated to the discounted expected value of profits from the match. Profits from a match take into account both the wage cost of that match as well as the capital rental cost for that match. This condition is a free-entry condition in the creation of vacancies.

2.3 Government

The government has a stream of purchases $\{g_t\}$ to finance using labor and capital income taxes and real debt. The flow budget constraint of the government is

$$\tau_t^n n_t w_t h_t + \tau_t^k (r_t - \delta) n_t k_t + b_t = g_t + R_t b_{t-1} \tag{16}$$

Note that capital taxation is net of depreciation.

2.4 Nash Bargaining

We assume that hours worked and the wage paid in any given job are simultaneously determined in a Nash bargain between the matched worker and firm. The choice of hours maximizes the joint surplus of the match, while the bargained wage payment divides the surplus between the worker and the firm. Details of the solution are given in Appendix A. Here we present only the outcome of the Nash bargain.

The choice of hours that maximizes the joint surplus from the match satisfies a standard labor optimality condition,

$$\frac{-v'(h_t)}{u'(c_t)} = (1 - \tau_t^n) g_h(k_t, h_t), \tag{17}$$

which states the firm's marginal revenue product must equal the household's marginal rate of substitution between consumption and leisure taking into account the distortionary labor tax. Thus, given any τ_t^n, the choice of hours is privately efficient because it maximizes the total surplus inside a match.

Bargaining over the wage payment yields

$$w_t h_t = \eta \left[g(k_t, h_t) - r_t k_t \right] + \frac{1-\eta}{1-\tau_t^n} \left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)} \right] + \frac{\eta\gamma}{q(\theta_t)} \left[\frac{\tau_{t+1}^n - \tau_t^n + \theta_t q(\theta_t)(1-\tau_{t+1}^n)}{1-\tau_t^n} \right], \tag{18}$$

where η is the bargaining power of the worker, $1 - \eta$ is the bargaining power of the firm, and h_t is determined by (17). The first term in square brackets on the right-hand-side is the firm's contemporaneous surplus (excluding wage payments) from consummating the match and is equal to output net of capital rental payments. The second term in square brackets is the worker's threat point in bargaining. If the worker walks away from the match, he would enjoy the utility value A of not working and avoid the disutility $v(h)$ from working (divided by $u'(c)$ to express in terms of goods). The third term in square brackets represents the saving on hiring costs that the firm enjoys when a job is created. In a model with either no distortionary taxation or a constant labor tax rate over time, this term reduces to $\eta\gamma\theta_t$. A time-varying tax rate thus drives an extra wedge into the bargaining process. This dynamic effect of taxes on bargaining may be an interesting one to study in future work.

Here, we limit our attention to steady-states, in which case the bargained wage payment reduces to

$$wh = \eta \left[g(k,h) - rk + \gamma\theta \right] + \frac{1-\eta}{1-\tau^n} \left[\frac{A}{u'(c)} - \frac{v(h)}{u'(c)} \right]. \tag{19}$$

In steady-state, the wedge created by τ^n works through its effect on the worker's threat point. For a given worker threat point, a rise in τ^n increases the wage payment. This simple mechanism is well-known in this class of models (see, for example, Pissarides p. 210-211) and is the key to our results. We provide more intuition, especially with regard to its implication for optimal capital taxation, in Section 5.1.4.

2.5 Matching Technology

Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a matching technology, $m(u_t, v_t)$, where u_t is the number of searching individuals and v_t is the number of posted vacancies. A match formed in period t will produce in period $t+1$ provided it survives exogenous separation at the beginning of period $t+1$. The evolution of total employment is thus given by

$$n_{t+1} = (1 - \rho^x)(n_t + m(u_t, v_t)). \tag{20}$$

2.6 Equilibrium

The household optimality conditions are summarized by (3), (7), and (8). We condense (3) and (7) along with the firm's optimality condition with respect to capital, (14), and the Nash solution for hours (17), into a present-value implementability constraint, given by

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \left[c_t - n_t h_t \left(g_h(k_t, h_t) - w_t - \frac{w_t}{g_h(h_t, k_t)} \frac{v'(h_t)}{u'(c_t)} \right) + \gamma v_t \right] =$$
$$u'(c_0) \left[R_{b,0} b_{-1} + \left(1 + \left(1 - \tau_0^k \right) (g_k(k_0, h_0) - \delta) \right) K_0 \right], \tag{21}$$

the derivation of which is presented in Appendix B. Along with the present-value implementability constraint, the conditions describing equilibrium are the no-arbitrage condition between government bonds and capital (8); the job-creation condition (15); the Nash solution for wage payments (18); the law of motion for employment (20); an identity restricting the size of the labor force to one,

$$n_t + u_t = 1; \tag{22}$$

and the resource constraint

$$c_t + N_{t+1} k_{t+1} - (1 - \delta) N_t k_t + g_t + \gamma u_t \theta_t = g(k_t, h_t). \tag{23}$$

Note that total costs of posting vacancies $\gamma u_t \theta_t$ are a resource cost for the economy, and we have made the substitution $v_t = u_t \theta_t$. Also note that we have made the substitution that aggregate capital K_t is related to per-match capital k_t by $K_t = N_t k_t$. The unknown processes are $\{c_t, n_{t+1}, h_t, k_{t+1}, u_t, \theta_t, w_t\}$.

3 Ramsey Problem

The problem of the Ramsey planner is to raise revenue for the government through labor and capital taxes in such a way that maximizes the welfare of the representative household, subject to the equilibrium conditions of the economy. In period zero, the Ramsey planner commits to a policy rule that maximizes the welfare of the representative household. Unlike the household, the Ramsey planner does have control over n_{t+1} and h_t (subject to the matching frictions and bargaining outcomes in the decentralized economy), and the planner's explicit consideration of the aggregate welfare of employed individuals versus unemployed individuals when making these choices is important for our results.

The Ramsey problem is thus to choose $\{c_t, n_{t+1}, h_t, k_{t+1}, u_t, \theta_t, w_t\}$ to maximize (2) subject to (14), (15), (18), (20), (21), (22), and (23). Because we are concerned only with steady-states, we consider only the Ramsey first-order conditions for $t > 0$. We assume that the time-zero Ramsey allocation is the same as the asymptotic steady-state Ramsey allocation, thus endogenizing the initial condition of the economy.[7] Throughout, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior. Once we have the Ramsey allocation, we back out the capital and labor tax rates that support the allocation using (8) and (17), respectively.

4 Model Parameterization

We characterize the Ramsey steady-state of our model numerically. Before turning to our results, we briefly describe how we parameterize our model. We assume that the instantaneous utility functions over consumption and hours worked take the forms

$$u\left(c_t\right) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{24}$$

and

$$v\left(h_t\right) = -\frac{\nu h_t^{1+\upsilon}}{1+\upsilon}. \tag{25}$$

The time unit of the model is meant to be a quarter, so we assume the subjective discount factor is $\beta = 0.99$, yielding an annual real interest rate of about four percent. We set the curvature parameter with respect to consumption to $\sigma = 1$, consistent with many macro models, and the curvature parameter with respect to hours to $\upsilon = 5$, which yields an intertemporal labor supply elasticity along the intensive margin of $1/5$. This value for υ is more in line with micro-estimates of labor supply elasticity than the typically lower values for υ used in macro models without an extensive margin. In macro models without an explicit extensive margin, lower values for υ (and thus higher values for the intertemporal elasticity of labor supply) are a stand-in for the unmodelled

[7]This adoption of the "timeless" perspective is innocuous here since we focus on only the steady-state rather than on transitional dynamics. We should also point out that, as is well-known in this literature, nothing guarantees a solution to the Ramsey problem, nor, if a solution exists, convergence to a steady-state. We do, however, numerically find a steady-state.

variations in the extensive margin. Because we have both an extensive and an intensive margin, we follow micro evidence in choosing v.

Our timing assumptions are such that production occurs after the realization of separations. Following the convention in the literature, we suppose that the unemployment rate is computed *before* the realization of separations. We set the probability of separation at $\rho^x = 0.10$, a typical value in this class of models. Thus, letting n denote the steady-state level of employment, $n(1-\rho^x)^{-1}$ is the employment rate, and $1 - n(1-\rho^x)^{-1}$ is the steady-state unemployment rate.

The production function is Cobb-Douglas,

$$g(k_t, h_t) = k_t^{1-\alpha} h_t^{\alpha}, \tag{26}$$

and we set labor's share to $\alpha = 0.7$. The quarterly depreciation rate is $\delta = 0.02$

The matching technology is also Cobb-Douglas,

$$m(u_t, v_t) = \psi u_t^{\xi_u} v_t^{1-\xi_u}, \tag{27}$$

with the elasticity of matches with respect to the number of unemployed set to $\xi_u = 0.40$, following Blanchard and Diamond (1989), and ψ a calibrating parameter that can be interpreted as a measure of matching efficiency.

The remaining parameter values are calibrated in a version of our model in which we shut down government spending. With zero government spending (and hence no need for distortionary taxes), the Hosios condition for the socially-optimal level of labor-market tightness is satisfied by setting the worker's power in Nash bargaining to the elasticity of matches with respect to the number of unemployed, so $\eta = \xi_u = 0.40$. We set $A = 0.40$, in line with much of the labor search literature. Because A is important for our results, though, we present results for alternative values as well.[8] With this baseline value for A, we calibrate ν so that the steady-state number of hours worked in a match in the model without government spending is $h = 0.30$. We calibrate γ and ψ to hit the matching rates $q(\theta) = 0.70$ and $\theta q(\theta) = 0.60$. The resulting values are $\gamma = 0.3728$ and $\psi = 0.66$ Finally, the steady-state value of government debt is assumed to be $b = 0$. While clearly not realistic, the assumption of zero debt facilitates the welfare comparisons between the Ramsey policy and alternative sub-optimal policies we conduct in Section 5.3. Our main results are not very sensitive to b.

Table 1 summarizes the calibration in the version of our model with no government spending and $A = 0.40$. When we vary either A or η in our experiments, we keep the parameter values listed in Table 1 fixed.

[8] Hagedorn and Manovksii (2005) argue that this commonly-used value may understate the true value of non-market activity. In addition to its usual identification with unemployment benefits, they argue that A should also take into account the value of home production.

Fixed parameters	β	σ	υ	α	δ	ξ_u	ρ^x
	0.99	1	5	0.70	0.02	0.40	0.10
Calibrated parameters	ν		γ		ψ		
	500		0.3728		0.66		

Table 1: Baseline calibration. Top panel shows parameter values set exogenously. Bottom panel shows parameter values calibrated in the model with zero government spending, no vacancy subsidy, $A = 0.40$, and $\eta = \xi_u = 0.40$.

5 Quantitative Results

After obtaining the dynamic first-order conditions of the Ramsey problem, we impose steady-state and numerically solve the resulting non-linear system. The policy variables τ^k and τ^n are then determined residually from the equilibrium conditions.

5.1 Optimal Policy at $\eta = \xi_u$

In an important contribution to the understanding of search models, Hosios (1990) shows that when the surplus generated by a match between a worker and a firm is split in such a way that each party is exactly compensated for its contribution to the formation of that match, the resulting amount of market tightness θ is socially optimal given the primitive search and matching frictions. In an environment with a Cobb-Douglas matching technology, Nash bargaining, and no distortionary taxes, this efficient sharing rule says that the worker's share of the surplus, our parameter η, must equal the share of workers in the matching function, ξ_u. We begin analyzing our model by demonstrating its underlying search efficiency when there is no government spending and the Hosios condition is satisfied. We then proceed to show that this underlying efficiency is disrupted once government spending must be financed by distortionary taxes.

5.1.1 Search Efficiency Without Government Spending

To demonstrate the underlying search efficiency in our model, we shut down government spending, set $\eta = \xi_u$, and use the other parameter values listed in Table 1, with the exception that we vary A. We vary A because this parameter is critical when there is government spending to finance. Figure 1 shows the optimal capital and labor tax rates are zero as functions of A. Thus, at the Hosios condition with no government spending to finance, there is no reason to impose any corrective taxes because the outcome is socially-optimal.

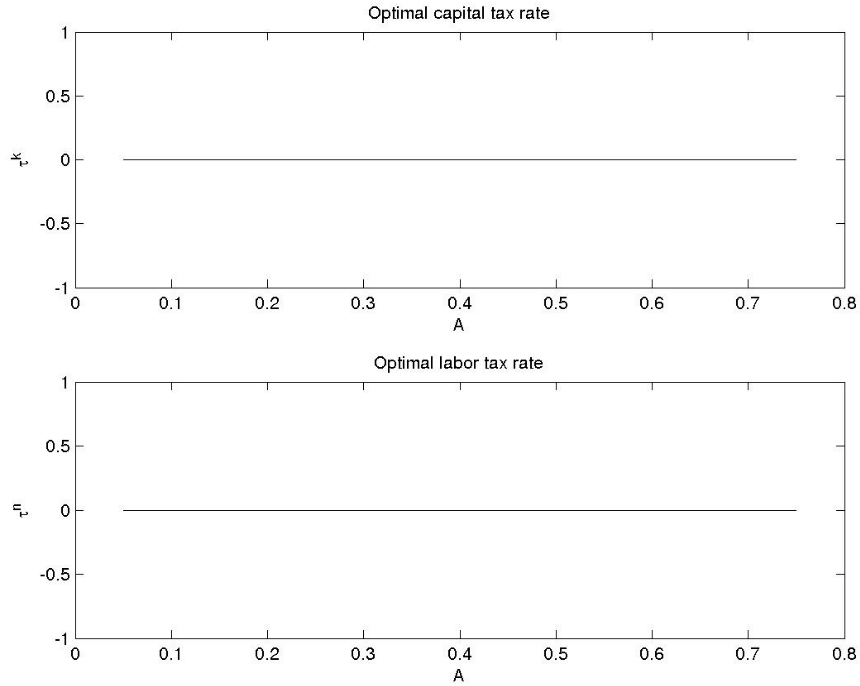

Figure 1: At $\eta = \xi_u$ and with zero government spending, optimal capital and labor tax rates as functions of A.

5.1.2 The Optimal Capital Tax and the Value of Non-Market Activity

Once there is government spending to finance, of course, all tax rates cannot be zero. To see how taxes optimally finance government purchases in our model, we fix $A = 0.40$, keep the parameter restriction $\eta = \xi_u$ intact, and vary government spending between zero and 50 percent of total output while again holding the rest of the parameters at their values in Table 1. Figure 2 presents the results of this experiment. The optimal labor tax rate rises from zero to near 80 percent as government spending rises from 0 to 50 percent of total output. However, the optimal capital tax rate falls from $\tau^k = 0$ to $\tau^k = -0.08$. This is in contrast to the celebrated zero-limiting-capital-tax result, which is independent of the level of government spending. In our model, the optimal policy involves a capital subsidy that grows with the share of government expenditures.

The capital subsidy is not due to any inefficiency in capital accumulation, however. Indeed, there is no wedge between the rental price and the marginal product of capital — we have $r = g_k$ always, as condition (14) states. Thus, the capital subsidy is trying to undo some other distortion in the economy. This other distortion is in labor-market tightness. Lacking an instrument that directly affects θ, the capital subsidy acts as a substitute.[9]

[9] Its ability to do so in an incomplete tax system is well-known — see Ljungqvist and Sargent (2004, p. 478).

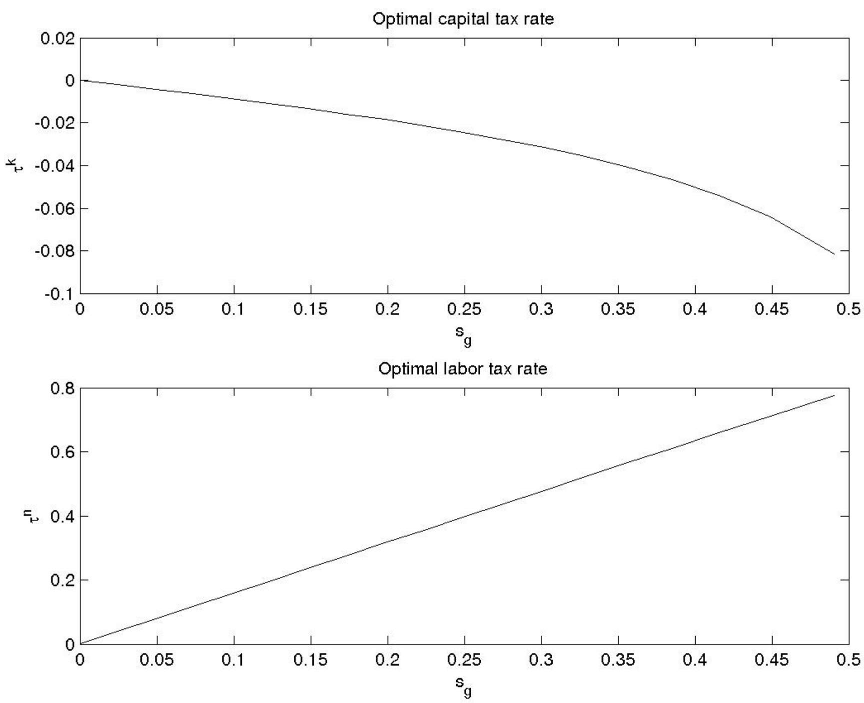

Figure 2: At $\eta = \xi_u$ and with $A = 0.40$, optimal capital and labor tax rates as functions of the share of government spending in total output, s_g.

5.1.3 A Vacancy Subsidy

We briefly demonstrate that the capital tax is in fact substituting for a more direct instrument. The two natural candidates for a direct labor-market instrument are a vacancy subsidy or an unemployment benefit. In Domeij (2005), the two instruments are perfect substitutes for each other, as might be expected because all that matters for search efficiency is market tightness θ, so that affecting either v or u can do the job. Thus, we will only consider a vacancy subsidy, which we introduce into our model in a straightforward way. We replace γ with $\gamma(1-\tau_t^s)$ in the firm's profit function and the resulting job-creation condition, and we introduce $\tau_t^s \gamma v_t$ in the government budget constraint, where τ_t^s is the vacancy subsidy rate. If $\tau_t^s > 0$, the firm receives a subsidy for each vacancy it creates, while if $\tau_t^s < 0$, the firm pays a tax for each vacancy. Note that the total vacancy subsidy adds to government purchases g_t and is now part of the optimal financing problem.

With this vacancy subsidy available, we re-do the previous exercise. The labor tax rate is virtually the same with or without the vacancy subsidy, as comparison of the middle panel of Figure 3 with the bottom panel of Figure 2 shows.[10] The optimal capital tax is now zero for any level of government spending, as the top panel of Figure 3 shows. The capital subsidy was therefore acting as a proxy for a vacancy subsidy, as the bottom panel confirms. The vacancy subsidy rate rises from zero to about 50 percent as government spending rises from zero to 50 percent of output. The fundamental distortion being caused by the financing of government spending is thus in the labor market in the form of too few vacancies relative to searching workers. That is, the role of the vacancy subsidy — and, when the vacancy subsidy is absent, the role of the capital subsidy — is to increase θ.

5.1.4 The Role of the Value of Non-Market Activity and the Goal of Optimal Policy

Despite Domeij's (2005) result to the contrary, an inefficiency in the labor market — revealed by either $\tau^s > 0$ or $\tau^k < 0$ — is not surprising even though all of our experiments are conducted with the typical parameter restriction that guarantees the Hosios condition for labor market efficiency satisfied. The typical parameter restriction $\eta = \xi_u$ does not generally guarantee socially-optimal labor-market tightness in a model with distortionary labor taxes. To see this, consider the term

$$\frac{1}{1-\tau^n}\left[\frac{A}{u'(c)} - \frac{v(h)}{u'(c)}\right] \tag{28}$$

in equation (19), which is the worker's threat point in Nash bargaining over the wage. *Ceteris paribus*, a rise in τ^n makes the value of leisure relatively more attractive to a worker because part

[10] τ^n is not identical for a given s_g in the bottom panel of Figure 2 and the middle panel of Figure 3 because the total vacancy subsidy $\tau^s\gamma v$ in general will not be identical to the total capital subsidy $\tau^k(r-g_k)kn$. In other words, achieving a given reduction in the distortion in the labor market in general requires transferring an amount that depends on the channel through which the transfer occurs.

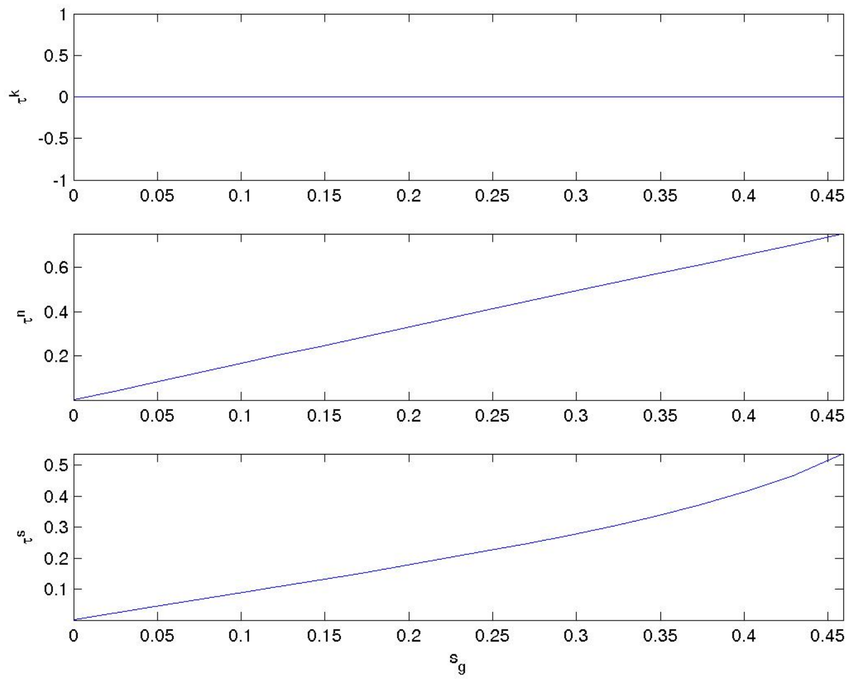

Figure 3: At $\eta = \xi_u$ and with $A = 0.40$, optimal capital and labor tax rates and vacancy subsidy rate as functions of the share of government spending in total output, s_g.

of his labor earnings are dissipated as taxes. In bargaining, the worker is able to leverage this higher threat point into a higher pre-tax wage. However, individual workers only care about their own welfare; they do not internalize how the outcome of the bargaining process affects the welfare of the unemployed. In particular, they fail to take into account the fact that a higher wage reduces firm profitability, thus reducing firms' incentives to post vacancies. When fewer vacancies are posted, it is more difficult for unemployed individuals to find jobs, which reduces total output. In setting optimal policy, the Ramsey planner internalizes this insider/outsider problem and corrects for it using the most efficient tax instrument available. A capital subsidy encourages capital accumulation, which raises the marginal product of a worker, which makes posting vacancies more attractive. This mechanism is behind the results in Figures 2 and 3.

The threat point (28) shows that A, the utility experienced by an unemployed individual, is important for the impact of τ^n on the labor market. To quantify its importance, we analyze our model along the A dimension. We now hold constant government spending at about 20 percent of total output, maintain the restriction $\eta = \xi_u$, and compute the optimal policy for various values of A. Figure 4 shows that when we shut down the vacancy subsidy, τ^k falls with A while τ^n rises with A. The intuition for the decline in τ^k comes directly from (28): a rise in A pushes up the bargained wage, which reduces firms' incentives to post vacancies. This incentive can be partially restored by using a capital subsidy. At about $A = 0.15$, the capital tax is zero, but this value for the utility value of leisure is much lower than typically used in the literature.

The capital subsidy in Figure 4 once again is used to indirectly affect labor market tightness, just as we found above. Re-introducing the vacancy subsidy, we have the optimal tax rates shown in Figure 5. The optimal vacancy subsidy rises as A rises, while the optimal capital tax is always zero.

To summarize our findings thus far: in contrast to Domeij (2005), we find that under the parameter restriction typically associated with the Hosios condition, the optimal capital tax is non-zero and is typically negative. The capital tax is used to indirectly correct a distortion that arises in the labor market because heterogeneity in the welfare of employed and unemployed individuals gives rise to an insider/outsider problem; workers do not internalize how the outcome of the wage bargaining process influences the job finding rate of the unemployed.

5.2 Optimal Policy Away From the Hosios Condition

In Section 5.1, we saw that even though the typical Hosios parameterization was in place, the Ramsey policy called for correcting an inefficiency in labor-market tightness, either directly through a vacancy subsidy or indirectly via a capital tax. The result stems from the fact that with distortionary taxes, $\eta = \xi_u$ does not deliver the Hosios condition; at $\eta = \xi_u$, the worker's bargaining

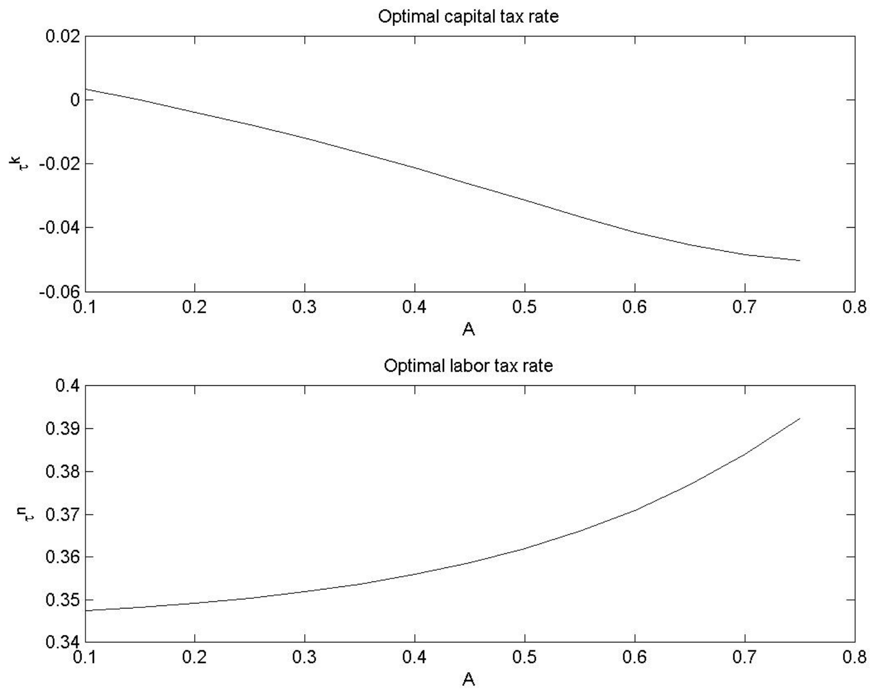

Figure 4: At $\eta = \xi_u$, the optimal capital tax rate and optimal labor tax rate as functions of A.

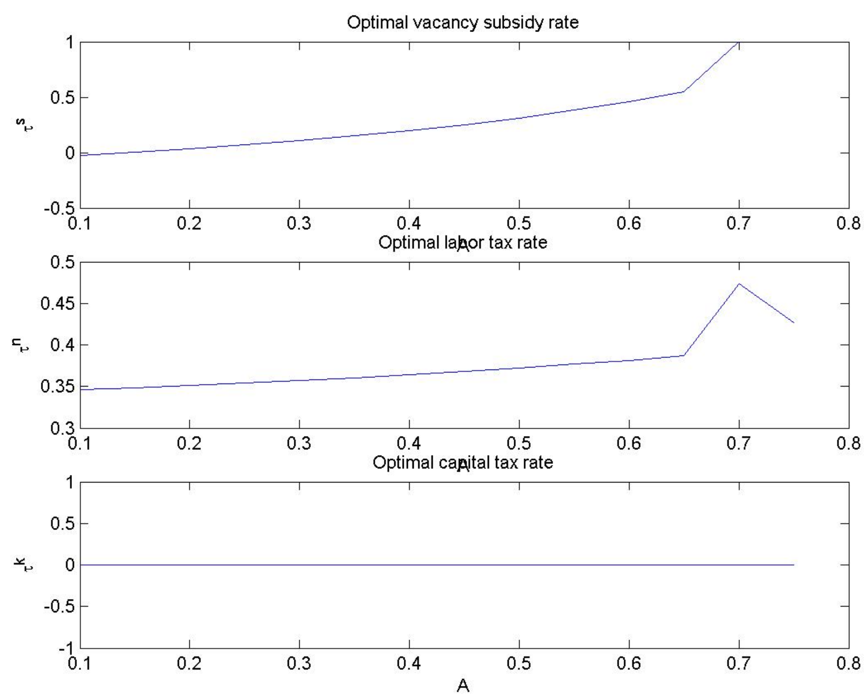

Figure 5: At $\eta = \xi_u$, the optimal vacancy subsidy rate, optimal labor tax rate, and optimal capital tax rate as functions of A.

power is effectively larger than his contribution to the formation of the match. That is, the effective η is larger than ξ_u because $\tau^n > 0$ makes leisure relatively more attractive, requiring the firm to offer a larger pre-tax wage payment to induce acceptance.

In models where all features of the economy except w and θ are parameters, Pissarides (2000, Chapter 9) shows that efficiency with distortionary taxes requires $\eta < \xi_u$. This result carries over to our more complicated model as well. It is straightforward to demonstrate that $\eta < \xi_u$ delivers efficiency in our model. Assuming no vacancy subsidy and holding government spending at about 20 percent of output, we plot in Figure 6 the optimal capital tax rates as a function of the bargaining power η for several values of A. For our benchmark $A = 0.40$, the workers' bargaining power at which a zero capital tax is optimal is around $\eta = 0.35$, smaller than $\xi_u = 0.40$. Figure 6 shows that A shifts the profile of τ^k with respect to η. For $A = 0.70$, which is in line with the value Hagedorn and Manovskii (2005) find in the presence of distortionary taxes, the capital tax is zero at about $\eta = 0.18$. For $A = 0.20$, much lower than typically used in the literature, τ^k is approximately zero at $\eta = \xi_u = 0.40$.[11]

We also plot in Figure 6 how τ^k varies with η in Domeij's (2005) model.[12] As he shows, $\tau^k = 0$ exactly at $\eta = \xi_u$. In our model, only a particular choice of the value of A makes this coincidence hold. In Domeij (2005), even though it is atomistic workers who make contact with firms and engage in work, the threat point in wage bargaining is a household-level threat point, rather than an individual-level threat point.[13] We interpret this as some type of collective bargaining — or, in other words, the lack of an insider-outsider problem in the bargaining process. Even though it is particular individuals who must work and experience the disutility of working, in Domeij's (2005) framework they all take into account only the welfare of the household (loosely speaking, union) when bargaining. This specification of bargaining has quite different implications than ours, because there is no individual-level utility gain term analogous to our $(A - v(h))$ that affects the bargain. Furthermore, it turns out that Domeij's (2005) setup results in a household-level threat point that endogenously adjusts in a way that exactly offsets any changes in the labor tax rate, ensuring that the Hosios condition always delivers socially-optimal market tightness despite $\tau^n > 0$.[14]

Thus, another way to understand the difference between our results is in the assumed bargaining protocols. Collective bargaining in Domeij's model delivers an exact coincidence between a zero capital tax and the typical Hosios parameterization, while individual-level bargaining in our model delivers no such coincidence. We think it is worthwhile to know the implications of the two different

[11] More precisely, $A = 0.147$ delivers $\tau^k = 0$ when $\eta = 0.40$.

[12] Based on our own implementation of his model. Our numerical results for his model match up with the results he presents in his Table 1.

[13] See Domeij (2005, equation 23).

[14] We have verified this in our own implementation of Domeij's (2005) model.

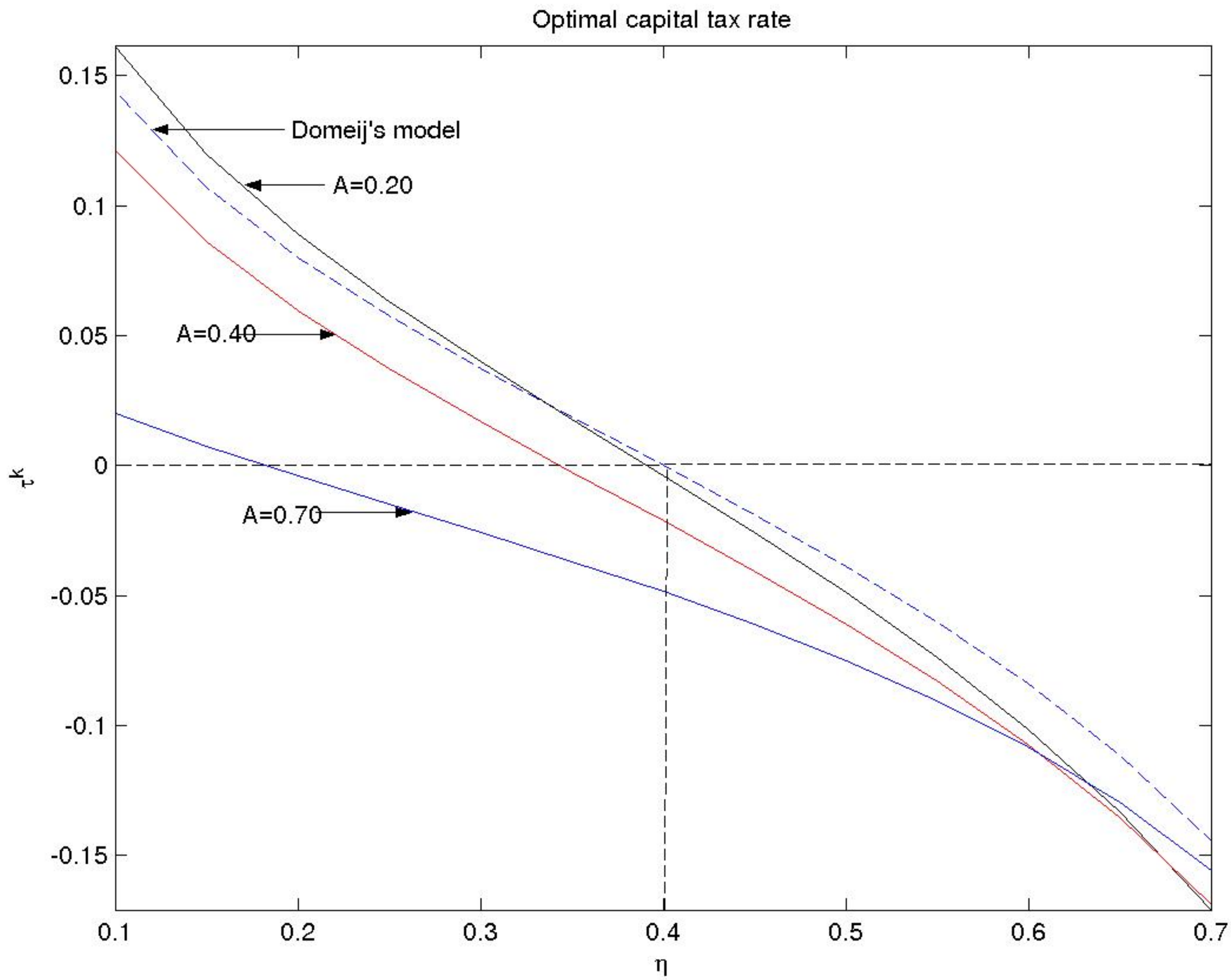

Figure 6: Optimal capital tax rate and optimal labor tax rate as functions of workers' bargaining power η in our model (for $A = 0.20$, $A = 0.40$, and $A = 0.70$) and our implementation of Domeij's model.

assumptions for optimal policy.

5.3 Welfare Loss of Zero Capital Tax

Examining Figure 6, one may think that it does not make much difference whether the optimal capital tax is exactly $\tau^k = 0$, as in Domeij's model, or simply near zero, as under the optimal policy in our model, at $\eta = 0.40$. We show here that in fact the welfare consequences, while small, are not negligible. We quantify the welfare loss of following a zero capital tax versus following the optimal policy for various values of A.

We answer this question in the following way. Holding the level of government spending fixed and $\eta = 0.40$, we compute steady-state welfare under the Ramsey policy for various values of A according to

$$(1-\beta)V^* = u(c^*) + n^* v(h^*) + (1-n^*)A, \tag{29}$$

where asterisks denote the steady-state Ramsey allocation and V^* denotes the resulting household welfare. We then solve a restricted version of the Ramsey problem in which τ^k is forced to be zero rather than left as a free policy variable. Requiring $\tau^k = 0$ means we must modify the Ramsey problem in the following way. To prevent any wedge between the intertemporal marginal rate of transformation and the intertemporal marginal rate of substitution (which is what a non-zero capital tax introduces), we add as a constraint on the Ramsey problem

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} - (1 - \delta + g_k(k_{t+1}, h_{t+1})) = 0, \tag{30}$$

which says that the gross return on bonds equals the gross return on capital with no tax wedge.[15] This restriction is simply the no-arbitrage condition (8) with $\tau^k_{t+1} = 0$ imposed.

Denote the allocation resulting from this restricted Ramsey problem with a bar over variables. We compute the welfare loss from the restricted Ramsey policy as the percentage increase 100ξ in consumption that the household requires in order to be just as well off as in the unrestricted Ramsey allocation — that is, we find the ξ such that

$$u(\bar{c}(1+\xi)) + \bar{n}v(\bar{h}) + (1-\bar{n})A = (1-\beta)V^*. \tag{31}$$

We follow the convention of computing welfare by assuming it is only consumption compensation that is required. As Figure 7 shows, the welfare loss of a policy with $\tau^k = 0$ increases with A, peaking around 0.3 percent of consumption at roughly $A = 0.70$. For our benchmark value $A = 0.40$, the welfare loss is 0.17 percent of consumption, which translates into roughly \$12 billion for the U.S. economy.

[15] See Chari and Kehoe (1999, p. 1679-1680) for more discussion on this type of restriction in a Ramsey problem.

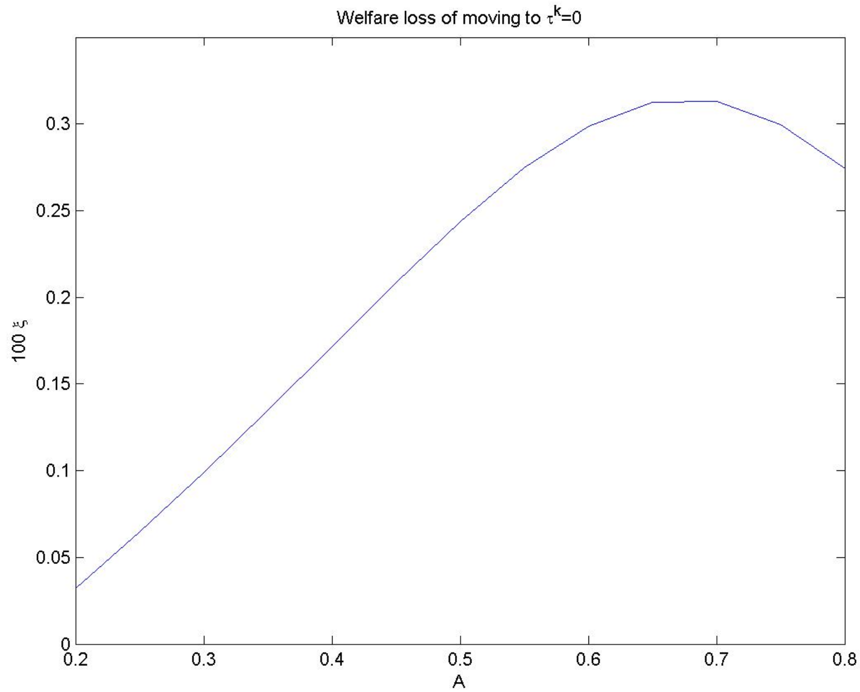

Figure 7: At $\eta = \xi_u$, the percentage increase in consumption required under a policy of $\tau^k = 0$ to make households just as well off as under the unrestricted Ramsey policy.

6 Conclusion

This paper shows that heterogeneity between unemployed and employed individuals matters for the optimal choice of fiscal policy. Our main result is that a non-zero capital tax is used to mitigate an externality that arises from search and matching frictions, one which cannot be corrected by the labor tax. In contrast to Domeij (2005), we find that this result holds even at a well-known parameter configuration that typically eliminates this matching externality. In our model, bargaining occurs between a firm and an individual worker who cares only about his own welfare. These workers do not internalize how the outcome of the bargaining process influences the welfare of the unemployed. As a result, differences in welfare across the two labor market states generates an insider/outsider problem that does not arise in Domeij's (2005) model. We find that the welfare loss of not taking into account this welfare heterogeneity when setting capital taxes is between 0.1 and 0.3 percent of consumption.

There are at least two directions in which this model could be extended in future research. First, heterogeneity enters our model through differences in the utility of employed versus unemployed individuals. While our results show that even this simple source of heterogeneity matters for optimal policy, it no doubt also is important to consider alternative sources of differences between employed and unemployed individuals. For example, we could move away from the consumption insurance assumed in this model, allowing individual consumption and capital holdings to differ across labor states. Moving in this direction, however, adds to the complexity of the problem by increasing the state space of the model.

Second, our results consider optimal steady state taxation and thus ignore dynamics. However, the solution to the wage bargaining problem implies that labor tax dynamics may actually be important for determining optimal fiscal policy. In particular, equation (18) shows that a time-varying tax rate drives an extra wedge into the division of the surplus created by a match. The way in which this wedge influences the optimal choice of fiscal policy may be interesting to study in future work.

A Derivation of Nash Bargaining Solution

Here we derive the Nash-bargaining solution between an individual worker and the firm for both the total wage payment and the number of hours worked. The value of working for an individual is

$$\mathbf{W_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)\left((1-\rho^x)\mathbf{W_{t+1}} + \rho^x \mathbf{U_{t+1}}\right)\right]. \tag{32}$$

The value of not working is

$$\mathbf{U_t} = \frac{A}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)\left(\theta_t q(\theta_t)(1-\rho^x)\mathbf{W_{t+1}} + (1-\theta_t q(\theta_t)(1-\rho^x))\mathbf{U_{t+1}}\right)\right], \tag{33}$$

where $\theta_t q(\theta_t) = m(u_t, v_t)/u_t$ is the probability that an unemployed individual finds a match. Note that because A is measured in utils, we divide by the marginal utility of consumption to express the utility of leisure in terms of goods.

The value of a filled job is

$$\mathbf{J_t} = g(k_t, h_t) - w_t h_t - r_t k_t + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)\mathbf{J_{t+1}}\right], \tag{34}$$

and we have, from the job-creation condition,

$$\frac{\gamma}{q(\theta_t)} = \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)\mathbf{J_{t+1}}\right]. \tag{35}$$

The firm and worker choose w_t and h_t to maximize the Nash product

$$\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta} \mathbf{J_t}^{1-\eta}, \tag{36}$$

with η the bargaining power of the worker. The first-order condition with respect to w_t is

$$\eta\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta-1}\left(\frac{\partial \mathbf{W_t}}{\partial w_t} - \frac{\partial \mathbf{U_t}}{\partial w_t}\right) J_t^{1-\eta} + (1-\eta)\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta} J_t^{-\eta}\frac{\partial \mathbf{J_t}}{\partial w_t} = 0. \tag{37}$$

With $\frac{\partial \mathbf{W_t}}{\partial w_t} = (1-\tau_t^n)h_t$, $\frac{\partial \mathbf{U_t}}{\partial w_t} = 0$, and $\frac{\partial \mathbf{J_t}}{\partial w_t} = -h_t$, the first-order condition gives the Nash sharing rule

$$\frac{\mathbf{W_t} - \mathbf{U_t}}{1-\tau_t^n} = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{38}$$

The labor tax drives a wedge in the Nash sharing rule, so net-of-taxes the worker receives a smaller share of the surplus than he would absent the tax.

Using the definitions of $\mathbf{W_t}$ and $\mathbf{U_t}$,

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} - A + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)\left((1-\rho^x)(1-\theta_t q(\theta_t))\mathbf{W_{t+1}} - (1-\rho^x)(1-\theta_t q(\theta_t))\mathbf{U_{t+1}}\right)\right]. \tag{39}$$

Combine terms on the right-hand-side to get

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)} + \beta\left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-\rho^x)(1-\theta_t q(\theta_t))(\mathbf{W_{t+1}} - \mathbf{U_{t+1}})\right]. \tag{40}$$

Using the sharing rule $\mathbf{W_t} - \mathbf{U_t} = \frac{(1-\tau_t^n)\eta}{1-\eta}\mathbf{J_t}$,

$$\mathbf{W_t} - \mathbf{U_t} = (1-\tau_t^n)w_t h_t + \frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)} + (1-\theta_t q(\theta_t)) \left(\frac{\eta}{1-\eta}\right) \beta \left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right) (1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}} \right], \tag{41}$$

or

$$\frac{\mathbf{W_t} - \mathbf{U_t}}{1-\tau_t^n} = w_t h_t + \frac{1}{1-\tau_t^n}\left[\frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)}\right] + \left(\frac{(1-\theta_t q(\theta_t))}{1-\tau_t^n}\right)\left(\frac{\eta}{1-\eta}\right)\beta E_t \left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right) (1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}} \right]. \tag{42}$$

Next, insert this in the sharing rule to get

$$w_t h_t + \frac{1}{1-\tau_t^n}\left[\frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)}\right] + \left(\frac{(1-\theta_t q(\theta_t))}{1-\tau_t^n}\right)\left(\frac{\eta}{1-\eta}\right)\beta \left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right) (1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}} \right] = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{43}$$

Using $\mathbf{J_t} = g(k_t, h_t) - w_t h_t - r_t k_t + \frac{\gamma}{q(\theta_t)}$ on the right-hand-side,

$$w_t h_t + \frac{1}{1-\tau_t^n}\left[\frac{v(h_t)}{u'(c_t)} - \frac{A}{u'(c_t)}\right] + \left(\frac{(1-\theta_t q(\theta_t))}{1-\tau_t^n}\right)\left(\frac{\eta}{1-\eta}\right)\beta \left[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right) (1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}} \right] = \frac{\eta}{1-\eta}\left(g(k_t, h_t) - w_t h_t - r_t k_t + \frac{\gamma}{q(\theta_t)}\right). \tag{44}$$

Solving for $w_t h_t$,

$$w_t h_t = \eta\left[g(k_t, h_t) - r_t k_t + \frac{\gamma}{q(\theta_t)}\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)}\right] - \eta\left(\frac{1-\theta_t q(\theta_t)}{1-\tau_t^n}\right)\beta\left[\frac{u'(c_{t+1})}{u'(c_t)}(1-\rho^x)(1-\tau_{t+1}^n)\mathbf{J_{t+1}}\right]. \tag{45}$$

Make the substitution $\mathbf{J_{t+1}} = g(k_{t+1}, h_{t+1}) - w_{t+1}h_{t+1} - r_{t+1}k_{t+1} + \frac{\gamma}{q(\theta_{t+1})}$ to write

$$w_t h_t = \eta\left[g(k_t, h_t) - r_t k_t + \frac{\gamma}{q(\theta_t)}\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)}\right] - \eta\left(\frac{1-\theta_t q(\theta_t)}{1-\tau_t^n}\right)\beta\left[\frac{u'(c_{t+1})}{u'(c_t)}(1-\rho^x)(1-\tau_{t+1}^n)\left(g(h_{t+1}) - w_{t+1}h_{t+1} + \frac{\gamma}{q(\theta_{t+1})}\right)\right]. \tag{46}$$

Next, combine the term involving $\gamma/q(\theta_t)$ in the first expression in square brackets with the third term on the right-hand-side. Rearranging, the wage payment can be written as

$$w_t h_t = \eta\left[g(k_t, h_t) - r_t k_t\right] + \frac{1-\eta}{1-\tau_t^n}\left[\frac{A}{u'(c_t)} - \frac{v(h_t)}{u'(c_t)}\right] + \frac{\eta\gamma}{q(\theta_t)}\left[\frac{\tau_{t+1}^n - \tau_t^n + \theta_t q(\theta_t)(1-\tau_{t+1}^n)}{1-\tau_t^n}\right], \tag{47}$$

which is expression (18) in the text.

Turning to the determination of hours worked in a match, the first-order-condition of the Nash product with respect to h_t is

$$\eta\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta-1}\left(\frac{\partial \mathbf{W_t}}{\partial h_t} - \frac{\partial \mathbf{U_t}}{\partial h_t}\right) J_t^{1-\eta} + (1-\eta)\left(\mathbf{W_t} - \mathbf{U_t}\right)^{\eta} J_t^{-\eta}\frac{\partial \mathbf{J_t}}{\partial h_t} = 0. \tag{48}$$

With $\frac{\partial \mathbf{W_t}}{\partial h_t} = (1-\tau_t^n)w_t + \frac{v'(h_t)}{u'(c_t)}$, $\frac{\partial \mathbf{U_t}}{\partial h_t} = 0$, and $\frac{\partial \mathbf{J_t}}{\partial h_t} = g_h(k_t, h_t) - w_t$, the first-order-condition can be written

$$\frac{\eta}{1-\eta}\mathbf{J_t}\left[(1-\tau_t^n)w_t + \frac{v'(h_t)}{u'(c_t)}\right] = \left(\mathbf{W_t} - \mathbf{U_t}\right)\left[w_t - g_h(k_t, h_t)\right]. \tag{49}$$

Substituing $\frac{\eta}{1-\eta}\mathbf{J_t} = \frac{\mathbf{W_t} - \mathbf{U_t}}{1-\tau_t^n}$, we have that hours are determined according to

$$\frac{-v'(h_t)}{u'(c_t)} = (1-\tau_t^n)g_h(k_t, h_t), \tag{50}$$

which is expression (17) in the text. Note this condition is unaffected by the wage. The labor tax rate can be expressed in terms of allocations as

$$\tau_t^n = 1 + \frac{1}{g_h(k_t, h_t)} \frac{v'(h_t)}{u'(c_t)}. \tag{51}$$

B Derivation of Implementability Constraint

The derivation of the implementability constraint follows the procedure laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). Start with the household flow budget constraint in equilibrium

$$c_t + K_{t+1} + b_t = (1-\tau_t^n)w_t h_t n_t + R_{b,t} b_{t-1} + \left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t + d_t. \tag{52}$$

Multiply by $\beta^t u'(c_t)$ and sum over dates,

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} + \sum_{t=0}^{\infty}\beta^t u'(c_t)b_t \quad = \quad & \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)R_{b,t}b_{t-1} \\ & + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)d_t.\end{aligned}$$

Use the household's Euler equation, $u'(c_t) = \beta[R_{b,t+1}u'(c_{t+1})]$, to substitute for $u'(c_t)$ in the term on the left-hand-side involving b_t,

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} + \sum_{t=0}^{\infty}\beta^{t+1} R_{b,t+1}u'(c_{t+1})b_t \quad = \quad & \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)R_{b,t}b_{t-1} \\ & + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t + \sum_{t=0}^{\infty}\beta^t u'(\end{aligned}$$

Canceling terms in the second summation on the left-hand-side with the second summation on the right-hand-side leaves only the time-zero bond position,

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} \quad = \quad & \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right] K_t \\ & + \sum_{t=0}^{\infty}\beta^t u'(c_t)d_t + u'(c_0)R_{b,0}b_{-1}.\end{aligned}$$

Again use $u'(c_t) = \beta[R_{b,t+1}u'(c_{t+1})]$ along with the no-arbitrage condition between government bonds and capital, $R_{b,t+1} = \left(1 + (r_t - \delta)\left(1 - \tau_t^k\right)\right)$, to substitute in for the second term on the left hand side involving K_{t+1}. Doing so leaves only the time-zero capital stock,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t h_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)d_t + u'(c_0)\left[R_{b,0}b_{-1} + \left(1 + (r_0 - \delta)\left(1 - \tau_0^k\right)\right)K_0\right]. \tag{53}$$

Now insert equilibrium profits, given by $d_t = n_t g(k_t, h_t) - w_t h_t n_t - r_t k_t n_t - \gamma v_t (1 - \tau_t^s)$ on the right hand side to yield

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[c_t - n_t g(k_t, h_t) + \tau_t^n w_t h_t n_t + r_t k_t n_t + \gamma v_t (1 - \tau_t^s)\right] = u'(c_0)\left[R_{b,0}b_{-1} + \left(1 + (r_0 - \delta)\left(1 - \tau_0^k\right)\right)K_0,\right] \tag{54}$$

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[c_t-\left(g\left(k_t,h_t\right)-\tau_t^n w_t h_t-g_k\left(k_t,h_t\right)k_t\right)n_t+\gamma v_t\left(1-\tau_t^s\right)\right]=u'(c_0)\left[R_{b,0}b_{-1}+\left(1+\left(g_k(k_0,h_0)-\delta\right)\left(1-\tau_0^k\right)\right)K_0\right] \tag{55}$$

where in the last expression we use the condition $r_t = g_k\left(k_t, h_t\right)$. Because g is homogeneous of degree one we have that

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[c_t-\left(g_h\left(k_t,h_t\right)-\tau_t^n w_t\right)n_t h_t+\gamma v_t\left(1-\tau_t^s\right)\right]=u'(c_0)\left[R_{b,0}b_{-1}+\left(1+\left(g_k(k_0,h_0)-\delta\right)\left(1-\tau_0^k\right)\right)K_0\right]. \tag{56}$$

Using the Nash solution for hours $\frac{-v'(h_t)}{u'(c_t)} = (1-\tau_t^n)g_h(h_t, k_t)$ we can write $g_h\left(k_t, h_t\right) - \tau_t^n w_t = g_h\left(k_t, h_t\right) - w_t - \frac{w_t}{g_h(h_t,k_t)}\frac{v'(h_t)}{u'(c_t)}$. Inserting this into the previous expression, we arrive at our form of the present value implementability constraint,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[c_t-n_t h_t\left(g_h\left(k_t,h_t\right)-w_t-\frac{w_t}{g_h(h_t,k_t)}\frac{v'(h_t)}{u'(c_t)}\right)+\gamma v_t\left(1-\tau_t^s\right)\right]=$$
$$u'(c_0)\left[R_{b,0}b_{-1}+\left(1+\left(g_k(k_0,h_0)-\delta\right)\left(1-\tau_0^k\right)\right)K_0\right],$$

presented in Section 3. Note that setting $\gamma = 0$ (no vacancy posting cost), $n_t = 1$ (no extensive margin), and using the result that in a Walrasian labor market $w_t = g_h(k_t, h_t)$, the implementabilty constraint becomes

$$\sum_{t=0}^{\infty}\beta^t\left[u'(c_t)c_t+v'(h_t)h_t\right]=u'(c_0)\left[R_{b,0}b_{-1}+\left(1+\left(g_k(k_0,h_0)-\delta\right)\left(1-\tau_0^k\right)\right)K_0\right], \tag{57}$$

which is identical to the household present-value budget constraint in a baseline (i.e., Lucas and Stokey (1983) or Chari and Kehoe (1999)) Ramsey model.

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