Relative Price Distortion and Optimal Monetary Policy in Open Economies

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Abstract

This paper addresses three issues on the conduct of monetary policy in open economies on the basis of a two-country model with Calvo-type sticky prices. Is the isomorphism of the optimal policy problems between closed and open economies robust to whether the foreign country is buffeted by cost-push shocks? How can we obtain a linear quadratic approximation that replicates the key results of the original optimal policy problem, especially when there is an analytical solution to the original problem in the presence of initial price dispersion? What are optimal policy recommendations for the central banks in open economies when both cost-push shock and initial price dispersion exist?

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Keywords: Cost-Push Shocks; Relative Price Distortion; Interdependence; Open Economy; Optimal Policy

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1 Introduction

Much of the recent literature in open macroeconomics has addressed the issue of how to conduct monetary policy in open economy models that include imperfect competition and nominal rigidities as mechanisms for non-neutralities of monetary policies. A very restricted set of examples include Benigno and Benigno (2004), Clarida, Gali and Gertler (2001, 2003) and Gali and Monacelli (2005). Furthermore, the recent welfare analysis of the monetary policy builds on linear-quadratic approximations to micro-founded optimization models with nominal rigidities, following Clarida, Gali, Gertler (1999) and Woodford (2003).

We focus on the following three issues on the optimal monetary policy for open economies based using a two-country model with Calvo-type sticky prices. First, we ask if the isomorphism of the optimal policy problems between closed and open economies can be preserved when the foreign country is subject to cost-push shocks. Second, we derive a linear quadratic approximation that replicates the key results of the original optimal policy problem, especially allowing for initial price dispersion. Third, we characterize the optimal policy recommendations for the central banks in open economies when both cost-push shocks and initial price dispersion exist.

In order to address these issues, our model maintains a simple structure which inherits from much of the recent open-economy literature—such as unit elasticity of substitution between domestic and foreign goods, complete asset market, and price setting in terms of producers' currency. The key difference from the existing literature, however, is that we explicitly allow for dynamic movements of relative price distortion responding to inflation and initial price dispersion, as done in a closed-economy setting by Yun (2005).

Our results can be summarized as follows. First, we demonstrate that the optimal monetary policy requires the home country to respond to output gaps of the foreign country, which can arise from foreign cost-push shocks. This is in contrast to Clarida, Gali and Gertler (2001, 2003), which argues for the isomorphism of the optimal policy problems between closed and open economies. The reason for such a difference is that their analysis exploits the assumption that cost-push shocks exist only in the home country, while our analysis permit cost-push shocks in both of the home and foreign countries.

The second issue we analyze is how one can formulate a linear-quadratic problem that replicate the optimal policy derived from the original economy without any approximations. In particular, given the set-up of the model analyzed above, optimal monetary policy implies that the optimal producer's price inflation rate is set to the growth rate of relative price distortion in a model with initial price dispersion. We derive this result from the original economy under the assumption that cost-push shocks do not exist in both of home and foreign countries. Since Yun (2005) shows that the same is true in a closed economy, our result implies that such characterization of the optimal inflation rate remains true for open and closed economies. We then present a linear-quadratic approximation to the original optimal policy problem, which preserves the key properties of the original analytic solution. In order to do so, we follow the approach taken in Woodford (2003), even though normalization for the measure of the relative price distortion differs from that of the conventional approximation.

Finally, we solve the linear quadratic optimization problem for the case where both of the initial relative price distortion and cost-push shock exist at home and aborad. In particular, with appropriately chosen short-run target levels of output gap and inflation, we present a generalized version of the optimal relationships between output gap and inflation rate analyzed in Clarida, Gali and Gertler (1999, 2001, 2002). Specifically, we demonstrate that—when the short-run target levels of output and inflation rate are defined as the optimal values under the optimal policy in the absence of cost-push shocks—the deviations of output gap and inflation rate from their short-run targets mimic the optimal policy rules discussed in Clarida, Gali and Gertler (1999, 2001, 2002).

The rest of the paper proceeds as follows. In section 2, we revisit models of Clarida, Gali and Gertler (2001, 2002) to explore consequences of cost-push shocks in home and foreign countries. In section 3, we derive a law of motion for a measure of relative price distortion under the Calvo pricing. We then present a linear quadratic optimization problem whose solution is equivalent to minimizing relative price distortion in each period. In section 4, we discuss the optimal policy problems for the case where both of the initial relative price distortion and cost-push shock exist at home and abroad. Section 5 concludes. The appendix provides the details on the economy and the linear-quadratic approximations.

2 Clarida, Gali and Gertler (2001, 2002) Revisited

We begin by briefly discussing the optimal policy problem of Clarida, Gali and Gertler (2001, 2002). They argue that the optimal monetary policy problem facing the central bank in an open economy is basically the same as that of a closed economy, though its parameters may be different. Specifically, Clarida, Gali and Gertler (2001, 2002) showed that the quadratic loss-function of the central bank in an open economy turns out to be

$$\sum_{t=0}^{\infty} \beta^t E_0[\frac{\pi_t^2}{2} + \omega \frac{x_t^2}{2}], \tag{2.1}$$

while the Phillips curve can be written as

$$\pi_t = \kappa x_t + u_t + \beta E_t[\pi_{t+1}]. \tag{2.2}$$

Here, π_t, x_t, and u_t are the domestic inflation rate, the output gap, and the cost-push shock of the home country. The parameters in the optimization problem is defined as

$$\omega = \frac{\kappa}{\epsilon}; \quad \kappa = \frac{(1-\alpha)(1-\alpha\beta)(\sigma(1-\gamma)+\gamma+\nu)}{\alpha}, \tag{2.3}$$

where ϵ is the demand elasticity of firms, γ is the population share of the foreign country, σ is the inverse of elasticity of inter-temporal substitution, ν is the inverse of elasticity of labor supply, and α is the fraction of firms that do not change their previous period's prices.[1]

Having described the optimal policy problem of Clarida, Gali and Gertler (2001, 2002), we now include cost-push shocks of the foreign country into their model.[2] The preference at period 0 of the representative household in the home

[1] The Phillips curve and loss function analyzed in this paper may have the same functional forms as those of closed economies. However, coefficients of inflation and output gap in these functions differ from those of closed economies. The Phillips curve equations in the present section are derived from log-linearization of pricing equations. In particular, the terms associated with relative price distortion are not included in the Phillips curve equation, because it is assumed that relative price distortion does not exist. In this section, we also follows this assumption in order to focus on the role of cost-push shocks. However, the Phillips curve equation in the next section allows for the presence of relative price distortion.

[2] Appendix B includes detailed derivation of the preference and the Phillips curve in this case.

country can then be approximated by

$$\sum_{t=0}^{\infty} \beta^t E_0[\frac{\pi_t^2}{2} + \omega \frac{x_t^2}{2} - \phi x_t^* x_t], \tag{2.4}$$

where ϕ is defined as

$$\phi = (\frac{\kappa}{\epsilon}) \frac{\gamma(1-\sigma)}{\gamma + \nu + \sigma(1-\gamma)} \tag{2.5}$$

The Phillips curve equation can be written as

$$\pi_t = \kappa x_t - \bar{\kappa} x_t^* + u_t + \beta E_t[\pi_{t+1}], \tag{2.6}$$

where x_t^* denote the output gap of the foreign country and $\bar{\kappa}$ is defined as

$$\bar{\kappa} = \frac{\gamma(1-\alpha)(1-\alpha\beta)(1-\sigma)}{\alpha}. \tag{2.7}$$

Compared with the optimal policy problem of Clarida, Gali and Gertler (2001, 2002), the difference is that the foreign output gap, x_t^*, is included in both our preference and Phillips curve. It may be worthwhile to discuss the reason why this difference takes place. First, it should be noted that zero inflation policy can achieve the first-best allocation if neither initial price dispersion at home nor cost-push shock in the foreign country exists and fiscal policy eliminates the distortion associated with monopolistic competition. This has been emphasized in recent literature such as Goodfriend and King (1997) and Woodford (2003). The same is true for open economies analyzed in this paper. Second, output gap is defined as the deviation of output from its first-best level. Hence, the presence of cost-push shocks is important to determine whether or not the optimal policy leads to a zero output gap. For example, the optimal policy leads to $x_t^* = 0$ without initial price dispersion or cost-push shock in the foreign country. However, the optimal policy in the foreign country leads to $x_t^* \neq 0$ with the cost-push shocks. This is the key difference from Clarida, Gali and Gertler (2001, 2002).

But it also should be noted that $\phi = 0$ when we have a unit elasticity of inter-temporal substitution. Besides, we can see that when $\phi = 0$, the two loss functions (2.1) and (2.4) turn out to be the same regardless of the value of x_t^*. As a result, so long as either households have a logarithmic utility function for consumption or cost-push shocks do not exist in the foreign country, the optimal policy problem facing the central banks in open economies is to minimize (2.1) subject to (2.2). In this case, the optimal policy problem for an open economy is isomorphic to that of a closed economy, as argued in Clarida, Gali and Gertler (2001, 2002). However, when foreign cost-push shocks cause nonzero foreign output gap, the appropriate loss function for the central bank is the one specified in (2.4). For this reason, we maintain throughout this section the assumption that cost-push shocks exist home and abroad.

In order to pursue the optimal policy, the central bank minimizes the loss function (2.4) subject to the Philips curve equation (2.6). When commitment is possible, the first-order conditions of the optimal policy problem can be summarized as follows:

$$x_t - x_{t-1} = -\frac{\kappa}{\omega}\pi_t + \frac{\phi}{\omega}(x_t^* - x_{t-1}^*). \tag{2.8}$$

Therefore, the optimal policy has the change in output gap adjust not only to the deviations of inflation from target but also to the change of the foreign output gap.[3] When the foreign output gap is assumed to be zero as in Clarida, Gali and Gertler (2001, 2002), the optimal policy rule that is implied by the first-order conditions is given by

$$x_t - x_{t-1} = -\frac{\kappa}{\omega}\pi_t. \tag{2.9}$$

Comparing (2.8) with (2.9), we can see that the presence of cost-push shocks in the foreign country along with non-unit elasticity of inter-temporal substitution would break an isomorphism between optimal policy problems in open and closed economies. This is in contrast to Clarida, Gali and Gertler (2001, 2002).

3 Refinement of optimal policy problems in open economies with relative price distortion

In the previous section, we have discussed consequences of symmetric presence of cost-push shocks in home and foreign country on the optimal policy. In this section, however, we ignore cost-push shocks in both of home and foreign countries. This is to focus on the role of relative price distortion in characterizing the optimal monetary policies in open economies. We begin by considering

[3] The optimal policy under discretion can be written as $x_t = -\frac{\kappa}{\omega}\pi_t + \frac{\phi}{\omega}x_t^*$.

the law of motion for the relative price distortion under Calvo pricing and then move on to a linear quadratic problem. The measure of relative price distortion shows up in our quadratic loss function.

3.1 Analytic solution to the original optimal policy problem with relative price distortion under the Calvo pricing

There are two types of domestic goods: intermediate goods and final goods. Domestic intermediate goods are sold only to domestic firms producing final goods, while domestic final goods can be purchased by both domestic and foreign households. In addition, the markets for final goods are perfectly competitive and the number of final goods producers in each country equals its population size. Moreover, each intermediate goods firm z produces a type z of intermediate goods indexed in a unit interval [0, 1] and sets its price as a monopolistic competitor.

Let Y_t denote the output level at period t of a final goods producer. We then assume that all domestic final goods are produced using the same technology:

$$Y_t = (\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz)^{\frac{\epsilon}{\epsilon-1}}, \tag{3.1}$$

where $\epsilon > 1$ and $Y_t(z)$ denotes the demand of a final goods producer for an intermediate goods z. Furthermore, since each final goods producer minimizes its cost of producing Y_t, taking intermediate goods prices as given, the demand curve of each intermediate goods z can be written as

$$Y_t(z) = (\frac{P_t(z)}{P_t})^{-\epsilon} Y_t, \tag{3.2}$$

where $P_t(z)$ denotes the price at period t of the intermediate goods z, and P_t denotes the price index of the domestic intermediate goods. Here, the price index for domestic intermediate goods are defined by

$$P_t = (\int_0^1 P_t(z)^{1-\epsilon} dz)^{\frac{1}{1-\epsilon}}. \tag{3.3}$$

We also define the relative price distortion as the part of output that is foregone because of price dispersion, following Goodfriend and King (1997), Woodford

(2003), and Yun (2005). The relative price distortion at period t, denoted by Δ_t, can be then measured as follows:

$$\Delta_t = \int_0^1 (\frac{P_t(z)}{P_t})^{-\epsilon} dz. \qquad (3.4)$$

Following Calvo (1983), we assume that a fraction of intermediate goods producers, $1-\alpha$, are allowed to choose a new optimal price at period t, $P_t^\star$, in each period $t = 0, 1, \cdots, \infty$. In addition, the other fraction of firms do not change their previous prices. Then, combining (3.3) with (3.4), we see that under the Calvo-type staggered price-setting, our measure of relative price distortion specified above turns out to be

$$\Delta_t = (1-\alpha)(\frac{1-\alpha\Pi_t^{\epsilon-1}}{1-\alpha})^{\frac{\epsilon}{\epsilon-1}} + \alpha\Pi_t^\epsilon\Delta_{t-1}, \qquad (3.5)$$

where Π_t is the ratio of price-levels at period t and $t-1$. It then follows from equation (3.5) that the current level of relative price distortion depends on the current rate of the producer price index inflation and the previous level of relative price distortion.[4]

Having described the law of motion for relative price distortion, we now explain how the relative price distortion works in the aggregate production relation. Each firm z employs labor to produce its product z using a linear production function:

$$Y_t(z) = A_t N_t(z), \qquad (3.6)$$

where $N_t(z)$ is the number of hours hired by firm z and $Y_t(z)$ denotes the output level of the domestic firm z. Next, substituting (3.2) into (3.6) and then aggregating the resulting equation linearly, the aggregate production function can be written as

$$Y_t = \frac{A_t}{\Delta_t} N_t, \qquad (3.7)$$

[4]See Woodford (2003) for a detailed discussion on how one can derive a quadratic loss function from the utility function of the household, in which a measure of price dispersion in the Calvo model is defined as a cross-section variance of logarithms of individual prices. Besides, see Schmitt-Grohé and Uribe (2004) for an explicit discussion on how to derive a measure of relative price distortion in the Calvo model, which is identical to the one used here.

where $N_t = \int_0^1 N_t(z)dz$. It is obvious from equation (3.7) that a rise in the relative price distortion increases the part of output that is foregone given the amount of hours worked and the level of technology. Each period, therefore, we seek an inflation rate that minimizes the measure of relative price distortion given the previous level of relative price distortion:

$$\min_{\Pi_t}\{(1-\alpha)(\frac{1-\alpha\Pi_t^{\epsilon-1}}{1-\alpha})^{\frac{\epsilon}{\epsilon-1}} + \alpha\Pi_t^{\epsilon}\Delta_{t-1}\}. \tag{3.8}$$

Substituting the first-order condition of this minimization problem into the law of motion for relative price distortion, we have the following relation between inflation and relative price distortion:

$$\Pi_t = \frac{\Delta_t}{\Delta_{t-1}}. \tag{3.9}$$

Taking logarithm to both sides of (3.9), we have

$$\pi_t = \delta_t - \delta_{t-1}. \tag{3.10}$$

where $\pi_t = \log P_t$ - $\log P_{t-1}$ and $\delta_t = \log \Delta_t$.

3.2 Linear quadratic approximation in the presence of relative price distortion

Now that we have shown that minimizing relative price distortion at period t requires the inflation rate to equal the logarithmic difference of relative price distortion at periods t and $t-1$, we move on to another interesting finding that the same solution can be obtained by minimizing a quadratic loss function, which is derived from the utility function of the household on the basis of the measure of relative price distortion. The solution of this minimization problem will be shown to be consistent with the Phillips curve in a decentralized setting.

3.2.1 Optimal monetary policy in the presence of relative price distortion

It is shown in appendix B that when we use the measure of relative price distortion (3.5), the preference at period 0 of the representative household in the home country can be approximated by

$$\sum_{t=0}^{\infty}\beta^t E_0[\frac{\pi_t^2}{2}+\frac{1-\alpha}{\alpha}\pi_t\delta_t+\omega\frac{x_t^2}{2}-\phi x_t^* x_t+\theta\delta_t x_t], \tag{3.11}$$

where θ is defined as

$$\theta=(\frac{\kappa}{\epsilon})\frac{1+\nu}{\gamma+\nu+\sigma(1-\gamma)}. \tag{3.12}$$

It is now worth discussing the reason why cross-products of relative price distortion with inflation and output gap show up in the loss function specified above. The relative price distortion terms are included in the loss function because of a different order approximation to the relative price distortion than the one used in Woodford (2003). It is shown in appendix B that the order of approximation residual is $O(||\delta,\xi||^3)$ in our paper while it is $O(||\delta^{\frac{1}{2}},\xi||^3)$ in Woodford (2003), where $||\xi||$ denotes a bound on the amplitude of exogenous shocks.

In order to replicate the solution to the minimization problem of relative price distortion specified in the previous section, we consider a static optimization problem, which minimizes the quadratic loss function specified in (3.11). The first-order condition for output gap is

$$\omega x_t-\phi x_t^*+\theta\delta_t=0. \tag{3.13}$$

The first-order condition for inflation rate is given by

$$\pi_t=-\frac{1-\alpha}{\alpha}\delta_t. \tag{3.14}$$

Meanwhile, when we make the first-order approximation to the measure of relative price distortion (3.5) around the steady state with constant prices, we have a deterministic AR(1) process of the form:

$$\delta_t=\alpha\delta_{t-1}. \tag{3.15}$$

Then, substituting (3.15) into (3.14) leads to the following equation:

$$\pi_t=\delta_t-\delta_{t-1}. \tag{3.16}$$

The optimal inflation rate at period t therefore turns out to be the logarithmic difference of relative price distortions at period t and $t-1$. Comparing (3.16) with (3.10), we can see that a static minimization of the loss function (3.11) leads to the rate of inflation at which relative price distortions are minimized over time.

To the extent that there is perfect exchange rate pass-through and fiscal policy can eliminate the distortion associated with monopolistic competition and openness, we find that the optimal inflation rate of the producer's price index equals the logarithmic change of relative price distortions. In closed economy, the optimal inflation rate equals the logarithmic change of relative price distortions as shown in Yun (2005). Thus, in the absence of cost-push shocks, characterization of optimal inflation rates in open and closed economies are isomorphic, in the sense that optimal inflation rates are expressed in terms of relative price distortion. However, it does not mean that optimal policy problems between closed and open economies are isomorphic, which can be easily confirmed by the loss function (3.11).

3.2.2 Decentralization

It will be shown that when the central bank has a loss function of the form (3.11), the Phillips curve equation is not a binding constraint, which in turn implies that the static minimization of the loss function (3.11) that ignores the Phillips curve equation is legitimate.

We begin with an equilibrium relation between the real marginal cost and output gap, which holds as results of decentralized decision-makings. We then substitute the optimal conditions for output gap and inflation rate into this equation so as to find the size of the real marginal cost, which supports the optimal output gap and inflation rate as equilibrium outcomes. First, putting the efficiency condition for labor supply, zero trade balance condition, and the aggregate production function together, we can see that the real marginal cost can be written as

$$\hat{mc}_t - \hat{p}_{ct} = (\gamma + \nu + \sigma(1-\gamma))x_t - (1-\sigma)\gamma x_t^* + \nu\delta_t, \tag{3.17}$$

where $\hat{mc}_t$ is the log-deviation of the real marginal cost from its steady state value and $\hat{p}_{ct}$ is the log-deviation of the ratio of the producer price index to the consumer price index from its steady state value. Meanwhile, given the definitions of parameters ϕ, ω, and θ, we can rewrite the optimal condition for output gap specified in (3.13) as follows:

$$(\gamma + \nu + \sigma(1-\gamma))x_t - (1-\sigma)\gamma x_t^* + (1+\nu)\delta_t = 0. \tag{3.18}$$

Comparing (3.17) with (3.18), we can see that attaining equation (3.13) as an equilibrium condition in a decentralized economy requires the real marginal cost to satisfy

$$\hat{m}c_t - \hat{p}_{ct} = -\delta_t. \tag{3.19}$$

Furthermore, when we log-linearize pricing equations of firms and then combine the resulting equations, the Phillips curve equation can be written as

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\hat{m}c_t - \hat{p}_{ct}) + \beta E_t[\pi_{t+1}]. \tag{3.20}$$

Then, we can see that substituting (3.15) and (3.16) into (3.20) leads to the same condition as specified in (3.19). This means that the optimal conditions for inflation and output gap (3.13) and (3.16) together with (3.15) satisfy the Phillips curve equation. Besides, recall that the optimal conditions for inflation and output gap (3.13) and (3.16) are derived from a planning problem that ignores the Phillips curve equation as a constraint. As a result, we can see that when the central bank has a loss function of the form (3.11), the Phillips curve equation is not a binding constraint.

4 Optimal monetary policy in the presence of relative price distortion and cost-push shock

In this section, we discuss the optimal monetary policy problems for the case where home and foreign countries have effects of foreign output gaps and relative price distortion. In the previous section, we have derived a quadratic loss function in the presence of relative price distortion. Since output gap of the foreign country can arise either because of cost-push shocks or because of relative price distortion or both, we can see that even when both of cost-push shocks and relative price distortion exist, the loss function of the central bank is still the same as the one with relative price distortion alone. But once cost-push shocks are included, we should take into account the short-run trade-off between inflation rate and output, which was absent in the previous section. In particular, the solution to the minimization problem analyzed in the previous section may not be achievable as a result of decentralized decision-makings in the face of cost-push shocks. Thus, when cost-push shocks are included, the Phillips curve equation becomes a constraint for the optimal policy problem. The optimal policy in this section, therefore, minimizes

$$\sum_{t=0}^{\infty}\beta^t E_0[\frac{\pi_t^2}{2}+\frac{1-\alpha}{\alpha}\pi_t\delta_t+\omega\frac{x_t^2}{2}-\phi x_t^* x_t+\theta\delta_t x_t], \tag{4.1}$$

subject to the following Phillips curve equation:

$$\pi_t=\kappa x_t-\bar{\kappa}x_t^*+\mu\delta_t+u_t+\beta E_t[\pi_{t+1}], \tag{4.2}$$

where μ is defined as

$$\mu=\frac{\nu\kappa}{\sigma+\nu}. \tag{4.3}$$

With commitment possible, the first-order conditions of the optimal policy problem can be summarized as follows:

$$\pi_t+\frac{1-\alpha}{\alpha}\delta_t+\lambda_t-\lambda_{t-1}=0, \tag{4.4}$$

$$\omega x_t-\phi x_t^*+\theta\delta_t-\kappa\lambda_t=0, \tag{4.5}$$

where λ_t is the Lagrange multiplier to the Phillips curve equation. Combining (4.4) with (4.5), the optimal relationship between output gap and inflation rate under commitment can be written as

$$x_t-\bar{x}_t-(x_{t-1}-\bar{x}_{t-1})=-\frac{\kappa}{\omega}(\pi_t-\bar{\pi}_t), \tag{4.6}$$

Here, $\bar{\pi}_t$ and $\bar{x}_t$ can be interpreted as the optimal inflation rate and output gap in the absence of cost-push shocks because $\bar{x}_t$ and $\bar{\pi}_t$ are defined as

$$\bar{\pi}_t=-\frac{1-\alpha}{\alpha}\delta_t, \tag{4.7}$$

$$\bar{x}_t=\frac{\phi}{\omega}x_t^*-\frac{\theta}{\omega}\delta_t. \tag{4.8}$$

It follows from equation (4.6) that the optimal output gap and inflation rate deviate from $\bar{x}_t$ and $\bar{\pi}_t$ because of the presence of cost-push shocks. Besides, $\bar{x}_t$ and $\bar{\pi}_t$ can be achieved under the optimal policy in the absence of cost-push shocks but they can not be attained in the presence of cost-push shocks. In this sense, we interpret $\bar{x}_t$ and $\bar{\pi}_t$ as the short-run optimal targets for output gap and inflation rate. Given this interpretation, the optimal policy rule (4.6) has the change in deviations of output gap from its optimal level adjusts to the deviation of producer's price inflation from its optimal level.[5]

In sum, our analysis has illustrated that even when relative price distortion and cost-push shock exist home and abroad, the optimal policy rules in open economies are not isomorphic to those in closed economies. For example, it is noteworthy that $\bar{x}_t$ depends on the output gap of the foreign country. Hence, when the home country follows the optimal policy rule specified in (4.6), a change in the output gap of the foreign country generates a change in the output gap of the home country.

But it should be noted that they are very similar. Specifically, our policy rules discussed above mimic those of Clarida, Gali and Gertler (1999, 2001, 2002), except that $\bar{x}_t$ and $\bar{\pi}_t$ are included. In particular, it is noteworthy that effects of openness and relative price distortion on optimal policy rules are summarized by the inclusion of short-run targets such as $\bar{x}_t$ and $\bar{\pi}_t$ in the optimal policy rules. As a result, whether or not the optimal policy rules in closed and open economies are isomorphic depends on the existence of $\bar{x}_t$ and $\bar{\pi}_t$ in the optimal policy rules for open economies.

5 Conclusion

We have demonstrated that, while ignoring relative price distortion, the assumption on the existence of cost-push shocks in the foreign country can play an important role in isomorphism of optimal policy rules between open and closed economies. In order to for the isomorphism to hold, it is necessary to assume that the home country is subject to cost-push shocks whereas the foreign country is not and the foreign output gap is zero. However, we illustrate that when home and foreign countries are subject to cost-push shocks, the output gap under the optimal policy responds to the foreign output gap unless households have unit elasticity of inter-temporal substitution.

We have also presented a linear quadratic approximation that replicates the optimal solutions of the original optimal policy problem in the presence

[5]The optimal relationship between output gap and inflation under discretion is given by $x_t - \bar{x}_t = -\frac{\kappa}{\omega}(\pi_t - \bar{\pi}_t)$.

of initial price dispersion. Our analysis complements the conventional linear quadratic approach to the welfare analysis of the monetary policy such as the work of Woodford (2003). Besides, we demonstrate that when home and foreign countries have initial price dispersion, the optimal output gap responds to the foreign output gap unless households have unit elasticity of inter-temporal substitution. This result holds true regardless of cost-push shocks in the two countries. The presence of initial price dispersion can break down the isomorphism of the optimal monetary policy problems between open and closed economies.

Finally, we have seen that optimal policy rules in open and closed economies are very similar, even though they are not isomorphic. The only difference between optimal policy rules in open and closed economies is that when there are initial price dispersion and cost-push shock, short-run targets of output gap and inflation rate for open economies are not zero and may be affected by foreign output gap. As a result, we can conclude that our results complement those of Clarida, Gali and Gertler (1999, 2001, 2002) because our policy rules mimic their policy rules except for short-run targets.

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Appendix

A Brief Summary of the Original Economy

We briefly highlight a two-country open economy version of the sticky price model with Calvo-type pricing. The preference at period 0 of the household h in the home country is summarized by

$$\sum_{t=0}^{\infty} \beta^t E_0 [\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t(h)^{1+\nu}}{1+\nu}], \tag{A.1}$$

where β is the time discount factor, $N_t(h)$ denotes the hours worked by household h, and C_t is a composite consumption index in per-capita terms. We also assume that each household h sells differentiated labor services to firms, following Clarida, Gali, and Gertler (2001). The demand for differentiated labor services is therefore given by

$$N_t(h) = (\frac{W_t(h)}{W_t})^{-v_t} N_t, \tag{A.2}$$

where v_t denotes time-varying elasticity of substitution between goods and N_t is a composite index of hours worked at period t. The composite consumption index is a Cobb-Douglas function of the form:

$$C_t = C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{A.3}$$

where $0 < \gamma < 1$, and $C_{H,t}$ and $C_{F,t}$ are domestic and foreign consumption goods, respectively. The corresponding consumer price index denoted by $P_{c,t}$ is then given by

$$P_{c,t} = \kappa^{-1} P_t^{1-\gamma} P_{F,t}^{\gamma}, \tag{A.4}$$

where $\kappa = (1-\gamma)^{1-\gamma}\gamma^{\gamma}$, P_t is the producer price index, and $P_{F,t}$ is the price index of imported goods.

The home country has a population share of $(1-\gamma)$ and the share of the foreign country is γ. In addition, the consumption and consumer price indices for the foreign country are symmetrically defined as

$$C_t^* = {C_{H,t}^*}^{1-\gamma} {C_{F,t}^*}^{\gamma}; \quad P_{c,t}^* = \kappa^{-1} {P_{H,t}^*}^{1-\gamma} {P_t^*}^{\gamma}, \tag{A.5}$$

where ‘ * ’ is used to represent quantities and prices in the foreign country.

The representative household maximizes (A.1) subject to the following flow budget constraints in each period $t = 0, 1, \cdots, \infty$:

$$C_t + E_t[Q_{t,t+1}\frac{B_{t+1}}{P_{c,t+1}}] = \frac{B_t}{P_{c,t}} + (1+\tau)\frac{W_t}{P_{c,t}}(\frac{W_t(h)}{W_t})^{1-v_t}N_t - T_t, \qquad \text{(A.6)}$$

where $Q_{t,t+1}$ is the stochastic discount factor used for computing the value at period t of a unit of consumption goods at period $t+1$, B_{t+1} is the nominal payoff at period $t+1$ of the portfolio held at period t, T_t is the real lump-sum tax, W_t is the nominal wage at period t, and τ is the subsidy rate at period t for labor supply. In particular, following the literature, the magnitude of the subsidy rate in each period will be determined endogenously to eliminate distortions associated with imperfect competition and openness in goods market, while the subsidy is funded by lump-sum tax imposed on households.

The first-order condition for labor supply is given by

$$N_t^{\nu} = (1+\tau)U_t\frac{W_t}{P_{c,t}}C_t^{-\sigma}, \qquad \text{(A.7)}$$

where U_t is defined

$$U_t = \frac{v_t - 1}{v_t}. \qquad \text{(A.8)}$$

The optimization conditions for contingent claims on future consumption goods lead to

$$Q_{t,t+1} = \beta(\frac{C_t}{C_{t+1}})^{\sigma}. \qquad \text{(A.9)}$$

Moreover, since households in the home and foreign countries have the same preference over consumption and labor as that described in (A.1), we have a symmetric set of first-order conditions from the households of the foreign country. Under a suitable normalization of initial conditions, the assumption of complete markets—though a simple asset market with only bonds would suffice—makes domestic consumption equal to foreign consumption:

$$C_t = C_t^*, \qquad \text{(A.10)}$$

for $t = 0, 1, \cdots \infty$.

There are two types of domestic goods: intermediate goods and final goods.

Domestic intermediate goods are sold only to domestic firms producing final goods, while domestic final goods can be purchased by both domestic and foreign households. In addition, the markets for final goods are perfectly competitive and the number of final goods producers in each country equals its population size. Moreover, each intermediate goods firm z produces a type z of intermediate goods indexed in a unit interval [0, 1] and sets its price as a monopolistic competitor.

Let Y_t denote the output level at period t of a final goods producer. In addition, all domestic final goods are produced using the same technology:

$$Y_t = (\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz)^{\frac{\epsilon}{\epsilon-1}}, \tag{A.11}$$

where $\epsilon > 1$ and $Y_t(z)$ denotes the demand of a final goods producer for an intermediate goods z. Each final goods producer minimizes its cost of producing Y_t, taking intermediate goods prices as given. The demand curve of each intermediate goods z is then given by

$$Y_t(z) = (\frac{P_t(z)}{P_t})^{-\epsilon} Y_t, \tag{A.12}$$

where $Y_t(z)$ is the demand at period t for intermediate goods produced by firm z, $P_t(z)$ denotes the price at period t of the intermediate goods z, and P_t denotes the price index of the domestic intermediate goods. In addition, the price index for domestic intermediate goods are defined by

$$P_t = (\int_0^1 P_t(z)^{1-\epsilon} dz)^{\frac{1}{1-\epsilon}}. \tag{A.13}$$

Having described the behavior of final goods firms, the analysis turns to the price setting of intermediate goods firms. Each firm z employs labor to produce its product z using a linear production function:

$$Y_t(z) = A_t N_t(z), \tag{A.14}$$

where $N_t(z)$ is the number of hours hired by firm z and $Y_t(z)$ denotes the output level of the domestic firm z. In addition, the labor market is assumed to be perfectly competitive and nominal wage is completely flexible.

Following Calvo (1983) and Yun (1995), a fraction of intermediate goods producers, $1-\alpha$, are allowed to choose a new optimal price at period t, $P_t^\star$, in each period $t = 0, 1, \cdots, \infty$. In addition, the other fraction of firms do not change their previous prices. Hence, the Calvo-type staggering allows one to rewrite the price index definition equation (3.2) as follows:

$$1 = (1-\alpha)(\frac{P_t^\star}{P_t})^{1-\epsilon} + \alpha \Pi_t^{\epsilon-1}, \tag{A.15}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ denotes the ratio of domestic producer's price level at period t to its level at period $t-1$.

In addition, the profit-maximization problem under the Calvo-type staggered price-setting can be written as

$$\max_{P_t^\star} \sum_{k=0}^{\infty} \alpha^k E_t[Q_{t,t+k}(\frac{P_{t+k}}{P_{c,t+k}}\frac{P_t^\star}{P_{t+k}} - mc_{t+k})(\frac{P_t^\star}{P_{t+k}})^{-\epsilon} Y_{t+k}], \tag{A.16}$$

where $mc_t = \frac{MC_t}{P_{c,t}}$. The first-order condition for $P_t^\star$ can be written as

$$\frac{P_t^\star}{P_t} = \frac{I_t}{K_t}, \tag{A.17}$$

where I_t and K_t are, respectively, defined as

$$K_t = \sum_{k=0}^{\infty} (\alpha\beta)^k E_t[C_{t+k}^{-\sigma} \frac{P_{t+k}}{P_{c,t+k}} (\frac{P_t}{P_{t+k}})^{1-\epsilon} Y_{t+k}], \tag{A.18}$$

$$I_t = \sum_{k=0}^{\infty} (\alpha\beta)^k E_t[C_{t+k}^{-\sigma} (\frac{P_t}{P_{t+k}})^{-\epsilon} \frac{\epsilon}{\epsilon-1} mc_{t+k} Y_{t+k}]. \tag{A.19}$$

We close the description of the model by considering equilibrium conditions to obtain a relationship between the aggregate consumption and production factor input. The aggregate market clearing for domestic and foreign goods, therefore, can be written as

$$(1-\gamma)Y_t = (1-\gamma)C_{H,t} + \gamma C_{H,t}^*, \tag{A.20}$$

$$\gamma Y_t^* = (1-\gamma)C_{F,t} + \gamma C_{F,t}^*. \tag{A.21}$$

It is well known that, in our setup of unit elasticity of substitution between home and foreign goods, the trade balance is zero:

$$P_t Y_t = P_{c,t} C_t \tag{A.22}$$

$$P_t^* Y_t^* = P_{c,t}^* C_t^* \tag{A.23}$$

Let S_t denote the terms of trade at period t of the home country, which is defined as the ratio of the price index of imported goods to that of exported goods, i.e. $S_t = \frac{P_{F,t}}{P_t}$. Then, dividing both sides of (A.23) by their corresponding sides of (A.22) and applying the law of one price to the resulting equation, one can obtain an expression of the terms of trade in terms of the ratio of the output level of home country to that of the foreign country:

$$S_t = \frac{Y_t}{Y_t^*}. \tag{A.24}$$

In addition, the consumption price index leads to $\frac{P_{c,t}}{P_t} = \kappa^{-1} S_t^{\gamma}$, while equation (A.22) implies $\frac{P_{c,t}}{P_t} = \frac{Y_t}{C_t}$. Then, combining these two equations results in $C_t = \kappa\ Y_t S_t^{-\gamma}$. Hence, substituting (A.24) into this equation, one can show that the per capita consumption in the home country is a Cobb-Douglas function of domestic and foreign final goods of the form:

$$C_t = \kappa Y_t^{1-\gamma} Y_t^{*\gamma}. \tag{A.25}$$

B Linear Quadratic Approximation

B.1 Derivation of our quadratic loss function

In this section, I use the approximation method of Woodford (2003) to derive a loss function that includes relative price distortion and foreign country's output gap. But I adopt a different order of approximation to the measure of relative price distortion than the one used in Woodford (2003). The reason for this is that the primary concern of this section is to display how one can replicate the optimal conditions from the optimal policy problem with the original utility function by solving an optimal policy problem with a quadratic loss function.

Before going further, notice that when either relative price distortion or cost-push shock or both exist in the foreign country, we have $x_t^* \neq 0$, where x_t^*

is the output gap of the foreign country. Besides, $C_t = \kappa Y_t^{1-\gamma} Y_t^{*\gamma}$. The utility function for consumption can be therefore written as

$$\frac{C_t^{1-\sigma}-1}{1-\sigma} = \frac{(C_t^o)^{1-\sigma}(X_t^{1-\gamma}X_t^{*\gamma})^{1-\sigma}-1}{1-\sigma}, \tag{B.26}$$

where C_t^o denotes the first-best level of consumption at period t, X_t is the ratio of the home country's output level to its first-best level and X_t^* is the ratio of the foreign country's output level to its first-best level. In the following, I focus on second-order Taylor expansions around the steady state with constant prices, in which $C = C^o$. The second-order Taylor expansion to the utility function for consumption is given by

$$\left(\begin{array}{c} \frac{C_t^{1-\sigma}}{1-\sigma} = \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}[(1-\gamma)(X_t-1) + \gamma(X_t^*-1) + \gamma((1-\sigma)\gamma-1)\frac{(X_t^*-1)^2}{2} \\ +(1-\gamma)\{((1-\gamma)(1-\sigma)-1)\frac{(X_t-1)^2}{2} + (1-\sigma)\gamma(X_t-1)(X_t^*-1)\}] + O(||\xi||^3), \end{array} \right) \tag{B.27}$$

where $||\xi||$ is a bound on the amplitude of exogenous shocks and $O(||\xi||^3)$ denotes the order of approximation residual.

Following Woodford (2003), one can express second-order approximations to X_t and X_t^* in terms of their logarithmic deviations from their steady state values as follows:

$$X_t - 1 = x_t + \frac{1}{2}x_t^2 + O(||\xi||^3);\ X_t^* - 1 = x_t^* + \frac{1}{2}x_t^{*2} + O(||\xi||^3) \tag{B.28}$$

where x_t $(= \log X_t)$ and x_t^* $(= \log X_t^*)$ denote output gaps of the home and foreign countries, respectively, both of which are defined as logarithmic deviations of X_t and X_t^* from their steady state values. Substituting (B.28) into (B.27) and then rearranging, we have

$$\left(\begin{array}{c} \frac{C_t^{1-\sigma}}{1-\sigma} = \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}[(1-\gamma)(x_t + \frac{(1-\gamma)(1-\sigma)x_t^2}{2}) \\ +\gamma(x_t^* + \frac{(1-\sigma)\gamma x_t^{*2}}{2}) + (1-\gamma)\gamma(1-\sigma)x_t x_t^*] + t.i.p. + O(||\xi||^3), \end{array} \right) \tag{B.29}$$

where $t.i.p.$ collects terms that are independent of monetary policy of the home country. Note that $t.i.p.$ includes the part of the home welfare that is affected by the foreign country's monetary policy:

$$g(x_t^*) = \gamma C^{1-\sigma}(x_t^* + \frac{(1-\sigma)\gamma x_t^{*2}}{2}).$$

Besides, a second-order Taylor expansion to the utility function of labor can be written as

$$\left(\begin{array}{c} \frac{1}{1+\nu}(\frac{\Delta_t Y_t}{A_t})^{1+\nu} = \frac{N^{1+\chi}}{1+\nu} + N^{1+\nu}[(X_t-1)+(\Delta_t-1) \\ +\nu(\frac{(X_t-1)^2}{2}+\frac{(\Delta_t-1)^2}{2}) + (1+\nu)(X_t-1)(\Delta_t-1)] + O(||\xi||^3). \end{array}\right) \quad \text{(B.30)}$$

Similarly, substituting (B.28) into (B.30) and then rearranging yields

$$\frac{N_t^\nu}{1+\nu} = \frac{N^{1+\nu}}{1+\nu} + N^{1+\nu}[x_t+\delta_t+\frac{1+\nu}{2}x_t^2+(1+\nu)x_t\delta_t]+t.i.p.+O(||\delta,\xi||^3), \quad \text{(B.31)}$$

where δ_t ($= \log \Delta_t$) is the logarithm of the measure of relative price distortion. Note that the order of approximation residual is $O(||\delta,\xi||^3)$ in this paper, while it is $O(||\delta^{\frac{1}{2}},\xi||^3)$ in Woodford (2003).

We now discuss the role of distorting labor income tax or employment subsidy in deriving a quadratic loss function. Note that the marginal rate of substitution between consumption and labor at the steady state is given by

$$(1-\gamma)C^{1-\sigma} = N^{1+\varphi}. \quad \text{(B.32)}$$

Subtracting (B.31) from (B.29) and then substituting (B.32) into the resulting equation, we can obtain a quadratic approximation to the instantaneous utility function of the representative household:

$$u_t - \bar{u} = -v[\delta_t + (\sigma+\nu+\gamma(1-\sigma))\frac{x_t^2}{2} - (1-\sigma)\gamma x_t^* x_t + (1+\nu)\delta_t x_t\}] + t.i.p. + \bar{O}, \quad \text{(B.33)}$$

where $v = (1-\gamma)C^{1-\sigma}$, u_t is the instantaneous utility level at period t of the home country, and $\bar{u}$ is its steady state utility level, and $\bar{O} = O(||\delta,\xi||^3)$. Similarly, utility function of the foreign country can be approximated as follows:

$$u_t^* - \bar{u}^* \approx -v^*[\delta_t^* + (\sigma+\nu+(1-\gamma)(1-\sigma))\frac{x_t^{2*}}{2} - (1-\gamma)(1-\sigma)x_t^* x_t + (1+\nu)\delta_t^* x_t^*\}], \quad \text{(B.34)}$$

where $v^* = \gamma C^{*1-\sigma}$, u_t^* is the instantaneous utility level at period t of the foreign country and $\bar{u}^*$ is its steady state utility level. Here, note that $t.i.p.$ includes the part of the foreign country's welfare that depends on the output gap of the home country:

$$g^*(x_t) = (1-\gamma)C^{1-\sigma}(x_t + \frac{(1-\gamma)(1-\sigma)x_t^2}{2}).$$

Next, I turn to the approximation of the measure of relative price distortion. A second-order Taylor expansion to equation (3.5) is

$$\delta_t = \alpha\delta_{t-1} + \frac{\alpha}{2}\delta_{t-1}^2 + \alpha\epsilon\pi_t\delta_{t-1} + \frac{\alpha\epsilon}{2(1-\alpha)}\pi_t^2 + O(||\delta,\xi||^3). \tag{B.35}$$

Besides, equation (B.35) can be rewritten as

$$\alpha\delta_{t-1} = \delta_t - \frac{\alpha}{2}\delta_{t-1}^2 - \alpha\epsilon\pi_t\delta_{t-1} - \frac{\alpha\epsilon\pi_t^2}{2(1-\alpha)} + O(||\delta,\xi||^3).$$

Hence, substituting this equation into (B.35) and rearranging, one can have an alternative representation of equation (B.35) as follows:

$$\delta_t = \alpha\delta_{t-1} + \frac{\alpha}{2}\delta_{t-1}^2 + \epsilon\pi_t\delta_t + \frac{\alpha\epsilon}{2(1-\alpha)}\pi_t^2 + O(||\delta,\xi||^3). \tag{B.36}$$

Integrating forward (B.36) from an initial value of δ_{-1} yields

$$\delta_t = \alpha^{t+1}\delta_{-1} + \sum_{k=0}^{t}\alpha^{t-k}(\frac{\alpha}{2}\delta_{k-1}^2 + \epsilon\pi_k\delta_k + \frac{\alpha\epsilon}{2(1-\alpha)}\pi_k^2) + O(||\delta,\xi||^3). \tag{B.37}$$

It follows from (B.37) that a discounted sum of logarithms of relative price distortions can be written as

$$\sum_{t=0}^{\infty}\beta^t\delta_t = \frac{\alpha\epsilon}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t(\frac{\pi_t^2}{2} + \frac{1-\alpha}{\alpha}\pi_t\delta_t) + n(\delta_{-1}) + O(||\delta,\xi||^3), \tag{B.38}$$

where $n(\delta_{-1})$ is defined as

$$n(\delta_{-1}) = \frac{\alpha\delta_{-1}}{1-\alpha\beta} + \frac{\alpha^2\delta_{-1}^2}{2(1-\alpha\beta)(1-\alpha^2\beta)}.$$

As a result, substituting (B.38) into (B.33), we have the following quadratic loss function of the central bank:

$$\Omega\sum_{t=0}^{\infty}\beta^t E_0[\frac{\pi_t^2}{2} + \frac{1-\alpha}{\alpha}\pi_t\delta_t + \omega\frac{x_t^2}{2} - \phi x_t^* x_t + \theta\delta_t x_t\}], \tag{B.39}$$

where parameters Ω, ω, ϕ, and θ are defined as

$$\Omega = \frac{v\alpha\epsilon}{(1-\alpha)(1-\alpha\beta)}; \omega = \frac{\kappa}{\epsilon}; \phi = \frac{\gamma(1-\sigma)\omega}{\gamma+\nu+\gamma(1-\sigma)}; \theta = \frac{(1+\nu)\omega}{\gamma+\nu+\gamma(1-\sigma)}. \tag{B.40}$$

Similarly, the quadratic loss function of the foreign country's central bank is given by

$$\Omega^* \sum_{t=0}^{\infty} \beta^t E_0[\frac{\pi_t^{*2}}{2} + \frac{1-\alpha}{\alpha}\pi_t{}^* \delta_t^* + \omega^* \frac{x_t^{*2}}{2} - \phi^* x_t^* x_t + \theta^* \delta_t^* x_t^*], \tag{B.41}$$

where $\Omega^* = \frac{v^* \alpha^* \epsilon}{(1-\alpha^*)(1-\alpha^* \beta)}$ and ω^*, ϕ^*, θ^* and κ^* are defined as

$$\omega = \frac{\kappa^*}{\epsilon}; \phi = \frac{(1-\gamma)(1-\sigma)\omega^*}{\gamma+\nu+(1-\sigma)(1-\gamma)}; \theta = \frac{(1+\nu)\omega^*}{\gamma+\nu+(1-\sigma)(1-\gamma)}, \tag{B.42}$$

$$\kappa^* = \frac{(1-\alpha^*)(1-\alpha^*\beta)(\gamma+\nu+(1-\sigma)(1-\gamma))}{\alpha^*}; \tag{B.43}$$

It is now noteworthy that as noted earlier, when either relative price distortion or cost-push shock or both exist in the foreign country, we have $x_t^* \neq 0$. Hence, quadratic loss functions derived above can be those for either the case where both of cost-push shock and relative price distortion exist or the case where relative price distortion alone exists.

B.2 The conventional quadratic loss function

Having described how to derive a loss function up to the error of $O(||\delta, \xi||^3)$, we now move onto the discussion of using errors of order $O(||\delta^{\frac{1}{2}}, \xi||^3)$ for derivation of the conventional loss function. This is consistent with the order of approximation used in Woodford (2003). Besides, we assume that the foreign country does not achieve its first-best allocation because of cost-push shocks.

It follows from equations (B.29) and (B.31) that the use of $O(||\delta^{\frac{1}{2}}, \xi||^3)$ for a quadratic approximation to the instantaneous utility function leads to

$$u_t - \bar{u} = -v[\delta_t + (\sigma+\nu+\gamma(1-\sigma))\frac{x_t^2}{2} - (1-\sigma)\gamma x_t^* x_t] + t.i.p. + O(||\delta^{\frac{1}{2}}, \xi||^3). \tag{B.44}$$

It is clear that if it is assumed that the foreign country achieves the first-best allocation, the use of $O(||\delta^{\frac{1}{2}}, \xi||^3)$ for a quadratic approximation to the instantaneous utility function leads to

$$u_t - \bar{u} = -v[\delta_t + (\sigma+\nu+\gamma(1-\sigma))\frac{x_t^2}{2}] + t.i.p. + O(||\delta^{\frac{1}{2}}, \xi||^3). \tag{B.45}$$

Comparing (B.45) with (B.44), we can see that cross-product terms between home and foreign output gaps are included because of the assumption that cost-push shocks prevent the foreign country from achieving its first-best allocation, especially when $O(||\delta^{\frac{1}{2}}, \xi||^3)$ is used as a order of approximation for a quadratic approximation to the instantaneous utility function. Similarly, the use of $O(||\delta^{\frac{1}{2}}, \xi||^3)$ for the utility function of the foreign country leads to

$$u_t^* - \bar{u}^* \approx -v^*[\delta_t^* + (\sigma + \nu + (1-\gamma)(1-\sigma))\frac{x_t^{2*}}{2} - (1-\gamma)(1-\sigma)x_t^* x_t], \quad \text{(B.46)}$$

Furthermore, a second-order Taylor expansion to equation (3.5) under the approximation error of $O(||\delta^{\frac{1}{2}}, \xi||^3)$ turns out to be

$$\delta_t = \alpha\delta_{t-1} + \frac{\alpha\epsilon}{2(1-\alpha)}\pi_t^2 + O(||\delta^{\frac{1}{2}}, \xi||^3). \quad \text{(B.47)}$$

Integrating forward (B.47) from an initial value of δ_{-1} yields

$$\delta_t = \alpha^{t+1}\delta_{-1} + \frac{\alpha\epsilon}{2(1-\alpha)}(\sum_{k=0}^{t}\alpha^{t-k}\pi_k^2) + O(||\delta^{\frac{1}{2}}, \xi||^3). \quad \text{(B.48)}$$

It follows from (B.48) that a discounted sum of logarithms of relative price distortions can be written as

$$\sum_{t=0}^{\infty}\beta^t\delta_t = \frac{\alpha\epsilon}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t(\frac{\pi_t^2}{2}) + O(||\delta^{\frac{1}{2}}, \xi||^3). \quad \text{(B.49)}$$

As a result, substituting (B.49) into (B.46), we have the following quadratic loss function of the central bank:

$$\Omega\sum_{t=0}^{\infty}\beta^t E_0[\frac{\pi_t^2}{2} + \omega\frac{x_t^2}{2} - \phi x_t^* x_t\}]. \quad \text{(B.50)}$$

This is identical to the loss function given in (2.4).

B.3 Deriving the Phillips curve equation

Setting $\frac{W_t}{P_{c,t}} = A_t\frac{MC_t}{P_{c,t}}$ in equation (A.7) and then substituting (A.25) into the resulting equation, we have the following equation:

$$\frac{mc_t}{mc} = U_t\Delta_t^{\nu}X_t^{\nu+\sigma(1-\gamma)}X_t^{*\sigma\gamma}, \quad \text{(B.51)}$$

where $mc_t = \frac{MC_t}{P_{c,t}}$. Combining $\frac{P_{c,t}}{P_t} = \frac{Y_t}{C_t}$ with (A.25) and then rearranging the resulting equation, we have the following equation

$$\frac{P_{c,t}}{P_t}\frac{\bar{P}_t}{\bar{P}_{c,t}} = (\frac{X_t}{X_t^*})^{\gamma}, \tag{B.52}$$

where $\frac{\bar{P}_{c,t}}{\bar{P}_t}$ is the value of $\frac{P_{c,t}}{P_t}$ at the first-best allocation. Hence, taking logarithms to both sides of the two equations and then summing the resulting equations leads to

$$\hat{m}c_t + \hat{p}_{c,t} = u_t + \nu\delta_t + (\nu + \gamma + \sigma(1-\gamma))x_t - (1-\sigma)\gamma x_t^* \tag{B.53}$$

where $u_t = \log U_t$, $\hat{m}c_t = \frac{mc_t}{mc}$, and $\hat{p}_{c,t} = \frac{P_{c,t}}{P_t}\frac{\bar{P}_t}{\bar{P}_{c,t}}$. Besides, log-linearizing (A.15), (A.17) (A.18) and (A.19) and then combining the resulting equations, we have

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\hat{m}c_t + \hat{p}_{c,t}) + \beta E_t[\pi_{t+1}]. \tag{B.54}$$

Combining these two equations, we have the following Phillips curve equation:

$$\pi_t = \kappa x_t - \bar{\kappa}\bar{x}_t + \mu\delta_t + u_t + \beta E_t[\pi_{t+1}]. \tag{B.55}$$

Here, it is not difficult to see that when $\delta_t = 0$ is assumed in equation (B.55), one can have the same Phillips curve equation as (2.6).

C Exact Solution to the Original Optimal Policy Problem in the Absence of Cost-Push Shock

In this section, we illustrate that when there is not cost-push shock, one can obtain an analytic solution to the original optimal policy problem without any approximation. Now, I turn to the optimal policy problem. Let $J^n(\Delta_{t-1}, \upsilon_t)$ represent the value function at period t in the Bellman equation for the optimal policy problem and υ_t denote the state vector at period t. Specifically, the optimal monetary policy solves the following optimization problem:

$$J^n(\Delta_{t-1}, \upsilon_t) = \max_{C_t, N_t, \Pi_t, \Delta_t}\{\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu} + \beta E_t[J^n(\Delta_t, \upsilon_{t+1})]\} \tag{C.1}$$

subject to

$$C_t = \kappa(\frac{A_t N_t}{\Delta_t})^{1-\gamma} Y_t^{*\gamma}. \tag{C.2}$$

$$\Delta_t = (1-\alpha)(\frac{1-\alpha\Pi_t^{\epsilon-1}}{1-\alpha})^{\frac{\epsilon}{\epsilon-1}} + \alpha\Pi_t^{\epsilon}\Delta_{t-1}, \tag{C.3}$$

taking as given an initial value for the measure of relative price distortion Δ_{-1}, and state vectors $\{\upsilon_t\}_{t=0}^{\infty}$ including $\{Y_t^*\}_{t=0}^{\infty}$. The first-order conditions for this optimization problem are then given by

$$(1-\gamma)C_t^{1-\sigma} = N_t^{1+\nu}, \tag{C.4}$$

$$(\frac{1-\alpha\Pi_t^{\epsilon-1}}{1-\alpha})^{\frac{1}{\epsilon-1}} = \Pi_t\Delta_{t-1}, \tag{C.5}$$

$$\phi_t = (1-\gamma)\frac{Y_t}{\Delta_t C_t^{\sigma}} + \alpha\beta E_t[\Pi_{t+1}^{\epsilon}\phi_{t+1}], \tag{C.6}$$

where ϕ_t denotes the Lagrange multiplier for the constraint (C.3) in period t.

Having described the first-order conditions of the optimal monetary policy problem, one can compute the optimal allocation $\{C_t, N_t, \Delta_t\}_{t=0}^{\infty}$ and the optimal producer price inflation rate $\{\Pi_t\}_{t=0}^{\infty}$ by solving equations (C.2), (C.3), (C.4) and (C.5) in each period $t = 0, 1, \cdots, \infty$, given an initial value of Δ_{-1} and exogenous variables. But it should be noted that the dynamical system consisting of two difference equations (C.3) and (C.5) is self-sufficient to determine the equilibrium dynamics of the domestic producer price inflation rate and measure of relative price distortion from period 0 onward for an appropriately chosen value of Δ_{-1}. This in turn implies that one can obtain a series of the optimal inflation targets without having to solve the entire set of the optimality conditions explained above. Specifically, solving equations (C.3) and (C.5) simultaneously yields a nonlinear solution of the form:

$$\Pi_t = (\alpha + (1-\alpha)\Delta_{t-1}^{\epsilon-1})^{\frac{-1}{\epsilon-1}}. \tag{C.7}$$

Under the optimal policy, the resulting dynamics for the relative price distortion is

$$\Delta_t = \Delta_{t-1}(\alpha + (1-\alpha)\Delta_{t-1}^{\epsilon-1})^{\frac{-1}{\epsilon-1}}, \tag{C.8}$$

and the choice of domestic producer price inflation can also be expressed as

$$\Pi_t = \frac{\Delta_t}{\Delta_{t-1}}. \tag{C.9}$$

It is clear from (C.8) that the absence of the initial price dispersion leads to $\Delta_t = 1$ for $t = 1, 2, \cdots, \infty$. It then follows from (C.9) that the optimal monetary policy stabilizes the price level completely if the initial price dispersion does not exist. However, if the initial price dispersion does exist, the optimal monetary policy allows for a gradual transition of the relative price distortion toward the steady state with no price dispersion.