Inflation Premium and Oil Price Volatility*

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Abstract

In this paper we establish a link between the volatility of oil price shocks and a positive expected value of inflation in equilibrium (*inflation premium*). In doing so, we implement the perturbation method to solve up to second order a benchmark New Keynesian model with oil price shocks. In contrast with log linear approximations, the second order solution relaxes certainty equivalence providing a link between the volatility of shocks and *inflation premium*. First, we obtain analytical results for the determinants of the level of *inflation premium*. Thus, we find that the degree of convexity of both the marginal cost and the phillips curve is a key element in accounting for the existence of a positive *inflation premium*. We further show that the level of *inflation premium* might be potentially large even when a central bank implements an active monetary policy. Second, we evaluate numerically the second order solution of the model to explain the episode of high and persistent inflation observed in the US during the 70's. We find, in contrast with Clarida, Gali and Gertler (QJE, 2000), that even when there is no difference in the monetary policy rules between the pre-Volcker and post-Volcker periods, oil price shocks can generate high inflation levels during the 70's through a positive high level of *inflation premium*. As by product, our analysis shows that oil price shocks along with a distorted steady state can generate a time-varying endogenous trade-off between inflation and deviations of output from its efficient level. The previous trade-off, once uncertainty is taking into account, implies that a positive level of *inflation premium* is an optimal response to oil price shocks.

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1 Introduction

In an influential paper, Clarida, Gali and Gertler (2000, hereafter CGG), have advanced the idea that the reduction of inflation in the US during the past two decades was explained mainly by the improvement in monetary policy and instead oil price shocks have played a minor role. CGG based their conclusions on the estimations of monetary policy reaction functions for two periods: pre and post Volcker[1]. Their estimations show that during the 70s the FED, on average, let the real short term interest rate to decline as anticipate inflation rose, whereas during the post Volcker period the monetary policy of the FED became more active, raising the real interest rate in response to an increase on inflation expectations. Similar evidence is found by Cogley and Sargent (2002) and Lubik and Schorfhedie (2004).

However, these results are in contrast with recent contributions by Sims and Zha (2005, SZ hereafter), Canova, Gambetti and Pappa (2005), Primiceri (2004), Gordon (2005) and Leeper and Zha (2003) who find no evidence of a substantial change in the reaction function of monetary policy after Volcker period[2]. Instead they find strong evidence of a sizable reduction of the volatility of the main business cycle driven forces across these two periods. The previous authors emphasize on the role of second moments of shocks in identifying changes in monetary policy regime and the consequent inflation dynamics. In particular, SZ pointed out that the fact that previous works did not allow for heteroskedasticity have biased their results towards finding significant shifts in coefficients in the monetary policy rule.

The discussion of whether monetary policy or oil price shocks was the main driving force of inflation dynamics goes vis-a-vis with the debate about the recessionary effects of either monetary shock or oil price shocks. For example, Bernanke, Gertler and Watson (1997) argue that monetary policy has played a larger role during 70´s in explaining the negative output dynamics. In particular, if the monetary policy would not have reacted that much during that period the negative effects over output could have been mitigated. On the other hand, Hamilton (2001) and Hamilton and Herrada (2004) find out that the previous authors results rely on a particular identification scheme, and on the contrary they find that a contractionary monetary policy played only a minor role on the contractions in real output, being oil prices the main source of shock[3].

CGG (2000) downweight the role of oil price shocks on the grounds that changes in oil prices can only generate temporary increases in the price level but not persistent increases in inflation. They argue that supply shocks in order to generate persistent increases in inflation should be accompanied by a accommodative monetary policy rule. However, as it was pointed out by SZ (2005) the importance of shocks in explaining the dynamics of inflation relies to a great extent on taking into account the second moments of shocks. Therefore, standard log-linear approximations of general equilibrium models are not suitable to analyze the interaction between monetary policy and second moment of shocks. For instance, CGG use a linear

[1] It refes to the appointment of Paul Volcker as Chairman of the Federal Reserve System.

[2] Orphanides (2001) shows that when real time data are used to estimate policy reaction functions, the evidence of a change in policy after 1980 is weak.

[3] Hamilton (2001), finds that the size of the effect that Bernanke, Gertler and Watson (1997) attribute to oil shocks is substantially smaller than that reported by other researchers, primarily due to their choice of a shorter lag length than used by other researchers.

model where certainty equivalence holds and, therefore, the potential interaction between the volatility of shocks and average level of inflation is neglected.

In this paper we try to reconcile these two streams of the literature in a tractable and unified framework. In particular, we propose a set up that generates a link between the volatility of oil price shocks and average level of inflation. Thus, we show that a standard New keynesian model, in which oil price shocks is the only source of fluctuations and enters as non-produced input in production, is capable to generate high persistent levels of inflation in response to oil shocks. Although the model is very similar to the one analyzed by CGG, the main difference is that we use a second-order solution for the rational expectations equilibrium, instead of a linear one. The second order solution, by relaxing certainty equivalence, generates a link between the volatility of shocks and the average level of inflation. This link permit the model to deliver an extra higher level of inflation compared to the level of inflation obtained in a log-linear model. We define this extra level of average inflation as the *level of inflation premium*[4].

We implement, both analytically and numerically, the second-order solution by using the Perturbation method in the line of Sims (2002), Schmitt-Grohé and Uribe (2004) and Kim and Kim (2005). We propose a novel strategy in implementing the perturbation method to obtain analytical solution for the level of *inflation premium* in equilibrium. Different from other paper in which the perturbation method is applied directly to the non-linear system of equations, we instead first approximate the model up to second order and then we apply the perturbation method to the approximated model. This strategy permit us to disentangle the key determinants of the *inflation premium* in equilibrium.

Our analytical findings provide a link between the volatility of oil price shocks and the *level of inflation premium*. The mechanism through which this link is established works as follows: first, since oil is used as an input in production, the volatility of oil prices affects expected marginal cost of firms. When the marginal cost function is convex in oil prices, expected marginal costs are increasing in the volatility of this shock inducing forward looking firms to charge higher prices optimally. We show that the necessary condition for a convex marginal cost is an elasticity of substitution between oil and labor smaller than one. Furthermore, the smaller the elasticity of substitution, the larger the convexity of the marginal cost function and consequently the larger the level of *inflation premium*. A second channel through which oil prices affect inflation is by the convexity of Phillips curve with respect to output[5]. To the extent that the Phillips curve is convex on output, higher volatility on output generated by oil price shocks delivers a positive premium in inflation[6]. And third, we show that the overall level of *inflation premium* in equilibrium depends crucially on the relative weight that the central bank puts on output with respect to inflation. In particular, the larger the weight on output fluctuations in the reaction function of the central bank (Taylor rule) the larger the level of *inflation premium*.

[4] The extra level of inflation generated by volatility is similar to the effect of consumption volatility on the level of average savings as in the literature of precautionary savings.

[5] Weise (1999) finds some evidence that indicates that the Phillips curve by the U.S might be convex. He finds that inflation is more responsive to demand shocks when economy is in a boom than when it is in recession.

[6] Castillo and Montoro (2005) explain in detail the determinants of the convexity of the new phillips curve under Calvo price-setting. They show that the convexity of the Phillips curve is cruciall in generating asymmetric responses of output to demand shocks.

For the numerical exercises we calibrate the model for the U.S. economy by considering that oil prices shocks has exhibited a change in its mean and volatility across the pre and post Volcker periods. Our results shows that oil prices are able to generate, through high levels of *inflation premium*, the persistent increase in the level of inflation observed during the 70s even when an active monetary policy is in place. Furthermore, we perform a counterfactual exercise by fitting the historical values of oil prices shocks into the model by considering an active Taylor rule for the whole sample. We find that the model can track fairly well the average values of inflation during the 70s. Hence, our paper provides support to the empirical findings of SZ (2005) that second moments of shocks might be important to understand the change in macroeconomic behavior observed in the US economy without relying in an accommodative monetary policy.

Additionally, we also perform robustness analysis by considering alternative specifications of the model. In particular we introduce real rigidities as in Blanchard and Gali (2005 hereafter, BG) and we find that these rigidities reduce the level of *inflation premium.* The main intuition of this results is as follows: real rigidities tend to smooth marginal costs since today's real wage depends of the previous period wage rate. Smoother marginal costs reduce the uncertainty that price setters face, therefore inducing them to set prices charging a lower premium. We also include demand shocks to the model and the quantitative results do not change much, in fact, demand shocks marginally increase the level of *inflation premium.*

Last but no least, we explore the implications of the level of *inflation premium* for the design of monetary policy. We show that *inflation premium* might emerge as an optimal response of the central bank to supply shocks. In particular, unlike BG, we find that when we allow for a distorted steady-state and the production function exhibits a constant elasticity of substitution, oil prices generate a meaningful endogenous trade-off between the stabilization of inflation and output gap.[7] This trade-off implies that is optimal for the central bank to partially react to oil price shocks and to allow on average higher levels of inflation[8]. In fact, the trade-off would imply that the behavior of inflation during the 70s might reflect not only a perfectly consistent monetary policy but an optimal one.

Finally, other authors have introduced the second order approach in closed and open economies, however, most of the work have mainly focused on welfare evaluations across different environments or stochastic processes but none of them have dealt with the implications of second order solution for the determination of the *inflation premium.* Thus, Benigno and Woodford (2004) implement the second order solution to evaluate optimal monetary and fiscal policy in a closed economy. Benigno and Benigno (2005) have used the second order approach to evaluate the optimal policy in a two-country model with complete markets[9]. Closer to our work are the recent papers by Evans and Hnatkovska (2005) and Castillo and Montoro (2005).

[7] These authors find that with a Coob-Douglas production function supply shocks do not generate a trade-off between the stabilization of inflation and output gap. In order to generate the trade-off, BG rely on some reduced form of real rigidities in the labor market.

[8] Benigno and Woodford (2005) have shown that even under a distorted steady state it is possible to find an accurate welfare measure in terms of output fluctuations with respect to an efficient level of output. Therefore, a meaningful trade-off arises once a well defined measure of efficient level is considered.

[9] Also, Ferrero (2005) extends Beningo and Woodford (2004) to a two country open economy model. In a similar direction, De Paoli (2004) evaluates welfare for the case of a small open economy model with home bias.

The first authors evaluate the role of uncertainty in explaining differences in asset holdings in a two-country model. The latter authors build up a model with non-homothetic preferences and show how asymmetric responses of output and inflation emerges from the interaction of a convex Phillips curve and a state dependent elasticity of substitution in a standard New keynesian model. On the other hand, Obstfeld and Rogoff (1998) develop an explicit stochastic NOEM model relaxing the assumption of certainty equivalence. Based on simplified assumptions, they obtain analytical solutions for the level exchange rate premium. Different from Obstfeld and Rogoff (1998) and the aforementioned authors in this paper we perform both a quantitative and analytical evaluation of the second order approximation of the New Keynesian benchmark economy in order to account for the level of *inflation premium* generated by oil price shocks.

The plan of the paper is as follows. Section 2 presents some stylized facts for the US economy, in particular we focus on the mean level of inflation for different subperiods. In section 3 we outline a benchmark New Keynesian model augmented with oil as a non produced input. Section 4 explains the mechanism at work in generating the level of *inflation premium.* In section 5 we report the numerical results. In section 6 we discuss the implications of the second order solution for monetary policy and finally in the last section we conclude.

2 Oil Shocks and the US economy

In the paper we focus on the role of oil shocks at driving the mean and volatility in inflation in the U.S. economy. Our hypothesis points out to the fact that changes in the data generating process in oil shocks, basically, changes in the mean and volatility of these shocks, can account for differences in inflation mean levels in the U.S. economy in two well known periods (see CGG 2000)[10]. We would like to highlight the role of change in volatility of oil prices as adding uncertainty in inflation and consequently higher means. Thus, in this section we report some unconditional moments, in particular, mean and standard deviations for the following variables: inflation, GDP gap[11], the three months T-bill and oil prices. The data are quarterly time series spanning 1970:1-2005:2. In order to calculate the moments we divide the 1970:1-2000:4 sample in two main subperiods: 1970:01 -1987:2, which corresponds to the Pre-Volcker period. The second period 1987:3-2000:4 corresponds to the Alan Greenspan´s one. Importantly, these subperiods are similar to the ones considered by CGG (2000) as a evidence of change in monetary policy in U.S. CGG associate the first period to one in which the central bank has adopted an accommodative policy and the second one in which the central bank responds more actively to expected inflation.

We obtain the data from the Haver USECON database (mnemonics are in parentheses). Our measure of the price level is the nofarm business sector deflator (LXNFI), the measure of GDP corresponds to the nonfarm business sector output (LXNFO), we use the quarterly average daily of the 3-month T-bill (FTB3) as the nominal interest rate, and finally our measure of oil prices is the Spot Oil Prices West Texas Intermediate (PZTEXP). We express output in percapita terms by dividing LXNFO by a measure of civilian noninstitutional population aged

[10] We interpret the mean level of inflation within each period as the level inflation premium.

[11] We measure GDP gap as the deviation of the log of output from a linear trend. We do this in order to be consistent with the definition of output used in the model.

above 16 (LNN) and oil prices are deflated by the nofarm business sector deflator.

In Figure 1 we plot the real oil prices against quarterly annualized inflation for the period 1970:1-2005:2. In the first sub-sample we observe a persistent initial increase in inflation vis-a-vis and increase in oil prices following the oil price shock in 1974. From 1980 on we observe a steadily decline in inflation accompanied by a persistent drop in oil prices. For the second sub-sample, we observe also a close co-movement between inflation and oil prices, thus from early nineties until 1999 it is observed a downward trend in both oil prices and inflation, whereas from 2000 on we observe a markedly upward trend in oil and a moderate increase in inflation. During the first sub-sample oil prices present larger and more persistent swings with respect to the second sub-sample. Thus, in the first period oil prices exhibit high both volatility and mean whereas in the second period the aforementioned moments have decreased significantly. The previous observation is confirmed by the statistics in Table 1. The standard deviation of real oil prices has decreased in half, from 0.57 to 0.20, similarly the mean in oil prices has decreased from 0.29 to 0.2.

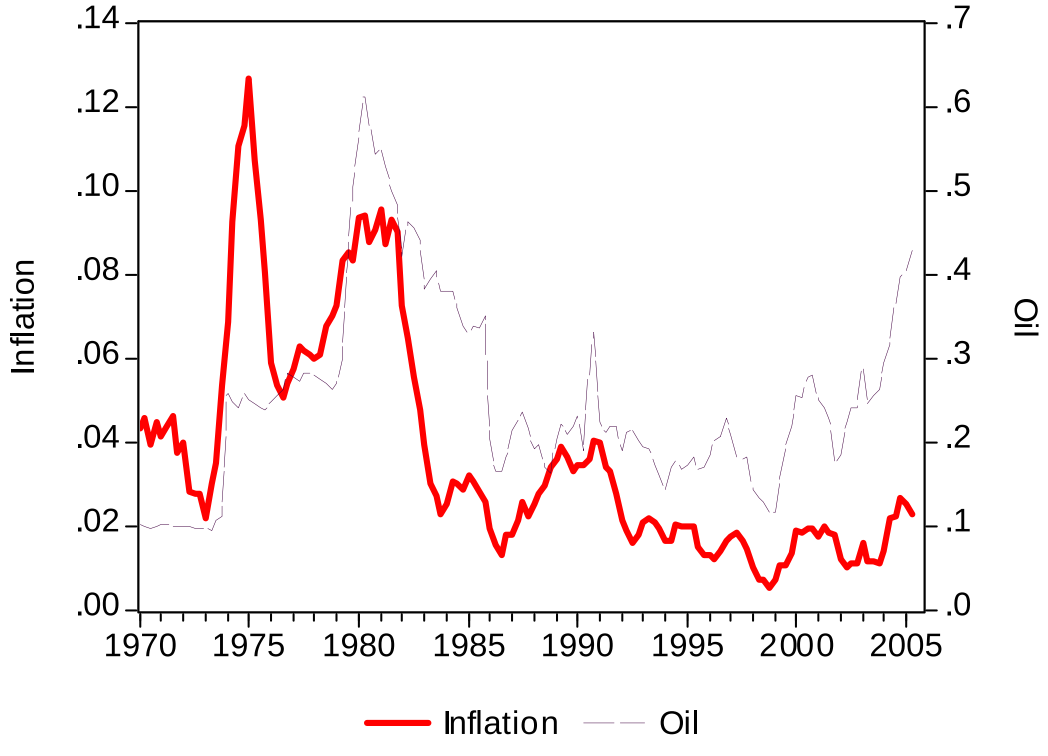

Figure1 Inflation and oil prices

Now we turn to analyze which might be the empirical implications of the change in volatility and mean in the oil price. Notice that by comparing the sub-samples we observe an important change in means and volatilities in inflation, GDP gap, and interest rates across sub-samples. Quarterly inflation standard deviation has decreased from 0.8% to 0.3% and the mean has moved from 1.4% to 0.5%, between the pre-Volcker and post-Volcker periods, respectively. Similarly, the three-month T-bill has decreased in both means and volatilities. Finally, GDP gap has decreased in volatility (from a standard deviation of 4.3% to 2.8%) and has experimented and increase in its mean (from -0.36% to -0.22%).

Table 1: Unconditional Moments

	Mean		Standard Deviation	
	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker
Inflation	1.38	0.53	0.80	0.29
GDP	−0.36	−0.22	4.3	2.8
T-Bills	1.91	1.34	0.71	0.36
Oil Prices	0.286	0.198	0.57	0.20

Pre-Volcker and post-volcker correspond to the period 70:1-87-:2 and 87:3-2000:4

In a nutshell, it seems that the change in oil prices dynamics have had a key effect over the mean of the main macroeconomic variables in U.S. from 70´s on. The numbers presented here are consistent with our story, which emphasizes in the change in inflation premium and uncertainty coming from oil prices as the main driving forces in explaining the level of *inflation premium* in the U.S. In the next section we present a microfoundated model able to establish the link between volatility and average inflation.

3 A New keynesian model with oil prices

The model economy corresponds to the standard New Keynesian Model in the line of CGG (2000). In order to capture oil shocks we follow BG (2005) by introducing a non-produced input M, represented in this case by oil. Q will be the real price of oil which is assumed to be exogenous.

3.1 Households

We assume the following period utility on consumption and labor

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu}, \tag{1}$$

where σ represents the coefficient of risk aversion and ν captures the inverse of the elasticity of labor supply. The optimizer consumer takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t} \tag{2}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the nominal gross interest rate , Γ_t is the share of the representative household on total nominal profits, and T_t are transfers from the government[12]. The first order conditions for the optimizing consumer´s problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\nu} = MRS_t \tag{4}$$

Equation (3) is the standard Euler equation that determines the optimal path of consumption. At the optimum the representative consumer is indifferent between consuming today or tomorrow, whereas equation (4) describes the optimal labor supply decision. We assume that labor markets are competitive and also that individuals work in each sector $z \in [0, 1]$. Therefore, L corresponds to the aggregate labor supply:

$$L = \int_0^1 L_t(z) dz \tag{5}$$

3.2 Firms

3.2.1 Final Good Producers

There is a continuum of final good producers of mass one, indexed by $f \in [0, 1]$ that operate in an environment of perfect competition. They use intermediate goods as inputs, indexed by $z \in [0, 1]$ to produce final consumption goods using the following technology:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

where ε is the elasticity of substitution between intermediate goods. Then the demand function of each type of differentiated good is obtained by aggregating the input demand of final good producers

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \tag{7}$$

where the price level is equal to the marginal cost of the final good producers and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} \tag{8}$$

and Y_t represents the aggregate level of output.

$$Y_t = \int_0^1 Y_t^f df \tag{9}$$

[12] In the model we assume that the government owns the oil endowment. Oil is produced in the economy at zero cost and sold to the firms at an exogenous price Q_t. The government transfers all the revenues generated by oil to consumers represented by T_t

3.2.2 Intermediate Goods Producers

There is a continuum of intermediate good producers. All of them have the following CES production function

$$Y_t(z) = \left[(1-\alpha)\left(L_t(z)\right)^{\frac{\psi-1}{\psi}} + \alpha\left(M_t\left(z\right)\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \tag{10}$$

where M is oil which enters as a non-produced input, ψ represents the intratemporal elasticity of substitution between labor-input and oil and α denotes the share of oil in the production function. We use this generic production function in order to capture the fact that oil has few substitutes[13], i.e $\psi < 1$. The oil price shock, Q_t, is assumed to follow an $AR(1)$ process in logs,

$$\log Q_t = \log \overline{Q} + \rho \log Q_{t-1} + \varepsilon_t \tag{11}$$

Where $\overline{Q}$ is the steady state oil price. From the cost minimization problem of the firm we obtain an expression for the real marginal cost given by:

$$MC_t(z) = \left[(1-\alpha)^{\psi} \left(\frac{W_t}{P_t} \right)^{1-\psi} + \alpha^{\psi}\left(Q_t\right)^{1-\psi} \right]^{\frac{1}{1-\psi}} \tag{12}$$

where $MC_t\left(z\right)$ represents the real marginal cost, W_t nominal wages and P_t the consumer price index. Notice that marginal costs are the same for all intermediate firms, since technology has constant return to scale and factor markets are competitive, i.e. $MC_t\left(z\right) = MC_t$. On the other hand, the individual firm´s labor demand is given by:

$$L_t^d(z) = \left(\frac{1}{1-\alpha} \frac{W_t/P_t}{MC_t} \right)^{-\psi} Y_t(z) \tag{13}$$

Intermediate producers set prices following a staggered pricing mechanism a la Calvo. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t} \right) = \frac{\mu E_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k \zeta_{t,t+k} MC_{t,t+k} F_{t+k}^{-\varepsilon} Y_{t+k} \right]}{E_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k \zeta_{t,t+k} F_{t+k}^{1-\varepsilon} Y_{t+k} \right]} \tag{14}$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the price markup, $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma}$ is the stochastic discount factor, $P_t^*\left(z\right)$ is the optimal price level chosen by the firm, $F_{t+k} = \frac{P_{t+k}}{P_t}$ the cumulative level of

[13] Notice that when $\psi = 1$, the production function colapses to the standadard cobb-douglas function as the one used by BG (2005). $Y_t(z) = \left(L_t(z)\right)^{1-\alpha} M_t^{\alpha}$.

inflation and Y_{t+k} is the aggregate level of output. The optimal price solves equation (14) and is determined by the average level of expected future marginal costs as follows[14]:

$$\left(\frac{P_t^*(z)}{P_t}\right) = \mu E_t\left[\sum_{k=0}^{\infty}\varphi_{t,t+k}MC_{t,t+k}\right] \tag{15}$$

Where:

$$\varphi_{t,t+k} = \frac{(\theta\beta)^k\,\zeta_{t,t+k}F_{t+k}^{-\varepsilon}Y_{t+k}}{E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\,\zeta_{t,t+k}F_{t+k}^{1-\varepsilon}Y_{t+k}\right]} \tag{16}$$

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, the price of the final good that minimize the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*(z)\right)^{1-\varepsilon} \tag{17}$$

Following Benigno and Woodford (2005), equations (14) and (17) can be written recursively introducing the auxiliary variables N_t and D_t :

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon} \tag{18}$$

$$D_t = Y_t\left(C_t\right)^{-\sigma} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon-1}D_{t+1}\right] \tag{19}$$

$$N_t = \mu Y_t\left(C_t\right)^{-\sigma}MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon}N_{t+1}\right] \tag{20}$$

Equation (18) comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $P_t^*(z)/P_t$. These three last equations summarize the recursive representation of the non linear Phillips curve.[15]

[14] In order to write the optimal price in a recursive form, we use the following change of variables following Benigno and Woodford (2005). We define N_t and D_t as follows:

$$N_t = E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\,\mu\zeta_{t,t+k}MC_{t,t+k}F_{t+k}^{-\varepsilon}Y_{t+k}\right]$$

$$D_t = E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\,\zeta_{t,t+k}F_{t+k}^{1-\varepsilon}Y_{t+k}\right]$$

Therefore, the optimal price of a typical firm can be written as:

$$\frac{P_t^*(z)}{P_t} = \frac{N_t}{D_t}$$

[15] Writing the optimal price setting in a recursive way is necessary in order to implement numerically or algebraically the perturbation method.

3.3 Monetary Policy

The central bank conducts monetary policy by targeting the nominal interest rate in the following way

$$R_t = \left[\overline{R} \left(\frac{E_t \Pi_{t+1}}{\overline{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi_y} \right] \tag{21}$$

The steady state values are expressed without time subscript and with and upper bar.

3.4 Market Clearing

In equilibrium labor, intermediate and final goods markets clear. Since there is no capital accumulation nor government sector, the economywide resource constraint is given by

$$Y_t = C_t \tag{22}$$

The labor market clearing condition is given by:

$$L_t^s = L_t^d \tag{23}$$

Where the demand for labor comes from the aggregation of individual intermediate producers in the same way as for the labor supply:

$$\begin{aligned} L^d &= \int_0^1 L_t^d(z)dz = \left(\frac{1}{1-\alpha} \frac{W_t/P_t}{MC_t} \right)^{-\psi} \int_0^1 Y_t(z)dz \\ L^d &= \left(\frac{1}{1-\alpha} \frac{W_t/P_t}{MC_t} \right)^{-\psi} Y_t \Delta_t \end{aligned} \tag{24}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well, implying that is not possible to use the usual representative firm assumption and it will appears this price dispersion term in the aggregate labor demand.

4 Inflationary Premium and Oil Price Shocks: The Mechanism

4.1 The second order solution and the inflation premium

In order to explain the determinants of the level of inflation premium we rely on a second order log approximation solution of the model. The second order solution delivers a link between volatility of price shocks and the means of endogenous variables[16]. As we mentioned in the introduction we define level of inflation premium as the extra level of inflation that arises in equilibrium once the second order solution is considered.[17]

[16] The model also delivers a premium in other endogenous variables, however, we focus on inflation.

[17] It is important to remark that this extra level of average inflation is part of the dynamic rational expectations equilibrium up to second order, and it can not be interpreted as a part of the steady state equilibrium. This second order effect on the level inflation is similar to the effect of the volatility of consumption on savings that is known in the literature as precautionary savings.

We follow the perturbation method implemented by Schmitt-Grohe and Uribe (2004) (S-GU, hereafter)[18]. This method permit us to solve the model algebraically and obtain closed form solution for the equilibrium level of inflation. Up to second order inflation in equilibrium can be written as quadratic polynomial in both the level and the standard deviation of oil prices:

$$\pi_t = \frac{1}{2} b_o \sigma_q^2 + b_1 q_t + \frac{1}{2} b_2 (q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{25}$$

where the b's are the unknown coefficients that we need to solve for and $O\left(\|q_t, \sigma_q\|^3\right)$ denotes terms on q and σ_q of order equal or higher than 3.[19]. Notice that the term $b_1 q_t$ corresponds to the policy function that we would obtain using any standard method for linear models (i.e. undetermined coefficients) whereas the additional elements of the policy function $\frac{1}{2} b_o \sigma_q^2 + \frac{1}{2} b_2 (q_t)^2$ account for the level of uncertainty (premium)[20]. This extra level of inflation has two components: $\frac{1}{2} b_o \sigma_q^2$, which is constant and, $\frac{1}{2} b_2 (q_t)^2$ which is time varying. We further express the dynamics of oil prices as:

$$q_t = \rho q_{t-1} + \eta \sigma_q e_t \tag{26}$$

where the oil shock has been normalized to have mean zero and standard deviation of one, i.e. $e_t \ iid \ \tilde{} \ (0, 1)$. We set $\eta = \sqrt{1-\rho^2}$ in order to express $V(q_t) = \sigma_q^2$. Given the previous policy functions the unconditional expected level of inflation is different from zero and positive if $b_o + b_2 > 0$, and is given by the following expression:

$$E(\pi) = \frac{1}{2} (b_o + b_2) \sigma_q^2 \tag{27}$$

In order to find the determinants of the level inflation premium, in the next section we use the algebraic solution of the perturbation method to express b_o and b_2 as functions of the deep parameters of the model.

4.2 Second order expansion of the model

Different from other papers which apply perturbation methods directly to the non-linear system of equations, we first approximate the model up to second order and then apply the perturbation method[21]. Our proposed approach has the advantage that makes easier to obtain clear analytical interpretation of the sources of the level of inflation premium.

[18] The perturbation method was originally develop by Judd (1998) and Collard and Julliard (2001). The fixed point algorithm proposed by Collard and Julliard (2001) introduces a dependence of the coefficients of the linear and quadratic terms of the solution with the volatility of the shocks. In contrast, the advantage of the algorithm proposed by S-GU is that the coefficients of the policy are invariant to the volatility of the shocks and the corresponding ones to the linear part of the solution are the same as those obtained solving a log linear approximated model, which makes both techniques comparable.

[19] Smith-Grohe and Uribe (2004) show that the quadratic solution does not depend neither on σ_q nor on $q_t \sigma_q$ i.e. they show that the coefficients in the solution for those terms are zero.

[20] Notice that these additional components also imply that the impulse response functions of inflation is asymmetric.

[21] Since a second order taylor expansion is an exact approximation up to second order of any non-linear equation, having the system expressed in that way would give the same solution as the system in its non-linear form.

Up to second order the model can be written as a system of two equations, the aggregate demand and the aggregate supply side. A detailed derivation is provided in Appendix B.

We denote variables in steady state with over bars (i.e. $\overline{X}$) and the log deviations around the steady state with lower case letters (i.e. x). The second order approximation of the aggregate supply can be written as follows[22]:

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t \left(2\left(1-\sigma\right) y_t + mc_t\right) + \frac{1}{2}\varepsilon \pi_t^2 + \beta E_t v_{t+1} + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{28}$$

where we have defined the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon - 1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}\left(1-\theta\beta\right)\pi_t z_t \tag{29}$$

v_t is an auxiliary variable that simplifies the second order expansion of the Phillips curve. The coefficient κ measures the standard linear effect of the marginal costs on inflation $\kappa = \frac{1-\theta}{\theta}\left(1-\theta\beta\right)$. On the other hand, z_t is a recursive auxiliary variable that comes from approximating the equations of the auxiliary variables N_t and D_t and has the following first order expansion

$$z_t = 2\left(1-\sigma\right) y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon - 1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{30}$$

We further can express the real marginal costs equation as the following second order equation on output and oil prices[23]:

$$mc_t = \chi\left(\nu+\sigma\right) y_t + \left(1-\chi\right) q_t + \chi v \widehat{\Delta}_t + \frac{1}{2}\left(1-\chi\right)\chi^2 \frac{1-\psi}{1-\alpha^F}\left(\left(\nu+\sigma\right) y_t - q_t\right)^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{31}$$

where:

$$\alpha^F = \alpha^{\psi}\left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi}, \quad \chi = \frac{1-\alpha^F}{1+v\psi\alpha^F}$$

we define α^F as the share of oil in the marginal costs and χ is a parameter that measures the impact of oil shocks over the marginal cost. $\widehat{\Delta}_t$ is the log-deviation of the price dispersion measure Δ_t, which is a second order function of inflation (see appendix B3 for details). It is important to note that even though the share of oil in the production function, α, can be small, its impact on marginal cost,α^F, can be magnified when oil has few substitutes (i.e. ψ is low, in general $\psi < 1$) [24]. Also, a permanent increase in oil prices, i.e. an increase in $\overline{Q}$, would make marginal cost of firms more sensitive to oil price shocks since it increases α^F . In the

[22] We follow Benigno and Woodford (2005) strategy when writting the AS as a second order approximation.

[23] The marginal costs can also be expressed as a function of the output gap: $x_t = y_t - y_t^F$, where y^F is the output under flexible prices.

$mc_t = \chi\left(\nu+\sigma\right) x_t + \frac{1}{2}\left(1-\chi\right)\left(\nu+\sigma\right)\frac{1-\psi}{1+\nu\psi\alpha^F}\left(\chi\left(\nu+\sigma\right) x_t^2 - 2x_t q_t\right)$

[24] For example, for the U.S. is estimated that oil share is in the order of 2%. However, it is estimated a elasticity of substitution of 0.56, which gives, assuming $\overline{Q} = \overline{W/P} = \overline{MC}$, $\alpha^F = (0.02)^{0.56} = 11\%$. This share would be even higher if we consider a high steady state value of oil,$\overline{Q}$.

case when $\psi = 1$, the Cobb-Douglas production function, the importance of oil is diminished since the amplifier effect of ψ is neglected [25].

Similarly, we approximate up to second order the aggregate demand and the monetary policy rule. The details can be found in appendix B. In the next sub-section we work on a tractable version of the model summarized in two equations on π_t and y_t.

4.3 Determinants of the Level of Inflation Premium

After combining the corresponding equations for the marginal costs, the policy rule, and the auxiliary variables v_t and z_t, the model can be written as a system of two second order equations on inflation and output:

$$\pi_t = \kappa_y y_t + \kappa_q q_t + \beta E_t \pi_{t+1} + \frac{1}{2}\omega_\pi \sigma_q^2 + \frac{1}{2}\kappa \left(\Omega_\pi + \Omega_{mc}\right) q_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \quad (32)$$

$$y_t = E_t\left(y_{t+1}\right) - \frac{1}{\sigma}\left(\left(\phi_\pi - 1\right) E_t \pi_{t+1} + \phi_y y_t\right) + \frac{1}{2}\omega_y \sigma_q^2 + O\left(\|q_t, \sigma_q\|^3\right) \quad (33)$$

where parameters $\kappa_y, \kappa_q, \Omega_{mc}, \Omega_\pi, \omega_\pi, \omega_y$ are defined in the appendix B.

The parameters $\{\Omega_{mc}, \Omega_\pi, \omega_\pi, \omega_y\}$ are the sources of the level of inflation premium and capture the interaction of nonlinearities of the model and volatility of oil price shocks. Note that if the aforementioned parameters were equal to zero the model collapses to a standard version of a New Keynesian model in log linear form.

The first parameter Ω_{mc} accounts for the convexity of marginal costs with respect to oil prices and depends, crucually, on the elasticity of substitution ψ. In order to illustrate this mechanism, a log quadratic approximation of the marginal cost function is useful:

$$mc_t = \left(1 - \alpha^F\right) w_t + \alpha^F q_t + \frac{1}{2}\alpha^F\left(1 - \alpha^F\right)\left(1 - \psi\right)\left(w_t - q_t\right)^2 + O\left(\|q_t, \sigma_q\|^3\right) \quad (34)$$

Notice that when $\psi < 1(\psi > 1)$, the second derivative of equation (34) with respect to oil, given by $\alpha^F\left(1 - \alpha^F\right)(1 - \psi)$, is positive (negative) and therefore the marginal cost function is convex (concave) with respect to oil prices. Thus, the parameter Ω_{mc} is positive (negative), and everything else equal, a positive (negative) inflation premium should be observed. The case when $\psi < 1$ captures the fact that when oil is difficult to substitute, an increase in the oil price triggers a more than proportional increase in marginal costs. Hence, when the marginal cost is convex in oil prices, the volatility of shocks induce forward looking firms to charge optimally higher prices. In Figure 2, in order to ilustrate the previous mechanism, we plot the relation between the level of inflation premium and the parameter ψ:

[25] Since oil has few substitutes an appealing functional form to capture this feature is the CES production function. This function offers flexibility in the calibration of the degree of substitution between oil and labor. Some authors that have included oil in the analysis of RBC models and monetary policy, have omitted this feature. For example, Kim and Loungani (1992) calibrate a production function for the U.S. that is Cobb-Douglas between labor and a composite of capital and energy, and they found that the impact of oil in the RBC for the U.S. economy is small given that oil has a small share on output. However, we argue that considering an elasticity of substitution lower than one would amplify the effects they found.

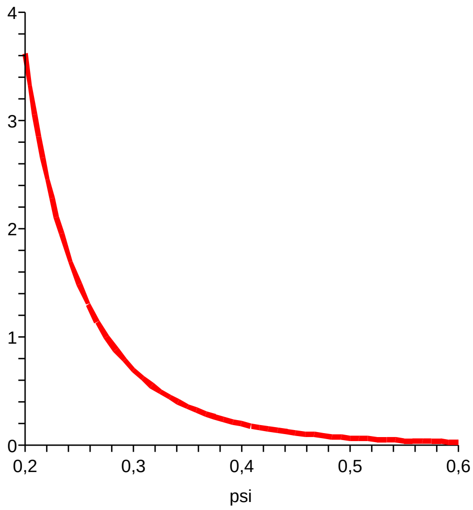

Figure 2: Level of Inflation Premium and ψ

The parameter Ω_π accounts for the convexity of the Phillips curve with respect to output. More precisely, when this parameter is positive, inflation is convex with respect to output and it follows that higher volatility on output, generated by oil price shocks, delivers a positive premium in inflation. As it is shown in appendix B, a necessary condition for the convexity of the Phillips curve on output is that

$$\left[2-\kappa_y\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\right]+\tau\nu>(2-\tau)\,\sigma \tag{35}$$

Since for benchmark calibrations $\left[2-\kappa_y\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\right]$ tends to be positive, a sufficient condition for the convexity of the Phillips curve is $v-\sigma$ sufficiently large since $\tau<1$[26]. Then in our model the convexity of the Phillips curve comes from the convexity of real wages with respect to output. Our assumption on consumption and leisure preferences imply that households tend to value more leisure as their income increases, making their labor supply less elastic. In this case, changes in real wages would generate lower increases in labor supply since the income effect dominates the substitution effect generated by the change in real wages[27].

The parameter ω_π captures the direct effect of volatility on future expected inflation, which is positive for any parametrization ($\omega_\pi>0$). Finally, ω_y, is negative and accounts for the standard precautionary savings effect.

The previous four parameters interact in general equilibrium to determine the overall level of inflation premium. The analytical solution obtained by the perturbation method implies the following expression for the overall expected level of inflation premium

$$E(\pi)=\frac{1}{2}(b_o+b_2)\,\sigma_q^2$$

[26] $\tau=\chi\left(1+(1-\psi)\frac{1-\chi}{1-\alpha^F}\right)$

[27] In an economy without oil (i.e. $\chi=1$ and $\psi=1$), the condition for the convexity becomes: $\left[2-\kappa_y\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon v\right)\right]+v>\sigma$

where

$$b_o + b_2 = \frac{\phi_y (b_2 + \omega_\pi) + \sigma\kappa_y (a_2 + \omega_y)}{\Delta_0} \tag{36}$$

$$b_2 = \left[\sigma \left(1 - \rho^2\right) + \phi_y\right] \frac{\kappa (\Omega_\pi + \Omega_{mc})}{\Delta_2} > 0 \tag{37}$$

and $\Delta_2, \Delta_0 > 0$[28]. If $b_o + b_2 > 0$ the model will deliver a positive premium. Notice that in order to warranty $b_2 > 0$ we need the phillips curve to be convex $\Omega_\pi > 0$ and the elasticity of substitution between labor an oil to be less than one, which implies that $\Omega_{mc} > 0$. Then, it follows from expression (36) that in order to obtain a positive level of inflation premium the following condition must hold:

$$\phi_y (b_2 + \omega_\pi) > -\sigma\kappa_y (a_2 + \omega_y) \tag{38}$$

where a_2,and $\omega_y < 0$.[29]

Furthermore, from equation (38) a necessary condition for a positive inflation premium ($E\pi > 0$) is $\phi_y > 0$. Hence, even so there exist several sources to generate the inflation premium, in general equilibrium, the policy of the central bank is key to determine the way in which uncertainty is distributed between output and inflation premiums. Notice that the larger ϕ_y the larger the level of inflation premium. Thus a central bank that cares only about inflation, in equilibrium, it would not generate a positive inflationary premium, instead all the uncertainty in the economy will be observed by a negative premium in output (reduction in output). In figure 3, we depict the relation between the level of inflation premium an the parameter ϕ_y.

[28] We define

$$\begin{aligned} \Delta_2 &= (\phi_\pi - 1) \rho^2 \kappa_1 + \left(1 - \beta\rho^2\right) \left[\sigma (1 - \rho)^2 + \phi_y\right] \\ \Delta_0 &= (\phi_\pi - 1) \kappa_1 + (1 - \beta) \phi_y \end{aligned}$$

then, $\Delta_0, \Delta_2 > 0$ to the extent that $\phi_\pi > 1$, which it is interpreted as an active monetary policy rule.

[29] The parameter a_2 is the coefficient of the quadratic part of the policy function of output.

$$a_2 = -\left[(\phi_\pi - 1) \rho^2\right] \kappa (\Omega_\pi + \Omega_{mc}) \frac{1}{\Delta_2} < 0$$

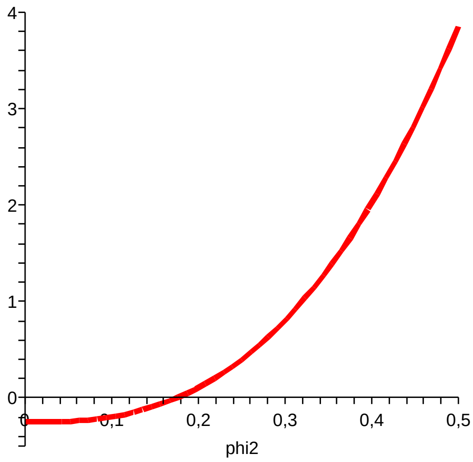

Figure 3: Level Inflation Premium and ϕ_y

Remarkably, the existence of these inflation and output premiums depend crucially on the existence of a trade-off between inflation and output. When the central bank does not face this trade-off, it is always possible to find a policy rule where the inflation premium in inflation is zero. The previous implication steams from the fact that the second order solution depends upon the log-linear one[30]. Therefore, in order to observe a positive inflation premium a necessary condition is the existence of an endogenous trade-off for the central bank. In section 6 we show how oil price shocks generate an endogenous trade-off which can make optimal for a central bank to have a positive inflation premium in equilibrium.

5 Some Numerical Experiments

In this section we explore the ability of the model to generate changes in the level of premium in the main variables of the model, in particular, in inflation. We use the Schimitt-Grohe and Uribe (2004) code which provides second order numerical solutions to a non-linear system.

5.1 Calibration

To calibrate the model we choose standard parameter values in the literature. We set a quarterly discount factor, β, equal to 0.99 which implies an annualized rate of interest of 4%. For the coefficient of risk aversion parameter, σ, we choose a value of 2 and the inverse of the elasticity of labour supply, v,is calibrated to be equal to 3, similar to those used in the *RBC* literature and consistent with the micro evidence. We choose a degree of monopolistic competition, ε, equal to 7.66. which implies a firm mark-up of 15%. The elasticity of substitution between oil and labor, ψ, is set equal to 0.59 as suggested by Kim and Loungani (1992) and

[30]In a log-linear solution, when the central bank does not face a meaningful trade-off between stabilizing inflation and output, the optimal policy implies both zero inflation and output gap.

we set $\alpha = 0.028$ so that the share of oil prices in the marginal cost is around 13%[31]. The probabilities of the Calvo lotteries is set equal to 0.66 which implies that the firms adjust prices every four quarters. To be consistent with our analytical solution we used a standard active taylor-type rule and we set $\phi_\pi = 1.5$ and $\phi_y = 0.5$[32]. Finally, the log of real oil price follows an $AR(1)$ stochastic process with $\rho_q = 0.95$ and standard deviation, $\sigma_\varepsilon = 0.14$ for the first sample and $\rho_q = 0.82$ and standard deviation, $\sigma_\varepsilon = 0.12$ for the second one. These processes imply standard deviations for real oil prices of 0.57 and 0.20 in each sample, respectively.

5.2 Explaining the U.S. Level of Inflation Premium with Oil Price Shocks

To clarify, the simulations that follow are a first step at exploring whether the mechanisms we have just emphasized have potential for explaining the level inflation-premium. We interpret oil as the main driven force of the level of inflation premium, although we are aware that in order to closely match the moments of other macro variables, additional shocks might be necessary. Thus, we intend to confront the data in line with the mechanism previously described. We do so by generating the unconditional mean of inflation implied by the calibrated model for the pre and post Volcker periods. The only difference in the calibration between these two periods is the assumption on the data generating process of oil. We fit an AR(1) process for oil prices in each period and find that both the persistence and the variance of oil price shocks have fallen from the first to the second period.

The key result is that we are able to generate a positive level of inflation premium which allows the model to mimic the average inflation level in the US in the pre-Volcker and post Volcker periods without relying on different monetary policy regimes across periods as it was suggested by *CGG*.

In this section we evaluate how the model does at capturing the conditional mean in the key macro variables, and in particular in inflation. In Table 2 we report the means of inflation, output gap and nominal interest rates for the two previously defined periods

Table 2: Unconditional Moments Generated by the Benchmark Model

	Pre and Volcker		Post Volcker	
	Simulated	Observed	Simulated	Observed
$\overline{\pi}$	**1.29**	**1.38**	**0.26**	**0.53**
$\overline{y}$	−1.30	−0.36	−0.27	−0.22
$\overline{R}$	1.28	1.91	0.26	1.34
σ_q	0.57	0.57	0.20	0.20

We compare the simulated benchmark economy with the observed data. The key result to highlight, is that the model can match very closely the mean of both inflation and output for the two sub-periods. Thus, inflation mean during the first period is 1.38% while the model

[31] Leduc and Sill (2004) have assumed a higher share of oil in the production function (0.34).

[32] Importantly, we have used the same taylor type rule for the overall sample. Values $\phi_\pi > 1$ and $\phi_y > 0$ are consistent with recent estimation using bayesian methods by Rabanal and Rubio-Ramirez (2005). Although the previous authors find out by using a shorter sample, from 1982 on, that both parameters are estimated to be higher with respect to the overall sample.

delivers a value of 1.29%. Similarly, for the second period we observe a mean inflation of 0.53% and the model predicts a value of 0.26%. The model is much less successful in matching the moments of the nominal interest rate and to a less extent of output. This might reflect the importance of productivity and demand shocks to account for the dynamics of these other variables.

To check the robustness of our results we calculate the moments under two extensions. First, we consider real rigidities in the line of BG (2005)[33] to account for a smooth adjustment of marginal costs. Second, we introduce demand shocks. The results are reported in the table 3, and show that our conclusions do not change much even when these additional features are taken into account.

Table 3: Unconditional Moments Generated by Alternative Models

	Pre and Volcker		Post Volcker	
	Real rigidities	Demand shocks	Real rigidities	Demand shocks
$\overline{\pi}$	**1.29**	**1.27**	**0.26**	**0.24**
$\overline{y}$	−1.34	−1.34	−0.29	−0.28
$\overline{R}$	1.32	1.23	0.26	0.22

Finally, we used a simple counterfactual exercise in order to factor the effect of the level of inflation premium. We use the policy functions derived in the previous section for inflation to simulate the inflation paths by fitting the historical sequence of oil price shocks into the previously discussed policy functions. The sequence of oil prices is obtained as the residuals of an $AR(1)$ model for the period 1970q1 to 2000q4[34]:

$$q_t = \underset{(39.0)}{0.96} q_{t-1} + 0.13\epsilon_t \tag{39}$$

The simulate series of inflation using the second and first order approximated solutions and the actual inflation series are plotted in figure 4. All series are expressed in annual rates. As it can be notice, the second order solution of the model can track pretty well the historical evolution of inflation. In particular, it can account for the sharp increase in inflation during the 70s and the persistent high inflation levels observed afterwards. The picture also explains why a linear approximated solution of the model might lead to conclude that oil price shocks can not explain the observed pattern in inflation during the 70s. The linear solution by eliminating any inflation premium component constantly underpredicts the mean level of inflation.

[33] We introduce real rigidities letting real wages to depend partially on the previous level of real wages as follows: $\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} MRS_t^{1-\gamma}$. See Felices (2005) and Rabanal (2004) for two different forms of micro-formalizations of real wage rigidities. We set $\gamma = 0.5$ for the simulations.

[34] We are aware that there are some non-linearities in oil prices, but in order to keep simple the analysis we have omitted any non-linear behavior in the data generating process of oil prices.

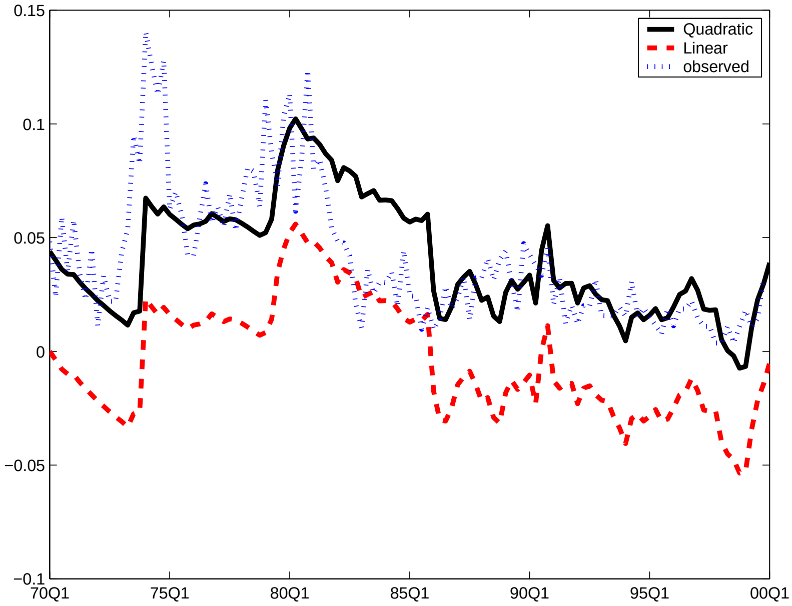

Figure 4: Simulated Inflation Paths for the US

Moreover, it is striking how the model can not only account for the behavior of inflation during the 70s, but also during second half of the 80s and the 90s, specially if we consider that our model uses only oil shocks to simulate the series. Thus, the impact of oil prices shocks does not depend on the change in the feedback monetary policy rules as it is suggested by *CGG*. Our results show that in the context of our model a supply shock can indeed induce higher levels of inflations under both periods. Therefore, contrary with CGG findings, the accommodative or passive reaction of the monetary policy must not be considered as a critical factor in the 1970´s period.

The only period where the simulated path of inflation and the observed one differs from each other is during the Volcker administration. During this period it seems more likely that negative monetary policy shocks were also in place in addition to the oil price shocks. Thus, our findings suggest that in order to match the data during the Volcker period, it is required a strong negative demand shock to compensate the high expected levels of inflation generated by oil price shocks.

The main argument of *CGG* to disqualify high oil prices as an explanation for persistent high levels of inflation during the 70s was that oil price shocks are only capable of producing one time increase on the price level but not persistent higher inflation levels. However, this intuition is only true in a world where uncertainty does not matter. Theoretically, as the previous section shows, when uncertainty is high and the model exhibits no linearities, a first order approximation to the rational expectations solution is very inaccurate since it omits the effects of inflation premium in the equilibrium dynamics. Inflation premium emerges in general equilibrium usually from the concavity of both the utility and the production functions. As we have shown in the section 3, with a CES production function, expected marginal costs are increasing on the volatility of wages and oil prices since the marginal cost is a convex function of both wages and oil prices. Higher expected marginal costs induce firms to set higher prices in a world where firms are forward looking.

In order to check the robustness of our results for our assumption of the monetary policy reaction function, we simulate the path of the short term interest rate using the same model and sequence of oil price shocks used previously. We want to show that our result does not depend critically on this assumption. In contrast with *CGG* and others who consider policy reaction functions that generate indeterminacy of the equilibrium for the pre-volcker period, instead we assume an active policy rule that guarantees determinacy. Moreover, our implied policy function is able to generate a sequence of nominal interest rates that match closely the observed one during the 70s. For the rest of the sample, the simulated interest rate systematically underpredicts the observed one. This might reflect the fact that in order to match the nominal interest rate additional sources of shocks are needed.

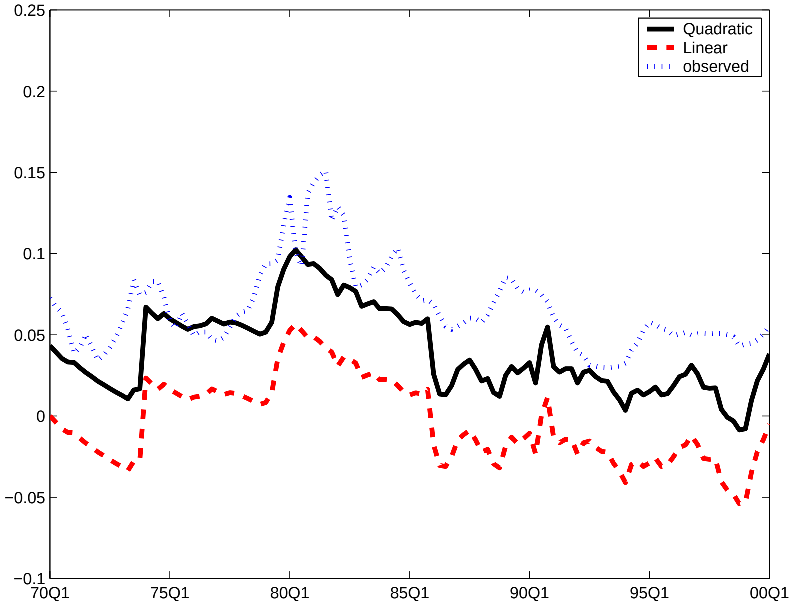

Figure 5: Simulated Nominal Interest Rate for US

As the previous pictures show the simulated inflation and interest rate paths follows closely the observed ones for the USA. In table 4 we provide further evidence of the ability of the model to capture de properties of the data showing that the means of the simulated series for inflation, output gap and interest rate matches closely their analogs for US data.

Table 5: Conditional Moments Generated by the Benchmark Model

	Pre and Volcker		Post Volcker	
	Simulate	Observed	Simulated	Observed
$\overline{\pi}$	**1.34**	**1.38**	**0.67**	**0.53**
$\overline{y}$	−0.85	−0.36	−0.44	−0.22
$\overline{R}$	1.33	1.9	0.651	1.3

6 Oil prices and Endogenous Trade-off

As BG (2005) pointed out the lack of meaningful trade-off between stabilization of inflation and the output gap is one of the drawbacks of standard New Keynesian models. Thus, the problem faced by the central bank becomes trivial, since full inflation stabilization becomes optimal regardless its cost in terms of output gap losses. This implication, denominated as the "*divine coincidence*" by BG, implies as well that no inflationary and output premiums are present[35].

Fortunately, in contrast to a standard NK models and BG (2005) specification, our benchmark model exhibits an endogenous trade-off generated by oil price shocks without relying on real rigidities. As it is shown in the appendix C, our economy can be written in terms of inflation and deviation of output with respect to its efficient level as follows:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^E\right) \tag{40}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y x_t + \mu_t \tag{41}$$

where

$$\mu_t = \frac{\kappa_q}{\kappa_y}\left(\frac{1}{\left(1-\alpha^E\right)\alpha^F}\right)\left(\alpha^F - \alpha^E\right) q_t$$

$x_t = y_t - y_t^E$ and y_t^E corresponds to the log deviations of the efficient level of output. The parameters α^E and α^F account for the share of oil prices on the marginal cost under the efficient and flexible price levels of output, respectively,

$$\alpha^E = \alpha^{\psi}\left(Q\right)^{1-\psi} \tag{42}$$

$$\alpha^F = \alpha^{\psi}\left(Q\mu\right)^{1-\psi} \tag{43}$$

In our model the endogenous trade off emerges from the combination of a distorted steady state and a CES production function. When the elasticity of substitution between oil and labor is equal to one,$\psi = 1$, the Coob-Douglas case as in BG, the trade off disappears, hence the flexible and efficient level of output only differ by a constant term, which in turn implies that $\alpha^E = \alpha^F$. In addition, when monopolistic competition distortion is eliminated, using a proportional subsidy tax, as in Woodford (2003), the trade-off is inhibited, since again $\alpha^E = \alpha^F$[36].

The trade-off in variances in turns will deliver a trade-off in means. Therefore, in our model the central bank also faces the dilemma of reducing the mean of inflation at the cost of reducing the average growth rate of the economy. As we have discussed previously, the policy rule of the central bank allows to split out the costs of high inflation and output premiums.

[35] Remember that the second order solution of the model depends upon the first order solution.

[36] Benigno and Woodford (2005) in a similar model but without oil price shocks have also found an endogenous trade-off by combining a distorted steady state with a government expenditure shock. In their framework, is the combination of a distorted steady state along with a non-linear aggregate budget constraint due to government expenditure. What is crucial to generate the endogenous trade-off. Similarly in our paper, is the combination of the distorted steady state and the non-linearity of the CES production function what delivers the endogenous trade-off.

The existence of this endogenous trade off implies that is optimal for the central bank to allow higher levels of inflation in response to supply shocks. Thus our results imply that the inflation behavior during the 70s no only might reflect a perfectly consistent monetary policy but an optimal one.

Furthermore, this trade-off depends, among other things, on the steady state level of oil prices, $\overline{Q}$, and on the elasticity of substitution between oil and labor, ψ. As equations (42) and (43) show, the trade-off is increasing (decreasing) on ψ, when $\psi < 1$ ($\psi > 1$)

$$\alpha^F - \alpha^E = \alpha^{\psi} \left(\overline{Q}^{1-\psi} \left(\mu^{1-\psi} - 1 \right) \right) \tag{44}$$

This results is particularly interesting since we can argue that during the 70s it was observed a permanent increase in the mean value of oil prices, which it might explain why the FED allowed unusual high levels of inflation.

We can then use our setup to show this latter point more formally. Let´s consider the following lost function as the objective to the central bank, as in Clarida, Gali and Gertler (1999),[37]

$$W = -\frac{1}{2} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\varpi x_t^2 + \pi_t^2 \right) \right] \tag{45}$$

and the Phillips curve defined in terms of efficient output gap (equation (41)). As CGG (1999) shows, the optimal policy under discretion implies a *"lean against the wind"* type of policy:

$$x_t = -\frac{\kappa_y}{\varpi} \pi_t \tag{46}$$

This type of policy implies a trade off between inflation and output gap since the variance of output gap and inflation are proportional

$$var(x_t) = \left(\frac{\kappa_y}{\varpi} \right)^2 var(\pi_t) \tag{47}$$

This trade-off allows to generate a policy frontier for the central bank that plots the optimal levels of the standard deviation of output and inflation for different values of the preference parameter ϖ. Notice that since κ_y depends on the elasticity of substitution of oil and labor, ψ, the policy frontier changes as this parameter changes. In particular, when the ψ is smaller, this is when oil and labor are very poor substitutes, the trade-off that the central bank faces deteriorates. For each percentage point of inflation the central bank has to increase by more the standard deviation of output.

[37] For a microfounded welfare function with oil prices and CES production function see Montoro (2005). For this paper we consider ϖ as preferences parameter that do not depend on the structural parameters of the model.

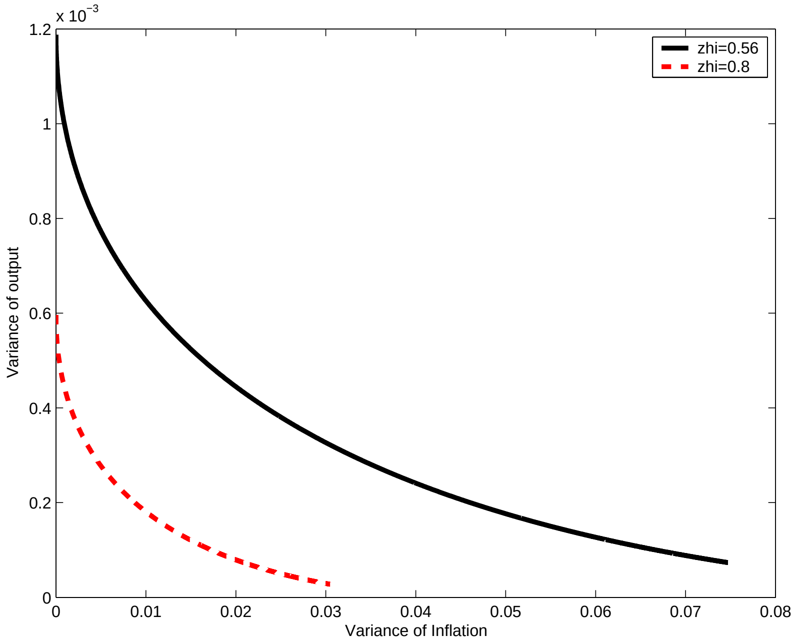

Figure 6: Policy Frontier and oil price elasticity of Substitution

7 Conclusions

Traditionally New Keynesian log-linear models have been used to match second order moments. However, they have the limitation that their solution implies certainty equivalence neglecting any role of uncertainty and volatility over the level of inflation. To the extent that uncertainty is important in real economies, a second order solution of the New Keynesian model is required to improve their fit of the data. In particular, this type of solution provides a link between volatility of shocks and the average values of endogenous variables offering a non-conventional way to analyze business cycles. In this paper we have taken this approach and show how the interaction between volatility and the convexity of both the marginal costs and the phillips curve improves the ability of a standard New keynesian model to explain the history of inflation in the USA.

The second order solution allow us to provide an additional element to the explanation suggested by CGG for the high inflation episode during the 70s. Our explanation puts at the center of the discussion the volatility of supply shocks, in particular oil price shocks. Contrary to what a linear solution implies, a second order solution establishes the link between volatility of oil prices and expected inflation what we called *inflation premium*. In the paper we show that a calibrated version of our model can match very closely the inflation behavior observed in the USA during both the pre-Volcker and post-Volcker periods. In particular we show that the high volatility of oil price shocks during the 70s implied an endogenous high level of *inflation premium* that can account for the high average inflation levels observed in US during that period . The analytical solution obtained by implementing the perturbation method shows that the existence of this *inflation premium* depends crucially on, first, the convexity of both the marginal costs and the Phillips curve and second, on the response of the monetary authority. The reaction of the central bank determines in equilibrium how higher volatility generated by oil price shocks is distributed between a higher average inflation and lower growth rate. Moreover, in order to observe a positive *inflation premium* it is required that the central bank partially reacts to supply shocks.

In addition, a standard result of the New Keynesian models is that they can not generate an endogenous trade-off for monetary policy, therefore in those models zero inflation and zero output gap is the optimal response of the Central Bank, consequently zero *inflation premium* becomes optimal. In this paper, we show that this result, denominated by Blanchard and Gali as the "Divine Coincidence" holds only under rather special assumptions: when the steady state coincides with the efficient one (there is no a distorted steady state) or when the production function has an elasticity of substitution equal to 1. Instead, we show that for the general case, allowing for a distorted along with a CES production function, oil price shocks are able to generate an endogenous cost push shock making the central bank problem a meaningful one.

This endogenous cost push shock generates a trade-off in means for the central bank. In this case the central bank can not reduce the average level of inflation without sacrificing output growth. We show that the optimal policy implies to partially accommodate oil price shocks and to let, on average, a higher level of inflation. Furthermore, this trade-off depends crucially on the share of oil in the production function, on the elasticity of substitution between oil and labor and on the average oil prices. Thus our results imply that the inflation behavior in the U.S. during the 70s not only might reflect a perfectly consistent monetary policy but an optimal one.

Our results can be extended in many directions. First, it will be worth to explore the effect of openness in *inflation premium*. Second, the analytical perturbation method strategy proposed in the paper can be used to capture the effects of change in a monetary policy regime over inflation. Finally, it will be worth also to explore the implications of other source of shocks in the determination in the level of *inflation premium*.

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A Equations of the Model

A.1 The system of equations

Using the the market clearing conditions that close the model, the dynamic equilibrium of the model described in section 2 is given by the following set of 8 equations:

$$\theta \left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon} \tag{48}$$

$$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon} N_{t+1}\right] \tag{49}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon-1} D_{t+1}\right] \tag{50}$$

$$1 = \beta E_t\left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}}\right] \tag{51}$$

$$R_t = \overline{R}\left(\frac{E_t\Pi_{t+1}}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y} \tag{52}$$

$$MC_t = \left[(1-\alpha)^{\psi}\left(W_t/P_t\right)^{1-\psi} + \alpha^{\psi}\left(Q_t\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \tag{53}$$

$$\frac{W_t}{P_t} = Y_t^{\sigma} L_t^{v} \tag{54}$$

$$L_t = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t \tag{55}$$

The first three equations represent the Phillips curve, which has been written recursively using the auxiliary variables N_t and D_t. The aggregate demand block is represented by the IS and the taylor rule. the last three equations describe the labor market equilibrium. We use this set of eight non-linear equations to obtaint numerically the second order solution of the model.

A.2 The deterministic steady state

The non-stochastic steady state of the endogenous variables is given by:

Inflation	$\overline{\Pi} = 1$
Auxiliary variables	$\overline{N} = \overline{D} = \overline{Y}/\left(1-\theta\beta\right)$
Interest rate	$\overline{R} = \beta^{-1}$
Marginal costs	$\overline{MC} = 1/\mu$
Real wages	$\overline{W/P} = \tau_y \frac{1}{\mu}\left(1-\alpha^F\right)^{\frac{1}{1-\psi}}$
Output	$\overline{Y} = \tau_y\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}}\left(1-\alpha^F\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$\overline{L} = \tau_l\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}}\left(1-\alpha^F\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

where

$$\alpha^F = \alpha^\psi \left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi} = \alpha^\psi \left(\mu\overline{Q}\right)^{1-\psi}$$

α^F is the share of oil in the marginal costs, τ_w and τ_y and τ_l are constants[38]. Notice that the steady state values of real wages, output and labor depend on the steady state ratio of oil prices with respect to the marginal cost. This implies that permanent changes in oil prices would generate changes in the steady state of this variables. Also, as the standard new-keynesian models, the marginal cost in steady state is equal to the inverse of the mark-up ($\overline{MC} = 1/\mu = (\varepsilon - 1)/\varepsilon$). Since monopolistic competition affects the steady state of the model, ouput in steady state is below the efficient level (i.e. when $\overline{MC} = 1$). We call to this feature a distorted steady state.

A.3 The flexible price equilibrium

The flexible price equilibrium of the endogenous variables is consistent with zero inflation in every period (i.e. .$\Pi_t^F = 1$). In this case marginal costs are constant, equal to its steady state value, and the other variables are affected by the oil shock.

Inflation	$\Pi_t^F = 1$
Interest rate	$1/R_t^F = E_t\left(\frac{1-\alpha^F\left(Q_{t+1}/\overline{Q}\right)^{1-\psi}}{1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}}\right)^{-\sigma\gamma}$
Marginal costs	$MC_t^F = 1/\mu$
Real wages	$W_t^F/P_t^F = \tau_y\frac{1}{\mu}\left(1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}$
Output	$Y_t^F = \tau_y\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}}\left(1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$L_t^F = \tau_l\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}}\left(1-\alpha^F\left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Notice that the flexible price equilibrium is not efficient, since there are distortions from monopolistic competition in the intermediate goods market (i.e. $MC_t^F > 1$).

[38] More precisely:

$$\tau_w = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}} \quad \tau_y = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}\frac{1+v}{\sigma+v}} \quad \tau_l = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}\frac{1-\sigma}{\sigma+v}}$$

A.4 The efficient price equilibrium

The efficient price equilibrium of the endogenous variables is consistent with zero inflation in every period and the monopolistic distortions are eliminated, that is $MC_t^E = 1$.

Inflation	$\Pi_t^E = 1$
Interest rate	$1/R_t^E = E_t \left(\frac{1-\alpha^E \left(Q_{t+1}/\overline{Q}\right)^{1-\psi}}{1-\alpha^E \left(Q_t/\overline{Q}\right)^{1-\psi}} \right)^{-\sigma\gamma}$
Marginal costs	$MC_t^E = 1$
Real wages	$W_t^E/P_t^E = \tau_y \left(1 - \alpha^E \left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}$
Output	$Y_t^E = \tau_y \left(1 - \alpha^E \left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$L_t^E = \tau_l \left(1 - \alpha^E \left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Where: $\alpha^E = \alpha^\psi \left(\overline{Q}\right)^{1-\psi} < \alpha^F$ is the share of oil in the marginal costs when the monopolistic distortions are eliminated. Notice that the efficient equilibrium of wages, output and labor differs from the flexible price equilibrium only through α^E.

A.5 The linear system

The model economy up to a linear approximation is represented by the following system of six equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{56}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{57}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{58}$$

$$mc_t = w_t + \alpha^F \left(q_t - w_t\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{59}$$

$$w_t = \sigma y_t + v l_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{60}$$

$$l_t = y_t + \psi\left(mc_t - w_t\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{61}$$

Equation(56) is the Phillips curve, the aggregate demand is represented by the IS (57) and the taylor rule (58), and the last three equations describe the labor market equilibrium.

B The second order approximation of the system

B.1 The AS equation

The Phillips curve with oil prices is given by the following three equations:

$$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon} \tag{62}$$

$$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t\left(\Pi_{t+1}\right)^{\varepsilon} N_{t+1} \tag{63}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t\left(\Pi_{t+1}\right)^{\varepsilon-1} D_{t+1} \tag{64}$$

B.1.1 The first order approximation of the Phillips Curve

The linear expansions of equations (62), (63) and (64):

$$\pi_t = \frac{(1-\theta)}{\theta}\left(n_t - d_t\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{65}$$

$$n_t = (1-\theta\beta)\left((1-\sigma) y_t + rmc_t\right) + \theta\beta E_t\left(\varepsilon\pi_{t+1} + n_{t+1}\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{66}$$

$$d_t = (1-\theta\beta)\left((1-\sigma) y_t\right) + \theta\beta E_t\left((\varepsilon-1)\pi_{t+1} + d_{t+1}\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{67}$$

To obtain the standard New-Keynesian Phillips Curve we need to solve for $n_t - d_t$ as a function of the real marginal costs and expected inflation. We substract equations (66) and (67):

$$n_t - d_t = (1-\theta\beta) mc_t + \theta\beta E_t\left(\pi_{t+1} + n_{t+1} - d_{t+1}\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{68}$$

Taking forward one period equation (65), we can solve for $n_{t+1} - d_{t+1}$:

$$\left(n_{t+1} - d_{t+1}\right) = \frac{\theta}{1-\theta}\pi_{t+1} + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{69}$$

Replace equation (69) in (68) and we obtain $n_t - d_t$ as a function of the real marginal costs and expected inflation:

$$n_t - d_t = (1-\theta\beta) mc_t + \frac{\theta}{1-\theta}\beta E_t\pi_{t+1} + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{70}$$

Replacing equation (70)in (62), we obtain:

$$\pi_t = \kappa rmc_t + \beta E_t\left(\pi_{t+1}\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{71}$$

where $\kappa = \frac{(1-\theta)}{\theta}(1-\theta\beta)$. This is the standard Phillips curve, inflation depends linearly on the real marginal costs and expected inflation.

B.1.2 The second order approximation of the Phillips Curve

Similarly to the previous subsection, we follow same steps as Benigno and Woodford (2005) to obtain the second order expansion for equations (62), (63) and (64):

$$\pi_t = \frac{(1-\theta)}{\theta}(n_t - d_t) - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{72}$$

$$n_t = (1-\theta\beta)\left(a_t + \frac{1}{2}a_t^2\right) + \theta\beta\left(E_t b_{t+1} + \frac{1}{2}E_t b_{t+1}^2\right) - \frac{1}{2}n_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{73}$$

$$d_t = (1-\theta\beta)\left(c_t + \frac{1}{2}c_t^2\right) + \theta\beta\left(E_t e_{t+1} + \frac{1}{2}E_t e_{t+1}^2\right) - \frac{1}{2}d_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{74a}$$

Where we have defined the auxiliary variables a_t,b_{t+1},c_t and e_{t+1} as:

$$\begin{array}{rr} a_t = (1-\sigma)\, y_t + mc_t & b_{t+1} = \varepsilon\pi_{t+1} + n_{t+1} \\ c_t = (1-\sigma)\, y_t & e_{t+1} = (\varepsilon - 1)\,\pi_{t+1} + d_{t+1} \end{array}$$

Substract equations (73) and ($74a$), and using the fact that $X^2 - Y^2 = (X-Y)(X+Y)$, for any two variables X and Y :

$$\begin{aligned} n_t - d_t \;=\; & (1-\theta\beta)(a_t - c_t) + \frac{1}{2}(1-\theta\beta)(a_t - c_t)(a_t + c_t) \\ & +\theta\beta E_t(b_{t+1} - e_{t+1}) + \frac{1}{2}\theta\beta E_t(b_{t+1} - e_{t+1})(b_{t+1} + e_{t+1}) \\ & -\frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{75}$$

Plugging in the values of a_t, b_{t+1}, c_t and e_{t+1} into equation (75), we obtain (89)

$$\begin{aligned} n_t - d_t \;=\; & (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t\,(2(1-\sigma)\, y_t + mc_t) \\ & +\theta\beta E_t(\pi_{t+1} + n_{t+1} - d_{t+1}) + \frac{1}{2}\theta\beta E_t(\pi_{t+1} + n_{t+1} - d_{t+1})((2\varepsilon - 1)\,\pi_{t+1} + n_{t+1} + d_{t+1}) \\ & -\frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{76}$$

Taking forward one period equation (72), we can solve for $n_{t+1} - d_{t+1}$:

$$n_{t+1} - d_{t+1} = \frac{\theta}{1-\theta}\pi_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}\frac{(\varepsilon-1)}{1-\theta}(\pi_{t+1})^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{77}$$

replace equation (77) in (76) and make use of the auxiliary variable $n_t + d_t = (1-\theta\beta)\, z_t$

$$\begin{aligned} n_t - d_t \;=\; & (1-\theta\beta)\, mc_t + \frac{1}{2}(1-\theta\beta)\, mc_t\,(2(1-\sigma)\, y_t + mc_t) \\ & +\frac{\theta}{1-\theta}\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)\, E_t\pi_{t+1} z_{t+1}\right] \\ & -\frac{1}{2}\frac{\theta}{1-\theta}(1-\theta\beta)\,\pi_t z_t + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{78}$$

Notice that we use only the linear part of equation (77) when we replace $n_{t+1} - d_{t+1}$ in the quadratic terms. Similarly, we make use of the linear part of equation (72) to replace $(n_t - d_t) = \frac{\theta}{1-\theta}\pi_t$ in the right hand side of equation (78).

Replace equation (78) in (72):

$$\begin{aligned}\pi_t &= \kappa mc_t + \frac{1}{2}\kappa mc_t \left(2\left(1-\sigma\right) y_t + mc_t\right) \\ &+\frac{\theta}{1-\theta}\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon - 1}{1-\theta} + \varepsilon\right) E_t\pi_{t+1}^2 + \left(1-\theta\beta\right) E_t\pi_{t+1}z_{t+1}\right] \\ &-\frac{1}{2}\left(1-\theta\beta\right)\pi_t z_t - \frac{1}{2}\frac{\left(\varepsilon-1\right)}{1-\theta}\left(\pi_t\right)^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right)\end{aligned} \tag{79}$$

for $\kappa = \frac{(1-\theta)}{\theta}\left(1-\theta\beta\right)$ as defined previously.

Define the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}\left(1-\theta\beta\right)\pi_t z_t \tag{80}$$

where $z_t = \left(n_t + d_t\right) / \left(1-\theta\beta\right)$ has the following linear expansion:

$$z_t = 2\left(1-\sigma\right) y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon - 1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{81}$$

Using the definition for v_t, equation (79)can be expressed as:

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2\left(1-\sigma\right) y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\left\|q_t, \sigma_q\right\|^2\right) \tag{82}$$

which is equation (29) in the main text.

B.2 The MC equation and the labor market equilibrium

The real marginal cost and the labor market is given by the following non-linear equations:

$$MC_t = \left[\left(1-\alpha\right)^{\psi}\left(W_t/P_t\right)^{1-\psi} + \alpha^{\psi}\left(Q_t\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \tag{83}$$

$$\frac{W_t}{P_t} = Y_t^{\sigma} L_t^{v} \tag{84}$$

$$L_t = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t \tag{85}$$

Which have the following second order expansion:

$$mc_t = \left(1-\alpha^F\right) w_t + \alpha^F q_t + \frac{1}{2}\alpha^F\left(1-\alpha^F\right)\left(1-\psi\right)\left(w_t - q_t\right)^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{86}$$

$$w_t = \nu l_t + \sigma y_t \tag{87}$$

$$l_t = y_t - \psi\,(w_t - mc_t) + \widehat{\Delta}_t \tag{88}$$

Where w_t and $\widehat{\Delta}_t$ are, respectively, the log of the deviation of the real wage and the price dispersion measure from their respective steady state. Notice that equations (87)and (88) are not approximations, but exact expresions.

Solving equations (87) and (88) for the equilibrium real wage:

$$w_t = \frac{1}{1+\nu\psi}\left[(\nu+\sigma)\,y_t + \nu\psi mc_t + v\widehat{\Delta}_t\right] \tag{89}$$

Plugging the real wage in equation (86) and simplifying:

$$\begin{aligned} mc_t &= \chi\,(\sigma+v)\,y_t + (1-\chi)\,(q_t) + \chi v\widehat{\Delta}_t \\ &\quad + \frac{1}{2}\frac{1-\psi}{1-\alpha^F}\chi^2\,(1-\chi)\,[(\sigma+v)\,y_t - q_t]^2 + O\left(\|q_t,\sigma_q\|^3\right) \end{aligned} \tag{90}$$

where $\chi = \left(1-\alpha^F\right)/\left(1+v\psi\alpha^F\right)$ .This is the equation (31) in the main text. This expression is the second order expansion of the real marginal cost as a function of output and the oil prices.

B.3 The price dispersion measure

The price dispersion measure is given by

$$\Delta_t = \int_0^1 \left(\frac{P_t\,(z)}{P_t}\right)^{-\varepsilon} dz \tag{91}$$

Since a proportion $1-\theta$ of intermediate firms set prices optimally, whereas the other θ set the price last period, this price dispersion measure can be written as:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*\,(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}\,(z)}{P_t}\right)^{-\varepsilon} dz \tag{92}$$

Dividing and multiplying by $(P_{t-1})^{-\varepsilon}$ the last term of the RHS:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*\,(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}\,(z)}{P_{t-1}}\right)^{-\varepsilon}\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} dz \tag{93}$$

Since $P_t^*\,(z)\,/P_t = N_t/D_t$ and $P_t/P_{t-1} = \Pi_t$, using equation (8) in the text and the definition for the dispersion measure lagged on period, this can be expressed as

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\,(\Pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}\,(\Pi_t)^{\varepsilon} \tag{94}$$

Which is a recursive representation of Δ_t as a function of Δ_{t-1} and Π_t.

We can show that a second order approximation of the price dispersion depends solely on second order terms on inflation. Take the second order taylor expansion to equation (91):

$$\widehat{\Delta}_t = -\varepsilon\int_0^1 (p_t\,(z) - p_t)\,dz + \frac{1}{2}\varepsilon^2\int_0^1 (p_t\,(z) - p_t)^2\,dz + O\left(\|q_t,\sigma_q\|^3\right) \tag{95}$$

similarly, take the second order taylor expansion to the definition of the price index, equation (8) :

$$0 = (1-\varepsilon)\int_0^1 (p_t(z) - p_t)\, dz + \frac{1}{2}(1-\varepsilon)^2 \int_0^1 (p_t(z) - p_t)^2 dz + O\left(\|q_t, \sigma_q\|^3\right) \tag{96}$$

Use equation (96) to eliminate the linear term in equation (95):

$$\widehat{\Delta}_t = \frac{1}{2}\varepsilon \int_0^1 (p_t(z) - p_t)^2 dz + O\left(\|q_t, \sigma_q\|^3\right) \tag{97}$$

From this expression we can see that the price dispersion measure is function only of second order terms. Using the fact that a proportion $1-\theta$ of individuals set prices optimally whereas the other θ set the price last period, equation (97) can be expressed as:

$$\widehat{\Delta}_t = \frac{1}{2}(1-\theta)\varepsilon (p_t^* - p_t)^2 + \frac{1}{2}\theta\varepsilon \int_0^1 (p_{t-1}(z) - p_t)^2 dz + O\left(\|q_t, \sigma_q\|^3\right) \tag{98}$$

The the first term of the RHS of equation (98) is:

$$\frac{1}{2}(1-\theta)\varepsilon (p_t^* - p_t)^2 = \frac{1}{2}\varepsilon \frac{\theta^2}{1-\theta}(\pi_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{99}$$

Where we have used the linear expansion of equation (17) in the text. Similarly, the second term of the RHS of equation (98):

$$\frac{1}{2}\theta\varepsilon \int_0^1 (p_{t-1}(z) - p_t)^2 dz = \frac{1}{2}\theta\varepsilon \int_0^1 \left[(p_{t-1}(z) - p_{t-1}) + (p_{t-1} - p_t)\right]^2 dz \tag{100}$$

$$= \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\theta\pi_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{101}$$

where we have made use of the equation (97) lagged one period for the definition of $\widehat{\Delta}_{t-1}$, the linear expansion of equations (17) and (8). Considering both terms, we obtain the second order approximation of Δ_t is:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{102}$$

Moreover, we can use equation (102) to write the infinite sum:

$$\sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_t = \theta \sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty} \beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right) \tag{103}$$

$$(1-\beta\theta)\sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_t = \theta\widehat{\Delta}_{t_o-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty} \beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right)$$

Dividing by $(1-\beta\theta)$ and using the definition of κ :

$$\sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_t = \frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1} + \frac{1}{2}\frac{\varepsilon}{\kappa}\sum_{t=t_o}^{\infty} \beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right) \tag{104}$$

The discounted infinite sum of $\widehat{\Delta}_t$ is equal to the sum of two terms, on the initial price dispersion and the discounted infinite sum of π_t^2.

B.4 The IS and the taylor rule

The non-linear equations for the IS and the taylor rule:

$$1 = \beta E_t \left[\left(\frac{Y_{t+1}}{Y_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{105}$$

$$R_t = \overline{R} \left(\frac{E_t \Pi_{t+1}}{\overline{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi_y} \tag{106}$$

which have the following a second order expansion:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} \left(r_t - E_t \pi_{t+1} \right) - \frac{1}{2} \sigma E_t \left[(y_t - y_{t+1}) - \frac{1}{\sigma} \left(r_t - \pi_{t+1} \right) \right]^2 + \left(\|q_t, \sigma_q\|^3 \right) \tag{107}$$

$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_y y_t \tag{108}$$

Replacing the linear solution of y_t inside the quadratic part of equation (108):

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} \left(r_t - E_t \pi_{t+1} \right) - \frac{1}{2} \sigma E_t \left[y_{t+1} + \frac{1}{\sigma} \pi_{t+1} - E_t \left(y_{t+1} + \frac{1}{\sigma} \pi_{t+1} \right) \right]^2 + \left(\|q_t, \sigma_q\|^3 \right) \tag{109}$$

Where $E_t \left[y_{t+1} + \frac{1}{\sigma} \pi_{t+1} - E_t \left(y_{t+1} + \frac{1}{\sigma} \pi_{t+1} \right) \right]^2$ is the variance of $y_{t+1} + \frac{1}{\sigma} \pi_{t+1}$.

B.5 The system in two equations

B.5.1 The AS

Replace the equation for the marginal costs (90) in the second order expansion of the Phillips curve (109)

$$\begin{aligned} v_t &= \kappa_y y_t + \kappa_q q_t + \kappa \chi v \widehat{\Delta}_t + \frac{1}{2} \varepsilon \pi_t^2 + \\ &+ \frac{1}{2} \kappa \left[c_{yy} y_t^2 + 2 c_{yq} y_t q_t + c_{qq} q_t^2 \right] + \beta E_t v_{t+1} + \left(\|q_t, \sigma_q\|^3 \right) \end{aligned} \tag{110}$$

where we have used the auxiliary variables: $v_t = \pi_t + \frac{1}{2} \left(\frac{\varepsilon - 1}{1 - \theta} + \varepsilon \right) \pi_t^2 + \frac{1}{2} \left(1 - \theta \beta \right) \pi_t z_t$

The linear coefficients are given by

$$\begin{aligned} \kappa_y &= \kappa \chi \left(\sigma + \nu \right) \\ \kappa_q &= \kappa \left(1 - \chi \right) \end{aligned}$$

and the quadratic part is given by the coefficientes:

$$\begin{aligned} c_{yy} &= c_{yy}^{\pi} + c_{yy}^{mc} \\ c_{yq} &= c_{yq}^{\pi} + c_{yq}^{mc} \\ c_{qq} &= c_{qq}^{\pi} + c_{qq}^{mc} \end{aligned}$$

We have separated the coefficientes from the source of the non-linearity, from the price setting (π) or the marginal costs (mc), which are the following:

$$\begin{array}{rr} c_{yy}^{\pi}=\chi\left(\sigma+\nu\right)\left[2\left(1-\sigma\right)+\chi\left(\sigma+\nu\right)\right] & c_{yy}^{mc}=\left(1-\psi\right)\frac{\chi^{2}(1-\chi)(\sigma+\nu)^{2}}{1-\alpha^{F}} \\ c_{yq}^{\pi}=\left(1-\chi\right)\left[2\left(1-\sigma\right)+\chi\left(\sigma+\nu\right)\right] & c_{yq}^{mc}=-\left(1-\psi\right)\frac{\chi^{2}(1-\chi)(\sigma+\nu)}{1-\alpha^{F}} \\ c_{qq}^{\pi}=\left(1-\chi\right)^{2} & c_{qq}^{mc}=\left(1-\psi\right)\frac{\chi^{2}(1-\chi)}{1-\alpha^{F}} \end{array}$$

Equation (110) is a recursive second order representation of the Phillips curve. However, we need to express the price dispersion in terms of inflation in order to have a the Phillips curve only as a function of output, inflation and the oil price shock. Equation (110) can also be expressed as the discounted infinite sum:

$$v_{t}=\sum_{t=t_{o}}^{\infty}\beta^{t-t_{o}}\left\{\kappa_{y}y_{t}+\kappa_{q}q_{t}+\kappa\chi v\widehat{\Delta}_{t}+\frac{1}{2}\varepsilon\pi_{t}^{2}+\frac{1}{2}\kappa\left[c_{yy}y_{t}^{2}+2c_{yq}y_{t}q_{t}+c_{qq}q_{t}^{2}\right]\right\}+\left(\left\|q_{t},\sigma_{q}\right\|^{3}\right)$$

make use of equation (104), the discounted infinite sum of $\widehat{\Delta}_{t}$, v_t becomes

$$v_{t}=\sum_{t=t_{o}}^{\infty}\beta^{t-t_{o}}\left\{\kappa_{y}y_{t}+\kappa_{q}q_{t}+\frac{1}{2}\varepsilon\left(1+\chi v\right)\pi_{t}^{2}+\frac{1}{2}\kappa\left[c_{yy}y_{t}^{2}+2c_{yq}y_{t}q_{t}+c_{qq}q_{t}^{2}\right]\right\}+\frac{\chi v\theta}{1-\beta\theta}\widehat{\Delta}_{t_{o}-1}+\left(\left\|q_{t},\sigma_{q}\right\|^{3}\right)$$

Which can be expressed in the following recursive way:

$$\begin{aligned} v_{t} &= \kappa_{y}y_{t}+\kappa_{q}q_{t}+\frac{1}{2}\varepsilon\left(1+\chi v\right)\pi_{t}^{2}+\frac{\chi v\theta}{1-\beta\theta}\widehat{\Delta}_{t_{o}-1} \\ &\quad +\frac{1}{2}\kappa\left[c_{yy}y_{t}^{2}+2c_{yq}y_{t}q_{t}+c_{qq}q_{t}^{2}\right]+\beta E_{t}v_{t+1}+\left(\left\|q_{t},\sigma_{q}\right\|^{3}\right) \end{aligned} \tag{111}$$

This is the second order expression of the Phillips curve as a function solely of ouput, inflation and the initial price distortion, which without any loss of the generality it can be assumed equal to zero for the analysis (i.e. $\widehat{\Delta}_{t_o-1}=0$).

Convexity of the AS The convexity/concavity of π_t with respect to y_t in equation (110) is measured by the partial derivative $\partial^2\pi_t/\partial y_t^2$:

$$\begin{aligned} \frac{\partial^{2}\pi_{t}}{\partial y_{t}^{2}} &= \frac{\kappa c_{yy}-\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\left(\partial\pi_{t}/\partial y_{t}\right)^{2}}{1+\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\pi_{t}} \\ &= \frac{\kappa c_{yy}-\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\left[\kappa\chi\left(\sigma+\nu\right)\right]^{2}}{1+\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\pi_{t}} \end{aligned} \tag{112}$$

Therefore, the AS equation is convex (concave) with respect to output if $\frac{\partial^2\pi_t}{\partial y_t^2}>(<)\,0$.

Since the denominator is positive, the covexity condition is:

$$\kappa\chi\left(\sigma+\nu\right)\left[2\left(1-\sigma\right)+\chi\left(\sigma+\nu\right)\left(1+\left(1-\psi\right)\frac{1-\chi}{1-\alpha^{F}}\right)\right]>\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\left[\kappa\chi\left(\sigma+\nu\right)\right]^{2} \tag{113}$$

This condition can be written such us:

$$\left[2-\kappa_y\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon\chi v\right)\right]+\tau\nu>(2-\tau)\,\sigma \tag{114}$$

where: $\tau=\chi\left(1+(1-\psi)\,\frac{1-\chi}{1-\alpha^F}\right)$ and $\kappa_y=\kappa\chi\,(\sigma+\nu)$ is the slope of the Phillips curve with respect to output. In the case of a closed economy without oil in the production function (i.e. $\chi=1$ and $\psi=1$), this condition becomes:

$$\left[2-\kappa_y\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon v\right)\right]+v>\sigma \tag{115}$$

Since for most calibrations $2-\kappa_y\left(\frac{\varepsilon-1}{1-\theta}-\varepsilon v\right)$ tend to be positive. A sufficient condition (but not necessary) for convexity of the Phillips curve in the case without oil is that: $v>\sigma$.

The AS premium components Equation (110) can be written in the following form:

$$\pi_t=\kappa_y y_t+\kappa_q q_t+\beta E_t\pi_{t+1}+\frac{1}{2}\omega_\pi\sigma_q^2+\frac{1}{2}\kappa\left(\Omega_\pi+\Omega_{mc}\right)q_t^2+O\left(\|q_t,\sigma_q\|^3\right) \tag{116}$$

where we can express the quadratic terms as:

$$\begin{aligned}\omega_\pi\sigma_q^2+\kappa\Omega_\pi q_t^2 \;=\;& \kappa\left[c_{yy}^\pi y_t^2+2c_{yq}^\pi y_t q_t+c_{qq}^\pi q_t^2\right]+\varepsilon\,(1+\chi v)\,\pi_t^2 \\ &+\left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)\left(\beta E_t\pi_{t+1}^2-\pi_t^2\right)+(1-\theta\beta)\left(\beta E_t\pi_{t+1}z_{t+1}-\pi_t z_t\right)\end{aligned} \tag{117}$$

and

$$\Omega_{mc}q_t^2=c_{yy}^{mc}y_t^2+2c_{yq}^{mc}y_t q_t+c_{qq}^{mc}q_t^2 \tag{118}$$

Equation (117) and (118) describe the inflation premium components coming from the price setting and the non-linear marginal costs, respectively. In equation (117) we have separated the components in those coming from the variance of the oil price (ω_π) and from those comming from the level (Ω_π). The inflation premium component from the marginal costs is affected only for the level, but not for the variance of the shocks.

In order to solve for ω_π,Ω_π and Ω_{mc} we can use the linear solution for output, inflation and the auxiliary variable z_t[39]

$$\begin{aligned} y_t &= a_1 q_t+O\left(\|q_t,\sigma_q\|^2\right)\\ \pi_t &= b_1 q_t+O\left(\|q_t,\sigma_q\|^2\right)\\ z_t &= c_1 q_t+O\left(\|q_t,\sigma_q\|^2\right)\end{aligned}$$

Additionally, we have the transition process for the oil price:

[39] From the linear expansion of the definition of z_t we can solve for c_1, where $c_1=\frac{1}{1-\theta\beta\rho}\left\{\left[2\,(1-\sigma)+\chi\,(\sigma+v)\right]a_1+(1-\chi)+\theta\beta\frac{2\varepsilon-1}{1-\theta\beta}\rho b_1\right\}$

$$q_t = \rho q_{t-1} + \eta\sigma_q e_t$$

where $e\tilde{}iid\,(0,1)$ and$\eta = \sqrt{1-\rho^2}$

Using the undetermined coefficients method, we can solve for the inflation premium components:

$$\omega_\pi = \beta\left[\left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)b_1^2 + (1-\theta\beta)\,b_1c_1\right] = \beta b_1^2 > 0 \tag{119}$$

$$\Omega_\pi = \left[c_{yy}^\pi a_1^2 + 2c_{yq}^\pi a_1 + c_{qq}^\pi\right] + \frac{\varepsilon(1+\chi v)}{\kappa}b_1^2 - \left(\frac{1-\beta\rho^2}{\kappa}\right)\left[\left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)b_1^2 + (1-\theta\beta)\,b_1c_1\right] \gtrless 0$$

$$\Omega_{mc} = c_{yy}^{mc}a_1^2 + 2c_{yq}^{mc}a_1 + c_{qq}^{mc} = (1-\psi)\,\frac{\chi^2(1-\chi)}{1-\alpha^F}\left[(\sigma+\nu)\,a_1 - q\right]^2 > 0$$

From simple observation of the coefficients, we can see that ω_π is positive and Ω_{mc} is positive when the elasticity of substitution between factors is lower than one (i.e. $\psi < 1$), whereas Ω_π can be either positive or negative.

B.5.2 The aggregate demand

Replace the policy rule (108) in the second order expansion of the IS (109). :

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left[(\phi_\pi - 1)\,E_t\pi_{t+1} + \phi_y y_t\right] \\ &\quad -\frac{1}{2}\sigma E_t\left[y_{t+1} + \frac{1}{\sigma}\pi_{t+1} - E_t\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)\right]^2 + O\left(\|q_t,\sigma_q\|^3\right) \end{aligned} \tag{120}$$

This can be expressed as:

$$y_t = E_t\,(y_{t+1}) - \frac{1}{\sigma}\left[(\phi_\pi - 1)\,E_t\pi_{t+1} + \phi_y y_t\right] + \frac{1}{2}\omega_y\sigma_q^2 + O\left(\|q_t,\sigma_q\|^3\right) \tag{121}$$

Where:

$$\omega_y\sigma_q^2 = -\sigma E_t\left[y_{t+1} + \frac{1}{\sigma}\pi_{t+1} - E_t\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)\right]^2 \tag{122}$$

Similar to the previous sub-section, the IS risk premium can be written as a function of the linear solution of inflation and output:

$$\omega_y = -\sigma\left(a_1 + \frac{1}{\sigma}b_1\right)^2 < 0 \tag{123}$$

Note that the risk premium component of the IS is negative, capturing precautionary savings due to output and inflation volatility.

B.5.3 The perturbation method

The policy functions of the second order solution for output and inflation can be written in the following form:

$$
\begin{aligned}
y_t &= \frac{1}{2}a_o\sigma_q^2 + a_1 q_t + \frac{1}{2}a_2\left(q_t\right)^2 + O\left(\left\|q_t,\sigma_q\right\|^3\right) \\
\pi_t &= \frac{1}{2}b_o\sigma_q^2 + b_1 q_t + \frac{1}{2}b_2\left(q_t\right)^2 + O\left(\left\|q_t,\sigma_q\right\|^3\right)
\end{aligned}
\qquad (124)
$$

where the $a\acute{}s$ and $b\acute{}s$ are the unknown coefficients that we need to solve for and $O\left(\left\|q_t,\sigma_q\right\|^3\right)$ denotes terms on q and σ_q of order equal or higher than 3. We express the dynamics of the oil price as:

$$q_t = \rho q_{t-1} + \eta\sigma_q e_t \qquad (125)$$

where the oil shock has been normalized to have mean zero and standard deviation of one, i.e. $e\tilde{}iid\left(0,1\right)$.Also, we set $\eta = \sqrt{1-\rho^2}$ in order to express $V\left(q_t\right) = \sigma_q^2$.

In order to solve for the 6 unknown coefficients, we use the following algorithm that consist in solving recursively for three systems of two equations. This allow us to obtain algebraic solutions for the unknown coefficients. We follow the following steps:

1. We replace the closed forms of the policy functions (124) and the transition equation for the shock (125) in the equations for the AS (116) and the AD (121).
2. **Solve for a_1 and b_1:** we take the partial derivatives with respect to q_t to the two equations of step 1, then we proceed to evaluate them in the non-stochastic steady state (i.e. when $q_t = 0$ and $\sigma_q = 0$). Then, the only unknowns left are a_1 and b_1 for two equations. We proceed to solve for a_1 and b_1 as function of the deep parameters of the model.
3. **Solve for a_2 and b_2:** similar to step 2, we take succesive partial derivatives with respect to q_t and q_t to the two equations of step 1 and we evaluate them at the non-stochastic steady state. Then, we solve for the unknowns a_2 and b_2.
4. **Solve for a_0 and b_0:** similar to steps 2 and 3, we take succesive partial derivatives with respect to σ_q and σ_q to the two equations of step 1 and we evaluate them at the non-stochastic steady state. Then, we solve for the unknowns a_0 and b_0. The solution for the coefficients is given by:

$$a_o = -\left(\phi_\pi - 1\right)\left[\left(b_2+\omega_\pi\right) - \sigma\left(1-\beta\right)\left(a_2+\omega_y\right)\right]\frac{1}{\Delta_0} \qquad b_o = -b_2 + \left[\phi_y\left(b_2+\omega_\pi\right) + \sigma\kappa_y\left(a_2+\omega_y\right)\right]\frac{1}{\Delta_0}$$

$$a_1 = -\left[\left(\phi_\pi - 1\right)\rho\right]\kappa_q\frac{1}{\Delta_1} < 0 \qquad b_1 = \left[\sigma\left(1-\rho\right)+\phi_y\right]\kappa_q\frac{1}{\Delta_1} > 0$$

$$a_2 = -\left[\left(\phi_\pi - 1\right)\rho^2\right]\kappa\left(\Omega_\pi + \Omega_{mc}\right)\frac{1}{\Delta_2} < 0 \qquad b_2 = \left[\sigma\left(1-\rho^2\right)+\phi_y\right]\kappa\left(\Omega_\pi + \Omega_{mc}\right)\frac{1}{\Delta_2} > 0$$

where we have defined the following auxiliary variables:

$$
\begin{aligned}
\Delta_0 &= \left(\phi_\pi - 1\right)\kappa_1 + \left(1-\beta\right)\phi_y \\
\Delta_1 &= \left(\phi_\pi - 1\right)\rho\kappa_y + \left(1-\beta\rho\right)\left[\sigma\left(1-\rho\right)+\phi_y\right] \\
\Delta_2 &= \left(\phi_\pi - 1\right)\rho^2\kappa_y + \left(1-\beta\rho^2\right)\left[\sigma\left(1-\rho\right)^2+\phi_y\right]
\end{aligned}
$$

where Δ_0, Δ_1, and Δ_2 are all positive.

C Endogenous Trade-off

From equation (90), we can derive linearly the marginal cost as function of output and oil price shocks, as follows:

$$mc_t = \frac{\left(1-\alpha^F\right)(\sigma+v)}{1+\upsilon\psi\alpha^F}y_t + \alpha^F\frac{(1+\upsilon\psi)}{1+\upsilon\psi\alpha^F}q_t + O\left(\|q_t,\sigma_q\|^2\right) \tag{C.1}$$

This equation can be also written in terms of parameters κ_y and κ_q, defined previously in the main text, as follows:

$$mc_t = \frac{\kappa_y}{\kappa}y_t + \frac{\kappa_q}{\kappa}q_t + O\left(\|q_t,\sigma_q\|^2\right) \tag{C.2}$$

Under flexible prices, $mc_t = 0$. Condition that defines the natural level of output in terms of the oil price shock :

$$y_t^F = -\frac{\kappa_q}{\kappa_y}q_t + O\left(\|q_t,\sigma_q\|^2\right) \tag{C.3}$$

Notice that in this economy the flexible price level of output does not coincide with the efficient one since the steady state is distorted by monopolistic competition The efficient level of output is defined as the level of output with flexible prices under perfect competition, we use equation (53) to calculate this efficient level of output under the condition that $\mu = 1$ as follows:

$$y_t^E = -\frac{\alpha^E}{(1-\alpha^E)}\frac{\left(1-\alpha^F\right)}{\alpha^F}\frac{\kappa_q}{\kappa_y}q_t + O\left(\|q_t,\sigma_q\|^2\right) \tag{C.4}$$

Where $\alpha^E = \alpha^\psi\left(\overline{Q}\right)^{1-\psi}$. This parameter can be also expressed in terms of the participation of oil under flexible prices as follows:

$$\alpha^E = \alpha^F\mu^{\psi-1}$$

Notice that when there is no monopolistic distortion or when $\psi = 1$ we have that $\alpha^E = \alpha^F$ and $y_t^E = y_t^F$.

Using the definition of efficient level of output, we can write the marginal costs equation in terms an efficient output gap, x_t. Where $x_t = \left(y_t - y_t^E\right)$ in the following way

$$mc_t = \frac{\kappa_y}{\kappa}\left(y_t - y_t^E\right) + \frac{1}{\kappa}\mu_t + O\left(\|q_t,\sigma_q\|^2\right) \tag{C.5}$$

Where

$$\mu_t = \kappa_y\left(1 - \frac{\alpha^F}{(1-\alpha^F)}\frac{\left(1-\alpha^E\right)}{\alpha^E}\right)y_t^E$$

Using equations (C.5) and (56), the Phillips curve can be written as follows:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa_y x_t + \mu_t + O\left(\|q_t,\sigma_q\|^2\right) \tag{C.6}$$

This equation corresponds to equation (41) in the main text. We can further write μ_t in terms of the oil price shocks using the definition of the efficient level of output:

$$\mu_t = \frac{\kappa_q}{\kappa_y}\left(\frac{\alpha^F - \alpha^E}{\left(1-\alpha^E\right)\alpha^F}\right)q_t$$

The dynamic IS equation can also be written in terms of the efficient output gap.

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^E\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.7}$$

where:

$$r_t^E = \sigma\left(1-\rho\right) y_t^E + O\left(\|q_t, \sigma_q\|^2\right)$$

which in turn can be written as follows:

$$r_t^E = -\sigma\left(1-\rho\right)\frac{\alpha^E}{\left(1-\alpha^E\right)}\frac{\left(1-\alpha^F\right)}{\alpha^F}\frac{\kappa_q}{\kappa_y} q_t + O\left(\|q_t, \sigma_q\|^2\right)$$

Notice that when there is no monopolistic distortion or when $\psi = 1$ we have that $\alpha^E = \alpha^F$, which implies that there is no an endogenous trade off.

$$\mu_t = 0 \ \forall t$$