Dynamic cointegration and Relevant Vector Machine: the relationship between gold and silver

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Abstract: We use the Relevant Vector Machine, a technique of supervised learning introduced by Tipping (2001), to conduct a dynamic cointegration analysis on the series of the price of gold and silver over the period 1971-2004. Unlike the results of traditional cointegration analysis, this study reveals that there is a dynamic long run relationship over the whole period.

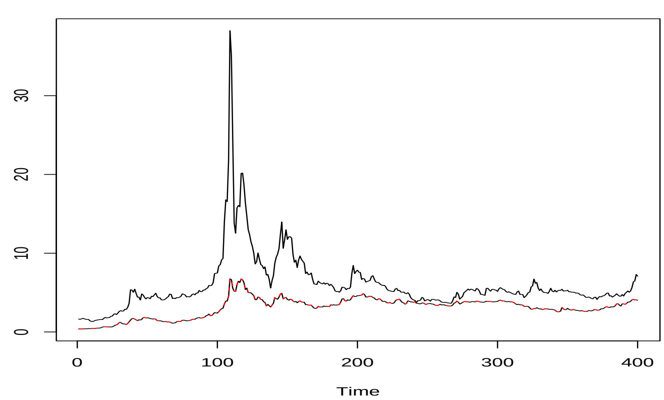
Keywords: Dynamic cointegration, Relevance Vector Machine

1. Introduction

In this paper we study the dynamic relationship between gold and silver (fig.1) over the period 1971-2004, that covers a very extensive range of economic conditions, political change in major producers and increased sophistication in asset markets generally.
Gold and silver have been actively traded for thousands of years and remain important and closely observed markets. They have historically been seen as close substitute for one another and this would suggest that the two markets share the same dynamics.

However there are also economic fundamentals that may drive the prices of gold and silver apart. While both are used extensively in industrial processes, there are significant differences between these uses. Therefore it seems that while they share a similar set of drivers they each also have important unique macroeconomic drivers (see for example

Figure 1: *Gold and silver time series (respectively in black and red)*



Dooley, Isard and Taylor, 1995).

In literature there are several attempts to test for the existence and stability of the gold silver relationship (among the others, Escribano and Granger, 1998). In this framework, we want to give further insight to the problem. We analyse the data set of monthly prices of gold and silver over 1971-2004 with an extremely recent approach to reveal the existence of a dynamic cointegration relationship (Pellizzari, Pizzi and Salmasi, 2005). The estimation of the cointegration coefficient is realized with a technique of supervised learning called Relevance Vector Machine or RVM (Tipping, 2001) that exploits a probabilistic Bayesian learning framework to derive accurate predictions models.
In the next section we describe dynamic cointegration and the RVM. while section 3 is devoted to the results of the application to the gold and silver data set.

2. Dynamic cointegration

The traditional concept of cointegration (Engle and Granger, 1987) assumes that the cointegrating coefficient is constant over time. If the coefficient is allowed to be non constant we have the following definition of dinamic cointegration:

Let $\mathbf{x}_t$ a $N \times 1$ vector, its components are dynamically cointegrated of order d, b, $CI(d, b)$ if *i)* the series in $\mathbf{x}_t$ are all $I(d)$ and *ii)* there exists a non zero vector $\alpha_\mathbf{t}$ such that $z_t = \alpha_\mathbf{t}\prime\mathbf{x_t}$ is $I(d-b)$, $b > 0$.

Let us consider the case of $N = 2$, the (dynamic) cointegrating relation is $z_t = x_{1t} - \alpha_t x_{2t}$. Several problems can be tackled: *i)* to determine α_t *ii)* to comprehend the structure of α_t *iii)* to develop valid estimation techniques.

If the time span is sufficiently large α_t can be approximated by $\alpha_t \cong \frac{x_{1t}}{x_{2t}}$. In general, we can assume that α_t depends on several variables, $\alpha_t = l(x_t, x_{t-1}, \ldots, y_t, y_{t-1}, \ldots, t)$, and its structure can be investigated by plotting $\alpha_\mathbf{t}$ versus the variables that presumably affect its dynamics. As far as the third problem is concerned, we propose the use of a Bayesian probabilistic framework for learning, known as Relevant Vector Machine (RVM), Tipping (2001). It is a model of identical functional form to the Support Vector Machine (SVM), among the others Vapnik (1998), but the training takes place in a Bayesian framework and predictive distributions of the outputs instead of point estimates are obtained.

Given a set of examples of input vectors $\{\mathbf{x_n}\}_{n=1}^N$ (training set) along with corresponding targets $\{t_n\}_{n=1}^N$, "learning" is the process of inferring a function $y(\mathbf{x})$ on which are based the predictions of t for previously unseen values of $\mathbf{x}$. The RVM makes predictions based on the function

$$y(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^{N} w_i K(\mathbf{x}, \mathbf{x}_i) + w_0$$

where $K(\mathbf{x}, \mathbf{x}_i)$ is a kernel function, w_i are the weights. One basis function is defined for each example in the training set.

The main feature of RVM (for further formal details, see Tipping, 2001) is represented by the introduction of a prior density function over the model weights governed by a set of hyperparameters, one associated with each weight, whose most probable values are iteratively estimated from the data. Sparsity is achieved because in practise we find that the posterior distibutions of many of the weights w_i are sharply peaked around zero. For this reason, one of the advantages in using RVM is that, with the same generalization performance, it utilises way fewer kernel functions than an equivalent SVM.

3. Real data analysis

The data set we examine comes from the Interational Financial Statistics. The two series are the monthly prices of gold and silver (US dollars per ounce) from the beginning of 1971 to april 2004. In the first 3 rows of table 1 we report the results of the ADF test on the series of gold (x_t), silver (y_t) and residuals of the traditional cointegrating regressions (z_t). As we can observe, there is no evidence of linear cointegration since the null hypothesis of unit root is always accepted.

Table 1: *ADF test*

Series	Test ADF	p-value
y_t	-2.5295 (lag order=7)	0.3536
x_t	-2.756 (lag order=7)	0.2580
z_t (linear coint.)	-2.6716 (lag order=7)	0.296
z_t (dynamic coint.)	-5.8458 (lag order=7)	0.01

After rescaling series x_t and y_t in $(1, 2)$ we determine $\frac{y_t}{x_t}$ (solid line in fig.2). Since the latter can be thought as $\alpha_t + \frac{z_t}{x_t}$, we employ the RVM approach to filter out α_t. It is assumed a gaussian prior distribution on weigths w_i, whereas for the $N + 1$ hyperparameters the distributions are non informative Gamma. We use a gaussian kernel and set σ, the inverse of the bandwidth parameter, equal to 5e-04.

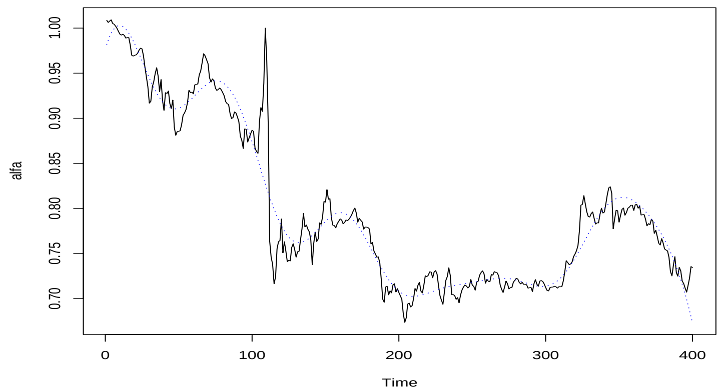
Table 2: *Relevant vectors with their time index and corresponding date*

Time index	1	51	52	100	132	162	189	222	280	343	400
Date	1:71	3:75	4:75	4:79	12:81	6:84	9:86	6:89	4:94	7:99	4:04
w_i	0.914	-4.627	5.101	0.6	-0.102	0.757	-0.114	0.549	0.533	0.623	0.549

The RVM algorithm finds 11 relevant vectors (tab. 2) and the α_t series, obtained filtering $\frac{y_t}{x_t}$, is represented with a dashed line in fig.2. An ADF test is performed on the dynamic cointegration residuals and the results are reported in the last row of table 1. As we can see, unlike the traditional cointegration case, in this dynamic case the unit root is rejected. This means that evidence of dynamic cointegration is found.

Interesting information is gathered by observing the time index of the relevant vectors (tab. 2). Some of them are located where the α_t series tends to exhibit a sort of level shift, see for instance the vector with index 100 which is located just before the well documented bubble in silver prices from roughly june 1979 to march 1980. Bearing in mind the relevant vectors indexes, we consider, in particular, two subperiods 1:71-4:79 and 4:79-4:04. The ADF test reveals in both of them evidence of linear cointegration (p-values, respectively, 0.06 and 0.08). The relevant vectors show also other possibilities to split the sample, but in those cases no linear cointegration is found after 4:94.

Figure 2: $\frac{y_t}{x_t}$ *(solid line) and* α_t *(dashed line) series*



This results show clearly how promising seems to be the use of RVM in the pioneering context of dynamic cointegration. Indeed, RVM exhibits interesting potentiality both in terms of accuracy when filtering the series and ability in revealing where are located the main change points of the long term dynamic.

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