A Ricardian Perspective on the Fiscal Theory of the Price Level

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Abstract

On the basis of a model on time consistent interaction of monetary and fiscal policy, we propose a positive theory of government debt and inflation. The basic take is that the long-term level of public liabilities can be explained as the endogenous outcome of a dynamic game played between two interacting macroeconomic policy makers: a central bank and a fiscal authority. We assume a "conservative" central bank that puts excessive weight on an inflationary loss term, but is also responsive to general economic conditions as measured by consumer welfare. On the other hand, the behavior of the fiscal authority is governed by its relative impatience, which we see as resulting from dynamic frictions in the political process. This gives rise to profligate fiscal policies and introduces a strategic conflict between the two authorities about the path of the economy. The Markov-perfect equilibrium outcome of the resulting dynamic game is a path of real debt that converges to a finite positive level and is associated with a steady state inflation bias. This inflation bias is the result of the fiscal authority gaining leverage over the nominal properties of the equilibrium allocation. Thus, our model can be seen as providing a game-theoretic foundation for the propositions made in the fiscal theory of the price level.

1 Introduction

During the last decades, normative proposals for the conduct of monetary policy have put increasing emphasis on inflation targets as a primary objective. Similarly, it is now an established consensus that central bank independence is an important institutional prerequisite for the success of monetary policy in achieving its goal of low and stable inflation. The view behind these

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two developments seems to be that a monetary authority can in principle successfully implement a targeted path for inflation, once its statutes equip it with an appropriate mandate for price stability and the independence of monetary policy choices is warranted. However, it is not so clear whether the sufficiency of an independent and properly incentivized central bank for price stability survives in settings where the interaction between monetary and fiscal policies plays an important role.

Indeed, as argued among others by Woodford (2001), the case for the separation of decision authority over monetary and fiscal policies is based on two central presumptions: First, that fiscal policy is not an important determinant of inflation; and second, that the effects of monetary policy on the government budget are neglegible. A setting, where both of these tenets may be violated, is given by an economy with a significant amount of outstanding government debt in nominal terms. While the second dimension is captured by the simple relationship that monetary policy, via its effect on the price level, affects the real value of outstanding public liabilities and thus the tightness of the intertemporal government budget constraint, the first dimension relies on a more controversial mechanism which has been stressed by the literature around what has become known as the fiscal theory of the price level.[1] Specifically, in a world where "non-Ricardian" policy regimes, i.e. policy rules which do not guarantee that the intertemporal government budget constraint is satisfied regardless of how government purchases and prices evolve, are possible, the fiscal theory establishes that the specification of fiscal policy matters for the consequences of monetary policy. This view has been criticized along various dimensions. Kocherlakota and Phelan (1999) condense the discussion into a single issue, the interpretation of the intertemporal budget constraint. According to these authors, the fiscal theory of the price level takes the intertemporal budget constraint as a mere *equilibrium condition* requiring that imbalances between the real value of government debt and future primary surpluses be corrected by adjustments in the price level that lead back to equilibrium. Conversely, the traditional view interprets the intertemporal budget constraint as a *constraint on policy*; according to this position, policy rules that do not satisfy the intertemporal budget constraint for *any* sequence of prices are not feasible and thus a misspecification.

Whereas the debate on the fiscal theory remains unsettled, the present paper adopts another approach which on the one hand is in line with the traditional view that admits only Ricardian policies, but on the other hand generates results similar to those proposed by the fiscal theory. To arrive there, we borrow from two distinct branches of the literature. The first one is given by the fiscalist approaches to the question of price level determination in dynamic general equilibrium economies mentioned above. Starting with the seminal contribution by Sargent and Wallace (1981), this literature has found that the behavior of fiscal policy may impose restrictions on what monetary policy can achieve and has identified the intertemporal government budget constraint as the crucial building block that makes monetary and fiscal policies interdependent. However, models of this sort are generally tacit about how the policies considered actually come about and whether they are sustainable. These issues are taken up in another branch of macroeconomic research which considers models of monetary and fiscal policy where policy choices are the result of explicit optimization exercises with well-defined constraints. The drawback with these contributions is that they are generally based on the assumption that there is only one entity which effectively decides about the complete set of

[1] A selection from the large set of papers that develop this theory includes e.g. Leeper (1991), Sims (1994) or Woodford (2001).

policy instruments. Alternatively, when the focus of their analyses is monetary (fiscal) policy, it is essentially assumed that fiscal (monetary) policy is absent or exogenously given to the model. The consequence is that such models offer only limited insights into dynamic monetary-fiscal interactions.

Against this background, we present a dynamic general equilibrium model of policy making that allows for two institutions commissioned with the conduct of policy. Specifically, we analyze a simple monetary general equilibrium economy with flexible prices and formalize a policy game between two independent authorities: a fiscal authority and a monetary authority. The starting point for our analysis is a related model proposed by Diáz-Giménez et al. (2004). These authors analyze from an optimal taxation perspective the burden that is caused by nominal debt in a dynamic economy without capital. With a particular specification of preferences[2] and with a single policy authority controlling monetary aggregates, their central findings are the following: As long as there are positive amounts of nominal government debt, the incentive under sequential policy implementation to reduce the stock of debt through unanticipated inflation creates the standard time inconsistency problem. In the rational expectations equilibrium, the incentive to generate unanticipated inflation increases the cost of the outstanding debt even if there are no unanticipated inflation episodes. Therefore, the optimal policy without commitment is to progressively deplete the outstanding stock of debt until the extra liability costs vanish. The authors' general message thus is that, with nominal debt and sequential policy making, the optimal policy (inflation) will not only depend on elasticities as in a standard model of Ramsey-optimal taxation, but also on the marginal gain from changing the value of the existing debt.

A companion paper, Niemann (2005), extends the framework from Diáz-Giménez et al. (2004) to a model featuring dynamic interaction between two benevolent authorities. The key finding there is that the decentralization of authority over the relevant policy variables, the supply of money balances and a linear consumption tax, can potentially coordinate the public's expectations in a way that has important implications for the dynamic evolution of the economy. In particular, the rational expectations equilibrium from the case of a single policy maker is no longer the only equilibrium, and the associated inflation bias can vanish even for positive levels of outstanding government debt. The reason for this result is that, although the two authorities share the same objective, the fact that there is an authority that is not subject to the monetary time inconsistency problem allows for coordination failure among the two independently operating agencies. As a consequence, the economy is in a situation of multiple (Markov-perfect) equilibria, and the equilibrium reported in Diáz-Giménez et al. (2004) is complemented by a welfare superior equilibrium that is not subject to the inflation bias arising in the single agency case and implements an entirely stationary allocation. The present paper differs from this normative benchmark in that we perturb the objective functions of the strategically interacting government authorities. Specifically, we assume a "conservative" central bank that puts excessive weight on an inflationary loss term, but is also responsive to general economic conditions as measured by consumer welfare. On the other hand, the behavior of the fiscal authority is governed by its relative impatience, which we see as resulting from dynamic frictions in the political process. This gives rise to myopic and profligate fiscal policies and introduces a strategic conflict between the two authorities about the path of the

[2] Martin (2004) generalizes the results presented in Diáz-Giménez et al. (2004) and illustrates how the particular specification affects the equilibrium outcomes.

economy. An immediate implication of this strategic conflict is that the perturbed game, unlike the normative benchmark discussed in Niemann (2005), is characterized by a unique Markov-perfect equilibrium allocation. In a nutshell, the reason is that the dynamic game being played by the two institutions is no longer of a pure coordination nature such that coordination failure can no longer be the source of multiple equilibria.

The strategic game proceeds within the framework of a dynamic general equilibrium model where government policies are implemented sequentially over time, but, in each period, the two authorities move simultaneously. Of course, the timing of events is crucial. In the literature, there seem to be conflicting views.[3] Although some authors, e.g. Beetsma and Bovenberg (1998), argue that fiscal policy is sluggish relative to monetary policy, we stick to a notion of simultaneous moves rather than formalizing the interaction in terms of a dynamic Stackelberg game, where, in each period, the fiscal authority moves first and the monetary authority follows. We justify this as follows: First, it is our objective to see whether and to what extent fiscal policy can gain leverage over monetary policy and the (nominal properties of the) final equilibrium outcomes. The crucial question then is one of how rigid the two authorities' commitment to a certain path of policy choices is. Therefore, a Stackelberg game does not seem to be the appropriate modelling choice since it allows for within-period commitment of the fiscal authority by construction. Moreover, it is our view of monetary policy that considerations related to the interaction with fiscal variables play only a minor role for "day-to-day" operations, but are essential in shaping policy over the medium and long run when also fiscal policy has some flexibility.

In settings where explicit commitment is not available, it has been investigated whether delegation of authority over policy decisions can help to improve upon the inferior outcomes when policy makers succumb to dynamically inconsistent incentives. Specifically, in the context of Barro-Gordon (1983a,b) type models where monetary policy faces the task of stabilizing output and inflation, the issue of delegation to decision makers with biased incentives has received much attention; Rogoff's (1985) weight-conservative central banker, inflation targets as proposed by Svensson (1997) and incentive contracts for central bankers as proposed by Walsh (1995) are probably the best-known examples. However, all these approaches completely abstract from fiscal policy or take it as exogenously given. The consequence is that these models fail to take into account the dynamic implications arising from the *interaction* of monetary and fiscal policies. Most importantly, while models along these lines provide important insights concerning the optimal design of stabilization policies, they completely ignore the intertemporal government budget constraint. It is this issue that we will focus on in this paper.

This makes it necessary to consider a dynamic general equilibrium model rather than a reduced form specification. While this comes at some cost in terms of modeling effort, there are a number of important advantages. First, our model allows for true policy interaction in the sense of a dynamic game with a non-trivial state variable played between the two authorities. Second, our model automatically comprises dynamic forward-looking behavior of all agents such that current economic outcomes are influenced by expectations about future policy. We analyze the Markov-perfect equilibrium (MPE) of the dynamic game played between the monetary and the fiscal authority. Our central finding for the considered case of a conservative central bank and an impatient fiscal authority is that the latter can strategically exploit the monetary

[3] Compare e.g. the discussion in Dixit and Lambertini (2003a).

authority's commitment problem, whereby the dynamic inconsistency of monetary policy stems from its incentives to surprise the public by inflation. This makes the inflation bias reappear even under interaction and has important implications for the dynamics of government debt. The reason for fiscal policy affecting inflation even in our otherwise monetarist world is that, although it has no effect on inflation, it has an important effect on the monetary authority's incentives to generate inflation. It is in this sense that our model is capable of generating results that are in line with the predictions of the fiscal theory of the price level; however, in contrast to the latter approach, we do not rely on assumptions concerning asymmetries in the two authorities' commitment power or on the off-equilibrium contingencies introduced by non-Ricardian fiscal policies.

The rest of the paper is organized as follows. The next section sets up the model and defines a competitive equilibrium for our economy. Then, the following section briefly lays out the structure of the policy game between the monetary and the fiscal authority. Section 4 contains a description of the equilibrium outcomes. While we are able to characterize the MPE of the game analytically, its quantitative implications must be analyzed by numerical methods. Finally, we discuss the institutional implications arising from our analyis before the paper concludes with a review of the related literature and some further remarks. Technical details and an outline of the numerical methods used are delegated to the Appendix.

2 The model

The basic object of our analysis a monetary dynamic general equilibrium model economy which is identical to the one in Diáz-Giménez et al. (2004). The economy is made up of a government sector and a private sector, and as in Lucas and Stokey (1983) there is no capital. The government sector consists of a monetary authority and a fiscal authority who take their decisions independently. The policy instrument controlled by the monetary authority is the supply of money M^g_{t+1} (throughout, the superscript g is used to distinguish an aggregate variable from an individual variable where necessary). The fiscal authority collects consumption taxes τ^c_t in order to finance an exogenously given stream of public expenditures g_t.[4] For simplicity, we let public spending be deterministic and constant over time such that $g_t = g$ for all $t \geq 0$. The two authorities interact via the consolidated budget constraint of the government sector. The monetary authority issues money as a liability of the fiscal authority; seignorage revenues accrue directly to the fiscal authority. Thus, we restrict attention to the public finance role of monetary policy in order to focus on the implications of decentralized decision power among the two independent institutions. Finally, we assume that the fiscal authority, besides its tax policy, issues nominal one-period bonds B^g_{t+1}, whereby the quantity of bonds traded is determined by the following flow budget constraint for the government sector which has to be satisfied for all $t \geq 0$:

$$M^g_{t+1} + B^g_{t+1} + P_t\tau^c_t c_t \geq M^g_t + B^g_t(1 + R_t) + P_t g \tag{1}$$

[4] What ultimately matters for the construction of our equilibria are the fiscal deficits over time. While the empirical evidence suggests that over short time horizons fiscal adjustments are brought about by changes in government spending rather than in taxation, it turns out that endogenizing taxation is conceptionally more straightforward than endogenizing spending. Therefore, we introduce fiscal discretion with respect to the size of deficits as stemming from variable taxation with government spending given exogenously.

Here, P_t is the price level prevailing at time t, while R_t is the nominal interest rate paid on the bonds issued at date $t-1$. The initial stock of money M_0^g and the initial debt liabilities $B_0^g(1+R_0)$ are given. However, we impose the additional consistency condition that, in equilibrium, there is no surprise inflation in the initial period; thus, by linking the nominal interest rate R_0 to the equilibrium rate of inflation in the first period, we prevent the authorities from taking advantage of the inelasticity of the amount of oustanding nominal balances M_0 and B_0 in the first period.

On the private side, the economy is inhabited by a continuum of identical infinitely-lived households whose preferences over sequences of consumption c_t and labor n_t can be represented by the following expression:

$$\sum_{t=0}^{\infty} \beta^t \left\{u(c_t) - v(n_t)\right\}, \tag{2}$$

where the discount factor β is strictly between 0 and 1. In what follows, we will assume $u(c_t) = log(c_t)$ and $v(n_t) = \alpha n_t$.[5] Each consumer faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - P_t(1+\tau_t^c)c_t + B_t(1+R_t) + W_t n_t, \tag{3}$$

where W_t is the nominal wage and B_{t+1} and M_{t+1} are nominal government debt and nominal money balances taken over from period t to period $t+1$. We assume that each consumer faces a no-Ponzi condition that prevents him from running explosive consumption/debt schemes, implying:

$$lim_{T\to\infty} \beta^T B_{T+1} = 0$$

As a shortcut for introducing a well-defined money demand we assume that the gross-of-tax consumption expenditure in period t must be financed using currency carried over from period $t-1$, which implies the following cash-in-advance (CIA) constraint:

$$M_t \geq P_t(1+\tau_t^c)c_t \tag{4}$$

The timing structure underlying this CIA constraint follows Svensson (1985) and requires that the goods market operates and closes before the asset market opens. This implies that unexpected monetary expansions, due to the inflationary pressure they cause, are distortionary because the nominal asset portfolio cannot be reshuffled in response to monetary innovations and only the money balances taken over from the previous period are available to facilitate current consumption purchases.

[5]The assumption of linear disutility of labor is made to sharpen the discussion, but implies also that the government sector cannot affect the real interest rate. In contrast, the assumption of log utility from consumption is essential because it allows a recursive formulation of the dynamic problem. Another implication of this assumption is that it allows to focus on the role of nominal debt as a source of time inconsistency rather than on the effects due to private holdings of nominal money balances. That is, we abstract from seignorage on base money and focus on the implications of changing the real value of nominal debt. Importantly, this focus is consistent with the situation in most developed economies where government debt is arguably more important than money holdings as a source of time inconsistent incentives. See also Nicolini (1998) for an instructive exposition of the nature of the time inconsistency of monetary policy and Martin (2004) for results with a more general specification of preferences.

The productive side of the model economy is very simple since there is no capital. In each period, labor n_t can be transformed into private consumption c_t or public consumption g_t at a constant rate, which we assume to be one. Then, the equilibrium real wage is $w_t \equiv \frac{W_t}{P_t} = 1$ for all $t \geq 0$, and aggregate feasibility is reflected by the following linear resource constraint:

$$c_t + g \leq n_t \tag{5}$$

Now, we are ready to define a competitive equilibrium for given government policy choices $\{\tau_t^c, M_{t+1}^g\}_{t=0}^{\infty}$.[6]

Definition 1 *A competitive equilibrium for this economy is composed of the government sector's policies $\{\tau_t^c, M_{t+1}^g, B_{t+1}^g, g\}_{t=0}^{\infty}$, an allocation $\{c_t, n_t, B_{t+1}, M_{t+1}\}_{t=0}^{\infty}$, and prices $\{R_{t+1}, P_t\}_{t=0}^{\infty}$ such that:*

1. *given $B_0^g(1 + R_0)$ and M_0^g, the policies and the prices satisfy the sequence of budget constraints of the government sector described in expression (1);*

2. *when households take $B_0(1+R_0)$, M_0 and prices as given, the allocation solves the household problem of maximizing (2) subject to the private budget constraint (3), the CIA constraint (4) and the no-Ponzi condition;*

3. *markets clear, i.e.: $B_t^g = B_t$, $M_t^g = M_t$, and g and the allocation satisfy the economy's resource constraint (5) for all $t \geq 0$.*

On the basis of our assumptions on household preferences, it is straightforward to show that in the competitive equilibrium allocation of this economy the household budget constraint (3) and the aggregate resource constraint (5) are both satisfied at equality. Moreover, the first order conditions of the Lagrangean representing the household's constrained optimization problem are both necessary and sufficient conditions to characterize the solution to the household problem. Finally, when $R_{t+1} > 0$, the CIA constraint (4) is binding, and the competitive equilibrium allocation for given government policies can be determined from the government budget constraint (1), the aggregate resource constraint (5) and the following conditions that must hold for all $t \geq 0$:

$$M_t = P_t(1 + \tau_t^c)c_t \tag{6}$$

$$\frac{u'(c_t)}{v'(n_t)} = (1 + R_t)(1 + \tau_t^c) \tag{7}$$

$$(1 + R_{t+1}) = \frac{v'(n_t)}{\beta v'(n_{t+1})} \frac{\bar{P}_{t+1}}{P_t}, \tag{8}$$

where in the last equation $\bar{P}_{t+1}$ is the price level that a household with rational expectations conditional on information at time t expects to prevail in period $t + 1$.

[6] In the following definition, we let B_{t+1}^g and g be part of the policy vector because they are associated with the government sector. However, it should be clear that g follows an exogenous process and that B_{t+1}^g is effectively determined by private demand.

3 The policy game

We now seek to find a time consistent policy rule that is sequentially optimal from the two authorities' perspectives. Here, our focus is on an environment where there is no explicit commitment technology. The ensuing inflationary bias due to (anticipated) incentives to create surprise inflation is dealt with by partial delegation to a "conservative" monetary authority whose objective function differs from the representative agent's welfare. Since the sequential decisions about policy are decentralized, we define a policy rule to be the combination of a fiscal and a monetary policy rule. Each of these latter rules is determined independently by the competent institution, respectively. We limit the analysis to Markov-stationary policy rules, where a policy rule is a mapping that returns policy outcomes as a function of the current states of the economy.[7] In order to identify the equilibrium policy rule, we therefore need to find the optimal time-invariant strategies in the strategic game between the two authorities. A formal description of the game-theoretic structure of the two authorities' interaction is given in Appendix A.3. Here, it suffices to mention that, since the two authorities choose their policies simultaneously and take the respective other authority's policy as well as the public's formation of expectations as given, the appropriate equilibrium concept for the stage game interaction is Nash. We denote the policy function by $\varphi(b^g) = (\varphi_f(b^g), \varphi_m(b^g))$, where $\varphi_f(b^g)$ and $\varphi_m(b^g)$ are the fiscal and monetary parts of the rule which give the respective policy instruments τ^c and M' as functions of the aggregate state $b^g \equiv \frac{B^g}{P_{-1}}$.[8]

The data of the economy introduced so far are sufficient to characterize a competitive equilibrium for a sequence of arbitrary policy choices. What is lacking to pin down these policy choices are (i) the preferences of the two policy making authorites as represented by their objective functions, and (ii) an appropriate definition of a game-theoretic equilibrium. We now turn to the former issue. Let $U(b, b^g; \varphi)$ be the lifetime utility enjoyed by a household with individual state b when the aggregate state is b^g and the policy rule employed by the two authorities is φ. The fiscal authority is impatient insofar as it tries to maximize the discounted sum of the household's period utilities $u(c_t) - v(n_t)$, but its discount factor $\delta < \beta$ is distorted downwards as compared to the representative household's discount factor. The fiscal objective function is:

$$\sum_{t=0}^{\infty} \delta^t \left\{ u(c_t) - v(n_t) \right\}$$

We see this modelling option as a shortcut to introduce politico-economic frictions into the model. Examples include electoral concerns or (fiscal) institutions that disperse the decision power over debt and deficits.[9] A divergence in the discount factors of the form $\delta < \beta$ then

[7] The implication is that history does not matter except via its influence on the current states. It is precisely this restriction that rules out reputational mechanisms.

[8] Strictly speaking, the aggregate state variable b^g falls short of a truly sufficient statistic since it does not inform about how, at the beginning of any period, nominal wealth is divided into money and debt. However, comparison of the results in Diáz-Giménez et al. (2004), who work with the aggregate state b^g, with those obtained by Martin (2004), who chooses $\frac{B}{M}$, reveals that the particular choice of the aggregate state does not affect the equilibrium dynamics of the model.

[9] In a context closely related to ours, Beetsma and Bovenberg (1999) introduce such frictions as the result of special-interest politics. Persson and Tabellini (2000), chapter 13, provide an extensive review of the politico-

reflects the systematic tendency towards myopic policy choices. We let $V(b^g;\varphi)$ denote the fiscal value function associated with a given aggregate state b^g and policy rule φ.

As regards the monetary authority, our starting point are the statutes of many independent central banks which ascribe importance to the task of curbing inflation or alternatively stabilizing the price level while they also refer to further indicators for general economic performance. For example, the "Protocol on the Statute of the European System of Central Banks and of the European Central Bank"[10] prescribes the following objectives for the European monetary authority (Article 2): *"... the primary objective of the E[uropean] S[ystem] [of] C[entral] B[anks] shall be to maintain price stability. Without prejudice to the objective of price stability, it shall support the general economic policies in the [European] Community with a view to contributing to the achievement of the objectives of the Community... The ESCB shall act in accordance with the principle of an open market economy with free competition, favouring an efficient allocation of resources ...".* We parametrize this by defining the monetary authority's objective function as follows:

$$\sum_{t=0}^{\infty}\beta^t\left\{-\gamma(\frac{P_t}{\bar{P}_t})^2+(1-\gamma)(u(c_t)-v(n_t))\right\}$$

Here, γ is a weight that balances the relative impacts on the monetary authority's payoff of general welfare (as measured by the representative household's lifetime utility) and a loss term resulting from unexpected price level deviations from its expected level. This specification is a particular interpretation of a weight-conservative central banker as discussed by Rogoff (1985). The monetary aversion against *surprise* inflation features both the reluctance to use the inflation tax as a lump-sum instrument and the debt management motive of not scaling up the real value of outstanding liabilities by downward deviations in the price level. This monetary objective captures two important points in line with real world evidence: First, the monetary authority has an explicit interest in price level stability (which is the rationale for delegating power to an independent monetary institution in the first place); and secondly, despite its specific mission, the monetary authority cares also about general economic conditions.[11] On the basis of this specification for period payoffs, we define the value function for the monetary authority as $W(b^g;\varphi)$.

The main goal is to identify a policy rule $\varphi(b^g)$ that is time consistent. This means that the authorities must not have an incentive to deviate from this rule when they choose their policy

economic literature on the accumulation of public debt. There, the main arguments evolve around the notion of "divided government" and political instability. The first issue can give rise to a dynamic common pool problem with too much spending occurring too soon or to delayed stabilization as a consequence of a war of attrition. The second line of research stresses the strategic calculus of governments who accumulate debt in order to increase their reelection probability or to affect incentive constraints faced by their successors or political opponents. While most of these models are formulated in terms of variable government spending, the obvious result with exogenous spending is that political incentives map into myopic policy choices which attach too much weight to the present as opposed to the future. A possible way of modelling such fiscal behavior would be to let the fiscal authority be engaged in quasi-geometric discounting. However, such a specification on its own gives rise to a dynamic game between the subsequent incarnations of the fiscal authority, which is sufficiently difficult to analyze already in isolation; compare e.g. Krusell, Kuruscu and Smith (2000, 2002). Therefore, we choose to model the bias towards the present as simply emerging from a lower discount factor.

[10] Protocol annexed to the Treaty establishing the European Community. See European Central Bank (2000).

[11] Compare also the literature on central bank contracts, e.g. Walsh (1995).

instruments simultaneously and sequentially over time. Finding such a policy rule involves three steps:[12]

1. Define the economic equilibrium for arbitrary policy rules φ. This allows to determine the representative household's welfare level as well as the authorities' value functions for arbitrary policy rules φ.

2. Define the optimal equilibrium policy π in the current period when future policies are determined by some arbitrary policy rule φ. Since the optimal current policy depends on the current states, this step determines the optimal current policy rule $\pi(\varphi)$, given a future rule φ.

3. Define the conditions under which the authorities will not deviate from the rule assumed for the future, i.e. impose time consistency on the policy rule. Time consistency will obtain if the policy rule assumed for the future is equal to the rule that is optimal in the current period (policy fixed point): $\varphi = \pi(\varphi)$.

With this structure the policy equilibrium can be represented recursively. Recall that in our deterministic model with constant government expenditure, the aggregate state is simply b_t^g;[13] the individual state is given by b_t. We now operationalize the three steps described above; details of the procedure are specified in the Appendix.[14]

3.1 Equilibrium for arbitrary policy rule

Conditional on a policy rule φ employed by the two authorities, a competitive equilibrium is defined in the usual way. In the rational expectations equilibrium, a fixed point between a perceived law of motion $G^e(b^g;\varphi)$ for the endogenous aggregate state variable b^g and the induced actual law of motion $G(b^g;\varphi)$ has to obtain. This allows us to recast the definition of a competitive equilibrium in a recursive manner.

Definition 2 *A recursive competitive equilibrium for given policies φ consists of a household value function $U(b, b^g;\varphi)$, (individual) decision rules $\{c(b, b^g;\varphi), n(b, b^g;\varphi), B'(b, b^g;\varphi), M'(b, b^g;\varphi)\}$ and an aggregate function $G^e(b^g;\varphi)$ such that:*

1. *households optimize, i.e. given the states (b, b^g), policies φ and a perceived law of motion $G^e(b^g;\varphi)$, the value function $U(b, b^g;\varphi)$ and the decision rules $\{c(\cdot), n(\cdot), B'(\cdot), M'(\cdot)\}$ solve the household problem;*

[12] The procedure has been developed by Klein, Quadrini and Ríos-Rull (2003) who apply it to analyze a model of international tax competition.

[13] Note that, despite the monetary authority's interest in inflation, P_{t-1} does not appear as an independent state variable. The reason for this is that we can substitute appropriately in the monetary authority's objective function to get rid of prices; compare the Appendix A.2.

[14] The equations presented in the following are derived from a primal approach to the authorities' problems; the respective problems are conditional on the other authority's policy rule as well as on private expectations as represented by the barred variables in the constraints. The primal approach reformulation of the relevant decision problems is done in Appendix A.2.

2. *the perceived law of motion is the actual law of motion, i.e. households are representative and form rational expectations:*

$$b' = b'^g = G^e(b^g; \varphi)$$

3. *the pursued policies are feasible, i.e. the consolidated budget constraint of the government sector is satisfied in every period:*

$$M'^g + B'^g + P\tau^c c = M^g + B^g(1+R) + Pg.$$

Thus, using the optimal household decisions in response to a policy rule φ, we can solve for the household value function $U(b, b^g; \varphi)$. By the same token, once the actual law of motion $G(b^g; \varphi)$ consistent with policy rule φ is determined, we can infer the fiscal value function, conditional on the policy rule φ:

$$\begin{aligned} V(b^g;\varphi) &= \{[log(c(b^g;\varphi)) - \alpha(c(b^g;\varphi)+g)] + \delta V(b'^g;\varphi)\} \\ \text{s.t.} \qquad b'^g &= c(b^g;\varphi) + \beta^{-1} b^g \frac{\bar{M}'(b^g)}{M'(b^g;\varphi)} + g - \frac{\beta}{\alpha} \end{aligned}$$

Similarly, for the monetary authority, we have:

$$\begin{aligned} W(b^g;\varphi) &= \left\{ -\gamma \left(\frac{\bar{c}(b^g)(1+\bar{\tau}^c(b^g))}{c(b^g;\varphi)(1+\tau^c(b^g;\varphi))} \right)^2 + (1-\gamma)[log(c(b^g;\varphi)) - \alpha(c(b^g;\varphi)+g)] + \beta W(b'^g;\varphi) \right\} \\ \text{s.t.} \qquad b'^g &= c(b^g;\varphi) + \beta^{-1} b^g \frac{c(b^g;\varphi)(1+\tau^c(b^g;\varphi))}{\bar{c}(b^g)(1+\bar{\tau}^c(b^g))} + g - \frac{\beta}{\alpha} \end{aligned}$$

3.2 Optimal current policy rule for given future policy rule

We look for a MPE where both authorities correctly anticipate the other one's policy function and take it as given. Clearly, the optimal control laws depend on each other, but in the MPE with simultaneous moves each authority ignores the influence that its choice exerts on the other authority's current choice. Then each authority faces a situation where its own current policy choice affects both its current payoff and its value from the next period onwards. The contemporaneous effect reflects the impact of this period's allocation and prices on the period payoff. The effect on the future value works through two channels both of which hinge on the real value of debt b'^g that results at the end of the current period as a consequence of the current policies implemented by the two authorities: First, household expectations are a function of b'^g; therefore, b'^g (together with g) pins down the nominal interest rate that households demand as a compensation for buying government debt. Secondly, with a given policy rule from tomorrow onwards, b'^g determines the evolution of the economy and thus the value generated for the authorities.

Let $\pi = (\pi_f, \pi_m)$ denote the current policy rule, and let $\varphi = (\varphi_f, \varphi_m)$ denote the future policy rule. Individual households take these rules as given. With the appropriate notational changes, a recursive competitive equilibrium for arbitrary current policy actions π followed by a future policy rule φ is then defined analogously to above recursive competitive equilibrium for given policies φ. Faced with a continuation policy rule φ, the authorities' problem consists of optimally determining their contemporaneous policies π.

Specifically, we have the following for the fiscal authority (variables affected by current policies π are denoted with a hat, and barred variables are fixed by predetermined private expectations):

$$\hat{V}(b^g;\pi,\varphi) = max_{\pi_f}\{[log(c(b^g;\pi)) - \alpha(c(b^g;\pi)+g)] + \delta V(b'^g;\varphi)\},$$

where the maximization is subject to the fiscal implementability constraint:

$$b'^g = c(b^g;\pi) + \beta^{-1}b^g\frac{\bar{M}'(b^g)}{M'(b^g;\pi_m)} + g - \frac{\beta}{\alpha}$$

For the monetary authority, we have:

$$\hat{W}(b^g;\pi,\varphi) = max_{\pi_m}\{-\gamma\left(\frac{\bar{c}(b^g)(1+\bar{\tau}^c(b^g))}{c(b^g;\pi)(1+\tau^c(b^g;\pi))}\right)^2 + (1-\gamma)[log(c(b^g;\pi)) - \alpha(c(b^g;\pi)+g)] + \beta W(b'^g;\varphi)\},$$

where the maximization is subject to the monetary implementability constraint:

$$b'^g = c(b^g;\pi) + \beta^{-1}b^g\frac{c(b^g;\pi)(1+\tau^c(b^g;\pi_f))}{\bar{c}(b^g)(1+\bar{\tau}^c(b^g))} + g - \frac{\beta}{\alpha}$$

Note that the authorities maximize directly over their current policies (π_f and π_m, respectively). But the authorities understand their policies' impact on the ensuing private allocation. This effect is captured by deriving the authorities' value functions from the private allocation which, in turn, is conditional on policies. The authorities make their current policy choices (out of the set of feasible policies) simultaneously, taking the other one's policy rule as given. The fiscal authority chooses π_f to maximize $\hat{V}$, given π_m, and the monetary authority chooses π_m to maximize $\hat{W}$, given π_f. This leads to the following definition:

Definition 3 *Given the functions $\varphi = (\varphi_f, \varphi_m)$, a Nash equilibrium of the policy game is a pair of functions $\{\pi_i^*(b^g;\varphi)\}_{i=f,m}$ such that (i) $\pi_f^*(b^g;\varphi)$ maximizes $\hat{V}(b^g;\pi,\varphi)$, given $\pi_m^*(b^g;\varphi)$, and (ii) $\pi_m^*(b^g;\varphi)$ maximizes $\hat{W}(b^g;\pi,\varphi)$, given $\pi_f^*(b^g;\varphi)$.*

By construction, the Nash equilibrium will consist of feasible policies. However, out of equilibrium, the payoffs may not be well-defined. For example, this will be the case for policy choices that are jointly inconsistent with a competitive equilibrium. Then, the question is what will happen out of equilibrium. Noting that the described environment and the rules according to which the two authorities interact in this environment fall short of the formal description of a game, we will nevertheless proceed to analyze the MPE outcomes.[15]

[15]Formally, the structure presented is a quasi-game. The problem is that the outcome and the associated payoffs are not well-defined if there is no feasible allocation satisfying a consistency condition defined by equation (15) in the Appendix. In such situations, the authorities' policy choices τ^c and M' are incompatible with a competitive equilibrium. In a related context, but with only one authority, a possible solution to this lack of formal structure has been suggested by Bassetto (2002a,b) who proposes the introduction of an explicit market microstructure and the adoption of a modified notion of government policy within a period as contingent strategy rather than as uncontingent plan.

3.3 Policy fixed point

Now, we can define the equilibrium time consistent policies:

Definition 4 *The policy functions $\varphi = (\varphi_f, \varphi_m)$ define time consistent policies if they are the Nash solution of the policy game when the two authorities expect φ to determine future policies. Formally: $\varphi_i(b^g) = \pi_i(b^g; \varphi)$, $i = f, m$.*

A MPE of the policy game described above is a profile of Markov strategies for the two authorities that yields a Nash equilibrium in every proper subgame. It is these time consistent policies φ that we are interested in.

4 Markov-perfect equilibrium outcomes

4.1 Necessary conditions

Before characterizing the MPE outcomes associated with the dynamic policy game, at a more fundamental level existence and uniqueness of such a MPE must be verified. As regards the former issue, we can build on an existence result in Niemann (2005) which establishes that a differentiable MPE in stationary strategies exists for the infinite-horizon game at hand. However, in contrast to the benchmark case discussed in Niemann (2005), multiplicity of equilibrium outcomes is not a concern here. The reason for this is that in the present context the two policy authorities' objectives are conflicting. In particular, as will become obvious in a moment, the fiscal authority's impatience forces monetary policy to be active in the sense of (partially) monetizing the fiscal deficits. This results in anticipated inflationary distortions at the margin and implies that there is no scope for the favorable coordination of the public's expectations that was the condition for sustaining the stationary non-inflationary MPE in the benchmark case.

Assuming differentiable Markov strategies, it is useful to present the two authorities' first order conditions with respect to their choice variables c and b' for a given continuation policy φ (step 2 above). These first order conditions, together with the implementability constraint, are necessary conditions for a MPE.[16] Specifically, in the rational expectations equilibrium, the implementability constraint, which relates the admissible choices of consumption c and end-of-period debt b', reads:

$$b' = c + \beta^{-1} b + g - \frac{\beta}{\alpha} \tag{9}$$

When taking their decisions, the authorities perceive private expectations as a given function of the aggregate state. The optimality conditions stated here are the ones obtained if we impose rational expectations by requiring that, in the equilibrium of the deterministic model, realizations and expectations must coincide. The fiscal authority's first order conditions are:

$$\frac{1}{c} - \alpha = -\delta V'(b'; \varphi) \tag{10}$$

$$V'(b'; \varphi) = \frac{\delta}{\beta} V'(b''; \varphi) \left[1 + \varepsilon_{M''}(b'; \varphi)\right] \tag{11}$$

[16] The relevant equations are derived in Appendix A.5.

The first equation demands that from the fiscal authority's perspective the marginal gain from increased current consumption be equal to the marginal cost of entering the next period with a higher stock of real debt. Although the current fiscal authority per se is not subject to a time inconsistency problem, it cannot commit future policy makers and hence needs to take the continuation play φ as given; this effect is captured by the dependence of the continuation value $V(\cdot)$ on φ. As in conventional optimal taxation problems, fiscal policy choices reflect a distortion smoothing motive. However, the second optimality condition reveals that fiscal activity is affected by the distorted discount factor. Since $\delta < \beta$, the fiscal authority will try to postpone distortions into the future as compared with the welfare-optimal intertemporal pattern. The interpretation of the second equation then is that the marginal value of increasing the real amount of future debt b' (LHS) is the sum of two effects (RHS): (i) a direct effect, as if debt was indexed, and (ii) an indirect expectational effect due to the resulting increase in the nominal interest rate. Here, $\varepsilon_{M''}(b';\varphi)$ is the elasticity of the public's expectations about future money expansions in response to changes in the outstanding stock of real debt and captures the interest rate distortions induced by rationally anticipated monetary expansions. Generally, $\varepsilon_{M''}(b';\varphi) \geq 0$ because the monetary authority's incentives to monetize outstanding government liabilities via the inflation tax are a non-decreasing function of the stock of outstanding government debt at the end of any given period.[17] Such incentives are anticipated and feed into increased nominal interest rates, which, in turn, constitute an opportunity cost of consumption due to the CIA constraint. That is, the second equation reveals that not only will the fiscal authority behave impatiently, but it will also take into account future incentive problems in formulating its distortion smoothing policy. The situation for the monetary authority is slightly different, as can be seen from its relevant first order conditions:

$$2\gamma\frac{1}{c} + (1-\gamma)\left(\frac{1}{c} - \alpha\right) = -\beta W'(b';\varphi)\left[1 + \beta^{-1} b \frac{1}{c}\right] \tag{12}$$

$$W'(b';\varphi) = 2\gamma\frac{\varepsilon_{M''}(b';\varphi)}{b'} + W'(b'';\varphi)\left[1 + \varepsilon_{M''}(b';\varphi)\right] \tag{13}$$

The first equation states that the monetary authority's preferences over consumption sequences are such that it tries to equate the marginal gain from higher consumption (LHS) due to (i) the implied lower surprise inflation and (ii) the direct welfare effect to the marginal cost associated with higher debt (RHS) resulting from the additional debt needed to finance consumption plus the additional debt resulting from the lower price level in the current period.[18] The second equation gives the marginal value of increasing b' (LHS) as the sum of two terms (RHS) both of which depend on the continuation play φ: (i) the implied effect via the inflationary loss term, and (ii) the welfare effect through the corresponding increase in the stock of real debt b' at the beginning of the following period; the latter effect, in turn, can again be decomposed into a direct component, as if debt was indexed, and an indirect expectational effect due to the resulting increase in the nominal interest rate.

We now solve for the allocation implemented as the outcome of the dynamic interaction among the sequence of monetary and fiscal policy makers. This necessitates numerical methods

[17] While this is no theoretical result, the claim has been confirmed by a numerical robustness check.

[18] This follows from the CIA constraint: Higher consumption leads to additional debt due to the lower price level in the current period that is needed to facilitate this extra consumption. This causes an additional cost of higher consumption because, rather than using the inflation tax which operates lump-sum on the outstanding liabilities, the intertemporal budget constraint has to be satisfied via the distortionary consumption tax.

the details of which are specified in Appendix A.7. In order to illustrate the dynamic evolution of the economy in the presence of nominal government debt, we will invoke a simple numerical example. For that purpose, we choose the following values for the parameters of our model economy: $\alpha = 0.45$, $\beta = 0.98$, $\gamma = 0.5$, $b_0 = 1.28$, $g = 0.5$. These parameter values draw largely on Díaz-Giménez et al. (2004) in order to make our results comparable to their ones. However, we choose the values for initial government debt and public spending more in line with recent OECD data.[19] Finally, the fiscal discount factor δ is set equal to 0.95 ($< \beta$), reflecting a moderately impatient fiscal behavior.

4.2 Economic outcomes and institutional implications

To understand the MPE outcomes of the policy game considered here, the insights from the benchmark case of $\delta = \beta$ and $\gamma = 0$, where the fiscal discount factor is not perturbed and the monetary authority is not inflation averse per se, are helpful. In this situation, both authorities share the representative household's preferences, but the monetary authority has access to a policy instrument that gives rise to dynamically inconsistent incentives. However, as shown in Niemann (2005), the decentralization of decision power among the two interacting authorities is an institutional arrangement that may help to overcome the time inconsistency problem plaguing monetary policy and the associated inflation bias. The key mechanism sustaining such a non-inflationary equilibrium is the fact that the reaction function, which pins down optimal fiscal policy, acts as an additional constraint on monetary policy choices. Consequently, since the optimal fiscal policy is not dynamically inconsistent, there is scope for a favorable coordination of the public's expectations, and given such expectations and a budget balancing fiscal policy, a benevolent monetary policy maker refrains from using the inflation tax. Loosely speaking, the point is that the decentralized decision power among the two authorities does not allow the monetary authority to substitute the distortionary consumption tax by the lump-sum inflation tax. The result is that the standard single-agency MPE, where the stock of debt is driven to zero[20] in order to economize on the extra expectational costs of outstanding nominal liabilities, is complemented by another MPE which implements a stationary allocation even in the presence of positive amounts of oustanding government debt and does not involve any more a systematic inflation bias.

Given the welfare reducing role of government debt in the benchmark model, the following question emerges: Why is there nominal debt at all, if there are no benefits from it, but an outstanding amount of debt depresses consumption because it has to be serviced via distortionary taxation and may additionally give rise to adverse expectational effects? A potential answer to this question can be given if we acknowledge that the fiscal authority's preferences are slightly perturbed. Indeed, if the fiscal authority discounts the future at a higher rate than the private households and the monetary authority do ($\delta < \beta$), then its preferred policy consistently shifts the distortions caused by taxation and inflation into the future. Hence, there emerges a strategic conflict between the two authorities about when to incur these distortions. This conflict can be summarized by the two authorities' differing preferences with respect to the path of the

[19] For 2003, the average of general government gross financial liabilities across the OECD countries was 76.0% of GDP, while the ratio of general government total outlays to GDP was at 40.7%; compare OECD (2004).

[20] The convergence to a zero debt level is an implication of the particular specification of preferences; see Martin (2004) for a generalization.

endogenous state variable b.[21]

From the fiscal authority's relevant first order condition (11), given the strict concavity of $V(\cdot)$, one sees that it favors an increasing (decreasing) path of b whenever $\frac{\delta}{\beta}\left[1+\varepsilon_{M''}(b';\varphi)\right] <(>)1$. Obviously, this result stems from the relative impatience inherent in fiscal policy making which is traded off against the output losses due to the crowding out of consumption via debt. In the long run, the model predicts a stationary level of debt b^* which is characterized by $\frac{\delta}{\beta}\left[1+\varepsilon_{M''}(b^*;\varphi)\right] = 1$. For $b < b^*$, the fiscal authority is not willing to balance the budget but rather tends to accumulate debt; for the monetary authority this means that the selection of the non-inflationary equilibrium necessarily breaks down. The reason for this lies in the monetary authority's motive to contain the accumulation of debt which is achieved by engineering some inflation in order to devaluate the stock of outstanding liabilities. Hence, the monetary incentives to generate surprise inflation reappear on the path of convergence from below to b^*. In a rational expectations equilibrium the public anticipates such inflation, and - abstracting from the interference by fiscal polcies - the path of real debt preferred by the monetary authority would be decreasing, a scenario similar to the one in Díaz-Giménez et al. (2004).

The policies preferred by the two authorities can be qualitatively characterized by inspection of their relevant optimality conditions. However, it is not as straightforward to anticipate the details of how the economy will evolve in equilibrium as an outcome of the dynamic policy interaction. Therefore, we resort to a numerical example which is parameterized as described above. The key results of this exercise are displayed in Figures 1 and 2. Figure 1 shows the dynamic evolution of the end-of-period stock of government debt b'. The stock of real debt grows at a decreasing rate until it converges to a debt ceiling at b^*. The increasing distortions associated with the accumulation of debt affect the pattern of consumption displayed in Figure 2. The debt ceiling is determined by the fiscal optimality condition (11); however, it is important to realize that this condition is contingent on the equilibrium policy rule φ and thus depends also on monetary policy.[22] In particular, it must be the case that at b^* the losses incurred due to inflation and the benefits from stabilizing the level of debt by monetizing fiscal deficits via the inflation tax are equal from the monetary authority's perspective. A closer analysis of the situation for the monetary authority reveals the following: On the one hand, monetary policy needs to inflate the economy in order to contain the accumulation of debt preferred by the fiscal authority; indeed, the monetary incentives to inflate the economy are an increasing function of the stock of debt. On the other hand, the fact that monetary policy is responsive to the level of debt makes the accumulation of debt increasingly unattractive because such incentives are anticipated by the public. Given that also the fiscal authority suffers from the extra distortions caused by these expectational effects, the fiscal authority will have an incentive not to let debt go out of hands.

The mechanism behind these dynamics is that the fiscal impatience undermines the monetary authority's ability to credibly sustain the zero-inflation competitive equilibrium that was available with purely benevolent interacting authorities. Nominal debt now bears liability costs beyond the costs due to future distortionary taxation to balance the intertemporal government budget. Consequently, the optimal monetary policy would be to gradually decumulate the debt

[21] The following analyis throughout assumes that the initial state b_0 takes a small positive value such that convergence to the steady state b^* proceeds from below.

[22] Indeed, the same argument can be made based on the monetary optimality condition (13), which is contingent on the fiscal behavior stipulated by the equilibrium rule φ.

until these extra liability costs vanish. However, the fiscal authority's bias towards the present implies a tendency to accumulate debt. These two effects balance each other at the steady state b^*. Hence, to a certain extent - namely up to the point where the gains from reducing the liability costs of debt by inflation equal the costs to the monetary authority due to actual inflation - fiscal policy indeed dominates monetary policy. Importantly, we can show that there cannot be a MPE involving zero inflation on the path of convergence from below to the steady state b^*. This is seen by inspection of the monetary authority's first order condition (13) which can be rewritten as follows:

$$W'(b';\varphi) - W'(b'';\varphi) = \varepsilon_{M''}(b';\varphi)\left[\frac{2\gamma}{b'} + W'(b'';\varphi)\right] \tag{14}$$

For a strictly concave $W(\cdot)$ and a rising path of real debt, the LHS of this equation is positive, while the RHS is positive only for $\varepsilon_{M''}(b';\varphi) > 0$ and $\frac{2\gamma}{b'} > -W'(b'';\varphi)$, i.e. if monetary policy reacts by an expansion in response to increases in the real stock of debt and the welfare costs of debt are not yet too large.[23] Thus, for $b < b^*$, at a non-inflationary candidate stage game equilibrium, the monetary authority has an incentive to deviate by increasing the money supply in order to prevent an excessive accumulation of debt. Since the model features a quantity relation between money supply and the price level, this establishes that any MPE with an impatient fiscal authority necessarily involves positive inflation on the path from below towards b^*. At the steady state b^*, this inflation persists as can be seen from (11) which prescribes $\varepsilon_{M''}(b^*;\varphi) = (\frac{\beta}{\delta} - 1) > 0$. With rational expectations, the inflation bias feeds directly into higher nominal interest rates. Therefore, since the nominal interest rate is an opportunity cost on holding money balances, and since carrying nonnegative amounts of currency is inevitable due to the CIA constraint, the adverse impact on welfare is immediate. We summarize our results in:

Proposition 1 *If $\delta < \beta$ and $0 < \gamma < 1$, then there is no non-inflationary (differentiable) Markov-perfect equilibrium; in other words, for any time consistent policy rule φ, there is an inflation bias.*

Against the background of this result, it is interesting to investigate how changes in the two authorities' preferences impinge on the properties of the equilibrium outcomes. First, consider the effect of a lower fiscal discount factor δ, holding β fixed. The induced rise in the ratio $\frac{\beta}{\delta}$ implies throughout the state space that $\varepsilon_{M''}(b';\varphi)$ is increased, as can be inferred from the fiscal optimality condition (11). This means that a more impatient fiscal authority triggers a monetary policy that must be more responsive to variations in the stock of debt. The consequence of this is that $W'(b';\varphi)$ becomes more negative since the associated money expansions are antipicated and accentuate the indirect liability costs of any given end-of-period amount b' of outstanding debt. Evaluating the monetary optimality condition (13) at the steady state implemented by the equilibrium policy reveals that $\left[\frac{2\gamma}{b^*} + W'(b^*;\varphi)\right]$ must be zero. With $W'(b';\varphi)$ being globally more negative, the only way this can be achieved is via a lower b^*. This establishes the following result:

Proposition 2 *Given β, a more impatient fiscal authority, characterized by a lower δ, triggers a more responsive monetary policy as measured by a higher $\varepsilon_{M''}(b';\varphi)$, but the steady state level of debt b^* implemented as the Markov-perfect equilibrium outcome is lower.*

[23] Recall that $\varepsilon_{M''}(b';\varphi) < 0$ has not been found to be a numerically relevant case.

The intuition for this proposition is as follows: A more impatient fiscal policy maker incurs higher deficits which - if an excessive accumulation of debt is to be prevented - must be monetized by money expansions. Since the increased fiscal impatience accentuates the monetary margin already for lower levels of debt and since the monetary authority is reluctant to use its instrument, the equilibrium features a more aggressive monetary policy that implements a lower long run level level of debt b^* in order to economize on the extra liability costs of outstanding debt.

Next, consider what happens if γ, the monetary authority's aversion against surprise inflation is increased. Again, $\varepsilon_{M''}(b^*;\varphi)$, the degree of monetary responsiveness at the steady state b^* is pinned down by $\frac{\beta}{\delta}$; since the latter is unchanged, the equilibrium value for $\varepsilon_{M''}(b^*;\varphi)$ must be influenced by two effects which neutralize each other: On the one hand, the commitment function of a higher γ leads to lower absolute values for both $\varepsilon_{M''}(b';\varphi)$ and $W'(b';\varphi)$ for any given end-of-period value b'. On the other hand, there is the effect via the steady state value b^* at which the relevant expressions are evaluated. Again, the steady state condition that $\left[\frac{2\gamma}{b^*} + W'(b^*;\varphi)\right]$ must be zero is helpful. Here, a higher γ is compensated for by a higher b^*; however, since an increase in the first argument of $W'(b^*;\varphi)$ simultaneously works to make this expression more negative, a less than proportionate increase in b^* is sufficient. While the intuition for this result is very similar to the one for the first parameter change discussed, the second part of the following proposition suggests an interesting institutional interpretation:

Proposition 3 *With a more inflation averse monetary authority, characterized by a higher γ, an impatient fiscal policy triggers a less responsive monetary policy as measured by a lower $\varepsilon_{M''}(b';\varphi)$, but the steady state level of debt b^* implemented as the Markov-perfect equilibrium outcome is higher.*

This proposition has the remarkable implication that a more "conservative" central bank, identified as a monetary authority that is more averse against the surprise use of its inflation tax instrument, will generally not be more successful in containing the accumulation of public debt. This theoretical finding is also confirmed numerically as evidenced by Figure 3 which compares the dynamic evolution of real debt for three economies; the basic parametrization is the same as in the initial numerical example, but the monetary authority's inflation aversion parameter γ varies from 0.5 to 0.7 to 0.9. Importantly, the following trade off arises: Monetary conservatism is a successful commitment device to constrain the monetary accommodation of fiscal profligacy, but on the other hand a higher stock of debt is accumulated in equilibrium. What happens is that at any given level of end-of-period debt b', the recourse to the inflation tax is lower; but since this is understood by the fiscal authority, it has an incentive to accumulate more debt. The reason is that the crowding out of consumption via debt will be less pronounced because monetary conservatism helps to economize on the extra liability costs of public debt.

The fact that the monetary time inconsistency problem can be strategically exploited by the fiscal authority even in case of an explicitly inflation averse monetary policy maker raises the question whether there are institutional arrangements that may help to mitigate the adverse welfare consequences. Obviously, in the present context fiscal constraints can play a role as an institutional complement to an otherwise ineffective "conservative" central bank. Within the framework considered, such constraints should first of all be designed to provide a ceiling to the maximum admissible amount of real debt. Alternatively, establishing a limit on fiscal deficits can help as an auxiliary device to constrain the accumulation of debt resulting from the fiscal authority's impatience. With a binding constraint on deficits, the long-run level of real debt would be lower, and the transition to the long-run steady state would proceed along a path featuring lower rates of inflation.

5 Related literature and concluding remarks

Fiscal discipline is often seen as a requirement for price stability, both in independent economies and monetary unions. In this paper, we have taken the view that policy makers are unable to commit to future policies. Hence, due to its power to inflate away the nominal debt of the government sector against the private sector, the monetary authority suffers from a time inconsistency problem. Against this background, we have analyzed the interaction between monetary and fiscal policy in a deterministic dynamic general equilibrium model. The contributions of this paper are of both conceptual and applied nature. On conceptual grounds, the paper has provided a method to characterize and compute the MPE in a dynamic general equilibrium economy with large interacting players who cannot commit to future policies but are bound by the requirement that their combined actions must be compatible with a competitive equilibrium of the economy. The paper has then applied this idea to the interaction of monetary and fiscal policy in the presence of nominal government debt. The central insight to be gained from the analytical and numerical results is that an impatient fiscal authority can strategically exploit the time inconsistency problem inherent in monetary policy making. This finding is reminiscent of what Chari and Kehoe (2004) establish in the context of a monetary union. However, the mechanism involved is different in our context. Whereas Chari and Kehoe build their analysis on a free-rider problem between the fiscal constituencies in a monetary union,[24] our starting point is a politico-economic friction that results in diverging preferences about the accumulation or decumulation of government debt. On the basis of this setup, our analysis proposes a positive theory of government indebtedness and inflation.

In this respect, the paper relates to a number of fiscalist approaches to the determination of the price level. According to one interpretation (Kocherlakota and Phelan, 1999), the key difference between such fiscal theories of the price level and the traditional monetarist view lies in the role of the fiscal authority's intertemporal budget constraint, which links the real value of debt to the present value of primary surpluses the fiscal authority will run in the future. In two recent studies, Bassetto (2002a,b) examines the fiscal theory of the price level from a game-theoretic perspective and addresses the issue of government commitment. Specifically, he pays close attention to the behavior of the economy out of equilibrium. With this approach, he is able to shed light on the nature of the restrictions on fiscal policy due to the intertemporal budget constraint. Taking as given some target policy, Bassetto asks two main questions: (i) Is the fiscal authority actually able to adhere to this targeted policy in all contingencies, i.e. also *off the equilibrium path*? (ii) If not, can the fiscal authority implement the targeted policy as a *unique equilibrium outcome*? His answer basically is that unconditional rules involving spending levels that exceed the tax revenue in some period are misspecifications, while the

[24]Other papers that address the rationale for fiscal rules on the basis of fiscal externalities in monetary unions include Beetsma and Bovenberg (1997, 1998, 1999), Beetsma and Uhlig (1999) or Dixit and Lambertini (2001, 2003b).

fiscal authority can implement any competitive equilibrium as a unique equilibrium.[25]

In this paper, we have restricted attention to what happens *on the equilibrium path*. In contrast to Bassetto, we specify objective functions for two separate government authorities and demand that these authorities must be willing to adhere to their policy rule in any subgame. While our approach suffers from the drawback that we are not able to completely characterize what happens off the equilibrium path,[26] we nevertheless are able to provide some important insights. We identify the incentives involved and develop a notion of dominance between the two authorities that is not assumed exogenously, but rather derived as an endogenous result of the primitives of the dynamic game. Specifically, it is shown under which conditions and to what extent fiscal policy can gain leverage over monetary outcomes. So, our approach generates results similar to those of the fiscal theory, but without relying on a reinterpretation of the intertemporal government budget constraint as a mere equilibrium condition, a view that has been subject to much criticism on theoretical grounds.[27]

Having discussed the relationship between the fiscal theory and our approach, it should be stressed that the methodology we use is more in the tradition of optimal policy models. The issue of the time inconsistency of optimal plans has first been identified by Kydland and Prescott (1977); subsequently, Barro and Gordon (1983b) have applied this framework to a positive theory of monetary policy making. In a paper closely related to ours, Díaz-Giménez et al. (2004) explore the implications of nominal government debt on optimal monetary policy. Since the contribution by Lucas and Stokey (1983) also fiscal policy has been the topic of further research; important contributions include Chari and Kehoe (1990), Klein and Ríos-Rull (2003) or Klein, Krusell and Ríos-Rull (2003). However, in spite of the institutional arrangements that we observe in most developed economies, when the focus of their analyses is monetary (fiscal) policy, all these papers essentially assume that fiscal (monetary) policy is absent or exogenously given to the model. Against this background, our innovation has been that we consider a setting with an inherent time inconsistency problem that gives rise to a dynamic policy game where monetary *and* fiscal policies are decided upon by two separate authorities.

So far, the interaction between monetary and fiscal policy in a dynamic framework with optimizing authorities seems to have been neglected in the literature. An exception is the recent work by Adam and Billi (2004) who investigate a sticky price economy where output is inefficiently low due to the market power of firms. Their paper is complementary to ours since the setup the authors consider is one where monetary policy controls the nominal interest rate, while fiscal policy decides about the provision of public goods and taxation is lump-sum. In contrast to our problem where only monetary policy is subject to a time inconsistency problem, their setup gives rise to a potential time inconsistency problem also for fiscal policy. Adam and Billi analyze the dynamic economy under varying assumptions on the degree of the authorities' commitment. Their basic finding is that the monetary time inconsistency problem is more severe than the fiscal one. Similar to our results, they propose a conservative central bank as an institutional arrangement that may mitigate the distortions associated with sequential

[25] Essentially, this means that the fiscal authority must comply with a budget constraint both on and off the equilibrium path, but has the power to select specific equilibria.

[26] The point is that our model is tacit about what happens if a pair of policy choices is incompatible with a competitive equilibrium, e.g. when the exogenous g cannot be financed. In such situations the crucial question is: How does the adjustment process work to restore equilibrium?

[27] Compare e.g. Kocherlakota and Phelan (1999), Buiter (2002) or Niepelt (2004).

policy making. Along the same lines, Dixit and Lambertini (2003a) consider monetary-fiscal interactions with a conservative central bank and varying degrees of commitment of the two authorities. Their analysis shows that monetary commitment is negated when fiscal policy is discretionary. This result is similar to our finding that, despite its inflation aversion, the monetary authority is unable to implement a zero-inflation equilibrium when the fiscal authority is impatient.

A number of questions remain open and should be addressed in future research. First, we have already mentioned that our model falls short of a complete game-theoretic specification of the economy. Particularly, outcomes off the equilibrium path or adjustments from there are not well-defined. A relevant scenario of this kind is a debt crisis where households simply refuse to buy government debt at any intertemporal price. Incorporating such a crisis in our model as a zero-probability event or explicitly along the lines of Cole and Kehoe (2000) would be a very interesting, if difficult, extension. Second, our model takes government spending to be exogenous. In the baseline model presented here, we assume a constant path of public expenditure. However, most projections for advanced economies[28] predict that government spending will rise in the future due to the pressures associated with ageing societies. Therefore, it would be a worthwhile exercise to investigate how a deterministic trend in government spending would affect the dynamic game played between monetary and fiscal policy. Finally, by considering only one-period bonds our paper abstracts from the maturity structure of government debt. Indeed, it has been demonstrated how a richer maturity structure can help to overcome the time inconsistency problem faced by policy makers.[29] In the context of the dynamics of the fiscal theory of the price level, Cochrane (1999) has demonstrated that in an environment, where the inflation tax would otherwise operate as a lump-sum instrument, the introduction of long-term debt has the effect of converting the inflation tax from a lump-sum into a distortionary source of revenues by pushing the inflation generated by tax cuts into the future. The question then is how this result carries over to our setup where there is strategic interaction between a monetary and a fiscal authority.

[28] Compare e.g. OECD (2002).

[29] Compare Lucas and Stokey (1983), Persson, Persson and Svensson (1987) as well as Calvo and Obstfeld (1990).

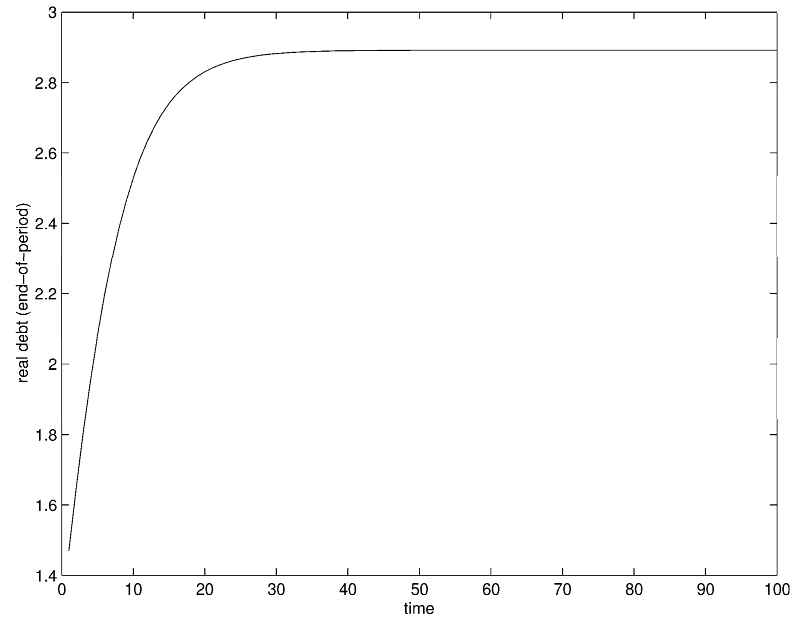

Figure 1: Path of real debt in benchmark example

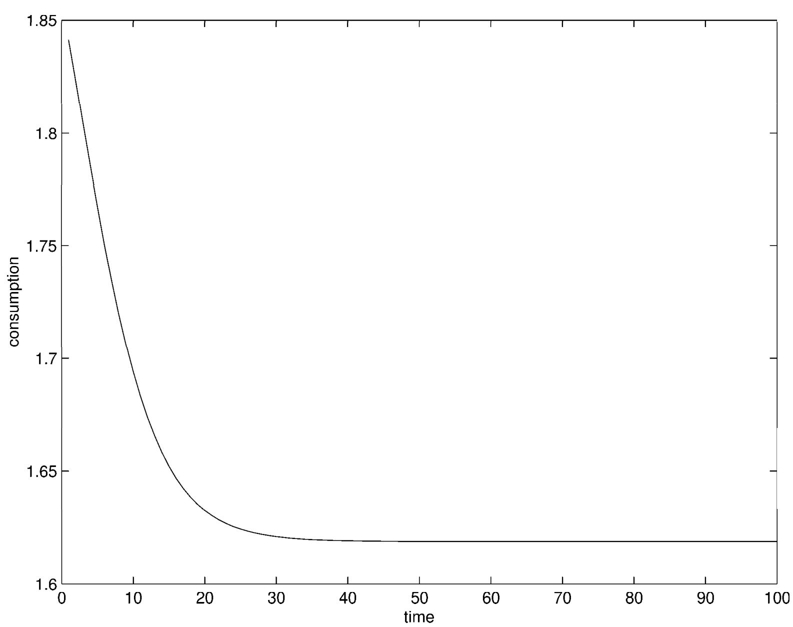

Figure 2: Path of consumption in benchmark example

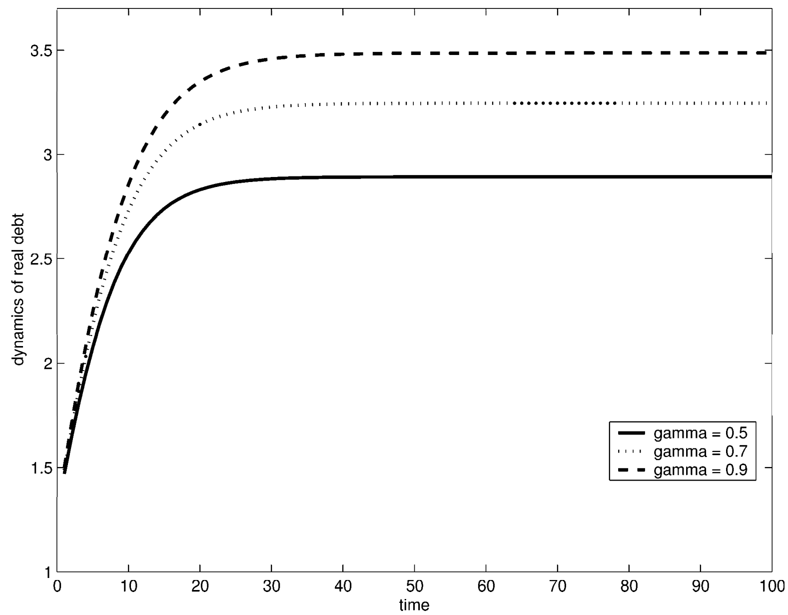

Figure 3: Debt dynamics for different degrees of monetary conservatism

A Appendix

A.1 Some notation

With a deterministic path of constant government expenditure g, the aggregate state is $b^g \equiv \frac{B^g}{P_{-1}}$; in the rational expectations equilibrium, the representative household's state b and the aggregate state b^g coincide. The current policy rule, which consists of a fiscal and a monetary rule for the present period, is $\pi(b) = (\pi_f(b), \pi_m(b))$. The future policy rule, which consists of a fiscal and a monetary rule from the next period onwards, is $\varphi(b) = (\varphi_f(b), \varphi_m(b))$. We abuse notation by letting the policy rules map the aggregate state into policy instruments (τ^c, M') whenever we are in the dual space (as it is done in the main text), while the mapping is into the allocation (c, b') whenever we are in the primal space (as in most part of the Appendix). As in the main text, barred variables refer to private expectations.

A.2 Objective functions and implementability constraint

To make a primal approach to the dynamic game operational, the authorities' objective functions and the constraints they face in their optimization problem have to be expressed exclusively in terms of allocation variables. This requires a number of substitutions which make use of the private equilibrium conditions (6), (7) and (8). Note that we use the household's first order conditions only at their expected values; this means that we do not yet impose rational expectations. This corresponds to the fact that, when facing their respective optimization problems, the authorities take private expectations as arbitrary given functions of the aggregate state.

Both authorities' value functions can be constructed directly from the allocation. It is convenient to start from the problem faced by the authorities when they assume that (i) the policy rule φ will govern the play from the next period onwards and that (ii) the current policy choice by the respective opponent (in terms of the true policy instrument) is π_{-i}. Then, for the fiscal authority, we have:

$$\hat{V}(b;\pi_m,\varphi) = max_{\pi_f}\left\{[log(c)-\alpha(c+g)] + \beta V(b';\varphi)\right\}$$

For the monetary authority, one obtains:

$$\hat{W}(b;\pi_f,\varphi) = max_{\pi_m}\left\{\left[-\gamma\left(\frac{\bar{c}(1+\bar{\tau}^c)}{c(1+\tau^c)}\right)^2 + (1-\gamma)(log(c)-\alpha(c+g))\right] + \beta W(b';\varphi)\right\}$$

For these value functions to make sense, they must be amended by the appropriate dynamic constraints. In their respective maximization problems, both authorities are constrained by a sequence of implementability constraints which internalize the fact that the private households react optimally to the government policies. We construct these implementability constraints by substituting the private equilibrium conditions (6), (7) and (8) into the consoldiated government budget constraint (1). Using the private optimality conditions at expected values and the CIA constraint at realized and expected values, we arrive at an equation which relates the set of implementable pairs (c, b') to the two authorities' policy choices:

$$b' = c\left(1-(1+\tau^c)\frac{M'}{M}\right) + \beta^{-1}b\frac{c(1+\tau^c)}{\bar{c}(1+\bar{\tau}^c)} + g$$

To let the two authorities face well-defined problems, we need to make sure that in each authority's optimization problem, the respective implementability constraint does not depend on the relevant authority's own policy instrument. For that purpose, consider the general implementability constraint faced by a policy maker who controls the *complete* set of policy instruments, i.e. τ^c and M':

$$b' = c + \beta^{-1}b\frac{c}{\bar{c}} + g - \frac{\beta}{\alpha}$$

In particular, note that this constraint differs from the policy-dependent constraint in that it is tacit about how an allocation is decentralized and it allows for free substitution of one policy instrument for the other. Now, comparison of the latter constraint and the policy-dependent implementability constraint derived above informs about the fact that in the case of two separate authorities an allocation is implementable if and only if the policy instruments together with the associated allocation satisfy the following consistency condition:

$$c(1+\tau^c)\frac{M'}{M} = \frac{\beta}{\alpha} \qquad (15)$$

This consistency condition (which must hold at expected and realized values) allows to derive two separate implementability constraints. Each of them is relevant for one of the two authorities and makes the implementability constraint contingent on the other authority's current choice of the policy instrument. Moreover, each implementability constraint is contingent on private expectations which the authorities perceive as given functions. For the fiscal authority, we obtain the following constraint:

$$b' = c + \beta^{-1} b \frac{\bar{M}'}{M'} + g - \frac{\beta}{\alpha} \tag{16}$$

For the monetary authority, we have:

$$b' = c + \beta^{-1} b \frac{c(1+\tau^c)}{\bar{c}(1+\bar{\tau}^c)} + g - \frac{\beta}{\alpha} \tag{17}$$

Note that in both cases, the implementability constraint's job is twofold: First, for a given value of the policy instrument chosen by the other authority, it imposes a joint restriction on compatible choices of c and b'; secondly, since the feasible set is contingent on the policy instrument chosen by the other authority, it establishes a one-to-one mapping between the resulting allocation (c, b') and the own policy instrument. However, note that there is an asymmetry in the two equations: In the implementability constraint faced by the fiscal authority, the choice variable c appears only once; conversely, in the monetary authority's implementability constraint, it appears twice. This reflects the fact that the fiscal authority's current policy choice has no direct influence on the contemporaneous price level P, whereas monetary policy, by manipulating the price level, can also affect the real value of the outstanding stock of government debt.

A.3 The economy as a game

Our model economy can be described as an infinite-horizon dynamic game of almost perfect information whose building block is a two-player simultaneous-moves stage game. The game is not of a repeated variety due to the presence of the state variable $b^g \equiv \frac{B^g}{P_{-1}}$. The players are the fiscal and the monetary authority, indicated by $i = f, m$ respectively. In each period, their actions are $a_f = \tau^c$ and $a_m = M'$. The action spaces for the two players can be assumed to be compact and time-invariant and are given by $A_f = [\tau^c_{min}, \tau^c_{max}]$ and $A_m = [M'_{min}, M'_{max}]$, where $\tau^c_{min} > -1$ and $M'_{min} > 0$; for future reference, we let $A_i = [a^i_{min}, a^i_{max}]$. Note that the action spaces do not depend on the state variable b^g for which the relevant state space is $I = [0, \bar{B}]$. In what follows, we restrict attention to stationary Markov strategies, where the actions taken by the respective players are functions of the state variable b^g only and otherwise do not depend on the past. The discounted stream of payoffs determines the players' objective functions $V(\cdot)$ and $W(\cdot)$, and the respective implementability constraints give rise to policy-dependent laws of motion for the state variable b^g. This completes the description of our model as a discounted Markov-stationary game with uncountable state and action spaces.

A.4 MPE - step 1: equilibrium for arbitrary policy rule

Specifying an arbitrary policy rule φ allows to calculate the value functions for the fiscal and monetary authorities resulting from this rule when the economy starts from aggregate state b. Specifically, for the fiscal authority, conditional on the rule φ, we get $V(b; \varphi)$ as the solution to:

$$V(b; \varphi) = \{[log(c(\varphi)) - \alpha(c(\varphi) + g)] + \delta V(b'; \varphi)\}$$

subject to the implementability constraint:

$$b' = c(\varphi) + \beta^{-1} b \frac{\bar{M}'(b)}{M'(\varphi)} + g - \frac{\beta}{\alpha} \tag{18}$$

The policy-dependent implementability constraint distinguishes between private expectations $\bar{M}'(b)$, which are a function of the aggregate state, and the implemented policy $M'(\varphi)$ as prescribed by the policy rule φ. Applying an envelope theorem to the continuation problem yields:

$$V'(b;\varphi) = \delta V'(b';\varphi)\left[\beta^{-1}\frac{\bar{M}'(b)}{M'(\varphi)} + \beta^{-1} b \frac{\frac{\partial \bar{M}'(b)}{\partial b}}{M'(\varphi)}\right] \tag{19}$$

In any rational expectations equilibrium, we have $\bar{M}'(b) = M'(\varphi)$, and therefore:

$$V'(b;\varphi) = \frac{\delta}{\beta} V'(b';\varphi)\left[1 + b\frac{\frac{\partial \bar{M}'(b)}{\partial b}}{\bar{M}'(b)}\right]$$

Defining $\varepsilon_{M'}(b;\varphi) \equiv \frac{\partial \bar{M}'(b)/\partial b}{\bar{M}'(b)/b}$, the elasticity of the private expectations with respect to monetary expansions $\bar{M}'(b)$ in response to changes in the stock of real debt b, we get:

$$V'(b;\varphi) = \frac{\delta}{\beta} V'(b';\varphi)\left[1 + \varepsilon_{M'}(b;\varphi)\right], \tag{20}$$

where we note that generally $\varepsilon_{M'}(b;\varphi) \geq 0$. Moreover, if $V(b;\varphi)$ is differentiable, an envelope theorem yields:

$$V'(b';\varphi) = -\lambda\delta^{-1}, \tag{21}$$

where the nonnegativity of λ, the Lagrange multiplier on the implementability constraint (18), implies that $V'(b';\varphi) \leq 0$.

Similarly, for the monetary authority, conditional on the rule φ, we get $W(b;\varphi)$ as the solution to:

$$W(b;\varphi) = \left\{\left[-\gamma\left(\frac{\bar{c}(b)(1+\bar{\tau}^c(b))}{c(\varphi)(1+\tau^c(\varphi))}\right)^2 + (1-\gamma)(log(c(\varphi)) - \alpha(c(\varphi)+g))\right] + \beta W(b';\varphi)\right\}$$

subject to the implementability constraint:

$$b' = c(\varphi) + \beta^{-1} b \frac{c(\varphi)(1+\tau^c(\varphi))}{\bar{c}(b)(1+\bar{\tau}^c(b))} + g - \frac{\beta}{\alpha} \tag{22}$$

The policy-dependent implementability constraint distinguishes between private expectations $\bar{c}(b)$, $\bar{\tau}^c(b)$, which are a function of the aggregate state, and the actual realizations $c(\varphi)$, $\tau^c(\varphi)$ as prescribed by the policy rule φ. Applying an envelope theorem to the continuation problem yields:

$$\begin{aligned} W'(b;\varphi) &= -2\gamma\left(\frac{\bar{c}(b)(1+\bar{\tau}^c(b))}{c(\varphi)(1+\tau^c(\varphi))}\right)\left(\frac{[\frac{\partial \bar{c}(b)}{\partial b}(1+\bar{\tau}^c(b)) + \bar{c}(b)\frac{\partial(1+\bar{\tau}^c(b))}{\partial b}]}{c(\varphi)(1+\tau^c(\varphi))}\right) \\ &+ \beta W'(b';\varphi)\left[\beta^{-1}\frac{c(\varphi)(1+\tau^c(\varphi))}{\bar{c}(b)(1+\bar{\tau}^c(b))} + \beta^{-1} b \frac{-c(\varphi)(1+\tau^c(\varphi))[\frac{\partial \bar{c}(b)}{\partial b}(1+\bar{\tau}^c(b)) + \bar{c}(b)\frac{\partial(1+\bar{\tau}^c(b))}{\partial b}]}{[\bar{c}(b)(1+\bar{\tau}^c(b))]^2}\right] \end{aligned}$$

In any rational expectations equilibrium, we have $\bar{c}(b) = c(\varphi)$ and $\bar{\tau}^c(b) = \tau^c(b)$, and therefore:

$$W'(b;\varphi) = -2\gamma\left(\frac{[\frac{\partial\bar{c}(b)}{\partial b}(1+\bar{\tau}^c(b)) + \bar{c}(b)\frac{\partial(1+\bar{\tau}^c(b))}{\partial b}]}{c(\varphi)(1+\tau^c(\varphi))}\right) + W'(b';\varphi)\left[1 - b\frac{[\frac{\partial\bar{c}(b)}{\partial b}(1+\bar{\tau}^c(b)) + \bar{c}(b)\frac{\partial(1+\bar{\tau}^c(b))}{\partial b}]}{\bar{c}(b)(1+\bar{\tau}^c(b))}\right]$$

Defining $\varepsilon_c(b;\varphi) \equiv \frac{\partial\bar{c}(b)/\partial b}{\bar{c}(b)/b}$, the elasticity of the private consumption plan $\bar{c}(b)$, and $\varepsilon_{\tau^c}(b;\varphi) \equiv \frac{\partial(1+\bar{\tau}^c(b))/\partial b}{(1+\bar{\tau}^c(b))/b}$, the elasticity of private expectations with respect to the consumption tax $(1+\bar{\tau}^c(b))$ in response to changes in the stock of real debt b, we get:

$$W'(b;\varphi) = -2\gamma\left(\frac{1}{b}\left[\varepsilon_c(b;\varphi) + \varepsilon_{\tau^c}(b;\varphi)\right]\right) + W'(b';\varphi)\left[1 - (\varepsilon_c(b;\varphi) + \varepsilon_{\tau^c}(b;\varphi))\right],$$

where we note that $\varepsilon_c(b;\varphi) \leq 0$ and $\varepsilon_{\tau^c}(b;\varphi) \geq 0$. Importantly, from the CIA constraint with M predetermined and a quantity relation between M' and P, it follows that $\varepsilon_{M'}(b;\varphi) = -(\varepsilon_c(b;\varphi) + \varepsilon_{\tau^c}(b;\varphi))$. Then, using this relation, we can substitute and get:

$$W'(b;\varphi) = 2\gamma\frac{\varepsilon_{M'}(b;\varphi)}{b} + W'(b';\varphi)\left[1 + \varepsilon_{M'}(b;\varphi)\right] \tag{23}$$

Moreover, if $W(b;\varphi)$ is differentiable, an envelope theorem yields:

$$W'(b';\varphi) = -\mu\beta^{-1}, \tag{24}$$

where the nonnegativity of μ, the Lagrange multiplier on the implementability constraint (22), implies that $W'(b';\varphi) \leq 0$.

A.5 MPE - step 2: Optimal current policy rule for given future policy rule

A.5.1 The fiscal problem

The current fiscal authority takes the predetermined private expectations $\bar{M}'(b)$ and the current monetary authority's policy $\pi_m = M'(b)$ as parametrically given and the continuation play $\varphi(b)$ as a given function of the aggregate state b. The problem for the fiscal authority is:

$$\hat{V}(b;\pi_m,\varphi) = max_{c,b'}\{[log(c) - \alpha(c+g)] + \delta V(b';\varphi)\} \tag{25}$$

subject to the implementability constraint:

$$b' \quad = \quad c + \beta^{-1}b\frac{\bar{M}'(b)}{M'(b)} + g - \frac{\beta}{\alpha}$$

The solution to this problem are policy functions $c_f(b;\pi_m,\varphi)$ and $b'_f(b;\pi_m,\varphi)$ for the fiscal authority. The first order condition with respect to c_f is:

$$\frac{1}{c} - \alpha = -\delta V'(b';\varphi) \tag{26}$$

From the envelope theorem, the optimal choice of b'_f must satisfy:

$$\hat{V}'(b;\pi_m,\varphi) = \delta V'(b';\varphi)\left[\beta^{-1}\frac{\bar{M}'(b)}{M'(b)} + \beta^{-1}b\frac{\frac{\partial \bar{M}'(b')}{\partial b}M'(b) - \frac{\partial M'(b)}{\partial b}\bar{M}'(b)}{M'(b)^2}\right]$$

In the rational expectations equilibrium, we have $\bar{M}'(b) = M'(b)$, and therefore:

$$\hat{V}'(b;\pi_m,\varphi) = \frac{\delta}{\beta}V'(b';\varphi)$$

Here, the expression on the RHS depends on the continuation policy φ; specifically we have from (20):

$$V'(b';\varphi) = \frac{\delta}{\beta}V'(b'';\varphi)\left[1+\varepsilon_{M''}(b';\varphi)\right] \tag{27}$$

A.5.2 The monetary problem

The current monetary authority takes the predetermined private expectations $\bar{c}(b)$, $\bar{\tau}(b)$ and the current fiscal authority's policy $\pi_f = \tau^c(b)$ as parametrically given and the continuation play $\varphi(b)$ as a given function of the aggregate state b. The problem for the monetary authority is:

$$\hat{W}(b;\pi_{-i},\varphi) = max_{c,b'}\left\{\left[-\gamma\left(\frac{\bar{c}(b)(1+\bar{\tau}^c(b))}{c(1+\tau^c(b))}\right)^2 + (1-\gamma)(log(c) - \alpha(c+g))\right] + \beta W(b';\varphi)\right\} \tag{28}$$

subject to the implementability constraint:

$$b' \;=\; c + \beta^{-1}b\frac{c(1+\tau^c(b))}{\bar{c}(b)(1+\bar{\tau}^c(b))} + g - \frac{\beta}{\alpha}$$

The solution to this problem are policy functions $c_m(b;\pi_f,\varphi)$ and $b'_m(b;\pi_f,\varphi)$ for the monetary authority. The first order condition with respect to c_m is:

$$\begin{aligned}-2\gamma\left(\frac{\bar{c}(b)(1+\bar{\tau}^c(b))}{c(1+\tau^c(b))}\right)\left(\frac{-\bar{c}(b)(1+\bar{\tau}^c(b))(1+\tau^c(b))}{[c(1+\tau^c(b))]^2}\right) + (1-\gamma)\left(\frac{1}{c}-\alpha\right) &= \\ -\beta W'(b';\varphi)\left[1+\beta^{-1}b\frac{(1+\tau^c(b))}{\bar{c}(b)(1+\bar{\tau}^c(b))}\right]&\end{aligned}$$

In the rational expectations equilibrium, we have $\bar{c}(b) = c(b)$ and $\bar{\tau}^c(b) = \tau^c(b)$, and therefore:

$$2\gamma\frac{1}{c} + (1-\gamma)\left(\frac{1}{c}-\alpha\right) = -\beta W'(b';\varphi)\left[1+\beta^{-1}b\frac{1}{c}\right] \tag{29}$$

From the envelope theorem, the optimal choice of b'_m must satisfy:

$$\begin{aligned}&\hat{W}'(b;\pi_f,\varphi) = \\ -\;& 2\gamma\left(\frac{\bar{c}(b)(1+\bar{\tau}^c(b))}{c(1+\tau^c(b))}\right)\left(\frac{[\frac{\partial \bar{c}(b)}{\partial b}(1+\bar{\tau}^c(b)) + \bar{c}(b)\frac{\partial(1+\bar{\tau}^c(b))}{\partial b}]c(1+\tau^c(b)) - \bar{c}(b)(1+\bar{\tau}^c(b))c\frac{\partial(1+\tau^c(b))}{\partial b}}{[c(1+\tau^c(\varphi))]^2}\right) \\ +\;& \beta W'(b';\varphi)\left[\beta^{-1}\frac{c(1+\tau^c(b))}{\bar{c}(b)(1+\bar{\tau}^c(b))} + \beta^{-1}b\frac{c\frac{\partial\tau^c(b)}{\partial b}\bar{c}(b)(1+\bar{\tau}^c(b)) - c(1+\tau^c(b))[\frac{\partial \bar{c}(b)}{\partial b}(1+\bar{\tau}^c(b)) + \bar{c}(b)\frac{\partial\bar{\tau}^c(b)}{\partial b}]}{[\bar{c}(b)(1+\bar{\tau}^c(b))]^2}\right]\end{aligned}$$

In the rational expectations equilibrium, we have $\bar{c}(b) = c(b)$ and $\bar{\tau}^c(b) = \tau^c(b)$ such that:

$$\hat{W}'(b;\pi_f,\varphi) = -2\gamma\left(\frac{\frac{\partial \bar{c}(b)}{\partial b}}{c}\right) + \beta W'(b';\varphi)\left[1 + b\frac{-\frac{\partial c}{\partial b}}{c}\right]$$

Again making use of the defining $\varepsilon_c(b) \equiv \frac{\partial \bar{c}/\partial b}{\bar{c}/b}$, we get:

$$\hat{W}'(b;\pi_f,\varphi) = \frac{-2\gamma}{b}\varepsilon_c(b) + W'(b';\varphi)\left[1 - \varepsilon_c(b)\right],$$

where we note that $\varepsilon_c(b) \leq 0$. Here, the term $W'(b';\varphi)$ on the RHS depends on the continuation policy φ; specifically we have from (23):

$$W'(b';\varphi) = 2\gamma\frac{\varepsilon_{M''}(b';\varphi)}{b'} + W'(b'';\varphi)\left[1 + \varepsilon_{M''}(b';\varphi)\right] \tag{30}$$

A.5.3 The system of equations

The set of necessary conditions characterizing the dynamic evolution of the economy as governed by the Nash equilibrium policy response $\pi(\varphi)$ to an arbitrary continuation policy φ is then given by equations (26), (27), (29), (30) and the following rational expectations version of the implementability constraint:

$$b' = c + \beta^{-1}b + g - \frac{\beta}{\alpha} \tag{31}$$

For a given future policy rule φ, the functions $V(b';\varphi)$ and $W(b';\varphi)$ as well as their derivatives and $\varepsilon_{M''}(b';\varphi)$ are determined. Making use of above envelope results and the consistency condition (15), we then have a system of six equations in the six unknown variables c, b', λ, μ, ε_{τ^c} and $\varepsilon_{M'}$. That is, the optimal current policy rule $\pi(\varphi)$, existence of which follows by standard arguments on the existence of Nash equilibrium (e.g. Theorem 2.7 in Vives, 1999), is uniquely defined with respect to the allocation (c, b') it implements in the current period and the *elasticities* of the two policy instruments; the decentralization of the equilibrium allocation in terms of levels of the policy instruments is indeterminate.

A.6 MPE - step 3: Policy fixed point

Finally, the time consistent MPE policy rule is found as the fixed point of the functional mapping $\pi : \varphi \to \pi(\varphi)$. Existence of such a fixed point follows from results in Niemann (2005).

A.7 Computational procedure

Traditionally, dynamic games have been attacked by linear-quadratic methods which allow for relative simple algorithms to find a solution. However, in many economic applications, linear-quadratic applications deliver highly inaccurate approximate solutions; see e.g. Miranda and Fackler (2002). Particularly, since linear-quadratic methods are local in nature, this is the case at points far away from the certainty-equivalent steady state or if the true payoff and transition functions are not well-approximated by second- and first-degree polynomials over the entire domain. Therefore, standard linear-quadratic methods do not seem appropriate to solve our model, the location of whose steady state in the state space we do not know initially. Projection methods have recently been proposed as an efficient way to solve functional equation problems, and, more specifically, dynamic games. The application to a primal approach problem in a general equilibrium context is, to our knowledge, new. The numerical algorithm to find the MPE of the dynamic policy game proceeds as follows:

1. Guess an arbitrary continuation policy rule φ.
2. For the given continuation policy φ, solve for the optimal current policy rule π; this is done by solving the system of equations collected in Appendix A.5.3. For a given future policy φ, this constitutes a system of six equations in the six unkowns c, b', λ, μ, ε_{τ^c} and $\varepsilon_{M'}$. We solve this system via a collocation method on a one-dimensional state space and obtain the current best response rule $\pi^*(\varphi)$.
3. Update the continuation policy by substituting the guess for φ by $\pi^*(\varphi)$.
4. Repeat steps 1 to 3 until $\pi^*(\varphi) = \varphi$.

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