Optimal Monetary Policy Response to Distortionary Tax Changes

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Abstract

We analyze the trade-offs faced by a monetary policy authority when the value added tax rate is increased. In the short run, such an increase acts as a cost push shock from the perspective of a central bank that is concerned with stabilizing the welfare relevant output gap. We develop a New Keynesian monetary model with real wage rigidity and consider the effects that obtain under a simple interest rate rule, on the one hand, and those that obtain under an optimal monetary policy from a timeless perspective (in the terminology of Woodford, 2003). The implications for the dynamic response of the economy differ strongly in the presence of real wage rigidity. While under a rule inflation is higher for about eight quarters, the optimal policy involves an adjustment that is about half as long, and is followed by a slight deflation. The reason is that this policy can be shown to include a commitment to maintain a certain price-level, which contains inflation expectations. In the absence of real wage rigidity, however, output would initially only fall by about half as much.

Keywords: Nominal and real rigidities, distortionary taxation, optimal monetary policy

1 Introduction

This paper analyzes the optimal monetary policy response to a change in the value-added tax in a New Keynesian dynamic stochastic general equilibrium (DSGE) model with rigid real wages. Because it constitutes a cost-push shock in the Phillips curve, such a tax increase confronts the monetary authority with a trade-off between output and inflation stabilization. We consider two types of systematic responses of a central bank. One is a simple Taylor-type interest rate rule, under which the nominal interest rate is adjusted proportional to deviations of inflation from target and to variations of the welfare-relevant output gap. The other type of response is a targeting rule that results from the optimal monetary policy under a timeless perspective.[1]

The framework we employ is the standard new Keynesian model with a value added tax on the revenue of the monopolistically competitive firms. In the baseline case, the revenue is reimbursed as a lump sum to the households, or equivalently, is used by the government on the same bundle of differentiated goods as that of households. Thus the only long-run effect of the tax is distortionary. Price stickiness is modelled as a quadratic cost of price adjustment on the gross price that firms charge to consumers. The model is complemented by introducing real wage rigidities in a simple manner following Krause and Lubik (2003) and Blanchard and Gali (2005). It is well-known that real rigidities add important inertia to the real marginal costs faced by firms, and hence affect inflation dynamics. We thus assume that current real wages are a weighted average between last period's wages (in discrete time) and the current marginal rate of substitution. Since real wages are prevented from fully adjusting downward after the fall in revenue due to the tax increase, the real marginal costs of firms rise. This confounds the policy problem, as the initial output response to a tax change is greatly amplified.

We find in the calibrated model that an interest rate rule implies an increase in inflation after the tax change, which slowly peters out. After about 8 quarters, inflation is back to steady state. Output falls on impact and then needs about the same time to revert back upwards to the new long-run level of output. However, this level is lower than initially, because the tax change raises the degree of distortion in the economy. Under an optimal monetary policy from the timeless perspective, the central bank commits to a price level (or price level path). This target makes sure that agents' inflation expectations are firmly anchored, such that stabilizing monetary policy can be conducted with a minimal loss in welfare. However, relative to the rule, output falls less on impact, falls further in the second quarter, but then quickly reverts back to the new steady state. From a welfare point of view, this

[1] See Woodford (2003).

outcome is be preferred as the adjustment of output and inflation is faster. Note that in the absence of real wage rigidity, the inflation response under both regimes is basically zero as the transition to the new long-run equilibrium can take place instantaneously.

The choice of optimal monetary policy involves two issues. First is to decide on the relevant notion of output gap, which the central bank aims to minimize along with inflation variations. This is reflected in the quadratic objective function of the central bank. The second issue is to find the correct, model-consistent objective function, in particular the determinants of the coefficients on the quadratic deviation of the output gap and the quadratic deviations of inflation from their respective target values.[2] This can only be determined from an, at least, second-order approximation of household utility, in order to obtain a linear-quadratic policy problem. We derive such an approximation. Loss functions specified in an ad-hoc fashion, often employ the gap between actual output and the natural rate of output. The latter is defined as the level of output that would obtain in the absence of price rigidities. However, the welfare-relevant output gap is given by the difference between actual output and the efficient level of output. In a world with distortions, the two concepts of the output gap differ, and it is variations of the second one that the central bank should consider. In both the case of a rule and that with optimal policy, we use the welfare-relevant output gap.[3]

The next section outlines the model and discusses the appropriate notion of the output gap. Section 3 derives the model-endogenous objective function, with respect to which optimal monetary policy under a timeless perspective is derived in section 4. Based on calibrated parameters, section 5 shows quantitative results, section 6 concludes.

[2] See also the discussion in Clarida, Gali, and Gertler (1999).

[3] The study by Nessen and Soderstrom (2001) is also concerned with different variants of monetary policy responses to (among other things) value-added tax increases. They, however, work within the class of simple rules which are chosen optimally with respect to different ad-hoc specifications of the loss function.

2 The model economy

2.1 Households, firms, government

The basic model is a version of the cashless economy as in Woodford (2003).[4] Households maximize the present value of utility

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\mu}}{1+\mu} \right]$$

with respect to consumption C and labor supply N, subject to the budget constraint

$$C_t + \frac{B_t}{P_t} = T_t + W_t N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + D_t.$$

Real household spending consists of consumption C and bond purchases where B is nominal bond holdings and P is the price level. Real income is the sum of a lump-sum transfer T, wage income WN, the interest income on bonds bought last period, and dividend income D. The nominal one-period interest rate is i. Consumption is a CES aggregate of differentiated products,

$$C_t = \left[\int_0^1 c_t(i)^{(\epsilon-1)/\epsilon} \, di \right]^{\epsilon/(\epsilon-1)},$$

which gives rise to demand functions for each individual good, with $\epsilon > 1$. The wage, prices, and interest rate are taken as given by households.

The monopolistically competitive firms maximize real profits

$$\Pi_{i0} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[\frac{P_{it}}{P_t} Y_{it} \frac{1}{1+\tau_t} - W_t N_t - \frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - \pi \right)^2 Y_t \right]$$

where $\pi_t = P/P_{-1}$ is the gross steady-state inflation rate, and $\frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - \pi \right)^2 Y_t$ is the quadratic cost of price adjustment, and τ denotes the value-added tax rate.

Profits are discounted with the household's subjective discount factor, and are maximized subject to the demand curve

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} Y_t$$

arising from consumer optimization. The linear production technology is given by

$$Y_{it} = A_t N_{it}.$$

where A is aggregate productivity.

[4] Our set up differs from his by introducing price rigidity via convex adjustment costs a la Rotemberg (1982) as opposed to using Calvo-type price setting.

The government collects the tax revenue which is reimbursed to the household in a lump sum in the same period,

$$T_t = \frac{\tau_t}{1+\tau_t} Y_t.$$

That is, the government runs a balanced budget each period. The economy is cashless, so there is no seignorage revenue.

The policy instrument at the disposal of the central bank is the one-period interest rate i. As discussed in more detail below, this will either be set according to a central bank reaction function or its evolution will be determined such that it supports the optimally chosen paths of inflation and output.

2.2 Optimality conditions and steady state

Household maximization yields a consumption Euler equation and a labor supply condition,

$$\begin{aligned} C_t^{-\sigma} &= \beta(1+i_t)E_t\left[\frac{1}{\pi_{t+1}} C_{t+1}^{-\sigma}\right], \\ W_t &= C_t^{\sigma} N_t^{\mu}. \end{aligned}$$

as well as the associated transversality condition. Firms' profit maximization results in a labor demand equation and a price setting equation. Real marginal revenue φ_t is the multiplier on the output constraint. In equilibrium, it equals real marginal costs,

$$\varphi_t = \frac{W_t}{A_t}.$$

In equilibrium, $P_{it} = P_t$, by symmetry. The resulting price setting equation then determines inflation as a function of expected inflation and current real marginal costs as well as the effect on revenue arising from changes in the tax:

$$\psi\left(\pi_t - \pi\right)\pi_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \psi\left(\pi_{t+1} - \pi\right)\pi_{t+1} \frac{Y_{t+1}}{Y_t} + \epsilon\varphi_t - (\epsilon - 1)\frac{1}{1+\tau_t}.$$

In steady state, when $\pi_t - \pi = 0$ for all t, real marginal cost equal

$$\varphi = \frac{\epsilon - 1}{\epsilon} \frac{1}{1+\tau}.$$

Using the fact that $A = 1$ in steady state, so that $N = Y$, and the fact that $C = Y$ in equilibrium, this implies a condition on steady state output

$$\varphi = W = C^{\sigma} N^{\mu} = Y^{\sigma+\mu}.$$

Real marginal cost being equal to the real wage in steady state, labor supply determines aggregate output. A higher tax rate implies a lower real wage and thus lower labor supply. This is the distortionary effect of the value added tax.

2.2.1 Log-linearized version of the model

The first-order log-linearization of the model's equilibrium conditions delivers the core equations of the standard New Keynesian model. In the following, a variable with "hat" denotes its log-deviations from its log steady state value, i.e.

$$\hat{Z}_t = \log Z_t - \log Z.$$

The consumption Euler equation linearizes to an expectational IS curve that gives output as a function of expected output and the ex-ante real interest rate,

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \widehat{\pi}_{t+1}).$$

The price setting equation becomes the familiar New Keynesian Phillips curve with one new component, the change in the tax rate,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \left(\widehat{\varphi}_t + \frac{\tau}{1+\tau} \widehat{\tau}_t \right),$$

where we define $\kappa = \frac{1}{\psi} \frac{\epsilon - 1}{1+\tau}$. Real marginal cost evolve as

$$\widehat{\varphi}_t = \widehat{W}_t - \widehat{A}_t,$$

and labor supply is

$$\widehat{W}_t = \sigma \widehat{Y}_t + \mu \widehat{N}_t.$$

Note also that $\widehat{Y}_t = \widehat{C}_t$ since the price adjustment costs are zero to a first order. Using the linearized production function, $\widehat{Y}_t = \widehat{N}_t + \widehat{A}_t$, the latter condition can be written as $\widehat{W}_t = (\sigma + \mu)\widehat{Y}_t - \mu \widehat{A}_t$ which will be used below.

One can see from the Phillips curve that the response of inflation after a tax change depends on the behavior of real marginal costs. These in turn depend on real wages. It will turn out that without real wage rigidity, real wages immediately fall to reach the new allocation, and thus real marginal cost fall by exactly same amount the tax increases inflationary pressure. Hence, inflation stays constant.

Finally, the stochastic processes for deviations of technology and the tax rate from their respective steady states are specified as

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_t^A$$

and

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + \epsilon_t^\tau,$$

respectively and ϵ_t^A and ϵ_t^τ are white noise shocks.[5] Tax changes are usually considered as permanent in nature. However, following the literature we approximate them

[5] Other exogenous shocks, e.g. to labor supply or the discount factor could be introduced but are not central to the problem considered here.

as a highly persistent process, i.e. ρ_τ will be smaller than but close to unity. Note that this can be interpreted as agents having approximately a no-change perception with respect to next period's tax rate, since

$$E_t\left(\hat{\tau}_{t+1}\right) \approx \hat{\tau}_t.$$

An alternative strategy would consist of modeling the tax rate evolution as a random walk. This, however, would make it necessary to rewrite the system with reference to this stochastic trend.

2.3 The natural rate of output and the efficient level of output

The natural rate of output is the equilibrium level of output that obtains in the absence of nominal price rigidities.[6] It can be found by maximizing household utility subject to the budget constraint and firms maximizing per period real profits $\left(\frac{P_{it}}{P_t}\right) Y_{it}\frac{1}{1+\tau_t} - W_t N_t$ subject to the demand and production functions. In this case, the real wage is equal to the marginal product of labor, including taxes, or in linearized form, $\widehat{W}_t = \widehat{A}_t - \frac{\tau}{1+\tau}\widehat{\tau}_t$. Equalizing to the marginal rate of substitution of labor above yields an equation for the natural rate of output,

$$\widehat{Y}_t^n = \frac{1+\mu}{\sigma+\mu}\widehat{A}_t - \frac{1}{\sigma+\mu}\frac{\tau}{1+\tau}\widehat{\tau}_t.$$

Note that the natural rate is not a constant, or, in a model with technological progress, on a smooth trend. Instead, the natural rate of output can fluctuate strongly in response to real shocks, as the technology and tax shocks included here. Corresponding to the expectational IS equation, the real equilibrium features an Euler equation given by

$$\widehat{Y}_t^n = E_t\widehat{Y}_{t+1}^n - \frac{1}{\sigma}\widehat{\rho}_t$$

where ρ is the natural real rate of interest, or in Woodford's terminology, the Wicksellian interest rate. Movements in the natural rate can be expressed in terms of changes in technology and taxes,

$$\hat{\rho}_t = \frac{\sigma}{\mu+\sigma}\left[(1+\mu)(\rho_A - 1)\hat{A}_t - \frac{\tau}{1+\tau}(\rho_\tau - 1)\hat{\tau}_t\right].$$

This level of output however, is not the first-best, efficient level that the central bank should aim to be 'close' to, and which would maximize household welfare. That output level is given by the allocation that would obtain in the absence of any distortion, including the one arising from taxes. It is given by

$$\widehat{Y}_t^* = \frac{1+\mu}{\sigma+\mu}\widehat{A}_t$$

and the associated Euler equation is

$$\widehat{Y}_t^* = E_t\widehat{Y}_{t+1}^* - \frac{1}{\sigma}\widehat{\rho}_t^*,$$

where

$$\widehat{\rho}_t^* = \frac{\sigma(1+\mu)}{\sigma+\mu}(\rho^A - 1)\hat{A}_t$$

is the real rate of interest associated with the efficient equilibrium. One can see that only technology shocks move the efficient level of output as they change the economy's production possibility frontier. Tax changes have no effect, even though they

[6] This is the definition of Woodford (2003) and others.

change the allocation of resources. The optimal monetary policy under commitment will however not aim to reach the efficient level of output all the time, as might be expected. However, the optimal response of the central bank to disturbances will be affected by the distortionary taxes and the monopolistic distortion, as we elaborate below.

2.4 The system in terms of the welfare-relevant output gap

As the central bank will want to target the efficient level of output, it is useful to transform the system in terms of the deviation of actual output from the efficient level of output. In the literature, the employed output gap is often that between actual output and the natural rate of output. This is only valid when the distortions in the economy are small, or ideally, offset by a subsidy by the government that offsets this distortion. However, this is not possible in our analysis that explicitly considers an additional distortion, namely the value added tax.

The definition of real marginal costs and the labor supply equation allow a transformation of the Phillips curve. Using the results from above, real marginal cost can be written as

$$\widehat{\varphi}_t = (\sigma + \mu)\widehat{Y}_t - (1 + \mu)\widehat{A}_t.$$

Substituting into the Phillips curve yields

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa \left((\sigma + \mu)\widehat{Y}_t - (1 + \mu)\widehat{A}_t + \frac{\tau}{1 + \tau}\widehat{\tau}_t \right).$$

Now use the expression of first best output to substitute out $(1 + \mu)\widehat{A}_t$. Therefore, defining $\widetilde{x}_t = \widehat{Y}_t - \widehat{Y}_t^*$,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \kappa(\sigma + \mu)\widetilde{x}_t + \kappa \frac{\tau}{1 + \tau}\widehat{\tau}_t$$

This expression shows clearly the role of changes of the value added tax as cost push shocks. It arises from the presence of a wedge between the natural rate and the efficient rate of output. Finally, the IS curve can be re-expressed accordingly

$$\widetilde{x}_t = E_t \widetilde{x}_{t+1} - \frac{1}{\sigma}(i_t - E_t \widehat{\pi}_{t+1} - \widehat{\rho}_t^*)$$

where $\widehat{\rho}_t^*$ is given above.

For a given interest rate, a tax increase increases inflation, and through the Fisher equation, lowers the real interest rate. This would raise the output gap, and hence output. However, when the interest rate rises by more than inflation, the output gap falls.

2.5 Introducing real wage rigidity

We introduce real wage rigidity that prevents instantaneous real adjustment after a tax shock. The real wage is formed as a convex combination of the previous period's real wage and the 'notional' real wage which is given by the marginal rate of substitution,[7]

$$\hat{W}_t = \gamma \hat{W}_{t-1} + (1-\gamma)\ \widehat{mrs}_t$$

where $\gamma \in [0,1]$ is the adjustment parameter. With $\gamma = 0$ we obtain the economy in its basic form as outlined above. The derivation of the equations describing the economy follow the same lines as in the case without real wage rigidity. The main point to observe is the form of the Phillips curve that now features a backward-looking term and an endogenous forecast error. We have

$$\begin{aligned} \widehat{\pi}_t &= \frac{\beta}{1+\gamma\beta} E_t \widehat{\pi}_{t+1} + \frac{\gamma}{1+\gamma\beta}\widehat{\pi}_{t-1} + \frac{1-\gamma}{1+\gamma\beta}\kappa\,(\mu+\sigma)\tilde{x}_t \\ &+ \frac{1}{(1+\gamma\beta)}\kappa\left[\frac{\tau}{1-\tau}\left(\hat{\tau}_t - \gamma\hat{\tau}_{t-1}\right) - \gamma\left(\hat{A}_t - \hat{A}_{t-1}\right)\right] + \eta_t \end{aligned}$$

with $\eta_t = \hat{\pi}_t - E_{t-1}\hat{\pi}_t$. The IS equation has the same form as before, with a suitable modification of the real interest rate.

3 Utility-based welfare criterion

The model is not closed without an equation determining the nominal interest rate. One possibility is to specify an interest rate rule that yields the interest rate as a function of endogenous variables, such as the targets inflation and the output gap. Often this is assumed in an ad hoc manner, when the focus is mainly on characterizing the economy's dynamic response to shocks. However, our goal here is to understand the optimal response of monetary policy to a value added tax change. Therefore, we need to start from first principles, derive an welfare criterion the central bank is to maximize and then find the optimal choice of the instrument.[8] The goal of the derivation that follows is a discounted loss function of the form

$$\sum_{t=0}^{\infty} \beta^t\, E_0 \mathcal{L}_t = \Omega \sum_{t=0}^{\infty} \beta^t\, E_0 \left[(\tilde{x}_t - x^*)^2 + \lambda \widehat{\pi}_t^2\right].$$

that approximates the stream of the household's utilities.

The general strategy is to take a second order approximation of the household's objective function in terms of aggregate variables the central bank is concerned

[7] Expressed in levels, the prevailing real wage is thus a geometric average of the two components.
[8] See in particular Benigno and Woodford (2005) and Walsh (2005).

about, such as output and inflation. Then this quadratic approximation together with the linearly approximated economic system constitute a linear-quadratic problem that can be solved with standard techniques. A little bit of judgement goes into the choice of variables. The problem is that the quadratic cost of inflation drop out in a linear system, so one wants to substitute it into the function of which a second order approximation is taken. A quadratic approximation of the whole system would of course circumvent this issue, but at the cost of increased complexity and computational burden.

With the aggregate resource constraint, derived from the household budget constraint in equilibrium, welfare can be expressed in terms of output and inflation. The income identity and production function are

$$\begin{aligned} Y_t &= C_t + \frac{\psi}{2}\left(\pi_t - \pi\right)^2 Y_t \\ Y_t &= A_t N_t \end{aligned}$$

where the second term in the first equation shows the resources lost from adjusting prices, and A and N are technology and labor input respectively. Solving for C and N allows substitution in the period utility:

$$U_t = \frac{\left[Y_t\left(1 - \frac{\psi}{2}\left(\pi_t - \pi\right)^2\right)\right]^{1-\sigma} - 1}{1-\sigma} - \frac{\left[Y_t/A_t\right]^{1+\mu}}{1+\mu}.$$

This is the function of which a second-order approximation is to be obtained. It is expressed in two variables, output and inflation.

It is convenient to first take a second-order Taylor approximation of utility in terms of absolute deviations, and then transform into deviations in terms of logarithms.[9] That is, use

$$\widetilde{Z}_t = Z_t - Z \approx Z\left(\widehat{Z}_t + \frac{1}{2}\widehat{Z}_t^2\right)$$

with

$$\widehat{Z}_t = \log Z_t - \log Z.$$

The quadratic approximation of the first term is

$$\frac{\left[Y_t\left(1 - \frac{\psi}{2}\left(\pi_t - \pi\right)^2\right)\right]^{1-\sigma} - 1}{1-\sigma} \approx Y^{1-\sigma}\left\{\widehat{Y}_t + \frac{1}{2}(1-\sigma)\widehat{Y}_t^2 - \frac{\psi}{2}\pi^2\widehat{\pi}_t^2\right\},$$

while the approximation of the second term is

$$\frac{\left[Y_t/A_t\right]^{1+\mu}}{1+\mu} \approx Y^{1+\mu}A^{-(1+\mu)}\left\{\widehat{Y}_t + \frac{1}{2}(1+\mu)\widehat{Y}_t^2 - (1+\mu)\widehat{Y}_t\widehat{A}_t\right\},$$

[9]See Walsh (2003).

where all terms of order higher than 2 have been dropped as they are small.

In the standard, distortion free case – i.e., without taxes and without real wage rigidity – the marginal rate of substitution for labor equals $N^\mu C^\sigma$ and the marginal product of labor is A. Hence $N^\mu C^\sigma = A$. Inserting the production function, and the fact that in steady state, $Y = C$, yields $Y^{\sigma+\mu} = A^{1+\mu}$ or

$$\frac{\epsilon-1}{\epsilon}Y^{1-\sigma} = (1-\Phi)Y^{1-\sigma} = Y^{1+\mu}A^{-(1+\mu)},$$

where

$$\Phi = 1 - \frac{\epsilon - 1}{\epsilon}.$$

Thus, the objective function becomes

$$\begin{aligned} U &\approx Y^{1-\sigma}\left[\left\{\widehat{Y}_t + \frac{1}{2}(1-\sigma)\widehat{Y}_t^2 - \frac{\psi}{2}\pi^2\widehat{\pi}_t^2\right\}\right. \\ &\quad \left. -(1-\Phi)\left\{\widehat{Y}_t + \frac{1}{2}(1+\mu)\widehat{Y}_t^2 - (1+\mu)\widehat{Y}_t\widehat{A}_t\right\}\right] \\ &= -Y^{1-\sigma}\frac{1}{2}(\sigma+\mu)\left[\left\{\widehat{Y}_t^2 - 2\left[\frac{\Phi + (1+\mu)\widehat{A}_t}{\sigma+\mu}\right]\widehat{Y}_t + \frac{\psi\pi^2}{\sigma+\mu}\widehat{\pi}_t^2\right\}\right. \\ &\quad \left. -\Phi\frac{1+\mu}{\sigma+\mu}\widehat{Y}_t^2 + 2\Phi\frac{1+\mu}{\sigma+\mu}\widehat{Y}_t\widehat{A}_t\right] \end{aligned}$$

The next step in the derivation depends on how large the distortion Φ is. If the distortion is sufficiently small, second order terms $\Phi\widehat{Y}_t^2$ and $\Phi\widehat{Y}_t\widehat{A}_t$ are small enough to be ignored.[10] So one can write a loss function

$$\text{Loss} \approx Y^{1-\sigma}\frac{1}{2}(\sigma+\mu)\left\{\widehat{Y}_t^2 - 2\left[\frac{\Phi + (1+\mu)\widehat{A}_t}{\sigma+\mu}\right]\widehat{Y}_t + \frac{\psi\pi^2}{\sigma+\mu}\widehat{\pi}_t^2\right\}.$$

The final step is to rewrite this expression in terms of the output gap. First, note that the log deviation of the natural rate of output, when prices are flexible, as well as the log deviation of the efficient level of output, is given by

$$\widehat{Y}_t^n = \widehat{Y}_t^* = \frac{1+\mu}{\sigma+\mu}\widehat{A}_t$$

That is, while the two differ in the steady state, they comove proportionally about the steady state. Namely, for the steady state, we have $Y = (1-\Phi)^{1/(\sigma+\mu)}$ and $Y^* = 1$, so that $\log(Y^*/Y) \approx \Phi/(\sigma+\mu) \equiv x^*$. Then the term

$$\widehat{Y}_t^2 - 2\left[\frac{\Phi + (1+\mu)\widehat{A}_t}{\sigma+\mu}\right]\widehat{Y}_t$$

can be written as

$$\widehat{Y}_t^2 - 2\frac{\Phi}{\sigma+\mu}\widehat{Y}_t - 2\widehat{Y}_t^*\widehat{Y}_t$$

[10] With this assumption, we follow Woodford (2003), chapter 6, section 3.2, and Walsh (2003), chapter 11.

and is approximately equal to

$$\left(\widehat{Y}_t - \widehat{Y}_t^* - \frac{\Phi}{\sigma+\mu}\right)^2$$

where a host of interaction terms independent of monetary policy have been ignored. This yields the period loss function of the central bank as:

$$\mathcal{L}_t = \overline{Y}^{1-\sigma}\frac{1}{2}(\sigma+\mu)\left[(\tilde{x}_t - x^*)^2 + \frac{\psi\pi^2}{\sigma+\mu}\widehat{\pi}_t^2\right],$$

with $\tilde{x}_t = \widehat{Y}_t - \widehat{Y}_t^*$. That is, for the discounted expected loss function anticipated in the beginning we have $\Omega = \bar{Y}^{1-\sigma}\frac{1}{2}(\sigma+\mu)$ and $\lambda = \psi\sigma^2/(\sigma+\mu)$.

While the monopolistic distortion is constant at all times, the distortion arising from taxes is time-varying. As for the steady state distortion, one obtains

$$\Phi = 1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\tau},$$

which will still assumed to be small. The natural rate of output will now differ from the efficient rate. However, the expression for the period loss function can be written in terms of efficient output in exactly the same way as before. Since the efficient level of output is not affected by the presence of real wage rigidity, the same result applies to that scenario as well.

4 Monetary policy

We will consider two types of monetary policy behavior in our model. The first is a simple interest rule of the Taylor type which however uses the 'appropriate' output gap. Thus, the model is closed by an equation for the one-period interest rate of the form

$$i_t = \phi_\pi\hat{\pi}_t + \phi_x\tilde{x}_t.$$

The second is optimal behavior under a timeless perspective as outlined in Woodford (2003). The central bank minimizes the loss function derived above subject to the constraints resulting from linearized optimality conditions of households' and firms' behavior, i.e. the linearized IS and Phillips curve equation.

For the case without real wage rigidity one obtains the Lagrangian

$$\Omega\sum_{t=0}^{\infty}\beta^t E_0\left\{\begin{array}{c}\left[(\tilde{x}_t - x^*)^2 + \lambda\widehat{\pi}_t^2\right] + \theta_t\left[\widetilde{x}_t - \widetilde{x}_{t+1} + \frac{1}{\sigma}(i_t - \widehat{\pi}_{t+1} - \widehat{\rho}_t^*)\right]\\ +2\phi_t\left[\widehat{\pi}_t - \beta\widehat{\pi}_{t+1} - \kappa(\sigma+\mu)\widetilde{x}_t - \kappa\frac{\tau}{1+\tau}\widehat{\tau}_t\right]\end{array}\right\}$$

where the Phillips curve and expectational IS curve have been added as constraints. The first order condition with respect to the interest rate yields

$$\theta_t \frac{1}{\sigma} = 0$$

which implies that the first constraint is not binding.[11] The other two first order conditions are

$$E_0[\lambda\hat{\pi}_t + \phi_t - \phi_{t-1}] = 0$$
$$E_0[(\tilde{x}_t - x^*) - \phi_t \kappa(\sigma + \mu)] = 0$$

for all t. From that one obtains

$$\widehat{\pi}_t = -\frac{1}{\kappa(\sigma + \mu)\lambda}(\tilde{x}_t - \tilde{x}_{t-1})$$

for all t. Thus, the optimal commitment solution is necessarily backward looking. The latter relationship, added to the IS and Phillips curve equations, leads to a full specification of the joint behavior of output, inflation and the interest rate. However, using the latter equation to eliminate inflation from the Phillips curve, one can also obtain an expectational difference equation for the optimal path of $\tilde{x}_t$. This in turn has a solution of the form

$$\tilde{x}_t = a_x \tilde{x}_{t-1} + b_x \hat{\tau}_t$$

with the coefficients being functions of the structural parameters. This finally allows to express inflation in terms of the lagged output gap and the tax shock. It should also be noted that optimal policy behavior from a timeless perspective can alternatively be expressed in terms of an interest rate reaction function. This would make the interest rate a function of the natural real rate of interest, expected inflation, the expected output gap, the lagged output gap, and the tax shock.

For the case with real wage rigidity, we have the same objective function of the central bank as shown above. However, since the Phillips curve has a different form in that case, the constraint in the Lagrangian changes. Knowing already that the IS equation need not be included as a constraint, we have for the Lagrangian,

$$\Omega \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \begin{array}{c} \left[(x_t - x^*)^2 + \lambda \widehat{\pi}_t^2\right] \\ +2\phi_t \left[\widehat{\pi}_t - \frac{\beta}{1+\gamma\beta} E_t \widehat{\pi}_{t+1} - \frac{\gamma}{1+\gamma\beta}\widehat{\pi}_{t-1} - \frac{1-\gamma}{1+\gamma\beta}\kappa(\mu + \sigma)\tilde{x}_t - H_t - \eta_t\right] \end{array} \right\},$$

where H_t collects terms with technology and tax shocks that will be irrelevant for the first order conditions.

The first order conditions now become

$$E_0\left[\lambda\hat{\pi}_t + \phi_t - \frac{1}{1+\gamma\beta}\phi_{t-1} - \beta\frac{\gamma}{1+\gamma\beta}\phi_{t+1}\right] = 0$$

[11] See Walsh (2003)

and

$$E_0 \left[(\tilde{x}_t - x^*) - \phi_t \frac{1}{1+\gamma\beta} \kappa(\sigma + \mu) \right] = 0$$

which implies that in this case, the optimal path of inflation chosen by the central bank satisfies

$$\pi_t = -\frac{1}{\kappa(\sigma+\mu)(1-\gamma)\lambda} \left[(1+\gamma\beta)\tilde{x}_t - \tilde{x}_{t-1} - \beta\gamma E_t \tilde{x}_{t+1} \right].$$

We thus obtain the interesting result that the backward- and forward-looking nature of the 'hybrid' Phillips curve arising from real wage rigidity is inherited by the expression for inflation in terms of the output gap. This is an implication of the optimal monetary policy under a timeless perspective.

5 Calibration and Results

We numerically solve for the rational expectations equilibrium of the linearized model along the lines of Sims (2002). For the solution, parameter values need to be assigned. We set most of them in line with the literature. The parameter ψ in the price adjustment cost function is chosen such that the parameter κ in the Phillips curve has about the same value as in models based on the Calvo pricing assumption. The shock ϵ_t^τ will be chosen such that it corresponds to an increase of the value-added tax rate from 16 to 19 percentage points. Time is measured in quarters. The structural parameter values are thus as follows

$$\begin{aligned} \beta &= 0.98; \quad \sigma = 2; \quad \mu = 20; \quad \epsilon = 11; \quad \psi = 80; \\ \gamma &= 0.9; \quad \tau = 0.16; \quad \rho_A = 0.9; \quad \rho_\tau = 1. \end{aligned}$$

For the simple interest rate reaction function we set $\phi_x = 0.5$ and $\phi_\pi = 1.5$.

By its very nature, the tax increase considered here, should be understood as being permanent. However, strictly speaking, the model being log-linearized around a non-stochastic steady state only allows for non-permanent – albeit possibly highly persistent – shocks. Setting the corresponding parameter ρ_τ in the tax process to a high value close to one, say 0.99, yields qualitatively the same results as setting it to 1.0.[12] Thus, we stick to the parameterization that corresponds to a permanent tax change.

The top panels of the following graphs document the reaction of inflation, actual output, and the one-period interest rate. The bottom panels show the output

[12]In particular we do not have 'discontinuities' when moving from the highly persistent to the scenario with a truly permanent tax change.

gap with respect to the efficient level of output (which is relevant for monetary policy), the output gap with respect to the natural rate of output (which is useful to understand the driving forces of inflation), and the natural rate of output itself.[13]

First consider Figure 1. It shows the case of a completely flexible labor market, that is, in the absence of real wage rigidity. The economy adjusts immediately to the new long-run level of output when the tax change occurs.[14] The reason is that the fall in the marginal revenue product of labor instantly translates into real wages. In turn, households reduce their labor supply, which, from the production function, leads to a fall in output. The new level of output is lower because of the increased tax distortion. The efficient output gap falls (i.e., becomes more negative) along with actual output. It does so by exactly that amount that compensates the tax shock in the Phillips curve, thereby leaving inflation unaffected. There is response of monetary policy.

[13]Note that we abuse notation here by denoting the welfare gap with x^*, not $\widetilde{x}$. This is not to be confused with the steady state deviation.

[14]Again, when writing expressions such as 'the new long-run level', we interpret highly persistent changes as if they were permanent.

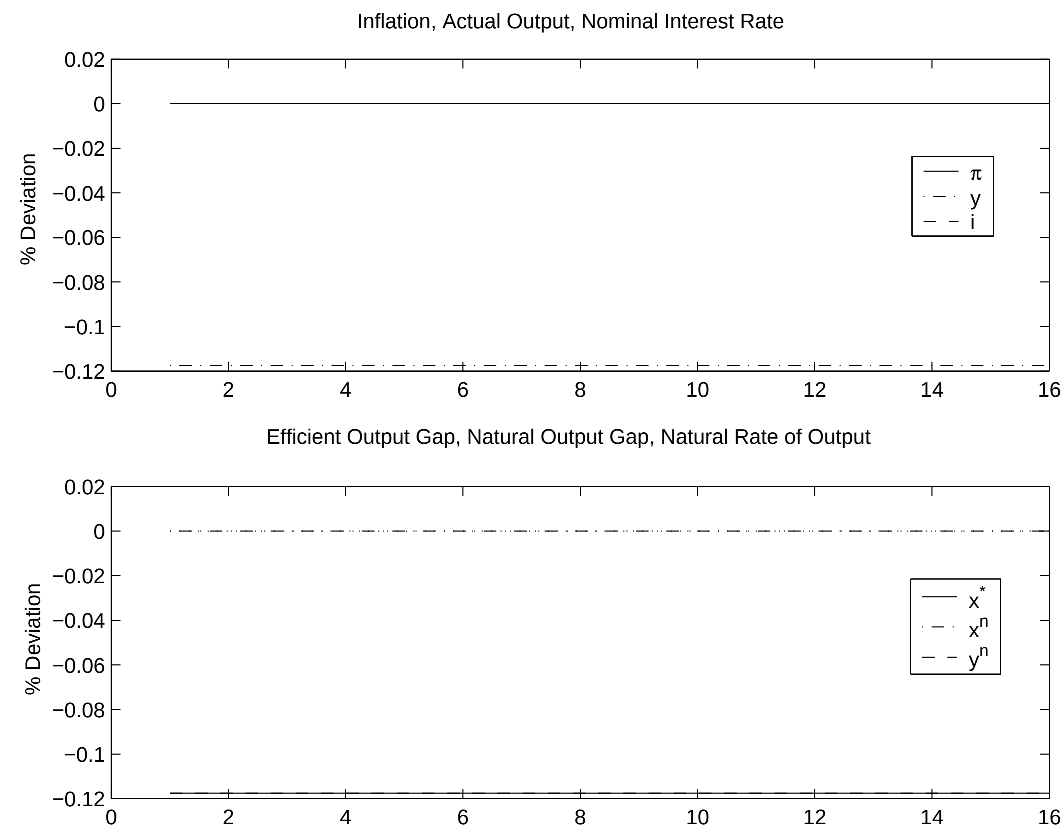

Figure 1: Effect of a tax change – with monetary policy following a simple rule; no real wage rigidity

These effects differ strongly in the presence of real wage rigidity, as shown in Figure 2. Inflation rises, and the output gap falls. The intuition is that the downward adjustment of the real wage is sluggish, so that, for firms, even though labor costs do fall, they rise relative to marginal revenue. This cost push induces firms to increase their prices, resulting in a rise in inflation. As real wages fall, labor supply is reduced. However, there is an additional effect due to the fall in labor demand after the tax rise. This further decreases the real wage. In fact, output – and with it labor input – need to fall by enough so that the real wage equals the new marginal revenue. With the assumed equation for the real wage adjustment, this can only happen with a strong fall in the marginal rate of substitution of labor. This, in turn, is only possible if output falls by enough. Hence the drop in output beyond the new long-run level. Overall, the tax shock in this case generates positive inflation for approximately two years.

The response of monetary policy is an increase in interest rates. The adjustment of the nominal rate follows the Taylor principle: it rises by more than inflation to bring about a rise in the real interest rate. Otherwise, inflation would rise by even more, and the model may not have a determinate equilibrium. Finally, a look at the lower panel further illustrates the driving forces of the adjustment. The natural rate of output y^n falls significantly and by more than actual output. This results in the rising output gap x^n, which causes inflation to rise. However, from the perspective of monetary policy, the welfare relevant output gap is that between actual output and the efficient level of output. This gap is much less volatile. Therefore, rather than pushing up the interest rate strongly to close the natural output gap x^n, the central bank has an incentive to lower interest rates, to give some stimulus to actual output. This effect of course is offset by the interest rate response to rising inflation.

The previous cases may well be a realistic description of central bank behavior (given the empirical success of simple Taylor rules). But it is not necessarily optimal from a welfare-theoretic point of view. As discussed earlier, monetary policy should be chosen optimally based on a welfare criterion that is based on the parameters that describe the structure of the model. Furthermore, this optimal choice of policy should be based on a commitment that the central will behave in a particular way.

Figure 3 shows the behavior of the economy in response to the tax increase under the central bank's commitment to the inflation-targeting criterion derived earlier. Remember that the central bank adjusts interest rates such that current inflation is inversely related to the change in the welfare relevant output gap, and positively related to expected changes in that output gap. We consider the case of real wage rigidity only, as the results for flexible wages are essentially the same as with the simple interest rate rule without real wage rigidity.

In contrast to the interest rule, the optimal monetary policy leads to an ad-

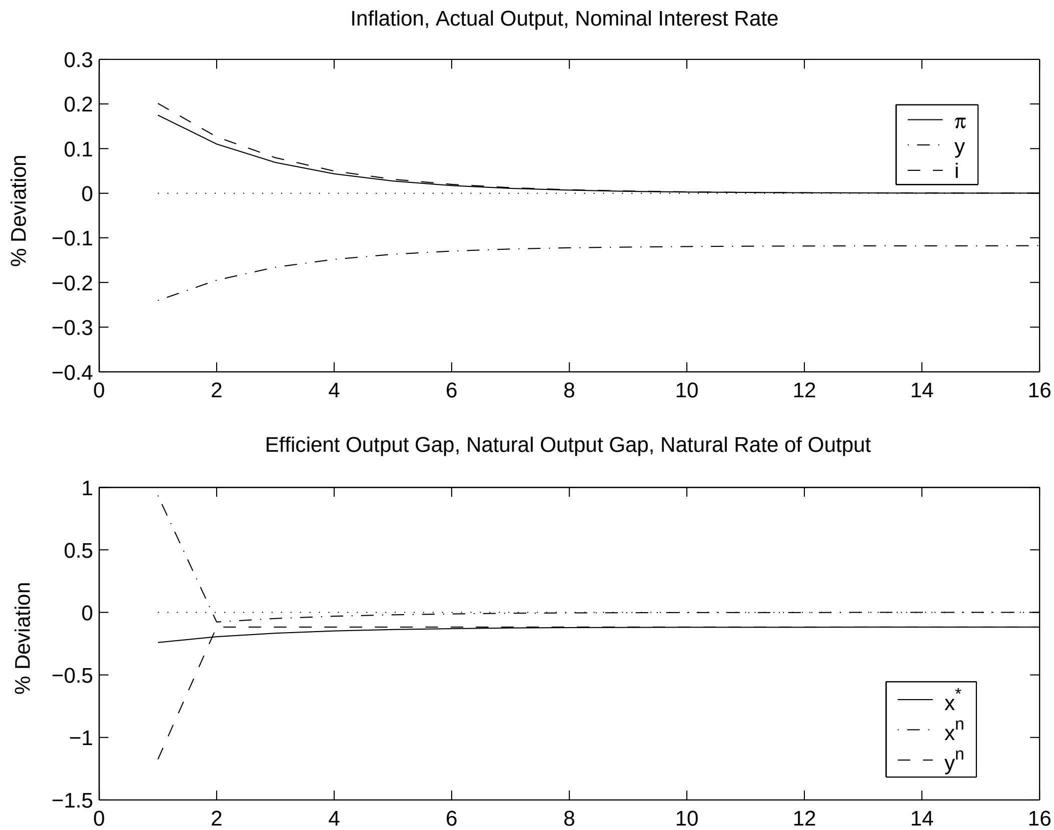

Figure 2: Effect of a tax change – with monetary policy following a simple rule; with real wage rigidity

justment of inflation that reverts much faster back to steady state after about four quarters. In fact, inflation even becomes negative for some time after that, reversing some of the price level increase that followed the higher inflation. This result is close to the finding of Woodford (2003) and others[15], that optimal monetary policy under commitment effectively involves price level targeting, rather than inflation targeting. This serves to most effectively anchor inflation expectations, which, via the New Keynesian Phillips curve, determine current inflation. Thus, the reduction in inflation can be brought about by a much lower increase in interest rates than before.

At the same time, the level of output appears to have its strongest decline in the second quarter, rather than the first, but finds its way up to the new, lower,

[15]See also Gali (2003) and Clarida, Gali, and Gertler (1999).

steady state after about 6 quarters. Even though barely noticeable, output is even slightly above the long-run level. Interestingly, the initial response of the interest rate is slightly negative rather than positive. This effect arises from the nature of the targeting rule.

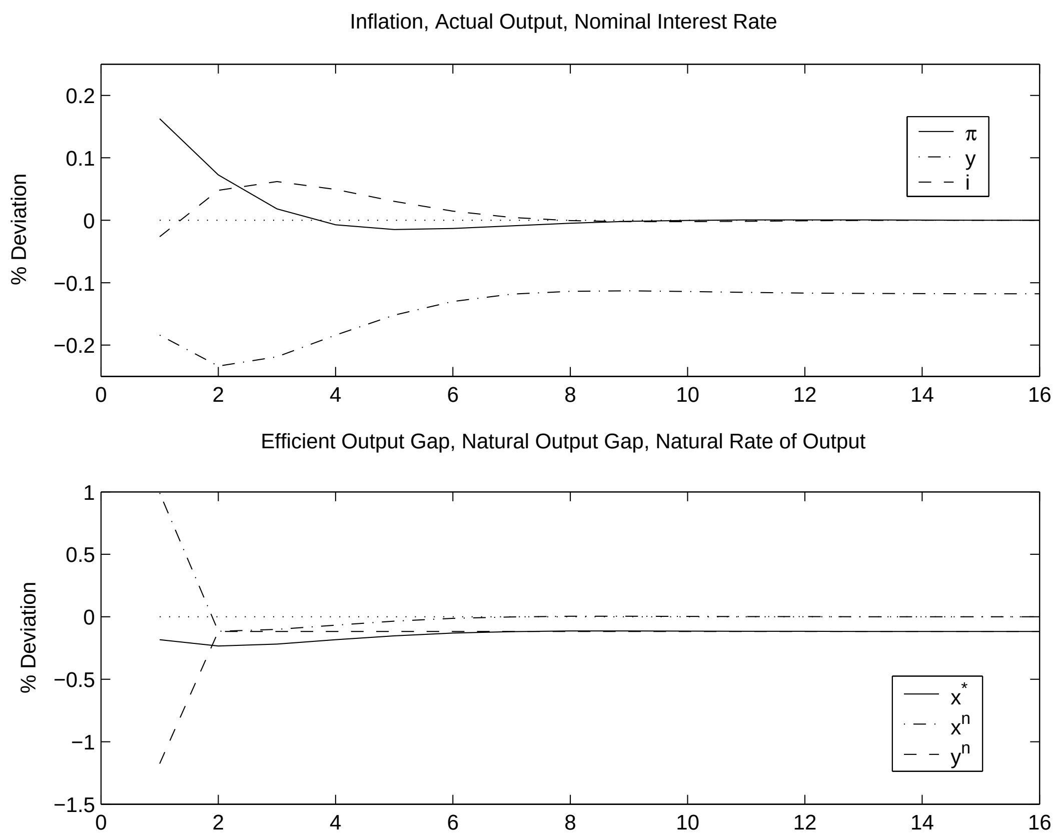

Figure 3: Effect of a tax change – with optimal monetary policy; with real wage rigidity

6 Conclusion

The optimal monetary policy response to a value added tax change is derived and contrasted with the behavior under a simple Taylor-type interest rate rule. The welfare-maximizing central bank commits to an inflation rule that helps manage expectations and thus allows a containment of inflation with lower cost. We have derived this result in a standard New Keynesian monetary model with distortionary taxes, which – in conjunction with real wage rigidity – generates a cost push shock term in the Phillips curve. Furthermore, it induces inflation inertia. This approach differs from the literature which assumes inflation inertia exogenously, and introduces cost push shocks in an ad hoc manner. While this may be suitable for characterizing the trade-off that monetary policy faces in stabilizing inflation and output, a stronger microfoundation as considered here allows a meaningful welfare analysis.

The analysis helps to understand key macroeconomic effects of changes in distortionary taxes. It also highlights the central role of real wage rigidity for the adjustment of the economy. If wages setters would accept the inevitability of the long-run change in output that arises from the tax increase, the adjustment would take place at even lower cost. In our analysis, we have held other aspects of fiscal policy constant, which may offset the short run effects of the tax change. We leave this to future work.

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