Exchange Rate Regimes, Determinacy, and Learnability in a Two-Block World Economy

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Abstract

We study how determinacy and learnability of global rational expectations equilibrium may be affected by monetary policy in a simple, two country, New Keynesian framework. The two blocks may be viewed as the U.S. and Europe, or as regions within the euro zone. We study cases in which optimal policies are being pursued country by country - where exhange rates are floating - as well as types of managed exchange rate regimes, including monetary union. This case is of strong practical relevance for monetary policy analysis in the euro area. We find that new concerns can arise in the analysis of classic topics such as the desirability of exchange rate targeting and monetary conditions indices (MCIs), the so-called ´corner solutions´ debate, and monetary union. *Keywords*: Monetary strategy, sunspot equilibrium, learning, monetary policy rules, new open economy macroeconomics, exchange rate regimes, monetary union, second generation policy coordination. JEL codes: E52.

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1 Introduction[1]

1.1 Overview

[UPDATE] New Keynesian macroeconomic models have become a workhorse for studying a variety of monetary policy issues in closed economy environments. An important component of this study has been the development of the idea that equilibrium determinacy and learnability may be significantly influenced by the monetary policy choices.[2] Recently, simple extensions of the New Keynesian model to open economy environments have been developed. Our primary concern in this paper is to provide an analysis of the extent to which the findings concerning determinacy and learnability for the closed economy New Keynesian framework may be altered when open economy considerations are brought to bear. Our learnability criterion is that of Evans and Honkapohja (2001).

Our approach to this question is to adopt a simple framework for a two-country world due to Clarida, Gali, and Gertler (2002). This framework provides one straightforward extension of the New Keynesian model to two countries and allows comparison to the more common single country and open economy analyses as special cases. We are able to make some progress analytically in showing how determinacy and learnability conditions depend on the actions of policymakers in each country, the conditions under which one policymaker can or cannot mitigate the threats of indeterminacy and learnability, and the degree to which international policy coordination may fail or succeed in delivering determinacy and learnability of worldwide equilibrium.

THINK ABOUT CASE WHERE WE HAVE COMPLETELY FLEXIBLE PRICES, DO WE THEN HAVE DETERMINACY? (SUGGESTED BY AZARIADES), SEND HIM FIRST DRAFT FOR COMMENTS

[1] The authors are grateful for helpful comments by Peter Sinclair, Bianca De Paoli, Jens Sondergaard, Jarkko Jaaskela, Maria Demertzis, Peter Vlaar and seminar participants at the Bank of England and De Nederlandsche Bank.

[2] See, for instance, Woodford (2003), Bullard and Mitra (2002), Evans and Honkapohja (2003a,b), and Preston (2003).

HELPFUL TO THINK ABOUT SOE IN ANALYSIS OF EXCHANGE RATE PEG, IS USEFUL SUB-CASE AS THEN $p^*_{C,t} = p^*_{F,t}$.

1.2 Recent related literature

[UPDATE, COPY/SHORTEN TEXT FROM BS (2005)]The seminal work on the New Keynesian framework and the role of monetary policy is Woodford (2003). We have chosen to follow the extension to a two-country environment proposed by Clarida, Gali, and Gertler (2002).

Monacelli (2002) OP FLASH DRIVE

Batini, Levine, and Pearlman (2004) study indeterminacy in a two-country New Keynesian model. Their focus is on the relationship between many-period forward-looking inflation targeting rules and indeterminacy conditions.

De Fiore and Liu (forthcoming, JEDC) study indeterminacy in a small open New Keynesian economy.

Bencivenga, Huybens, and Smith (2001) study the relationship between dollarization and endogenous volatility (i.e., indeterminacy). Their model emphasizes financial intermediation and the degree of capital market integration.

Pappa (2004) studies optimal monetary policy in a two country New Keynesian model and concludes that there may be little gain from policy coordination. She does not focus on the determinacy and learnability issues emphasized here.

Bullard and Schaling (2005) study how determinacy and learnability of global rational expectations equilibrium may be affected by monetary policy in a simple, two country, New Keynesian framework. The two blocks may be viewed as the U.S. and Europe, or as regions within the euro zone. We seek to understand how monetary policy choices may interact across borders to help or hinder the creation of a unique rational expectations equilibrium worldwide which can be learned by market participants. They study cases in which optimal policies are being pursued country by country as well as some forms of cooperation. They find that open economy considerations may alter conditions for determinacy and learnability relative to closed economy analyses, and that new concerns can arise in the analysis of classic topics such as the desirability of exchange rate targeting and monetary policy cooperation.

De Paoli (2005) OP FLASHDRIVE

Sondergaard (200x) OP FLASHDRIVE

Llosa and Tueste (2005), [UPDATE WITH OCTOBER 2005 VERSION] building on Bullard and Mitra (2002) evaluate the expectational stability (E-stability) of rational expectations equilibrium in a simple New Keynesian small open economy model. In particular, they extend Bullard and Mitra (2002) results of E-stability for a closed economy to an open economy framework by evaluating Taylor-type rules. The main results are the following: a) the stability conditions under learning in open economies are isomorphic to the closed economy counterpart when the central bank targets contemporaneously either domestic price inflation or CPI inflation; b) the problem of instability under learning becomes more serious in open economies when the central bank reacts actively to expected consumer price inflation (CPI). Thus, unlike Bullard and Mitra (2002) the Taylor principle does not necessarily induce both determinate and learnable rational expectation equilibria, where the degree of openness plays a crucial role c) it is easier for the economy to get into an unstable region when a central bank reacts to expected movements in the exchange rate along with an actively reaction to expected CPI inflation.

[INTEGRATE LITERATURE BY JENSEN ON DETERMINACY AND OPTIMALITY]

1.3 Organization

In this part of the project we want to extend the Bullard and Schaling (2005) paper (hereafter BS) in several ways.

First, they only look at the polar cases of completely fixed and completely flexible exchange rates. In this project we will analyze the intermediate case of ‘dirty floating’, that is the case where the government attaches some weight to the exchange rate - in addition to targeting inflation. So, this case could

Flexible exchange rates (and instrument rules) , all regimes **symmetric**	**Fixed** exchange rates (and targeting rules)	**Managed** exchange rates (and instrument rules) , all regimes **asymmetric**
3.1 Simple Taylor-type rules (PPI-based Taylor rules or CDI rules)	4 Symmetric: monetary union	6.1 The LT CEX rule (in line with LT´s SOE)
3.2 Instrument rules with international variables (CPI-based Taylor rules or CCPI rules)	5 Asymmetric: an exchange rate peg	6.2 The LT FEX rule (in line with LT´s SOE)
3.3 IFB Taylor rules (FDI rules)		6.3 Corner solutions and MCI´s
3.4 Inflation forecast based rules with international variables (FCPI rules)		6.4 A floating but managed exchange rate (targeting rule is special case of a ´generalized´ instrument rule)

be seen as flexible exchange rate targeting.

We plan to do this – among other things – via extending BS with inflation forecast based rules (IFB rules). We expect to find that Llosa and Tuesta's (2005) inflation forecast based rules with international variables (what they call FCPI rules) and managed exchange rate rules (what they call FEX and CEX rules) can be analyzed as special cases of BS – either as part of the small open economy (SOE) framework (that is implicit in BS) or the two-country set-up.

Further, we plan to extend the BS paper further by analyzing a new symmetric fixed exchange rate regime, namely the one corresponding with a monetary union. This case is expected to be of strong practical relevance for monetary policy analysis in the euro area.

[Detailed organization to be provided here.] Plaatje is WORD tabel, die is vervolgens binnen WORD gekopieerd en geplakt (paste special) als picture (niet enhanved meta file), dit plaatje is vervolgens ´gewoon´ (via windows clipboard) ingeplakt in SWP.

Beweging in het paper is van symmetrische naar asymmetrische regimes, parallel lopend aan een verhaallijn die begint met flexibele koersen en eindigt met managed floating. Er tussenin zit de case van vaste wisselkoersen, waarbij wij een symmetrisch (EMU) en een asymmetrisch (peg) probleem analyseren. Met betrekking tot de dimensie instrument vs targeting rules is de beweging van instrument via targeting rules terug naar de (generalized) instrument rule.

2 A two-country New Keynesian model

2.1 Overview

We employ the two-country model of Clarida, Gali, and Gertler (2002). This model is one natural extension of the closed economy New Keynesian model to the open economy case in which two large economies are interacting, and so it provides a good starting point for the analysis of determinacy issues in the open economy.[3] Our purpose is not to develop new aspects of this model *per se*, but to use the model to try to understand some of the main considerations that might arise concerning determinacy and learnability in the world economy, when determinacy can be importantly influenced by policymakers. Accordingly, we merely present the main equations of the Clarida, Gali, and Gertler (2002) framework here. We provide most of the detail of the derivation of these equations in the Appendix A, in order to communicate to the interested reader the nature of the assumptions underlying the analysis.

2.2 Environment

The model economy is log-linearized about a steady state, as derived in Appendix A, and is given by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma_0^{-1} \left[r_t - E_t \pi_{t+1} - \overline{rr}_t \right], \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_0 \tilde{y}_t + u_t \tag{2}$$

where $\kappa_0 = \gamma(\sigma - 1)$, $\sigma_0 = \sigma - \kappa_0$, $\kappa = \sigma - \kappa_0 + \phi$, $\lambda_0 = \delta\kappa$, and $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. The variable $\tilde{y}_t$ represents the domestic output gap, π_t

[3] There is a great deal of related work [mention related work here.] We think it would be interesting to analyze determinacy and learnability issues in these related frameworks.

represents domestic producer price inflation, and r_t represents the the (log of) the short term nominal interest rate. Here $r\bar{r}_t$ is the domestic natural real interest rate (conditional on foreign output), given by

$$r\bar{r}_t = \sigma_0 E_t \Delta \bar{y}_{t+1} + \kappa_0 E_t \Delta y^*_{t+1} \tag{3}$$

The stochastic term u_t follows an $AR(1)$ process given by

$$u_t = \rho u_{t-1} + \epsilon_t \tag{4}$$

The symbol E_t is the standard expectations operator.[4] The equations (1) and (2) have five fundamental parameters: The household discount factor β, a parameter controlling the curvature in preferences over consumption σ, a parameter controlling the curvature in preferences over leisure ϕ, the mass of agents or degree of openness γ, and the probability that a firm will be able to change its price today θ, which we sometimes refer to as the degree of price stickiness. The foreign economy is described analogously as

$$\tilde{y}^\star_t = E^\star_t \tilde{y}^\star_{t+1} - \sigma_0^{\star -1} \left[r^\star_t - E^\star_t \pi^\star_{t+1} - \overline{rr}^\star_t \right], \tag{5}$$

$$\pi^\star_t = \beta E^\star_t \pi^\star_{t+1} + \lambda^\star_0 \tilde{y}^\star_t + u^\star_t \tag{6}$$

where $\kappa^\star_0 = (1-\gamma)(\sigma - 1)$, $\sigma^\star_0 = \sigma - \kappa^\star_0$, $\kappa^\star = \sigma - \kappa^\star_0 + \phi$, $\lambda^\star_0 = \delta \kappa^\star$, and $\delta = [(1-\theta)(1-\beta\theta)]/\theta$.

In these equations $\tilde{y}^\star_t$ is the foreign output gap, $\pi^\star_t$ is foreign producer price inflation, and $r^\star_t$ is foreign nominal interest rate. The operator $E^\star_t$ indicates rational expectations of the participants in the foreign economy, in what follows to simplify notation we use E_t instead of $E^\star_t$, but resort back to $E^\star_t$when necessary (e.g. in case of learning). Here $r\bar{r}^*_t$ is the foreign natural real interest rate (conditional on domestic output), given by

$$r\bar{r}^*_t = \sigma^*_0 E^*_t \Delta \bar{y}^*_{t+1} + \kappa^*_0 E^*_t \Delta y_{t+1} \tag{7}$$

The stochastic term $u^\star_t$ is assumed to follow an $AR(1)$ process given by and

$$u^\star_t = \rho u^\star_{t-1} + \epsilon^\star_t \tag{8}$$

[4]Later in the paper we will modify this and the related operator $E^\star$ for the analysis of learning.

with $|\rho| < 1$. In equations (5) and (6), the fundamental parameters β, σ, ϕ, γ, and θ are all the same, in keeping with the fact that the preferences and technologies in the two economies are the same. The only difference is that γ in (1) and (2) has been replaced by $1-\gamma$ in (5) and (6).

The *nominal* exchange rate e_t obeys CPI-based, or 'aggregate' PPP (see Appendix A for more details), and is therefore given by

$$e_t = (p_{C,t} - p^*_{C,t}) = (p_t + \gamma s_t) - (p^*_t - \{(1-\gamma)s_t\}) = p_t - p^*_t + s_t \qquad (9)$$

where p_t is shorthand for $p_{H,t}$ being the domestic producer price level, and p^*_t is shorthand for the foreign PPI $p^*_{F,t}$. $p_{C,t}$ and $p^*_{C,t}$ stand for the home and foreign CPI, respectively.

Finally, we obtain a simple expression linking the terms of trade to movements in the output gap:

$$s_t = (\tilde{y}_t - \tilde{y}^*_t) + (\bar{y}_t - \bar{y}^*_t) = (\tilde{y}_t - \tilde{y}^*_t) + \bar{s}_t \qquad (10)$$

where $\bar{s}_t$ is the natural level of the terms of trade.

The open economy effects in this model come through the composite parameters κ_0 and $\kappa^\star_0$. In the special case where period utility is logarithmic in consumption, so that $\sigma = 1$, the effects vanish as $\kappa_0 = \kappa^\star_0 = 0$. In this special case, each economy evolves as if it were an isolated, closed economy. In addition, the special cases where either $\gamma \to 0$ or $\gamma \to 1$ respectively place all the mass of agents in the home or the foreign economy. In these cases, the home or foreign economy behaves as if it were an isolated, closed economy, while the partner behaves as if it were a small open economy.[5]

The two-country model involves the short term nominal interest rates r_t and $r^\star_t$. In the remainder of the paper we will analyze the model under different scenarios for how these interest rates are determined by policymakers. We will begin with a simple specification that produces simple intuition, and

[5]Clarida, Gali and Gertler (2001) and Gali and Monacelli (2002) analyze the case of a small open economy (SOE) using a similar framework to the one employed here. The special case of a SOE can be inferred from our model by setting the share of home goods in the foreign consumption basket equal to zero. Using the notation of this paper, we would set $(1-\gamma^*) = 0$ in $c^*_t = (1-\gamma^*)c^*_{H,t} + \gamma^* c^*_{F,t}$.

later move to more complicated optimal policy specifications under a variety of assumptions on the nature of the optimization policymakers undertake.

3 Flexible exchange rates

3.1 Simple Taylor-type rules (PPI-based Taylor rules or CDI rules)

3.1.1 The dynamic system

As in Bullard and Schaling (2005) in this section we simply assume that the policymakers in each country follow Taylor-type policy rules given by

$$r_t = \varphi_\pi \pi_t + \varphi_y \tilde{y}_t \tag{11}$$

for the domestic economy, and by

$$r_t^\star = \varphi_\pi^\star \pi_t^\star + \varphi_y^\star \tilde{y}_t^\star \tag{12}$$

for the foreign economy.

[MENTION THAT HERE WE FOCUS ON RULES BASED ON CONTEMPORANEOUS DATA , AND THAT THIS CASE CORRESPONDS WITH WHAT LT (2005) CALL THE DOMESTIC INFLATION (CDI) RULE.]

By substituting (11) and (12) into equations (1) and (5), we can write the four equation system as follows. First, define $\mathcal{Z}_t = [\tilde{y}_t, \pi_t, \tilde{y}_t^\star, \pi_t^\star]'$ along with $\mathcal{V}_t = [\overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star]'$. Then write the system as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B} E_t \mathcal{Z}_{t+1} + \mathcal{X} \mathcal{V}_t \tag{13}$$

where $\mathcal{A}_0 = 0$,[6]

$$\mathcal{B} = \begin{bmatrix} B_{11} & \mathbf{0} \\ \mathbf{0} & B_{22} \end{bmatrix},$$

$$B_{11} = \frac{1}{\sigma_o + \varphi_y + \lambda_o \varphi_\pi} \begin{bmatrix} \sigma_o & 1 - \beta\varphi_\pi \\ \lambda_o \sigma_o & \lambda_o + \beta\left(\sigma_o + \varphi_y\right) \end{bmatrix},$$

[6] This is consistent with Bullard and Mitra (2002) as we allow for constant terms.

$$B_{22} = \frac{1}{\sigma_o^{\star} + \varphi_y^{\star} + \lambda_o^{\star}\varphi_\pi^{\star}} \begin{bmatrix} \sigma_o^{\star} & 1 - \beta\varphi_\pi^{\star} \\ \lambda_o^{\star}\sigma_o^{\star} & \lambda_o^{\star} + \beta\left(\sigma_o^{\star} + \varphi_y^{\star}\right) \end{bmatrix},$$

where $\mathbf{0}$ is a 2×2 matrix of zeroes, and

$$\mathcal{X} = \begin{bmatrix} X_{11} & \mathbf{0} \\ \mathbf{0} & X_{22} \end{bmatrix},$$

with

$$X_{11} = \begin{bmatrix} \sigma_o^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$X_{22} = \begin{bmatrix} \sigma_o^{\star,-1} & 0 \\ 0 & 1 \end{bmatrix},$$

and where $\mathcal{V}_t$ follows a vector $AR(1)$ process with serial correlation given by the scalar ρ.

3.1.2 Determinacy

Because the four variables in this system are free in the terminology of Blanchard and Kahn (1980), we require all eigenvalues of $\mathcal{B}$ to be inside the unit circle for determinacy. Since $\mathcal{B}$ is block diagonal, this requirement means that the eigenvalues of B_{11} and B_{22} must be inside the unit circle. From a version of Proposition 1 in Bullard and Mitra (2002), this implies that the following two conditions must hold for determinacy in this system:

$$\lambda_o(\varphi_\pi - 1) + (1 - \beta)\varphi_y > 0 \tag{14}$$

and

$$\lambda_o^{\star}(\varphi_\pi^{\star} - 1) + (1 - \beta)\varphi_y^{\star} > 0. \tag{15}$$

These conditions are versions of the Taylor principle for each country and depend on the household discount factor β, which is assumed to be the same in the two countries, on the policy parameters in the Taylor-type rules in the two countries, and on the composite parameters λ_o and $\lambda_o^{\star}$. We can write the composite parameters as

$$\begin{aligned} \lambda_o &= \delta\left[\sigma + \phi - \gamma(\sigma - 1)\right], \\ \lambda_o^{\star} &= \delta\left[\sigma + \phi - (1 - \gamma)(\sigma - 1)\right]. \end{aligned}$$

Thus the conditions (14) and (15) can be written as

$$\delta\left[\sigma+\phi-\gamma\left(\sigma-1\right)\right]\left(\varphi_{\pi}-1\right)+\left(1-\beta\right)\varphi_{y}>0 \tag{16}$$

and

$$\delta\left[\sigma+\phi-\left(1-\gamma\right)\left(\sigma-1\right)\right]\left(\varphi_{\pi}^{\star}-1\right)+\left(1-\beta\right)\varphi_{y}^{\star}>0. \tag{17}$$

The term in brackets is positive, so that if $\varphi_y = \varphi_y^\star = 0$, the conditions state that each central bank has to move nominal interest rates more than one-for-one in response to deviations of inflation from target. We have several remarks on conditions (16) and (17).

First, the conditions are not the same except in the special case where policies are identical (in the sense that $\varphi_\pi = \varphi_\pi^\star$ and $\varphi_y = \varphi_y^\star$) and $\gamma = 1/2$, which would be interpreted as the case that the two economies are equally open.[7] Otherwise, the degree of openness differs and this translates into a difference in the two conditions. This means in particular that identical policy in the two countries, in the sense of identical values for the Taylor-type policy rule coefficients, may be enough to meet one determinacy condition but not the other.

Second, the policy parameters from a single country can only influence one of the two conditions. Thus policymakers from each country must separately meet conditions for determinacy: Determinacy conditions are met, in some sense, "country by country."[8] If one country fails, then worldwide equilibrium is indeterminate, and one country cannot "make up for" the failure of a second country to meet appropriate conditions.

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[7] If $\gamma = \frac{1}{2}$, there is no home bias in consumption. More generally, for $\gamma < \frac{1}{2}$, both country H and country F consumers will demand relatively more H goods than F goods, while for $\gamma > \frac{1}{2}$, both sets of consumers will demand relatively more F goods than H goods. Likewise, $1-\gamma$ corresponds to the share of country F consumption allocated to goods imported from country H, and so is a measure of openness for that country. This has the important implication that as γ increases country H becomes more open and F moves towards more autarky ($1-\gamma$ declines).

[8] This is due to the combination of flexible exchange rates and purchasing power partity which keeps countries perfectly insulated from foreign instabilities transmitted via the terms of trade. With a fixed exchange rate this will no longer be the case.

3.1.3 Learnability

We now turn to the learnability of rational expectations equilibrium for cases where that equilibrium is unique. We allow the expectations in equation (13) to initially be different from rational expectations.[9] The MSV solution of equation (13) is given by

$$\mathcal{Z}_t = \bar{\mathcal{A}} + \bar{\mathcal{C}}\mathcal{V}_t$$

where the conformable matrix $\bar{\mathcal{A}}$ is null and

$$\bar{\mathcal{C}} = (I - \rho\mathcal{B})^{-1}\mathcal{X}.$$

We endow agents with a *perceived law of motion*

$$\mathcal{Z}_t = \mathcal{A} + \mathcal{C}\mathcal{V}_t \tag{18}$$

where $\mathcal{A}$ and $\mathcal{C}$ are conformable. Using this perceived law of motion and assuming time t information $(1, \overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star)'$ we can calculate

$$E_t\mathcal{Z}_{t+1} = \mathcal{A} + \mathcal{C}\rho\mathcal{V}_t.$$

Substituting this into equation (13) yields the *actual law of motion*

$$\begin{aligned} \mathcal{Z}_t &= \mathcal{B}(\mathcal{A} + \mathcal{C}\rho\mathcal{V}_t) + \mathcal{X}\mathcal{V}_t \\ &= \mathcal{B}\mathcal{A} + (\mathcal{B}\mathcal{C}\rho + \mathcal{X})\mathcal{V}_t. \end{aligned}$$

We then define a map T from the perceived law of motion to the actual law of motion as

$$T(\mathcal{A}, \mathcal{C}) = (\mathcal{B}\mathcal{A}, \mathcal{B}\mathcal{C}\rho + \mathcal{X}).$$

Expectational stability is attained if the differential equation

$$\frac{d}{d\tau}(\mathcal{A}, \mathcal{C}) = T(\mathcal{A}, \mathcal{C}) - (\mathcal{A}, \mathcal{C})$$

[9] Preston (2003) considers deriving the fundamental equations of models in this class assuming agents are learning. Under his interpretation of the microfoundations, the equations are altered and long-horizon forecasts matter. We think it would be interesting to carry out an analysis of this type for the open economy case.

is locally asymptotically stable at $\left(\bar{\mathcal{A}}, \bar{\mathcal{C}}\right)$. Results in Evans and Honkapohja (2001) establish that under weak conditions, expectational stability governs stability in the real-time learning dynamics.

We use Proposition 10.3 in Evans and Honkapohja (2001) to calculate the condition for expectational stability. According to the proposition, the condition for expectational stability is that the real parts of the eigenvalues of the matrices $\mathcal{B}$ and $\rho\mathcal{B}$ are less than unity. Because $0 \leq \rho < 1$, this means that the we need only check the real parts of the eigenvalues of $\mathcal{B}$. Also, because of the block diagonality of $\mathcal{B}$, the expectational stability condition can be calculated country by country, that is, via B_{11} and B_{22}, and by a version of Proposition 2 in Bullard and Mitra (2002) yields conditions (16) and (17). This means that both countries must meet the open economy version of the Taylor principle in order for the world equilibrium to be learnable. It also means that the conditions for determinacy are the same as the conditions for learnability in the special case where both countries follow simple Taylor-type instrument rules. This is known not to be true in general in models in this class with alternative instrument rules, but it provides a good benchmark.[10]

In this calculation we have proceeded as if all actors in the world economy possessed the entire information set $\left(1, \overline{rr}_t, u_t, \overline{rr}_t^{\star}, u_t^{\star}\right)'$. This is a natural assumption in a domestic economy setting, but it may not be as natural in a multi-country setting. It means that all actors in all countries are keeping track of all state variables worldwide. Instead, one might assume that domestic residents have the information set $\left(1, \overline{rr}_t, u_t\right)'$ and that foreign residents have the information set $\left(1, \overline{rr}_t^{\star}, u_t^{\star}\right)'$. One would then postulate perceived laws of motion for each set of agents as

$$\begin{aligned} Z_{11,t} &= A_{11} + C_{11} V_{11,t}, \\ Z_{22,t} &= A_{22} + C_{22} V_{22,t}, \end{aligned}$$

where $Z_{11} = \left[\tilde{y}_t, \pi_t\right]'$, $Z_{22} = \left[\tilde{y}_t^{\star}, \pi_t^{\star}\right]'$, $V_{11} = \left[\overline{rr}_t, u_t\right]'$, and $V_{22} = \left[\overline{rr}_t^{\star}, u_t^{\star}\right]'$

[10] An example of a case in which determinacy and learnability conditions do not coincide is when the policy authorities use a Taylor-type policy rule but react to lagged information on inflation and the output gap. See Bullard and Mitra (2002). For a wider variety of Taylor-type instrument rules in a similar model, see Llosa and Tuesta (2005).

and where A_{11}, A_{22}, C_{11}, and C_{22} are conformable matrices. Because of the block diagonality in the system, proceeding in this way will yield the same conditions for expectational stability. However, this may not be the case for other systems, as we discuss below.

3.2 Instrument rules with international variables (CPI-based Taylor rules or CCPI rules)

HERE WE TALK ABOUT THE FACT THAT BS (2005) ANALYZE INSTRUMENT RULES WITH INTERNATIONAL VARIABLES NAMELY TAYLOR RULES FEATURING CPI INFLATION INSTEAD OF DOMESTIC INFLATION. LT (2005) CALL SUCH RULES CONSUMER INFLATION TAYLOR RULES OR CCPI RULES.

3.3 IFB Taylor rules (FDI rules)

Resultaten in deze paragraaf zijn een speciaal geval van die uit de paragraaf over IFB rules met international variables (FCPI rules). Namelijk als in deze laatste par $\varphi_s = \varphi_s^* = 0$ dan reduceren de FCPI rules tot IFB Taylor rules.

3.4 Inflation forecast based rules with international variables (FCPI rules)

[TEXT BELOW IS BASED ON SECTION 1 OF COMMENTS NOTE]

Perhaps we can check whether the Llosa-Tuesta result that the problem of instability under learning becomes more serious in open economies when the central bank reacts actively to expected consumer price inflation (CPI) carries over to the Bullard Schaling (2005) environment. Then, in the section on instrument rules with international variables, we should investigate the properties of what Llosa and Tuesta call (FCPI) rules:

(13') =LT FCPI rule

(16') =LT FCPI rule

Rules (13') and (16') above are examples of 'inflation-forecast-based' rules (see Batini et al.) which are of significant interest because similar reaction functions are used for the Quarterly Projection Model of the Bank of Canada. Rule (13') corresponds with rule (34) from Llosa and Tuesta (2005, p. 11).

The first thing to note is that using the definition of CPI inflation, rule (13') can be rewritten as

(13')

so in the context of Bullard and Schaling (2005) [= CGG (2002)] the FCPI rule is basically what LT call a domestic inflation Taylor (FDI) rule, augmented with an expected terms of trade term . Using the solution for the terms of trade

where

the rule can be written as

(13')

or

(13') =

LT FCPI rule

where . Note that rule (13') looks very similar to rule (19) in Bullard-Schaling. The only difference is that instead of contemporaneous data we have forward expectations on the right-hand-side.

For the foreign country we get

(16') =LT FCPI rule

Now both central banks can be viewed as responding not only to expected domestic economic events, but also to expected foreign conditions in the form of foreign output gaps.

I suggest we investigate determinacy and learnability of the dynamic system conditional on (13') and (16') in order to see whether our results are consistent with TL. There again we can investigate special cases, such as (i) one economy is large and (ii) both economies have positive mass.

p. 2 What is exactly the difference between determinacy and real determinacy?

On whether or not policy rules guarantee determinacy, can you please add the Bullard-Schaling (2005) paper to the references on two-country environments?

p. 3 I guess that LT's analytical result that the Taylor principle is a necessary and sufficient condition for uniqueness of equilibrium under a CPI target – which they claim is a novel finding for determinacy in open economies – for contemporaneous data rules is similar to the results of Bullard and Schaling (2005) in their section 3 (on instrument rules, there the rules are also in the form of contemporaneous data rules). In BS the Taylor principle is an open economy Taylor principle (parameters adjusted for openness). Is this also the case in LT? If the principle is unchanged from the closed economy –in spite of targeting the CPI – what would be the intuition for that?

p. 3 I guess LT's result that rules that respond to future forecasts of consumer inflation might not be desirable is similar to what we know about the (in) determinacy ensuring properties of IFB rules along the lines of e.g. Batini et al.

p. 3 LT say that 'The striking result is that unlike BM (2002), forward-looking policy rules that react aggressively to CPI inflation with little or no reaction to the output gap, do not necessarily induce both determinate and learnable REE'. But, BM (2002) is a closed economy model, therefore then there should be no difference between PPI and CPI inflation. Is there something I misunderstand here?

4 Monetary Union

In this section we analyze the case of a monetary union. That is, we assume the two countries have joined a monetary union, so there is a fixed exchange rate ($e_t = 0$) and monetary policy is set in one of the member states (either home or foreign) for the common monetary area (say the euro zone) as a whole. We analyze this case by assuming that both countries have symmetric economic structures (i.e. all structural parameters are identical and the countries are of equal size). This is useful, as it then allows us to exploit a method introduced by Aoki (1981). This procedure enables the dynamics of the system to be decoupled into (1) averages and (2) differences of the relevant variables. Not only does this render the analysis tractable, but it also helps provide insight into the analysis.

4.1 Averages system

More specific, the analysis can be simplified by defining the averages and differences for any variable X, say according to

$$X^A \equiv \frac{1}{2}(X + X^*) \tag{19}$$

$$X^D \equiv X - X^* \tag{20}$$

In this section we analyze the averages system, the differences system is discussed subsequently.

Suppose interest rates are set by a single monetary authority for the entire two-country region. For ease of exposition we assume that $\gamma = 0.5$.

In this section we assume that the central bank sets policy optimally. This means that the nominal interest rate is inferred from an explicit optimization exercise.[11] We investigate the benchmark case of discretion[12] and consider a wide range of implementation strategies of the first-order condition along the lines of Evans and Honkapohja (2003) (hereafter EH) that may or may not provide determinacy of rational expectations equilibrium.

Then, the monetary authority minimizes the welfare loss for the union in terms of average inflation and the average output gap, or

$$\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{2} [\left(\pi_\tau^A - \pi^{AT}\right)^2 + \alpha_0 \left(\tilde{y}_\tau^A - \tilde{y}^{AT}\right)]^2 \tag{21}$$

with $\Lambda \equiv \frac{\xi}{\delta}$and $\alpha_0 \equiv \frac{\delta\kappa}{\xi} = \frac{\lambda_0}{\xi}$. The minimization is subject to

$$\tilde{y}_t^A = \hat{E}\tilde{y}_{t+1}^A - \sigma_0^{-1}(r_t^A - E_t\pi_{t+1}^A - r\bar{r}_t^A) \tag{22}$$

[11]For a recent discussion about target and instrument rules see Svensson (2003), and the response to Svensson by McCallum and Nelson (2004).

[12]For a discussion of determinacy issues for optimal rules in a closed economy where the timing protocol is commitment, see Giannoni and Woodford (2002a, b).

$$\pi_t^A = \beta \hat{E}_t \pi_{t+1}^A + \lambda_0 \tilde{y}_t^A + u_t^A \tag{23}$$

$$\overline{rr}_t^A = \sigma_0 \hat{E}_t \Delta \bar{y}_{t+1}^A + \kappa_0 \hat{E}_t \Delta y_{t+1}^A \tag{24}$$

$$u_t^A = \rho u_{t-1}^A + \epsilon_t^A \tag{25}$$

As said we assume that all structural parameters in the home and foreign country are the same. More specific, we assume that $\delta = \delta^*, \beta = \beta^*$, $\sigma = \sigma^*$, which together with $\gamma = 0.5$ implies that $\sigma_0 = \sigma_0^*$,$\kappa_0 = \kappa_0^*$ and $\lambda_0 = \lambda_0^*$. Notice, that the optimization problem faced by the common monetary authority is isomorphic to the one faced by the home and foreign central banks in the Section on the non-cooperative equilibrium. If like there we assume that the common central bank lacks a commitment technology then we can derive the expectations based optimal rule for the common monetary area as

$$r_t \ - \overline{rr}_t^A = \delta_{00}^A + \delta_{\pi 0} \hat{E}_t \pi_{t+1}^A + \delta_{y0} \hat{E}_t \tilde{y}_{t+1}^A + \delta_{u0} u_t^A \tag{26}$$

where the coefficients are

$$\delta_{00}^A = -\frac{\sigma_0 \left(\lambda_0 \pi^{AT} + \alpha_0 \tilde{y}^{AT} \right)}{\alpha_0 + \lambda_0^2}, \ \delta_{\pi 0} = \frac{\alpha_0 + \lambda_0^2 + \sigma_0 \lambda_0 \beta}{\alpha_0 + \lambda_0^2}, \ \delta_{y0} = \sigma_0,$$

$$\delta_{u0} = \frac{\lambda_0 \sigma_0}{\alpha_0 + \lambda_0^2}. \tag{27}$$

Determinacy of the aggregate union-wide equilibrium depends on the usual determinacy conditions. That is, by substituting the equilibrium interest rate (26) into the aggregate IS curve (22), and combining the resulting expression with the aggregate Phillips curve (23) we get the system

$$z_t^A = c + B^A z_{t+1}^{Ae} + e^A u_t^A, \tag{28}$$

where $z_t^A = \left[\tilde{y}_t^A, \pi_t^A \right]'$, $e^A = [-\sigma_o^{-1} \delta_{00}^A, -\sigma_o^{-1} \delta_{00}^A \lambda_0]'$, $c = \left[-\sigma_0^{-1} \delta_{u0}, 1 - \lambda_0 \sigma_0^{-1} \delta_{u0} \right]'$, and

$$B^A = \begin{bmatrix} 0 & -\sigma_0^{-1} \left(\delta_{\pi 0} - 1 \right) \\ 0 & \beta - \lambda_0 \sigma_0^{-1} \left(\delta_{\pi 0} - 1 \right) \end{bmatrix}. \tag{29}$$

We see that for the aggregate system - for the monetary union as a whole - a unique rational expectations equilibrium exists since $0 < \beta - \lambda_0 \sigma_0^{-1} \left(\delta_{\pi 0} - 1 \right) \Longleftrightarrow \frac{\beta \alpha_0}{\alpha_0 + \lambda_0^2} < 1$.

4.2 Differences system

Let's now look at the implications of the common monetary policy for economic performance in member states. A covenient way to do this is to define the corresponding differences system. If we do this then we get the following system:

$$\tilde{y}_t^D = \hat{E}\tilde{y}_{t+1}^D - \sigma_0^{-1}(r_t^D - E_t\pi_{t+1}^D - r\bar{r}_t^D), \tag{30}$$

$$\pi_t^D = \beta\hat{E}_t\pi_{t+1}^D + \lambda_0\tilde{y}_t^D + u_t^D, \tag{31}$$

$$u_t^D = \rho u_{t-1}^D + \epsilon_t^D, \tag{32}$$

$$\overline{rr}_t^D = \sigma_0\hat{E}_t\Delta\bar{y}_{t+1}^D - \kappa_0\hat{E}_t\Delta y_{t+1}^D, \tag{33}$$

$$r_t^D - E_t\pi_{t+1}^D = E_t\Delta e_{t+1} - E_t\pi_{t+1}^D \tag{34}$$

$$s_t = \tilde{y}_t^D + \bar{y}_t^D = \tilde{y}_t^D + \bar{s}_t. \tag{35}$$

Under a fixed nominal exchange rate our UIP equation (34) (in real terms) becomes $r_t^D - E_t\pi_{t+1}^D = -E_t\pi_{t+1}^D$, substituting this into the differenced IS curve (30) and combining the resulting expression with the Phillips curve for the inflation differential (31), we can write down the system as

$$\tilde{y}_t^D = \hat{E}\tilde{y}_{t+1}^D - \sigma_0^{-1}(-E_t\pi_{t+1}^D - r\bar{r}_t^D) \tag{36}$$

$$\pi_t^D = (\beta + \lambda_0\sigma_0^{-1})\hat{E}_t\pi_{t+1}^D + \lambda_0\hat{E}\tilde{y}_{t+1}^D + \lambda_0\sigma_0^{-1}r\bar{r}_t^D + u_t^D \tag{37}$$

However, so far we have not used the fact that via the solution for the terms of trade (35) we have $p_t^* - p_t = \tilde{y}_t^D + \bar{s}_t$. Normalizing the time $t-1$ home and foreign price levels at zero, we can write $\pi_t^* - \pi_t = \tilde{y}_t^D + \bar{s}_t$, or $\pi_t^D = -\tilde{y}_t^D - \bar{s}_{t.}$Leading this expression and taking expectations at time t, we get

$$E_t\pi_{t+1}^D = -E_t\tilde{y}_{t+1}^D - E_t\bar{s}_{t+1.} \tag{38}$$

After combining (38) with the expressions for the inflation and output gap differentials, we can write the differences system in state-space notation as $z_t^D = B^D z_{t+1}^{De}$, where $z_t^D = \left[\tilde{y}_t^D, \pi_t^D\right]$, the constants and shock-terms are omitted and

$$B^D = \begin{bmatrix} 1 - \sigma_0^{-1} & 0 \\ 0 & \beta + \lambda_0(\sigma_0^{-1} - 1) \end{bmatrix}. \tag{39}$$

It is easy to see that the following two conditions must hold for determinacy in this system:

$$\sigma_0^{-1} > 0 \tag{40}$$

and

$$\sigma_0^{-1} < \frac{\lambda_0 + (1-\beta)}{\lambda_0} \tag{41}$$

If $\gamma = 0.5$, as is the case here (we have complete symmetry), the first condition boils down to $\frac{2}{1+\sigma} > 0$, which is satisfied no matter what value σtakes. The second condition can be rewritten as $\beta < 1 - \lambda_0(\frac{1-\sigma}{1+\sigma})$, which is also satisfied for sensible parameter values. [MAAR IS DIT RESULTAAT NIET VOLLEDIG HET GEVOLG VAN GAMMA = 0.5 ?ZODAT DAN HET DIFFERENCES SYSTEEM NIET MEER ECHT RELEVANT IS, REAL WORLD EXAMPLE IS DAT ER NU RUZIE IS TUSSEN ECB EN NCB´S OVER WIE NATIONAL TERMS OF TRADES (RERS/COMPETITIVENESS) MAG UITREKENEN]In our two-country model, both the averages and differences systems must be determinate for the world economy (here the common monetary area, or monetary union) to have a unique equilibrium.[13] It follows that the monetary union is characterized by determinacy. This result can be summarized in the form of

Proposition 1 *For all parameter values the REE that corresponds to the optimal monetary policy for EMU under discretion implemented via the expectations based rule is determinate.*

4.3 Learnability of monetary union

JBB Please do learnability of expectations based optimal rule (26). This is standard Evans Honkapohja (2003) and can be copied from there.

5 An exchange rate peg

[13]See also Batini , Levine and Pearlman (2004) on this point.

5.1 Overview

In this section we suppose the home country targets its nominal exchange rate e *vis-a-vis* the foreign country. We assume the foreign economy sets its monetary policy based on its own domestic considerations. The home country gives up its domestic monetary autonomy in return for "importing monetary stability" from the foreign, anchor country.

This is a leading example of an *asymmetric* exchange rate regime, as only the anchor country's variables matter for its interest rate (depending on the nature of the policy adopted there), and the home country simply sets its interest rate to ensure it realizes a fixed exchange rate. The home country in setting policy takes foreign monetary conditions into account, but the foreign country need not incorporate the home country's conditions in its own monetary policy stance. This arrangement is similar to the regimes adopted by some European countries prior to economic and monetary union and to the present peg of the Chinese renminbi to the U.S. dollar.In this section we focus on the case where each policymaker sets monetary policy autonomously [UPDATE AS US COULD INDEED OPTIMIZE INSTEAD OF INSTRUMENT RULE, THEN WE HAVE NASH, IS THIS SENSIBLE EMPIRICALLY?], that is, here we study the non-cooperative case. [UPDATE think about Stackelberg protocols in context of (dirty) pegging]. Clarida, Gali and Gertler (2002) also study cooperation in the context of their New Keynesian model and are thus part of what Canzoneri, Cumby and Diba (CCD) (2004) call second generation models of policy coordination. CCD state that the gains from coordination are larger in second generation models than in first generation models. For a survey of the lessons from the first generation literature see Nolan and Schaling (1996).

5.2 The policy problem

The home country minimizes

$$(1-\gamma)\,\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{2} [\left(e_\tau - e^T\right)]^2 \tag{42}$$

The minimization is subject to

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma_0^{-1}\left[r_t - E_t\pi_{t+1} - \overline{rr}_t\right] \tag{43}$$

$$\pi_t = \beta E_t\pi_{t+1} + \lambda_0\tilde{y}_t + u_t \tag{44}$$

$$\bar{rr}_t = \sigma_0\hat{E}_t\Delta\bar{y}_{t+1} + \kappa_0\hat{E}_t\Delta y^*_{t+1} \tag{45}$$

$$u_t = \rho u_{t-1} + \epsilon_t \tag{46}$$

$$e_t = e_{t-1} + s_t - s_{t-1} + \pi_t - \pi^*_t \tag{47}$$

$$s_t = (\tilde{y}_t - \tilde{y}^*_t) + \bar{s}_t \tag{48}$$

For ease of exposition we normalize the initial levels of the nominal exchange rate and terms of trade at zero ($e_{t-1} = s_{t-1} = 0$), so that

$$e_t = s_t + \pi_t - \pi^*_t \tag{49}$$

In what follows we normalize the exchange rate target at zero ($e^T = 0$).[14] It can be easily seen from (42) that the first-order condition then becomes $e_t = 0$, which combined with (49) implies

$$s_t = -(\pi_t - \pi^*_t) \tag{50}$$

The intuition behind (50) is the following. The nominal exchange rate obeys CPI-based PPP and - after appropriate normalization - is given by $e_t = \pi_t - \pi^*_t + s_t$. Now, in order to prevent fluctuations in e_t , the home central bank should manipulate the trems of trade s_t - which it can affect via the domestic output gap - in such a way as to offset the GDP deflator based inflation differential, so we have (50).

Since the terms of trade can be affected by the domestic output gap - which in turn is affected by the home nominal interest rate - the CB should try to achieve a level of the home output gap given by

$$\tilde{y}_t = -(\pi_t - \pi^*_t) + \tilde{y}^*_t - \bar{s}_t \tag{51}$$

Equation (51) is obtained by substituting the expression for the terms of trade into the first-order condition and rearranging.

[14]This leaves open the question of how to deal with revaluations under a pegged regime.

5.3 The policy rule

We know that home GDP inflation, π_t, is governed by the Phillips curve (44). So, after substituting (44) into (51), we get home's optimal monetary policy rule in terms of its indirect control $\tilde{y}_t$:

$$\tilde{y}_t = -\frac{\beta}{1+\lambda_0} E_t \pi_{t+1} + \frac{1}{1+\lambda_0}(\pi_t^* + \tilde{y}_t^*) - \frac{1}{1+\lambda_0}(\bar{s}_t + u_t) \tag{52}$$

Of course, the optimal level of the output gap above -required to keep the terms of trade at a level where it exactly offsets the GDP deflator based international inflation differential and thus keeps the nominal exchange rate fixed - is in turn achieved by moving the domestic nominal interest rate (the direct control) appropriately. The home interest rate reaction function can be obtained by combining the result (52) with the IS curve (43):

$$r_t - \bar{rr}_t = \delta_0^{''} + \delta_{\pi 0}^{'} \hat{E}_t \pi_{t+1} + \delta_{y0} \hat{E}_t \tilde{y}_{t+1} + \delta_{\pi *} \pi_t^* + \delta_{y*} \tilde{y}_t^* + \delta_{u0}^{'} u_t \tag{53}$$

where the coefficients are

$$\delta_0^{''} = \frac{\sigma_0}{1+\lambda_0}\bar{s}_t, \;\; \delta_{\pi 0}^{'} = \frac{(1+\lambda_0)+\sigma_0\beta}{1+\lambda_0}, \delta_{y0} = \sigma_0,$$

$$\delta_{\pi *} = \delta_{y*} = -\delta_{u0}^{'} = -\frac{\sigma_0}{1+\lambda_0}. \tag{54}$$

The rule above describes the optimal home monetary reaction function that implements its monetary policy of pegging the exchange rate to the foreign anchor country. Note that this has a similar form as the expectations based optimal rule (**??**) under flexible inflation targeting (assuming discretion) as suggested by Evans and Honkapohja (2003). NOTE THAT THERE ARE OTHER WAYS TO IMPLEMENT THE FOC $e_t = 0$ THAT MAY IN FACT RESULT IN INDETERMINACY, WHO? **ES**

5.4 The dynamic system, determinacy, and learnability

Whether or not a fixed exchange rate ensures determinacy of the home country's rational expectations equilibrium - and for that matter whether we also have determinacy in the foreign economy - depends on how the anchor (foreign) country implements monetary policy, and on the international spill-over effects on the home country.

The system can be solved in the usual way. We start by substituting home's policy rule (53) into the home IS curve (43) and combining the resulting expression with home's Phillips curve (44). Then, we get the following expressions for home output and inflation:[15]

$$\tilde{y}_t = -\sigma_0^{-1}\delta_0^{''} - \sigma_0^{-1}(\delta_{\pi 0}^{'} - 1)E_t\pi_{t+1} - \sigma_0^{-1}\delta_{u0}^{'}u_t - \sigma_0^{-1}\delta_{\pi *}(\pi_t^* + \tilde{y}_t^*) \qquad (55)$$

$$\pi_t = [\beta - \lambda_0\sigma_0^{-1}(\delta_{\pi 0}^{'} - 1)]E_t\pi_{t+1} - \lambda_0\sigma_0^{-1}\delta_0^{''} + (1 - \lambda_0\sigma_0^{-1}\delta_{u0}^{'})u_t - \lambda_0\sigma_0^{-1}\delta_{\pi *}(\pi_t^* + \tilde{y}_t^*) \qquad (56)$$

Here the dependence of home's economic outcomes on the foreign macroeconomy is evident from the presence of the terms $\pi_t^*$and $\tilde{y}_t^*$ in the home semi-reduced form IS and Phillips curves.

Whether or not a fixed exchange rate regime is compatible with determinacy of worldwide rational expectations equilibrium depends on how the foreign, anchor country implements monetary policy, and on any international spillover effects on the home country. We make the assumption that the foreign, anchor country is inward-looking, and concerned only about reacting to developments in its own economy. We proceed with the most straightforward assumption, namely that the foreign inflation country follows a simple Taylor-type policy rule. This allows us to easily study cases where the foreign, anchor monetary authorities are pursuing policies either consistent or inconsistent with determinacy and learnability of worldwide rational expectations equilibrium.

The world economy can again be written in standard form. The matrix $\mathcal{B}$ is given by

$$\mathcal{B} = \begin{bmatrix} B_{11} & B_{12} \\ \mathbf{0} & B_{22} \end{bmatrix}$$

[15] Where we have used that $\delta_{\pi *} = \delta_{y*}$.

where B_{22} is the matrix associated with a simple Taylor rule in use in the foreign country. The eigenvalues there will depend on whether the foreign country is following the open economy version of the Taylor principle or not, as discussed earlier in the paper. The eigenvalues of B_{11} will also have to be less than unity for determinacy. This matrix is given by

$$B_{11} = \begin{bmatrix} 0 & \sigma_o^{-1}\left(1-\delta'_{\pi,0}\right) \\ 0 & \beta+\sigma_o^{-1}\lambda_o\left(1-\delta'_{\pi,0}\right) \end{bmatrix}.$$

The eigenvalues are zero and

$$v = \frac{\beta}{1+\lambda_o} < 1.$$

We conclude that determinacy holds under maintained assumptions provided the foreign, anchor monetary authorities are following the Taylor principle. Learnability holds under the same conditions.[16]

One may be able to imagine scenarios under which this result would break down, if the foreign, anchor economy had some other policy. But this result suggests there need not be anything intrinsically unstable in the use of an exchange rate peg.

6 Managed exchange rates

6.1 The LT CEX rule

[HERE WE FOLLOW COMMENTS NOTE SECTION 3]

p. 3 LT point out that Lubik and Schorfheide (2003) have found that Canada does include the nominal exchange rate in its policy rule. Note that Bullard and Schaling (2005) have pointed out that if central banks follow PPP rules (either of the Uribe (2003) or Zanna (2004) types) in the CGG (2002) model this is equivalent to strict targeting of the nominal exchange

[16] Since $\mathcal{B}$ is upper triangular, we have (see Golub and van Loan (1996, p. 311) $\lambda(\mathcal{B}) = \lambda(B_{11}) \cup \lambda(B_{22})$. This means that we have determinacy of the world economy if the eigenvalues of $B_{11}$and B_{22} lie inside the unit circle; otherwise the equilbrium will be indeterminate.

rate, i.e. this is equivalent to a nominal exchange rate peg. Suppose CB's have a partial interest in following such a rule, e.g. the home CB follows the generalized Uribe instrument rule

(24')

or the corresponding Zanna rule

(27')

We can now use different values for the feedback parameter , we restrict its value such that . The case where corresponds with the simple Taylor rule analyzed in section 3 of the Bullard Schaling paper. In the limiting case where the rules collapse to the strict Uribe and Zanna rules respectively.

Under conditions were the nominal exchange rate already obeys PPP - which is the case in CGG (2002) and Bullard and Schaling (2005) - the generalized instrument rules above can be written as the managed exchange rate rule

(24")

where the parameter captures the endogenous response of the central bank to level of the nominal exchange rate.

If we normalize the initial ( ) level of the nominal exchange rate at zero then this rule can be written as

(24"')

and then has a similar form as Llosa and Tuesta's managed exchange rate rule (CEX rule). However, here – unlike in Llosa and Tueste (2005) – the central bank responds to PPI inflation, not CPI inflation. Then the parameter captures the endogenous response of the central bank to changes in the nominal exchange rate.

6.2 The LT FEX rule

[HERE WE FOLLOW COMMENTS NOTE SECTION 2]

p. 3 LT find that forward looking managed exchange rate (FEX) rules can easily induce E-unstable equilibria. I think that result would carry over to the Bullard-Schaling (2005) environment (both in terms of indeterminacy and E-instability). The intuition would be that we know from the work of

Batini et al. that IFB rules are generally bad news for determinacy. Now, in the context of the BS (2005) model LT's FEX rule (in BS notation) would be

(LT FEX rule)

Here, like LT, we let the CB respond to CPI inflation instead of domestic inflation. So, if this rule collapses to what they call the FCPI rule (see above).

Now, in the Bullard Schaling model – as in CGG (2002) – the nominal exchange rate obeys PPP, so that

and

so that

plugging this into the rule above we get

which can be rewritten as

using the definitions of the home and foreign CPI inflation rates from BS we get

or

In turn, using the definition of the terms of trade, this can be further be rewritten as

or

(LT FEX rule)

Now, this rule looks very similar to rule (13') = LT FCPI rule. There is one difference: compared with equation (13') = LT FCPI rule we have an extra term on the RHS, namely the expected foreign domestic inflation rate (with a negative sign). Note that if the LT FEX rule collapses exactly to rule (13') = LT FCPI rule. SO THIS SECTION GENERALIZES RESULTS FROM FCPI RULES (SECTION 3.4) BY INCLUDING HOME AND FOREIGN EXPECTED PPI INFLATION

6.3 Corner solutions and MCI´s

[TEXT BELOW IS BASED ON THE 24.01.06 SLIDES]

Managed exchange rates: dirty floating

Suppose the home central bank follows the managed exchange rate rule

(5)

where the parameter captures the endogenous response of the central bank to level of the nominal exchange rate. Now, the higher the value of the feedback parameter the more weight the central bank attaches to the nominal exchange rate.

Towards a monetary conditions index (MCI)

If we normalize the initial ( ) level of the nominal exchange rate at zero then rule (5) can be written as

(5´)

Note that if we also normalize at zero and redefine the nominal exchange rate as units of foreign currency per units of home currency, (5´) in turn can be written as

(7)

where

That is as a rule that specifies a monetary conditions index (MCI).

The latter concept was developed at the Bank of Canada and has been used there more extensively than elsewhere. [HERE PUT PAPERS RBA AND OTHER PAPER ON MCI´S IN REFS, PAPERS ARE ON FLASH DRIVE]

The Bank's MCI was a weighted sum of changes in the nominal Canadian 90-day commercial paper interest rate and a nominal G-10 bilateral trade-weighted exchange rate index.

More on the MCI

The general expression for the MCI is:

where and are the respective weights on the interest rate and the exchange rate. The weights on the interest rate and exchange rate reflect their estimated relative effects on Canadian output.

The Bank of Canada used weights of 3 to 1, interest rate to exchange rate.

That is, a one percentage point increase in the interest rate induces three times the change in the Bank's MCI as would a 1% appreciation of the Canadian dollar. Thus, in terms of the above formula the Bank set and .

If the relative weight on the exchange rate is zero, the MCI is identical to the first difference of the nominal interest rate, i.e. if and then .

Normalizing the initial level of the nominal interest rate at zero, we can then write

Then we have the ´corner solution´ [Fischer (2001)] of a completely floating exchange rate.

In the opposite case where the relative weight on the exchange rate becomes very large, i.e. if and , then the model collapses to or . Normalizing the initial level of the nominal exchange rate at zero, we can then write

This is the other corner solution of a completely fixed exchange rate.

Monetary strategy: MCIs and corner solutions

The implication of the above analysis is that in this paper we can have also an analytical critique of the use of MCI's in monetary policy.

In general, some values of will and some values will not ensure determinacy/learnability. This is what we mean with the "conditions for monetary conditions" in the title of this presentation.

For example, one practical monetary strategy question is whether a ´three to one´ (or ´four to one´) rule for monetary policy will ensure determinacy.

In addition we can link the debate on corner solutions with the determinacy debate.

According to Fischer only corner solutions are sustainable; intermediate regimes do not survive. We can now think about this in determinacy/learnability terms.

The model provides a new way of answering this age old question; some values of will and some will not ensure determinacy (and learnability).

6.4 A floating but managed exchange rate

[HERE WE FOLLOW DETERMINACY AND MANGED FLOATING EXCHANGE RATES SECTION FROM NOTE]. Now, the higher the value of the feedback parameter in the managed exchange rate rule

(24")

the more weight the central bank attaches to the nominal exchange rate. Of course in the limiting case where this rule collapses to the nominal exchange rate peg analyzed in section 5 of the Bullard Schaling paper. As pointed out there it is crucial for determinacy how the exchange rate peg is implemented. In order to have consistency between what we do here and there for the case where we let the peg be implemented by the rule

This means that in the latter case the CB will choose the nominal interest rate in such a way that it implements the 'targeting rule' , that is we assume policy is set in accordance with rule (47) from Bullard and Schaling (2005).

In order to analyze the case of a flexible but managed exchange rate we let the central bank follow the instrument rule

which can be rewritten as

or

(x.x)

Note that and . This can also be re-stated in terms of what Obstfeld (1998) has called the 'open economy trilemma': this refers to the inability of monetary authorities to simultaneously maintain fixed exchange rates, open capital markets and independent domestic monetary policy – that is with an open capital account they cannot have 100 percent domestic monetary autonomy and at the same time a 100 percent fixed exchange rate. As, in this model we do have perfect international risk-sharing (reflected in UIP holding), the authorities can opt for one of the two 'corner solutions' (Fischer ()), that is fixed (no domestic monetary autonomy) or floating exchange rates (complete domestic autonomy), corresponding with the cases of and respectively.

However, as we have shown above another possibility to solve this dilemma is to 'do a bit of both'; to trade-off domestic monetary autonomy and fixing the exchange rate, that is to choose a regime that can be characterized as dirty floating, the case where . So, it is not as simple as choosing two out of three (one must go). Given the fact that one has an open capital account one can also choose a combination of domestic monetary autonomy and exchange rate targeting.

How does the debate on corner solutions link with the determinacy debate? According to Fischer only corner solutions are sustainable; intermediate regimes do not survive. Think about this in determinacy/learnability terms.

It still remains to be seen what is the best choice (in welfare terms) for the value of . One can choose one thing ex ante, but whether this is the right thing ex post remains to be seen. Here we already have one new way of answering this age old question; some values of will and some will not ensure determinacy (and learnability). The next question is what value will maximize utility?

From the above, we can infer that for and assuming the foreign economy follows a simple Taylor rule where the coefficients do not satisfy the open economy Taylor principle, the home country is perfectly insulated from the foreign mayhem as long as

which is the determinacy condition under flexible exchange rates for the home country from section 3 of Bullard and Schaling.

Conversely, in the opposite case where and the anchor misbehaves the home economy is completely exposed to the misbehaving partner economy. The question is what happens ceteris paribus to the domestic economy for the more general case where ? This would be the case of a managed but flexible exchange rate; say a managed float. The intuition seems to be that under those circumstances more exchange rate management, or a larger emphasis on exchange rate targeting would be bad as it tends to push the domestic economy in the direction of indeterminacy (given the fact that the anchor is assumed to misbehave). Table x.x summarizes).

7 Conclusion

[To be completed.]

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8 Appendix A: derivation of log-linearized model

The world economy is composed of two countries, the domestic and the foreign economy (where the latter is indicated by an asterisk). Each country is populated by a continuum of households, or agents, and when we refer to individual households we use the index h. Households in both countries are continuously distributed within the range of 0 to 1, and consume differentiated goods produced by firms of both countries. The home country household is continuously distributed within the range of 0 to $1-\gamma$ ($h \in [0, 1-\gamma]$), and the foreign country household within the range of $1-\gamma$ to 1 ($h^* \in [1-\gamma, 1]$), where $0 < 1-\gamma < 1$. Then, $1-\gamma$ and γ also represent the home and foreign populations, respectively. There is no population growth. Apart from this difference in *size* (if $\gamma \neq 1/2$) the two countries are otherwise the same in terms of preferences and technologies. This means that *all structural (deep) parameters are the same across countries.*

Markets are complete, which will ensure that consumption is equated across households at each date. There are the same number of final goods producing firms in each country as there are households. Final goods producers are perfectly competitive and take intermediate goods and labor as inputs. Each country has an intermediate goods producing sector. This sector is monopolistically competitive, so that each firm produces a differentiated product. Nominal prices are sticky in the intermediate goods sector in the sense of Calvo (1983). The number of intermediate goods producers in each country is normalized to unity.

8.1 Households

The households live forever and maximize utility defined over consumption and leisure. A representative household in the home country maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t) - V\left[N_t(h)\right] \right\}, \tag{57}$$

where

$$U\left(C_t\right) - V\left[N_t(h)\right] = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi}, \tag{58}$$

with $\sigma > 0$ and $\phi > 0$. In this expression, C_t is an index of consumption and $N_t\left(h\right)$ represents the hours of labor supply of household h. Similarly, for country $*$ preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U^*(C_t^*) - V^*\left[N_t^*(h^*)\right] \right\}, \tag{59}$$

where

$$U^*\left(C_t^*\right) - V^*\left[N_t^*(h^*)\right] = \frac{(C_t^*)^{1-\sigma}}{1-\sigma} - \frac{N_t^*(h^*)^{1+\phi}}{1+\phi}. \tag{60}$$

The consumption index of home (H) and foreign (F) goods is given by

$$C_t = C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{61}$$

This consumption basket is associated with a consumption price index given by[17]

$$P_{C,t} = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma} = k^{-1} P_{H,t} S_t^{\gamma} \tag{62}$$

where the *terms of trade*, the relative price of foreign to domestic goods measured in domestic currency, is defined as $S_t \equiv P_{F,t}/P_{H,t}$ and $k \equiv (1-\gamma)^{(1-\gamma)} \gamma^{\gamma}$.

Similarly, for country $*$ we have

$$C_t^* = (C_{H,t}^*)^{1-\gamma} (C_{F,t}^*)^{\gamma} \tag{63}$$

$$P_{C,t}^* = k^{-1} (P_{H,t}^*)^{1-\gamma} (P_{F,t}^*)^{\gamma} = k^{-1} P_{F,t}^* (\frac{1}{S_t})^{1-\gamma} \tag{64}$$

Note that here we assume – in line with the two countries being the same or *symmetric* in terms of preferences and technology - that there is *no heterogeneity of consumption baskets between countries* as the share of H goods in the foreign consumption basket $1-\gamma^* = 1-\gamma$, equals the share of H goods in the home basket. As we shall show below this assumption (along with the law of one price (84) and (85)) is necessary for aggregate, or CPI-based, PPP to hold.

If $\gamma = \frac{1}{2}$, there is no home bias in consumption. More generally, for $\gamma < \frac{1}{2}$, both home country and country $*$ consumers will demand relatively more H goods than F goods, while for $\gamma > \frac{1}{2}$, both sets of consumers will demand relatively more F goods than H goods. When the consumption price indices for country H and country F goods are equal, the parameter γ corresponds to the share of home country consumption allocated to goods imported from country $*$, and so it provides one natural measure of *openness*.[18] Likewise, $1-\gamma$ corresponds to the share of country $*$ consumption allocated to goods imported from the home country, and so is a measure

[17] The consumption-based price index P_c is defined as the minimum expenditure required to buy one unit of the composite good C, given the prices of the home and foreign goods. We use the symbol P_C for the consumption price index, instead of P as used by CGG (2002). This in order to highlight the difference between domestic (producer price) and consumer price (CPI) inflation.

[18] Note that CGG (2002) work with Cobb-Douglas preferences, as equation (61) can be seen to be equal to $\lim_{\hat{\eta} \to 1} [(1-\gamma)^{1/\hat{\eta}} C_{H,t}^{\hat{\eta}-1/\hat{\eta}} + \gamma^{1/\hat{\eta}} C_{F,t}^{\hat{\eta}-1/\hat{\eta}}]^{\hat{\eta}/\hat{\eta}-1}$, where $\hat{\eta}$ is the elasticitity

of openness for that country. This has the important implication that as γ increases the home country becomes more open and $*$ moves towards more autarky ($1-\gamma$ declines).

8.2 Budget constraints

We follow Clarida, Gali, and Gertler (2002) and express the sequence of household budget constraints in nominal terms. We let $W_t(h)$ represent the nominal wage associated with the labor supply $N_t(h)$ of household h, $h \in [0, 1-\gamma]$ in the home country, and $W_t^*(h^*)$ represent the nominal wage associated with the labor supply $N_t^*(h*)$ of household $h^*, h^* \in [1-\gamma, 1]$ in country $*$. We denote by D_{t+1} the payoff of the portfolio purchased at t, with $\hat{Q}_{t,t+1}$ the corresponding stochastic discount factor. Finally, let T_t denote lump sum taxes and Γ_t denote lump sum profits accruing from ownership of intermediate goods firms. Then the maximization of (57) takes place subject to the sequence of budget constraints

$$P_{C,t}C_t + E_t\{\hat{Q}_{t,t+1}D_{t+1}\} = W_t(h)N_t(h) + D_t - T_t + \Gamma_t \tag{65}$$

for $t = 0, ..., \infty$ in the home country,and

$$P_{C,t}^*C_t^* + E_t^*\{\hat{Q}_{t,t+1}^*D_{t+1}^*\} = W_t^*(h^*)N_t^*(h^*) + D_t^* - T_t^* + \Gamma_t^*$$

for $t = 0, ..., \infty$ in country $*$.

8.2.1 Labor supply

Each household in each country is a monopolistically competitive supplier of labor and faces a constant elasticity demand function for its services.

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\eta} N_t, \tag{66}$$

of substitution between H and F goods, and the composite consumption index is a CES function. For more details see Obstfeld and Rogoff (1996, pp. 221-225). The CES specification is used by e.g. CGG (2001), Pappa (2004) and Monacelli (2005). The corresponding CES price index is $P_{c,t} = [(1-\gamma)P_{H,t}^{1-\hat{\eta}} + \gamma P_{F,t}^{1-\hat{\eta}}]^{1/1-\hat{\eta}}$.

where N_t is per capita employment in the home country and

$$W_t \equiv \left(\frac{1}{1-\gamma} \int_0^{1-\gamma} W_t(h)^{1-\eta_t} dh \right)^{1/(1-\eta_t)} \tag{67}$$

is the associated aggregate wage index in the domestic economy. The elasticity of labor demand, η_t, is the same across workers, but may vary over time. This particular demand curve evolves from a production technology that has labor input a CES aggregate of individual household labor hours, as described below. For the foreign country we get:

$$N_t^*(h^*) = \left(\frac{W_t^*(h^*)}{W_t^*} \right)^{-\eta^*} N_t^*, \tag{68}$$

where N_t^* is per capita employment in country $*$ and

$$W_t^* \equiv \left(\frac{1}{\gamma} \int_{1-\gamma}^{1} W_t^*(h^*)^{1-\eta_t^*} dh^* \right)^{1/(1-\eta_t^*)} \tag{69}$$

is the associated aggregate wage index in country $*$.

The first-order condition for labor supply reflects the household's market power

$$\frac{W_t(h)}{P_{C,t}} = (1 + \mu_t^w) N_t(h)^{\phi} C_t^{\sigma}, \tag{70}$$

where $\mu_t^W = 1/\eta_t - 1$ is the optimal wage markup.[19] Wages are perfectly flexible, implying the absence of any endogenous variation in the wage markup arising from wage rigidities. On the other hand, we allow for exogenous variation in the wage markup arising from shifts in η_t, interpretable as exogenous variation in workers' market power. Note that because wages are flexible, all workers will charge the same wage and gave the same level of hours. Thus we can write

$$W_t(h) = W_t \tag{71}$$

$$N_t(h) = N_t \tag{72}$$

[19]In loglinear terms we get $w_t - p_{C,t} = (\phi n_t + \sigma c_t) + \mu_t^w$.

for all $h \in [0, 1-\gamma]$ and all t.

For the foreign country we have:

$$\frac{W_t(h^*)}{P^*_{C,t}} = (1+\mu_t^{w*})N_t^*(h^*)^{\phi}(C_t^*)^{\sigma}, \tag{73}$$

where $\mu_t^{*W} = 1/\eta_t^* - 1$ is the optimal wage markup, [20]and

$$\begin{aligned} W_t^*(h^*) &= W_t^* \\ N_t^*(h^*) &= N_t^* \end{aligned} \tag{74}$$

for all $h^* \in [1-\gamma, 1]$ and all t.

8.3 Consumption and international risk sharing

The first-order necessary conditions for consumption allocation and intertemporal optimization are standard:

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_{C,t}}\right)^{-1} C_t, \tag{75}$$

$$C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_{C,t}}\right)^{-1} C_t, \tag{76}$$

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_{C,t}}{P_{C,t+1}}\right) = \hat{Q}_{t,t+1}. \tag{77}$$

Let R_t denote the gross nominal yield on a one-period discount bond. Then by taking the expectation on each side of equation (77) we obtain the following Euler equation:[21]

$$1 = \beta R_t E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_{C,t}}{P_{C,t+1}}\right)\right\}, \tag{78}$$

where $R_t^{-1} = E_t\{\hat{Q}_{t,t+1}\}$ is the price of the discount bond.

[20] In loglinear terms we get $w_t^* - p_{c,t}^* = (\phi n_t^* + \sigma c_t^*) + \mu_t^{w*}$.

[21] We use the symbol $\hat{Q}_{t,t+1}$ for the stochastic discount factor as in our model Q_t stands for the CPI-based real exchange rate.

A symmetric set of first-order conditions holds for citizens of the foreign country:

$$C^*_{H,t} = (1-\gamma)\left(\frac{P^*_{H,t}}{P^*_{C,t}}\right)^{-1} C^*_t, \tag{79}$$

$$C^*_{F,t} = \gamma\left(\frac{P^*_{F,t}}{P^*_{C,t}}\right)^{-1} C^*_t \tag{80}$$

In particular, given the international tradability of state-contingent securities, the intertemporal efficiency condition can be written as:

$$\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\left(\frac{P^*_{C,t}}{P^*_{C,t+1}}\right) = \hat{Q}^*_{t,t+1} = \hat{Q}_{t,t+1}\left(\frac{E_{t+1}}{E_t}\right) \tag{81}$$

[22]Then by taking the expectation on each side of equation (81) we obtain

$$\beta E^*_t\left\{\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\left(\frac{P^*_{C,t}}{P^*_{C,t+1}}\right)\right\} = E^*_t\left\{\hat{Q}_{t,t+1}\left(\frac{E_{t+1}}{E_t}\right)\right\} = (R^*_t)^{-1}$$

[23]

Substituting (62) into (75), we get

$$C_{H,t} = (1-\gamma)\, k^{-1} S^{\gamma}_t C_t \tag{82}$$

For the foreign country we get

$$C^*_{H,t} = (1-\gamma)\, k^{-1} S^{\gamma}_t C^*_t \tag{83}$$

Therefore, *local* and *foreign* demand for local goods is given by (82) and (83). Hence export demand for the home good rises when the terms of trade depreciate (i.e. the price P_H falls relative to P_F), and when the overall consumption basket in country $*$ increases.

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[22]See for instance Chari, Kehoe and McGrattan (2002).

[23]Note that UIP holds if $E_t = E^*_t$, so that $E^*_t\left\{\hat{Q}_{t,t+1}\left(\frac{E_{t+1}}{E_t}\right)\right\} = (R^*_t)^{-1} = E_t\left\{\hat{Q}_{t,t+1}\left(\frac{E_{t+1}}{E_t}\right)\right\} = R^{-1}_t E_t\left\{\left(\frac{E_{t+1}}{E_t}\right)\right\}$. So, that in logs we get $r_t = r^*_t + E_t\Delta e_{t+1}$. In Monacelli (2005) the nominal exchange rate affects the expected future path of the deviations from the law of one price, and in turn the equilibrium path of inflation and the output gap.

8.4 Pass-through, the real exchange rate, and purchasing power parity

CGG (2002) building on CGG (2001), Corsetti and Pesenti (2001) and Gali and Monacelli (2002) [hereafter GM] - and in line with the seminal 'new-open economy macroeconomics' model by Obsfeld and Rogoff (1995) - assume that the law of one price holds, implying that

$$P_{F,t} = E_t P^*_{F,t} \tag{84}$$

and

$$P_{H,t} = E_t P^*_{H,t} \tag{85}$$

where E_t is the nominal exchange rate (defined as units of home currency per unit of foreign currency). Thus, firms in both countries set their export prices at the foreign-currency equivalents of their domestic sales prices, based on producer's currency pricing (PCP).[24] The Table below illustrates.

	In domestic economy	In country $*$
Price of good H	$P_{H,t}$	$P^*_{H,t}$, where $P^*_{H,t} = \frac{P_{H,t}}{E_t}$
Price of good F	$P_{F,t}$, where $P_{F,t} = E_t P^*_{F,t}$	$P^*_{F,t}$

Multiplying the foreign CPI and the nominal exchange rate E_t we get:

$$E_t P^*_{c,t} = E_t k^{-1} (P^*_{H,t})^{1-\gamma} (P^*_{F,t})^{\gamma} \tag{86}$$

Combining this expression with the relation (84) between the domestic and foreign currency prices of good F yields

$$E_t P^*_{c,t} = E_t k^{-1} (P^*_{H,t})^{1-\gamma} (\frac{P_{F,t}}{E_t})^{\gamma} = E_t^{1-\gamma} k^{-1} (P^*_{H,t})^{1-\gamma} (P_{F,t})^{\gamma} \tag{87}$$

Using the link (85) between the domestic and foreign currency prices of good H results in

$$E_t P^*_{c,t} = E_t^{1-\gamma} k^{-1} (\frac{P_{H,t}}{E_t})^{1-\gamma} (P_{F,t})^{\gamma} = k^{-1} (P_{H,t})^{1-\gamma} (P_{F,t})^{\gamma} = P_{C,t} \tag{88}$$

[24]Note that the two-country model of Batini, Levine and Pearlman (2004) also assumes producer-price based PPP, i.e. prices in home and foreign blocks are linked by (85) and (84).

So, the assumption of symmetric consumer preferences across countries, i.e. the same degree of bias towards H-goods in both countries ($\gamma = \gamma^*$) along with the law of one price implies CPI-based purchasing power parity. Put, differently the (consumption price based) real exchange rate equals 1, or

$$Q_t \equiv \frac{E_t P^*_{C,t}}{P_{C,t}} = 1 \Longleftrightarrow E_t = \frac{P_{C,t}}{P^*_{C,t}}, \tag{89}$$

so that the nominal exchange rate equals the ratio of the home and foreign CPI.

In addition, using the law of one price, (75) and (76) can be combined to yield:

$$\frac{C_{H,t}}{C_{F,t}} = \frac{(1-\gamma)\, P_{F,t}}{\gamma P_{H,t}} = (\frac{1-\gamma}{\gamma}) S_t = \frac{(1-\gamma) E_t P^*_{F,t}}{\gamma P_{H,t}} \tag{90}$$

Note that in the CGG (2002) model CPI-based PPP does not imply a constant terms of trade. The terms of trade $S_t = P_{F,t}/P_{H,t}$ - which under the law of one price is identical with the PPI-based real exchange rate $E_t P^*_{F,t}/P_{H,t}$ – moves in response to shocks in the global economy. Thus, we do not have equivalence of *domestic (producer price) inflation* and *CPI inflation*, i.e. $P_{C,t} \neq P_{H,t}$ and $P^*_{C,t} \neq P^*_{F,t}$, this is important as a deterioration of home's terms of trade/real depreciation (higher S) drives a wedge between the price paid by home consumers $P_{C,t}$ and the local currency price set by firms in the home market $P_{H,t}$.

More specific, it is interesting to point out that our model implies that the degree of *real exchange rate pass-through* - here defined as the effect of a producer price (PPI) based real exchange rate depreciation on consumer prices - can be simply inferred from the definition for the home CPI. It is easy to see that the degree of pass-through, for the home country is equal to γ: a 1 percent *real* depreciation increases home CPI inflation by γ percent. This result can be demonstrated as follows. First, using the assumption of PPP in the definition of the terms of trade implies that $S_t \equiv P_{F,t}/P_{H,t} = E_t P^*_{F,t}/P_{H,t}$.

Using this result in (62) taking logs and first differences yields[25]

$$\pi_{C,t} = \pi_{H,t} + \gamma \Delta s_t \tag{91}$$

Similarly, the degree of pass-through for the foreign country is $-(1-\gamma)$. This can be seen from the fact that in logs we have

$$\pi^*_{C,t} = \pi^*_{F,t} - (1-\gamma)\Delta s_t \tag{92}$$

If the home currency depreciates by 1 percent in real terms, the foreign currency appreciates by 1 percent in real terms, which lowers the foreign CPI by $1-\gamma$ percent.

Note that in the CGG (2002) framework *nominal exchange rate pass-through* - i.e the effect of a nominal depreciation on *import prices* - is very high (in fact 100 percent) - in line with the Law of one Price holding - as both countries set their export prices at the foreign-currency equivalents of their domestic sales prices, based on producer's currency pricing (PCP). This can be easily seen from equations (84) and (85). The degree of nominal exchange rate pass-through defined as the effect of a nominal depreciation on *consumer prices* is also 100 percent, as according to (89) we also have CPI based PPP.

In the small open economy (SOE) models of CGG (2001) GM (2002) equation (63) should be modified as there the foreign country - the rest of the world (ROW) - is essentially a closed economy and therefore does not consume any H goods. Using using the notation of this paper, there we have $\gamma \neq \gamma^*$, in fact in CGG (2001) and GM (2002) we have $\gamma^* = 1$, implying $C^*_t = C^*_{F,t}$. Then, from (64) we have $P^*_{c,t} = P^*_{F,t}$ and equivalence between domestic and CPI inflation holds for the ROW.[26] Then, the terms of trade for the domestic economy (in logs) is given by $s_t = p_{F,t} - p_{H,t}$, combining this

[25] Note that the above equation holds independently of the degree of pass-through. See also Monacelli (2005, pp. 6-7).

[26] The size of the small open economy is negligible relative to the rest of the world, an assumption that allows the treatment of the latter *as if* it was closed economy. More precisely, Monacelli (2005) points out that this is a world of two asymmetric countries in which one is small relative to the other (whose equilibrium is in the limit taken as exogenous). This kind of setup allows the explicit modeling of the role of financial markets and risk sharing and to overcome a typical problem of a unit root in consumption

with the law one price $e_t = p_{F,t} - p^*_{F,t}$ yields $s_t = e_t + p^*_{F,t} - p_{H,t} = e_t + p^*_{C,t} - p_{H,t}$. Using this result in the definition of the (CPI based) real exchange rate implies

$$q_t = s_t + p_{H,t} - p_{C,t} = (1-\gamma)s_t \tag{93}$$

Thus, in CGG (2001) and GM (2002) the CPI-based (log) real exchange rate in the domestic economy is proportional to the (log) terms of trade, with the proportionality coefficient being an inverse function of the degree of openness. Therefore we can conclude that in the frameworks by CGG (2001) and GM (2002) there is one reason, and one reason only why we may not have what me may call 'aggregate PPP'; that is a non-constant CPI-based real exchange rate. It can be easily seen from equation (93) that this reason is the *heterogeneity of consumption baskets* between the small open economy and the rest of the world, an effect captured by the term $(1-\gamma)s_t$, as long as $\gamma < 1$. For $\gamma \to 1$, in fact, the two aggregate consumption baskets coincide (as then both economies do not import any goods from the other country, as CGG (2001) and GM (2002) already assume that $\gamma^* = 1$). In that case relative price variations are not required in equilibrium. In the present paper - in line with CGG (2002) - (93) collapses to $q_t = 0$ as we *do* have aggregate PPP, and there is *no relation* between the CPI-based real exchange rate and the terms of trade s_t (which is identical with the PPI-based real exchange rate $s_t = e_t + p^*_{F,t} - p_{H,t}$). The reason is that throughout the paper it is assumed that $\gamma = \gamma^*$, that is *no heterogeneity of consumption baskets* between the domestic and foreign economies.[27]

Monacelli (2005) also works with a SOE model but assumes that there are *deviations from the law of one price*, so that equation (84) (in logs) is

that characterizes traditional small open economy models with incomplete markets. See Schmitt-Grohe and Uribe (2002) for a discussion on how to 'close small open economy models'.

[27] More specific, if we assume that $\gamma \neq \gamma^*$, equations (63) and (64) generalize to $C^*_t = (C^*_{H,t})^{1-\gamma^*}(C^*_{F,t})^{\gamma^*}$ and $P^*_{C,t} = (k^*)^{-1}(P^*_{H,t})^{1-\gamma^*}(P^*_{F,t})^{\gamma^*} = (k^*)^{-1}P^*_{F,t}(\frac{1}{S_t})^{1-\gamma^*}$. Applying the same algebraic manipulations as in the main text we get a generalized version of equation (88): $E_t P^*_{c,t} = E_t^{1-\gamma^*}(k^*)^{-1}(\frac{P_{H,t}}{E_t})^{1-\gamma^*}(P_{F,t})^{\gamma^*} = (k^*)^{-1}(P_{H,t})^{1-\gamma^*}(P_{F,t})^{\gamma^*} \neq P_{C,t}$.

generalized to

$$p_{F,t} = p^*_{F,t} + e_t - \psi_{F,t} \tag{94}$$

where $\psi_{F,t}$ denotes the deviation of the world price $p^*_{F,t} + e_t$ from the *domestic* currency price of imports, and is a measure of the deviation from the law of one price.[28] He calls $\psi_{F,t}$ the *law-of-one price gap* (l.o.p. gap). In line with the SOE models of CGG (2001) and GM (2002) he also works with the equivalence between foreign domestic and CPI inflation $P^*_{c,t} = P^*_{F,t}$.Then, equation (94) reduces to

$$p_{F,t} = p^*_{C,t} + e_t - \psi_{F,t} \tag{95}$$

Combining this pricing relation (95) with the definition for the terms of trade implies $p^*_{C,t} = s_t - e_t + \psi_{F,t} + p_{H,t.}$ Substituting this expression in the definition of the CPI-based real exchange rate results in a generalized version of the proportionality relation between the real exchange rate and the terms of trade (93) found by CGG (2001) and GM (2002):

$$q_t = (1 - \gamma)s_t + \psi_{F,t} \tag{96}$$

Thus, in Monacelli (2005) there are two sources of deviations from CPI-based PPP (the latter would require q_t to be equal to zero). The first lies in the heterogeneity of consumption baskets between the SOE and the ROW, an effects captured by the term $(1 - \gamma)s_t$, as long as $\gamma < 1$. The second source of deviation from PPP is due to the deviation from the law of one price, captured by movements in $\psi_{F,t}$. Monacelli claims that allowing for incomplete pass-through yiels three important implications for the design of monetary policy. First, the analysis of an open economy becomes fundamentally different from a closed economy. Thus, the policy problem for the central bank is no longer *isomorphic* to the one it would face if it were a closed economy (as is the case in CGG (2001, 2002)). Second, productivity-driven deviations from the law of one price assume the interpretation of endogenous cost-push

[28] However he assumes that equation (85) still applies; i.e. the *export* price of the domestic good, $P^*_{H,t}$, is determined by the law of one price.

shocks. This marks a distinction from some of the recent literature (based on the prototype Calvo sticky price model with perfect pass-through) that, in order to generate a meaningful policy trade-off, has typically resorted to ad-hoc cost-push shocks as exogenous shifters of the Phillips curve. Third, by generating a real policy trade-off incomplete pass-through allows one to contrast the features of the optimal policy program under commitment to the one under discretion.In Monacelli's framework the critical element of the optimal commitment policy (relative to discretion) is in the possibility, through the exchange rate (which is a forward-looking variable), to affect the expected future path of the deviations from the law of one price, and in turn the equilibrium paths of inflation and the output gap.[29]

In the CGG (2002) model we have *symmetric* international price setting behavior; namely symmetric PCP, this implies that the price that foreigners pay for home goods, and the price that home residents pay for foreign goods fluctuates when the nominal exchange rate changes. Conversely, under pricing to market (PTM) producers set the price in the consumers' currency. In the PTM model, the firm chooses two different prices – one for residents of its own country, and for residents of the other country. The price charged by the home (∗) firm to the home (∗) residents is the same as in the PCP model. The Table below illustrates.

	In domestic economy	In country $*$
Price of good H	$P_{H,t}$	$P^*_{H,t}$
Price of good F	$P_{F,t}$	$P^*_{F,t}$

As a consequence, prices consumers face do not respond to exchange rate changes. Put differently, the terms of trade $S_t = P_{F,t}/P_{H,t}$ does not respond to nominal exchange rate changes. PTM models were introduced by Devereux and Engel (1998, 2002), Betts and Devereux (2000) and Chari, Kehoe and McGrattan (2002) among others. Obstfeld and Rogoff (2000) question the validity of the PTM model by noting that, in the real world, the exchange-rate pass-through is not zero and a depreciation of a nation's

[29] For more details see Monacelli (2005, p. 4).

currency - in line with PCP - actually worsens its terms of trade.[30]

The papers by Obstfeld and Rogoff (O-R) (1995) and Devereux and Engel (1998) are esssentially mirror images, as the former assumes that both countries adopt PCP, whereas the latter assumes that all firms set export prices according to PTM. Thus, both papers work with a *symmetric* international pricing regime. In fact, it is also possible to analyze the case of *asymmetric* international pricing behavior. This is done in an interesting paper by Otani (2002). He expands a model by Betts and Devereux (2000) and assumes that a fraction s of domestic firms and a fraction s^* of foreign firms adopt PTM price setting.[31] Otani states that one might wonder if the assumption of exogenously predetermined percentages of PTM firms in the domestic and foreign countries might be implausible under changes in macroeconomic policies - i.e. under changes in the policy regime - and/or structural economic change. Devereux and Engel (2001) analyze this issue and show that exporters generally set prices in the currency of the economy with the most credible monetary policy. Therefore, it is plausible that firms in the country with credible monetary policies adopt PCP, whereas firms in the country with less credible monetary policies adopt PTM. For example, relatively small emerging market (EM) economies may set their export prices based on the large country's currency under a floating regime, as is the case with respect to trade between the U.S. and Asian economies.[32] Since, in our model we assume symmetric PCP it is more natural to interpret the home

[30] In recent work, Campa and Goldberg (2002) estimate import pass-through elasticities for a range of OECD countries. They find that: (i) the degree of pass-through is partial in the short-run and becomes gradually complete only in the long-run, (ii) the sensitivity of prices to exchange rate movements is much larger at the *wholesale* import stage. Their results imply a rejection of both the PCP and PTM assumptions.

[31] His model includes both the O-R model ($s = s^* = 0$) and Betts and Devereux (2000) ($s = s^* > 0$) as special cases. He shows that in his model - in general - CPI-based PPP does not hold when prices are rigid. In the O-R model the latter holds under both flexible and rigid prices.

[32] Note that this can be seen as a way of insulating the smaller economies' terms of trade from nominal exchange rate changes under flexible exchange rates. A more drastic way of doing this would be for the small economy to peg its currency to that of its larger trading partner. This case is analyzed in section 5 of the paper.

and foreign economies as two large blocks, say the U.S. and Europe.

Note that the distinction between PCP and PTM is only relevant under *flexible exchange rates*. For example, in section 5 of the paper we analyze the case of *fixed exchange rates*, brought about by the home country pegging its currency to the foreign or anchor country.[33] Then, both countries' choice of invoicing currency, i.e. PCP or PTM becomes irrelevant since there is essentially a single currency.

8.5 Firms

8.5.1 Final goods producers

Each final goods firm in the home country uses a continuum of intermediate goods to produce output, according to the following CES technology:

$$Y_t = \left(\int_0^1 Y_t(f)^{(\xi-1)/\xi} df \right)^{\xi/(\xi-1)} \tag{97}$$

where Y_t denotes aggregate output, while $Y_t(f)$ is the input produced by intermediate goods firm f. Both variables are normalized by population size $1-\gamma$, i.e. they are expressed in *per capita* terms. Profit maximization, taking the price of the final good as given, implies the set of demand equations:

$$Y_t(f) = (\frac{P_{H,t}(f)}{P_{H,t}})^{-\xi} Y_t \tag{98}$$

as well as the domestic price index

$$P_{H,t} = (\int_0^1 P_{H,t}(f)^{1-\xi} df)^{1/(1-\xi)}. \tag{99}$$

For the foreign country we get:

$$Y_t^* = \left(\int_0^1 Y_t^*(f^*)^{(\xi-1)/\xi} df^* \right)^{\xi/(\xi-1)}, \tag{100}$$

[33] A more radical approach would be for an emerging nations to give up its domestic currency and adopt an advanced nation's currency as legal tender. This policy option has received the name of "official dollarization," even if the advanced country's currency is other than the dollar.

$$Y_t^*(f^*) = (\frac{P_{F,t}(f^*)}{P_{F,t}})^{-\xi} Y_t^*, \tag{101}$$

$$P_{F,t} = (\int_0^1 P_{F,t}(f^*)^{1-\xi} df^*)^{1/(1-\xi)} \tag{102}$$

8.5.2 Intermediate goods producers

Each intermediate goods firm produces output using a technology that is linear in labor input, $N_t(f)$ (also normalized by population size), as follows:

$$Y_t(f) = A_t N_t(f), \tag{103}$$

where A_t is an exogenous technology parameter. The labor used by each firm is a CES composite of individual household labor, as follows:

$$N_t(f) = (\frac{1}{1-\gamma} \int_0^{1-\gamma} N_t(h)^{(\eta_t - 1)/\eta_t} dh)^{\eta_t/(\eta_t - 1)} \tag{104}$$

Aggregating across optimizing intermediate goods firms yiels the market demand curve for household labor given by equation (66), where the technological parameter η_t is the wage elasticity of hours demand. Because in equilibrium each household charges the same wage and supplies the same number of hours, we can treat the firm's decision problem over total labor demand as just involving the aggregates $N_t(f)$ and W_t. We also assume that each firm receives a subsidy of τ percent of its wage bill.

In addition, intermediate goods firms set prices on a staggered basis as in Calvo (1983), where θ is the probability a firm keeps its price fixed in a given period and $1-\theta$ is the probability it changes it, where probability draws are i.i.d over time. Firms that do not adjust their price simply adjust output to meet demand (assuming they operate in a region with a non-negative net markup). In either case, choosing labor to minimize costs conditional on output yields:

$$MC_t = \frac{(1-\tau)(W_t/P_{H,t})}{A_t} = \frac{(1-\tau)(W_t/P_{C,t})S_t^{\gamma}}{kA_t}, \tag{105}$$

where MC_t denotes real marginal cost. Observe that, given the constant returns technology and the aggregate nature of shocks, real marginal cost is the same across firms.

Firms that are able to choose their price optimally in period t choose the reset price $P^0_{H,t}$ to maximize the following objective:

$$E_t \sum_{j=0}^{\infty} \theta^j \hat{Q}_{t,t+j} Y_{t+j}(f)(P^0_{H,t} + P_{H,t+j} MC_{t+j}) \qquad (106)$$

subject to the demand curve (98). The solution to this problem implies that firms set their price equal to a discounted stream of expected future nominal marginal cost

$$E_t \sum_{j=0}^{\infty} \theta^j \hat{Q}_{t,t+j} Y_{t+j}(f)(P^0_{H,t} - (1+\mu^p) P_{H,t+j} MC_{t+j}) = 0 \qquad (107)$$

Note that if a firm was able to freely adjust its price each period, it will choose a constant markup over marginal cost, i.e., $\theta = 0$ implies

$$\frac{P^0_{H,t}}{P_{H,t}} = (1+\mu^p) MC_t \qquad (108)$$

Finally, the law of large numbers implies that the domestic price index evolves according to

$$P_{H,t} = [\theta(P_{H,t-1})^{1-\xi} + (1-\theta)(P^0_{H,t})^{1-\xi}]^{1/(1-\xi)} \qquad (109)$$

For the foreign country we get:

$$Y^*_t(f^*) = A^*_t N^*_t(f^*), \qquad (110)$$

where the labor used by each firm is

$$N^*_t(f^*) = (\frac{1}{\gamma} \int_{1-\gamma}^{1} N^*_t(h^*)^{(\eta_t - 1)/\eta_t} dh^*)^{\eta_t/(\eta_t - 1)}. \qquad (111)$$

Choosing labor to minimize costs conditional on foreign output yields:

$$MC^*_t = \frac{(1-\tau)(W^*_t/P^*_{F,t})}{A^*_t} = \frac{(1-\tau)(W^*_t/P^*_{C,t}) S_t^{-1+\gamma}}{k A^*_t}, \qquad (112)$$

Similarly foreign firms that are able to choose their price optimally in period t choose the reset price $P^0_{F,t}$ to maximize the objective:

$$E^*_t \sum_{j=0}^{\infty} \theta^{*j} \hat{Q}_{t,t+j} Y^*_{t+j}(f^*)(P^0_{F,t} + P_{F,t+j} MC^*_{t+j}), \tag{113}$$

subject to the demand curve (101). Similarly, the solution implies

$$E^*_t \sum_{j=0}^{\infty} \theta^{*j} \hat{Q}_{t,t+j} Y^*_{t+j}(f^*)(P^0_{F,t} - (1+\mu^{p*}) P_{F,t+j} MC^*_{t+j}) = 0 \tag{114}$$

Finally, the foreign producers price index (PPI) evolves according to

$$P_{F,t} = [\theta^*(P_{F,t-1})^{1-\xi} + (1-\theta^*)(P^0_{F,t})^{1-\xi}]^{1/(1-\xi)} \tag{115}$$

where $\theta^* = \theta$.

8.6 Equilibrium

We begin by characterizing the equilibrium conditional on output. How the model is closed depends on the behavior of prices and monetary policy. We first characterize the flexible price equilibrium, for which an exact solution is available, and then turn to the case of staggered price setting, for which an approximate solution is available.

The law of one price (84) and (85), which implies $P_{C,t} = E_t P^*_{C,t}$ for all t, in conjunction with equations (**??**) and (77), and a suitable normalization of initial conditions yields:

$$C_t = C^*_t \tag{116}$$

for all t.[34]

[34] Following Gali and Monacelli (2002, p. 6), this can be seen from combining (77) and (81), this yields $\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma} (\frac{P_{C,t}E_t}{P_{C,t}}) * (\frac{P_{c,t+1}}{P^*_{c,t+1}E_{t+1}})$. Or using Q_t for the CPI-based real exchange rate, $\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma} (\frac{Q_t}{Q_{t+1}})$, or $\left(\frac{C_{t+1}}{C^*_{t+1}}\right)^{-\sigma} = \left(\frac{C_t}{C^*_t}\right)^{-\sigma} (\frac{Q_t}{Q_{t+1}})$. It then follows (after iterating) that: $C_t = \vartheta C^*_t Q_t^{1/\sigma}$ where $\vartheta \equiv \left[\left(\frac{C_0}{C^*_0}\right)^{-\sigma} Q_0\right]^{-1/\sigma}$.Under suitable initial conditions and under aggregate PPP, $Q_0 = \vartheta = 1$, implying that $C_t = C^*_t$.

Goods market clearing in the home and foreign countries implies

$$(1-\gamma)Y_t = (1-\gamma)C_{H,t} + \gamma C^*_{H,t} \tag{117}$$

$$\gamma Y^*_t = (1-\gamma)C_{F,t} + \gamma C^*_{F,t} \tag{118}$$

Remember that Y_t, $C_{H,t}$,$C_{F,t}$,Y^*_t,$C^*_{H,t}$ and $C^*_{F,t}$ were formulated in *per capita* terms. Thus, in order to equate *aggregate* output and demand these variables have to be multiplied by population sizes.

The demand curves for home and foreign goods by home citizens, equations (75) and (76), respectively along with the analogues for foreign citizens (79) and (80) and the law of one price imply that according to (89) the CPI-based real exchange rate is unity:

$$Q_t \equiv \frac{E_t P^*_{C,t}}{P_{C,t}} = 1 \Longleftrightarrow E_t = \frac{P_{C,t}}{P^*_{C,t}}$$

It then follows (after also taking into account equations (117) and (118)) that the (nominal) trade balance is zero within each country (as nominal production equals nominal absorption or expenditure)

$$P_{H,t}Y_t = P_{C,t}C_t \tag{119}$$

$$P^*_{F,t}Y^*_t = P^*_{C,t}C^*_t \tag{120}$$

In turn, combining equations (62) and (119) implies an open economy aggregate demand schedule that relates domestic per capita output, per capita consumption, and the terms of trade or PPI-based real exchange rate, $S_t = P_{F,t}/P_{H,t} = P^*_{F,t}E_t/P_{H,t}$ as follows:

$$Y_t = k^{-1}C_t S_t^{\gamma} \tag{121}$$

with

$$S_t = \frac{Y_t}{Y^*_t} \tag{122}$$

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FORTHCOMING 2006 IN JIMF]. The aggregate demand curve for the foreign country can be found by combining equations (64) and (120) which yields:

$$Y_t^* = k^{-1} C_t^* S_t^{-1+\gamma} \tag{123}$$

Observe that equations (121) and (122) and the consumption Euler equation (78) determine domestic output demand, conditional on foreign output and the path of the real interest rate. This can be easily demonstrated. The consumption Euler equation (78) can be written as

$$C_t = E_t C_{t+1} * \left(\beta^{-1} R_t^{-1} \frac{E_t P_{C,t+1}}{P_{C,t}} \right)^{\sigma^{-1}} \tag{124}$$

Using (121), we know that $C_t = kY_t S_t^{-\gamma}$, and hence $E_t C_{t+1} = kE_t \left\{ Y_{t+1} S_{t+1}^{-\gamma} \right\}$. Using these expressions in (124) we get

$$kY_t S_t^{-\gamma} = kE_t \left\{ Y_{t+1} S_{t+1}^{-\gamma} \right\} * \left(\beta^{-1} R_t^{-1} \frac{E_t P_{C,t+1}}{P_{C,t}} \right)^{\sigma^{-1}} \tag{125}$$

which can be rewritten as

$$Y_t = E_t \left\{ Y_{t+1} \left(\frac{S_{t+1}}{S_t}\right)^{-\gamma} \right\} * k^{-1} \left(\beta^{-1} R_t^{-1} \frac{E_t P_{C,t+1}}{P_{C,t}} \right)^{\sigma^{-1}} \tag{126}$$

Combining this expression with the solution for the terms of trade (122) yields an expression that determines domestic output demand, conditional on foreign output and the path of the real interest rate. In log-linearized form this expression is the famous open economy IS equation (160).

In addition, (121) and (122) can be combined to yield an expression for consumption as a function of domestic and foreign output:

$$C_t = k(Y_t)^{1-\gamma} (Y_t^*)^{\gamma} \tag{127}$$

For the foreign country we also get (127) as according to (116), we have $C_t = C_t^*$.

On the supply side, notice that

$$N_t = \int_0^1 N_t(f) df = \frac{Y_t}{A_t} \int_0^1 (Y_t(f)/Y_t) df$$

and (98) can be combined to yield the aggregate production function

$$Y_t = \frac{A_t N_t}{V_t} \tag{128}$$

where

$$V_t \equiv \int_0^1 (\frac{P_{C,t}(f)}{P_{C,t}})^{-\xi} df \geq 1 \tag{129}$$

For the foreign country we get

$$N_t^* = \int_0^1 N_t^*(f^*) df^* = \frac{Y_t^*}{A_t^*} \int_0^1 (Y_t^*(f^*)/Y_t^*) df^*, \tag{130}$$

which can be combined with (101) to give the foreign aggregate production function

$$Y_t^* = \frac{A_t^* N_t^*}{V_t^*} \tag{131}$$

where

$$V_t^* \equiv \int_0^1 (\frac{P_{C,t}^*(f^*)}{P_{C,t}^*})^{-\xi} df^* \geq 1 \tag{132}$$

Combining the labor supply and demand relations, equations (70) and (105) respectively, and then using the aggregate demand schedule (121) and the aggregate production function (128) to eliminate C_t and N_t yields the following expression for real marginal cost:

$$\begin{aligned} MC_t &= (1-\tau)(1+\mu_t^w) \frac{k^{-1} N_t^{\phi} C_t^{\sigma} S_t^{\gamma}}{A_t} \\ &= (1-\tau) k^{\sigma-1} (1+\mu_t^w) A_t^{-(1+\phi)} Y_t^{\kappa} (Y_t^*)^{\kappa_0} V_t^{\phi}, \end{aligned} \tag{133}$$

where $\kappa = \frac{\partial MC_t}{\partial Y_t}$ and $\kappa_0 = \frac{\partial MC_t}{\partial Y_t^*}$ are the elasticities of marginal cost with respect to domestic and foreign output, given by

$$\kappa \equiv \sigma(1-\gamma) + \gamma + \phi = \sigma + \phi - \kappa_0 \tag{134}$$

and

$$\kappa_0 \equiv \sigma\gamma - \gamma = \gamma(\sigma - 1) \tag{135}$$

For the foreign country we get:

$$\begin{aligned} MC_t^* &= (1-\tau)(1+\mu_t^{w*})\frac{k^{-1}(N_t^*)^{\phi}(C_t^*)^{\sigma}S_t^{-(1-\gamma)}}{A_t^*} \\ &= (1-\tau)k^{\sigma-1}(1+\mu_t^{w*})(A_t^*)^{-(1+\phi)}Y_t^{\kappa_0^*}(Y_t^*)^{\kappa^*}(V_t^*)^{\varphi}, \end{aligned} \tag{136}$$

where $\kappa^* = \frac{\partial MC_t^*}{\partial Y_t^*}$and $\kappa_0^* = \frac{\partial MC_t^*}{\partial Y_t}$ are the elasticities of foreign marginal cost with respect to foreign and domestic output,[35] given by

$$\kappa^* \equiv \sigma + \varphi - \kappa_0^* \tag{137}$$

and

$$\kappa_0^* \equiv (1-\gamma)(\sigma-1) \tag{138}$$

Note that it is also possible to write down an expression for home and foreign marginal cost in *non-isomorphic form*. Then, one does not eliminate the terms of trade via equation (122).Then, equations (133) and (136) become

$$MC_t = (1-\tau)(1+\mu_t^w)k^{(\sigma-1)}Y_t^{(\phi+\sigma)}V_t^{\phi}A_t^{-(1+\phi)}S_t^{\gamma(1-\sigma)} \tag{139}$$

and

$$MC_t^* = (1-\tau)(1+\mu_t^{w*})k^{(\sigma-1)}(Y_t^*)^{(\phi+\sigma)}(V_t^*)^{\phi}(A_t^*)^{-(1+\phi)}S_t^{(1-\gamma)(\sigma-1)} \tag{140}$$

where $\kappa = \frac{\partial MC_t}{\partial Y_t}$ and $\hat{\kappa} = \frac{\partial MC_t}{\partial S_t}$are the elasticities of marginal cost with respect to domestic output and the terms of trade, given by

$$\kappa \equiv \sigma + \phi \tag{141}$$

and

$$\hat{\kappa} \equiv \gamma(1-\sigma) \tag{142}$$

Similarly, $\kappa^* = \frac{\partial MC_t^*}{\partial Y_t^*}$and $\hat{\kappa}^* = \frac{\partial MC_t^*}{\partial S_t}$ are the elasticities of foreign marginal cost with respect to foreign output and the terms of trade, given by

$$\kappa^* \equiv \sigma + \varphi \tag{143}$$

[35] Note that in the subsequent loglinearization of (133) and (136) it is assumed that $V_t = V_t^* = 1$. This has to do with the fact that the symmetry assumption used by CGG (2002, p. 888) whereby all firms choose the same level of output.

and

$$\hat{\kappa}^* \equiv (1-\gamma)(\sigma-1) \tag{144}$$

As pointed out by CGG (2002, p. 887), the *international transmission effects* of monetary policy depend critically on how the open economy affects the behavior of marginal cost, as summarized by the two key elasticities κ and κ_0. Note first that the sign of κ_0, the elasticity of marginal cost with respect to foreign output, is *ambiguous.* There are two effects of a change in Y_t^*on MC_t that work in opposite directions: A rise in Y_t^*causes the terms of trade to appreciate (S_t falls) which, holding constant domestic consumption reduces domestic marginal cost, as equation (133) suggests, as reflected by the term $-\gamma$ in (135) (or by the term γin (142)). The reason that marginal cost falls is that the real PPI-based appreciation lowers the CPI and hence according to (70) domestic *nominal* wages. In turn, given the level of $P_{H,t}$ the movement in the terms of trade then lowers *real* wages $W_t/P_{H,t}$ which (in line with (105)) reduces marginal cost. Obviously, this partial effect is stronger, the stronger the effect of the terms of trade on the CPI, that is the more open the domestic economy, that is the higher the parameter γ. In this sense, an expansionary foreign monetary policy that raises Y_t^*(induced say by a lower foreign nominal interest rate) is a *locomotive policy* as it tends to raise both output at home and abroad. At the same time, however, holding constant domestic output, the increase in Y_t^* raises domestic consumption due to risk sharing, leading to an increase in marginal cost (since the rise in $C_t$increases the marginal rate of substitution between consumption and leisure). The latter wealth effect, captured by the term $\sigma\gamma$ in (135), dominates the terms of trade effect if $\sigma > 1$ (implying $\kappa_0 > 0$); and vice versa if $\sigma < 1$ (implying $\kappa_0 < 0$).

In turn, the impact of the open economy on κ, the elasticity of marginal cost with respect to domestic output, depends inversely on κ_0. An increase in Y_t , for example causes a depreciation in the terms of trade, raising MC_t. That effect is captured by the term γ in (134). Due to risk sharing, however, consumption increases by proportionately less than the increase in domestic output, which works to dampen the increase in marginal cost (relative to the closed economy), as reflected by the term $\sigma(1-\gamma)$ in (134).Finally, there is a third channel, also found in the closed economy, through which domestic output variations affect marginal cost, and which works through the effect on employment and the disutility of labor, as captured by the term φ. As pointed out by CGG (2002, p. 887), in the end whether openness increases or decreases the elasticity of marginal cost with respect to domestic output (relative to the closed economy) depends on the size of σ. Again, the wealth effect dominates the terms of trade effect when $\sigma > 1$ (implying $\kappa_0 > 0$). In this instance, the open economy's aggregate marginal cost schedule is flatter than its closed economy counterpart (i.e. since $\kappa_0 > 0$, κ is below its value for a closed economy (given by $\sigma + \varphi$), holding constant the preference parameters σ and φ.

CGG (2002) emphasize that in the knife-edge case of logarithmic utility ($\sigma = 1$), the terms of trade and risk sharing effects cancel, in this instance, there are no direct effects of the open economy on marginal cost: $\kappa_0 = 0$ and $\kappa = \sigma + \varphi$, exactly as for a closed economy.

8.6.1 Equilibrium under flexible prices

We consider an equilibrium with flexible prices where the wage markup is fixed at its steady-state value $1 + \mu^w$. We focus on this case because we would like to define a measure of the natural level of output that has the feature that cyclical fluctuations in this construct do not refelct variations in the degree of efficiency (hence we shut off variation in the wage markup). This approach also makes sense if we think of variations in the wage markup as standing in for wage rigidity.

In addition, we make the distinction between the equilibrium that arises when prices are flexible at home, taking foreign output as given, and the one that arises when prices are flexible across the globe. Following CGG (2002) we refer to the former as the '*domestic* flexible price equilibrium' and the latter as (just) the 'flexible price equilibrium'.The distinction between these concepts is highly relevant when CGG (2002) compare the Nash versus cooperative equilibria.

8.6.2 The domestic flexible price equilibrium

Let a variable with an upper bar (e.g. $\bar{X}_t$)denote its value when prices are flexible at home, but foreign output is taken as exegenously given (independently of how it is determined). We proceed to characterize the domestic flexible price equilibrium, as follows.

Under flexible prices, all firms set their price equal to a constant markup over marginal cost, as implied by condition (108).Symmetry, further, implies that all firms choose the same price. Imposing the restriction $P^0_{H,t}/P_{H,t} = 1$ on equation (108) implies that in the flexible price equilibrium, real marginal cost is constant and given by

$$\bar{MC} = \frac{1}{1+\mu^p} \tag{145}$$

where we use the bar to denote the domestic flexible price equilibrium value of a variable. Symmetry of prices further implies that all firms use the same level of output, inducing $\bar{V}_t = 1$. Hence from equation (128),

$$\bar{Y}_t = A_t \bar{N}_t. \tag{146}$$

Furthermore, using (145) and fixing the wage markup at its steady state then permits us to use equation (133) to solve for the natural level of output in the domestic flexible price equilibrium

$$\bar{Y}_t = \left(\frac{k^{1-\sigma} A_t^{1+\phi} (Y_t^*)^{-\kappa_0}}{(1-\tau)(1+\mu^w)(1+\mu^p)} \right)^{1/\kappa} \tag{147}$$

Note that the impact of foreign output Y_t^* on $\bar{Y}_t$ depends on the sign of κ_0. If $\kappa_0 < 0$ (implying that MC_t is decreasing in foreign output Y_t^*), then $\bar{Y}_t$ varies positively with Y_t^*; and vice versa if $\kappa_0 > 0$. With $\kappa_0 = 0$, $\bar{Y}_t$ depends only on domestic economic factors.

For the foreign country the natural level of output - taken home output as given - is given by

$$\bar{Y}_t^* = \left(\frac{k^{1-\sigma} (A_t^*)^{1+\phi} (Y_t)^{-\kappa_0^*}}{(1-\tau)(1+\mu^{w*})(1+\mu^p)} \right)^{1/\kappa^*} \tag{148}$$

8.6.3 Flexible price equilibrium

We obtained the domestic natural level of ouput, $\bar{Y}_t$, by taking foreign output as exogenously given. As discussed by CGG (2002), it is also useful to define the natural level of output $\bar{\bar{Y}}_t$, that arises when prices are flexible worldwide:

$$\bar{\bar{Y}}_t = \left(\frac{k^{1-\sigma} A_t^{1+\phi} (\bar{\bar{Y}}_t^*)^{-\kappa_0}}{(1-\tau)(1+\mu^w)(1+\mu^p)} \right)^{1/\kappa} = \bar{Y}_t \left(\frac{\bar{\bar{Y}}_t^*}{Y_t^*} \right)^{-\kappa_0/\kappa} \tag{149}$$

We now express the system with sticky prices as a loglinear approximation about the steady state that determines behavior conditional on a path for the nominal interest rate. We use lower case variables to denote log deviations from the deterministic steady state.

From equation (121), aggregate demand is given by

$$y_t = c_t + \gamma s_t, \tag{150}$$

where from the Euler equation (78), aggregate consumption evolves according to

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma}(r_t - E_t \{\pi_{H,t+1}\} - \gamma E_t \{\Delta s_{t+1}\}), \tag{151}$$

where r_t is the nominal rate of interest, $\pi_{H,t+1}$ is the rate of domestic inflation from t to $t+1$. In what follows we use the symbol π_t as shorthand for $\pi_{H,t}$. For the foreign country we get

$$c_t^* = E_t^* \left\{c_{t+1}^*\right\} - \frac{1}{\sigma}(r_t^* - E_t^* \left\{\pi_{F,t+1}^*\right\} + (1-\gamma) E_t \{\Delta s_{t+1}\}) \tag{152}$$

and

$$y_t^* = c_t^* - (1-\gamma) s_t. \tag{153}$$

From equation (122), the terms of trade is given by

$$s_t = y_t - y_t^* \tag{154}$$

On the *supply side*, the first-order approximation to the aggregate production function (128) implies

$$y_t = a_t + n_t \tag{155}$$

Further, combining the loglinearized price setting rule (107) with the price index (109) yields

$$\pi_t = \delta mc_t + \beta E_t \{\pi_{t+1}\}, \tag{156}$$

where $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. Let $\tilde{y}_t = y_t - \bar{y}_t$ denote the domestic output gap, i.e. the gap between output and thedomestic natural level. Then, from the loglinearized version of the expression for marginal cost (133) and the production function (155), we obtain[36]

$$mc_t = \kappa \tilde{y}_t + \mu_t^w, \tag{157}$$

where from equation (147):

$$\bar{y}_t = \kappa^{-1}[(1+\phi)a_t - \kappa_0 y_t^*] \tag{158}$$

For the foreign country we get

$$\bar{y}_t^* = (\kappa^*)^{-1}[(1+\phi)a_t^* - \kappa_0^* y_t]. \tag{159}$$

8.7 The isomorphic loglinearized model

As pointed out by CGG (2002, p. 889), it is straightforward to collapse the system into IS[37] and Phillips curve[38]-type equations that determine $\tilde{y}_t$ and π_t conditional on the path of r_t:

$$\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - \sigma_0^{-1}[r_t - E_t \{\pi_{t+1}\} - r\bar{r}_t] \tag{160}$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda_0 \tilde{y}_t + u_t \tag{161}$$

with

$$\sigma_0 = \sigma - \kappa_0, \tag{162}$$

[36] The loglinearized version of (133) - taking into account that $V_t = 1$ and hence $v_t = \ln(V_t) = 0$ - is $mc_t = \mu_t^w - (1+\varphi)a_t + \kappa y_t + \kappa_0 y_t^*$. Rewriting (158) as $a_t = \frac{\bar{y}_t}{\kappa^{-1}(1+\varphi)} + \frac{\kappa_0 y_t^*}{(1+\varphi)}$. Substituting this expression into the expression for mc_t above, we get (157).

[37] Equation (160) can be simply derived by combining (150) and (151), using equation (171) and the fact that $y_t = \tilde{y}_t + \bar{y}_t$.

[38] The Phillips curve is obtained upon substituting (157) into (156).

$$\lambda_0 = \delta\kappa \tag{163}$$

and $u_t = \delta\hat{\mu}_t^w$, and where $r\bar{r}_t$ is the domestic natural real interest rate (conditional on foreign output), given by

$$r\bar{r}_t = \sigma_0 E_t\{\Delta\bar{y}_{t+1}\} + \kappa_0 E_t\{\Delta y^*_{t+1}\} \tag{164}$$

In addition, we assume that the 'cost push shock' u_t obeys the following stationary first-order process:

$$u_t = \rho u_{t-1} + \epsilon_t \tag{165}$$

with $0 < \rho < 1$, and where ϵ_t is white noise.

For the *foreign country* we get

$$\tilde{y}^\star_t = E^\star_t \tilde{y}^\star_{t+1} - \sigma_0^{\star-1}\left[r^\star_t - E^\star_t \pi^\star_{t+1} - \overline{rr}^\star_t\right], \tag{166}$$

$$\pi^*_t = \beta E^*_t\left\{\pi^*_{t+1}\right\} + \lambda^*_0 \tilde{y}^*_t + u^*_t, \tag{167}$$

$$r\bar{r}^*_t = \sigma^*_0 E^*_t \Delta\bar{y}^*_{t+1} + \kappa^*_0 E^*_t\{\Delta y_{t+1}\}, \tag{168}$$

$$u^\star_t = \rho u^\star_{t-1} + \epsilon^\star_t \tag{169}$$

with $\pi^*_t = \pi^*_{F,t}$ is shorthand for foreign producer price inflation, $\sigma^*_0 = \sigma - \kappa^*_0$, $\kappa^*_0 \equiv (1-\gamma)(\sigma-1)$, $\kappa^\star = \sigma - \kappa^\star_0 + \phi$, $\lambda^\star_0 = \delta\kappa^\star$, and where $u^*_t = \delta\hat{\mu}^{w*}_t$, $\lambda^* = \delta\kappa^*$. Note that in general, $\lambda_0 \neq \lambda^*_0$ as $\kappa \neq \kappa^*$ because $\kappa_0 \neq \kappa^*_0$. The latter has to do with the fact that $\kappa_0 = \gamma(\sigma-1)$, whereas $\kappa^*_0 \equiv (1-\gamma)(\sigma-1)$. Only for the special case that $\gamma = 0.5$ we have that $\kappa_0 = \kappa^*_0$, and hence $\kappa = \kappa^*, \lambda_0 = \lambda^*_0$. The operator $E^\star_t$ indicates rational expectations of the participants in the foreign economy, in what follows to simplify notation we use E_t instead of $E^\star_t$, but resort back to $E^\star_t$ when necessary (e.g. in case of learning).

As we have shown earlier, because of non-heterogeneity of consumption baskets between the two countries and the law of one price the *nominal* exchange rate e_t obeys CPI-based or 'aggregate' PPP, and is therefore given by

$$e_t = (p_{C,t} - p^*_{C,t}) = (p_t + \gamma s_t) - (p^*_t - \{(1-\gamma)s_t\}) = p_t - p^*_t + s_t \tag{170}$$

where p_t is shorthand for $p_{H,t}$ being the domestic price level, and p_t^* is shorthand for $p_{F,t}^*$.

Finally, we obtain a simple expression linking the terms of trade to movements in the output gap:

$$s_t = (\tilde{y}_t - \tilde{y}_t^*) + (\bar{y}_t - \bar{y}_t^*) = (\tilde{y}_t - \tilde{y}_t^*) + \bar{s}_t \tag{171}$$

where $\bar{s}_t$ is the natural level of the terms of trade.

Equations (160), (161), (164), (165), (166), (167), (168), (169), (170) and (171) define the model that we work with in the main text. Together they define a system of ten equations and as many endogenous variables, namely $\tilde{y}_t, \tilde{y}_t^\star, \pi_t, \pi_t^*, r\bar{r}_t, r\bar{r}_t^*, u_t, u_t^\star, e_t$ and s_t.Of course, crucial exogenous variables are the home and foreign nominal interest rates r_t and $r_t^\star$. In the main text of the paper, these will be modelled by both *instrument* and *target* rules.

8.8 The non-isomorphic loglinearized model

Note that the framework above which is the CGG (2002) model is isomorphic to that of the *closed economy*.[39] The reason is that the key equations can be formulated without reference to home and foreign CPI inflation. Moreover, in the modelling of targeting rules for nominal interest rates, typically the latter are derived from an explicit optimization exercize whereby the central bank trades off output gap stabilization and inflation rate stabilization, where the latter in an open economy should be captured by *CPI inflation*, in accordance with, for example

$$(1-\gamma)\,\Lambda^{"} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{2} [\left(\pi_{C,\tau} - \pi_C^T\right)^2 + \alpha_0^{"}\left(\tilde{y}_\tau - \tilde{y}^T\right)]^2 \tag{172}$$

However in CGG (2001, 2002) we know that the 'terms of trade gap' $s_t - \bar{s}_t$ is proportional to the outgput gap $\tilde{y}_t$. Accordingly, under certain conditions[40], based on a second order approximation of the houshold's utility

[39] Anther important isomorphic model is the CGG (2001), which is a small open economy (SOE) framework.

[40] One of which is to take the level of foreign output as exogenous.

function, it is possible to collapse the policy objective (172) to

$$(1-\gamma)\,\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{2} [\left(\pi_\tau - \pi^T\right)^2 + \alpha_0 \left(\tilde{y}_\tau - \tilde{y}^T\right)]^2 \tag{173}$$

where we have used equation (171) to fold $s_t - \bar{s}_t$ into $\tilde{y}_t$, and equation (91) to map $\pi_{C,t}$ into π_t and Δs_t.

Thus, the monetary policy problem for the two-country world has - from the perspectives of both economies - the same form as in the closed economy studied by Clarida, Gali and Gertler (1999). More specific, the functional forms are the same, but the (reduced) form coefficients in those relations differe from the ones in the closed economy form as they embody open economy effects, as is evident from (134), (135) and (162).

Note however, that since the functional forms are equivalent it is also possible to work with (172). Of course, then the constraints should also explicitly incorporate home and foreign CPI inflation. Accordingly, we can write down the corresponding *non-isomorphic* model as:

$$\tilde{y}_t = E_t\left\{\tilde{y}_{t+1}\right\} - \sigma^{-1}[r_t - E_t\left\{\pi_{C,t+1}\right\}] - \gamma E_t\left\{\Delta s_{t+1}\right\} + E_t\left\{\Delta \bar{y}_{t+1}\right\} \tag{174}$$

The non-isomorphic IS curve now involves two new variables: the (expected depreciation of) the terms of trade and (expected) CPI inflation. As a consequence, instead of the coefficient σ_0^{-1}we now have simply σ^{-1}. Also instead of the variable $r\bar{r}_t$ we get $E_t\left\{\Delta \bar{y}_{t+1}\right\}$. This equation can be simply derived by combining (150), (151) and (91), and using that $y_t = \tilde{y}_t + \bar{y}_t$.

The home *CPI inflation rate* is now given by

$$\pi_{C,t} = \pi_t + \gamma \Delta s_t, \tag{175}$$

whereas the Phillips curve remains as before [NOTE ZIET ER NIET NON-ISOMORPHIC UIT VANWEGE LAMDA0 TERM]:

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \lambda_0 \tilde{y}_t + u_t, \tag{176}$$

with $\lambda_0 = \delta\kappa$, $\kappa \equiv \sigma(1-\gamma) + \gamma + \phi = \sigma + \phi - \kappa_0$, $\kappa_0 \equiv \sigma\gamma - \gamma = \gamma(\sigma - 1)$.

The cost-push shock, u_t, still obeys:

$$u_t = \rho u_{t-1} + \epsilon_t, \tag{177}$$

with $0 < \rho < 1$, and where ϵ_t is white noise.

For the *foreign country* we get

$$\tilde{y}_t^* = E_t \left\{\tilde{y}_{t+1}^*\right\} - \sigma^{-1}[r_t^* - E_t^* \left\{\pi_{C,t+1}^*\right\}] + (1-\gamma)E_t^* \left\{\Delta s_{t+1}\right\} + E_t^* \left\{\Delta \bar{y}_{t+1}^*\right\} \tag{178}$$

$$\pi_{C,t}^* = \pi_t^* - (1-\gamma)\Delta s_t$$

$$\pi_t^* = \beta E_t^* \left\{\pi_{t+1}^*\right\} + \lambda_0^* \tilde{y}_t^* + u_t^* \tag{179}$$

$$u_t^\star = \rho u_{t-1}^\star + \epsilon_t^\star \tag{180}$$

As before, the *nominal* exchange rate e_t obeys CPI-based or 'aggregate' PPP, and is therefore given by

$$e_t = e_{t-1} + (\pi_{C,t} - \pi_{C,t}^*) \tag{181}$$

Finally, we have the usual expression linking the terms of trade to movements in the output gap:

$$s_t = (\tilde{y}_t - \tilde{y}_t^*) + (\bar{y}_t - \bar{y}_t^*) = (\tilde{y}_t - \tilde{y}_t^*) + \bar{s}_t \tag{182}$$

where $\bar{s}_t$ is the natural level of the terms of trade. Note that in the non-isomorphic model $\sigma^* = \sigma$, and open economy effects only affect the supply-side elasticities $\lambda_0, \lambda_0^*, \kappa, \kappa^*, \kappa_0$ and κ_0^*.

9 Appendix B: determinacy conditions in a symmetric two-country world

We start with the averages and differences systems in state-space form:

$$z_t^A = B^A z_{t+1}^{Ae}, \text{and} \tag{183}$$

$$z_t^D = B^D z_{t+1}^{De} \tag{184}$$

where $z_t^A = \left[\tilde{y}_t^A, \pi_t^A\right]'$ and $z_t^D = \left[\tilde{y}_t^D, \pi_t^D\right]'$. We can infer the state-space form for $z = [\tilde{y}_t, \pi_t]'$ and $z_t^* = [\tilde{y}_t^*, \pi_t^*]'$by using that $z = z^A + \frac{1}{2} z^D$ and $z^* = z^A - \frac{1}{2} z^D$. Then, for the home country we get

$$z_t = B^A E_t z_{t+1}^A + \frac{1}{2} B^D E_t z_{t+1}^D \tag{185}$$

which can be rewritten as

$$z_t = B^A [\frac{1}{2} E_t z_{t+1} + \frac{1}{2} E_t z_{t+1}^*] + \frac{1}{2} B^D [E_t z_{t+1} - E_t z_{t+1}^*] \tag{186}$$

or

$$z_t = \frac{B^A + B^D}{2} E_t z_{t+1} + \frac{B^A - B^D}{2} E_t z_{t+1}^* \tag{187}$$

Following a similar approach for the foreign country we get

$$z_t^* = \frac{B^A - B^D}{2} E_t z_{t+1} + \frac{B^A + B^D}{2} E_t z_{t+1}^* \tag{188}$$

Define $z_t' = [z_t, z_t^*]'$, then we have

$$z_t^{'} = \mathcal{B} E_t z_{t+1}' \tag{189}$$

where$\mathcal{B}$ is a 2 x 2 matrix given by

$$\mathcal{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, \text{ and the sub-matrices } B_{ij} \text{are also 2 x 2 and given by} \tag{190}$$

$$B_{11} = B_{22} = \frac{B^A + B^D}{2} \tag{191}$$

$$B_{12} = B_{21} = \frac{B^A - B^D}{2} \tag{192}$$

For this system to be determinate we need both of the eigenvalues of $\mathcal{B}$ to be inside the unit circle.