Scenario-Generation Methods for Public Debt Management

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The management of the public debt is of paramount importance for any country. From the mathematical viewpoint, it can be seen as a stochastic optimal control problem with a number of constraints imposed by national and supranational regulations and by market practices.

The Public Debt Management Division of the Italian Ministry of Economy asked some time ago our Institute to face such problem and in particular to determine the composition of the portfolio issued every month which minimizes a predefined *objective function* [1].

In a very simplified form the problem can be described as follows. The Italian Treasury Department issues ten different types of securities including one with floating rate. Securities differ in the maturity (or expiration date) and in the rules for the payment of interests. Short term securities (those having maturity up to two years) do not have coupons. Medium and long term securities (up to thirty years) pay cash dividends, every 6 months, by means of coupons. The problem is to find a strategy for the selection of public debt securities that minimizes the expenditure for interest payment (according to the ESA95 criteria) and satisfies, at the same time, the constraints on debt management. The cost of future interest payments depends on the future value of the term structure (roughly speaking when a security expires or a coupon is paid, there is the need to issue a new security whose cost depends on the term structure at expiration time). This is the reason why there is the need to generate scenarios of future interest rates.

Note that the goal is not to *forecast* the *actual* future term structure but to generate a set of realistic possible scenarios. For our purposes, the scenarios should cover a wide range of possible outcomes of future term-structures in order to provide a reliable estimate of the possible distribution of future debt charges. This distribution is useful in a risk-management setting to estimate, for instance, a sort of *Value at Risk* (VaR) of the selected issuance policy.

In our approach, each rate of the term structure is the result of the sum of two components. The first component is determined by the evolution of the European Central Bank (ECB) official rate r^{ECB}. The second component is represented by the fluctuations having null correlation with the ECB rate.

In mathematical terms, this means that each rate r^h is decomposed as

$$r_t^h = \alpha_h r_t^{ECB} + p_t^{h,\perp} \tag{1}$$

where r_t^{ECB} is the linear interpolation of the ECB rate and α_h is the correlation coefficient between r^h and r^{ECB}, given by

$$\alpha_h = \frac{\mathbb{E}((r^{ECB} - \mathbb{E}(r^{ECB})) \cdot (r^h - \mathbb{E}(r^h)))}{\sqrt{\mathrm{Var}(r^{ECB}) \cdot \mathrm{Var}(r^h)}} .$$

By construction, the time series $p_t^{h,\perp}$ has null correlation with r^{ECB}. The factors α^h are different for each maturity and a larger value of α^h means a larger correlation with the ECB rate. Numerical tests show that, as expected, longer maturities are less correlated with the ECB official rate.

The problem of simulating a new term structure is decomposed in two sub-problems: *i)* the simulation of the ECB rate r^{ECB} and *ii)* the simulation of the fluctuations $p_t^{h,\perp}$.

For the simulation of the ECB official rate we resort to a simple model that couples the evolution of the European Inflation and the ECB official rate. The input data set is represented by the monthly observations of the European Inflation from January 1, 1999 up to day and the ECB official rate for the same period. The simulation of the inflation is required also because the Italian Treasury issues a security whose coupon is bound to the European Inflation.

The process that represents the evolution of the inflation π is the following:

$$\pi_t = \pi_{t-1} + s_t \times \delta_t$$

where δ_t is a discrete random variable selected according to the distribution of the absolute value of the increments of the historical inflation and s_t is a random "sign" variable that assumes value 1.0 with probability p_t or -1.0 with probability $(1 - p_t)$. If there is no *trend*, $p_t = 0.5$.

The model simulates the changes of the ECB rate as a response function to the observation of the behaviour of the inflation. The observation period is set equal to six months. If there is a net change of inflation that exceeds a predefined threshold Δ_{thr} in this time frame, there is a change of the ECB rate. If the inflation grows, there is obviously a raise of the official rate whereas if the inflation diminishes, the official rate decreases.

The change of the official rate is either 25 or 50 basis points with equal probability. This choice is coherent with historical data of past ECB interventions. There are lower bounds for both the ECB official rate and the inflation.

The changes of the ECB official rate determine a feedback on the evolution of the inflation. Basically this means a modification of the trend. If there is a change of the ECB official rate at time t_c equal to $\delta_{t_c}^{ECB}$, then the probability p associated to the sign of the inflation increments changes as follows:

$$p = p_{t_c} * (1 - (\delta_{t_c}^{ECB} / r_{t_c}^{ECB}))$$

where $r_{t_c}^{ECB}$ is the ECB official rate a time t_c.

As to the simulation of the fluctuations $p_t^{h,\perp}$, we employ two different techniques. The first one is based on the Principal Component Analysis (PCA). The second one is based on the following multi-dimensional extension of the Cox-Ingersoll-Ross (CIR) model

$$dp_t^{h,\perp} = k_h(\mu_h - p_t^{h,\perp})dt + \sqrt{p_t^{h,\perp}} \sum_{j=1}^{M} \sigma_{hj} dB_t^j \quad \text{for } h = 1, \ldots, M$$

where $B_t = (B_t^1, \ldots, B_t^M)$ is an M-dimensional Brownian motion representing M sources of randomness (M is the number of possible maturities).

References

[1] Adamo M *et al.,* 2004, Optimal Strategies for the Issuances of Public Debt Securites, *International Journal of Theoretical and Applied Finance* 7(7), 805-822.