Breaking trend panel unit root tests

Pui Sun Tam[*]

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Abstract

This paper proposes Lagrange Multiplier based panel unit root tests allowing for structural breaks through simple extensions of existing group mean and combination tests. The proposed tests are more general than those previously suggested. They consider potential breaks in the intercept, in the slope, and both. A desirable property of the tests is their flexibility to accommodate heterogeneous break types across cross-sections in a panel. Response surfaces to approximate finite sample distributions of the underlying test statistics required to implement the panel tests are provided. The tests are analyzed for the case when the break dates are known and for the case when they are endogenously determined. A bootstrap test is further suggested to deal with cross-sectional dependency. The proposed tests are applied to two major macroeconomic variables, per capital gross domestic product and consumer prices of OECD countries.

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[*] Faculty of Business Administration, University of Macau, Av. Padre Tomás Pereira, Taipa, Macau, China. Telephone: 853-397-4756; Fax: 853-838-320; Email: pstam@umac.mo.

1. Introduction

Notwithstanding their frequent use in applied works, most panel unit root tests in the literature ignore the presence of potential structural breaks. Erroneous omission of structural breaks may induce deceptive inference, as evidenced by Perron (1989) for the univariate case, and Murray and Papell (2000), Carrion-i-Silvestre *et el*. (2002), Breitung and Candelon (2005) and Im *et el*. (2005, hereafter ILT) in the panel setting. However, only a handful of studies, including Murray and Papell (2000), Carrion-i-Silvestre *et el*. (2002), Papell (2002), Tzavalis (2002), Bai and Carrion-i-Silvestre (2004), Breitung and Candelon (2005) and ILT, have attempted to develop panel unit root tests with structural breaks. This paper addresses this concern and proposes extensions to the ILT test, and the combination tests of Maddala and Wu (1999) and Choi (2001) originally considered for the no break case to account for structural breaks.

The ILT test is noticeable among the prevailing panel unit root tests with structural breaks. It is a group mean test that combines individual exogenous intercept break tests, developed by Amsler and Lee (1995, hereafter AL), across cross-sections in a panel. Based on the Lagrange Multiplier (LM), or score, principle advanced by Schmidt and Phillips (1992, hereafter SP), the test allows for breaks under both the null and alternative hypotheses, so that rejection of the null unambiguously implies trend stationarity. It also permits a high degree of heterogeneity across cross-sections in a panel. However, two deficiencies of the test can be heeded. First, it only caters for intercept breaks, but for trending variables, it is possible for a break to affect the intercept of the trend function, as well as its slope. Erroneous determination of the break type may lead to incorrect conclusions (see Montañés *et al.*, 2005). Second, it is devised under the assumption of known breaks, which is strongly criticized for the

problem of pre-testing and data-mining (see Christiano, 1992). Thus, ILT apply the minimum LM test of Lee and Strazicich (2003) to select the break date for each individual, and then combine the minimum LM tests as in the known break case, but do not provide finite sample performance of the resulting panel unit root test.

This paper contributes in the following respects. First, we extend the ILT test to incorporate breaks in the intercept and slope, and pure breaks in the slope. Second, we show that the ILT test with minimum LM break selection exhibits severe over-rejections of the null. As a remedy, we apply the minimum sum of squared residuals (SSR) test of Nunes (2004) to select the break date for each time series. Notice that the ILT test, by combining test statistics, only has limited ability to permit heterogeneous break type across cross-sections in a panel. Thus, we also extend the combination tests to account for structural breaks. Since combination tests are based on combining *p*-values from unit root tests applied to individual time series in a panel, they permit the use of a different unit root test on a different time series in the same panel. Consequently, combination tests permit full flexibility to accommodate heterogeneous break types across cross-sections.

We also simulate finite sample distributions of individual statistics required for computing the extended ILT and combination tests, and summarize the simulation results using response surfaces. We highlight in passing the importance of using finite sample distributions simulated under different lag order assumptions, which has not been thoroughly examined for panel unit root tests in the literature. We further suggest employing the bootstrap approach to compute the panel unit root tests when the cross-sectional independence assumption is relaxed.

The plan of this paper is as follows. Section Two details the testing methodology. Section Three deals with practical implementation issues of the proposed tests.

Section Four reports Monte Carlo simulation results, Section Five provides empirical applications, and Section Six concludes.

2. Testing Methodology

2.1 Model Framework

We consider the following general data generating process (DGP) for each cross-section in a panel

$$y_{it} = \mu_i + Z_{it}\delta_i + u_{it}, \quad u_{it} = \alpha_i u_{i,t-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim iidN(0,\sigma_i^2), \quad i = 1,\cdots,N, \quad t = 1,\cdots,T, \tag{1}$$

where Z_{it} is the nonstochastic process excluding the intercept term, δ_i is the vector of coefficients corresponding to Z_{it}, u_{i0} is fixed, and *iid* means independently and identical distributed. Thus, individual specific intercepts, time trend coefficients, structural break coefficients, serial correlation structure, first-order autoregressive parameters and break locations are allowed.[1] No contemporaneous correlations among cross-sections are assumed, that is, ε_{it} is independent of ε_{js} when $i \neq j$ for all t and s. The models of interest are specified as follows

(Model 0) $Z_{it} = t$, $\delta_i = \delta_{1i}$,

(Model 1) $Z_{it} = (t, DU_{it})$, $\delta_i = (\delta_{1i}, \delta_{2i})'$,

(Model 2) $Z_{it} = (t, DU_{it}, DT_{it})$, $\delta_i = (\delta_{1i}, \delta_{2i}, \delta_{3i})'$,

(Model 3) $Z_{it} = (t, DT_{it})$, $\delta_i = (\delta_{1i}, \delta_{3i})'$,

where $DU_{it} = 1$ if $t > TB_i$ and 0 otherwise, $DT_{it} = t - TB_i$ if $t > TB_i$ and 0 otherwise,

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[1] With serially correlated errors taking the general form, ε_{it} follows a stationary and invertible autoregressive moving average process $A_i(L)\varepsilon_{it} = B_i(L)v_{it}$, where $A_i(L)$ and $B_i(L)$ are finite order polynomials in the lag operator L with all roots lying outside the unit circle and $v_{it} \sim iidN(0,\sigma_{vi}^2)$, so that ε_{it} can be approximated by an autoregressive process of order k_i, $B_i(L)^{-1}A_i(L)\varepsilon_{it} = C_i(L)\varepsilon_{it} = v_{it}$, with $C_i(L) = \sum_{j=1}^{k_i} c_j L^j$.

TB_i $(1 < TB_i < T)$ being the break date, also defined as $TB_i = \lambda_i T$ $(0 < \lambda_i < 1)$, and λ_i being the break fraction. Model 0 does not account for structural breaks. Model 1 allows for a change in the intercept of the trend function, Model 2 allows for a change in the intercept and slope, whereas Model 3 permits a change in the slope.

The null and alternative hypotheses are

H_0: $\alpha_i = 1$, $i = 1, \cdots, N$,

H_1: $\alpha_i < 1$, $i = 1, \cdots, N_1$, and $\alpha_i = 1$, $i = N_1 + 1, \cdots, N$, $\qquad\qquad$ (2)

where for test consistency, it is assumed N_1 / N is a constant as $N \to \infty$. Under the null of $\alpha_i = 1$, DGP (1) can be expressed as

$$\Delta y_{it} = \Delta Z_{it} \delta_i + \varepsilon_{it}, \quad t = 2, \cdots, T, \quad y_{i1} = \mu_i + Z_{i1} \delta_i + u_{i0} + \varepsilon_{i1}. \qquad (3)$$

Based on the LM, or score principle, SP suggest unit root testing for an individual using the regression equation

$$\Delta y_{it} = \phi_i \widetilde{S}_{i,t-1} + \Delta Z_{it} \gamma_i + \sum_{j=1}^{k_i} c_{ij} \Delta \widetilde{S}_{i,t-j} + e_{it}, \quad t = k_i + 2, \cdots, T. \qquad (4)$$

where $\phi_i = \alpha_i - 1$, $\widetilde{S}_{it} = y_{it} - \widetilde{\mu}_{ui} - Z_t \widetilde{\delta}_i$, $t = 2, \cdots, T$, with $\widetilde{S}_{i1} = 0$, $\widetilde{\delta}_i$ are the restricted MLEs of δ_i obtained by a regression of the null equation (3), and $\widetilde{\mu}_{ui} = y_{i1} - \mu_i - Z_{i1} \widetilde{\delta}_i$, the restricted MLE of $\mu_{ui} (\equiv \mu_i + u_{i0})$. Equation (4) takes the augmented form to allow for autocorrelated errors as in AL. The LM statistic is computed as the t-statistic for testing $\phi_i = 0$. We denote the individual LM statistics under Models 0, 1, 2 and 3 as τ_{iT}^0, $\tau_{iT}^1(\lambda_i)$, $\tau_{iT}^2(\lambda_i)$ and $\tau_{iT}^3(\lambda_i)$ respectively.

2.2 ILT Tests

ILT first consider Model 0. Under the null as $T \to \infty$ for fixed N, each τ_{iT}^0 converges

to the SP distribution, with finite mean E_i^0 and variance V_i^0. [2] With the cross-sectional independence assumption, the τ_{iT}^0's are *iid*. Thus, the panel unit root test under Model 0 is given by

$$\sqrt{N}\,\frac{(\bar{\tau}_{NT}^0 - E_i^0)}{\sqrt{V_i^0}}, \qquad\qquad (5)$$

where $\bar{\tau}_{NT}^0 = (1/N)\sum_{i=1}^{N}\tau_{iT}^0$. The statistic (5) converges to a standard normal distribution as T and N grow large in sequence by the Lindeberg-Lévy central limit theorem (CLT) under the null, and diverges to negative infinity under the alternative. ILT also consider Model 1 with exogenous known breaks. As shown by AL, under the null as $T \to \infty$ for fixed N, each $\tau_{iT}^1(\lambda_i)$ converges to the SP distribution, since ΔDU_{it} in equation (4) equals one at only one point and is thus asymptotically negligible. The panel unit root test under Model 1 with exogenous known breaks is

$$\sqrt{N}\,\frac{(\bar{\tau}_{NT}^1(\lambda) - E_i^0)}{\sqrt{V_i^0)}}, \qquad\qquad (6)$$

where $\bar{\tau}_{NT}^1(\lambda) = (1/N)\sum_{i=1}^{N}\tau_{iT}^1(\lambda_i)$, with $\lambda = (\lambda_1, \cdots, \lambda_N)$.

We extend the ILT testing methodology to Model 2 with intercept and slope breaks and Model 3 with slope breaks only. For Model 2, each $\tau_{iT}^2(\lambda_i)$ has a limiting distribution that depends on the break location, with finite mean E_i^2 and variance V_i^2, under the null as $T \to \infty$ for fixed N, according to Lee and Strazicich (2003) and Nunes (2004).[3] Cross-sectional independence implies the $\tau_{iT}^2(\lambda_i)$'s are *iid*. Thus, we

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[2] The SP distribution is given by $-(1/2)\left[\int_0^1 \underline{V}_i(r)^2\,dr\right]^{-1/2}$, with $\underline{V}_i(r) = V_i(r) - \int_0^1 V_i(r)dr$, a demeaned Brownian bridge, $V_i(r) = W_i(r) - rW_i(1)$, a standard Brownian bridge, and $W_i(r)$, a standard Brownian motion.

[3] The distribution is given by $-\frac{1}{2}\left[\int_0^1 \underline{V}_i(\lambda_i,r)^2\,dr\right]^{-1/2}$, where $\underline{V}_i(\lambda_i,r)$ denotes the residuals from the projection of $V_i(\lambda_i,r)$ onto the orthogonal complement of $dz(\lambda_i,r) = \{1, du(\lambda_i,r)\}$,

formulate the panel unit root test under Model 2 with exogenous known breaks as

$$\Gamma^2(\lambda) = \frac{\sqrt{N}[\bar{\tau}_{NT}^2(\lambda) - E_i^2]}{\sqrt{V_i^2}},$$ (7)

where $\bar{\tau}_{NT}^2(\lambda) = (1/N)\sum_{i=1}^{N}\tau_{iT}^2(\lambda_i)$. For Model 3, under the null as $T \to \infty$ for fixed N,

each $\tau_{iT}^3(\lambda_i)$ has a limiting distribution that depends on the break location, which is

the same as that for $\tau_{iT}^2(\lambda_i)$, since inclusion of ΔDU_{it} in equation (4) is asymptotically

irrelevant. Therefore, the panel unit root test under Model 3 with exogenous known

breaks can be devised likewise as

$$\Gamma^3(\lambda) = \frac{\sqrt{N}[\bar{\tau}_{NT}^3(\lambda) - E_i^3]}{\sqrt{V_i^3}},$$ (8)

where $\bar{\tau}_{NT}^3(\lambda) = (1/N)\sum_{i=1}^{N}\tau_{iT}^3(\lambda_i)$, E_i^3 and V_i^3 denote the mean and variance

respectively of the limiting distribution of $\tau_{iT}^3(\lambda_i)$.[4]

Another extension of the ILT test concerns the endogenous determination of

break dates. ILT propose selecting the break date for each individual time series with

the minimum break selection criterion, that is, the break date is selected where the LM

statistic is minimized. Like the exogenous break test, the minimum LM test with an

intercept break, denoted as $\tau_{iT}^1(\lambda_i(\tau_{iT}^1))$, converges in distribution to the SP test

according to Nunes (2004). The panel unit root test under Model 1 using minimum

LM break selection is calculated as in (6), where $\bar{\tau}_{NT}^1(\lambda(\tau_T^1)) = (1/N)\sum_{i=1}^{N}\tau_{iT}^1(\lambda_i(\tau_{iT}^1))$, with

$\lambda(\tau_T^1) = (\lambda_1(\tau_{1T}^1), \cdots, \lambda_N(\tau_{NT}^1))$, replaces $\bar{\tau}_{NT}^1(\lambda)$. However, the minimum LM test is

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$du(\lambda_i, r) = 1$ if $r > \lambda_i$ and 0 otherwise, $V_i(\lambda_i, r) = W_i(r) - \dfrac{rW_i(\lambda_i)}{\lambda_i}$ if $r \le \lambda_i$, and

$V_i(\lambda_i, r) = W_i(r) - \dfrac{rW_i(\lambda_i)}{\lambda_i} - (r - \lambda_i)\left[\left(\dfrac{W_i(1) - W_i(\lambda_i)}{1 - \lambda_i} - \dfrac{W_i(\lambda_i)}{\lambda_i}\right)\right] = W_i(r) - W_i(\lambda_i) - \dfrac{r - \lambda_i}{1 - \lambda_i}[W_i(1) - W_i(\lambda_i)]$

if $r > \lambda_i$.

[4] Since the limiting distributions of $\tau_{iT}^2(\lambda_i)$ and $\tau_{iT}^3(\lambda_i)$ are the same, $E_i^2 = E_i^3$ and $V_i^2 = V_i^3$. However, because finite sample distributions of the two statistics differ as shown in Section 3.1, we opt for the use of different notations under Models 2 and 3.

dominated by Nunes' (2004) proposed minimum SSR test, where the break date is selected according to the estimator of TB_i (or λ_i), that is, the date (or break fraction) that minimizes the SSR in an ordinary least squares (OLS) regression of the null equation (3). The minimum SSR test with an intercept break, denoted as $\tau_i^1(\hat{\lambda}_i)$, converges in distribution to the SP test. This test is appealing since it selects the correct break date almost perfectly, and it possesses better finite sample properties than the minimum LM test. We calculate the panel unit root test under Model 1 using minimum SSR break selection by replacing $\bar{\tau}_{NT}^1(\lambda)$ with $\bar{\tau}_{NT}^1(\hat{\lambda}) = (1/N)\sum_{i=1}^{N} \tau_{iT}^1(\hat{\lambda}_i)$, where $\hat{\lambda} = (\hat{\lambda}_1, \cdots, \hat{\lambda}_N)$, in (6).

Application of the minimum LM and SSR break selection criteria to individual time series gives the test statistics $\tau_{iT}^2(\lambda_i(\tau_{iT}^2))$ and $\tau_{iT}^2(\hat{\lambda}_i)$ respectively under Model 2, and $\tau_{iT}^3(\lambda_i(\tau_{iT}^3))$ and $\tau_{iT}^3(\hat{\lambda}_i)$ respectively under Model 3. Since these criteria choose the correct break date asymptotically as proved in Nunes (2004), limiting distributions of $\tau_{iT}^2(\lambda_i(\tau_{iT}^2))$ and $\tau_{iT}^2(\hat{\lambda}_i)$ coincide with that of $\tau_{iT}^2(\lambda_i)$, whereas limiting distributions of $\tau_{iT}^3(\lambda_i(\tau_{iT}^3))$ and $\tau_{iT}^3(\hat{\lambda}_i)$ are the same as $\tau_{iT}^3(\lambda_i)$. Therefore, the panel unit root tests under Models 2 and 3 using the two break selection criteria can be formulated likewise as in Model 1 through appropriate standardization of the simple averages of the corresponding individual test statistics.

2.3 Combination Tests

Maddala and Wu (1999) and Choi (2001) consider panel unit root testing under Model 0. Applying a one-sided left-tailed unit root test with no break to an individual time series, the asymptotic *p*-value of the resulting test statistic G_{iT}, denoted as Π_i, is defined as $\Pi_i = F(G_{iT})$, where $F(\cdot)$ denotes the distribution function of G_{iT} when

$T \to \infty$. The four combination tests are the Fisher's inverse chi-square test P, its modified version P_m, the inverse normal test Z and the logit test L

$$P = -2\sum\nolimits_{i=1}^{N} \ln(\Pi_i) . \tag{9}$$

$$P_m = -\frac{1}{\sqrt{N}} \sum\nolimits_{i=1}^{N} [\ln(\Pi_i) + 1)] . \tag{10}$$

$$Z = \frac{1}{\sqrt{N}} \sum\nolimits_{i=1}^{N} \Phi^{-1}(\Pi_i) , \tag{11}$$

$$L = \sqrt{g} \sum\nolimits_{i=1}^{N} \ln\left(\frac{\Pi_i}{1 - \Pi_i}\right) , \tag{12}$$

where $\Phi(\cdot)$ stands for a standard normal distribution function, and $g = [3(5N+4)]/[\pi^2 N(5N+2)]$. With cross-sectional independence, $\Pi_i \sim iidU[0,1]$ as $T \to \infty$ for fixed N under the null, so that $-2\ln(\Pi_i) \sim iid\chi^2(2)$, $\Phi^{-1}(\Pi_i) \sim iidN(0,1)$ and $\ln[\Pi_i/(1-\Pi_i)]$ have the logistic distribution with mean zero and variance $\pi^2/3$. Consequently, $P \sim \chi^2(2N)$, $Z \sim N(0,1)$, and approximately, $L \sim t(5N+4)$, with $t(\cdot)$ being the Student's t-distribution function. Furthermore, as T and N grow large sequentially, P_m, Z and $L \approx \sqrt{3/(\pi^2 N)} \sum\nolimits_{i=1}^{N} \ln[\Pi_i/(1-\Pi_i)]$ converge to a standard normal distribution by the CLT. Under the alternative, P and P_m diverge to positive infinity, whereas Z and L diverge to negative infinity.

It is easy to see that the above testing procedure can readily be adapted to accommodate structural breaks. To this end, the DGP for each cross-section is first specified with the inclusion of appropriate structural break dummies, depending on the type of structural break. Next, the relevant unit root test with a break is applied to each time series, and the corresponding p-values obtained. These p-values are then combined in accordance with (9) to (12) for panel unit root testing. It is noteworthy that because combination tests combine p-values, they are non-parametric in nature,

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so that they allow for a different unit root test applied on a different individual time series with individual specific nonstochastic process, which can be specified under Model 0, 1, 2 or 3. Therefore, in effect, they permit full flexibility to accommodate heterogeneous break type across cross-sections in a panel. Such desirable property does not pertain to the ILT test, which combines individual statistics, so that its ability to accommodate heterogeneous break types is constrained by the limiting behavior of these individual statistics.

3. Practical Implementation Issues

3.1 Finite Sample Distribution

To implement the proposed tests, it is necessary to make available distributions of τ_{iT}^0, $\tau_{iT}^2(\lambda_i)$ and $\tau_{iT}^3(\lambda_i)$ and to obtain p-values, means and variances. Since the limiting distributions are in general poor approximations in finite samples, we simulate finite sample distributions of the LM statistics for 31 sample sizes, with T = {30, 33, 35, 37, 40, 43, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100, 125, 150, 175, 200, 225, 250, 275, 300, 350, 400, 450, 500, 750, 1000, 1250}. Nine known break fractions are considered, with λ = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9} when a break is allowed. Since finite sample distributions also depend on the procedure employed to select k, two ways of determining k are considered for each T under Model 0 and each combination of T and λ under other models. The first way assumes k to be fixed at 0, 5 and 8, whereas the second selects k by the general-to-specific t-sig method recommended by Ng and Perron (1995), with the upper bound k_{max} set at 5 and 8.

The number of replications (I) used is as follows. When k = 0, 5, 8 and k_{max} = 5, I = 50,000, 30,000 and 20,000 for $T \leq 100$, $100 < T \leq 500$, $T > 500$. When k_{max} = 8, I = 30,000, 20,000 and 10,000 for $T \leq 100$, $100 < T \leq 500$, and $T > 500$. In each iteration,

T+100 observations are generated according to a driftless random walk with *N(0,1)* errors while the initial value is set at zero, and the first 100 observations discarded to reduce the initial value effect.[5] With the generated data, LM statistics are computed according to equation (4), and 399 equally spaced percentiles (from which *p*-values can be obtained by linear interpolation as in Choi, 2001), means and variances of the statistics obtained and stored. All simulations are programmed in GAUSS 6.0.

Simulation results are summarized using estimated response surface regressions. Under Models 0 and 1, the response surface regression equation takes the form

$$q(T) = \beta_0 + \beta_1 \frac{1}{T} + \beta_2 \frac{1}{T^{3/2}} + \beta_3 \frac{1}{T^2} + \beta_4 \frac{1}{T^3} + \omega \,, \tag{13}$$

where *q* denotes the distribution quantiles, means or variances, and ω is the error term. Under Models 2 and 3, the response surface regression equation is

$$q(T,\lambda) = g_0(T) + g_1(T)\lambda^{3/2} + g_2(T)\lambda^2 + g_3(T)\lambda^3 + g_4(T)\lambda^4 + \beta_5 \frac{1}{T} + \beta_6 \frac{1}{T^2} + \omega \,, \tag{14}$$

where $g_j(T) = \beta_{j0} + \beta_{j1} \frac{1}{T^{1/4}} + \beta_{j2} \frac{1}{T^{1/3}} + \beta_{j3} \frac{1}{T^{1/2}} + \beta_{j4} \frac{1}{T^{2/3}}$, *j* = 0, 1, 2, 3, 4. It is worth noting that under Model 1, the break fraction is found to be statistically insignificant in explaining the simulated quantiles, means and variances under all lag order assumptions. The parameters in (13) and (14) are estimated using OLS. However, since the errors are heteroskedastic, heteroskedasticity-consistent standard errors are computed using the jackknife covariance matrix estimator of MacKinnon and White (1985). The choice of response surface terms is motivated by the overall fit of the regression as measured by the coefficient of determination (R^2), small standard error of regression ($\hat{\sigma}_\omega$), small mean absolute error (mean $|\hat{\omega}|$), small maximum absolute error (max $|\hat{\omega}|$), statistical significance of regressors and parsimony.

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[5] The individual exogenous break LM-type tests are exactly invariant to the break magnitudes under the null when the break is correctly placed according to Nunes (2004). Furthermore, without loss of generality, the intercept and time trend coefficient are set to zero, and the error variance set to one.

Response surface regression results are reported in Tables 1 to 3. To conserve space, we only present results for $k = 0, 5$ and $k_{max} = 5$, and among the 399 pecentiles, the 5% quantile for illustration. With the reported estimates, the 5% quantiles, means and variances of the LM statistics for selected sample sizes and break fractions are computed and displayed in Table 4. Some interesting observations are revealed. First, response surface estimates provide good approximations to distributions of the LM statistics. Overall, the estimates track closely those simulated results available in the literature. Second, finite sample distribution of $\tau_{iT}^1(\lambda_i)$ does not differ significantly from that of τ_{iT}^0 for any lag order choice. This suggests finite sample distribution of τ_{iT}^0 can also be used to compute the panel statistics under Model 1. In contrast, larger discrepancies are found between finite sample distributions of $\tau_{iT}^2(\lambda_i)$ and $\tau_{iT}^2(\lambda_i)$, although they converge to the same limiting distribution. Third, finite sample distributions of the LM statistics under all models depend on the lag selection method employed. Using distributions of the LM statistics when $k = 0$ as benchmarks, distributions of the statistics when $k > 0$ are shifted to the right, whereas distributions of the statistics when $k_{max} > 0$ are shifted to the left. This draws attention to the use of appropriate distributions of the individual statistics in computing panel statistics under different lag order assumptions.

3.2 Cross-Sectional Dependency

The panel unit root tests discussed assume cross-sectional independence in a panel. Different methods for tackling cross-sectional dependency have been proposed in the literature. In this paper, we adopt the bootstrap approach as it can cater for cross-correlations of the general form

$$\varepsilon_{it} = \varphi_i f_t + \upsilon_{it} , \quad f_t \sim iidN(0,1) , \quad \upsilon_t \sim iidN(0,\Omega_\upsilon) , \tag{15}$$

where f_t is an unobserved stationary common factor, φ_i is the factor loading that measures the heterogeneous impact of the common factor on each individual, $\upsilon_t = (\upsilon_{1t}, \cdots, \upsilon_{Nt})'$ and Ω_υ is not diagonal. Furthermore, in analyzing finite sample data, bootstrap helps to obtain approximations to finite sample distributions of the panel statistics for more accurate statistical inference drawn.

The bootstrap procedure used is a recursive one based on resampling residuals from OLS estimation of the null model of each time series in a panel. Notice that computation of the panel statistics requires the use of distributions of the underlying individual statistics. Following conventional practice, finite sample distributions of individual statistics can be simulated as described in Section 3.1. An alternative method to obtain finite sample approximations is through the use of bootstrapped distribution of the individual statistic of interest, which is specific to the observed time series under study via a bootstrap procedure similar to that outlined above. Details of the bootstrap procedures are documented in the appendix.

4. Monte Carlo Simulation Results

We conduct Monte Carlo simulation experiments to assess finite sample performance of the panel unit root tests. Data are generated with N cross-sections and $T+100$ time series observations for each cross-section according to DGP (1) with the following parameter values: $\mu_i = 0$, $\delta_{1i} = 0$ $\delta_{2i} = 5$, $\delta_{3i} = 1$, $u_{i0} = 0$, and $\sigma_i^2 = 1$ for all i. The break is set to occur at the middle of each time series. In analyzing size and power properties, $\alpha_i = 1$ and $\alpha_i = 0.9$ respectively for all i. The first 100 observations for each cross-section are discarded to reduce the initial value effect.

Three experiments are conducted. In the first experiment, we analyze performance of the panel unit root tests under all models using different break

selection criteria while assuming no serial correlation and no cross-sectional dependency. Results are carried out for $N = \{10, 25, 50\}$ and $T = \{25, 50\}$, with 3,000 iterations for each combination of N and T. In the second experiment, we introduce serial correlation while breaks are assumed exogenously known so that the effects of using different lag orders are not obscured by the influence of break selection. The disturbances ε_{it} are generated by an AR(1) process $\varepsilon_{it} = 0.3\varepsilon_{i,t-1} + v_{it}$, $v_{it} \sim iidN(0,1)$. In the third experiment, we consider cross-sectional dependency. The disturbances ε_{it} are generated according to (15), with $\varphi_i \sim U[-1,3]$, which represents strong cross-correlations. To ensure the covariance matrix Ω_v be symmetric, positive definite and nonsingular, it is generated as in Chang (2002). Simulation results are obtained for $N = \{10, 25\}$ and $T = \{25, 50\}$. In assessing the performance of the bootstrap test, we use 500 Monte Carlo replications and 500 bootstrap replications.

Distributions of individual LM statistics used in calculating the panel tests are simulated using the conventional approach as described in Section 3.3. Bootstrapped distributions of the LM statistics for use in the bootstrap test are simulated out of 5,000 replications. In computing the panel statistics under Model 1, distribution of τ_{iT}^0 is employed. All tests are conducted at the 5% nominal level. Size-adjusted powers are presented for fair comparison. All simulations are programmed in GAUSS 6.0. In what follows, we only report results for Models 1 and 2. Results for Models 0 and 3 are similar to those for Models 1 and 2 respectively.

In the first experiment, sizes of all tests are reasonably close to the 5% nominal level under the assumption of exogenous breaks. The inverse normal and logit tests exhibit most stable size and possess highest power among all tests. When break dates are endogenously determined with the minimum SSR criterion, the performance of all tests is comparable to that for the exogenous known break case. However, when break

dates are determined with the minimum LM criterion, all tests suffer from severe size distortions. The difference may be attributed to the higher frequency of true break date selection with the minimum SSR criterion. More importantly, finite sample distribution of $\tau_{iT}^1(\hat{\lambda}_i)$ and $\tau_{iT}^2(\hat{\lambda}_i)$ are in line with that of τ_{iT}^0 and $\tau_{iT}^2(\lambda_i)$ respectively. In contrast, finite sample distribution of $\tau_{iT}^1(\lambda_i(\tau_{iT}^1))$ and $\tau_{iT}^2(\lambda_i(\tau_{iT}^2))$ are shifted to the left of τ_{iT}^0 and $\tau_{iT}^2(\lambda_i)$.[6] Consequently, selecting break dates by the minimum LM criterion and then combining the resulting minimum LM test evidence across individuals to compute panel statistics using distribution of τ_{iT}^0 under Model 1 and distribution of $\tau_{iT}^2(\lambda_i)$ under Model 2 is rendered inappropriate. Thus, we drop the minimum LM break selection criterion in subsequent discussion.

In the second experiment with serial correlation, we consider both the fixed lag case and the sequential lag selection case. Results for the fixed lag case are displayed in Table 6. With AR(1) errors, the true lag orders are 1 for all i. When $k_i = 0$ for all i are used, all sizes are reported to be zero. Size performance of the panel tests when using lag orders equal to the true ones is similar to that when using lag orders greater than the true ones. Under Model 1, sizes are in general not far from the nominal size of 5%, but combination tests (except the modified inverse chi-square test) have better size property than the ILT test overall. Under Model 2, all tests suffer from some size distortions when T is small relative to N, with the inverse chi-square test attaining the best size property. For all model types, power decreases with higher lag order, with the inverse normal and logit tests exhibiting the highest power overall. Thus, with serial correlation, it is desirable to have T large relative to N, so that N/T does not diverge as both N and T approach infinity simultaneously as mentioned in ILT.

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[6] The critical values of $\tau_{iT}^1(\lambda_i(\tau_{iT}^1))$ are substantially more negative than those of τ_{iT}^0 in finite samples according to simulation results in Nunes (2004) for the one-break case and Lee and Strazicich (2003) for the two-break case.

When the lag order is selected, we set the maximum lag order at $T^{1/3}$. Two methods of computing the panel tests are used. The first method entails first determining the optimal lag for all i and then calculating panel statistics using distributions of individual statistics simulated with corresponding fixed lag order as in ILT. For the second method, panel statistics are calculated using distributions of individual statistics simulated with corresponding lag selection method. With the first method, all tests exhibit size distortions as shown in the upper panel of Table 7. ILT relate such kind of size distortions to the accuracy of true lag order selection. We attribute the size distortion to the use of inappropriate finite sample distribution of individual statistics, since finite sample distributions of individual statistics with endogenously determined lag order differ much from those with fixed lag order as shown in Section 3.1. This argument is evidenced by the fact that when distributions of individual statistics simulated with corresponding lag selection method are used to compute the panel statistics, the panel tests no longer suffer from severe over-rejections according to results in the lower panel of Table 7.

Data are cross-correlated in the third experiment. We find that panel tests taking no account of cross-sectional dependency suffer from size-distortions as shown in the upper panel of Table 8. Size biases accumulate as N grows, but are not attenuated by increasing T. All panel tests become less powerful in the presence of cross-sectional dependency. Results in the lower panel of Table 8 demonstrate the bootstrap approach helps to correct for size distortions of the panel tests in the presence of cross-correlations. The inverse normal test possesses the highest overall power.[7]

We draw the following conclusions from the simulation results. First, the inverse

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[7] Results shown in the lower panel of Table 8 are those using bootstrapped distributions of individual statistics to compute the panel tests. Contrasting the use of conventional and bootstrapped distributions of individual statistics according to unreported results, size performance of the bootstrap test in either case is comparable, whereas power of the bootstrap test rise faster as T increases for fixed N using the latter.

normal and logit tests perform best in terms of size and power, and are therefore recommended for empirical applications. Second, in endogenizing the break dates instead of assuming exogenously known, the minimum SSR selection criterion is recommended for its high frequency of true break date selection and good size and power properties of the resulting panel tests. Third, bootstrap is an effective way to correct for size distortions even in the presence of strong cross-correlations which take on a general structure.

5. Empirical Applications

Since the seminal paper by Nelson and Plosser (1982), econometricians are increasingly interested in testing whether macroeconomic time series are integrated or trend stationary. We apply the proposed panel unit root tests using the minimum SSR break selection criterion to the study of two frequently investigated macroeconomic variables, per capita GDP and consumer prices, of OECD countries for the period 1960-2003 and 1949-2004 respectively.[8] The logarithms of these series are analyzed.

To carry out unit root testing with the panel data, individual LM regression for each country is first conducted with a 10% trimming at both ends of the time series. The optimal lag order is selected using the general-to-specific t-sig method with k_{max} = 8. The optimal break model type for each time series is selected following the procedure of Alba and Papell (1995). The most general model, Model 2, is first estimated. If both the estimated level and slope break coefficients are significant, Model 2 is applied. If only the slope coefficient is significant, Model 3 is used. If only the level coefficient is significant, Model 1 is employed. If both coefficients are not

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[8] Data for per capita GDP measured in PPP for 25 OECD countries are obtained from the OECD Health Data 2005. These GDP data are expressed in constant 2000 prices using GDP deflators taken from the World Development Indicators 2005. Data for consumer prices for 23 OECD countries, with 2000 as the base year, come from the International Financial Statistics April 2005.

significant, the no break situation is considered. Test results for each country are presented in the upper panel of Table 9. It can be noted that Model 0 is rarely selected for individual time series, thus justifying the allowance for structural breaks in unit root testing. Furthermore, Models 2 and 3 are more frequently selected than Model 1, which signifies the importance of accounting for a break in the slope. Comparing the computed individual statistics with the bootstrapped critical values obtained according to the procedure outlined in the appendix using 5,000 replications, it is found that the unit root null for per capita GDP is rejected for three countries and that for consumer prices for five countries, at conventional significance levels.

The null hypothesis of cross-sectional independence is further tested with the Pesaran (2004) test. The test statistic is $\sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\vartheta}_{ij}\right)$, where $\hat{\vartheta}_{ij}$ is the sample estimate of the pairwise correlation of the residuals, which follows a standard normal limiting distribution. The computed test statistics are 15.666 for per capita GDP, and 21.454 for consumer prices, thus we reject the null of cross-sectional independence at conventional significance levels for both variables.

In the presence of cross-sectional correlations, the bootstrap approach as described in Section 3.2 is employed. The bootstrapped critical values of the panel statistics are obtained out of 5,000 replications. Panel unit root test results using bootstrapped distributions of individual statistics are presented in the lower panel of Table 9.[9,10] The joint unit root null for per capita GDP is not rejected, thus supporting the non-stationary nature of the variable. As for consumer prices, results provide evidence of trend-stationarity. The joint unit root null is rejected at the 5% level using

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[9] Results using conventional distributions, are qualitative the same and are therefore not reported.

[10] For the group mean test, when asymptotic critical values are used for hypothesis testing, its flexibility in accommodating heterogeneous break type across cross-sections is restricted as discussed in Section 2.2. Accommodating full flexibility like the combination tests will result in nonstandard distribution, the critical values of which can be obtained using bootstrap.

the inverse chi-square test and its modified version, and at the 1% level using other panel tests.

6. Conclusion

This chapter proposes LM-type group mean and combination tests for panel unit root testing with structural breaks. The proposed tests have the following advantages. First, they are modeled with breaks under both the null and alternative hypotheses. Second, they are able to account for breaks in the intercept, in the slope and both. Third, they take into account the heterogeneity nature of cross-sections in a panel by allowing for individual specific intercepts, time trend and structural break coefficients, serial correlation structure, first-order autoregressive parameters, break locations and break types. Simulation results show that two of the combination tests, namely the inverse normal and logit tests, have the best size and power properties. Furthermore, in endogenizing the break date, the minimum SSR criterion is preferred to the more commonly employed minimum LM criterion. Finally, bootstrap is shown to be effective in correcting for size distortions arising from a relaxation of the cross-sectional independence assumption. The proposed panel unit root tests with structural breaks are applied to study the time series behavior of per capita GDP and consumer prices for OECD countries. Empirical results lend support to the nonstationary nature of the former and the trend stationary behavior of the latter.

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Appendix

A. Bootstrapping critical values for panel unit root test

(a1) Select the null model for each individual time series in the observed panel data according to null equation (3) with the incorporation of serial correlation

$$\Delta y_{it} = \Delta Z_{it}\delta_i + \sum_{j=1}^{k_i} c_{ij}\Delta y_{it-j} + v_{it} \, . \tag{A1.1}$$

The optimal k_i is determined by the general-to-specific *t-sig* method. Where a break is allowed, the optimal k_i is first determined at each possible break point, then all possible regressions with the optimal k_i estimated, and the break date selected where the Schwarz Bayesian Criterion is minimized.[11]

(a2) Estimate the selected null model, defined at the selected break date $\hat{TB}_i$ and the corresponding optimal $\hat{k}_i$, for every individual with the observed data by OLS and obtain the parameter estimates $\hat{\delta}_i$, $\hat{c}_{i1}$, $\cdots$, $\hat{c}_{i\hat{k}_i}$ together with the regression residuals $\hat{v}_{it}$, $t = \hat{k}_i + 2, \cdots, T$. The residuals are then rescaled by the multiplicative factor $\sqrt{l_i/(l_i - o_i)}$, where l_i is the number of observations used in estimation and o_i is the number of parameters estimated, to obtain $\breve{v}_{it}$, $t = \hat{k}_i + 2, \cdots, T$.

(a3) Construct RP number of pseudo panel samples. For each pseudo panel sample, resample $\breve{v}_t = (\breve{v}_{1t}, \cdots, \breve{v}_{Nt})$ randomly with replacement T times to obtain $v_t^{**} = (v_{1t}^{**}, \cdots, v_{Nt}^{**})$, $t = 1, \cdots, T$.[12] For each individual time series, construct the first-differenced series

$$\Delta y_{it}^{**} = \Delta Z_{it}\hat{\delta}_i + \sum_{j=1}^{\hat{k}_i} \hat{c}_{ij}\Delta y_{it-j}^{**} + v_{it}^{**} \, . \tag{A1.2}$$

The initial starting values for all individuals together $(y_{-\hat{k}}^{**}, \cdots, y_0^{**})$, with $\hat{k} = \max_i(\hat{k}_i)$ and $y_j^{**} = (y_{1j}^{**}, \cdots, y_{Nj}^{**})$, $j = -\hat{k}, \cdots, 0$, are obtained by block resampling. The observed data are divided into $(T - \hat{k})$ overlapping blocks each with $(\hat{k} + 1)$ time observations per individual, and one block is randomly selected with replacement to serve as the initial values. Partial sums of Δy_{it}^{**} give the level series

$$y_{it}^{**} = y_{i0}^{**} + \sum_{j=1}^{t} \Delta y_j^{**} \, . \tag{A1.3}$$

(a4) Calculate the bootstrapped individual statistic of interest according to equation (4) for each time series in each pseudo panel sample. Construct the bootstrapped panel statistic of interest with these bootstrapped individual statistics for each pseudo panel sample. The *p*-values, means and variances used to compute the panel statistic are obtained in two ways. The first is using conventional distributions of the individual statistics simulated as described in Section 3.1. The second is using bootstrapped distributions of the individual statistics simulated in steps (b4) and (b5) below. [In step (b5), "observed individual statistic" is replaced by bootstrapped individual statistic of interest in this step.] A sorted vector of these RP bootstrapped panel statistics forms the bootstrapped distribution of the panel statistic under the null, from which bootstrapped critical values of the panel statistic are obtained.

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[11] The estimated break date for each individual here is the estimate under the null, conditional on the selected null model. This may differ from the break date generating the statistic for unit root testing.
[12] Resampling the residuals as a vector for fixed *t* for all individuals simultaneously help to preserve the cross-sectional dependency structure of the time series.

B. Bootstrapping critical values, mean, variance and p-vale for individual unit root test

(b1) Same as step (a1).

(b2) Same as step (a2).

(b3) Construct RU number of pseudo samples for each individual time series. First, resample $\bar{v}_{it}$, $t = \hat{k}_i + 2, \cdots, T$, randomly with replacement T times to obtain v_{it}^*, $t = 1, \cdots, T$. Then construct the first-differenced series Δy_{it}^*

$$\Delta y_{it}^* = \Delta Z_{it} \hat{\delta}_i + \sum_{j=1}^{\hat{k}_i} \hat{c}_{ij} \Delta y_{it-j}^* + v_{it}^* . \tag{A2.1}$$

The initial starting values $\left(y_{i(-\hat{k}_i)}^*, \cdots, y_{i0}^* \right)$ are obtained by random draw with replacement one block from the $(T - \hat{k}_i)$ overlapping blocks of the observed individual time series, each with length $(\hat{k}_i + 1)$. Partial sums of Δy_{it}^* gives the level series

$$y_{it}^* = y_{i0}^* + \sum_{j=1}^{t} \Delta y_j^* . \tag{A2.2}$$

(b4) Calculate the bootstrapped individual statistic of interest according to equation (4) for each pseudo sample. A sorted vector of these RU bootstrapped statistics forms the bootstrapped distribution of the individual statistic under the null, from which bootstrapped critical values, mean and variance can be computed.

(b5) Obtain *p*-value for the individual statistic as #{bootstrapped individual statistic in step (b4) ≤ observed individual statistic}/RU, where observed individual statistic is the test statistic calculated according to equation (4) using the observed data y_{it}.

Table 1. Response surface estimates under Models 0 and 1.

Coefficients & statistics	5% quantile	Mean	Variance	5% quantile	Mean	Variance	5% quantile	Mean	Variance
				Model 0					
		$k = 0$			$k = 5$			$kmax = 5$	
β_0	-3.0175	-1.9663	0.3328	-3.0152	-1.9702	0.3323	-3.0220	-1.9711	0.3323
β_1	-5.8713	-1.4399	1.2707	-12.007	1.2539	-1.6262	-27.319	-9.5769	2.9874
β_2				244.2308	54.9976	47.1178	84.6323	45.4104	96.9871
β_3	206.8227	79.463		-1189.27	-263.008	-182.136	-114.944	-228.564	-650.862
β_4	-4387.95	-1650.10	254.337	6922.293	3006.236	3810.814	-457.827	2112.922	7034.919
R^2	0.9426	0.8550	0.9666	0.7731	0.997	0.9964	0.9967	0.9977	0.9978
$\hat{\sigma}_\omega$	0.0088	0.0020	0.0029	0.0089	0.0033	0.0030	0.0099	0.0034	0.0033
Mean $|\hat{\omega}|$	0.0065	0.0015	0.0023	0.0063	0.0024	0.0021	0.0069	0.0022	0.0025
Max $|\hat{\omega}|$	0.0166	0.0040	0.0051	0.0189	0.0071	0.0071	0.0239	0.0080	0.0066
				Model 1					
		$k = 0$			$k = 5$			$kmax = 5$	
β_0	-3.0250	-1.972	0.3290	-3.0262	-1.9727	0.3324	-3.0067	-1.9648	0.3280
β_1	-3.3665	-0.2814		2.1107	4.3835	-1.8594	-37.3495	-1..3793	10.7862
β_2			1.9455			53.9962	196.9223	46.6379	-39.0829
β_3	-15.2599	-4.9768	-26.1806	60.5926	28.4646	-226.803	-353.439	-115.870	
β_4			711.7967	-3149.09	910.8902	4806.765	-3937.18		3370.686
R^2	0.9572	0.7169	0.978	0.7745	0.997	0.9964	0.9977	0.9987	0.9978
$\hat{\sigma}_\omega$	0.0080	0.0027	0.0026	0.0078	0.0032	0.0030	0.0076	0.0023	0.0030
Mean $|\hat{\omega}|$	0.0064	0.0021	0.0020	0.0062	0.0025	0.0023	0.0058	0.0017	0.0024
Max $|\hat{\omega}|$	0.0228	0.0057	0.0073	0.0199	0.0076	0.0103	0.0324	0.0075	0.0092

Notes: k is the fixed lag order used in regression. *kmax* is the upper bound of the lag order used when the lag order is determined endogenously in regression. R^2 is the coefficient of determination. $\hat{\sigma}_\omega$ is the estimated standard error of the regression. Mean $|\hat{\omega}|$ gives the mean absolute error of the response surface predictions versus simulated values, whereas Max $|\hat{\omega}|$ denotes the maximum absolute error.

Table 2. Response surface estimates under Model 2.

Coefficients & statistics	5% quantile	Mean	Variance	5% quantile	Mean	Variance	5% quantile	Mean	Variance
	$k = 0$			$k = 5$			$kmax = 5$		
β_{00}	-3.3650	-1.1379	1.7187	27.3195	25.8458	-2.0058	-3.0710	9.7186	9.4432
β_{01}	44.4158	-30.5227	-76.6409	-1295.14	-1272.73	31.214	46.4865	-528.316	-444.315
β_{02}	-115.328	58.2877	171.8811	2666.469	2697.047	0.6939	-135.189	1112.130	966.3929
β_{03}	153.4493	-46.8127	-189.566	-2454.34	-2657.94	-160.030	211.5373	-1077.17	-1011.46
β_{04}	-91.3441	19.9652	102.3988	1089.328	1281.366	174.8088	-144.674	505.0783	522.7624
β_{10}	-144.416	9.8848	96.2237	5043.148	1864.997	-2516.03	1364.446	1108.409	-440.979
β_{11}	194.9303	-50.8515	-144.274	-14748.7	-4530.52	7664.678	-4580.11	-2909.38	1863.976
β_{12}	-123.979	39.2899	94.9235	12566.42	3607.005	-6620.57	4051.674	2385.562	-1737.80
β_{13}	8.6147	-3.9079	-7.1657	-2028.96	-501.342	1099.268	-701.415	-355.360	328.4013
β_{14}				163.7555	33.901	-91.3299	60.3871	26.2779	-30.394
β_{20}	267.6613	-67.0487	-206.017	-11189.3	-3636.31	5652.536	-3376.03	-2382.51	1200.230
β_{21}	-352.534	176.0439	304.5694	32671.04	8508.458	-17211.4	11134.86	6137.928	-4800.21
β_{22}	221.271	-129.740	-200.065	-27837.2	-6672.37	14867.01	-9820.72	-5007.69	4428.554
β_{23}	-14.8551	12.0473	15.0541	4496.330	892.3936	-2469.15	1692.846	738.219	-823.765
β_{24}				-363.185	-57.0283	205.2491	-145.346	-53.9591	75.3743
β_{30}	-222.633	108.3534	200.8407	10628.67	2914.369	-5338.17	3599.857	2177.166	-1331.23
β_{31}	289.0128	-230.955	-294.495	-30989.2	-6385.68	16268.12	-11742.6	-5466.67	5176.935
β_{32}	-179.1358	166.3468	193.5095	26401.66	4863.656	-14054.4	10339.79	4425.527	-4748.78
β_{33}	11.6137	-14.8942	-14.5764	-4264.66	-599.826	2334.945	-1778.57	-641.362	875.5189
β_{34}				344.5736	33.3175	-194.162	152.5529	45.9403	-79.5811
β_{40}	102.5449	-50.5124	-92.5182	-4486.28	-1116.54	2194.693	-1608.60	-894.804	581.519
β_{41}	-136.764	104.7302	136.7118	13083.22	2330.573	-6699.70	5254.857	2214.726	-2270.93
β_{42}	85.4693	-75.2058	-90.0777	-11146.6	-1732.32	5789.019	-4630.08	-1783.52	2084.434
β_{43}	-5.6627	6.7000	6.8265	1800.4	198.0079	-961.858	797.4862	255.2959	-384.635
β_{44}				-145.447	-9.3079	79.9713	-68.4952	-17.9992	34.9807
R^2	0.9948	0.9985	0.9876	0.9836	0.9973	0.9742	0.9958	0.9986	0.9824
$\hat{\sigma}_\omega$	0.0126	0.0076	0.0045	0.0295	0.0123	0.0153	0.0193	0.0098	0.0122
Mean $|\hat{\omega}|$	0.0091	0.0047	0.0034	0.0211	0.0085	0.0102	0.0145	0.0064	0.0081
Max $|\hat{\omega}|$	0.0406	0.0330	0.0131	0.0919	0.0488	0.0683	0.0608	0.0406	0.0556

Note: See notes to Table 1.

Table 3. Response surface estimates under Model 3.

Coefficients & statistics	5% quantile	Mean	Variance	5% quantile	Mean	Variance	5% quantile	Mean	Variance
		$k = 0$			$k = 5$			$kmax = 5$	
β_{00}	-3.6489	-1.0263	1.5889	20.6507	24.2763	0.7546	-7.3648	8.8846	10.9748
β_{01}	54.1158	-34.6824	-70.2494	-1009.39	-1200.64	-82.297	231.4024	-491.442	-505.236
β_{02}	-134.429	66.3181	158.0253	2075.780	2543.678	230.2551	-518.086	1034.938	1087.692
β_{03}	170.8513	-52.9362	-175.257	-1904.81	-2504.96	-360.121	569.1555	-1003.16	-1112.59
β_{04}	-100.053	21.8433	95.0905	843.6152	1206.332	254.358	-305.243	470.6864	559.7532
β_{10}	-98.8721	20.6314	83.7165	3964.255	1625.891	-2005.91	940.451	928.9196	-169.342
β_{11}	110.7526	-71.2776	-120.512	-11608.5	-3857.91	6204.270	-3378.63	-2381.14	1075.579
β_{12}	-66.3714	53.4469	78.4884	9889.661	3039.158	-5381.95	3037.506	1933.028	-1066.47
β_{13}	3.939	-5.0916	-5.7999	-1596.05	-411.060	900.8314	-540.825	-281.311	220.0619
β_{14}				128.7458	26.7041	-75.418	47.6957	20.2049	-21.6496
β_{20}	160.6153	-83.0126	-172.289	-8738.81	-3047.66	4561.337	-2422.73	-1913.30	645.5827
β_{21}	-153.626	207.2012	241.1837	25516.64	6852.940	-14077.5	8411.401	4757.615	-3180.23
β_{22}	84.8515	-151.480	-156.322	-21731.1	-5274.11	12206.33	-7514.88	-3824.61	3046.567
β_{23}	-3.7291	13.8861	11.4296	3506.088	669.7494	-2042.02	1325.277	544.2838	-599.969
β_{24}				-282.858	-39.2396	170.9282	-116.076	-38.0125	57.2519
β_{30}	-104.676	110.5854	154.7544	8164.847	2249.812	-4339.52	2626.101	1609.825	-867.474
β_{31}	68.4929	-237.668	-209.027	-23777.9	-4523.50	13389.79	-8925.23	-3802.48	3806.860
β_{32}	-27.5136	171.4427	134.6985	20240.56	3291.837	-11608.3	7943.570	2999.463	-3576.47
β_{33}	-0.8227	-15.3833	-9.7235	-3263.09	-349.683	1941.576	-1392.84	-407.503	684.6781
β_{34}				263.0989	13.3254	-162.504	121.5016	26.6848	-64.0576
β_{40}	42.1844	-48.6371	-65.9632	-3394.06	-798.256	1784.613	-1167.34	-615.333	408.474
β_{41}	-23.8674	102.6427	87.8002	9885.146	1442.354	-5516.85	3965.465	1396.757	-1754.30
β_{42}	7.7864	-74.0234	-56.4739	-8413.63	-983.449	4783.890	-3529.84	-1082.90	1641.357
β_{43}	0.7234	6.6388	4.0601	1355.890	79.0682	-800.303	619.1879	140.462	-312.262
β_{44}				-109.263	0.1825	66.9814	-54.0416	-8.5451	29.0811
R^2	0.9947	0.9985	0.9892	0.9826	0.9976	0.9773	0.9966	0.9987	0.9869
$\hat{\sigma}_\omega$	0.0123	0.0074	0.0041	0.0292	0.0113	0.0137	0.0168	0.0090	0.0103
Mean $\|\hat{\omega}\|$	0.0088	0.0047	0.0032	0.0212	0.0078	0.0095	0.0123	0.0057	0.0071
Max $\|\hat{\omega}\|$	0.0427	0.0320	0.0137	0.0836	0.0518	0.0578	0.0553	0.0356	0.0479

Note: See notes to Table 1.

Table 4. Comparison of response surface estimates and simulated results in the literature.

T	Lag order	λ	Measure	Simulated results in literature		Response surface estimates			
				Source	Value	Model 0	Model 1	Model 2	Model 3
25/50/1,000	k=0		5% quantile	SP[a] (Model 0)	-3.18/-3.11/-3.02	-3.202/-3.087/-3.023	-3.184/-3.098/-3.028		
38	k=0		mean/variance	ILT[b] (Model 0)	-1.979/0.371	-1.979/0.371	-1.983/0.375		
38	k=5		mean/variance	ILT (Model 0)	-1.829/0.430	-1.830/0.434	-1.821/0.444		
56	k=0		mean/variance	ILT (Model 0)	-1.976/0.358	-1.976/0.357	-1.979/0.360		
56	k=5		mean/variance	ILT (Model 0)	-1.884/0.377	-1.883/0.379	-1.880/0.383		
62/111	k=0		5% quantile	AL[c] (Model 1)	-3.09/-3.06	-3.077/-3.057	-3.083/-3.055		
1,000	k=0	0.3/0.5/0.8	5% quantile	Nunes[d] (Models 2, 3)	-3.59/-3.66/-3.48			-3.598/-3.658/-3.490	-3.596/-3.658/-3.491
38	k=0/5		5% quantile			-3.109/-2.986	-3.124/-2.986		
38	kmax=5		5% quantile			-3.480	-3.456		
38	kmax=5		mean/variance			-2.149/0.502	-2.145/0.507		
38	k=0	0.3/0.5/0.8	5% quantile					-3.751/-3.828/-3.603	-3.739/-3.826/-3.638
38	k=5	0.3/0.5/0.8	5% quantile					-3.338/-3.840/-3.423	-3.333/-3.801/-3.484
38	kmax=5	0.3/0.5/0.8	5% quantile					-4.121/-4.342/-4.027	-4.098/-4.311/-4.073
38	k=0	0.5	mean/variance					-2.680/0.393	-2.678/0.390
38	k=5	0.5	mean/variance					-2.586/0.532	-2.572/0.382
38	kmax=5	0.5	mean/variance					-3.054/0.528	-3.030/0.519

Notes: [a] SP denotes Schmidt and Phillips (1992). [b] ILT denotes Im, Lee and Tieslau (2005), where effective number of observations is used instead of actual number of observations here. [c] AL denotes Amsler and Lee (1995). [d] Nunes denotes Nunes (2004).

Table 5. Sizes and size-adjusted powers of panel unit root tests under Models 1 and 2.

N	T	Γ Size	Γ Power	P Size	P Power	Z Size	Z Power	L Size	L Power	P_m Size	P_m Power
						Model 1					
Exogenous											
10	25	0.059	0.103	0.053	0.097	0.050	0.115	0.053	0.108	0.069	0.097
	50	0.055	0.378	0.046	0.304	0.047	0.429	0.045	0.418	0.059	0.304
25	25	0.057	0.155	0.054	0.135	0.050	0.179	0.051	0.182	0.061	0.135
	50	0.054	0.730	0.043	0.620	0.048	0.765	0.048	0.752	0.051	0.620
50	25	0.052	0.271	0.049	0.237	0.042	0.308	0.046	0.304	0.057	0.237
	50	0.056	0.943	0.049	0.862	0.046	0.968	0.047	0.965	0.056	0.862
Minimum LM											
10	25	0.641	0.094	0.556	0.096	0.623	0.099	0.612	0.098	0.606	0.096
	50	0.593	0.270	0.528	0.229	0.563	0.294	0.559	0.269	0.576	0.229
25	25	0.926	0.127	0.883	0.120	0.927	0.134	0.925	0.134	0.898	0.120
	50	0.895	0.500	0.854	0.441	0.876	0.557	0.879	0.523	0.874	0.441
50	25	0.997	0.196	0.991	0.183	0.998	0.213	0.997	0.200	0.992	0.183
	50	0.993	0.816	0.988	0.724	0.992	0.856	0.992	0.822	0.989	0.724
Minimum SSR											
10	25	0.059	0.100	0.053	0.090	0.051	0.108	0.052	0.105	0.069	0.090
	50	0.053	0.391	0.045	0.315	0.045	0.437	0.044	0.423	0.058	0.315
25	25	0.058	0.166	0.054	0.141	0.050	0.193	0.051	0.194	0.061	0.141
	50	0.055	0.712	0.041	0.619	0.048	0.762	0.048	0.750	0.050	0.619
50	25	0.053	0.259	0.049	0.215	0.044	0.294	0.047	0.286	0.058	0.215
	50	0.056	0.931	0.048	0.849	0.045	0.961	0.046	0.960	0.054	0.849
						Model 2					
Exogenous											
10	25	0.055	0.082	0.049	0.075	0.046	0.085	0.045	0.084	0.063	0.075
	50	0.055	0.169	0.048	0.148	0.049	0.177	0.046	0.187	0.059	0.148
25	25	0.059	0.078	0.054	0.079	0.058	0.082	0.057	0.082	0.064	0.079
	50	0.058	0.282	0.049	0.245	0.050	0.307	0.051	0.298	0.060	0.245
50	25	0.055	0.105	0.049	0.099	0.049	0.118	0.053	0.106	0.055	0.099
	50	0.057	0.463	0.044	0.382	0.046	0.492	0.045	0.498	0.052	0.382
Minimum LM											
10	25	0.896	0.062	0.806	0.066	0.917	0.066	0.903	0.062	0.847	0.066
	50	0.932	0.127	0.867	0.127	0.935	0.135	0.925	0.127	0.893	0.127
25	25	0.999	0.068	0.993	0.071	1.000	0.066	1.000	0.068	0.996	0.071
	50	1.000	0.191	0.998	0.177	1.000	0.207	1.000	0.187	0.999	0.177
50	25	1.000	0.091	1.000	0.092	1.000	0.090	1.000	0.088	1.000	0.092
	50	1.000	0.295	1.000	0.257	1.000	0.320	1.000	0.293	1.000	0.257
Minimum SSR											
10	25	0.055	0.081	0.049	0.074	0.046	0.084	0.045	0.082	0.063	0.074
	50	0.055	0.165	0.048	0.150	0.049	0.177	0.046	0.188	0.059	0.150
25	25	0.059	0.081	0.053	0.082	0.057	0.083	0.056	0.083	0.064	0.082
	50	0.060	0.281	0.051	0.243	0.050	0.305	0.050	0.298	0.062	0.243
50	25	0.056	0.102	0.049	0.101	0.050	0.114	0.052	0.108	0.055	0.101
	50	0.059	0.443	0.045	0.382	0.047	0.481	0.046	0.490	0.052	0.382

Notes: Γ is the group mean test following Im, Lee and Tieslau (2005), whereas P, Z, L and P_m are the combination tests, namely the inverse chi-square, inverse normal, logit and modified inverse chi-square tests, respectively, following Maddala and Wu (1999) and Choi (2001).

Table 6. Sizes and size-adjusted powers of panel unit root tests under Models 1 and 2 (exogenous breaks) with serial correlation, fixed lag in regression.

N	T	Γ		P		Z		L		P_m	
		Size	Power	Size	Power	Size	Power	Size	Power	Size	Power
						Model 1					
k = 0											
10	25	0.000	0.112	0.000	0.097	0.000	0.114	0.000	0.123	0.000	0.097
	50	0.000	0.379	0.000	0.276	0.000	0.444	0.000	0.460	0.000	0.276
25	25	0.000	0.166	0.000	0.138	0.000	0.188	0.000	0.199	0.000	0.138
	50	0.000	0.727	0.000	0.557	0.000	0.787	0.000	0.803	0.000	0.557
50	25	0.000	0.273	0.000	0.209	0.000	0.321	0.000	0.336	0.000	0.209
	50	0.000	0.945	0.000	0.804	0.000	0.974	0.000	0.979	0.000	0.804
k = 1											
10	25	0.052	0.104	0.046	0.094	0.047	0.112	0.046	0.111	0.059	0.094
	50	0.060	0.286	0.047	0.248	0.050	0.311	0.051	0.293	0.062	0.248
25	25	0.063	0.141	0.060	0.122	0.054	0.152	0.057	0.145	0.067	0.122
	50	0.063	0.570	0.051	0.471	0.047	0.624	0.051	0.607	0.062	0.471
50	25	0.057	0.184	0.057	0.152	0.057	0.194	0.055	0.197	0.065	0.152
	50	0.067	0.819	0.051	0.717	0.057	0.867	0.060	0.859	0.058	0.717
k = 2											
10	25	0.061	0.081	0.054	0.084	0.058	0.074	0.058	0.077	0.069	0.084
	50	0.062	0.249	0.056	0.191	0.053	0.265	0.054	0.252	0.069	0.191
25	25	0.065	0.106	0.062	0.091	0.060	0.113	0.063	0.106	0.073	0.091
	50	0.062	0.469	0.053	0.379	0.052	0.514	0.052	0.506	0.064	0.379
50	25	0.073	0.126	0.071	0.114	0.066	0.127	0.068	0.130	0.080	0.114
	50	0.063	0.743	0.049	0.639	0.051	0.783	0.051	0.776	0.054	0.639
						Model 2					
k = 0											
10	25	0.000	0.074	0.000	0.065	0.000	0.070	0.000	0.066	0.000	0.065
	50	0.000	0.165	0.000	0.125	0.000	0.163	0.000	0.162	0.000	0.125
25	25	0.000	0.078	0.000	0.072	0.000	0.074	0.000	0.081	0.000	0.072
	50	0.000	0.273	0.000	0.196	0.000	0.295	0.000	0.300	0.000	0.196
50	25	0.000	0.126	0.000	0.094	0.000	0.133	0.000	0.128	0.000	0.094
	50	0.000	0.466	0.000	0.318	0.000	0.506	0.000	0.518	0.000	0.318
k = 1											
10	25	0.096	0.067	0.078	0.063	0.095	0.065	0.092	0.063	0.094	0.063
	50	0.076	0.106	0.063	0.100	0.070	0.114	0.070	0.106	0.081	0.100
25	25	0.148	0.060	0.114	0.059	0.158	0.062	0.156	0.061	0.131	0.059
	50	0.091	0.172	0.069	0.163	0.087	0.169	0.088	0.171	0.082	0.163
50	25	0.219	0.076	0.160	0.073	0.252	0.083	0.239	0.081	0.174	0.073
	50	0.116	0.255	0.085	0.211	0.115	0.273	0.114	0.258	0.094	0.211
k = 2											
10	25	0.118	0.051	0.093	0.054	0.110	0.053	0.108	0.054	0.121	0.054
	50	0.082	0.092	0.065	0.090	0.078	0.100	0.072	0.097	0.081	0.090
25	25	0.179	0.045	0.153	0.040	0.176	0.047	0.175	0.046	0.171	0.040
	50	0.093	0.142	0.070	0.133	0.093	0.140	0.090	0.137	0.083	0.133
50	25	0.264	0.057	0.217	0.059	0.266	0.060	0.256	0.057	0.235	0.059
	50	0.135	0.193	0.092	0.174	0.135	0.206	0.134	0.205	0.103	0.174

Note: See notes to Table 5. k is fixed lag order used in regression for each individual time series.

Table 7. Sizes and size-adjusted powers of panel unit root tests under Models 1 and 2 (exogenous breaks) with serial correlation, lag selection in regression.

N	T	Γ		P		Z		L		P_m	
		Size	Power	Size	Power	Size	Power	Size	Power	Size	Power
					Distribution of individual statistic with fixed lag						
					Model 1						
10	25	0.174	0.076	0.170	0.069	0.133	0.074	0.138	0.074	0.201	0.069
	50	0.127	0.212	0.117	0.186	0.096	0.243	0.105	0.222	0.142	0.186
25	25	0.253	0.121	0.264	0.111	0.188	0.118	0.202	0.122	0.289	0.111
	50	0.169	0.416	0.167	0.340	0.127	0.453	0.139	0.432	0.189	0.340
50	25	0.365	0.128	0.389	0.114	0.274	0.138	0.294	0.132	0.412	0.114
	50	0.211	0.656	0.224	0.576	0.157	0.705	0.164	0.685	0.240	0.576
					Model 2						
10	25	0.567	0.067	0.497	0.064	0.560	0.059	0.550	0.063	0.544	0.064
	50	0.319	0.078	0.275	0.078	0.301	0.076	0.296	0.077	0.319	0.078
25	25	0.892	0.078	0.841	0.070	0.897	0.074	0.894	0.075	0.865	0.070
	50	0.567	0.101	0.497	0.103	0.555	0.098	0.554	0.097	0.533	0.103
50	25	0.995	0.038	0.987	0.036	0.995	0.030	0.995	0.033	0.989	0.036
	50	0.827	0.141	0.763	0.126	0.827	0.140	0.821	0.130	0.781	0.126
					Distribution of individual statistic with lag selection						
					Model 1						
10	25	0.031	0.077	0.031	0.070	0.021	0.078	0.024	0.084	0.038	0.070
	50	0.038	0.211	0.032	0.182	0.028	0.249	0.028	0.226	0.043	0.182
25	25	0.010	0.122	0.016	0.115	0.009	0.124	0.008	0.124	0.020	0.115
	50	0.025	0.420	0.023	0.354	0.017	0.452	0.018	0.451	0.028	0.354
50	25	0.009	0.132	0.012	0.117	0.007	0.143	0.007	0.133	0.015	0.117
	50	0.018	0.669	0.015	0.576	0.009	0.723	0.009	0.709	0.019	0.576
					Model 2						
10	25	0.066	0.044	0.058	0.046	0.059	0.044	0.059	0.044	0.074	0.046
	50	0.045	0.089	0.041	0.088	0.042	0.089	0.040	0.088	0.051	0.088
25	25	0.069	0.047	0.071	0.041	0.055	0.049	0.058	0.051	0.082	0.041
	50	0.038	0.120	0.035	0.119	0.039	0.118	0.038	0.118	0.043	0.119
50	25	0.070	0.055	0.071	0.058	0.055	0.054	0.057	0.051	0.078	0.058
	50	0.031	0.178	0.025	0.160	0.031	0.193	0.031	0.178	0.033	0.160

Note: See notes to Table 5.

Table 8. Sizes and size-adjusted powers of panel unit root tests under Models 1 and 2 with cross-sectional dependence.

N	T	Γ		P		Z		L		P_m	
		Size	Power	Size	Power	Size	Power	Size	Power	Size	Power
					No account for cross-sectional dependency						
					Model 1						
Exogenous											
10	25	0.166	0.088	0.133	0.088	0.176	0.089	0.166	0.088	0.148	0.088
	50	0.170	0.200	0.131	0.187	0.177	0.218	0.165	0.208	0.147	0.187
25	25	0.266	0.088	0.218	0.083	0.293	0.095	0.284	0.091	0.227	0.083
	50	0.273	0.211	0.218	0.189	0.297	0.231	0.289	0.217	0.228	0.189
Minimum SSR											
10	25	0.162	0.081	0.127	0.078	0.163	0.089	0.156	0.085	0.143	0.078
	50	0.162	0.183	0.123	0.171	0.166	0.196	0.159	0.185	0.140	0.171
25	25	0.258	0.083	0.209	0.079	0.280	0.087	0.273	0.084	0.218	0.079
	50	0.266	0.193	0.213	0.178	0.285	0.209	0.278	0.198	0.223	0.178
					Model 2						
Exogenous											
10	25	0.180	0.065	0.141	0.061	0.191	0.064	0.180	0.065	0.159	0.061
	50	0.178	0.115	0.134	0.102	0.182	0.117	0.170	0.116	0.152	0.102
25	25	0.272	0.058	0.226	0.058	0.301	0.060	0.291	0.058	0.236	0.058
	50	0.273	0.101	0.214	0.093	0.292	0.105	0.279	0.102	0.225	0.093
Minimum SSR											
10	25	0.142	0.063	0.114	0.059	0.142	0.063	0.137	0.062	0.132	0.059
	50	0.144	0.099	0.111	0.095	0.142	0.105	0.135	0.102	0.125	0.095
25	25	0.230	0.065	0.195	0.062	0.246	0.061	0.235	0.064	0.204	0.062
	50	0.230	0.090	0.192	0.087	0.239	0.095	0.229	0.090	0.202	0.087
					Account for cross-sectional dependency using bootstrap						
					Model 1						
Exogenous											
10	25	0.046	0.093	0.052	0.098	0.048	0.092	0.050	0.088	0.052	0.098
	50	0.038	0.170	0.038	0.162	0.036	0.184	0.038	0.172	0.038	0.162
25	25	0.047	0.081	0.052	0.092	0.042	0.076	0.046	0.078	0.052	0.092
	50	0.039	0.175	0.042	0.166	0.040	0.196	0.038	0.182	0.042	0.166
Minimum SSR											
10	25	0.059	0.100	0.058	0.098	0.060	0.102	0.058	0.098	0.058	0.098
	50	0.049	0.191	0.048	0.170	0.048	0.204	0.050	0.206	0.048	0.170
25	25	0.062	0.115	0.066	0.128	0.064	0.114	0.062	0.120	0.066	0.128
	50	0.054	0.212	0.052	0.190	0.052	0.220	0.054	0.210	0.052	0.190
					Model 2						
Exogenous											
10	25	0.061	0.070	0.064	0.076	0.054	0.064	0.056	0.066	0.064	0.076
	50	0.036	0.107	0.042	0.098	0.038	0.112	0.038	0.106	0.042	0.098
25	25	0.048	0.058	0.048	0.060	0.048	0.060	0.046	0.058	0.048	0.060
	50	0.047	0.111	0.052	0.112	0.046	0.118	0.048	0.110	0.052	0.112
Minimum SSR											
10	25	0.062	0.068	0.062	0.068	0.058	0.074	0.062	0.064	0.062	0.068
	50	0.048	0.111	0.050	0.104	0.044	0.112	0.044	0.112	0.050	0.104
25	25	0.056	0.062	0.058	0.068	0.058	0.066	0.060	0.062	0.058	0.068
	50	0.054	0.123	0.054	0.116	0.054	0.130	0.056	0.124	0.054	0.116

Note: See notes to Table 5.

Table 9. Outcomes of univariate and panel unit root tests for real per capita GDP in PPP, 1960-2003 and consumer prices, 1949-2004.

	Break model	Test results Statistic	k	TB	Bootstrapped critical values 1%	5%	10%	Break model	Test results Statistic	k	TB	Bootstrapped critical values 1%	5%	10%
		Per capita GDP							*Consumer prices*					
Univariate unit root tests														
Australia	2	-3.153	6	1981	-5.086	-4.373	-4.018	3	-3.761	8	1990	-5.014	-4.282	-3.916
Austria	1	-1.386	6	1992	-4.144	-3.440	-3.080	3	-4.091*	5	1984	-5.223	-4.440	-4.054
Belgium	1	-1.735	6	1974	-4.208	-3.479	-3.134	2	-4.266**	8	1973	-5.106	-4.221	-3.783
Canada	2	-3.245	1	1981	-5.191	-4.400	-4.042	3	-2.970	6	1965	-4.985	-4.161	-3.691
Denmark	3	-3.191	3	1973	-4.860	-4.130	-3.763	3	-4.149*	6	1985	-4.769	-4.205	-3.818
Finland	2	-2.567	1	1990	-5.276	-4.418	-3.982	3	-2.814	6	1990	-4.924	-4.101	-3.677
France	2	-3.775	1	1974	-4.825	-4.123	-3.796	3	-3.338	7	1985	-7.141	-5.118	-4.526
Germany	2	-3.513	8	1990	-6.381	-5.212	-4.687							
Greece	2	-1.599	2	1973	-4.687	-3.958	-3.533	3	-3.318	5	1972	-4.858	-4.163	-3.841
Iceland	2	-3.065	7	1986	-5.943	-5.048	-4.595	1	-3.063	4	1982	-7.833	-6.485	-5.700
Ireland	1	-1.724	1	1996	-4.262	-3.448	-3.111	0	-2.742	4		-4.459	-3.752	-3.371
Italy	1	-0.500	6	1974	-3.771	-2.987	-2.637	3	-2.891	3	1972	-5.770	-4.830	-4.351
Japan	2	-1.785	8	1973	-5.322	-4.466	-3.956	1	-2.190	8	1973	-4.185	-3.398	-3.014
Luxembourg	1	-2.244	1	1974	-4.168	-3.490	-3.120	0	-2.564	8		-4.572	-3.706	-3.275
Mexico	3	-2.127	0	1981	-4.983	-4.293	-3.955	3	-2.648	3	1973	-4.811	-4.024	-3.648
Netherlands	3	-2.441	1	1974	-4.762	-4.088	-3.713	3	-3.312	6	1982	-5.055	-4.339	-4.001
New Zealand	1	-1.408	6	1990	-4.357	-3.520	-3.100	1	-2.971	4	1979	-4.620	-3.784	-3.407
Norway	2	-1.984	0	1999	-4.350	-3.639	-3.272	3	-4.185**	6	1989	-4.851	-4.110	-3.746
Portugal	2	-4.653*	3	1974	-5.921	-4.796	-4.308	2	-6.155***	6	1973	-5.545	-4.731	-4.284
Spain	3	-3.174	3	1974	-6.066	-5.054	-4.508	3	-2.385	6	1986	-4.916	-4.168	-3.788
Sweden	3	-3.993*	5	1970	-4.890	-4.178	-3.813	2	-2.042	4	1990	-4.828	-4.021	-3.679
Switzerland	2	-1.626	6	1974	-4.778	-3.980	-3.632	3	-2.036	8	1993	-5.147	-4.353	-3.966
Turkey	1	-3.290	7	1975	-4.907	-4.147	-3.723							
United Kingdom	1	-2.797	1	1979	-4.575	-3.780	-3.389	2	-3.372	3	1974	-5.052	-4.292	-3.956
United States	1	-3.998**	1	1981	-4.174	-3.423	-3.092	2	-2.591	5	1966	-4.992	-4.057	-3.668
Panel unit root tests														
Γ		-0.330			-2.567	-1.726	-1.348		-2.668***			-2.661	-1.776	-1.404
P		50.150			76.741	68.188	63.469		70.372**			73.456	63.842	59.686
Z		-0.107			-2.432	-1.749	-1.341		-2.871***			-2.529	-1.758	-1.392
L		-0.158			-2.479	-1.757	-1.348		-2.851***			-2.593	-1.750	-1.390
Pm		0.015			2.674	1.819	1.347		2.541**			2.863	1.860	1.427

Notes: See notes to Table 5. Break model, k and TB are the selected break model, lag order and break date for each individual time series. ***, ** and * represent significance at the 1%, 5% and 10% level respectively.