Exchange Rate Variability in the Small Open Economy with Currency Substitution

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Abstract

An important feature of transition economies such as the Central and Eastern European countries is the so-called phenomenon of dollarization. It is of particular interest since extensive currency substitution not only makes domestic monetary and fiscal policies less effective, it also makes active exchange rate intervention more dangerous. In this respect, the adoption of new exchange rate regimes is a topic particularly crucial for those countries who wish to join the EU. In this paper we study a small open economy model with frictions, whose main distinctive feature is the introduction of foreign real money balances in a representative agent utility function. The equilibrium for the economy is presented by a highly non-linear multiequational system solved numerically up to a second order approximation. The model is calibrated to Czech Republic for which we could use the evidence on currency substitution collected by the Austrian National Bank. Welfare effects of different exchange rate regimes are taken into account.

Keywords: currency substitution, Small Open Economy, exchange rate policy.

JEL codes: F41, E52, D58.

1 Introduction

In this paper we consider two features of international economics: nominal exchange rate variability and dollarization. The first one has been blamed for limiting gains from international trade and for lowering welfare. The creation of the euro as well as the adoption of managed exchange rate regimes in many countries are partly due to the desire to reduce the nominal exchange rate risk. Recently, in the stream of New Open Economy Macroeconomics[1] a number of papers begun to improve the literature on optimal currency areas by formaliz-

[1] See Lane (2001) for a survey of the New Open Economy Macroeconomics.

ing Mundell's analysis of the welfare implications of exchange rate risk[2]. This paper extends this literature by considering an economy characterised by currency substitution. The main aim is to see how this phenomenon interacts with exchange rate variability and which kind of exchange rate policy is preferable in its presence.

Our model is a variant of Bergin and Tchakarov (2003) and shares basic features of models proposed in the literature of New Open Economy Macroeconomics. Contrary to Bergin and Tchakarov (2003) which is a two country model, in this paper we study a small open economy to which we add currency substitution by the introduction of the possibility of two currency types circulation in the economy. The model can be applied to study not only the standard "well-behaved" economies, but also those like many of Latin American countries (LAC) and Central and Eastern European countries.

In fact, an important feature of such economies is the so-called phenomenon of dollarization[3], i.e. when foreign currency performs traditional functions of domestic money as store of value (*asset/liability dollarization*), unit of account (*price dollarization*), and medium of exchange (*currency substitution*)[4]. The Federal Reserve and the Treasury have information on these subjects from various sources such as U.S. Customs reports, shipment data from overseas banknote wholesalers and published proxies for those shipments. Moreover, there are estimates based on in-country surveys from dollar–using countries, national surveys of domestic currency holdings, and a variety of empirical models developed by the Federal Reserve and others that estimate[5] overseas flows or holdings based on realistic assumptions concerning international currency usage. Table 1 shows some preliminary results from such estimates[6]. In the first column we show results from currency surveys conducted by the Treasury and Federal Reserve between 1997 and 2002. The other two columns report computations by Feige (2003) based on new data collected by United States Customs Service under the Currency and Foreign Transactions Reporting Act and data from a survey on 5 Europeans countries commissioned by the Austrian National Bank

[2] Among others, Obstfeld and Rogoff (2001), Devereux and Engel (2000), Bacchetta and Van Wincoop (2000) and Bergin and Tchakarov (2004). This last one can be considered as one of the main references for our work.

[3] An early survey on the problem of dollarization/currency substitution is presented by Calvo and Vegh (1992) for developing countries. Savastano, M. A. (1992) provides similar insightful studies on Latin America. Surveys of theoretical and empirical problems and developments in the field include a paper by Giovannini, A. and B. Turtelboom (1994) and the book by Mizen, P. and E. J. Pentecost (1996).

[4] Dollarization is *official* when a nation adopts de jure the currency of a foreign nation to wholly replace its domestic currency. This is also known as *full dollarization*. *Partial dollarization*, also defined as *unofficial* or de facto *dollarization*, occurs when individuals and firms voluntarily choose to use a foreign currency as a substitute for some of the monetary services of the domestic currency. This paper deals with partial dollarization and more precisely with currency substitution.

[5] Various estimations of US currency in circulation outside the United States can be found in a series of papers and reports. See, for example, U.S. Treasury (2000, 2003), Judson and Porter (2001, 1996), Porter and Weinbach (1999), Porter (1993).

[6] Unfortunately, because of lack of data, the table does not include estimates for countries (particularly from Latin America) known to be dollarized economies.

(ONB).

As expected, the dollarization degree tends to be higher in economies that have experienced high rates of inflation and/or exchange rate crisis, even when these occured much earlier. Some of these countries officially dollarized their economies (e.g. Argentina and Ecuador), but an economy can be heavily dollarized even in absence of official dollarization (e.g. Russia, Ukraine).

Not only the phenomenon is widespread but it also proves to be quite persistent since, because of hysteresis effects and habit persistence[7], the amount of foreign real money balances rarely falls to negligible levels even after a successful stabilisation of the economy. Given the size and diffusion of the phenomenon, it can be interesting to understand what are the economic policy implications of dollarization and, more precisely, of currency substitution.

Currency substitution has fiscal consequences. Foreign cash transactions reduce the costs of tax evasion and facilitate participation in the "underground" economy. The larger the size of unreported economy the larger the macroeconomic distortion. This weakens the government's ability in formulating a proper macroeconomic policy and deepens fiscal deficits. Moreover, extensive currency substitution not only makes domestic monetary and fiscal policies less effective, it also makes active exchange rate intervention more dangerous. In this respect, the adoption of new exchange rate regimes is a topic particularly crucial for those countries who wish to join the EU. In fact, accession countries are required not only to meet the Maastricht convergence criteria but also to participate in the ERM-II (Exchange Rate Arrangement between the Euro area and EU members outside the Euro area).

Schmitt-Grohé and Uribe (2001) have recently contributed to the debate analysing the costs of *full dollarization*. By means of an optimizing model of a small open economy calibrated to the Mexican economy, the authors compare the welfare costs of economic fluctuations under alternative monetary policies (*full dollarization* in the form of fixed exchange rate, inflation targeting, money growth rate pegs, or devaluation rate rules). Strictly speaking the paper is not about *dollarization*. This is not an omission of small account since the fact that the country is already partly dollarized can affect the welfare effects of *full dollarization*.

This paper differs from Schmitt-Grohé and Uribe's (2001) in the fact that it constructs a model which allows for *partial dollarization* in its form of *currency substitution* in order to investigate what are its main implications in terms of welfare.

Nominal rigidities are modelled using price adjustment costs as in Rotemberg (1982) while market imperfections are considered through monopolistic competition in the intermediate goods sector. The presence of nominal rigidities allows for non-neutral monetary policy effects, while the presence of market imperfections (namely, monopolistic competition in the intermediate goods sector) allows for non trivial pricing decisions and makes the output demand-determined in the short run. Abstracting from the domestic asset holding, financial market

[7] On this topic see among others Uribe (1997).

incompleteness is captured by considering only foreign assets in a model of debt-elastic interest-rate risk premium as in Schmitt-Grohé and Uribe. Finally, dollarization is accounted for by the introduction of foreign real money balances in a representative agent utility function.

The model is calibrated to Czech Republic which, as an accession country, has to fulfill ERM-II requirements and, as table 1 and graph[8] 1 show, appears to experience some degree of currency substituton.

The paper is organized as follows. Section 2 illustrates the model. Section 3 presents the calibration. Section 4 contains results of the numerical solution. Section 5 concludes.

2 A Small Open Economy Model

We consider a small open economy (SOE) composed of infinitely-lived individuals and of a continuum of firms whose shares are owned by the consumers. Agents have the possibility to use foreign currency. Use of foreign money as a mean of savings (asset substitution) and a mean of transaction (currency substitution) can be justified for countries with high inflation and unstable economy, or for countries with incomplete financial sector, i.e. in developing or transition economies. We model the 'dollarised' economy by allowing two monies in the utility function. Saving is possible by holding domestic/foreign money and foreign nominal bonds.

As for the production part, there are two types of home produced goods: final (X) and intermediate (Y). Final good is nontradable. Intermediate good is used as an input for home (X_H) and foreign (X_H^*) production. Intermediate goods are also produced abroad: imported intermediates are called X_F. The final good sector is perfectly competitive, while the intermediate sector is characterised by nominal rigidities in the form of monopolistic competition and adjustment costs à la Rotemberg (1982). Capital and labor are the inputs in the intermediate sector. However, intermediate inputs only are required for the production of final goods.

Finally, we assume that financial markets of our SOE are incomplete in the sense that state-contingent securities are not available.

2.1 Households

Our small open economy model is inhabited by a representative household whose istantaneous utility function takes the form[9]

$$U(C_t, N_t, m_t, m_t^*) = u(C_t) - v(N_t) + h(m_t, m_t^*) \tag{1}$$

where C_t is a consumption good, N_t denotes hours of labor, ε_t is the nominal exchange rate, $m_t \equiv \frac{M_t}{P_t}$ stands for real home currency holdings and $m_t^* \equiv \frac{\varepsilon_t M_t^*}{P_t}$

[8] The author is indebeted to the Austrian Nationa Bank (ONB) for providing its survey estimates. A description of the databe can be found in Stix (2001).

[9] For simplicity of notation, in wha follows we drop the household index (j).

is real foreign currency holdings. Following part of the literature on currency substitution we introduce real money balances in the utility function[10]. Feenstra (1986) demonstrates a functional equivalence between using real balances as an argument of the utility function and entering money into liquidity costs which appear in the budget constraint. The representative household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t) - v(N_t) + h\left(\frac{M_t}{P_t}, \frac{\varepsilon_t M_t^*}{P_t}\right) \right\} \tag{2}$$

As in Imrohoglu (1994) money services are produced by using a combination of domestic and foreign real balances in a constant elasticity of substitution (CES) production function

$$h(\cdot) = \chi_t \left[\alpha \left(\frac{M_t}{P_t}\right)^{1-\sigma_3} + (1-\alpha)\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^{1-\sigma_3} \right]^{\frac{1}{1-\sigma_3}} \tag{3}$$

where $\alpha \in (0,1)$, $\sigma_3 > 0$ and $\chi_t > 0$. This is a convenient functional form that separates the elasticity of currency substitution $1/\sigma_3$, from the share, $(1-\alpha)$, of foreign real balances in the production of domestic liquidity services[11]. The liquidity services parameter, χ, is assumed to change over time according to an AR(1) process in order to allow for money demand shocks.

We assume that households have access to simple foreign assets B_t^*, whose interest rate is, however, different from the one for rest of the world (ROW) households and it is described by (45). The household determines capital accumulation K_{t+1} which involves a constant rate of depreciation $\delta \in (0,1)$ per period and is subject to convex adjustment costs. Specifically, the law of motion of K_{t+1} is given by

$$K_{t+1} = (1-\delta) K_t + i_t - AC_{I,t} \tag{4}$$

where, following Rotemberg (1982), we have a quadratic adjustment cost that depends upon a parameter φ_I

$$AC_{I,t} \equiv \frac{\varphi_I}{2} \left[\frac{K_{t+1} - K_t}{K_t}\right]^2 K_t \tag{5}$$

Note that if the price adjustment cost parameter $\varphi_I = 0$ the model collapses to a flexible price specification. Also note that in steady state the price adjustment costs are equal to zero.

The representative household faces a sequence of budget constraints of the form

$$\begin{aligned} P_t C_t + P_t \left[K_{t+1} - (1-\delta) K_t\right] + P_t AC_{I,t} + M_t + \varepsilon_t M_t^* + \varepsilon_t B_t^* \leqslant (1+i_{t-1})\varepsilon_t B_{t-1}^* \\ + M_{t-1} + \varepsilon_t M_{t-1}^* + W_t N_t + D_t + P_t r_t K_t - T_t \end{aligned} \tag{6}$$

[10] See among others Calvo (1985) and Imrohoroglu (1994).

[11] This specification delivers a steady state with positive foreign real money balances as long as $\alpha > 0$. In the appendix we consider another possible specification for the $h(\cdot)$ function, namely, the one proposed by Obstfeld and Rogoff (1995). In this case steady state foreign money holdings are positive according to parameters measuring its costs and gains and the expected devaluation rate.

$\forall t$, where T_t denotes lump-sum taxes/transfers and D_t is profit from owned domestic intermediate firms. All variables are expressed in units of domestic currency.

The right hand side of the budget constraint gives the available resources as the sum of gross return on the bond holding, initial money holdings, labour income, profits from intermediates in tradeables, revenues from renting capital, less government taxation . These resources are used cover consumption, investment and to acquire the next period money balances and new bond holdings. Notice that M_{t-1} denotes the quantity of nominal money balances acquired during period t and carried over into period $t+1$.

Control variables are total consumption C_t, capital accumulation K_{t+1}, domestic nominal money holdings M_t, foreign nominal money holdings M_t^*, working hours N_t, nominal bond holdings denominated in foreign currency B_t^*. Money does appear in both the budget constraint and the utility function, so that money holdings can affect the paths of consumption and current account balance through the path of prices.

In what follows we let period utility take the form

$$U\left(C_t, N_t, m_t, m_t^*\right) = \frac{C_t^{1-\sigma_1}}{1-\sigma_1} - \frac{N_t^{1+\sigma 2}}{1+\sigma_2} + \chi_t \left[\alpha \left(m_t\right)^{1-\sigma_3} + (1-\alpha)\left(m_t^*\right)^{1-\sigma_3}\right]^{\frac{1}{1-\sigma_3}} \tag{7}$$

2.1.1 Optimality Conditions

The household chooses the set of stochastic processes $\{C_t, K_{t+1}, M_t, M_t^*, N_t, B_t^*\}_{t=0}^{\infty}$so to maximize 2 subject to 6 and some borrowing limit that prevents from engaging in Ponzi-type schemes, taking as given the sequences $\{P_t, \varepsilon_t, i_{t-1}, W_t, r_t\}$ .The associated optimality conditions are

(1)→Euler equation

$$\beta\left(1+i_t\right) E_t\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma_1}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)\right\} = 1 \tag{8}$$

(2)→Labor Supply in Intermediate Good Production, N_t; is given by a standard intratemporal optimality condition

$$\left(N_t\right)^{\sigma_2} = \frac{W_t}{P_t} C_t^{-\sigma_1} \tag{9}$$

This equation is the labour-leisure trade-off condition that comes from utility maximization with respect to wages. It ensures that marginal disutility of the additional factor supply (due to leisure foregone) on the left hand side is compensated by an extra unit of marginal utility of consumption, such that an extra unit of labour supply can buy at the real factor price.

(3)→Capital Accumulation

$$\left(1+\varphi_I \frac{K_{t+1}-K_t}{K_t}\right) = \beta E_t\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma_1}\left(r_{t+1}+(1-\delta)+\left(\frac{\varphi_I}{2}\frac{K_{t+2}^2-K_{t+1}^2}{K_{t+1}^2}\right)\right)\right\} \tag{10}$$

The remaining relevant optimality conditions differ according to which specification of the $h(\cdot)$ function we consider. With the Imrohoglu specification

(4)→Domestic Money Demand

$$\chi_t\alpha\left(\frac{M_t}{P_t}\right)^{-\sigma_3}\left[\alpha\left(\frac{M_t}{P_t}\right)^{1-\sigma_3}+(1-\alpha)\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^{1-\sigma_3}\right]^{\frac{\sigma_3}{1-\sigma_3}}=C_t^{-\sigma_1}-\beta E_t\left[C_{t+1}^{-\sigma_1}\left(\frac{P_t}{P_{t+1}}\right)\right] \tag{11}$$

(5)→Foreign Money Demand

$$\chi_t(1-\alpha)\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^{-\sigma_3}\left(\alpha\left(\frac{M_t}{P_t}\right)^{1-\sigma_3}+(1-a)\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^{1-\sigma_3}\right)^{\frac{\sigma_3}{1-\sigma_3}}=C_t^{-\sigma_1}-\beta E_t\left\{C_{t+1}^{-\sigma_1}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)\right\} \tag{12}$$

In the ROW a representative household faces a problem identical to the one outlined above. We assume that the size of the SOE is negligible relative to the ROW, which allows us to treat the latter as if it was a closed economy.

2.2 Firms

For the supply side we adopt a structure similar to the one in Romer (1990). There is a final good sector which is perfectly competitive and non tradeable, while the tradeable intermediate good is characterized by monopolistic competition.

2.2.1 Final goods sector

Because the production function is homogeneous of degree one, final output can be described in terms of the actions of a single, aggregate, price-taking firm. The firms are perfectly competitive, the output is determined as

$$X_t = \left[\gamma^{\frac{1}{\rho}}\left[X_{H,t}\right]^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}}\left[X_{F,t}\right]^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{13}$$

$X_{H,t}$ is a home produced intermediate good, $X_{F,t}$ is imported intermediate good, both used in the production of domestic final good. Parameter ρ will determine the elasticity of substitution between home and foreign goods, while γ will determine the ratio of imports to GDP.

We design the firm's problem using the budget separation method.

1) Inter-input allocation. The firms choose inputs quantity $X_{H,t}$ and $X_{F,t}$ to solve the following PMP (Profit Maximization Problem).

$$\underset{X_{H,t},\,X_{F,t}}{max} \qquad P_t \cdot X_t - P_{H,t}X_{H,t} - P_{F,t}X_{F,t} \tag{14}$$

subject to 13 and where P_t is price index taken as given (perfect competition)

$$P_t = \left\{\gamma\left[P_{H,t}\right]^{1-\rho} + (1-\gamma)\left[P_{F,t}\right]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \tag{15}$$

The FOC are the following

$$X_{H,t}: \quad P_t\left[\frac{\rho}{\rho-1}\left(X_t\right)^{\frac{1}{\rho}}\frac{\rho-1}{\rho}\gamma^{\frac{1}{\rho}}\left(X_{H,t}\right)^{-\frac{1}{\rho}}\right]-P_{H,t}=0$$

$$X_{F,t}: \quad P_t\left[\frac{\rho}{\rho-1}\left(X_t\right)^{\frac{1}{\rho}}\frac{\rho-1}{\rho}\left(1-\gamma\right)^{\frac{1}{\rho}}\left(X_{F,t}\right)^{-\frac{1}{\rho}}\right]-P_{F,t}=0$$

Rearranging, we get

$$X_{H,t}=\gamma\left(\frac{P_{H,t}}{P_t}\right)^{-\rho}X_t \tag{16}$$

$$X_{F,t}=(1-\gamma)\left(\frac{P_{F,t}}{P_t}\right)^{-\rho}X_t \tag{17}$$

2) Intra-basket allocation. In turn, each basket of intermediate goods is composed of a continuum of different varieties indexed by j. The corresponding Home Intermediate Good Index and Foreign Intermediate Good Index are given accordingly as

$$X_{H,t}=\left[\int_0^1 X_{H,t}(j)^{\frac{\phi-1}{\phi}}\,dj\right]^{\frac{\phi}{\phi-1}} \tag{18}$$

$$X_{F,t}=\left[\int_0^1 X_{F,t}(l)^{\frac{\phi-1}{\phi}}\,dl\right]^{\frac{\phi}{\phi-1}} \tag{19}$$

The parameter ϕ will determine the mark-up price over the marginal cost.

The Home and Foreign Intermediate Price indices are

$$P_{H,t}=\left\{\int_0^1\left[P_{H,t}(j)\right]^{1-\phi}dj\right\}^{\frac{1}{1-\phi}} \tag{20}$$

$$P_{F,t}=\left\{\int_0^1\left[P_{F,t}(l)\right]^{1-\phi}dl\right\}^{\frac{1}{1-\phi}} \tag{21}$$

Proceeding as in the previous step, the cost minimization gives the following intra-basket demands

- home demand for domestic intermediates

$$X_{H,t}(j)=\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi}X_{H,t} \tag{22}$$

- home demand for imports

$$X_{F,t}(l)=\left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\phi}X_{F,t} \tag{23}$$

2.2.2 Foreign sector

In the rest of the world a representative household, final and intermediate good firms, face problem identical to the ones outlined above. Allocations and prices are denoted with an asterisk.

Thus, the final good production is

$$X_t^* = \left[(\gamma^*)^{\frac{1}{\rho}} \left[X_{H,t}^* \right]^{\frac{\rho-1}{\rho}} + (1-\gamma^*)^{\frac{1}{\rho}} \left[X_{F,t}^* \right]^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{24}$$

where the parameters and variables have an interpretation similar to the previous one. Note that as $\gamma^* \to 0$ the SOE intermediate good does not enter in the production of the final good of the rest of the world.

Following the same lines as above, optimality conditions yield

- demand for the SOE produced intermediate good (exports)

$$X_{H,t}^* = \gamma^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\rho^*} X_t^* \tag{25}$$

- demand for the intermediate good produced in the rest of the world

$$X_{F,t}^* = (1-\gamma^*) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\rho^*} X_t^* \tag{26}$$

And so

- foreign demand for exports:

$$X_{H,t}^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\phi^*} X_{H,t}^* \tag{27}$$

- foreign demand for their own goods:

$$X_{F,t}^*(l) = \left(\frac{P_{F,t}^*(l)}{P_{F,t}^*} \right)^{-\phi^*} X_{F,t}^* \tag{28}$$

In addition, we assume that there are no barrires to trade such that the Law of One Price (LOP) holds for each good, implying that the prices of importables and exportables, $P_{F,t}(l)$ and $P_{H,t}(j)$,are linked to the respective world prices, $P_{F,t}^*(l)$ and $P_{H,t}^*(j)$, by the relationships

$$P_{F,t}(l) = \alpha \varepsilon_t P_{F,t}^*(l) \tag{29}$$

$$P_{H,t}(j) = \alpha \varepsilon_t P_{H,t}^*(j) \tag{30}$$

for all i, where ε_t is the nominal exchange rate (the price of foreign currency in terms of home currency), and $P_{F,t}^*(l)$ is the price of foreign good denominated in foreign currency. Integrating over all goods we obtain

$$P_{F,t} = \alpha \varepsilon_t P_{F,t}^* \tag{31}$$

$$P_{H,t} = \alpha \varepsilon_t P_{H,t}^* \tag{32}$$

2.2.3 Intermediate goods sector

The market is populated by a continuum of firms acting as monopolistic competitors, since intermediate goods substitute imperfectly for one another as inputs to producing the final good. During period t, the representative intermediate goods-producing firm rents capital $K_t(j)$ and hires $N_t(j)$ units of labor, in order to produce $Y_t(j)$ units of intermediate good according to the production function given by:

$$Y_t(j) = A_t K_t^{\theta}(j) N_t^{1-\theta}(j) \tag{33}$$

where A_t is a technology shifter common to all firms.

During each period t, the representative intermediate goods-producing firm sets a nominal price $P_{H,t}(j)$, subject to requirenment that it satisfies the representative finished goods-producing firm's demand.

The existence of an economy-wide competitive factor market implies that all firms will pay the same rental rate r_t and the same nominal wage W_t. This also implies that all firms face a common nominal marginal cost (in particular, independent of the level of individual output) that we denote by MC_t

$$MC_t = \frac{(r_t P_t)^{\theta} W_t^{1-\theta}}{A_t \theta^{\theta} (1-\theta)^{1-\theta}} \tag{34}$$

Each firm faces a quadratic cost of price adjustment as in Rotemberg (1982) and given by[12]

$$AC_{P,t}(j) \equiv \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 Y_t(j) \tag{35}$$

where π_H is the gross steady state rate of inflation in the intermediate sector. Note that if the price adjustment cost parameter $\varphi_P = 0$ the model collapses to a flexible price specification. Also note that in steady state the price adjustment costs are equal to zero.

Cost minimization implies the following efficiency condition for the choice of labor input and capital

$$P_t r_t K_t = \frac{\theta}{1-\theta} W_t N_t \tag{36}$$

The cost of price adjustment makes the firm's problem dynamic. Assuming no price discrimination, each firm chooses price $P_{H,t}(j)$ and outputs $X_{H,t}(j), X_{H,t}^*(j)$ in order to maximize its total market value, i.e.

$$\max_{P_{H,t}(j)} E \sum_{t=0}^{\infty} R_t D_t(j) \tag{37}$$

[12] This form is mutuated from Ireland (2004). Following Bergin and Tchakarov (2004) we assume the same adjustment cost for goods sold domestically and goods exported. Bergin (2003) has different adjustment costs.

where

$$D_t(j) = P_{H,t}(j)X_{H,t}(j) + \alpha\varepsilon_t P^*_{H,t}(j)X^*_{H,t}(j) - r_t P_t K_t(j) - W_t N_t(j) - P_{H,t}AC_{P,t}(j), \tag{38}$$

subject to production technology 33 and final sector demands 22 and 27.

Since firms are assumed to be owned by the representative household, they value future payoffs according to the household's intertemporal marginal rate of substitution in consumption and so the pricing kernel used to value random datre $t+n$ payoffs is

$$R_t = \beta^t C_t^{-\sigma_1} \tag{39}$$

Assuming a symmetric equilibrium, where all firms are identical

$$X_{H,t}(j) = X_{H,t} \qquad K_t(j) = K_t \qquad N_t(j) = N_t \qquad P_{H,t}(j) = P_{H,t} \tag{40}$$

the optimization problem implies the following pricing behaviour[13]

$$\begin{aligned} &X_{H,t}\left(1-\phi+\phi\tfrac{MC_t}{P_{H,t}}+\phi\kappa_t-\varphi_P\tfrac{\pi_{H,t}}{\pi_H}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]\right)+\\ &X^*_{H,t}\left(1-\phi^*+\phi^*\tfrac{MC_t}{P_{H,t}}+\phi^*\kappa_t-\varphi_P\tfrac{\pi_{H,t}}{\pi_H}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]\right)+\\ &\qquad+\varphi_P E_t\left\{\tfrac{R_{t+1}}{R_t}\tfrac{\pi^2_{H,t+1}}{\pi_H}\left[\tfrac{\pi_{H,t+1}}{\pi_H}-1\right](X_{H,t+1}+X^*_{H,t+1})\right\}=0 \end{aligned} \tag{41}$$

where

$$\kappa_t(j) \equiv \frac{AC_{P,t}(j)}{Y_t(j)} = \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 \tag{42}$$

Assuming $\phi = \phi^*$ (i.e., the SOE has the same price elasticity of demand as the ROW) and using the market clearing condition 59 the pricing condition becomes

$$\begin{aligned} &P_{H,t} = \tfrac{\phi}{(\phi-1)}\left(MC_t + P_{H,t}\kappa_t\right)\\ &+P_{H,t}\tfrac{\varphi_P}{(\phi-1)}\tfrac{\pi_{H,t}}{\pi_H}\left[1-\tfrac{\pi_{H,t}}{\pi_H}\right]\\ &+P_{H,t}\tfrac{\varphi_P}{(\phi-1)}E_t\left[\tfrac{R_{t+1}}{R_t}\left(\tfrac{\pi^2_{H,t+1}}{\pi_H}\left[\tfrac{\pi_{H,t+1}}{\pi_H}-1\right]\right)\tfrac{Y_{t+1}}{Y_t}\right] \end{aligned} \tag{43}$$

As usual if $\varphi_P = 0$ (i.e. no price adjustment costs) the above pricing condition boils down to:

$$P_{H,t} = \frac{\phi}{\phi - 1}MC_t = \mu MC_t \tag{44}$$

where $\mu = \frac{\phi}{\phi-1}$ denotes the desired (constant) markup value. Hence a representative firm chooses the price for its differentiated product as a constant markup over the marginal cost. This stems from the imperfect competition feature of the market. In fact, as $\phi \to \infty$ in the case of perfectly competitive output markets, $P_{H,t} = MC_t$, which is the usual pricing condition of a firm acting as a price taker.

[13] See the appendix for the complete derivation.

Hence, in presence of price adjustment costs, price-setting will deviate from the simple markup rule by some additional terms. First, the resource cost of setting a price (κ_t). Then a backward looking component measuring firms reluctancy to change prices because of menu costs. Finally a forward looking component reflecting the price that if the firm expects the need to change prices further in the next period, it will tend to change the price more today so to minimize future adjustment costs[14].

2.2.4 Financial markets

If state-contingent securities are not available we could think that the domestic household has access simply to non-state contingent nominal bonds. Following Obstfeld and Rogoff (1996), we defend this assumption on the basis that it would be strange to analyze imperfections and rigidities in goods markets along with complete financial markets. The small open economy model with incomplete asset markets features a steady state that depends on initial conditions. SOE that face an exogenous world interest rate display nonstationarity dynamics in response to stationary exogenous shocks. Because of this, the solution of the log-linearized model may not be a valid approximation to the exact nonlinear model. Hence, in order to impose stationarity we study a model with a debt-elastic interest-rate premium[15]. It means that the interest rate at which households can borrow internationally differs from the world interest rate by the risk premium.

In our model, domestic agents are assumed to face an interest rate, i_t, that is increasing in the country's net foreign debt, which we denote by F_t. Specifically, i_t is given by

$$i_t = i_t^* + \Psi\left(F_t\right) \tag{45}$$

where i^* denotes the world interest rate and $\Psi\left(\cdot\right)$ is a country-specific interest rate premium. The function $\Psi\left(\cdot\right)$ is assumed to be strictly increasing and $F_t \equiv -\frac{\varepsilon_t}{P_{H,t}}B_t^*$. This stationarity inducing technique has been used, among others, in recent papers by Ravenna and Natalucci (2002), Mendoza and Uribe (2000), and Schmitt-Grohé and Uribe (2001, 2002). In this model, domestic agents are assumed to face an interest rate that is increasing in the country's net foreign debt (F). To see why this device induces stationarity, let $\Psi\left(F\right)$ denote the steady state premium over the world interest rate paid by domestic residents, and F the steady state stock of foreign debt. Then in the steady state the Euler equation implies that

$$\beta\left(1+i\right) = \beta[1+i^* + \Psi\left(F\right)] = \pi \tag{46}$$

[14] Rotemberg pricing is, by now, quite common in the literature. Alternative means to introduce price stickiness are the Calvo (1983) and Yun (1996) pricing models.

[15] To induce stationarity several options are available: endogenous discounting, adjustment costs for the accumulation of foreign debt or the specification of debt-elastic risk premia. Schmidt-Grohé and Uribe (2003) find that all of the options deliver virtually identical results at business cycle frequencies.

This expression determines the steady-state net foreign asset position as a function of β, i^*, π and the parameters that define the premium function $\Psi(F)$ only.

Following Schmitt-Grohé and Uribe (2002) we use the following functional form for the risk premium

$$\Psi(F_t) \equiv \psi_2(e^{F_t - \bar{F}} - 1) \tag{47}$$

where ψ_2 and $\bar{F}$ are constant parameters. $\bar{F}$ is chosen to be equal to the desired steady state value of net foreign debt. To ensure that at busines cycle frequencies the model behaves as if the interest rate premium were constant, in the numerical experiments we set the debt elasticity ψ_2 very close to zero.

2.3 Government

We will consider two different monetary regimes. In order to compare the welfare effects of exchange rate variability we assume the following money growth rule[16]

$$\log M_t = \log M_{t-1} + \rho_m(\varepsilon_t - \overline{\varepsilon}) \tag{48}$$

This rule permits a *fixed exchange rate* regime for ρ_m set to a large negative value, or alternatively a *flexible exchange rate* regime, for ρ_m set near zero. It is assumed that the monetary authority can commit to set this parameter at a time invariant value[17]. Moreover, policies are specified in such a way that they give rise to the same nonstochastic steady state.

For simplicity we assume the following goverment's budget constraint

$$M_t = M_{t-1} + T_t \tag{49}$$

The assumed fiscal policy implies that the government rebates seignorage revenues to the public through lump-sum transfers. Note that in presence of currency substitution such revenues are smaller[18].

2.4 Terms of trade and some identities

We define terms of trade as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{50}$$

then the price indeces can be rewritten as

$$\frac{P_t}{P_{H,t}} = \left\{\gamma + (1-\gamma) S_t^{1-\rho}\right\}^{\frac{1}{1-\rho}} \equiv g(S_t), \tag{51}$$

$$\frac{P_t}{P_{F,t}} = \left\{\gamma S_t^{\rho-1} + (1-\gamma)\right\}^{\frac{1}{1-\rho}} = \frac{g(S_t)}{S_t} \tag{52}$$

[16] See Obstfeld and Rogoff (2001) among the others.

[17] This because we are just interested in comparing steady states under different economic policies and not in characterizing an optimal policy.

[18] In this paper we do not adress the dollarization's implications for seignorage revenue. On this issue see, for example, Schmitt-Grohé and Uribe (1999).

and in terms of inflation:

$$\pi_t^{1-\rho} = \frac{\gamma + (1-\gamma) S_t^{1-\rho}}{\gamma + (1-\gamma) S_{t-1}^{1-\rho}} \pi_{H,t}^{1-\rho} \tag{53}$$

Moreover

$$\frac{X_{H,t}}{X_t} = \gamma \left(\left\{ \gamma + (1-\gamma) S_t^{1-\rho} \right\}^{\frac{1}{1-\rho}} \right)^{\rho} = \gamma \left[g(S_t) \right]^{\rho} \tag{54}$$

$$\frac{X_{F,t}}{X_t} = (1-\gamma) \left(\left\{ \gamma S_t^{\rho-1} + (1-\gamma) \right\}^{\frac{1}{1-\rho}} \right)^{\rho} = (1-\gamma) \left[S_t g(S_t) \right]^{\rho} \tag{55}$$

2.5 Market Clearing and Equilibrium

We now turn to the description of a symmetric equilibrium with an initial level of net foreign assets equal to zero, $B_0 = 0$. In the symmetric equilibrium, all firms behave identically and all households behave identically, therefore, one can work with a single representative household and a single representative firm. We can drop the superscript notation in future references so that in the symmetric equilibrium we have:

$$X_{H,t}(j) = X_{H,t} \quad Y_t(j) = Y_t \quad N_t(j) = N_t \quad P_{H,t}(j) = P_{H,t} \quad D_t(j) = D_t \tag{56}$$

We assume that there are no government and domestic bonds. Since the goods produced in the SOE represent a negligible fraction of the world's consumption basket, we can consider the rest of the world is an approximately closed economy with

$$P_t^* \approx P_{F,t}^*, \qquad \pi_t^* \approx \pi_{F,t}^* \tag{57}$$

In equilibrium aggregate supply is equal aggregate demand, therefore

$$g(S_t) X_t = g(S_t) \{C_t + [K_{t+1} - (1-\delta) K_t] + AC_{I,t}\} + \frac{\varphi_P}{2} \left[\frac{1}{\pi_H} \frac{P_{H,t}}{P_{H,t-1}} - 1 \right]^2 \cdot X_t \tag{58}$$

$$Y_t = X_{H,t} + X_{H,t}^* \tag{59}$$

$$N_t^s = N_t^d \tag{60}$$

$$M_t = M_{t-1} + T_t \tag{61}$$

To deal with the non stationary nominal variables in the system, we consider stationary variables expressed in real terms such as $F_t \equiv -\frac{\varepsilon_t}{P_{H,t}} B_t^*$ (net foreign asset position), $m_t^* \equiv \frac{\varepsilon_t M_t^*}{P_t}$, (real foreing money holdings) and $w \equiv \frac{W_t}{P_{H,t}}$ (real wages).

A stationary rational expectation equilibrium[19] is a set of stationary stochastic processes $\{C_t, N_t, K_{t+1}, F_t, m_t, m_t^*, \varepsilon_t, i_t, r_t, w_t, X_t, Y_t, X_{H,t}, X_{F,t}, S_t, \pi_t, \pi_{H,t}{}_{t=0}^{\infty}\}$ satisfying 8-12, 16-17, 32-36, 41-59, and 89 given exogenous processes $\{\chi_t, A_t\}_{t=0}^{\infty}$ and initial values for K_0, F_0, S_0.

[19] The full description of the equilibrium as well as a particular steady state, are presented in the appendix.

3 Calibration

The shocks to technology and money demand are calibrated at standard values and are distributed as follows[20]

$$
\begin{array}{c}
\left[\log\left(A_t\right)-\log\left(\overline{A}\right)\right]=\rho_a\left[\log\left(A_{t-1}\right)-\log\left(\overline{A}\right)\right]+v_{at} \\
\left[\log\left(\chi_t\right)-\log\left(\overline{\kappa}\right)\right]=\rho_\chi\left[\log\left(\chi_{t-1}\right)-\log\left(\overline{\chi}\right)\right]+v_{\chi t} \\
v_{at}\sim N\left(0,0.01^2\right) \qquad v_{\chi t}\sim N\left(0,0.03^2\right) \\
\rho_a=0.9 \qquad \rho_\chi=0.99
\end{array}
\tag{62}
$$

For the calibration to the Czech Republic we borrow values from Natalucci and Ravenna (2002).

3.1 Preferences

The discount factor, β, is set equal to 0.99 and we interpret a period as one quarter. Particularly interesting for us are the money demand parameters: α and σ_3. As for the elasticity of intertemporal substitution of money services $(1/\sigma_3)$ empirical studies find a wide range of estimates: from 0.39 in Chari et al. (1998a) to 0.05 in Mankiw and Summers (1986). For the benchmark calibration we choose a intermediate value for elasticity, $\sigma_3 = 5$, and we calibrate $\alpha = 0.4$ so that the *dollarization index* of the economy (the share of foreing currency on total currency in circulation) matches the estimates. Natalucci and Ravenna (2002) use a utility function with the log of consumption implying an elasticity of consumption, σ_1, equal to 1. Instead our specification allows for different values of this parameter. Empirical studies estimate that the income elasticity of real money demand (σ_1/σ_3) equal to 1 and so we set $\sigma_1 = 4$. Parameter σ_2 is set equal to 2 implying an elasticity of labour supply equal to $(1/2)$.

3.2 Technology

The quaterly depreciation rate, δ, is set to 0.025. Following Bergin and Tchakarov (2003) the price adjustment cost, φ_P, is set at 50, and investment adjustment cost, φ_I, at 4.

The elasticity of substitution, ρ, between imported intermediate good, X_F, and domestic intermediate good, X_H, is set equal to 0.5. Parameter γ, the share of domestic intermediate good, X_H, in the production of final output can serve as a proxy for the openness of the economy. Hence, it describes the level of a small open economy's dependence on the rest of the world. The degree of monopolistic competition, ϕ, is set at 11 implying a markup of 10%. Assuming the tradable sector to be capital-intensive, the capital share in production, θ, is set at 0.67.

3.3 Government Policy

Following Schmitt-Grohé and Uribe (2002) we set $\bar{F} = 0$, and $\psi_2 = 0.000742$.

[20] See Bergin an Tchakarov (2004) for a discussion of such parametrization.

4 Solution Method and welfare measure

Since the equilibrium is a highly nonlinear system of equations, it is not possible to obtain a solution in its closed form. Hence the model is solved numerically applying Matlab codes (more precisely, DYNARE package[21]). Being interested in welfare implications of different monetary policies, the model is solved numerically up to the second order of approximation. In fact, contrary to the standard methodology which relies upon first order approximations, second order solution enables us to take into account both the direct and indirect effects of variability on welfare. This means that we can compare welfare across policies that do not have first-order effects on the model's deterministic steady state.

As usual in this literature[22] we measure welfare costs of business cycles associated with a particular monetary regime by the permanent shift in steady state consumption required to achieve the same utility, i. e. we find how much steady state consumption the household is ready to give up in order to be indifferent between the corresponding constant sequences of consumption and hours and the equilibrium stochastic processes for these two variables associated with the monetary policy under consideration. More precisely, welfare is computed from the portion of utility excluding real money balances and, μ, the cost of business cycles under a particular hypothesis, is given by

$$U\left[(1+\mu)\,C, N\right] = E\left[U\left(C_t, N_t\right)\right] \tag{63}$$

where C and N are deterministic steady state consumption and labour, while the left hand side is a measure of the welfare. There are two ways of computing welfare: unconditional and conditional[23].

Unconditional welfare is computed as a second order Taylor expansion of the utility function around the deterministic steady state, i.e.

$$\begin{aligned} U\left[(1+\mu)\,C, N\right] &= U - C^{1-\sigma_1} E\left(\hat{C}_t\right) + \\ &\quad -\frac{1}{2}\sigma_1 C^{1-\sigma_1} var\left(\hat{C}_t\right) - N^{1+\sigma_2} E\left(\hat{N}_t\right) - \frac{1}{2}\sigma_2 N^{1+\sigma_2} var\left(\hat{N}_t\right) \end{aligned} \tag{64}$$

where variables with a hat indicate deviations from steady state.

Conditional welfare takes into account the transition dynamics due to the implementation of the policy rule[24]

$$U\left[\left(1+\mu^{cond}\right) C, N\right] = (1-\beta)\sum_{t=0}^{\infty} \beta^t E_0 U\left(C_t, N_t\right) \tag{65}$$

Conditional welfare is more appropriate for policy experiments and this is the

[21] See Collard and Juillard (2000), Juillard (2004).

[22] Lucas (1987).

[23] Bergin and Tchakarov (2003)

[24] We do not address the time inconsistency problem because we are not interested in characterizing an optimal policy but just in comparing the effects of different policies.

measure we use in what follows. Hence

$$\mu^{cond} = \frac{1}{C}\left\{(1-\sigma_1)\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t E_0 U\left(C_t, N_t\right) + \frac{N^{1+\sigma 2}}{1+\sigma_2}\right]\right\}^{\frac{1}{1-\sigma_1}} - 1 \quad (66)$$

4.1 Welfare

Using the benchmark calibration, we compare the effects of exchange rate variability in two settings: one without currency substitution and one with currency substitution. The welfare costs, μ, are comparable across policies because they give rise to the same nonstochastic steady state.

It occurs to us at least one caveat: across the welfare analysis we assumed relevant parameters such as α and σ_3 invariant. We are aware that this is not an innocuous assumption since agent's behaviour, expecially with respect to the preferences over currency substitution can change according to the turbulence of the economy. A more elaborate model would set α as a function of current or past inflation and/or devaluation rate. This will allow for hysteresis and enrich the dinamics of the model. Nevertheless, currency substitution proves to be a persistent phenomenon even after stabilization policies have been successfully implemented and α can be considered exogenously constant for a certain period. In our opinion this assumption can be satisfied by the Czech Republic which is the one we are considering in this paper. Becasue of this and for simplicity we decided to take α as given to a constant level. Future extensions can include the endogeneization of the degree of dollarization.

Results of the welfare analysis are reported in table 2.

Overall, business cycles costs range between 1% and 2%. As expected exchange rate variability is costly in both scenarios, even if the gains from adopting a fix exchange rate policy are quite small. Surprisingly, business cycles costs are larger in the economy without currency substitution. Allowing agents to have access and use the foreing money helps to hedge against the exchange rate risk (the relative gain from switching from flexible exchange rate to fix exchange rate is smaller in presence of currency substitution) and, in general, against shocks[25]. Finally, most if not all of the costs is due to the technology shock.

We can conclude that for the Czech economy, as we modeled it, adopting a fixed exchange rate can be welfare improving (0.02% of gain).

4.2 Sensitivity analysis and simulations

In this section we perform some sensitivity analysis and stochastic simulations in order to assess the behaviour of the dollarized economy.

First, we try to asses the sensitivity of the stochastic welfare to changes in the share parameter α. In particular we wish to examine how the degree of dollarization, measured by (α), affets the welfare.

[25] This result is in line with a previous work of us (Colantoni and Kaminska (2004)) where we found that dollarization might be welfare improving.

As figure 1 shows in a fix exchange rate regime welfare remains constant as the degree of dollarization increases. Under flexible exchange rates, welfare is constantly lower than the one with fix exchange rates and fluctuates as α increases. For $\alpha = 0$ and $\alpha = 1$, i.e. one currency, welfare is better. It is best for α around 65% and worse for α around 35%. When looking at the volatilities of the devaluation and inflation rates we see that they are constant under fix exchange rates (see figure 2). As expected, when allowed to float the exchange rate is more volatile and its variability fluctuates, decreasing and increasing as the dollarization degree increases. As for inflation, for low and high degrees of dollarizaiton its volatility is smaller in a flexible exchange rate regime. For values of α between 55% and 70% it is higher (see figure 3).

Next, we try to asses the sensitivity of the stochastic welfare to changes in elasticity of substitution $(1/\sigma_3)$.

As figure 4 shows, under fix exchange rate, welfare appears to be not sensible to changes in the elasticity of substitution of liquidity services. On the contrary, with flexible exchange rate, welfare is worse and decreasing in $(1/\sigma_3)$. When looking at the volatilities of the devaluation and inflation rates we see that they are constant under fix exchange rates. As expected, when allowed to float the exchange rate is more volatile and its variability increases in the elasticity of substitution parameter (see figure 5). As for inflation, at lower levels of elasticity, inflation volatility is smaller in a flexible exchange rate regime. It increases as the elasticity increases and then decreases to the same level of fix exchange rate regime (see figure 6).

Finally, in figures 7 and 8, using the benchmark calibration, we present the dynamic paths of some variables of relevance in the curency substitution scenario under the two policies.

A given random shock to current money demand will have smaller overall effects since smaller changes in the exchange rate and interest rate would be required to bring money market back into equilibrium. If shocks originate in money markets then fixed exchange rates provide more stability, but if shocks are mainly real, floating exchange rates are superior in stabilizing.

5 Conclusions

The paper has built a small open economy model with frictions in order to examine the interaction between exchange rate variability and currency substitution. In particular we examined quantitatively the welfare effects of exchange rate risk in presence of currency substitution. In order to do so the model has been solved numerically up to the second order approximation. That is a novelty of this work. In fact, standard methodology relies upon log-linear approximations, which would miss many of the indirect implications of risk on welfare. Our measure of welfare is conditional since takes into account the transition dynamics due to the implementation of the policy rule.

As expected, exchange rate variability is costly even if welfare effects appear to be quite small. Surprisingly, when comparing the dollarized economy with one with no currency substitution, business cycles costs are larger in the economy without currency substitution. Allowing agents to have access and use the foreing money helps to hedge against the exchange rate risk (the relative gain from switching from flexible exchange rate to fix exchange rate is smaller in presence of currency substitution) and, in general, against shocks.

If, as in Schmitt-Grohé and Uribe (2004), we interpret fix exchange rate as *full dollarization* then we have to conclude together with them that also in presence of currency substitution full dollarization is preferable than flexible exchange rates. Nevertheless the presence of currency substitution somewhat mitigates the negative effects of exchange rate risk. Again, this is so because it makes easier for people to switch from one currency to the other in order to contrast negative movements in the relative price of moneys.

6 Appendix

6.1 The Obstfeld and Rogoff specification

As alternative specification for the $h(\cdot)$ function, we can consider[26]:

$$h\left(\frac{M_t}{P_t}, \frac{\varepsilon_t M_t^*}{P_t}\right) = \frac{\gamma_3}{1-\sigma_3}\left[\frac{M_t}{P_t} + a_1\frac{\varepsilon_t M_t^*}{P_t} - \frac{a_2}{2}\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^2\right]^{1-\sigma_3} \tag{67}$$

where $a_1 > 1-\beta$ and all parameters are larger than zero[27]. Based on Obstfeld and Rogoff (1996) this functional form rationalizes the legal restrictions on foreign currency use.

Notice that two types of money enter the utility function separately. This assumption insures that money holdings do not directly affect marginal rates of intertemporal substitution of consumption. A major argument for sticking to the assumption of money services separability is the desire to maintain some level of analytical tractability of the model.

Then, the optimality conditions change as follows:

(4a)→Domestic Money Demand

$$\gamma_3\left(\frac{M_t}{P_t} + a_1\frac{\varepsilon_t M_t^*}{P_t} - \frac{a_2}{2}\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^2\right)^{-\sigma_3} = C_t^{-\sigma_1} - \beta C_{t+1}^{-\sigma_1}\frac{P_t}{P_{t+1}} \tag{68}$$

(5a)→Foreign Money Demand

The quadratic functional form assumed yields the following foreign currency demand equation

$$m_t^* \equiv \frac{\varepsilon_t M_t^*}{P_t} = \frac{1}{a_2}\left(a_1 - 1 + \frac{\frac{\varepsilon_{t+1}}{\varepsilon_t} - 1}{r_t}\right) \tag{69}$$

for interior equilibria, where $\frac{\varepsilon_t M_t^*}{P_t}$ is positive. In the case $\frac{1}{a_2}(a_1 - 1 + \frac{1}{r_t}(\frac{\varepsilon_{t+1}}{\varepsilon_t} - 1)) < 0$ the foreign currency holding would be equal to zero. Agents will hold the foreign currency if SOE experiences a high enough nominal depreciation. Or, in other words, the demand for the real foreign money balances responds positively to an increase in the rate of depreciation of the exchange rate $\triangle\varepsilon_{t+1} \equiv \frac{\varepsilon_{t+1}}{\varepsilon_t}$. If a_2 is small enough then very small changes in the exchange rate can induce high demand for foreign currency.

Consequently, steady state conditions become:

$$\gamma_3\left[m + m^*\left(a_1 - \frac{a_2}{2}m^*\right)\right]^{-\sigma_3} = C^{-\sigma_1}\left(\frac{\pi-\beta}{\pi}\right) \tag{70}$$

---

[26] In a previous model (Colantoni and Kaminska (2004)) we used this specification and ran a numerical exercise similar to the one described in this paper.

[27] Note that with this specification the utility from foreign money is increasing only up to a certain amount, and then decreasing.

$$m_t^* = \max\left\{\frac{1}{a_2}\left(a_1 - 1 + \frac{\triangle\varepsilon - 1}{r}\right), 0\right\} \tag{71}$$

One last consideration on steady state foreing money demand

$$m^* = \max\left\{\frac{1}{a_2}\left(a_1 - 1 + \frac{\triangle\varepsilon - 1}{r}\right), 0\right\} \tag{72}$$

Intuitively, if there are no economic incentives to hold foreign currency, i.e. the rate of depreciation of exchange rate is 1, and there are non-zero costs of holding or using foreign currency (e.g. due to foreign exchange market fees or fines for evading government regulations) no rational economic agent will hold foreign currency balances in such a steady-state (we assume that $a_1 - 1 < 0$).

6.2 Computing the Phillips Curve

$\max\limits_{P_{H,t}(j)} E_0 \sum_{t=0}^{\infty} R_t D_t(j)$,

where

$$\begin{aligned} D_t(j) &= P_{H,t}(j)X_{H,t}(j) + \alpha\varepsilon_t P_{H,t}^*(j)X_{H,t}^*(j) - r_t P_t K_t(j) \\ &\quad - W_t N_t(j) - P_{H,t} AC_{P,t}(j), \end{aligned} \tag{73}$$

$$AC_{P,t}(j) \equiv \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 Y_t(j) \tag{74}$$

and subject to

$$X_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} X_{H,t} \tag{75}$$

$$X_{H,t}^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\phi^*} X_{H,t}^* \tag{76}$$

$$Y_t(j) = A_t K_t^{\theta}(j) N_t^{1-\theta}(j) \tag{77}$$

$$Y_t(j) = X_{H,t}(j) + X_{H,t}^*(j) \tag{78}$$

Recall that the optimal input demand implies the following efficiency condition

$$P_t r_t K_t = \frac{\theta}{1-\theta} W_t N_t \tag{79}$$

and substituting in the production function

$$\begin{aligned} Y_t(j) &= A_t K_t^{\theta}(j)\left(\frac{1-\theta}{\theta}\frac{P_t r_t K_t}{W_t}\right)^{1-\theta} \\ &= A_t K_t(j)\left(\frac{1-\theta}{\theta}\frac{P_t r_t}{W_t}\right)^{1-\theta} \end{aligned} \tag{80}$$

hence

$$K_t(j) = \frac{Y_t(j)}{A_t}\left(\frac{\theta}{1-\theta}\frac{W_t}{P_t r_t}\right)^{1-\theta} \tag{81}$$

Moreover, by the effiency condition

$$W_t N_t = \frac{1-\theta}{\theta} P_t r_t K_t \tag{82}$$

and so

$$\begin{aligned} r_t P_t K_t(j) + W_t N_t(j) &= \frac{1}{\theta} P_t r_t K_t \\ &= \frac{1}{\theta} P_t r_t \frac{Y_t(j)}{A_t}\left(\frac{\theta}{1-\theta}\frac{W_t}{P_t r_t}\right)^{1-\theta} \\ &= \frac{(P_t r_t)^{\theta}(W_t)^{1-\theta}}{A_t \theta^{\theta}(1-\theta)^{1-\theta}} Y_t(j) \end{aligned} \tag{83}$$

Finally, using the market clearing condition and the given definition of marginal cost

$$r_t P_t K_t(j) + W_t N_t(j) = MC_t \cdot \left(X_{H,t}(j) + X^*_{H,t}(j)\right) \tag{84}$$

So, using the demand functions for $X_{H,t}(j)$ and $X^*_{H,t}(j)$ the problem becomes

$$\begin{aligned} \max_{P_{H,t}(j)} E_0 \sum_{t=0}^{\infty} R_t \Bigg\{ & \left(P_{H,t}(j) - MC_t - P_{H,t}\kappa_t(j)\right)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} X_{H,t} \\ & + \left(\alpha\varepsilon_t P^*_{H,t}(j) - MC_t - P_{H,t}\kappa_t(j)\right)\left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\phi^*} X^*_{H,t} \Bigg\} \end{aligned} \tag{85}$$

where we defined

$$\kappa_t(j) \equiv \frac{AC_{P,t}(j)}{Y_t(j)} = \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 \tag{86}$$

Assuming no price discrimination (i.e. $P_{H,t}(j) = \alpha\varepsilon_t P^*_{H,t}(j)$), the first order condition becomes:

$$\begin{aligned} & \tfrac{\partial}{\partial P_{H,t}(j)} \textstyle\sum_{t=0}^{\infty} R_t \left\{ \left(P_{H,t}(j) - MC_t - P_{H,t}\tfrac{\varphi_P}{2}\left[\tfrac{1}{\pi_H}\tfrac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2\right)\left[\left(\tfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} X_{H,t} + \left(\tfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi^*}\right.\right. \\ & = R_t\left(1 - \tfrac{P_{H,t}}{P_{H,t-1}(j)}\tfrac{1}{\pi_H}\varphi_P\left[\tfrac{1}{\pi_H}\tfrac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]\right)\left[\left(\tfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} X_{H,t} + \left(\tfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi^*} X^*_{H,t}\right] \\ & + R_t\left(P_{H,t}(j) - MC_t - P_{H,t}\tfrac{\varphi_P}{2}\left[\tfrac{1}{\pi_H}\tfrac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2\right)\left[-\tfrac{\phi}{P_{H,t}}\left(\tfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi-1} X_{H,t} - \tfrac{\phi^*}{P_{H,t}}\left(\tfrac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi^*-}\right. \\ & + E_t\left\{R_{t+1}\left(\tfrac{P_{H,t+1}}{P^2_{H,t}(j)}\tfrac{P_{H,t+1}(j)}{\pi_H}\varphi_P\left[\tfrac{1}{\pi_H}\tfrac{P_{H,t+1}(j)}{P_{H,t}(j)} - 1\right]\right)\left[\left(\tfrac{P_{H,t+1}(j)}{P_{H,t+1}}\right)^{-\phi} X_{H,t+1} + \left(\tfrac{P_{H,t+1}(j)}{P_{H,t+1}}\right)^{-\phi^*} X^*_{H,t+1}\right]\right\} \end{aligned}$$

In a symmetric equilibrium, where all firms are identical we have:

$$X_{H,t}(j) = X_{H,t} \qquad K_t(j) = K_t \qquad N_t(j) = N_t \qquad P_{H,t}(j) = P_{H,t} \tag{87}$$

Hence, we can rewrite the Phillips Curve:

$$\begin{aligned}
&R_t\left(1-\varphi_P\tfrac{\pi_{H,t}}{\pi_H}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]\right)\left[X_{H,t}+X^*_{H,t}\right]\\
&-\tfrac{R_t}{P_{H,t}}\left(P_{H,t}-MC_t-P_{H,t}\tfrac{\varphi_P}{2}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]^2\right)\left[\phi X_{H,t}+\phi^* X^*_{H,t}\right]\\
&+E_t\left\{R_{t+1}\left(\varphi_P\tfrac{\pi^2_{H,t+1}}{\pi_H}\left[\tfrac{\pi_{H,t+1}}{\pi_H}-1\right]\right)\left[X_{H,t+1}+X^*_{H,t+1}\right]\right\}=0
\end{aligned}$$

Assuming $\phi = \phi^*$

$$\begin{aligned}
&R_t\left(1-\varphi_P\tfrac{\pi_{H,t}}{\pi_H}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]\right)\left[X_{H,t}+X^*_{H,t}\right]\\
&-\phi\tfrac{R_t}{P_{H,t}}\left(P_{H,t}-MC_t-P_{H,t}\tfrac{\varphi_P}{2}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]^2\right)\left[X_{H,t}+X^*_{H,t}\right]\\
&+E_t\left\{R_{t+1}\left(\varphi_P\tfrac{\pi^2_{H,t+1}}{\pi_H}\left[\tfrac{\pi_{H,t+1}}{\pi_H}-1\right]\right)\left[X_{H,t+1}+X^*_{H,t+1}\right]\right\}=0
\end{aligned}$$

from which

$$\begin{aligned}
&R_t\left(1-\varphi_P\tfrac{\pi_{H,t}}{\pi_H}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]-\phi+\phi\tfrac{MC_t}{P_{H,t}}+\phi\tfrac{\varphi_P}{2}\left[\tfrac{\pi_{H,t}}{\pi_H}-1\right]^2\right)\left[X_{H,t}+X^*_{H,t}\right]\\
&+E_t\left\{R_{t+1}\left(\varphi_P\tfrac{\pi^2_{H,t+1}}{\pi_H}\left[\tfrac{\pi_{H,t+1}}{\pi_H}-1\right]\right)\left[X_{H,t+1}+X^*_{H,t+1}\right]\right\}=0
\end{aligned}$$

and so

$$\begin{aligned}
P_{H,t} &= \tfrac{\phi}{(\phi-1)}\left(MC_t+P_{H,t}\kappa_t\right)\\
&+P_{H,t}\tfrac{\varphi_P}{(\phi-1)}\tfrac{\pi_{H,t}}{\pi_H}\left[1-\tfrac{\pi_{H,t}}{\pi_H}\right]\\
&+P_{H,t}\tfrac{\varphi_P}{(\phi-1)}E_t\left[\tfrac{R_{t+1}}{R_t}\left(\tfrac{\pi^2_{H,t+1}}{\pi_H}\left[\tfrac{\pi_{H,t+1}}{\pi_H}-1\right]\right)\tfrac{Y_{t+1}}{Y_t}\right]
\end{aligned} \tag{88}$$

6.3 The Symmetric Equilibrium

To deal with the non stationary nominal variables in the system, we consider stationary variables expressed in real terms such as $F_t \equiv -\frac{\varepsilon_t}{P_{H,t}}B^*_t$ (net foreign asset position), $m^*_t \equiv \frac{\varepsilon_t M^*_t}{P_t}$, (real foreing money holdings) and $w \equiv \frac{W_t}{P_{H,t}}$ (real wages).

Using the terms of trade, we can summarise the model by the following equations, where we already used the equatins for *profits, final good production, labour market clearing, current account balance, intermediate production technology*

$$g(S_t)(m_t^*-F_t)=-g(S_t)F_{t-1}\frac{\Delta\varepsilon_t}{\pi_t}(1+i_t)+g(S_t)\frac{\Delta\varepsilon_t}{\pi_t}m_{t-1}^*+ \\ +Y_t(1-\frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}}{P_{H,t-1}}-1\right]^2)-g(S_t)\{C_t+[K_{t+1}-(1-\delta)K_t]+AC_{I,t}\} \tag{89}$$

$$i_t = i_t^* + \psi_2(e^{F_t-\bar{F}} - 1) \tag{90}$$

$$\beta\,(1+i_t)\,E_t\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma_1}\left(\frac{\Delta\varepsilon_{t+1}}{\pi_{t+1}}\right)\right\} = 1 \tag{91}$$

$$C_t^{\sigma_1}N_t^{\sigma_2}g(S_t) = w_t \tag{92}$$

$$\left(1+\varphi_I\frac{K_{t+1}-K_t}{K_t}\right)=\beta E_t\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma_1}\left(r_{t+1}+(1-\delta)+\left(\frac{\varphi_I}{2}\frac{K_{t+2}^2-K_{t+1}^2}{K_{t+1}^2}\right)\right)\right\} \tag{93}$$

$$\chi_t\alpha(m_t)^{-\sigma_3}\left[\alpha(m_t)^{1-\sigma_3}+(1-\alpha)(m_t^*)^{1-\sigma_3}\right]^{\frac{\sigma_3}{1-\sigma_3}}=C_t^{-\sigma_1}-\beta E_t\left[C_{t+1}^{-\sigma_1}(\pi_{t+1})^{-1}\right] \tag{94}$$

$$\chi_t(1-\alpha)(m_t^*)^{-\sigma_3}\left[\alpha(m_t)^{1-\sigma_3}+(1-\alpha)(m_t^*)^{1-\sigma_3}\right]^{\frac{\sigma_3}{1-\sigma_3}}=C_t^{-\sigma_1}-\beta E_t\left[C_{t+1}^{-\sigma_1}\left(\frac{\Delta\varepsilon_{t+1}}{\pi_{t+1}}\right)\right] \tag{95}$$

$$X_{H,t} = \gamma\,[g(S_t)]^{\rho}\,X_t \tag{96}$$

$$X_{H,t}^* = \gamma^*\,(S)^{\rho^*}\,X_t^* \tag{97}$$

$$X_{F,t} = (1-\gamma)\left[\frac{g(S_t)}{S_t}\right]^{\rho}X_t \tag{98}$$

$$g(S_t)X_t=g(S_t)\{C_t+[K_{t+1}-(1-\delta)K_t]+AC_{I,t}\}+\frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}}{P_{H,t-1}}-1\right]^2\cdot X_t \tag{99}$$

$$X_{H,t}\left(1-\phi+\phi\frac{MC_t}{P_{H,t}}+\phi\kappa_t-\varphi_P\frac{\pi_{H,t}}{\pi_H}\left[\frac{\pi_{H,t}}{\pi_H}-1\right]\right)+ \\ X_{H,t}^*\left(1-\phi^*+\phi^*\frac{MC_t}{P_{H,t}}+\phi^*\kappa_t-\varphi_P\frac{\pi_{H,t}}{\pi_H}\left[\frac{\pi_{H,t}}{\pi_H}-1\right]\right)+ \\ +\varphi_P E_t\left\{\frac{R_{t+1}}{R_t}\frac{\pi_{H,t+1}^2}{\pi_H}\left[\frac{\pi_{H,t+1}}{\pi_H}-1\right](X_{H,t+1}+X_{H,t+1}^*)\right\}=0 \tag{100}$$

$$\pi_t^{1-\rho} = \frac{\gamma+(1-\gamma)\,S_t^{1-\rho}}{\gamma+(1-\gamma)\,S_{t-1}^{1-\rho}}\,\pi_{H,t}^{1-\rho} \tag{101}$$

$$Y_t = X_{H,t} + X_{H,t}^* \tag{102}$$

$$\log M_t = \log M_{t-1} + \rho_m\,(\varepsilon_t - \overline{\varepsilon}) \tag{103}$$

$$m_t = m_{t-1}\,(\pi_t)^{-1} + t_t \tag{104}$$

All above mentioned equations present the system of nonlinear equations, which has to be solved numerically for general paths of exogenous variables.

7 A particular Steady State

We now turn to the description of a symmetric equilibrium where all exogenous variables, including domestic money stock, are constant.

We characterize the perfect foresight steady state of our small open economy model, taking X^*, X_H^* and π^* as given and setting $\phi = \phi^*$ and $A = 1$. Moreover, following the solution procedure outlined in Lane (1999a), we normalize the endowment of a traded (intermediate) good in the steady-state in such a way that $P_{H,t} = P_{F,t}$. Note that in this case $S = g(S) = 1$. Finally, we set (β) such that the subjective discount factor is equal to the world interest rate, that is,

$$i^* = \frac{\pi^*}{\beta} - 1 \tag{105}$$

or, equivalently

$$\beta(1 + i^*) = \pi^* \tag{106}$$

We present the steady state of the economy by the following equations and the budget constraint can be written as[28]

$$\left(1 - \frac{\Delta\varepsilon}{\pi}\right)[m^* - (1+i)F] = Y - C \tag{107}$$

$$i^* = i \tag{108}$$

$$\pi = \beta(1+i)\Delta\varepsilon \tag{109}$$

$$w_t = C^{\sigma_1} N_t^{\sigma_2} \tag{110}$$

$$\beta[r + (1-\delta)] = 1 \tag{111}$$

$$\chi\alpha(m)^{-\sigma_3}\left[\alpha(m)^{1-\sigma_3} + (1-\alpha)(m^*)^{1-\sigma_3}\right]^{\frac{\sigma_3}{1-\sigma_3}} = C^{-\sigma_1}\left(\frac{\pi-\beta}{\pi}\right) \tag{112}$$

$$\chi(1-\alpha)(m^*)^{-\sigma_3}\left[\alpha(m)^{1-\sigma_3} + (1-\alpha)(m^*)^{1-\sigma_3}\right]^{\frac{\sigma_3}{1-\sigma_3}} = C^{-\sigma_1}\left(\frac{\pi-\beta\Delta\varepsilon}{\pi}\right) \tag{113}$$

$$X_H = \gamma X \tag{114}$$

$$X_H^* = \gamma^* X^* \tag{115}$$

$$X_F = (1-\gamma)X \tag{116}$$

$$X = C + \delta K + \frac{\varphi_P}{2}\left[\frac{1}{\pi_H} - 1\right]^2 \cdot X \tag{117}$$

$$\frac{\phi}{\phi-1}\left(\frac{r^\theta w^{1-\theta}}{A\theta^\theta(1-\theta)^{1-\theta}} + \frac{\varphi_P}{2}\left[\frac{1}{\pi_H} - 1\right]^2\right) = 0 \tag{118}$$

$$\pi_H = \Delta\varepsilon\pi_H^* \tag{119}$$

$$Y = X_H + X_H^* \tag{120}$$

[28] We use variables without time subscripts to refer to steady state values

$$\pi = \pi_H \tag{121}$$

$$m(1 - \pi^{-1}) = t \tag{122}$$

Note, that in the case of $F = \bar{F}$ (i.e., the steady-state of net foreign assets is equal to its desired level), it follows that in the steady state the interest rate premium is nil and the relationship between inflations and interest rates are

$$i^* = i$$

$$\pi = \Delta\varepsilon\pi^* \tag{123}$$

Note that the steady state is considered in its general form, so that we can use them for different cases, For example, for the case of the constant money supply $M = const$, the price level is also constant, and so the inflation in steady state is $\pi^* = \pi = 1$. From the system it follows that $\Delta\varepsilon = 1,\ t = 0,\ i = i^*$.

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Table 1: Estimates of Foreign Currency held as Cash

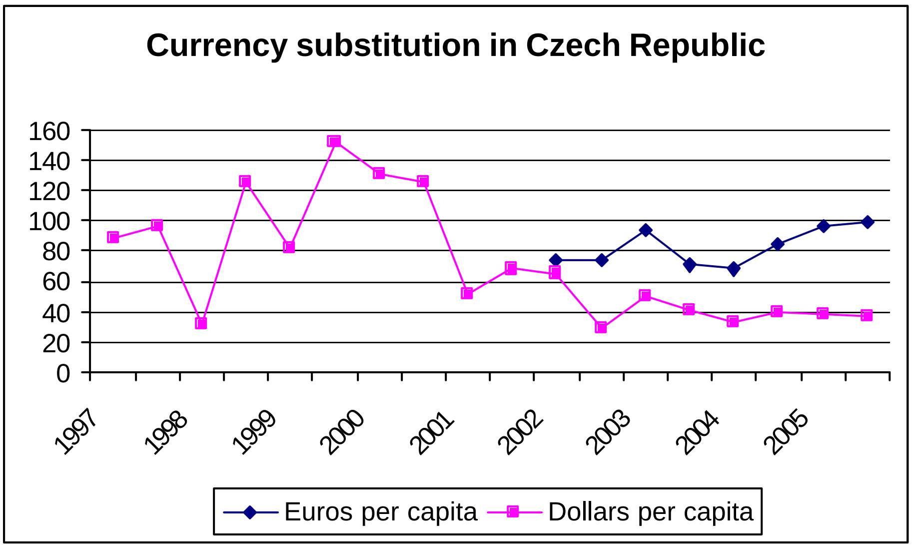
Country	GDP held in the form of U.S. currency (%) 1997-2002	Foregin Currency to Total Currency (%) 2001	$ Per Capita FCC 2001
Albania		14	46
Argentina	17.5		
Armenia		62	55
Azerbaijan		82	169
Belarus	5.8	34	17
Bulgaria	2.8	41	125
Cambodia	25.2		
Chile	0.4		
China,P.R.: Mainland	0.9		
China,P.R.:Hong Kong	1.2		
Croatia		35	117
Czech Republic		21	129
Dominica	3.9		
Ecuador	7.3		
Egypt	0.4		
El Salvador	7.5		
Estonia		59	414
Georgia		79	123
Hungary		6	25
Indonesia	0.3		
Kazakhstan		95	1024
Korea	2.3		
Kyrgyz Republic		48	20
Latvia	5.5	79	1209
Lithuania	3.6	11	25
Macedonia, FYR		5	5
Mexico	0.6		
Paraguay	0.6		
Peru	3.8		
Philippines	1.0		
Poland	0.4	27	93
Romania	0.8	55	61
Russia	10.0	87	903
Slovak Republic		28	123
Slovenia		54	329
South Africa	3.1		
Thailand	0.1		
Turkey	2.6		
Ukraine		64	131
Vietnam	2.7		

Sources. First Column: US Treasury Department (2003). Second and third column: Feige (2003)

Table 2:
The Welfare Costs[1] of Business Cycles under Alternative Monetary Policy Regimes

Case	**Shock**		
	Both	Monetary	Technology
No Currency Substitution			
flex	−0.0230	0	−0.0230
fix	−0.0175	0	−0.0175
Currency Substitution			
flex	−0.0110	$\simeq -0$	−0.0110
fix	−0.0108	$\simeq -0$	−0.0108

[1] Measured as share of deterministic steady state consumption



Graph 1: Source: author's calculations based on Austrian Central Bank Survey data.

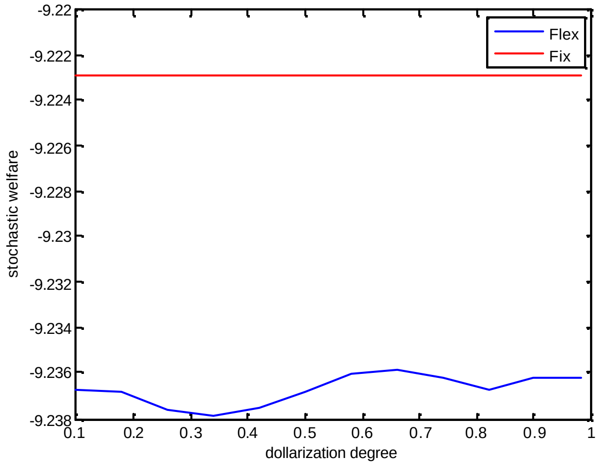


Figure 1: Stochastic Welfare with CS

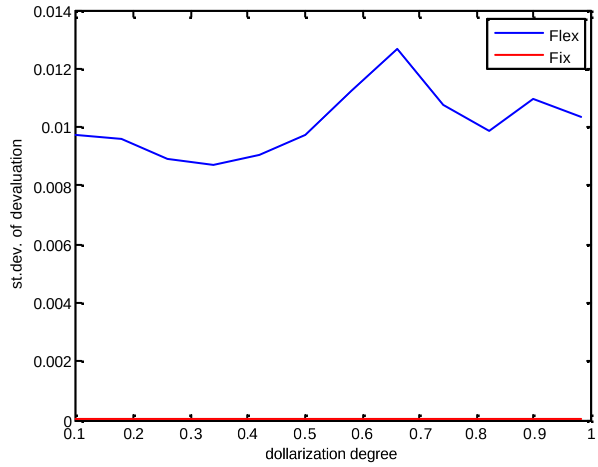


Figure 2: Standard deviation of devaluation rate in presence of CS

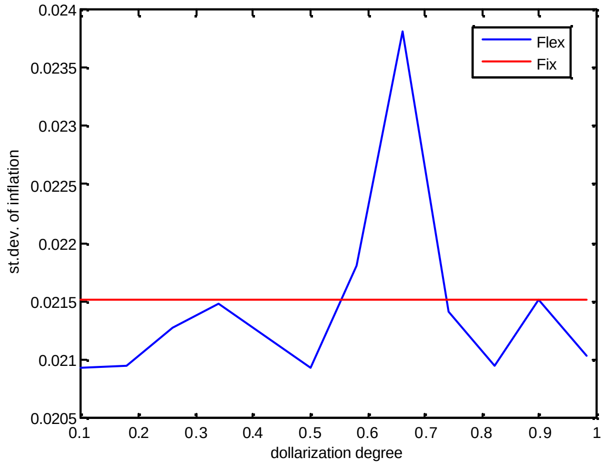


Figure 3: Standard deviation of Inflation with CS

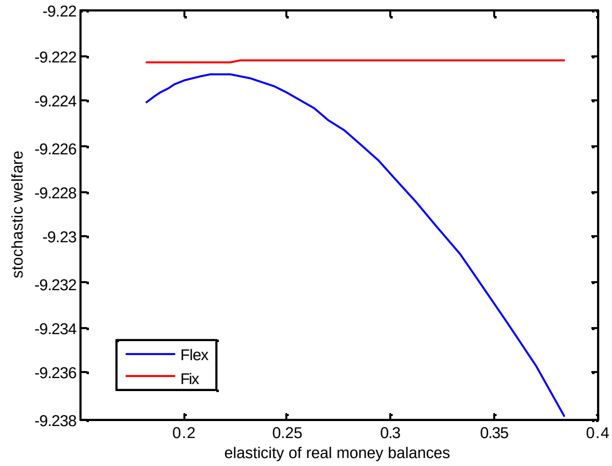


Figure 4: Stochastic welfare with CS

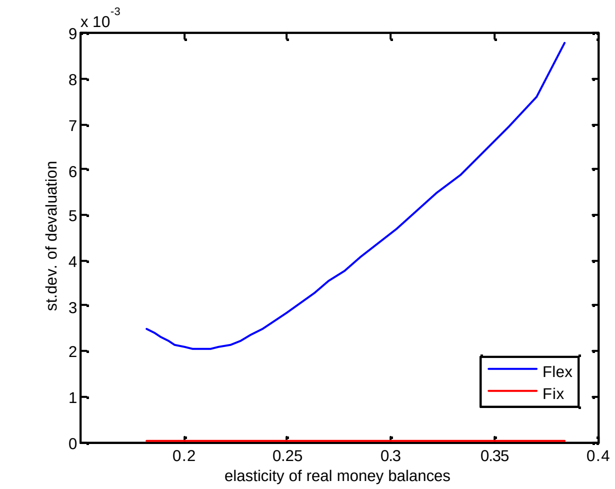


Figure 5: Standard deviation of devaluation rate in presence of CS

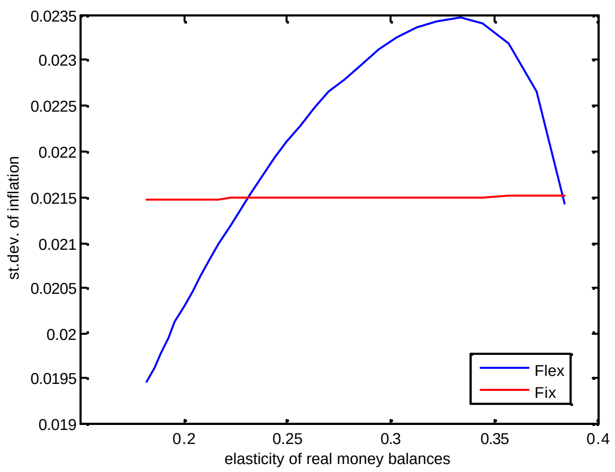


Figure 6: Standard deviation of Inflation with CS

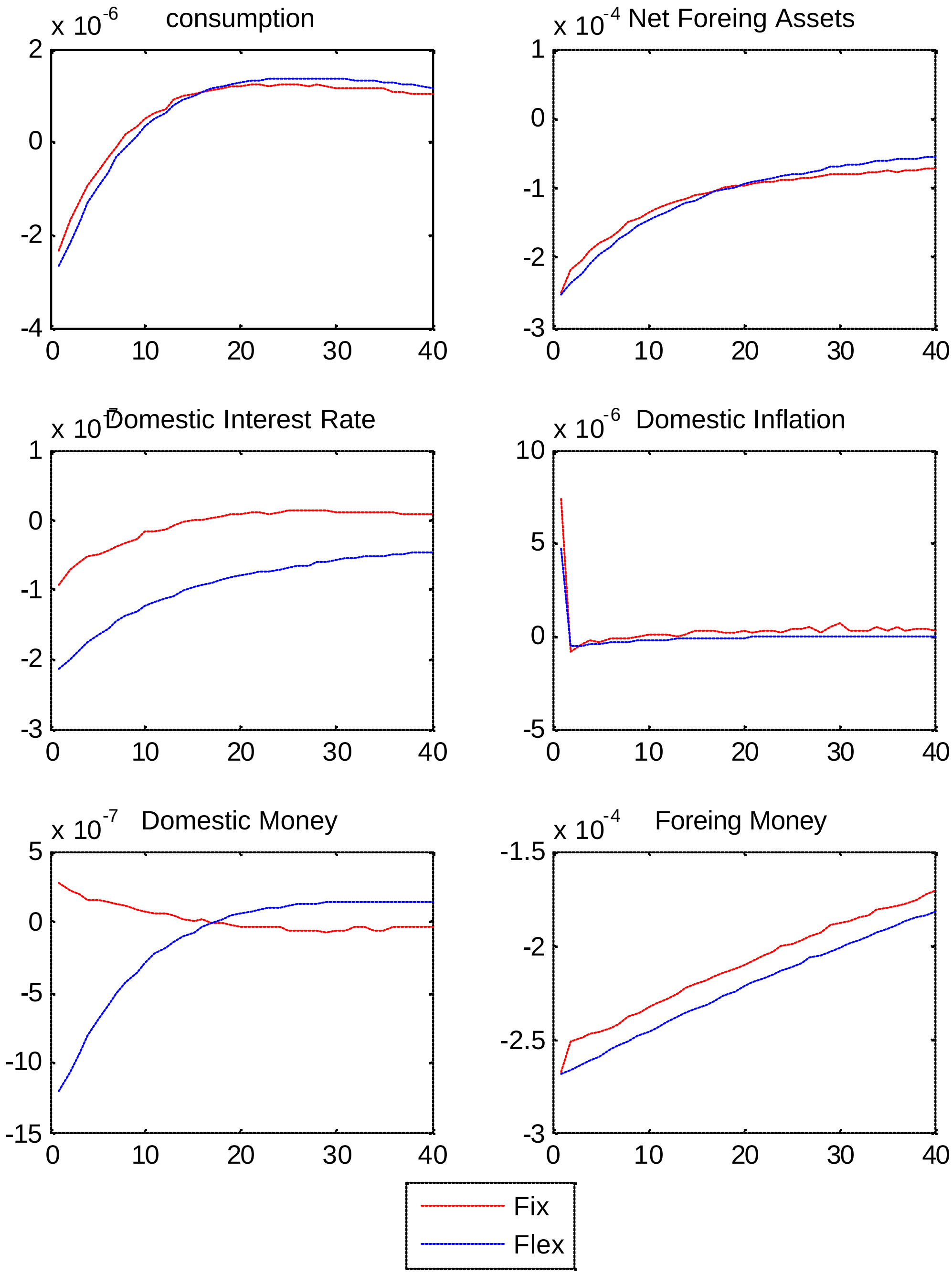


Figure 7: Shock to Money Demand

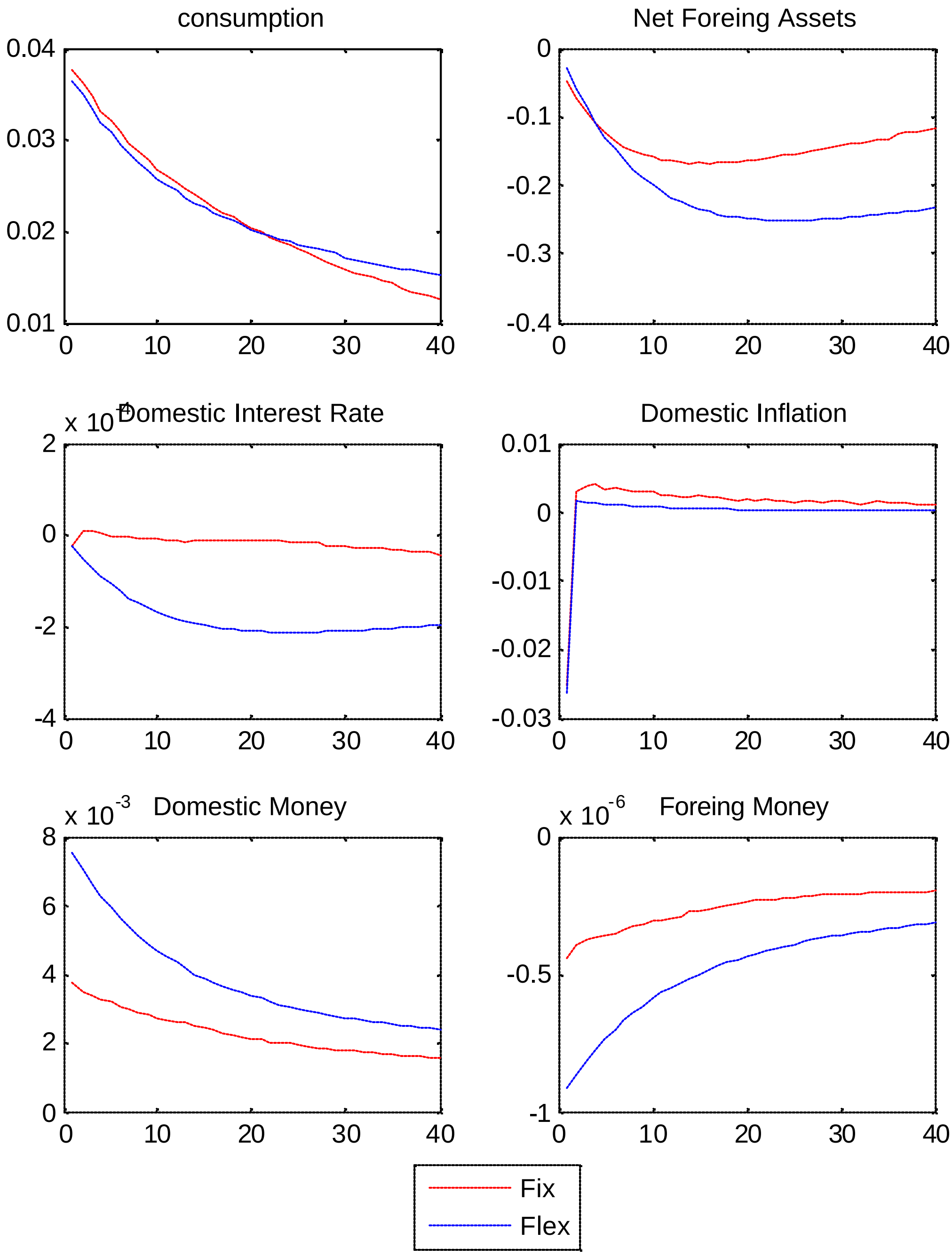


Figure 8: Shock to Technology