MODELING THE STRATEGIC TRADING OF ELECTRICITY ASSETS

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ABSTRACT:

We analyze how strategic asset trading can be used to gain competitive advantage. In the case of electricity markets, companies seek to improve the value of their generating portfolios by acquiring, or selling, power plants. Accordingly, we derive the basic determinants of plant value, explaining how a particular productive asset may have different values for different firms. From this, we develop an evolutionary model to understand how market structure interacts with strategic asset trading to increase the competitive advantage of firms, and furthermore, how this depends upon the actual price-setting microstructure in the wholesale market itself.

KEYWORDS: Competitive advantage, computational learning, auctions, asset trading, simulation, electricity markets

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1. INTRODUCTION

A substantial modelling challenge in understanding the strategic behaviour of competing firms is concerned with elucidating principles that prescribe their mutual trading of physical productive assets. A fundamental question in this context is why the same productive asset in the same product market should apparently be worth more to one company than another, and thereby motivate such a trade. In a general sense, of course, aspects of why companies buy assets from others have been considered from many organisational and financial perspectives including the prospects of operational synergies, scale economics, opportunistic accounting, different attitudes to risk or corporate ambition, all of which have been extensively researched. In this paper we focus specifically upon the motivation of firms to acquire assets selectively in order to be able to gain new capabilities and influence the market price within an evolutionary setting. This requires analysis of the influence of the market price-setting process upon the evolution of the accumulated productive asset positions of competing firms, and therefore reflects dynamic feedback within the classic market structure, conduct and performance paradigm. A typical example of this, which we will model in detail, is revealed in the new competitive electricity markets, where power plants have been extensively traded, especially in the US and UK, sometimes repeatedly, by strategic players seeking to adjust their portfolios of generating units to achieve better performance in the electricity wholesale markets.

All capital intensive industries manifest a co-evolution of market structure and performance, but what makes electricity particularly intriguing in this respect is the instantaneous, non-storable nature of the product, delivered into a market with low demand-elasticity, high requirements for security of supply and wide seasonal variations. This means that electricity is provided from an economic and technical mix of baseload, mid-merit and peaking plant, which in turn operate for a decreasing fraction of the year[1], but any of which may set the market clearing price at various times of the day and year. This

[1] Baseload plant will have higher capital, lower operational costs and run all of the time, except for maintenance periods, whilst daily peaks in demand will be additionally be met from the technically more flexible, lower capital cost, but higher operational cost peaking plant.

feature raises the strategic issue of whether the natural tendency, when these markets are competitive, is for competing companies to evolve towards becoming diversified players, with a mix of different kinds of technology assets, or niche players seeking to be more dominant in the base, mid or peaking segments of the market for power. Furthermore, this issue may be complicated by the market design, for example, to the extent that an administered market is introduced, based upon a single clearing price for all power at a particular point in time (e.g., the compulsory Pool auction model) or a multi-clearing process that facilitates discriminatory prices for the base, mid and peaking market segments (eg voluntary, continuous bilateral forward trading). As an explicit formulation, this strategic response of market structure evolution to the market price setting process, has received very little formal analysis.

In several general respects this links to the discussion of strategic factor markets (Barney, 1986), as the plant trading occurs through a market for physical assets as a basis for creating sustainable competitive advantage in the product market. With plant trading there are both asset mass and interrelatedness efficiencies (Dierickx and Cool, 1989), since, by acquiring a plant a firm may enter a market segment only available to that technology (such as baseload, mid or peak electricity). Thus, there is a path dependency in the emergence of a resource base through either niche concentration or dynamic capability (Leonard-Barton, 1992; Teece et al., 1997), and in the creation of asymmetric competition (Midgley et al, 1997) in these different market segments.

In the electricity sector, power plant trading is evidently conditioned by, and in turn conditions, the conduct, i.e., pricing behaviour, in the daily spot markets for electricity. Published research has analysed the *mandated* divestment of assets by anti-trust authorities to reduce market power (e.g., Green and Newbery, 1992; Borenstein and Bushnell, 1999; Day and Bunn, 2001; Bunn and Oliveira, 2003), but mandated divestment is quite a different issue from *voluntary* plant trading. To address the latter, a modelling framework is needed that captures the trading of assets as *an endogenous response* to performance in the daily market(s) for electricity.

Moreover, from the very fact that plant trades occur, it is clear that in a trading game all trading takes place in off-equilibrium states of the industry, and therefore the trajectory toward equilibrium is as important as the equilibrium itself[2]. Thus, in order to explain plant trading one needs to look at evolutionary models that incorporate path dependencies, learning and adaptation (e.g., Nelson and Winter, 1982; Nelson, 1991; Fudenberg and Levine, 1998). In order to model plant trading in electricity markets, the market simulation model developed in this paper incorporates two main components: a plant trading game and an electricity market game. The plant trading game represents the interaction between electricity companies that trade generating plants. The electricity market game formulates the daily electricity market prices, and hence plant valuations, by assuming Cournot players.

Before developing the plant trading game, section 2 develops some key theoretical background results, and then section 3 examines the relation between plant value and capacity withholding, which is the main driver of plant trading under Cournot competition. Section 4 develops the plant trading game itself and the different algorithms used to simulate learning, adaptation and trading. The paper concludes with an application of the model to the full England & Wales electricity market.

2. MARKET MECHANISMS FOR ELECTRICITY TRADING

In developing an adequate model of competition in electricity markets, in order to capture some of the key issues which drive plant trading, the representation needs to reflect some of the well-known stylised facts. These include 1) A generator's supply function is generally increasing and step-shaped, 2) A generator may receive different prices for his generation from different plants, even if these are identical, 3) Different generators may price the same type of plant differently, and 4) A generator aims at maximising the value of his portfolio of plants as a whole. A variety of competitive models have been applied to electricity spot markets including auction theory (e.g. Bunn and Oliveira, 2001; Nicolaisen et al., 2001), supply function equilibria (Green and Newbery, 1992), Bertrand games (e.g., Bunn and Oliveira, 2003), or Cournot games (e.g., Allaz and Vila 1993; Borenstein and Bushnell, 1999; Wei and Smeers, 1999; Hobbs, 2001). Mainly for reasons of computational efficiency,

[2] An equilibrium is a state of the industry in which no plant trade occurs: the value of a plant for its owner is not less than its value for any other player in the industry.

the development of simple intuition, and fits with the required stylized facts, the analysis in this paper models the electricity market as a Cournot game where each player decides how much to generate from each plant he owns. We explore two variations of this Cournot formulation. The first is a single-clearing Cournot game in which there is a single clearing price for each hour of the day. This model simulates a game where each player defines how much to sell at each hour (for different levels of demand), given the portfolio of plant owned. The second is a multi-clearing Cournot game in which there are different clearing prices for different markets, at a certain time of the day. In the multi-clearing mechanism, each player decides how much to offer from each one of his plants in the different markets, given the durations, the different demand functions and the structure of his portfolio. Both of these variants are motivated by the active discussion in the industry on the relative merits of mandatory Pool-based auctions *versus* voluntary continuous bilateral trading (eg Bower and Bunn, 2000).

Thus, in the Cournot game we consider a generator seeking to maximise the value of his portfolio of power plants as a whole. Each player *i* chooses his output $Q_{i,L}$ in market *L*, which is characterised by a certain demand, where the definition of each market can be adapted to the specific needs of the issue to be analysed. Let $C_{i,L}$ stand for the marginal cost of player *i*, A_L, α_L represent the intercept and slope of the inverse demand function, and D_L stand for the duration of market *L*. Further, let $K_{i,L}$ stand for player *i*'s total available capacity in market *L*. In this case, $C_{i,L}$ is assumed locally constant for a given plant, but it may be different for the different plants owned by a player. Thus, $C_{i,L}$ will generally be a step-function, which makes the optimisation problem computationally hard. In both models, the start-up costs and ramp rates are not explicitly taken into account; this is a simplifying assumption that has also been used in several other studies of electricity markets (e.g., Ramos at al., 1998; Borenstein et al. 1999, 2002). However, these technical constraints are implicit in defining the capability of a plant to access a given market segment. We exogenously define, for each plant, the market segment into which it can sell. This simplification decreases complexity from a non-linear to a linear complementarity problem (e.g. Ramos et al, 1998; Wei and Smeers, 1999; Hobbs, 2001).

The single-clearing market mechanism (uniform auction) permits only a single clearing price for any given trading period and therefore a player receives the same price, P_L, for the electricity generated by any plant selling in market L, as defined by the trading periods (usually hourly).

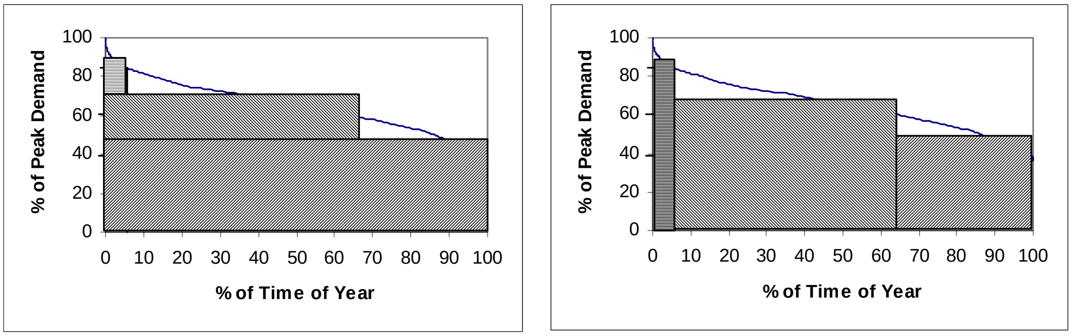
Thus, for a player i, the profit (π_i) maximisation problem is represented by equations (2.1).

$$
\begin{aligned}
&\max \pi_i = \sum_L \left(P_L - C_{i,L}\right) Q_{i,L} D_L \\
&st. \\
&P_L = A_L - \alpha_L . \sum_i Q_{i,L},\ \forall L \\
&Q_{i,L} \leq K_{i,L},\ \forall L \\
&Q_{i,L} \geq 0,\ \forall L
\end{aligned}
\tag{2.1}
$$

Assuming hourly trading periods, if the model is specified using annual load durations, then $\sum_L D_L = 365 \times 24$, whereas if the model uses a daily load profile, then $\sum_L D_L = 24$.

Alternatively, we compare this with the multi-clearing mechanism to capture the case of bilateral electricity markets, in which each generator has the possibility of selling the electricity of its different plants in different market segments. Thus, when continuous forward trading is active, the baseload, shoulder, and peak plants tend to sell electricity over different timescales at different prices. In modelling this we follow Elmaghraby and Oren (1999) and Borenstein et al. (1995), and aim to capture the interaction between different markets and technologies in defining the value of a plant. Thus, for a player i, the profit (π_i) maximisation problem is similar to the one presented in equations (2.1), with the difference that at each time t (in which a capacity $K_{i,}$) the available capacity is the sum of the available capacity in each market segment, as in equation (2.2), where L now refers to segments such as baseload, shoulder and peaking.

$$
K_i = \sum_L K_{i,L} \tag{2.2}
$$



(a) Multi-clearing (b) Single-clearing

FIGURE 2.1**:** A load duration comparison of the Clearing Mechanisms.

In Figure 2.1 we illustrate the two clearing mechanisms by using an annual load duration curve. In the multi-clearing mechanism (2.1.a) at any time there can be more than one price, and the horizontal slicing reflects the markets for baseload, shoulder and peak electricity. In contrast, in the single-clearing mechanism (2.1.b) at any given time the market pays a single price, and the vertical slicing reflects the durations of an equivalent three segments of time when the single market price would be set by baseload, shoulder and peaking plant.

At this stage it is useful to clarify some principles related to the optimal use of plant within a trading portfolio.

PROPOSITION 2.1: *In a Cournot game with capacity constraints, a player owning plants with different marginal costs* $C_1 < .. < C_j < .. < C_P$*, offers the capacity of plant j+1 only if he does not withhold capacity from plant j.* [Proof in Appendix].

Thus in order for a more expensive plant to be offered by a Cournot player, every available cheap plant needs to have been called first.

PROPOSITION 2.2: *Let* $P_L = A_L - \alpha_L . \sum_g^P Q_{g,L}$ *represent the inverse residual demand function of a market L. In a Cournot game, a player owning plants with different marginal costs* $C_1 < .. < C_j < .. < C_P$ *and not selling any electricity generated by plants j+1, j+2, … in that market,* *i.e.*, $\sum_{g=j+1}^{P} Q_{g,L} = 0$, *profitably withholds capacity of plant j if and only if* $\frac{1}{\alpha_L}\left(A_L - C_{j,L}\right) < 2.K_{j,L} + \sum_{g=1}^{j-1} Q_{g,L}$. [Proof in Appendix]

THEOREM 2.1: *In a Cournot game, a player owning plants with different marginal costs* $C_1 < .. < C_j < ..C_g .. < C_P$ *cannot profitably transfer load from a plant j to a plant j+g, for any g, via capacity withholding.* [Proof in Appendix]

We now extend the implications of this "merit order" in capacity withholding to plant trading by showing that the introduction of a plant in the portfolio has the potential to change the value of the other plants in the same portfolio. Let $Q_{(j+z,0),L}$ $\left(Q_{(j+z,1),L}\right)$ and $Q_{(j,0),L}$ $\left(Q_{(j,1),L}\right)$ represent, respectively, the load of plants *j+z* and *j*, respectively before (after) the transaction, in market *L*. Further, let P_0 $\left(P_1\right)$ represent the electricity market price before (after) the transaction of plant *i*.

THEOREM 2.2: *Assume that in a Cournot game, a player owns plants with different marginal costs* $C_1 < ... < C_j ... < C_P$. *This player can profitably reduce the load of any plant j+z, for any* $z > 1$ *by acquiring a plant with marginal cost* $C_{j,L}$ *only if*

$$\sum_L \left(\frac{A_L - C_{j+z,L}}{\alpha_L} \right) < 2\sum_L Q_{(j+z,0),L} + \sum_L \left(Q_{(j,1),L} - Q_{(j,0),L} \right) + \sum_L \left(\sum_{\substack{g=1 \\ g \neq j}}^{j+z-1} Q_{g,L} \right).$$

[Proof in Appendix]

Hence, Theorem 2.2 extends Theorem 2.1 by showing that not only is load transfer against the merit order not an optimal strategy for a Cournot player, but also by showing that if a player acquires (sells) cheaper technologies he drives out (drives in) the more expensive ones. Thus, Theorem 2.2 builds a bridge between the electricity market and the plant trading game. Section 3 presents a detailed analysis of this behaviour.

3. PLANT VALUE AND MARKET STRUCTURE DYNAMICS

This section presents an analysis of how different players compute the value of a plant differently. We show how the resource base of the power plant portfolio determines competitive advantage and that it is misleading to accept the common intuition that the value of a plant is a function of the cash flows it generates. A plant that generates negative cash flows may still have a positive contribution to the value of the portfolio, and thus a player may pay a positive price for it, or, if selling, he may receive a positive price for that plant. We also show how this depends upon the market microstructure and that, compared to the single-clearing mechanism, the multi-clearing mechanism may lead to less concentrated markets.

Define the following variables for a plant *j* owned by a player *i*. *V(j,i):* economic value of plant *j*. *OP(j,i)*: operational profit of plant *j*. *PC(j,i)*: portfolio contribution of plant *j*. $C_{j,L}$: total cost of plant *j* in market *L*. $Q_{j,L}$: load of plant *j* in market *L*. $K_{j,L}$: available capacity of plant *j* for market *L*. $Q_{(-j,i),L}$: load of player *i*'s plants, with exception of plant *j*, in market L. $Q_{-i,L}$: load of all the plants *not* owned by player *i*, in market *L*. $P_{L,F}$*:* market price in market *L* when a plant *j* offers its full capacity in *L*. $P_L = A_L - \alpha_L . \sum_i Q_{i,L}$: inverse residual demand function in a market *L*.

DEFINITION 3.1: *For a certain player i and plant j:*

(a) $OP(j,i) = \sum_L D_L \left(A_L - \alpha_L . Q_{-i,L} - \alpha_L . Q_{(-j,i),L} - \alpha_L . Q_{j,L} \right) . Q_{j,L} - \sum_L D_L . C_{j,L}$;

(b) The operational profit of player i is:

$$OP(i) = \sum_j \sum_L D_L \left(A_L - \alpha_L . Q_{-i,L} - \alpha_L . Q_{(-j,i),L} - \alpha_L . Q_{j,L} \right) . \left(Q_{j,L} + Q_{(-j,i),L} \right) - \\ - \sum_j \sum_L D_L . \left(C_{j,L} + C_{(-j,i),L} \right) .$$

The portfolio contribution of a plant represents the change in profit due to a reduction of the output of this plant when compared to its full capacity. Let $\max\left(\Delta Q_{j,L}\right)$ represent the maximum possible increase of generation from plant *j* for market *L*, given the capacity constraints.

DEFINITION 3.2 (Portfolio Contribution): *Defining the current market price in market L as* $P_L = A_L - \alpha_L.Q_{-i,L} - \alpha_L.Q_{(-j,i),L} - \alpha_L.Q_{j,L}$ *, and the price that would result from offering the remaining capacity in that market as* $P_{L,F} = A_L - \alpha_L.Q_{-i,L} - \alpha_L.Q_{(-j,i),L} - \alpha_L.Q_{j,L} - \alpha_L.\max\left(\Delta Q_{j,L}\right)$*, it follows that* $PC(j,i) = \sum_L D_L.\left(P_L - P_{L,F}\right).Q_{(-j,i),L}$ *, and thus*

$$PC(j,i) = \sum_L \alpha_L.\max\left(\Delta Q_{j,L}\right).Q_{(-j,i),L}.D_L \ .$$

DEFINITION 3.3 (Economic Value): $V(j,i) = OP(j,i) + PC(j,i)$.

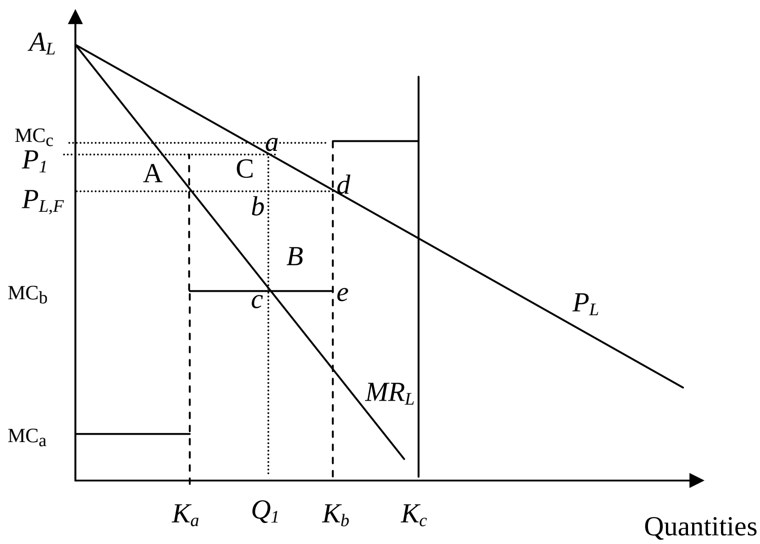


FIGURE 3.1: Example Illustrating the Concept of Portfolio Contribution.

Hence, the economic value of a plant, *V(j,i)*, as a function of the operational profit generated by that plant plus its portfolio contribution, is the maximum price a player is willing to pay for it. Figure 3.1 illustrates the concepts of economic value and profit contribution of a plant. Suppose that a player i holds three plants *a*, *b*, *c*, which have marginal costs MC_a, MC_b and MC_c, and available capacities $K_{a,L}$, $K_{b,L}$ and $K_{c,L}$, respectively. Further, let $P_L = A_L - \alpha_L.\sum_j Q_j$ and $MR_L = A_L - 2.\alpha_L.\sum_j Q_j$ describe, respectively, the inverse residual demand function and the marginal revenue of player *i*, in a market *L*. In this example, player *i* generates Q_1 units at a price P_1, and therefore he reduces the operational value of plant *b* by area (*B*-*C*) and increases the value of his portfolio by *(A+C-B*). Area *A* represents the portfolio contribution of plant *b*.

Next, Proposition 3.1.a) shows that when a player *i* sells the full capacity of a plant *j*, the operational profit of this plant is an upper bound on its value. Further, Proposition 3.1.b) shows that when a player *i* withholds some of the capacity of a plant *j*, the portfolio contribution of this plant is positive only in markets where the clearing price is higher than this plant's marginal cost. Furthermore, Proposition 3.2 shows that a plant has different value in different types of portfolio. Particularly, a marginal plant is more valuable in bigger portfolios.

PROPOSITION 3.1: (a) *Let* $Q_{j,L} = K_{j,L}$ *, then* $PC(j,i) = 0$. (b) *Let in a market L* $Q_{j,L} < K_{j,L}$ *then* $PC(j,i) > 0$ *only if* $MC_j < P_L$. [Proof in Appendix]

PROPOSITION 3.2: *In a Cournot game, the value of a plant j is a function of the portfolio of plant* $C_1 < .. < C_j < .. < C_P$ *to which it belongs. (a) In portfolios with larger total output, the portfolio contribution of a marginal plant to the portfolio is greater than in portfolios with small output. (b) Thus, in portfolios with larger total output the generation of a marginal plant tends to be lower than in portfolios with smaller output.* [Proof in Appendix]

Thus, it follows that the main market drivers are captured by Theorem 2.2, which shows that a player that buys a more efficient plant can profitably withhold capacity of his more expensive plants, and Proposition 3.2 which shows that the value of a plant is a function of the type of portfolio to which it belongs. Clearly, a player only buys a plant that has a positive value, and is only able to do so from a player who values it less. Let *s* and *b* represent the seller and a buyer of a plant *j*, and $V(j,s)$ and $V(j,b)$ represent the respective valuations of this plant. Further, for a player *a* and a plant *j* generating $Q_{j,a}$, let $OP_{j,a}(Q_{j,a})$ and $PC_{j,a}(Q_{j,a})$ represent, respectively, the operational profit and the portfolio contribution of this plant. Next Theorem 3.1 shows that for any plant *j* traded by two players *s* and *b* the weighted average of the buyer's output is higher than the weighted average of the seller's output.

THEOREM 3.1: *For any plant j,* $V\left(j,b\right) > V\left(j,s\right)$ *only if*

$$\sum_L D_L.\left(P_L - P_{L,F}\right).Q_{(-j,b),L} > \sum_L D_L.\left(P_L - P_{L,F}\right).Q_{(-j,s),L}$$. [Proof in Appendix]

Given the expectations regarding other players' behaviour, the only possible plant transactions are the ones in which the buyer expects to reduce the output of the plant bought. Theorem 3.2 shows that if for any plant *j* the buyer's residual output is higher than the seller's residual output, in the markets where *j* can sell, than plant trade implies a reduction of *j*'s output.

THEOREM 3.2: *For any plant j such that* $V\left(j,b\right) > V\left(j,s\right)$: $Q_{(-j,b),L} > Q_{(-j,s),L}$ if and only if $Q_{(j,b),L} < Q_{(j,s),L}$. [Proof in Appendix]

Further, Theorem 3.3 proves that a player whose residual output is lower than the current owner's residual output cannot buy plant *j*.

THEOREM 3.3: *For any plant j such that* $Q_{(-j,b),L} < Q_{(-j,s),L}$: $V\left(j,b\right) < V\left(j,s\right)$. [See Appendix]

Thus, it follows from Theorems 3.2 and 3.3 that every trade implies a reduction of the output of the plant being traded as in Corollary 3.1.

COROLLARY 3.1: *For any plant j*: $V\left(j,b\right) > V\left(j,s\right)$ *only if* $Q_{(j,b),L} < Q_{(j,s),L}$. [See Appendix]

We now return to the issue of market microstructure. While capacity withholding within the single-clearing mechanism rewards all the plants selling at a certain time, capacity withholding within in the multi-clearing mechanism, at a certain time, only benefits the plants selling in the market from which the capacity is withheld.

THEOREM 3.4: *The multi-clearing mechanism, in the long run, leads to a level of market concentration and electricity prices lower than the ones achieved by the single-clearing mechanism*. [Proof in Appendix]

In summary, the analysis in sections 2 and 3 provides a rationale for plant trading in electricity markets by showing that the value of a given plant is potentially different for different players, and that a player can influence the value of the plants in his portfolio. However, in order to model plant trading several other issues need to be tackled. In the multi-stage game the portfolio structure of the different players changes over time. Due to plant trading the stage game changes over time, therefore, a simple model of learning in games cannot deal with this type of problem. The complexity of the coordination problem implies that possibly second best transactions will take place, and thus forecasting structural evolution is quite challenging and furthermore, the seller and the potential buyers have different information regarding the value of a given plant (even if the past behaviour of this plant is common knowledge). In the next Section we present the plant trading game which tackles these issues, by developing a dynamic game that simulates how boundedly rational Cournot players can trade electricity plants.

4. PLANT TRADING GAME

The plant trading game is a multi-stage game of incomplete information where in the first stage each player chooses the amount of capacity he wants to hold from each different technology and, in a second-stage, he specifies the quantity of generation he wants to sell in the market. It is noteworthy that this game is not just a repetition of a single-stage game: the structure of the market changes as players buy and sell plants, thus the mapping of payoffs of the single-stage game changes as well. Therefore, the plant trading game represents a search mechanism in the space of possible market structures. In this game, the search dynamics are a function of the strategic decisions of each player in the industry.

In a game with N players and M plants, $\Omega = \left[(W_1, K_1, C_1), \ldots, (W_i, K_i, C_i), \ldots, (W_M, K_M, C_M) \right]$ describes the state of the game, i.e., the ownership structure of the industry. In Ω, the triples (W_i, K_i, C_i) represent the owner (W_i), the capacity (K_i), and the cost (C_i) of a plant *i*. Further, let the vector A^i represent player *i*'s actions and $\boldsymbol{A} = \left\{ A^1, A^2, \ldots, A^N \right\}$ represent an ordered vector of the actions of all the N players. Note that any given instance of $\boldsymbol{A}$ represents a transition between the states of the industry (possibly no trade, or more than one simultaneous trades).

PROPOSITION 4.1: *In the plant trading game, each player, at each stage of the game, can play M+1 different actions. Thus there are* $(M+1)^N$ *possible transitions between states and* N^M *possible states of the industry.* [Proof in Appendix]

Note that the number of transitions between states and the number of possible states of the industry are an exponential function of the number of players and the number of plants, respectively. The implication of this is striking: in order for a player to analyse all possible transitions between states S stages ahead, he has to analyse $(M+1)^{SN}$ possible combinations. Moreover, as plant trading implies a bilateral agreement between a buyer and a seller for the same plant, this is a very hard coordination problem. In the evolutionary simulation algorithm which we develop to provide insights into this game, we specify five main stages: *Initialization, Identification, Adaptation, Plant Trading* and *Updating the State of Game*. During *Initialization* the Cournot model is solved for the initial market structure and the value of each plant is computed, then the *Identification* procedure allows each player to infer a model representing how the system is behaving and identifies the plants that will be offered, most probably, in the next trading round. In *Adaptation*, each player computes his best response to the inferred model by using an adaptive best behaviour. Then, possibly, two of the players *Trade* a plant. Finally, the algorithm *Updates the State of the Game.*

TABLE 4.1: Identification Algorithm

D^i: Perceived outcomes of the player's actions in the path of his automaton, $D^i \equiv \{0,1\}$.

Σ^i : Set of actions a^i available to player *i*

A_t^i : Set of actions actually bid by player *i*, in state *t*, with size *W; such that* $A_t^i \subseteq \Sigma^i$

T_t^i : Plausibility Table, a one-dimensional table of dimension *M (number of plants)*

θ : Plausibility cut-off parameter, $0 \le \theta \le 1$

$S \equiv$ All prefixes of D^i with a length less of equal than *K*> *W;* $s\left(a^i\right) \in S$

At stage zero initialise $\left(S, T_0^i\right)$: $\forall a^i \in \Sigma^i, s\left(a^i\right) = [1,1,...,1], T_0^i\left(s\left(a^i\right), \theta\right) = 1$.

1. At any given stage *t* and for each player *i*:

1.a) For each possible action update the string of perceived outcomes of the past

$$D_t^i\left(a_t^i\right) = \begin{cases} 0 \leftarrow Trade\_not\_possible \\ 1 \leftarrow Trade\_possible \end{cases}$$

$$\forall a_t^i \in \Sigma^i, s_t^i = \phi\left(s_{t-1}^i, D_t^i\left(a_t^i\right)\right)$$

1.b) Compute $p_{i,t}^a$ the percentage of time each action is expected to be successful

Let $d_j \in s_t^i$ represent a perceived outcome in string s_t^i, such that $d_j \in \{0,1\}$.

$$\forall a_t^i \in \Sigma^i, p_{i,t}^a = \frac{\sum_{j=1}^{K} d_j}{K}$$

1.c) Let $\tau_{a,t}$ represent the perceived outcome of action *a*, such that $\tau_{a,t} \in \{0,1\}$:

$\forall \tau_{a,t} \in T_t^i, \tau_{a,t} = \Phi^i\left(p_{i,t}^a, \theta\right)$.

2. The update operator (ϕ)

Let $D_t^i\left(a_t^i\right)$ represent the expected outcome of action a_t^i, and let $s_{t-1}^i = [d_1, d_2, ..., d_K]$ represent the vector of the past outcomes of action a^i:

$$s_t^i = \left[d_2, ..., d_K, D_t^i\left(a_t^i\right)\right].$$

3. The forecast operator $\Phi\left(p, \theta\right) = \begin{cases} 1 \leftarrow p \ge \theta \\ 0 \leftarrow p < \theta \end{cases}$

Table 4.1 presents a summary of the *Identification* algorithm. A player infers a model of how the system behaves by keeping in memory the results of each one of his actions $\left(A_t^i\right)$ in the last *K* periods. These results, D^i, are trade-possible (1) or not trade-possible (0). Further, a player is able to infer the results of actions that he did not take $\left(\Sigma^i \setminus A_t^i\right)$ by analysing if-then-else scenarios. The actions actually submitted to an auction, A_t^i, influence the perception the other players hold on the system's behaviour, while actions not submitted do not.

Each player *i* then updates a Plausibility Table T_t^i, which forecasts, for every possible action, if there is a possibility of trade (1) or not (0). This plausibility is computed using a cut-off parameter θ, and acts as passive inertia[3] that discards actions that are not plausible. Thus, given the *K*-string of possible-events associated with each action, a player computes the percentage of time it would be possible for a trade to have happened, $p_{i,t}^a$, and, if $p_{i,t}^a \geq \theta$ this action is considered to be a plausible trade.

Further, this identification process obeys two rules necessary for the rational behaviour of a certain player. The first rule is consistency: the model identified by each player has to be consistent, i.e., the same action, in a certain state, always leads to the same new state. The second rule is completeness, i.e., a player builds a model that forecasts the outcome of every possible action.

Table 4.2 presents the *Adaptation* algorithm, which represents how players learn and adapt in a dynamic environment. A player learns a model defining the behaviour of the environment and then he adapts his behaviour in order to maximise his long-term profit in that context. This algorithm applies three principles in order to model rational behaviour: passive inertia, active inertia, and best response behaviour. Passive inertia reflects the cost of changing. Active inertia represents the conduct of a player that imposes his behaviour to others. Finally, best response behaviour is the attitude of a player maximising the value of his portfolio in stable environments.

[3] Passive inertia is the behaviour of a player who decides not to act due to a lack of confidence in the model learned, instead waiting for further information

TABLE 4.2: Adaptation Algorithm

Σ^i: Set of actions available to player *i*. a_t^i : action of player *i* at time *t*.

A_t^i: Set of *W* actions actually bid by player *i*, in stage t, *such that* $A_t^i \subseteq \Sigma^i$.

T_t^i : Plausibility Table, vector of dimension *M (number of plants)*.

Ω_t : State of the industry at time *t*. V_t^i : Value of *i*'s portfolio at time *t*.

$\mu(\Omega_t, a_t^i)$: utility (profit or reward) of player *i* at time *t*, for a given action a_t^i in state Ω_t.

ρ_i : Discount factor for agent *i*, $0 \le \rho_i \le 1$.

r: random generated number from a uniform distribution, such that $r \in [0,1]$.

w_t^i : inertia variable such that $w_t^i \in [0,1]$, at time *t*. *h*: number of steps of look-ahead.

(a,i) : Plant *a* is owned by player *i*

1. Each player *i* decides to adapt

1.a) Applies the Inertia principle, for a given w_t^i

$$\begin{cases} Z_t^i = BR(\Omega_t, T_t^i, \rho_i) \leftarrow r \ge w_t^i \\ Z_t^i = A_{t-1}^i \leftarrow r < w_t^i \end{cases}$$

1.b) Algorithm Best-Response $Z_t^i = BR(\Omega_t, T_t^i, \rho_i)$:

Compute the optimal policy, Z_t^i:

$$\forall t = 1, \ldots, h$$

$$Z_t^i = \arg\max_{a_t^i} \left[u(\Omega_t, a_t^i) + \rho_i V_{t+1}^i (\Omega_{t+1}, T_{t+1}^i) \right]$$

$$s.t.$$

$$T_1^i = T_0^i, \Omega_1 = \Omega_0$$

$$\forall \tau_{j,t+1} \in T_{t+1}^i, \tau_{j,t+1} = \delta^i(\tau_{j,t}, a_t^i)$$

$$\Omega_{t+1} = \{\Omega_t \setminus (a,i)\} \bigcup \{(a,j)\}$$

$$\delta^i(\tau_{j,t}, a_t^i) = \begin{cases} \tau_{j,t} \leftarrow j \ne a_t^i \\ 0 \leftarrow j = a_t^i \end{cases}$$

2. Complete Adaptation Model

If $\# Z_t^i < W$

Let $\Lambda_t^i = \{a_t^i : \tau_{j,t} \in T_t^i, \tau_{j,t} = 0\}$

$$\overline{Z_t^i} = BR(\Omega_t, \Lambda_t^i, \rho_i)$$

else $\overline{Z_t^i} = \{ \}$

3. Define the set of actions to bid into the auction

$$A_t^i = Z_t^i \cup \overline{Z_t^i}$$

Step 2 in Table 4.2 refers to the active inertia construct. When a player adapts his behaviour, first he is constrained by the behaviour of the others but, at the same time, he is able to constrain their behaviour. A player that uses active inertia imposes his behaviour by two different ways. First, by choosing among the proposed trades the one he is interested in, he is able, particularly in the case where he owns the plant in discussion, to influence the plausibility of other player's actions. Second, by choosing to propose some new actions and holding to them, a player increases the plausibility of these trades, influences the other players' perceptions and "persuades" them to adapt to his own behaviour. This is the plant trading game: a battle to gain credibility, to coordinate behaviour and to gain influence, in order to do the trades that improve a player's long-term performance.

Given the set of plausibly successful actions $\left(T_t^i\right)$ a player *i* computes the set of actions $\left(Z_t^i\right)$ that would improve the value of his portfolio. However, if the number of these actions $\left(\# Z_t^i\right)$ is less than the maximum number a player can submit to the auction, he may bid some additional trading proposals $\left(\overline{Z_t^i}\right)$ that he perceives to be the most profitable, albeit having a low plausibility.

In step 1.b), in order to estimate the value of state $V_{t+1}^i\left(\Omega_{t+1}, T_{t+1}^i\right)$, for each player *i*, without actually solving the Cournot game with capacity constraints (which needs to be solved in each possible node of the game), each player needs a theoretical model enabling the estimation of the value of each plant in his portfolio. Using the theory presented in sections 2 and 3, a player is able to compute these values. The analysis is split into the buyer's and the seller's problem as a seller knows how to best place his plant, whereas a buyer may decide to change the market position of the plant being traded in order to adapt it to his own portfolio.

For a given *seller*, the value of a plant equals the operational profit at time *t*, which he knows, plus the portfolio contribution of that plant. Proposition 3.1 describes how the portfolio contribution is computed. A plant using its full available capacity has null portfolio contribution and a plant that withholds capacity from a certain market has a portfolio contribution directly proportional to its owner's total load in that market. In the single-clearing mechanism, a player knows, for each one of his plants, the quantity withheld from each market. However, in the multi-clearing mechanism, a player cannot calculate, in a straightforward way, the quantity withheld from each market. In this case, a rational player withholds capacity from the market in which that given plant receives the highest portfolio contribution.

In order to calculate the maximum possible increase of generation from plant *j* in market *L* a player identifies the potential portfolio contribution of the given plant in each market using the criterion $Loss\ of\ Portfolio = -2\alpha_L D_L Q_{(-j,i),L}$. Thus, the quantity withheld from each market is $\max\left(\Delta Q_{j,L}\right)$. See equation (4.1).

$$\max\left(\Delta Q_{j,L}\right) = \begin{cases} K_j - \sum_L Q_{j,L} \leftarrow \forall L, L': 2\alpha_L D_L Q_{(-j,i),L} \geq 2\alpha_{L'} D_{L'} Q_{(-j,i),L'} \\ 0 \leftarrow otherwise \end{cases} \tag{4.1}$$

In contrast, the evaluation of a plant by a potential *buyer* is slightly more complex. He needs to compute the portfolio contribution and the expected operational profit of this plant as most probably a buyer's advantage arises from the possibility of repositioning the plant. First, a buyer computes the load of this plant in each market. This is done by maximising the operational profit of player *i*, (see Definition 3.1.b)), by using as decision variable the production of plant *j*. Again, this implies the solution of a non-linear optimisation problem every time a buyer evaluates a plant. However, the dimension of this problem is small, and its solution is easy, even if solved repeatedly. Second, the buyer faces the same problem as the seller with regard to the computation of the portfolio contribution. Given a certain assignment of load to each market, the buyer computes the portfolio contribution of this plant using equation (4.1).

The plant trading is organised in a single-call auction (Cason and Friedman, 1997). Table 4.3 describes the trading auction algorithm.

TABLE 4.3: The Trading Auction

a: Asset being auctioned
i, j: players offering (attempting to sell) or bidding (attempting to buy) assets in an auction
$P_{a,t}$: Transaction price of asset a at time t
$B_{a,i}$: Price bid by player i attempting to buy asset a
$O_{a,i}$: Price offered by player i attempting to sell asset a
T_a: Set of all possible trades for asset a
B_a: Set of all acceptable bids for asset a
T: Set of the all winning trades (at the most one per asset)

1. For every asset a find T_a

$$T_a = \left\{ \left(B_{a,i}, O_{a,j} \right) : i \neq j, B_{a,i} > O_{a,j} \right\}$$

$$B_a = \left\{ B_{a,i} : \left(B_{a,i}, O_{a,j} \right) \in T_a \right\}$$

2. Find T:

For every asset a find the winning trade $\left(B^{+}_{a,i}, O^{+}_{a,j} \right)$:

$$O^{+}_{a,j} = O_{a,j} \text{ and } B^{+}_{a,i} = \sup B_a$$

Find the set of all winning trades:

$$T = \bigcup_a \left(B^{+}_{a,i}, O^{+}_{a,j} \right)$$

3. Find the asset to be traded $\left(B^{*}_{a,i}, O^{*}_{a,j} \right)$

Let g stand for a function from T into R :

$$g = \left\{ \left(\left(B_{a,i}, O_{a,j} \right), G_a \right) \in T \times R \mid G_a = B_{a,j} - O_{a,j} \right\}$$

The asset to be traded is the one with the largest difference between offer and bid prices

$$\left(B^{*}_{a,i}, O^{*}_{a,j} \right) = \underset{\left(B_{a,i}, O_{a,j} \right)}{argmax}\ g$$

4. Compute the transaction price

Let $B^{++}_{a,z}$ represent the second highest bid for asset a:

$$B^{++}_{a,z} = \sup \left\{ B_a \setminus B^{*}_{a,i} \right\}$$

$$P_{a,t} = \max \left(\frac{B^{*}_{a,i} + O^{*}_{a,j}}{2}, B^{++}_{a,z} \right)$$

There is a separate auction for every plant, simultaneously. First, a trade is possible only if simultaneously there are one or more buyers and a seller, and the price offered by the buyers is higher than the seller's bid. Second, for each plant a, the algorithm computes the transaction price at time t ($P_{a,t}$) by calculating the simple average of the seller's bid price and the buyers' highest offered price. Then, $P_{a,t}$ equals the maximum of this simple average and the second highest offered price. Third, as the seller would not pay to sell, admissible trades are only those ones where the transaction price is positive (we assume a zero cost of closing-down a plant).

After computing the transaction price of each plant, the auctioneer chooses which transaction takes place at time *t*. It is possible to have more than one trade per iteration, and indeed, this would not hurt any of the theoretical properties of the model. However, this would imply that the jumps between successive states of the industry would be wider and the evaluation error for each plant would be higher. Thus, the restriction of only one plant traded at a time is to ensure a smoother adjustment trajectory.

After any successful trade, the algorithm computes a *New State of the Game.* Moreover, even if there is no trade, the probabilities associated with active inertia principle still need to be updated. Table 4.4 describes the algorithm for this stage.

Each player updates the capacities and marginal costs iteratively, taking into account the past performance of each plant. A player offers in a given market the generation of every plant with a marginal cost[4] lower than his marginal plant in this market[5].

[4] Note that the marginal cost of a given player is the highest one among all the plants he submits to a given market (Ramos at al., 1998; Borenstein et al. 1999, 2002).

[5] Moreover, a player may offer the generation of a plant in a certain market even when its marginal cost is higher than this player's marginal cost. However, this is only possible if in the previous iteration the player did not offer the generation of this plant in this market and additionally, if the marginal cost of this plant is lower than the player's marginal cost in any subsequent markets (assuming that the markets are organised in increasing order by the clearing price) to which the player did not offer any capacity of the given plant. Note, if the generation capacity was offered but the plant did not run, then it is not "offered" in this market (see Proposition 2.1).

TABLE 4.4: Update State of the Game

---

a: Any given plant that may be auctioned;

:= represents a process of iterative updating.

(a,i) : Plant *a* is owned by player *i;* $not(a,i,t)$: Plant *a* is not owned by player *i*, at time *t*

i, *j*: player *i* sells sells plant *a* to player *j* in an auction

$P_{L,t}$: Electricity price in market *L*, at time *t*

Ω_t : State of the industry at time *t*

$C_{(a,i),L}$: marginal cost of plant *a*, owned by player *i*, for market *L*

$K_{(a,i)}$: available capacity of asset *a*, owned by player *i*

$K_{(a,i),L}$: capacity of asset *a* offered in market *L* in the previous iteration

$C_{i,L}$: marginal cost of player *i* in market *L*

$K_{i,L}$: capacity of player *i* assigned to market *L*

w^i: inertia variable such that $w^i \in [0,1]$; $\sigma \in]0,1[$ is the parameter for inertia updating

$OP(a,i)$, $OP(i)$: Operational profit of plant *a* and player *i*, respectively

D_L : duration of market *L;*

$Q_{(a,i),L}$: total generation of plant *i* sold in market *L*

1. Update state of the industry Ω_t

$$\Omega_{t+1} = \{\Omega_t \setminus (a,i)\} \bigcup \{(a,j)\}$$

$$w_{t+1}^z = \begin{cases} w_t^z.\sigma \leftarrow otherwise \\ \;1 \quad \leftarrow \quad z = i, j \end{cases}$$

2. Update cost structure and capacities bid in each auction:

$\forall L, \forall i :$

$K_{i,L} := 0$

$\forall a, K_{(a,i),L} := 0$

2.1 For all available asset *a*

$$if\ C_{(a,i),L} \le C_{i,L} \text{ or } \left[not(a,i,t), C_{(a,i),L} \le C_{i,L+1}\right]$$

$$K_{i,L} := K_{i,L} + K_{(a,i)}$$

$$C_{i,L} := \max\left[C_{i,L}, C_{(a,i),L}\right]$$

$$K_{(a,i),L} := K_{(a,i)}$$

$$if\ multi\text{-}clearing\ and\ if\ K_{(a,i),L} > 0\ then\ K_{(a,i)} := 0$$

3. Solve Cournot game

4. Compute value of plant

$\forall i, a :$

$$OP(a,i) = \sum_L \left[\left(P_{L,t} - C_{(a,i),L}\right).Q_{(a,i),L}.D_L\right]$$

$$OP(i) = \sum_{(a,i)} OP(a,i)$$

---

5. EXEMPLIFICATION OF THE PLANT TRADING GAME

This section applies the above model to the real example of simulating the structural evolution of the England and Wales (E&W) electricity market under different initial conditions and market mechanisms. We specify the structure of the E&W electricity market as it was in 2000 (see Table 5.1[6]). These experiments simulated trading at a genset level (137 gensets) distributed among 24 different players. This leads to the existence of approximately 1.22E+189 possible states of the industry and 2.27E+51 possible transitions, at every stage of the game.

TABLE 5.1: Generation Industry[7]

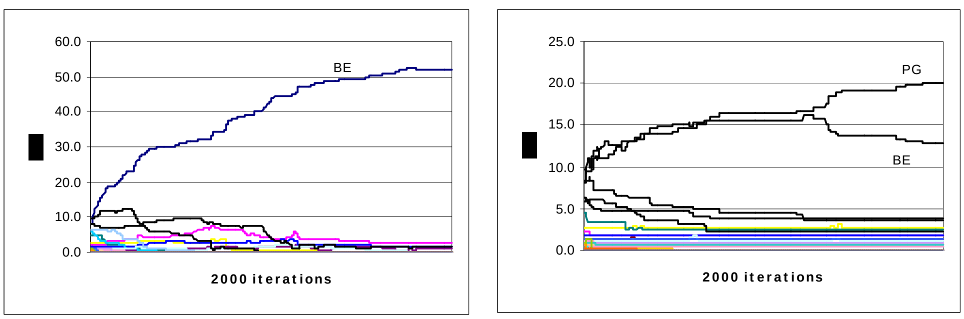
Capacity of each Company (% of Total, 59 GW) in 2000				
	Total	Nuclear	Large Coal+CCGT	Small Coal +OCGT + OIL + Pump. Storage
PG	16.5		19.7	24.9
NP	13.9		16.3	22.5
BE	12.4	54.0	4.9	
Edison	10.6		10.1	30.7
TXU	9.7		11.6	14.7
AES	7.8		10.1	6.8
EDF	4.7	17.3	2.0	
Magnox	3.9	19.9		
Others	20.5	8.8	25.3	0.4
Total GW	59.1	11.4	40.7	7.0

[6] Please refer to the UK Electricity Association (1999, 2000a,b,c).

[7] The generation capacity owned by each player was split into three categories, taking into account the degree of flexibility and running times of each technology: Nuclear plants, baseload technology running continuously. Large Coal and CCGT, the shoulder technologies. Small Coal, OCGT, Oil and Pumped storage, the peak plants. Thus BE had 54% of the Nuclear generation capacity installed in E&W (and 4.9% of the shoulder capacity), while AES owned both shoulder (10.1% of shoulder capacity) and peak plant (6.8% of peak capacity).

The demand elasticities used were 0.5, 0.35 and 0.25 respectively for the baseload, shoulder and peak market. The choice of these is consistent with those used previously, eg Wei and Smeers (1999) use 0.4 and 0.53 for residential and industrial clients respectively, in simulating the Belgium, France, Germany and Italy market; whilst Ramos et al. (1998) use an elasticity of 0.6 in simulating the Spanish market. The annual load durations for the shoulder and peak markets were specified as 5500 (5000) and 500 (500) hours for the multi (single) clearing mechanism. The duration of the baseload was specified as 8760 and 3260, respectively for the multi and single-clearing mechanisms. All the experiments presented in this section simulated 2000 iterations in each different scenario.

Figure 5.1 presents the results of the first set of experiments under the single and multi clearing mechanisms. Even though we are grounding this simulation on the actual E & W system, we are, of course, not seeking to imply any corporate forecasts, but only illustrate the attractors in the system[8].



(a) (b)

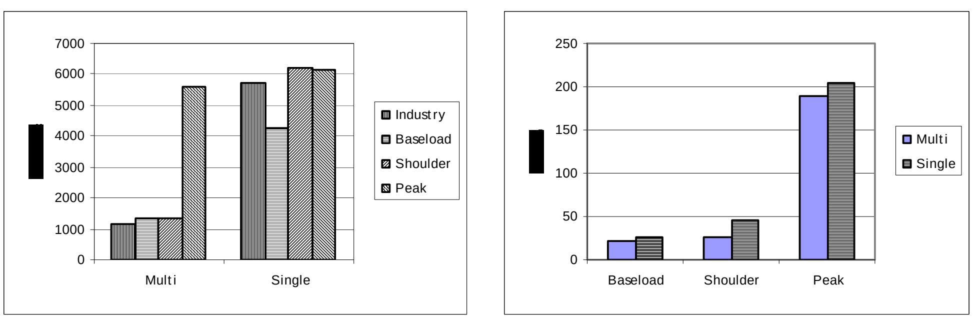
FIGURE 5.1: Capacity by player. Experiment with 24 players. (a) Single-clearing mechanism. (b) Multi-clearing mechanism.

As shown in Figure 5.1.a), BE becomes the incumbent player in a monopoly with a competitive fringe. BE was the dominant baseload generator it bought the shoulder and peak plants in order to increase the value of its baseload portfolio, and thus became the dominant company. But in Figure 5.1.b), PG

[8] Both BE and PG actually evolved in different ways for extraneous reasons outside the scope of this analysis. These included the cost of nuclear liabilities for BE and the lack of vertical integration by BE at a time when value migrated from the generating to the retail supply end of the supply chain. Such factors were not explored in this stylised model, and were not part of the research design of the simulations.

and BE are the dominant players in the multi-clearing mechanism, and we see a lower concentration level in the multi-clearing mechanism, as in this case capacity withholding is less profitable than in the single-clearing mechanism. Figure 5.2.a) shows that those under the single-clearing mechanism the concentration indices are higher than under multi-clearing mechanism. This high concentration translates itself into higher electricity prices (Figure 5.2.b)).

Since the starting values of the concentration indices under the different types of clearing mechanism were the same, these results imply that the dynamics of the single-clearing mechanism leads to higher market concentration. This can be seen in more detail by looking at the evolution of BE's and PG's market shares in the multi-clearing mechanism. Figure 5.3 shows that PG became a dominant player in the peak technologies (small-coal, pumped-storage, OCGT and oil) and BE became a dominant player by enlarging their dominant position in the baseload technologies (nuclear and big-coal). The comparison of the two experiments here is consistent with the previous analysis that the multi-clearing mechanism tends to lead to lower market concentrations than the single-clearing mechanism, and is an interesting new twist on the relationship of the evolved resource base to the market microstructure.



(a) (b)

FIGURE 5.2: Concentration and Prices. Experiment with 24 players. (a) HHI Concentration Indices: Multi (Single) represents the concentration index in the multi-clearing (single-clearing) mechanism. (b) Electricity Prices (in the Baseload, Shoulder and Peak markets) presented as a function of Clearing-Mechanism.

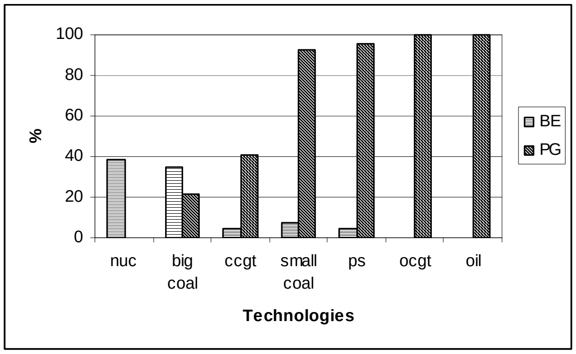


FIGURE 5.3: BE's and PG's Capacity Shares**,** it presents an analysis by Technology for the Multi-Clearing Mechanism.

Many other experiments have been used to validate this model by testing its results under different market structures. This validation process showed that when players behave as price takers, no trading occurs as the players generate the market clearing quantities. Further, in a simulation of the monopoly situation with potential new entrants, there was no entry, and the prices were always the monopoly ones. Both these results are reassuring for model credibility.

6. CONCLUSIONS

We have developed insights into the process of asset trading between competitive players from both stylised Cournot analysis and comprehensive computational simulation, modelling by the structure – conduct – performance feedback process within an evolutionary setting. By looking in detail at the trading of electricity generating plants, the impact of the market microstructure on the emergent asset resource base of the firms provided evidence of a new dimension in understanding the characteristics of strategic factor markets. In specific terms we identified clearly why different players can value the same asset differently, that a player can modify the value of some of his plants by acquiring another plant, that the cash flow generated by a given plant represents only a lower bound on its contribution to the value of the portfolio, and that the cash flow of a plant generating its full capacity is the upper bound on the value of this plant. Marginal plants are more valuable in portfolios with a large total output, but players cannot profitably transfer load from a cheap to an expensive plant by withholding capacity other than that from a more expensive plant. We determined that in Cournot competitive environments plant trading increases market concentration, with the prices and concentration levels in the single-clearing mechanism being higher than those in the multi-clearing alternative.

The methodological challenges in developing an evolutionary computation model for these purposes were substantial, as this is a game where players rarely interact, where the stage-game evolves over time, and where coordination is a hard problem. A solution to this was achieved through the development of identification and adaptation algorithms that enable players to coordinate behaviour for plant trading. The adaptation algorithm uses path dependent information to tackle the non-linear best-response problem both for buyers and sellers, taking into account the asymmetries of the two types of players. An auction algorithm chooses which plant will be traded and the transaction price, at any given time. The iterative updating of the model is a non-linear and clearly path dependent process. The feasibility of the large scale application of this type of modelling was demonstrated with an application to the full generating system in England and Wales, and as such, its demonstration offers a valuable bridge between stylised theoretical analysis of market structure evolution and the impact of detailed market microstructure effects.

APPENDIX

PROOF OF PROPOSITION 2.1: Assume that a plant *j+1* can offer in market *L*. Decomposing the non-linear cost components of each firm then the operational profit function of a given player is $\pi_0 = \sum_j \left(P_L - C_j\right).Q_{j,L}.D_L$. The proof follows by contradiction. Assume for all L $Q_{j,L} < K_{j,L}$ and $Q_{j+1,L} > 0$. If $Q_{j+1,L} \geq K_{j,L} - Q_{j,L}$ the player can improve his profit by transferring $K_{j,L} - Q_{j,L}$ load units to plant *j*; therefore $Q_{j,L} = K_{j,L}$, reaching a contradiction. Alternatively, if $Q_{j+1,L} < K_{j,L} - Q_{j,L}$ the player can improve his profit by transferring $Q_{j+1,L}$ units of load to plant *j*; thus, $Q_{j+1,L} = 0$, again reaching a contradiction. Q.E.D.

PROOF OF PROPOSITION 2.2: Decompose the operational profit into its components in different markets, and assume the available capacities for each market as given (in both models). Then player

i's profit is $\pi_{i,L} = \sum_{g=1}^{P} \left(P_L - C_{g,L}\right).Q_{g,L}.D_L$ and as $\sum_{g=j+1}^{P} Q_{g,L} = 0$ it follows that $\pi_{i,L} = \sum_{g=1}^{j} \left(P_L - C_{g,L}\right).Q_{g,L}.D_L$. Further, by definition of inverse residual demand $\pi_{i,L} = A_L.D_L.\sum_{g=1}^{j} Q_{g,L} - \alpha_L.D_L.\sum_{g=1}^{j}\sum_{z=1}^{j} Q_{g,L}.Q_{z,L} - D_L.\sum_{g=1}^{j} Q_{g,L}.C_{g,L}$. Therefore, by the optimality conditions $A_L.D_L - \alpha_L.D_L.\sum_{g=1}^{j-1} Q_{g,L} - 2.\alpha_L.D_L.Q_{j,L} - D_L.C_{j,L} = 0$. Hence, the optimal load for plant j is $Q_{j,L} = -\frac{1}{2}\sum_{g=1}^{j-1} Q_{g,L} + \frac{1}{2\alpha_L} A_L - \frac{1}{2\alpha_L} C_{j,L}$, which is less than $K_{j,L}$ if and only if $\frac{1}{\alpha_L}\left(A_L - C_{j,L}\right) < 2K_{j,L} + \sum_{g=1}^{j-1} Q_{g,L}$. Q.E.D.

PROOF OF THEOREM 2.1: From Proposition 2.2 it follows that a player offers load from plant $j+1$ only if he sells the full capacity of plant j. Further, Proposition 2.1 implies that it is possible to withhold the capacity of plant j only if the player does not offer generation from plants $j+1$, $j+2$, …, $j+P$. Hence, a player cannot profitably transfer load from cheaper to expensive plants.

Q.E.D.

PROOF OF THEOREM 2.2: By acquiring a plant j the inverse residual demand of player i for each market L shifts and can be represent as $P_L = A_L + \alpha_L Q_{(j,0),L} - \alpha_L \sum_{g=1}^{P} Q_{g,L}$. Therefore, by definition of marginal plant $\sum_{g=z+1}^{P} Q_{g,L} = 0$ and $\sum_{g=1}^{j+z-1} Q_{g,L} = \sum_{\substack{g=1 \\ g \neq j}}^{j+z-1} Q_{g,L} + Q_{(j,1),L}$, and by the optimisation conditions, it follows that

$$Q_{(j+z,1),L} = -\frac{1}{2}\sum_{g=1}^{j-1} Q_{g,L} + \frac{1}{2\alpha_L}\left(A_L + \alpha_L.Q_{(j,0),L}\right) - \frac{1}{2\alpha_L} C_{j,L} \text{ and that}$$

$$Q_{(j+z,1),L} = -\frac{1}{2}\sum_{\substack{g=1 \\ g \neq j}}^{j-1} Q_{g,L} - \frac{1}{2} Q_{(j,1),L} + \frac{1}{2} Q_{(j,0),L} + \frac{1}{2\alpha_L}\left(A_L - C_{j,L}\right).$$

Then, as $Q_{(j+z,1),L} \le Q_{(j+z,0),L}$ by adding up $Q_{j,L}$ for all L it follows that

$$\sum_L \left(\frac{A_L - C_{j+z,L}}{\alpha_L} \right) < 2\sum_L Q_{(j+z,0),L} + \sum_L \left(Q_{(j,1),L} - Q_{(j,0),L} \right) + \sum_L \left(\sum_{\substack{g=1 \\ g \ne j}}^{j+z-1} Q_{g,L} \right). \qquad \text{Q.E.D.}$$

PROOF OF PROPOSITION 3.1: (a) Let $Q_{j,L} = K_{j,L}$, it follows that $\max\left(\Delta Q_{j,L}\right) = 0$ by Definition 3.2, and therefore $PC\left(j,i\right) = \sum_L \alpha_L.\max\left(\Delta Q_{j,L}\right).Q_{(-j,i),L}.D_L = \sum_L 0 = 0$. Since, by Definition 3.3, $V\left(j,i\right) = OP\left(j,i\right) + PC\left(j,i\right)$, it follows that $V\left(j,i\right) = OP\left(j,i\right)$, and therefore, $OP\left(j,i\right)$ is an upper bound on the value of plant *j*. (b) Let $Q_{j,L} < K_{j,L}$ and assume that for any market L $MC_j \ge P_L$. Thus, by Definition 3.2, $P_{L,F} \le P_L$ and $MC_j \ge P_{L,F}$. Therefore, by the definition of marginal profit and Definition 3.1, $Q_{j,L} = 0$ and $\max\left(\Delta Q_{j,L}\right) = 0$, which, by Definition 3.2, implies that $PC_j = 0$. Hence, this proves by contrapositive that $PC\left(j,i\right) > 0$ only if $MC_j < P_L$ for at least one market *L*.

Q.E.D.

PROOF OF PROPOSITION 3.2: By Definition 3.3 the economic value of a certain plant $V\left(j,i\right) = OP\left(j,i\right) + PC\left(j,i\right)$ can be decomposed into Operational Profits and Portfolio Contribution. Assume $MC_j < P_L$. (a) By definition $PC(j,i) = \sum_L D_L.\left(P_L - P_{L,F}\right).Q_{(-j,i),L}$, and thus the larger $Q_{(-j,i),L}$ is the larger is the profit contribution of plant *j*. (b) By Definition 3.1.b) and by deriving the optimality condition for a certain output $Q_{j,L}$, it follows that the marginal loss of a portfolio is $-2\alpha_L D_L Q_{(-j,i),L}$. Thus, by the optimality conditions, the larger $Q_{-j,i}$ is the smaller is the total output of a marginal plant *j*. Q.E.D.

PROOF OF THEOREM 3.1: First assume that the trade occurred and therefore $V\left(j,b\right) > V\left(j,s\right)$, which is equivalent to $OP_{j,b}\left(Q_{j,b}\right) - OP_{j,s}\left(Q_{j,s}\right) + PC_{j,b}\left(Q_{j,b}\right) - PC_{j,s}\left(Q_{j,s}\right) > 0$. Since for the seller, by definition of optimum behaviour, $OP_{j,s}\left(Q_{j,s}\right)+PC_{j,s}\left(Q_{j,s}\right)\geq OP_{j,s}\left(Q_{j,b}\right)+PC_{j,s}\left(Q_{j,b}\right)$, it follows that

$OP_{j,b}\left(Q_{j,b}\right)-OP_{j,s}\left(Q_{j,b}\right)+PC_{j,b}\left(Q_{j,b}\right)-PC_{j,s}\left(Q_{j,b}\right)>0$. As $OP_{j,b}\left(Q_{j,b}\right)=OP_{j,s}\left(Q_{j,b}\right)$, it follows that $PC_{j,b}\left(Q_{j,b}\right)-PC_{j,s}\left(Q_{j,b}\right)>0$. Replacing the profit contribution by its definition, it follows that $\sum_L D_L.\left(P_L-P_{L,F}\right).Q_{(-j,b),L}>\sum_L D_L.\left(P_L-P_{L,F}\right).Q_{(-j,s),L}$. Q.E.D.

PROOF OF THEOREM 3.2: The profit of any player in a market *L*, for any player *i* owning P plants, is $\pi_{i,L}=\sum_{g=1}^{P}\left(P_L-C_{g,L}\right).Q_{(g,i),L}.D_L$. By the optimality conditions presented in the proof of Proposition 2.2 and adapting the notation to deal with a buyer and a seller, it follows that the optimal load for plant *j* is $Q_{(j,i),L}=-\frac{1}{2}\sum_{g=1}^{j-1}Q_{(g,i),L}+\frac{1}{2\alpha_L}A_L-\frac{1}{2\alpha_L}C_{j,L}$, or equivalently,

$Q_{(j,i),L}=-\frac{1}{2}Q_{(-j,i),L}+\frac{1}{2\alpha_L}A_L-\frac{1}{2\alpha_L}C_{j,L}$. Thus, $Q_{(j,b),L}<Q_{(j,s),L}$ is equivalent to

$-\frac{1}{2}Q_{(-j,b),L}+\frac{1}{2\alpha_L}A_L-\frac{1}{2\alpha_L}C_{j,L}<-\frac{1}{2}Q_{(-j,s),L}+\frac{1}{2\alpha_L}A_L-\frac{1}{2\alpha_L}C_{j,L}$ and hence $Q_{(-j,b),L}>Q_{(-j,s),L}$.

Q.E.D.

PROOF OF THEOREM 3.3: The proof follows by contradiction. Suppose that plant j was traded between players *b* and *s* and therefore $V\left(j,b\right)>V\left(j,s\right)$. Then, as $Q_{(-j,b),L}<Q_{(-j,s),L}$ from Theorem 3.2 it follows that $Q_{(j,b),L}>Q_{(j,s),L}$. As $V\left(j,b\right)>V\left(j,s\right)$ is equivalent to

$OP_{j,b}\left(Q_{j,b}\right)+PC_{j,b}\left(Q_{j,b}\right)>OP_{j,s}\left(Q_{j,s}\right)+PC_{j,s}\left(Q_{j,s}\right)$, by the optimality conditions it follows that

$OP_{j,b}\left(Q_{j,b}\right)+PC_{j,b}\left(Q_{j,b}\right)>OP_{j,s}\left(Q_{j,b}\right)+PC_{j,s}\left(Q_{j,b}\right)$ and thus, by definition of operational profit

$PC_{j,b}\left(Q_{j,b}\right)>PC_{j,s}\left(Q_{j,b}\right)$. However, as $Q_{(-j,b),L}<Q_{(-j,s),L}$ and $Q_{(j,b),L}>Q_{(j,s),L}$ from definition of profit contribution it follows that $PC_{j,b}\left(Q_{j,b}\right)<PC_{j,s}\left(Q_{j,s}\right)$. Hence, if $Q_{(-j,b),L}<Q_{(-j,s),L}$ then $V\left(j,b\right)<V\left(j,s\right)$, therefore *b* and *s* do not trade plant *j*. Q.E.D.

PROOF OF COROLLARY 3.1: From Theorem 3.3 it follows that in states of the industry where $Q_{(-j,b),L} < Q_{(-j,s),L}$ there is no trade as $V(j,b) < V(j,s)$. Moreover, Theorem 3.2 specifies that if $Q_{(-j,b),L} > Q_{(-j,s),L}$ and trade does happen, i.e., $V(j,b) > V(j,s)$, then $Q_{(j,b),L} < Q_{(j,s),L}$. Q.E.D.

PROOF OF THEOREM 3.4: Let $Q_{(-j,i),t}$ represent the residual quantity sold by player *i* at time *t*, and let $Q_{(-j,i),L}$ stand for the quantity sold by player *i* in market *L*, at time *t*. By Definition 7.2, $PC(j,i) = \sum_{L} \alpha_L . \max\left(\Delta Q_{j,L}\right) . Q_{(-j,i),L} . D_L$, it follows that a plant only has a positive portfolio contribution in markets where some other plants of the same player are also selling, i.e., $Q_{(-j,i),L} > 0$. Since, at any given time, by definition of single clearing mechanism $Q_{(-j,i),t} = Q_{(-j,i),L}$ and by definition of multi-clearing mechanism $Q_{(-j,i),t} = \sum_{L} Q_{(-j,i),L}$, it follows that $Q_{(-j,i),L} \leq Q_{(-j,i),t}$, therefore by Proposition 7.4.b) there is less pressure for market concentration and capacity withholding. Q.E.D.

PROOF OF PROPOSITION 4.1: At every stage of the game: (a) A player may try to buy a plant that he does not own, sell a plant that he owns, or keep the same portfolio. Therefore, a player has one possible action per each one of the *M* plants in the industry, and an extra one which is to do nothing. Hence, he can play *M+1* possible actions. (b) The number of possible transitions between states is the Cartesian product of the possible actions of each player, hence $(M+1)^N$. (c) The state of the industry is described by the ownership of each plant. Since each plant may be owned by each one of the *N* players, the number of possible states of the industry is the Cartesian product of the possible owners of each plant, hence N^M. Q.E.D.

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