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Extending the difference-in-differences (DID) to settings with many treated units and same intervention time: Model and Stata implementation

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Outline

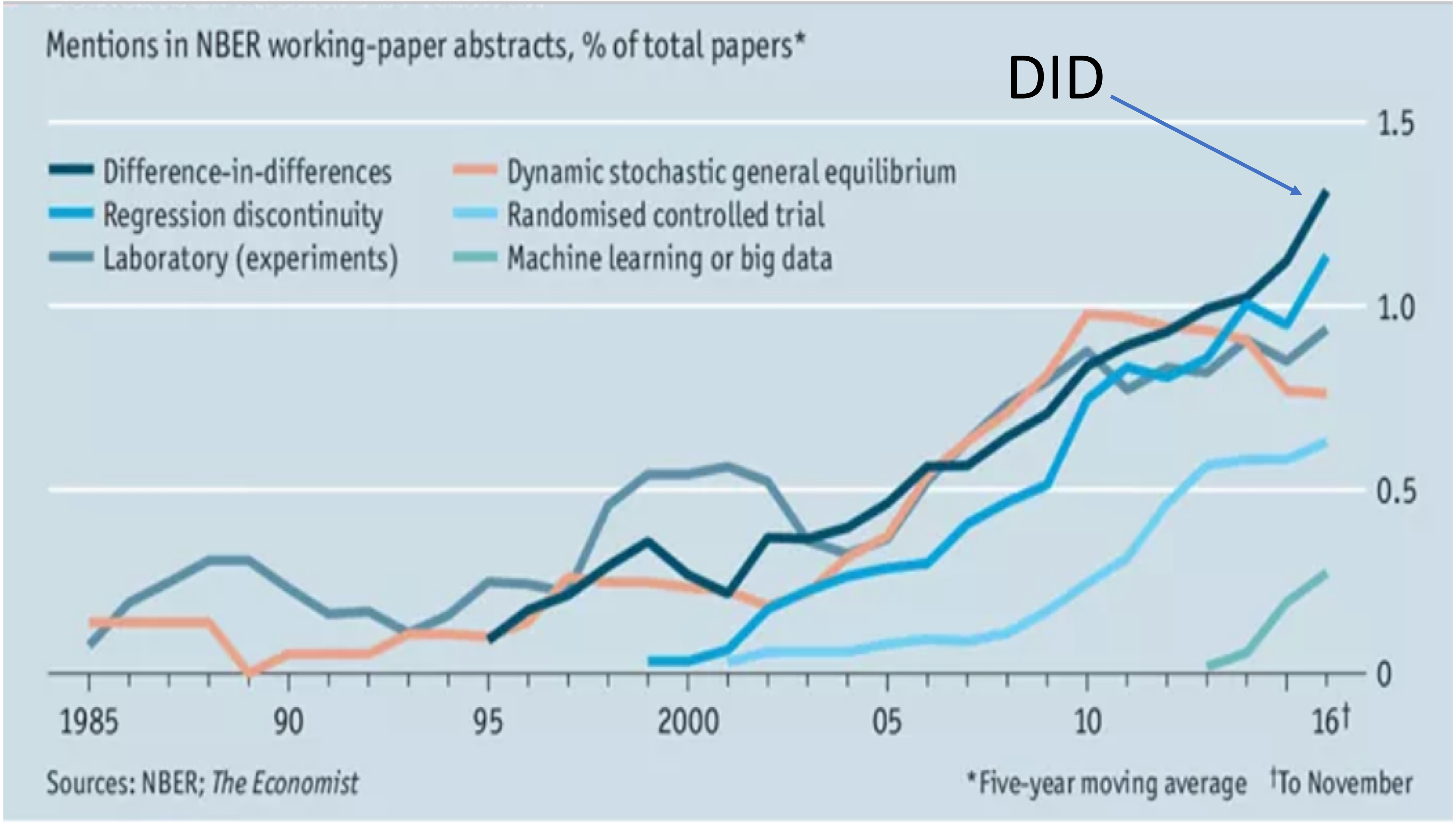
- Providing an original model extending the **Difference-In-Differences (DID)** to the case of a **binary treatment** having a **time-fixed** nature

- Overcoming the **Synthetic Control Model** **limitations** on **inference**

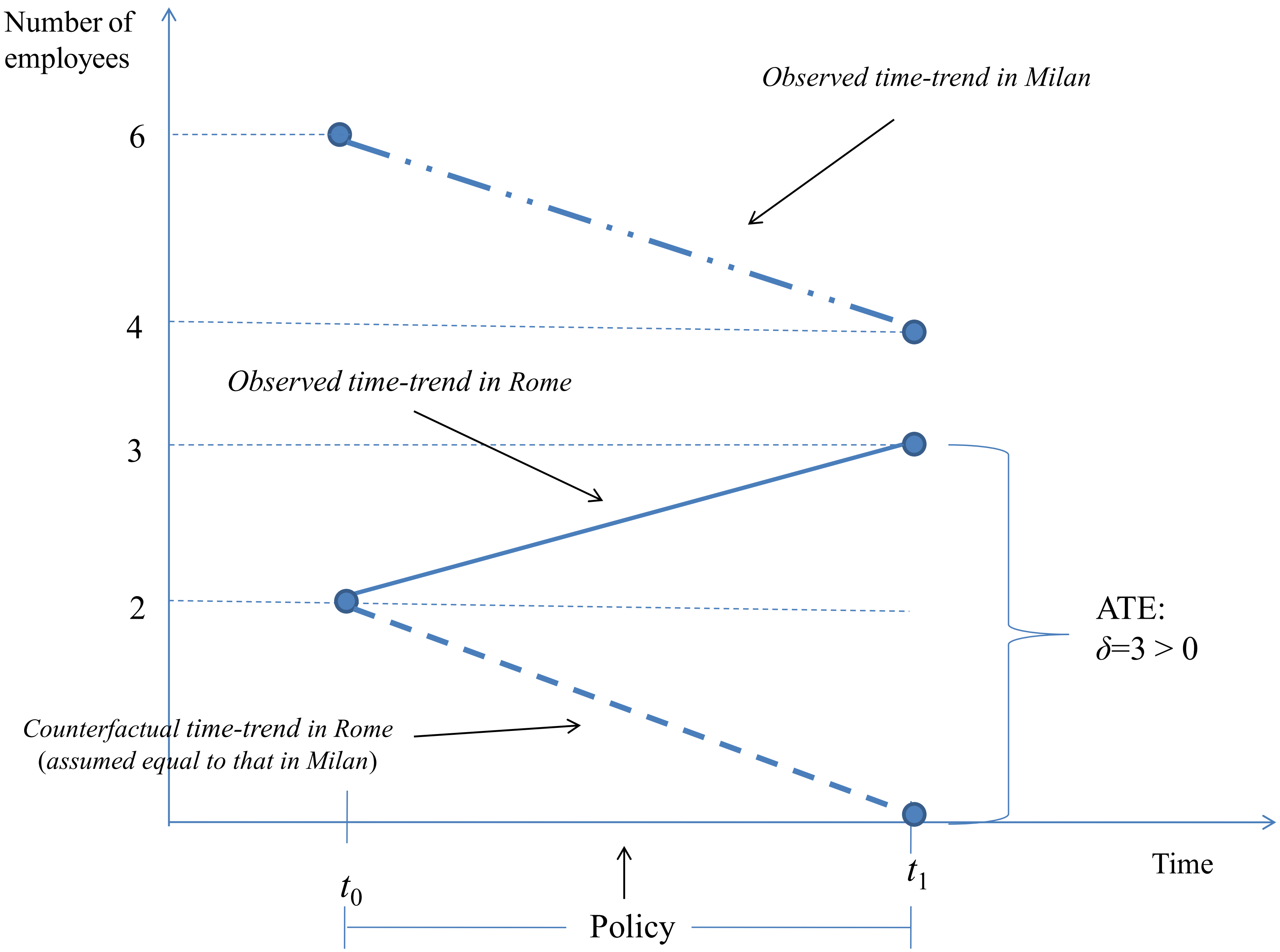
- Providing a **test** the **common-trend** assumption

- Presenting **`tfdiff`**: **Stata routine** to implement this model

Diffusion of the DID in Economics



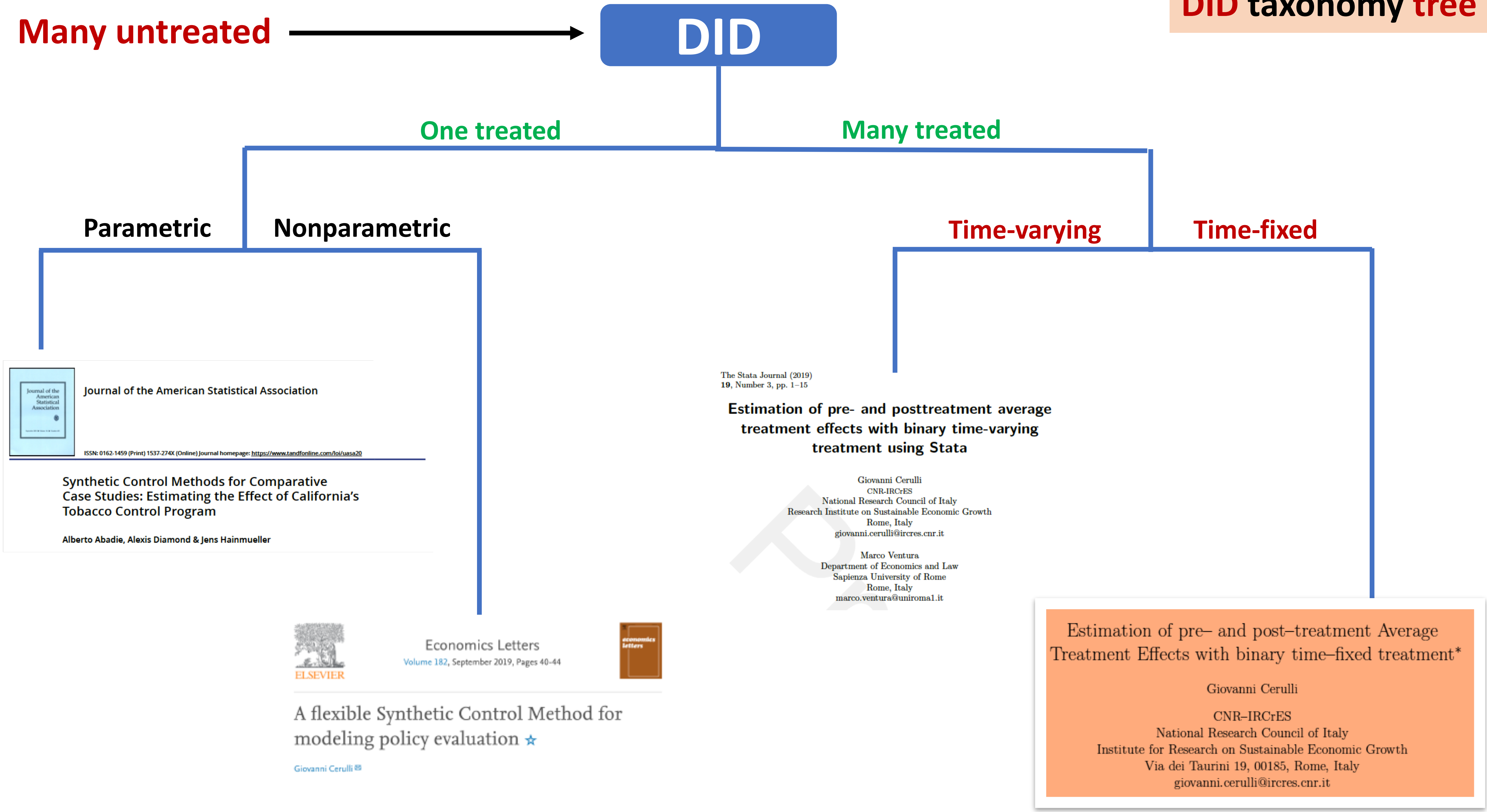
The basics of DID



DID modelling: a taxonomy

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">TREATMENT TIMING</td></tr>
<tr><td>TIME-FIXED</td><td>TIME-VARYING</td></tr>
<tr><td rowspan="2">NUMBER OF TREATED UNITS</td><td>ONE</td><td>Synthetic Control Method
(Abadie et al., 2010; Cerulli, 2019)</td><td>?</td></tr>
<tr><td>MANY</td><td>TFDIFF
(Cerulli, 2019)</td><td>TVDIFF
(Autor, 2003; Cerulli and Ventura, 2019)</td></tr>
</table>

DID taxonomy tree



Modeling

TFDIFF

The TFDIFF model and its Stata implementation

- **Generalization** of the Difference–in–Differences estimator in a longitudinal data setting

- Treatment is **binary** and **fixed at a given time**

- Many **pre– and post–intervention periods** are assumed available

- **Stata routine** implementing this model in an **automatic** way:

    - graphical representation of the estimated causal effects

    - Testing ***parallel-trend*** assumption for the necessary condition of the identification of causal effects

Some examples where the TFDIFF model can come in handy

Economics

In 2001, some European countries have adopted a common currency, the Euro. We would like to know whether this important economic reform has had an impact on adopters by comparing their economic performance over time with that of countries that did not adopt the Euro

Medicine

At a given point in time, some patients affected by too high blood pressure were exposed to a new drug developed to be more effective than previous ones in stabilizing blood pressure. We are interested in assessing the effect of this new drug by comparing follow-up blood pressure of treated people with that of a placebo group. We might be also interested in detecting effect duration over the follow-up time span

Environment

A group of regions decide to sign an agreement for reducing CO_2 emissions by promoting solar energy solutions. After some years, we are interested in assessing whether the level of CO_2 emissions in those regions is sensibly lower than the emissions in regions that did not sign the agreement

TFDIFF counterfactual setting

Longitudinal dataset with a **time-fixed treatment** at ***t* = 01**.

1	2	3	4	5	6	7	8	9	10	11	12	13
id	t	w	D_0	D_1	D_2	y_{00}	y_{01}	y_{02}	y	y^1	y^0	$(y^1 - y^0)$
1	00	1	1	0	0	$y_{1,00}$	0	0	$y_{1,00}$	$y^1_{1,00} = y_{1,00}$	$y^0_{1,00} = .$	.
1	01	1	0	1	0	0	$y_{1,01}$	0	$y_{1,01}$	$y^1_{1,01} = y_{1,01}$	$y^0_{1,01} = .$	.
1	02	1	0	0	1	0	0	$y_{1,02}$	$y_{1,02}$	$y^1_{1,02} = y_{1,02}$	$y^0_{1,02} = .$	.
2	00	0	1	0	0	$y_{2,00}$	0	0	$y_{2,00}$	$y^1_{2,00} = .$	$y^0_{2,00} = y_{2,00}$	.
2	01	0	0	1	0	0	$y_{2,01}$	0	$y_{2,01}$	$y^1_{2,01} = .$	$y^0_{2,01} = y_{2,01}$	.
2	02	0	0	0	1	0	0	$y_{2,02}$	$y_{2,02}$	$y^1_{2,02} = .$	$y^0_{2,02} = y_{2,02}$	.

- Columns 11 and 12 set out the potential outcomes
- Column 13 shows the treatment effect.

Econometric set-up

According to Table 1, we can write the observed outcome $y_{i,t}$ using the following identity:

$$y_{i,t} = y_{i,00} + (y_{i,01} - y_{i,00}) \cdot D_{1,it} + (y_{i,02} - y_{i,00}) \cdot D_{2,it} \tag{1}$$

Given $y^w_{i,t}$, we can define the average treatment effect conditional on t as:

$$\text{ATE}(t) = \text{E}(y^1_{i,t} - y^0_{i,t}|t) \tag{2}$$

Average treatment effect at time *t*

Potential outcome representation - 1

We assume the **potential outcomes** to take on this form ($w = 0,1$):

$$y_{i,t}^{w} = \mu_t^{w} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \varepsilon_{i,t}^{w} \qquad (3)$$

where μ_t^w is a time-varying component dependent on the treatment status, θ_i an idiosyncratic fixed–effect, λ_t a time fixed–effect, $g(\mathbf{x}_{it})$ a component depending on the characteristics of the unit i at time t, and $\varepsilon_{i,t}^w$ a random shock with finite variance and zero mean.

By taking the mean of Eq. (3), we have:

$$\mathrm{E}[y_{i,t}^{w}|w, t, \theta_i, \mathbf{x}_{it}] = \mu_t^{w} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) \qquad (4)$$

which entails that:

$$\mathrm{ATE}(t) = \mu_t^1 - \mu_t^0$$

Average treatment effect at time *t*

Potential outcome representation - 2

The potential outcomes have a similar expression as the one of the observed outcome in Eq. (1). Thus, for both the treated and untreated status, we have:

$$y^1_{i,t} = \mu^1_{00} + (\mu^1_{01} - \mu^1_{00}) \cdot D_{1,it} + (\mu^1_{02} - \mu^1_{00}) \cdot D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + e^1_{it} \tag{6}$$

$$y^0_{i,t} = \mu^0_{00} + (\mu^0_{01} - \mu^0_{00}) \cdot D_{1,it} + (\mu^0_{02} - \mu^0_{00}) \cdot D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + e^0_{it} \tag{7}$$

where $e^w_{it} = \varepsilon^w_{i,00} + (\varepsilon^w_{i,01} - \varepsilon^w_{i,00}) \cdot D_{1,it} + (\varepsilon^w_{i,02} - \varepsilon^w_{i,00}) \cdot D_{2,it}$, for $w = 0, 1$.

Using the Rubin's potential outcome identity, i.e.:

$$y_{i,t} = y^0_{i,t} + w_{i,t} \cdot (y^1_{i,t} - y^0_{i,t}) \tag{8}$$

Baseline regression

By substitution, we get:

$$y_{i,t} = \mu_{00}^{0} + (\mu_{01}^{0} - \mu_{00}^{0}) \cdot D_{1,it} + (\mu_{02}^{0} - \mu_{00}^{0}) \cdot D_{2,it} + (\mu_{00}^{1} - \mu_{00}^{0}) \cdot w_{i,t} +$$
$$w_{i,t} \cdot [(\mu_{01}^{1} - \mu_{01}^{0}) - (\mu_{00}^{1} - \mu_{00}^{0})] \cdot D_{1,it} +$$
$$w_{i,t} \cdot [(\mu_{02}^{1} - \mu_{02}^{0}) - (\mu_{00}^{1} - \mu_{00}^{0})] \cdot D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it}$$

$$\downarrow$$

$$y_{it} = \mu + \beta_1 D_{1,it} + \beta_2 D_{2,it} + \phi \cdot w_{it} + \gamma_1 w_{it} D_{1,it} + \gamma_2 w_{it} D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it}$$

Baseline **fixed-effect** regression

Recovering ATE(t) from the baseline regression

$$y_{it} = \mu + \beta_1 D_{1,it} + \beta_2 D_{2,it} + \phi \cdot w_{it} + \gamma_1 w_{it} D_{1,it} + \gamma_2 w_{it} D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it}$$

Causal effects

$$\mathrm{ATE}(00) = \phi$$

$$\mathrm{ATE}(01) = \phi + \gamma_1$$

$$\mathrm{ATE}(02) = \phi + \gamma_2$$

Generalization to (*T*+1) times

A generalization of the previous model to $(T+1)$ times, i.e. $\mathcal{T} = \{0, 1, 2, \dots, T-1, T\}$ with t^* the year of treatment is as follows:

$$y_{it} = \mu + \beta_1 D_{1,it} + \cdots + \beta_T D_{T,it} + \phi \cdot w_{it} + \gamma_1 w_{it} D_{1,it} + \cdots + \gamma_T w_{it} D_{T,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it} \quad (14)$$

with

$$\mathrm{ATE}(0) = \phi$$
$$\mathrm{ATE}(1) = \phi + \gamma_1$$
$$\dots$$
$$\mathrm{ATE}(t^*) = \phi + \gamma_{t^*}$$
$$\dots$$
$$\mathrm{ATE}(T) = \phi + \gamma_T$$

Causal effects over time

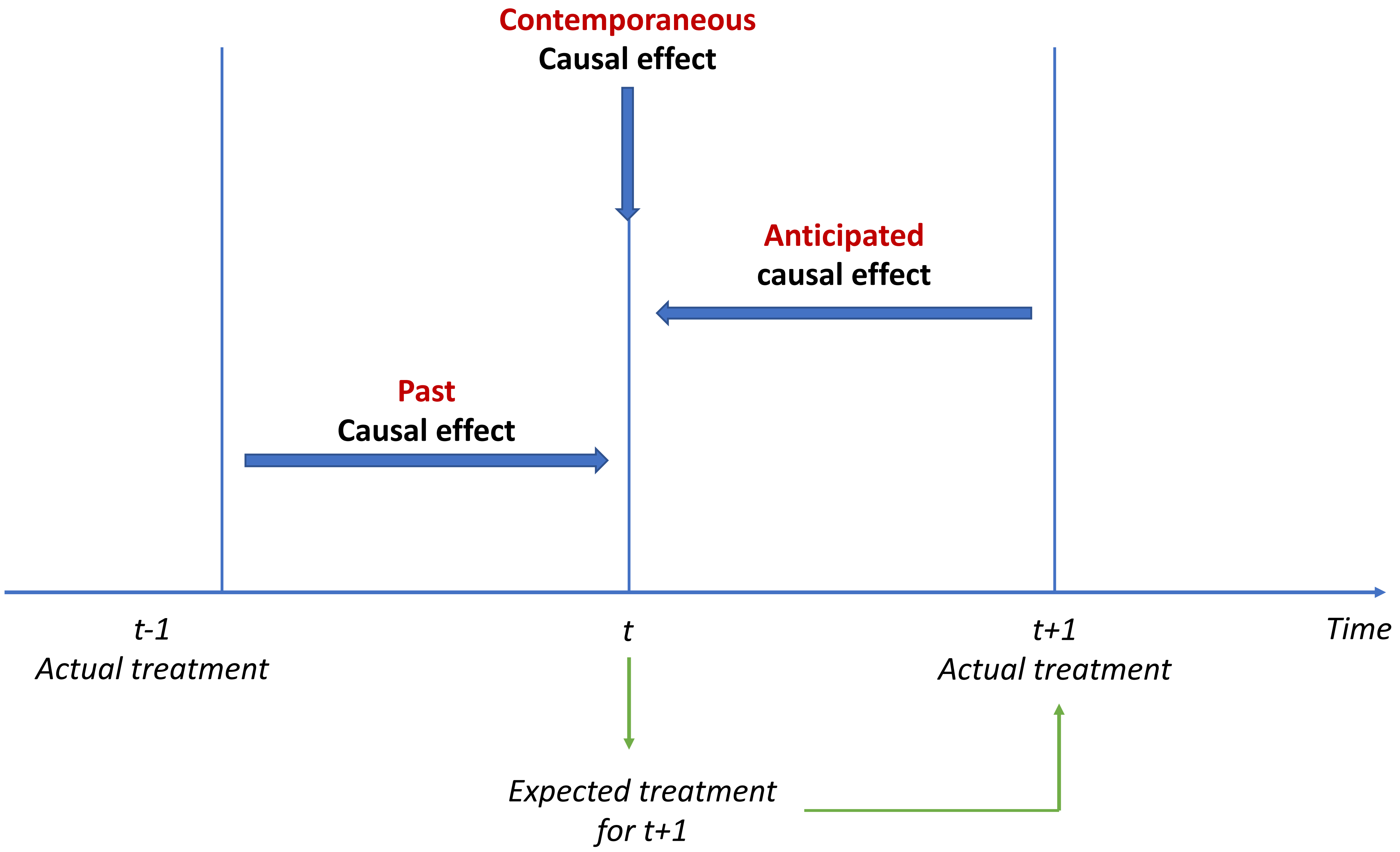
Testing the common–trend assumption

The previous model allows for a simple test for the common–trend assumption. This can be done by accepting this null:

$$H_0 : \phi = \gamma_1 = \cdots = \gamma_{(t^*-1)} = 0$$

If accepted, this test leads to accept the presence of a common-trend between treated and untreated units, although the test is valid only under no–anticipatory behaviors.

Anticipation effect



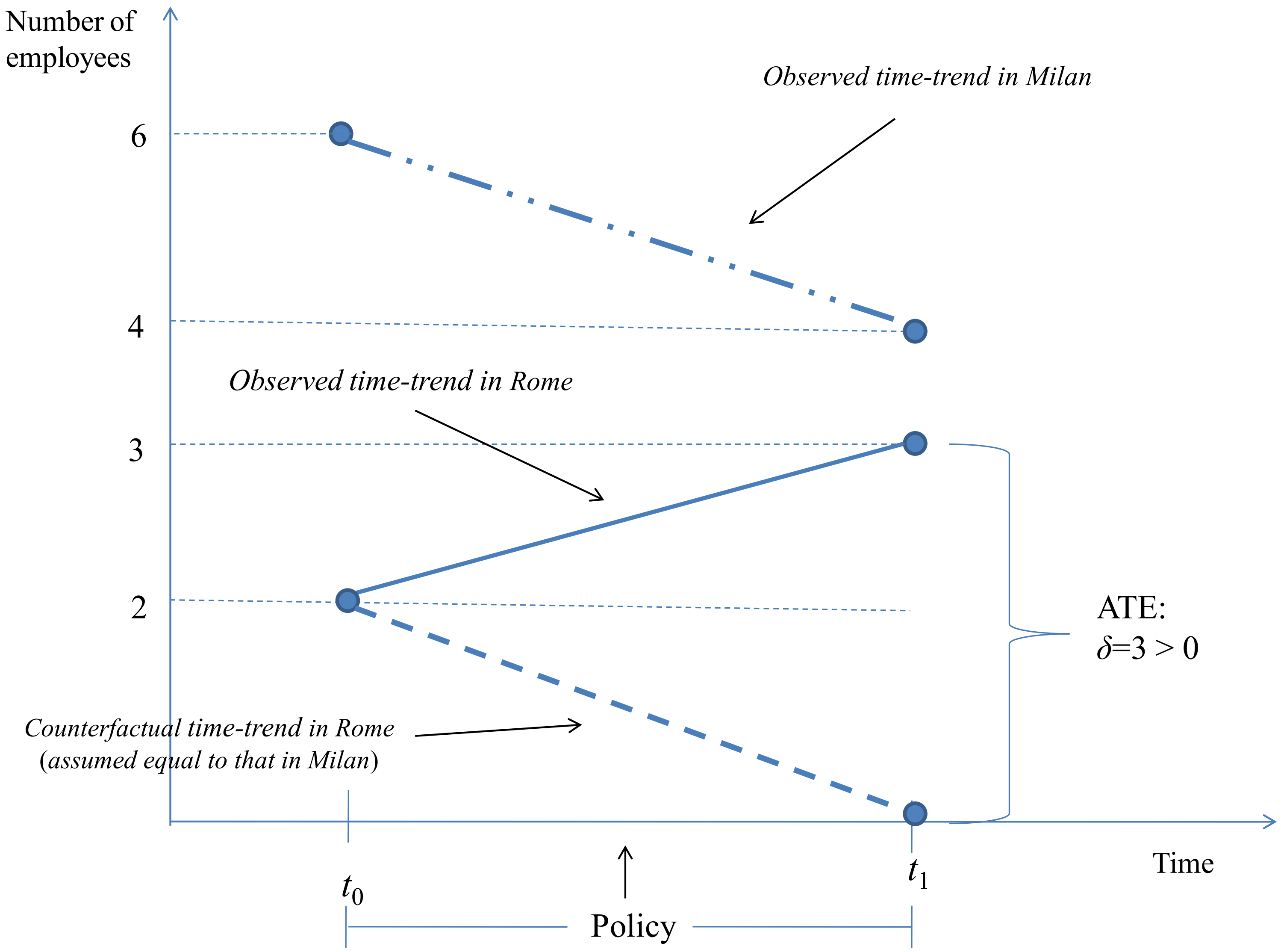
Testing the parallel-trend (or common-trend) assumption

- The **common-trend** assumption is at the basis of DID identification

- In general it is **untestable**

- If a sufficiently long times-series is available, the common-trend can be “assessed”, under **no-anticipatory effects**

Common–trend assumption: basis for DID to identify ATEs



Stata implementation via `tfdiff`

Stata syntax of `tfdiff`

`tfdiff` *outcome treatment* [*varlist*] [*if*] [*in*] [*weight*], **model**(*modeltype*)
t(*year*) [**test_tt graph save_graph**(*graphname*) **vce**(*vcetype*)]

fweights, *iweights*, and *pweights* are allowed;
where:

- *outcome*: is the target variable measuring the impact of treatment.
- *treatment*: is the binary treatment variable taking 1 for treated, and 0 for untreated units.
- *varlist*: is the set of pre-treatment (or observable confounding) variables.

Options of `tfdiff`

Options

- **model**(*modeltype*) specifies the estimation model, where modeltype must be one out of these two alternatives: "fe" (fixed effects), or "ols" (ordinary least squares). It is always required to specify one model.

- **t**(*year*) specifies the year of treatment

- **test_tt** allows for performing the parallel–trend test using the time–trend approach. The default is to use the leads.
- **graph** allows for a graphical representation of results. It uses the coefplot command implemented by Jann (2014).
- **save_graph**(*graphname*) permits to save the graph as *graphname*.
- **vce**(*vcetype*) allows for robust and clustered regression standard errors in model's estimates.

Simulation of the TFDIFF model

Time span: 2000-2020

Number of years: 21

Year of treatment: 2010

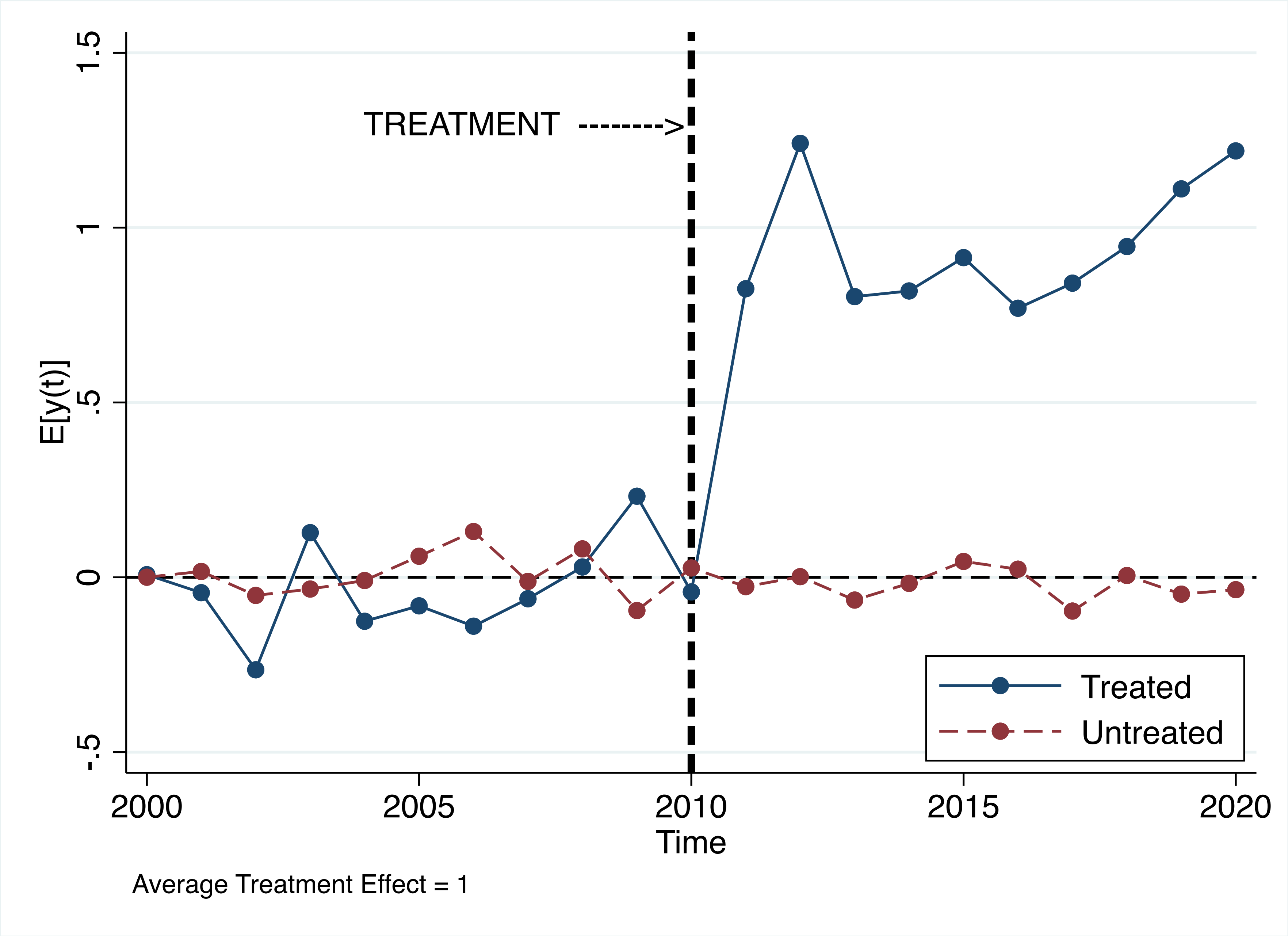
Treated units: 21 (441)

Untreated units: 79 (1,659)

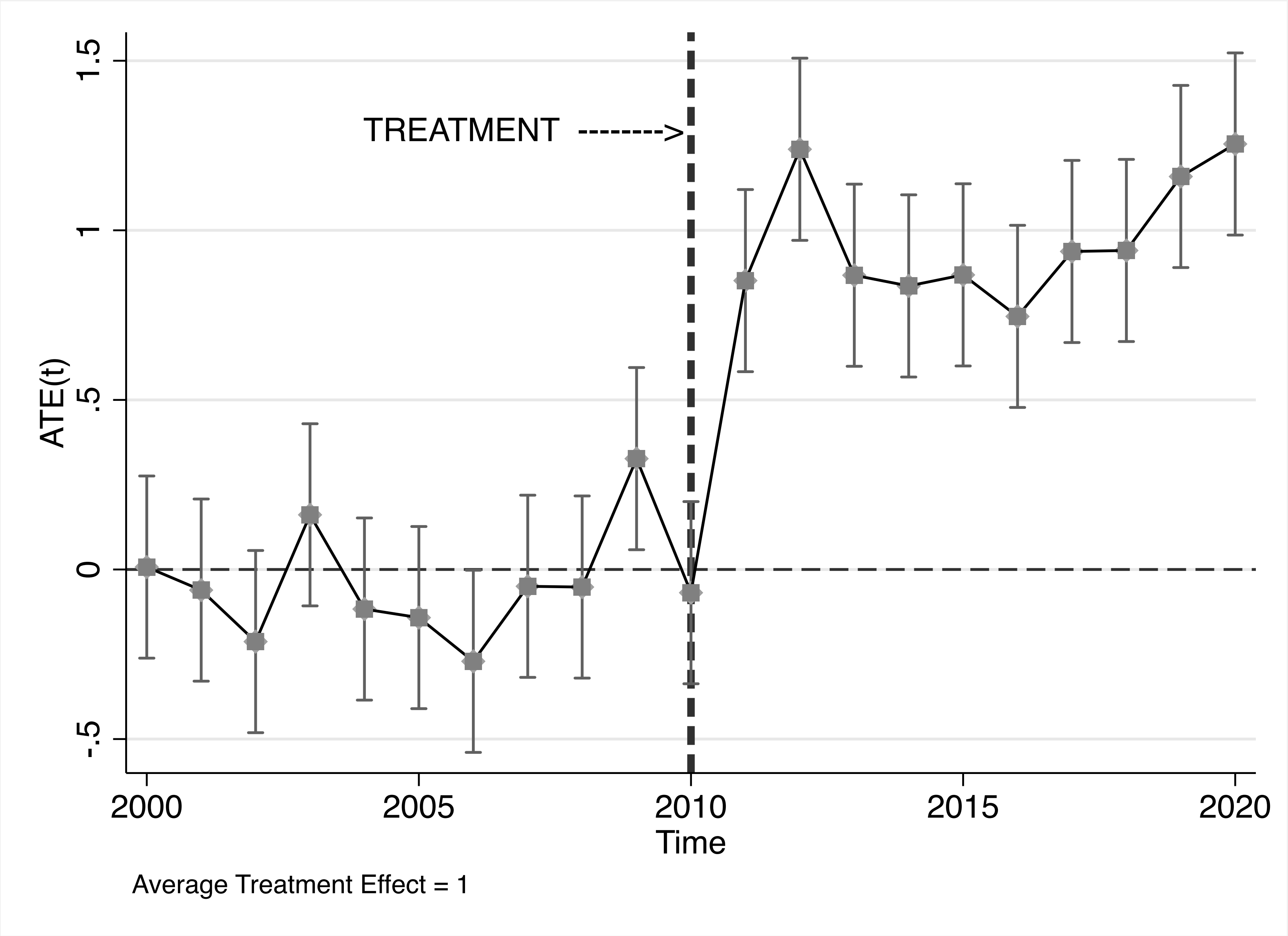
N = 100 (2,100)

Outcome: Rate of GDP growth

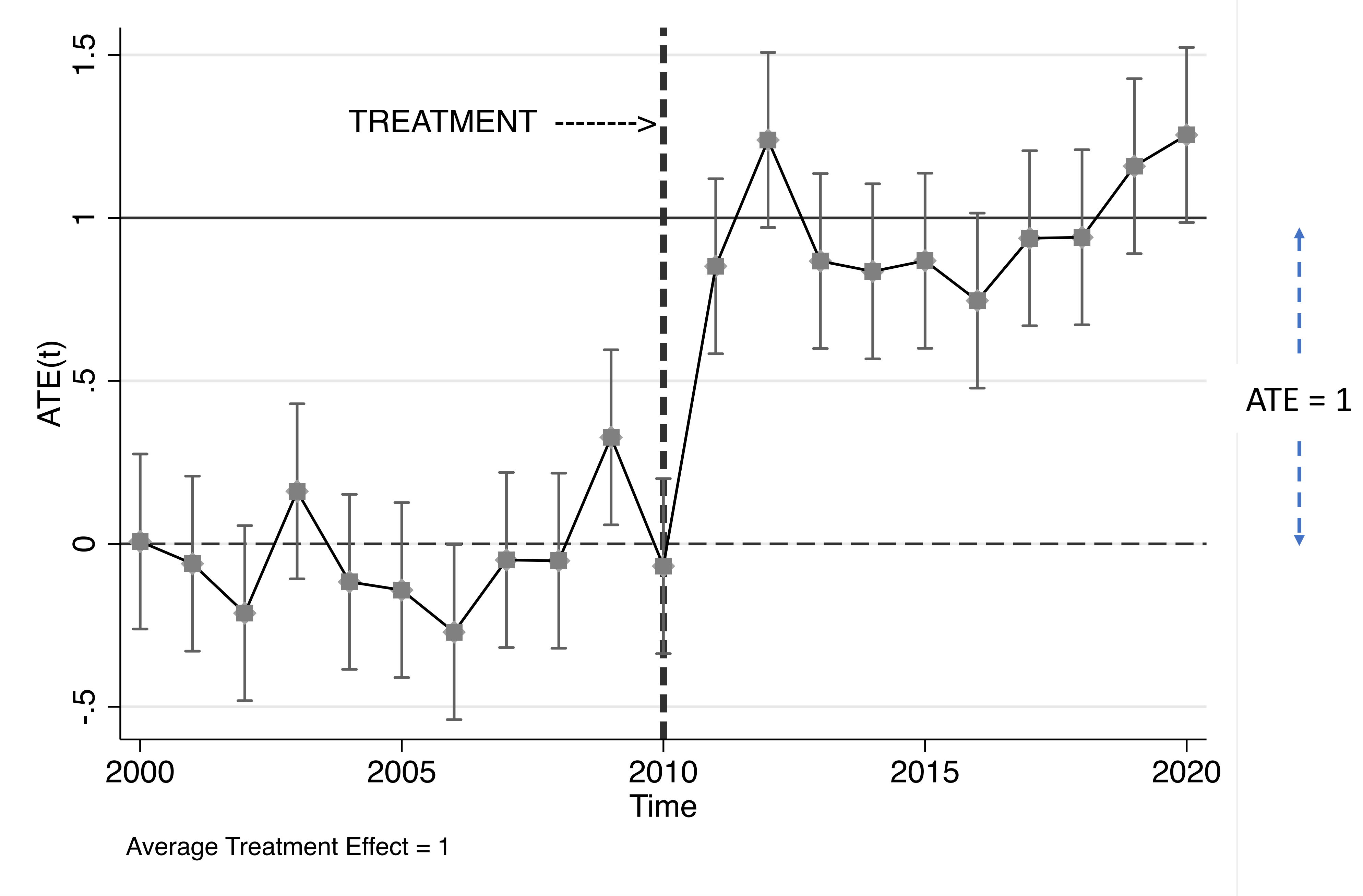
Potential Outcomes Means: estimates for ATE = 1



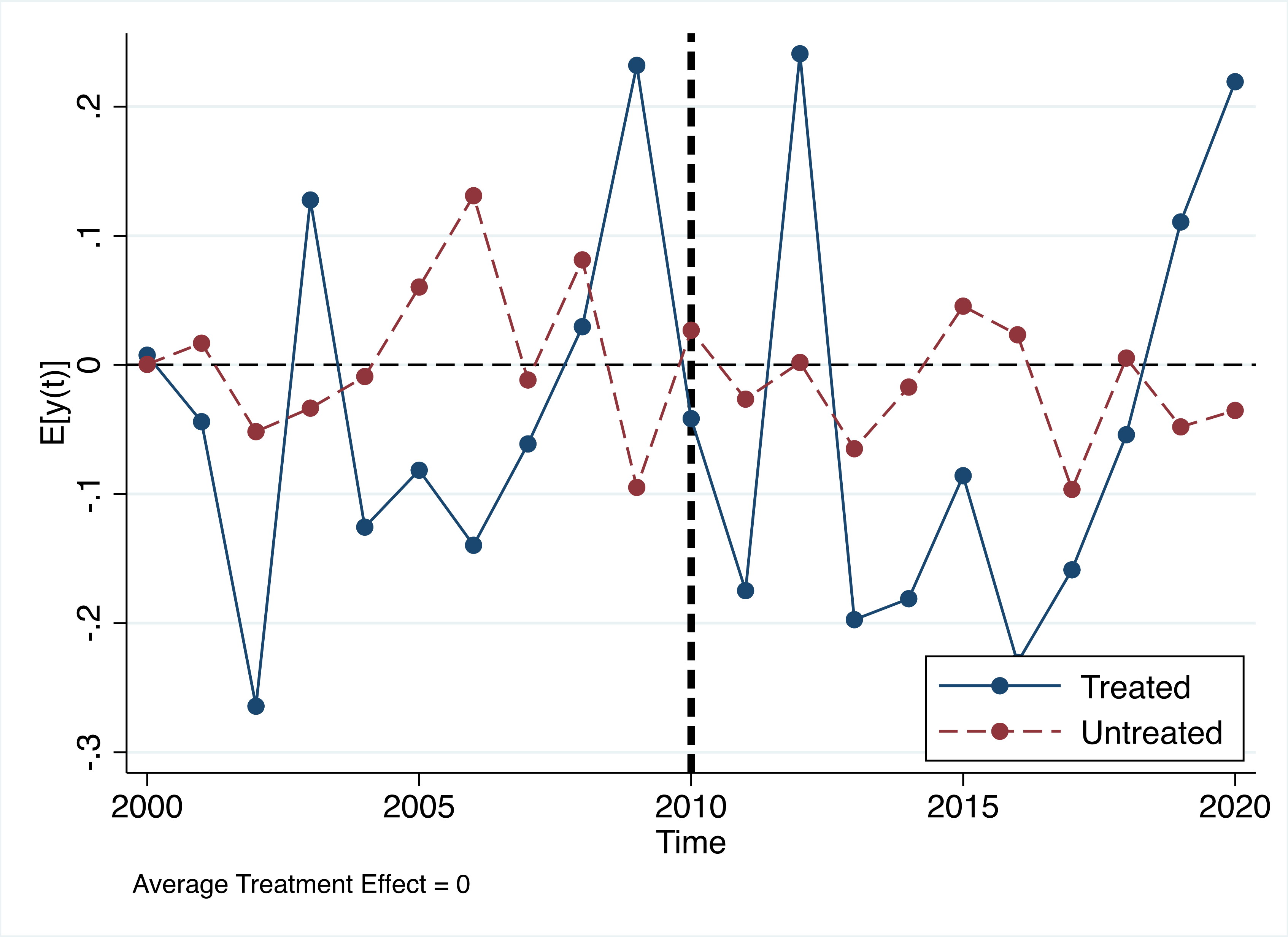
Average Treatment Effects at each *t*: estimates for ATE = 1



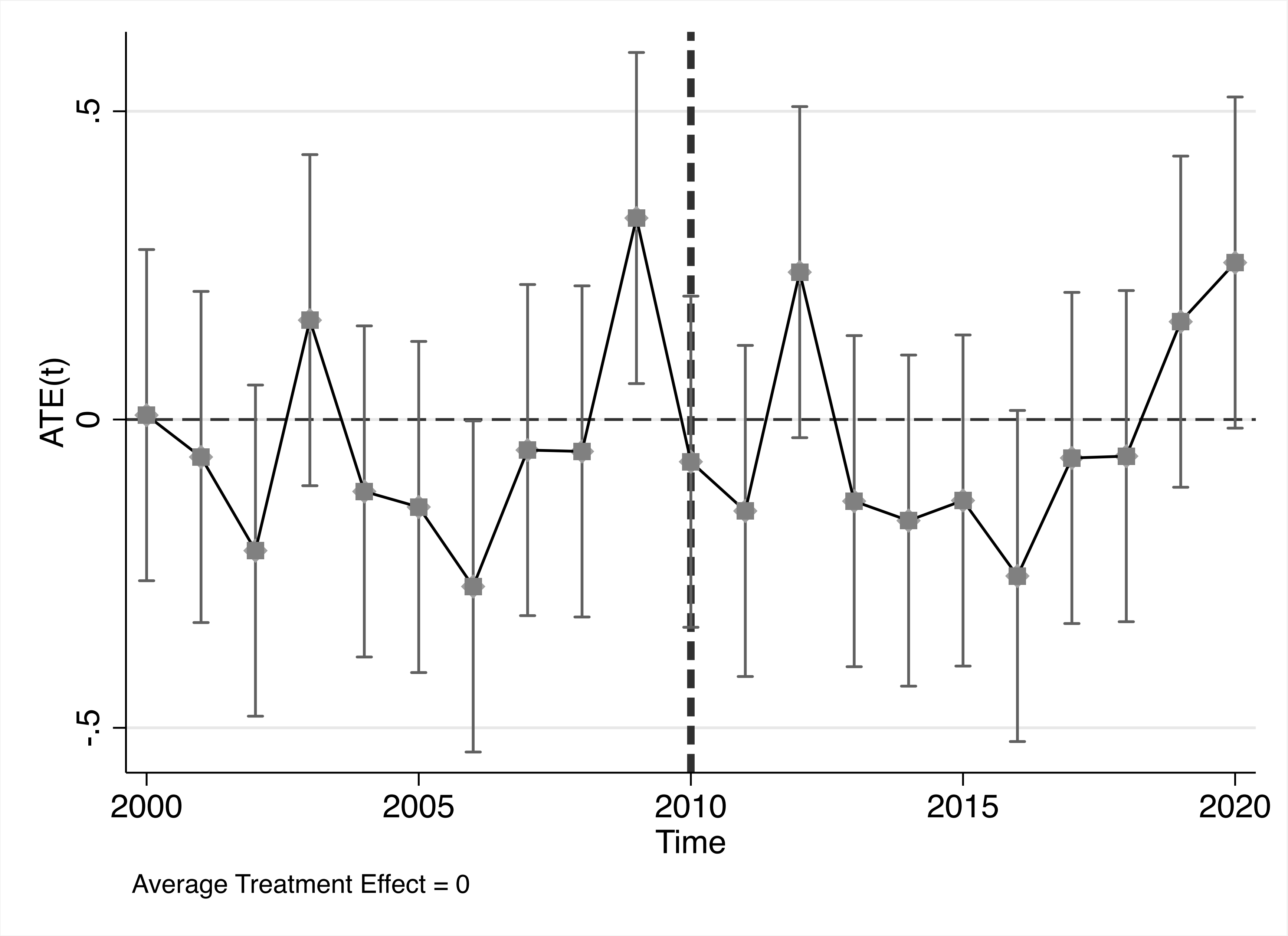
Average Treatment Effects at each *t*: estimates for ATE = 1



Potential Outcomes Means: estimates for ATE = 0 (*no-effect setting*)



Average Treatment Effects at each *t*: estimates for ATE = 0 (*no-effect setting*)



Conclusions

- The model **TFDIFF** accommodates a large set of treatment/policy situations in several fields of application

- Compared to the SCM which considers only **one treated unit**, **TFDIFF** uses **many treated units** and provides a more **robust inference** on the causal effects over time – i.e. ATE(t) - than SCM

- Under *no-anticipation*, **TFDIFF** provides a straightforward way to test the common-trend

- The Stata command I developed – **`tfdiff`** – is simple to use and provides a nice graphical representation of the results

References

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