The matching problem using Stata

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I. Motivation

- ❑ 21,167 people received a kidney in 2018 (USA), 2,855(ROK)

  ≈ **58 Kidney transplants each day**

- • 6,446 from living donor. (Some through kidney exchanges)
- ❑ Kidney waiting list: over 103,029 (23,591, ROK)
- ❑ 4,537 people died while waiting(2014)

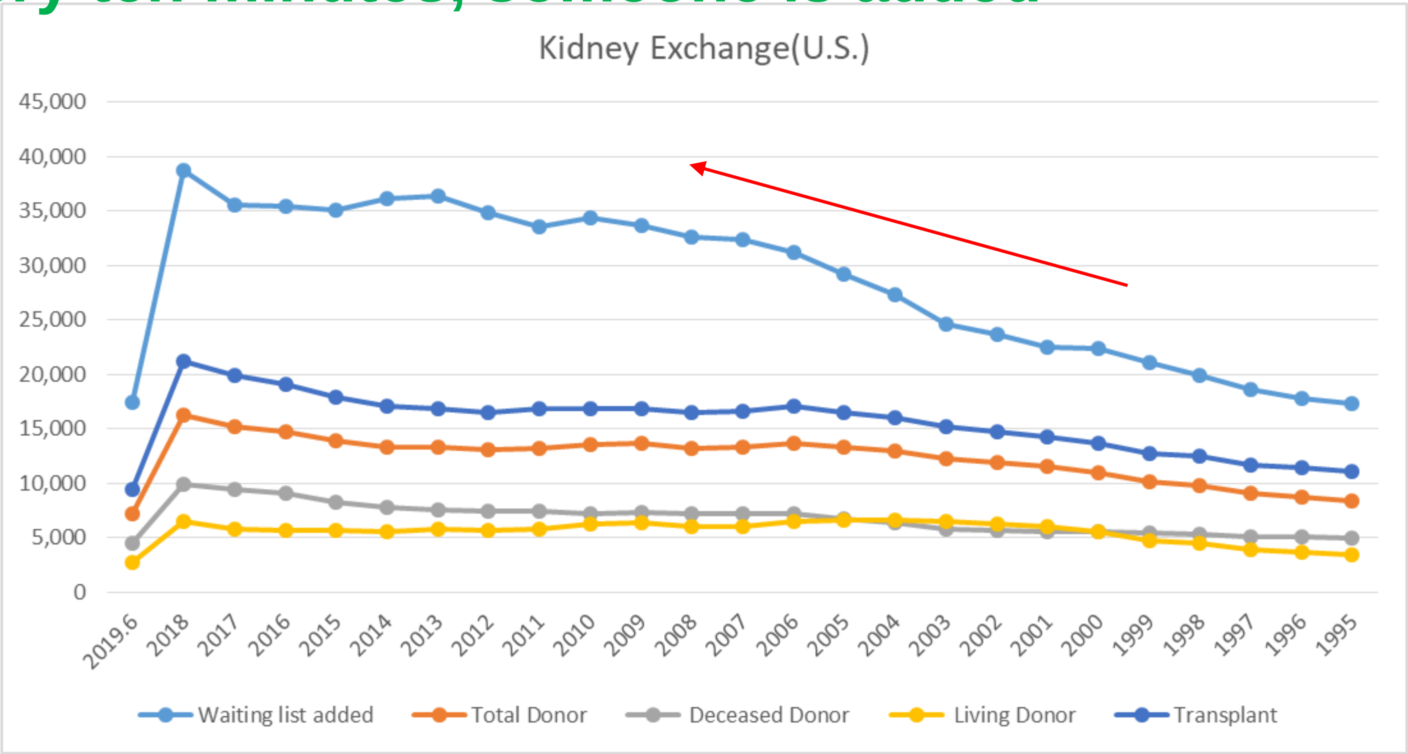
  ≈ **13 people die each day while waiting**

* Data source : Organ Procurement and Transplantation Network(OPTN)

I. Motivation

- 38,791 added to the national Kidney transplant waiting list in 2018 (17,397 added as of June 30 in 2019)

≈ **Every ten minutes, someone is added**



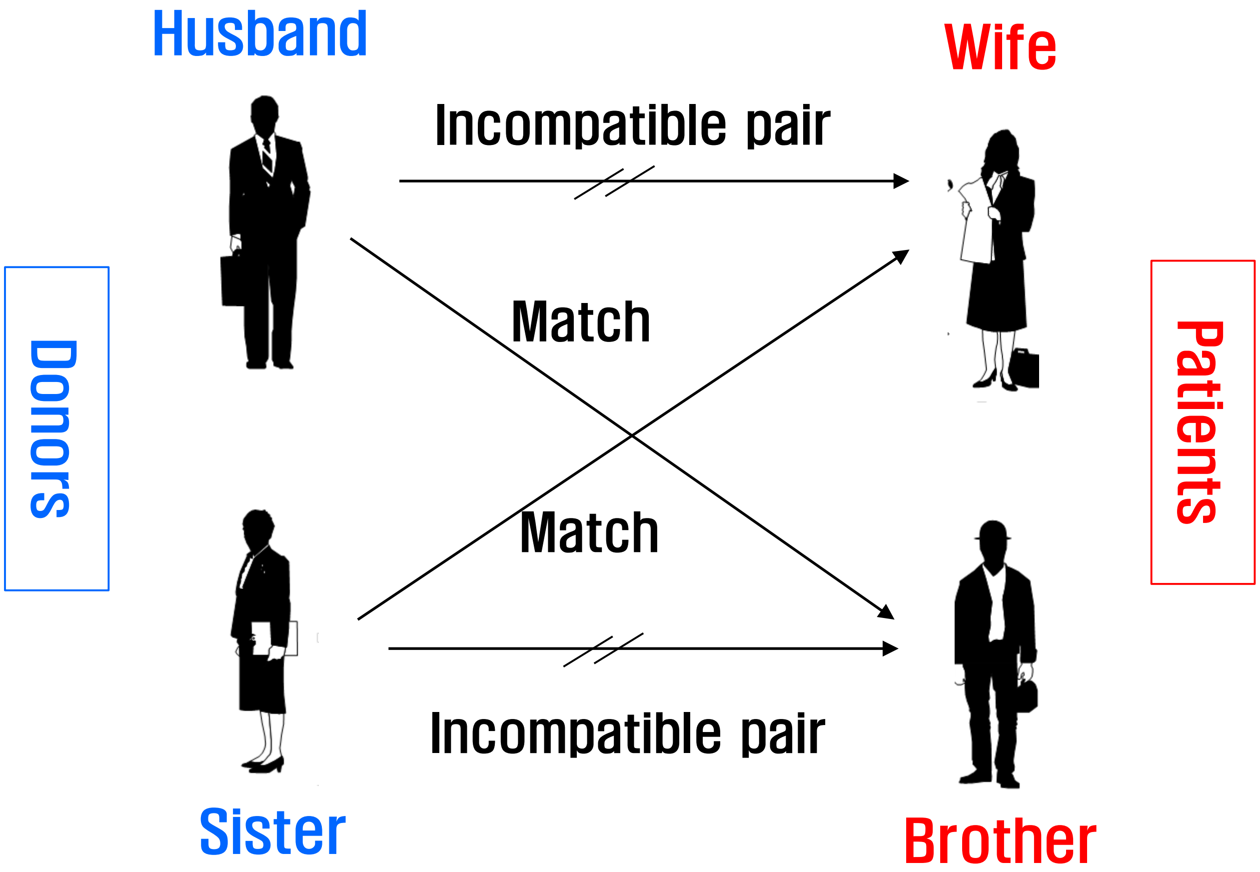
Matching organs. Saving lives.

* Data source : Organ Procurement and Transplantation Network(OPTN)

I. Motivation

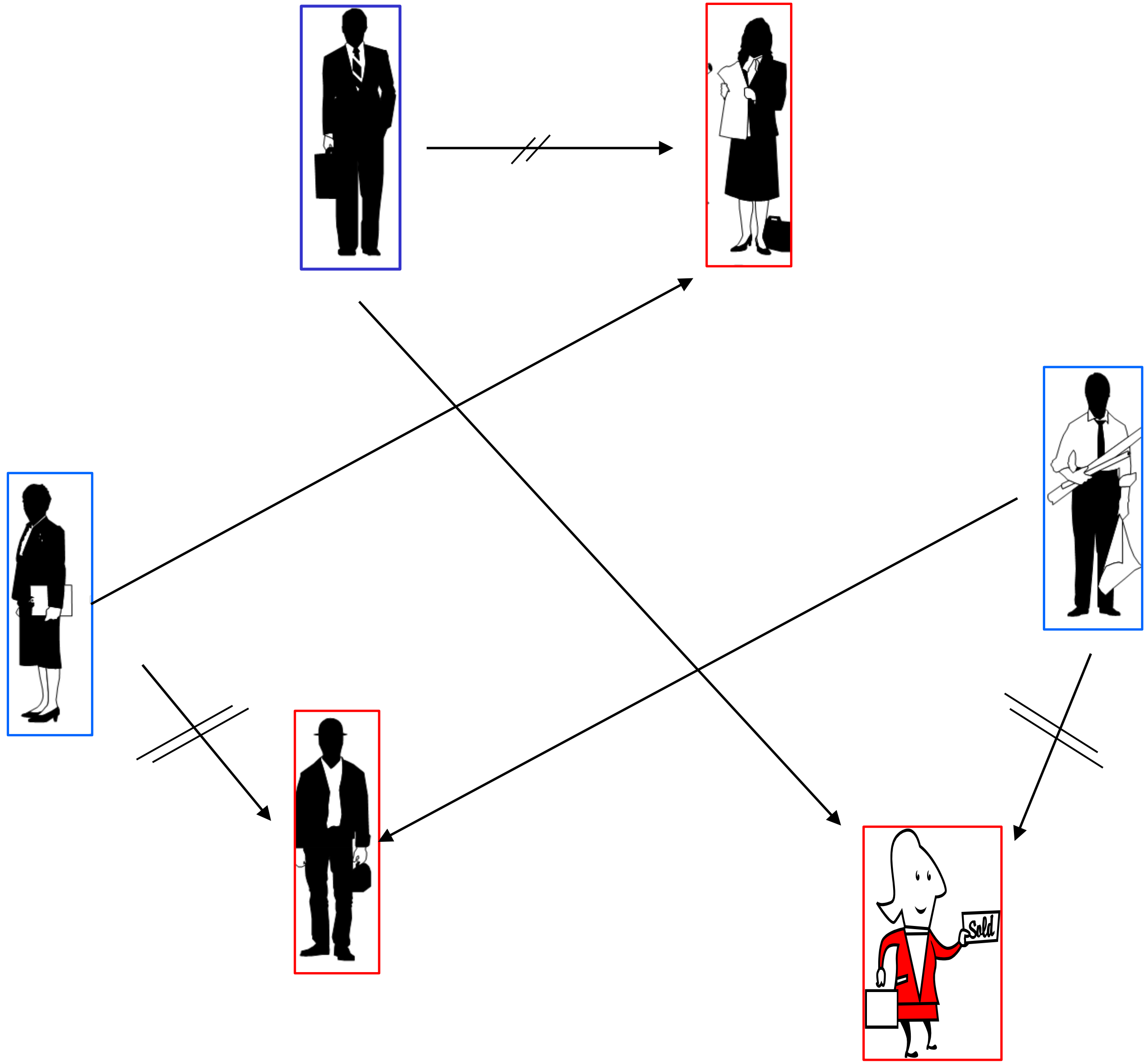
❑ Kidney Exchange (Living donor kidney matching) Types

- Two-way exchange(all surgeries must be executed simultaneously)



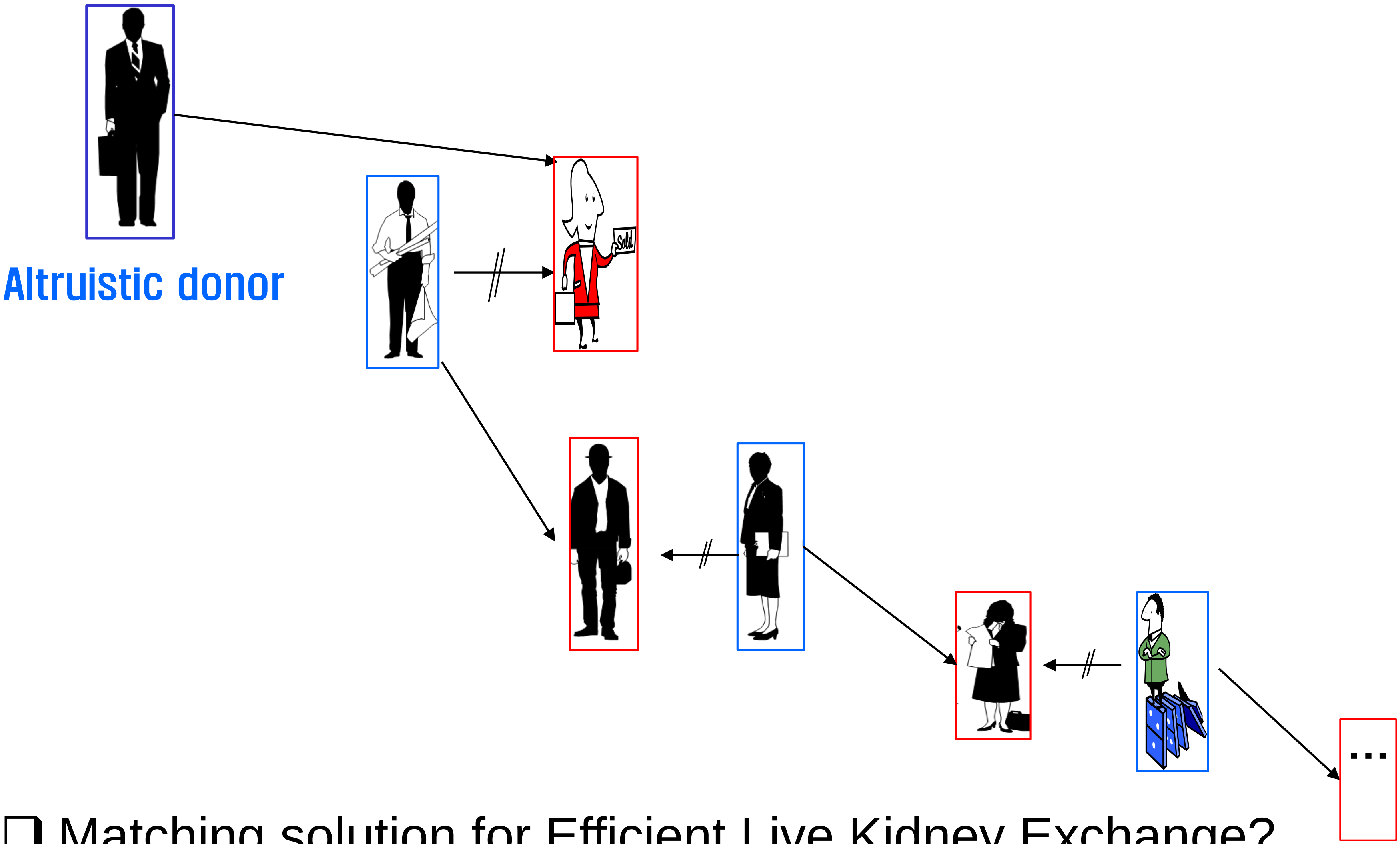
I. Motivation

- Three-way exchange(all surgeries must be executed simultaneously)



I. Motivation

- Chain(Simultaneous surgeries not required)

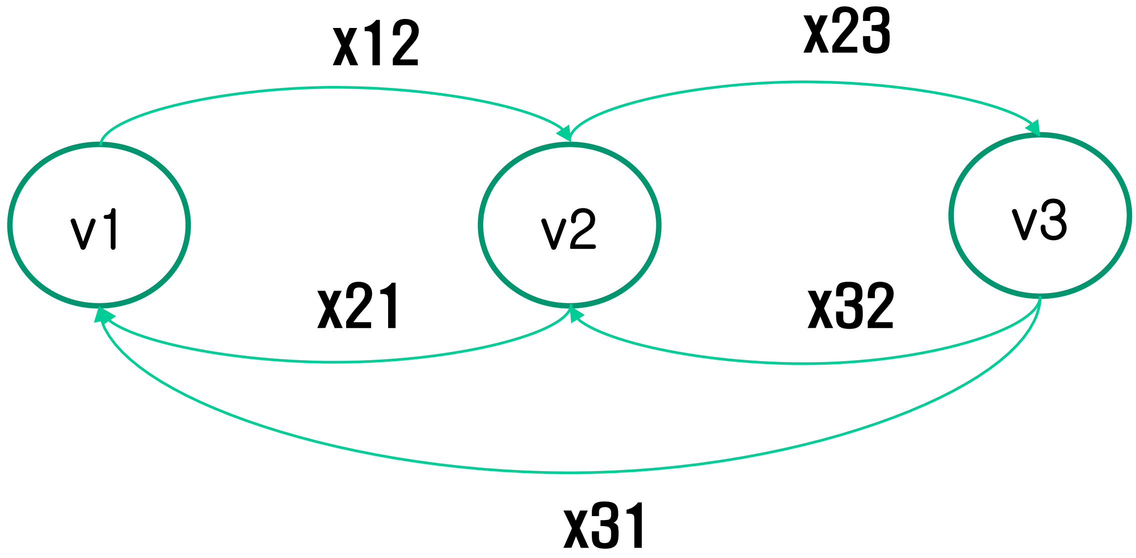


❑ Matching solution for Efficient Live Kidney Exchange?

II. Paired Live Kidney Exchange: exemplary solution

❑ Problem Considered

❍ Find maximum matching sets with certain cycle constraints(considering simultaneous surgery capacity).



- v : incompatible donor-patient pair
- xij : takes value 1 if matched and included in cycle. Otherwise 0. The same weight for xij is assumed.
- k : maximum number of cycle allowed

II. Paired Live Kidney Exchange: exemplary solution

❍ Problem Formulation

$$\text{Max} \sum_{i,j \in N} x_{i,j} \quad (1)$$

s.t.

$$\sum_{j \in N} x_{i,j} \leq 1 \quad \forall i, j \in N \quad (2)$$

$$\sum_{j \in N} x_{i,j} = \sum_{j \in N} x_{j,i} \; \forall i \in N \quad (3)$$

$$x_{i1i2} + x_{i2i3} + \cdots + x_{ikik+1} \leq k - 1 \quad (4)$$

II. Paired Live Kidney Exchange: exemplary solution

❍ Problem Arrangement

(1) $z = x_{12} + x_{21} + x_{23} + x_{32} + x_{31}$

(2)

$x_{12} \leq 1$

$x_{21} + x_{23} \leq 1$

$x_{32} + x_{31} \leq 1$

(3)

$x_{21} + x_{31} = x_{12}$

$x_{12} + x_{32} = x_{21} + x_{23}$

$x_{32} = x_{32} + x_{31}$

(4)

$x_{12} + x_{23} + x_{31} \leq 2$

$x_{12} + x_{21} \leq 2$

$x_{23} + x_{32} \leq 2$

II. Paired Live Kidney Exchange: exemplary solution

- Solution using the user written Command “lp”
  - Data Input

x12	x21	x23	x32	x31	rel	rhs
1	0	0	0	0	<=	1
0	1	1	0	0	<=	1
0	0	0	1	1	<=	1
-1	1	0	0	1	=	0
1	-1	-1	1	0	=	0
0	0	1	-1	-1	=	0
1	0	1	0	1	<=	2
1	1	0	0	0	<=	2
0	0	1	1	0	<=	2

II. Paired Live Kidney Exchange: exemplary solution

- Program Syntax

  lp varlists [if] [in] [using/] [, rel(varname) rhs(varname) min max intvars(varlist) tol1(real) tol2(real) <u>sav</u>ing(*filename*)]

  – rel(*varname*) specifies the variable with the relationship symbols. The default option is rel.
  – rhs(*varname*) specifies the variable with constants in the right hand side of equation. The default option is rhs.
  – min and max are case sensitive. min(max) is to minimize(maximize) the objective function.
  – intvars(*varlist*) specifies variables with integer value.
  – tol1(real) sets the tolerance of pivoting value. The default value is 1e-14. tol2(real) sets the tolerance of matrix inverse. The default value is 2.22e-12.

II. Paired Live Kidney Exchange: exemplary solution

❍ Result: lp with maximization option.

. lp x12 x21 x23 x32 x31,max intvars(x12 x21 x23 x32 x31) rel(rel) rhs(rhs)

Input Values:

	z	x12	x21	x23	x32	x31	s1	s2	s3	s4	s5	a1	a2	a3	rhs
r1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1
r2	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1
r3	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1
r4	0	-1	1	0	0	1	0	0	0	0	0	1	0	0	0
r5	0	1	-1	-1	1	0	0	0	0	0	0	0	1	0	0
r6	0	0	0	1	-1	-1	0	0	0	0	0	0	0	1	0
r7	0	1	0	1	0	1	0	0	1	0	0	0	0	0	2
r8	0	1	1	0	0	0	0	0	0	1	0	0	0	0	2
r9	0	0	0	1	1	0	0	0	0	0	1	0	0	0	2

LP Results: options(max)

	z	x12	x21	x23	x32	x31	s1	s2	s3	s4	s5
opt_val	1	1	1	0	0	0	0	1	1	0	2

- The solution maximizes the total number of transplants performed. Two way matching solutions are possible and (x12, x21) is one of the solution.
- Different weights for xij can result different solutions.

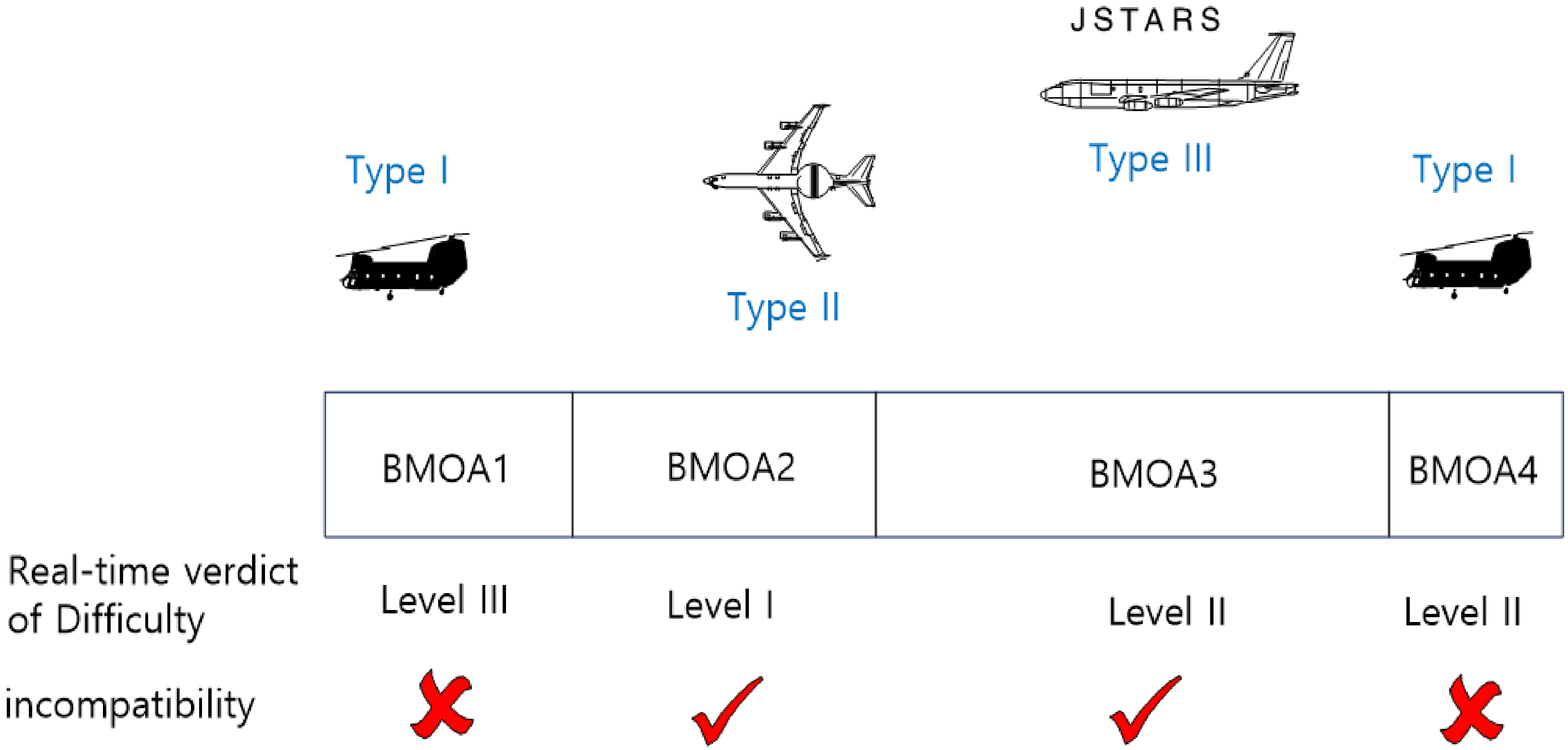
III. Remarks

- Remarks
  - Attempt for matching problem to determine the efficient live kidney matching set is valuable and the following information are generally required.
    - a list of altruistic donators
    - a list of patient–donor pairs
    - the compatibility information between all donors and patients
    - the “weight,” or priority, of each potential transplant, and
    - a bound on the maximum cycle length.

III. Remarks

❑ Remarks

❍ Real-time matching of target with assets?



❍ Some theoretical topics of matching problem in the reference.

References

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Thank You !