netivreg: Estimation of Peer Effects in Endogenous Social Networks

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Motivation

- Estimation of network effects is becoming increasingly common.
  - Interest on structural coefficients: endogenous peer effects and contextual effects
  - Estimate treatment effects and spillovers under interference.
- Exogenous network formation is a commonly used assumption in empirical work.
- Recent methods allowing for the presence of network endogeneity require explicit structural restrictions on the network formation process.

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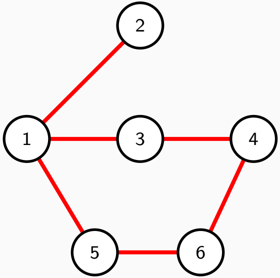
- Recent methods allowing for the presence of network endogeneity require explicit structural restrictions on the network formation process.
- **Research Question:** can the *multiplex network data structure* help with the treatment of identification issues?

Preview of the Results

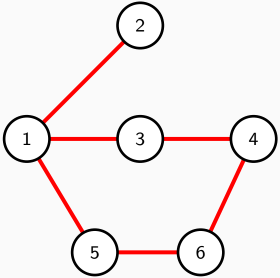
- Propose novel instruments based on the topology of multiplex networks to construct the estimator.
- Provide new identification results for peer/contextual effects that generalize existing methods by accounting for potential endogenous network formation.
- Computationally easy to implement estimator that is consistent and asymptotically normal.
- Stata implementation: `netivreg`.

Framework



- Contextual Effects (interference): i's outcome depends on the characteristics of other units.
- Endogenous Peer Effects (multiplier).

$$y_i = \alpha + \beta \sum_{i \neq j} W_{i,j} y_j + \delta \sum_{i \neq j} W_{i,j} x_j + \gamma x_i + \varepsilon_i.$$



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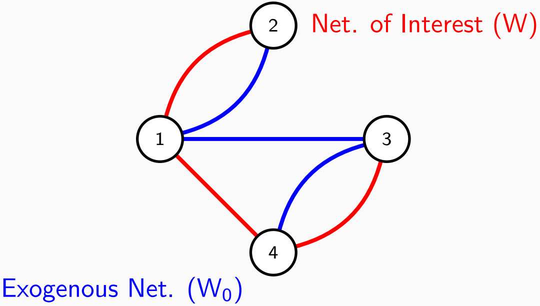
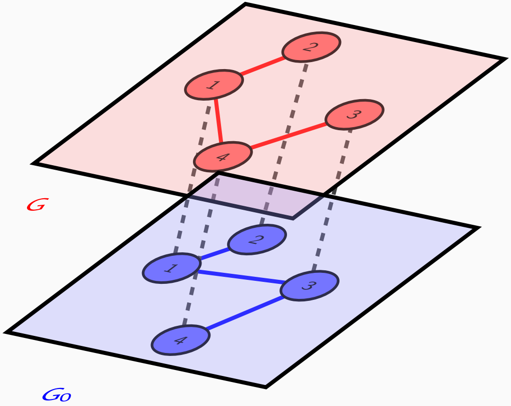
Objective: identify and consistently estimate the parameters $(\alpha, \beta, \gamma, \delta)$.

Main Challenges: Observational Data

- Simultaneity of the peer effects regressors (reflection problem)
- The decision of forming a peer connection can be correlated with unobserved characteristics or there could exists common shocks (correlated effects)
- The network structure could induce correlation between X and ε (unobserved homophily)

$$y = \alpha^0 \boldsymbol{\iota} + \beta^0 Wy + \delta^0 WX\boldsymbol{\delta}^0 + X\boldsymbol{\gamma}^0 + \varepsilon, \text{ with } \mathbb{E}[\varepsilon \mid W, X] \neq 0 \text{ and } \mathbb{E}[\varepsilon \mid W_0, X] = 0.$$





Ideal Experiment

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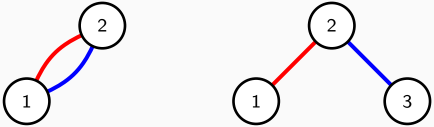
- Only the fact that two individuals share a classrooms does not necessarily generate social effects.
- It is possible to observe a relevant network (for example friendship) defining W.
- This method can be used to causally estimate network friendship effects.

Main Identifying Assumptions

1. Monolayer Linear model and Bi-layer multiplex network data $\mathcal{M} = 2$ (W and W_0).
2. Conditional distribution $\mathcal{F}(\varepsilon \mid X, \mathcal{M})$ is such that $\mathbb{E}[\varepsilon|W, X] \neq 0$ and $\mathbb{E}[\varepsilon|W_0, X] = 0$.
3. The networks generating the adjacency matrices W and W_0 are correlated in the sense that it is possible to find connections in common ($E_0 \cap E_1 \neq 0$) and distance two paths that change edge type ($(i, j) \in E_0$ and $(j, k) \in E_1$).



Identification

Let $\mathbf{\Pi}$ be the projection coefficients from a regression of WS on $\mathsf{W}_0\mathsf{S}$, where $\mathsf{S} = [\mathsf{y} \quad \mathsf{X}]$.

<u>Theorem:</u>

Let Assumptions 1, 2, 3, and $\gamma^0(\pi_{11}\beta^0 + \pi_{12}\delta^0) + \pi_{21}\beta^0 + \pi_{22}\delta^0 \neq 0$ hold. If the matrices I, W_0, $\mathsf{W}_0{}^2$ are linearly independent, then the parameters $\alpha^0, \beta^0, \gamma^0$ and δ^0 are identified.

Remark

Note that this is a generalization of the identification result in Proposition 1 of Bramoullé et al. (2009, JoE), i.e., if $\mathsf{W}_0 = \mathsf{W}$, one has $\mathbf{\Pi} = \mathsf{I}$, and the condition reduces to $\gamma^0\beta^0 + \delta^0 \neq 0$ and the matrices I, W and W^2 being linearly independent.

Rank Condition

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$$y = \alpha^0 \boldsymbol{\iota} + \mathrm{W}\mathrm{S}\boldsymbol{\theta}^0 + \mathrm{X}\boldsymbol{\gamma}^0 + \boldsymbol{\varepsilon} \quad \text{for} \quad \mathrm{S} = [\mathrm{y} \quad \mathrm{X}] \quad \text{and} \quad \boldsymbol{\theta}^0 = [\beta^0 \quad \boldsymbol{\delta}^0]$$
$$y = \alpha^0 \boldsymbol{\iota} + \mathrm{W}_0\mathrm{S}\boldsymbol{\theta}^* + \mathrm{X}\boldsymbol{\gamma}^0 + \mathrm{e}, \quad \text{for} \quad \boldsymbol{\theta}^* = \Pi\boldsymbol{\theta}^0.$$

Estimation Procedure

1. Estimate Π by OLS (WS on $\mathrm{W}_0\mathrm{S}$).
2. 2SLS of $[\boldsymbol{\iota}, \mathrm{X}, \mathrm{W}_0\mathrm{y}, \mathrm{W}_0\mathrm{X}]$ with instrument $\mathrm{Z} = \left[\boldsymbol{\iota}, \mathrm{X}, \mathrm{W}_0{}^2\mathrm{X}, \mathrm{W}_0\mathrm{X}\right]$. Calculate $\widehat{\boldsymbol{\theta}} = \widehat{\Pi}^{-1}\widehat{\boldsymbol{\theta}}^*$.
3. IV of $\left[\boldsymbol{\iota}, \mathrm{X}, \widehat{\mathrm{W}\mathrm{y}}, \widehat{\mathrm{W}\mathrm{X}}\right]$ with instruments $\widehat{\mathrm{Z}}^* = \left[\boldsymbol{\iota}, \mathrm{X}, \left[E\left(\mathrm{W}_0\mathrm{y}|\mathrm{X}, \mathrm{W}_0\right), \mathrm{W}_0\mathrm{X}\right]\widehat{\Pi}\right]$.

Estimator and Properties

$$\widehat{\psi}_{G3SLS} = \left(\widehat{\mathrm{Z}}^{*\top}\widehat{\mathrm{D}}\right)^{-1}\widehat{\mathrm{Z}}^{*\top}\mathrm{y},$$

$$\sqrt{n}(\widehat{\psi}_{G3SLS} - \psi) \xrightarrow{d} N(0, \mathrm{V}_{\psi})$$

Stata Implementation

Empirical Application: Specification

W: Coauthors - W_0: Alumni

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W: Coauthors - W_0: Alumni

$$y_{i,r,t} = \alpha + \beta \sum_{j \neq i} w_{\ell;i,j,t} y_{j,r,t} + \sum_{j \neq i} w_{\ell;i,j,t} \widetilde{x}_{j,r,t}^{\top} \delta + x_{\ell;i,r,t}^{\top} \gamma + \lambda_r + \lambda_t + \lambda_0 + \varepsilon_{i,r,t}$$

Peer Effects (β)

log(# Citations)

Contextual Effects (δ)

Editor

Different Gender

Direct Effects (γ)

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Authors

Pages

References

Fixed Effects (λs)

Journal

Year

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Empirical Application: Specification

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```
netivreg lcitations editor diff_gender n_pages n_authors n_references isolated
(edges = edges0), wx(diff_gender editor) cluster(c_coauthor) first second
```

Data Structure

Stata 16 Capabilities: (1) Python Integration for Sparse Matrices and (2) Multiframes

W (Coauthors)

	source	target
4	5	478
5	5	665
6	5	705
7	8	113
8	8	133
9	8	177
10	8	189
11	8	639
12	8	658
13	10	356
14	10	527
15	11	26
16	11	639
17	13	213
18	13	428

W_0 (Alumni)

	source	target
4	4	136
5	4	407
6	5	10
7	5	95
8	5	97
9	5	130
10	5	144
11	5	152
12	5	161
13	5	194
14	5	301
15	5	324
16	5	357
17	5	383
18	5	416

(y, X)

	id	lcitations	editor	diff_gender	isolated	n_pages	n_authors	n_references	journal	year
4	21	2.302585	0	0	1	15	2	27	aer	2000
5	31	3.806663	0	0	1	21	1	39	aer	2000
6	38	3.555348	0	0	1	17	2	31	aer	2000
7	51	3.583519	0	0	1	20	1	48	aer	2000
8	59	3.988984	0	0	1	17	2	50	aer	2000
9	68	2.197225	0	0	1	15	2	31	aer	2000
10	76	2.197225	0	0	0	11	2	18	aer	2000
11	86	3.218876	0	0	0	24	1	32	aer	2000
12	96	4.836282	0	0	0	24	2	57	aer	2000
13	105	4.691348	0	0	1	25	2	40	aer	2000
14	122	4.770685	0	0	1	30	1	30	aer	2000
15	139	3.850147	0	0	1	16	1	49	aer	2000
16	144	2.564949	0	0	0	21	2	16	aer	2000
17	151	4.26268	1	0	0	21	3	36	aer	2000
18	162	2.890372	0	0	1	26	1	26	aer	2000

Step1: Projection Coefficients

$$WS = W_0 S\Pi + U,$$

Projection of W on W0

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
W_lcitations						
W0_lcitations	.4956186	.0321772	15.40	0.000	.4324379	.5587993
W0_diff_gender	.0127121	.5132519	0.02	0.980	-.9950719	1.020496
W0_editor	.0085967	.7897166	0.01	0.991	-1.542033	1.559227
W_diff_gender						
W0_lcitations	.137265	.0033955	40.43	0.000	.1305979	.1439321
W0_diff_gender	.1422822	.0541602	2.63	0.009	.0359371	.2486273
W0_editor	.0325262	.0833338	0.39	0.696	-.131102	.1961544
W_editor						
W0_lcitations	.4249148	.0025367	167.51	0.000	.419934	.4298957
W0_diff_gender	.1027705	.0404624	2.54	0.011	.0233214	.1822195
W0_editor	.1367464	.0622576	2.20	0.028	.0145019	.2589909

Step2: 2SLS estimation

2SLS of $[\iota, X, W_0 y, W_0 X]$ with instrument $Z = [\iota, X, W_0{}^2 X, W_0 X]$

2SLS Regression

Number of obs =	729
Wald chi2(62) =	-1.1e+17
Prob > chi2 =	1.0000
R-squared =	0.1317
Root MSE =	1.846

lcitations	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
W_y						
lcitations	.9496092	.5481734	1.73	0.084	-.126744	2.025962
X						
diff_gender	.2224841	.1317096	1.69	0.092	-.0361313	.4810994
editor	.1691513	.1181452	1.43	0.153	-.06283	.4011327
n_pages	.0282953	.0048171	5.87	0.000	.0188369	.0377538
n_authors	.0747385	.0603238	1.24	0.216	-.043709	.1931859
n_references	.0119404	.0025597	4.66	0.000	.0069143	.0169665
isolated	-.2131575	.0942419	-2.26	0.024	-.3982041	-.0281109

Step3: Optimal Instrument

IV of $\left[\iota, X, \widehat{Wy}, \widehat{WX}\right]$ with instruments $\widehat{Z}^* = \left[\iota, X, [E(W_0 y | X, W_0), W_0 X]\, \widehat{\Pi}\right]$

```
Network IV Regression                                  Number of obs  =        729
Number of clusters (c_coauthor) =     575             Wald chi2(62)  =  6.5e+16
                                                      Prob > chi2    =   0.0000
                                                      R-squared      =   0.1723
                                                      Root MSE       =    1.339
                             (Std. Err. adjusted for 575 clusters in c_coauthor)
```

lcitations	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
W_y						
lcitations	.5200772	.3616317	1.44	0.151	-.1899963	1.230151
X						
diff_gender	.218709	.1305651	1.68	0.094	-.0376592	.4750771
editor	.1733642	.1157379	1.50	0.135	-.0538902	.4006187
n_pages	.0288947	.0044187	6.54	0.000	.0202184	.0375709
n_authors	.0719403	.0597035	1.20	0.229	-.0452891	.1891696
n_references	.0119892	.0025599	4.68	0.000	.0069628	.0170156
isolated	-.2230689	.0897056	-2.49	0.013	-.3992083	-.0469295

Conclusion

- Identification of a linear-in-means model with endogenous network.
- Computationally simple estimation that uses two-layered multiplex network structure with Stata implementation.
- Robust to different types of network endogeneity. It does not require to model unobserved heterogeneity and network formation.

Appendix

Distortions Induced by Social Effects: Consumption Examples

- If individuals care about **status** (conspicuous consumption models), the proportion of conspicuous consumption may increase with respect to other goods.
- If conspicuous consumption is considered wasteful, peer effects might have noticeable welfare consequences.
- Savings may differ from the optimal in an attempt to keeping up with the peers.

▸ Empirical Work

Aggregate Effects: Consumption Example

- Unanticipated tax changes to the rich might have aggregate consequences.
- If individuals who are not affected by the shock change their consumption after observing changes in consumption of the rich, the shock can spread through the network.
- Social multipliers depend on the size of the endogenous peer effects and the connectedness of the affected groups.

▸ Empirical Work

Angrist's (2014) Critique: Group Regressions

- **Reflection Problem:** a regression of individual outcomes on <u>group</u> mean outcomes is tautological.

- **Correlated Effects:** even the leave-one-out estimator does not provide information of human behavior. "Like students in the same school, households from the same village are similar in many ways".

- **Mechanical Relationship:** the coefficient on group averages in a multivariate model of endogenous peer effects does not reveal the action of social forces. He interprets the vale $1/(1-\beta)$ as approximately the ratio of the 2SLS to OLS estimands for the effect of individual covariates on outcomes (using dummy groups as instruments).

▸ Motivation

Angrist's (2014) Critique: Network Regressions

- Start by a saturated model $E[y_i \mid x_i] = \gamma_0 + \gamma_1 x_i$ satisfying $E[u_i \mid x_i] = 0$, for $u_i \equiv y_i - \gamma_0 - \gamma x_i$.

- Individuals are ordered from left to right. Each person i is connected only with the individual to her left $i-1$. Friends are only similar on unobservables: $u_i = \beta u_{i-1} + \varepsilon_i$.

- The outcome can be written in a linear-in-means (lmm) model form:

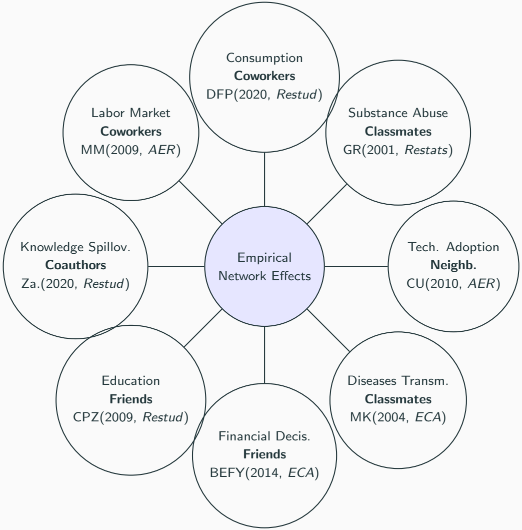
$$y_i = \gamma_0(1-\beta) + \beta y_{i-1} + \gamma x - \beta\gamma x_{i-1} + \varepsilon_i$$

- **Flaw in Angrist's example:** let $\delta = -\beta\gamma$ to write this model exactly as a lmm. Note that $\delta + \gamma\beta = 0$ so that the outcome equation can be written as (for $\alpha = \gamma_0(1-\beta)$)

$$y_i = \frac{\alpha}{1-\beta} + \gamma x_i + v_i$$

Motivation

Different Network Effects



▸ Motivation

Critique to Randomly Assigned Groups

- In principle, randomization of peers would guarantee identification in a monolayer linear in means model where endogenous network formation is ruled out.
- It can completely eliminate the problem of unobserved common variables.
- However, if individuals endogenously form groups (homophily), there can be a subsequent resorting. If resorting happens faster than the effects of social interactions, identification is not possible.
- Even with random peers, researchers face a classical problem of **omitted variables** when trying to estimate contextual effects ($\mathbb{E}[x_i \varepsilon_j \mid w_{i,j} = 1] \neq 0$).

▸ Literature

Multilayers Networks in Economics

Labor Supply

- Sisters, Cousins and Neighbors networks (NST (2018, *AEJ*))

Consumption

- Coworker and Spouses networks (DFP (2020, *Restud*))

Education

- Friendship network in t and $t-1$ (GI (2013, *JBES*))
- Roommates, classmates, Study-mate, Friendship networks (CL (2015))
- Siblings and Classmates networks (NR (2017, *JAE*))

Publication Outcomes

- Coauthors, Alumni and Same Advisor networks (EHJS (2020))

▸ Multilayer

Microfoundations

- The monolayer linear model of interest corresponds with the best response of a Bayesian Game of Social Interactions as proposed by Blume, Brock, Durlauf and Jayaraman (2015, JPE).

- Quadratic utility with social pressure or strategic complementarities

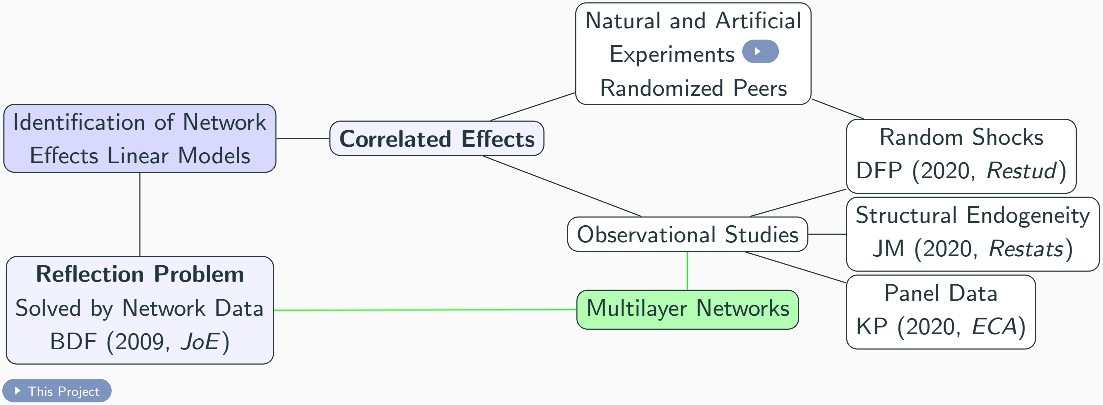
$$U_i\left(\omega_i, \omega_{-i}\right) = \left(\gamma x_i + z_i + \delta \sum_j c_{ij} x_j\right)\omega_i - \frac{1}{2}\omega_i^2 - \frac{\phi}{2}\left(\omega_i - \sum_j a_{ij}\omega_j\right)^2$$

- In their model endogeneity arises because an individual i, observing that he is connected to j, make an inference about the value of z_j that is dependent on x_j. Then, x_j will be correlated with ε_i in my equation of interest.

- Their critique of instrumental variable is that if individual i observe the instruments v_j, he can use it to predict z_j which will induce correlation between ε_i and the instrument.

- Our instrument is based on x_r of individuals r connected to i in a network that is independent of the individuals' utilities. Therefore x_r is not useful to predict z_j. Model

Positioning the Research Agenda in the Literature



Assumptions

Assumption 1

There exists a $n \times n$ adjacency matrix W_0 such that: $\mathbb{E}[v|x, W_0] = 0$

Assumption 2

Let Π be the be the full-rank matrix of coefficients from the system regression

$$WS = W_0 S \Pi + U,$$
$$E\left[U | W_0 y, W_0, X\right] = O.$$

where $\mathbb{E}[S^\top w_{0;i} w_{0;i}{}^\top S] > 0$. Furthermore, the first row of $\mathbf{\Pi}$ is such that $\pi_{11}\beta + \pi_{12}\delta < 1/\lambda_{max}$, where λ_{max} is the largest eigenvalue of W_0.

Identification

Rank Condition

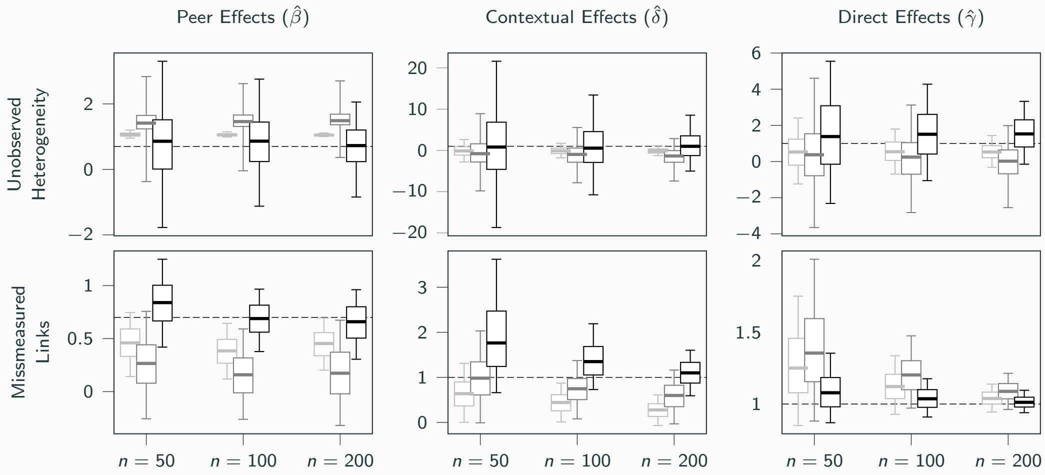
- Given that $\text{rank}(\Pi) \leq \min\{\text{rank}(E[S^\top w_{0;i} w_{0;i}^\top S]^{-1}), \text{rank}(E[S^\top w_{0;i} w_i^\top S])\}$, a necessary condition for $\text{rank}(\Pi) = k + 1$ is that $\text{rank}(E[S^\top w_{0;i} w_i^\top S]) = k + 1$ which would be equivalent to the **relevance** condition in the classical Instrumental Variable literature.

- For large enough sample, this condition imposes some restriction on the matrix $W_0 W$. This matrix contains the connections in common across the two networks in the main diagonal, and length two paths that change color in the off- diagonal.

- It cannot be zero so there have to be enough connections in common and indirect triads that change colors. This is a way to think about the **correlation** between the two matrices.

▸ Identification

Mote Carlo Experiments



Empirial Application: Data

- 1,628 articles published in the *American Economic Review*, *Econometrica*, the *Journal of Political Economy*, and the *Quaterly Journal of Economics* between 2000 and 2006. Source: RePEc, Scopus, and Journal Websites.

- Employment, education, and research interest information for 1,985 unique authors and 42 unique editors (37 of which also published papers in thee journals in this time period). Source: Web scrapping/text mining and Colussi (2018, ReStat).

- *Co-authorship* ($\ell = 1$) and *Alumni* ($\ell = 0$) networks are constructed for all 2,027 scholars.

Articles i and j are connected in network W_ℓ if at least one of the authors of article i shares a professional connection of type ℓ with at least one of article j's authors.

▸ Empirical Application