Using the package `hettreatreg` to interpret OLS estimates under treatment effect heterogeneity

Tymon Słoczyński

Brandeis University

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Treatment Effects

Consider the problem of identifying and estimating the effects of **a binary variable** ("treatment") on some outcome of interest.

There is a large number of methods that allow for flexible estimation of **average treatment effects** under conditional independence/unconfoundedness, including matching, regression adjustment, and doubly robust estimators.

In **Stata**: `teffects nnmatch`, `teffects ra`, `teffects ipwra`, etc.

Question

However, as noted by Imbens (2015), many applied researchers continue to use **OLS** to estimate a simple linear model:

$$Y = \alpha + \tau D + \beta X + \upsilon,$$

where Y – outcome, D – treatment, and X – other covariates. (In Stata: `regress`.) A major limitation of this approach is that the model above assumes that the effect of D is homogeneous.

Question: what is the appropriate **interpretation** of the OLS estimand in this model, τ, **if treatment effects are in fact heterogeneous**?

I provide a new answer to this question in my paper, **"Interpreting OLS Estimands When Treatment Effects Are Heterogeneous: Smaller Groups Get Larger Weights"** (forthcoming in the *Review of Economics and Statistics*, `https://doi.org/10.1162/rest_a_00953`).

Another answer to this question was provided by Angrist (1998). As I show in my paper, the two interpretations are compatible with each other.

New Answer

My answer: if we express τ as $\tau = w \cdot \tau_{\text{ATT}} + (1 - w) \cdot \tau_{\text{ATU}}$, where τ_{ATT} and τ_{ATU} are the average treatment effects on the treated (ATT) and untreated (ATU), then w is **inversely related** to $\text{P}(D = 1)$.

The more units get treatment, the less weight is placed on the average treatment effect on the treated.

In fact, one might wish to be estimating the average treatment effect (ATE):

$$\tau_{\text{ATE}} = \text{P}(D = 1) \cdot \tau_{\text{ATT}} + \text{P}(D = 0) \cdot \tau_{\text{ATU}},$$

but instead we estimate

$$\tau \simeq \text{P}(D = 0) \cdot \tau_{\text{ATT}} + \text{P}(D = 1) \cdot \tau_{\text{ATU}}.$$

Theory

I develop my theoretical contribution in two steps:

1. A general **decomposition of the OLS estimand** (into two components) which is based on very mild regularity conditions.
2. A new **interpretation** of the OLS estimand which places **causal labels** (ATT and ATU) on the two components above under additional assumptions:
   - conditional independence/unconfoundedness (this is standard);
   - linearity of potential outcomes in the LPM propensity score (this is quite strong but dates back to Rosenbaum and Rubin (1983)).

Notation

The unconditional probability of treatment:

$$\rho = \mathrm{P}\left(D = 1\right).$$

The LPM propensity score:

$$p\left(X\right) = \mathrm{L}\left(D \mid 1, X\right) = \alpha_{p} + \beta_{p} X.$$

Interpretation of OLS

Suppose that the assumptions above (and regularity conditions) are satisfied. Then, the OLS estimand can be written as

$$\tau = w_1 \cdot \tau_{\text{ATT}} + w_0 \cdot \tau_{\text{ATU}},$$

where

$$w_1 = \frac{(1-\rho) \cdot \text{Var}\left[p(X) \mid D=0\right]}{\rho \cdot \text{Var}\left[p(X) \mid D=1\right] + (1-\rho) \cdot \text{Var}\left[p(X) \mid D=0\right]}$$

and

$$\begin{aligned} w_0 &= 1 - w_1 \\ &= \frac{\rho \cdot \text{Var}\left[p(X) \mid D=1\right]}{\rho \cdot \text{Var}\left[p(X) \mid D=1\right] + (1-\rho) \cdot \text{Var}\left[p(X) \mid D=0\right]}. \end{aligned}$$

Bias from Heterogeneity

Under the same assumptions, we can provide a simple expression for **the difference between the OLS estimand and ATE (or ATT)**. Namely,

$$\tau - \tau_{\text{ATE}} \quad = \quad \delta \cdot \left(\tau_{\text{ATU}} - \tau_{\text{ATT}}\right),$$

where $\delta = \rho - w_1 = w_0 - (1 - \rho)$.

Also,

$$\tau - \tau_{\text{ATT}} \quad = \quad w_0 \cdot \left(\tau_{\text{ATU}} - \tau_{\text{ATT}}\right).$$

Note: if $\text{Var}\left[p\left(X\right) \mid D = 1\right] = \text{Var}\left[p\left(X\right) \mid D = 0\right]$, then $\delta = 2\rho - 1 = 2 \cdot (\rho - 0.5)$ and $w_0 = \rho$.

The Package `hettreatreg`

The package `hettreatreg`, available at SSC, implements the main results from my paper.

To download the package, type

```
. ssc install hettreatreg, all
```

The package is distributed together with an illustrative dataset, based on LaLonde (1986) and Dehejia and Wahba (1999). (See also Smith and Todd (2005), Angrist and Pischke (2009), and many others.) The file combines the treated group from the National Supported Work (NSW) Demonstration and the comparison group from the Current Population Survey (CPS). To load the dataset, type

```
. use nswcps, clear
```

My examples in this presentation are based on this dataset.

Replicating Angrist and Pischke (2009)

One of the estimates in Table 3.3.3 in Angrist and Pischke (2009) can be replicated in the following way:

```
. regress re78 treated age-re75, vce(robust)
```

Linear regression

Number of obs	=	16,177
F(10, 16166)	=	1718.20
Prob > F	=	0.0000
R-squared	=	0.4762
Root MSE	=	7001.7

re78	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
treated	793.587	618.6092	1.28	0.200	-418.9555	2006.13
age	-233.6775	40.7162	-5.74	0.000	-313.4857	-153.8692
age2	1.814371	.5581946	3.25	0.001	.7202474	2.908494
educ	166.8492	28.70683	5.81	0.000	110.5807	223.1178
black	-790.6086	197.8149	-4.00	0.000	-1178.348	-402.8694
hispanic	-175.9751	218.3033	-0.81	0.420	-603.8738	251.9235
married	224.266	152.4363	1.47	0.141	-74.52594	523.0579
nodegree	311.8445	176.414	1.77	0.077	-33.9464	657.6355
re74	.2953363	.0152084	19.42	0.000	.2655261	.3251466
re75	.4706353	.0153101	30.74	0.000	.4406259	.5006447
_cons	7634.344	737.8143	10.35	0.000	6188.146	9080.542

Using hettreatreg

help for **hettreatreg**

Title

hettreatreg — Diagnostics for linear regression when treatment effects are heterogeneous

Syntax

hettreatreg *indepvars* [*if*] [*in*], **outcome**(*varname*) **treatment**(*varname*) [*options*]

options	Description
Required	
outcome(*varname*)	designate an outcome variable
treatment(*varname*)	designate a binary treatment variable
Optional	
noisily	display model estimation output
vce(*vcetype*)	*vcetype* may be **ols**, **robust**, **cluster** *clustvar*, **bootstrap**, **jackknife**, **hc2**, or **hc3**; default is **ols**

Using hettreatreg

```
. hettreatreg age-re75, o(re78) t(treated)

"OLS" is the estimated regression coefficient on treated.

   OLS  =  793.6

P(d=1)  =  .011
P(d=0)  =  .989

    w1  =  .983
    w0  =  .017
 delta  =  -.971

   ATE  =  -6751
   ATT  =  928.4
   ATU  =  -6840

OLS = w1*ATT + w0*ATU = 793.6
```

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OLS = w1*ATT + w0*ATU = 793.6
```

Using `hettreatreg`

Interpretation: only 1.1% of the sample is treated; according to my theoretical results, this implies that the weight on ATT is close to one and the weight on ATU is close to zero.

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OLS = w1*ATT + w0*ATU = 793.6
```

Using `hettreatreg`

Interpretation: indeed, the weight on ATT is 0.983 and the weight on ATU is 0.017.

The value of $\hat{w}_0$ also implies that the difference between the OLS estimate and $\hat{\tau}_{\text{ATT}}$ is equal to 1.7% of the difference between $\hat{\tau}_{\text{ATU}}$ and $\hat{\tau}_{\text{ATT}}$.

Similarly, the value of $\hat{\delta}$ implies that the difference between the OLS estimate and $\hat{\tau}_{\text{ATE}}$ is equal to 97.1% of the difference between $\hat{\tau}_{\text{ATT}}$ and $\hat{\tau}_{\text{ATU}}$ (note the change in sign).

The bottom line is that the OLS estimate in this application is **roughly interpretable as ATT but definitely not as ATE**.

Needless to say, the interpretation of the OLS estimate will be different in other applications. (And you can use `hettreatreg` to examine this!)

Using hettreatreg

```
. hettreatreg age-re75, o(re78) t(treated)

"OLS" is the estimated regression coefficient on treated.

   OLS  =  793.6

P(d=1)  =  .011
P(d=0)  =  .989

    w1  =  .983
    w0  =  .017
 delta  =  -.971

   ATE  =  -6751
   ATT  =  928.4
   ATU  =  -6840

OLS = w1*ATT + w0*ATU = 793.6
```

Using `hettreatreg`

Interpretation: the difference between ATT and ATU appears to be large; thus, the OLS estimate and $\hat{\tau}_{\text{ATE}}$ are very different—given that $\hat{\delta}$ is close to one in absolute value.

Another Way to Obtain These Estimates

Begin by estimating the LPM propensity score:

```
. quietly regress treated age-re75, vce(robust)
. predict pscore
```

Another Way to Obtain These Estimates

```
. teffects ra (re78 pscore) (treated), vce(robust)

Iteration 0:   EE criterion =  1.770e-21
Iteration 1:   EE criterion =  9.442e-25

Treatment-effects estimation                    Number of obs     =     16,177
Estimator      : regression adjustment
Outcome model  : linear
Treatment model: none
```

re78	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
ATE						
treated (1 vs 0)	**-6750.701**	**1278.03**	**-5.28**	**0.000**	**-9255.593**	**-4245.808**
POmean						
treated 0	**14738.87**	**76.5959**	**192.42**	**0.000**	**14588.74**	**14888.99**

Another Way to Obtain These Estimates

```
. teffects ra (re78 pscore) (treated), vce(robust) atet

Iteration 0:   EE criterion =  1.770e-21
Iteration 1:   EE criterion =  1.508e-26

Treatment-effects estimation                    Number of obs     =     16,177
Estimator      : regression adjustment
Outcome model  : linear
Treatment model: none
```

re78	Coefficient	Robust std. err.	z	P>|z|	[95% conf. interval]	
ATET						
treated (1 vs 0)	**928.3604**	**670.6135**	**1.38**	**0.166**	**-386.0178**	**2242.739**
POmean						
treated 0	**5420.783**	**375.2868**	**14.44**	**0.000**	**4685.234**	**6156.332**

Conclusions

In many applications **the OLS estimates might not be close to any of the standard average treatment effects of interest**—solely because of treatment effect heterogeneity.

The weight which is placed by OLS on the average effect on each group (treated or untreated) is inversely related to its proportion.

The ability of OLS to provide a meaningful estimate of various average treatment effects is heavily data dependent.

The underlying decomposition is implemented in the package `hettreatreg`; in particular, **some useful diagnostics are available:**

- If interested in ATT, estimate w_0 or even simply report $\hat{\rho}$.
- If interested in ATE, estimate δ or even simply report $2\hat{\rho} - 1$.

More details and an additional application are available in "Interpreting OLS Estimands When Treatment Effects Are Heterogeneous: Smaller Groups Get Larger Weights," *REStat* (forthcoming), `https://doi.org/10.1162/rest_a_00953`. Please cite this paper when using `hettreatreg`.