GMM for Discrete Choice Models: A Capital Accumulation Application

Russell Cooper, John Haltiwanger and Jonathan Willis

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Abstract

This paper studies capital adjustment costs. Our goal here is to characterize these adjustment costs, which are important for understanding both the dynamics of aggregate investment and the impact of various policies on capital accumulation.

Our estimation strategy searches for parameters that minimize *ex post* errors in an Euler equation. This strategy is quite common in models for which adjustment occurs in consecutive periods. Here, we extend that logic to the estimation of parameters of dynamic optimization problems in which non-convexities lead to extended periods of investment inactivity.

1 Introduction

This paper studies capital adjustment costs. As in the recent literature, our model incorporates various forms of capital adjustment costs intended to capture the rich nature of capital adjustment at the plant-level. Our goal here is to characterize these adjustment costs, which are important for understanding both the dynamics of aggregate investment and the impact of various policies on capital accumulation.

Our estimation strategy searches for parameters which minimize *ex post* errors in an Euler equation. This strategy is quite common in models for which adjustment occurs in consecutive periods. Here, following Pakes (1994) and Aguirregabiria (1997), we extend that logic to the estimation of parameters of dynamic optimization problems in which non-convexities lead to extended periods of investment inactivity. We do so in the context of the capital adjustment problem.

This paper thus makes two contributions. First, we obtain parameter estimates for capital adjustment costs. Second, we obtain these estimates using a new methodology.

The paper begins by specifying the dynamic optimization problem at the plant-level. This problem is used to generate the Euler equation that underlies our empirical analysis. The empirical strategy is then laid-out in some detail. We provide some results using simulated data to guide us in terms of the choice of instruments and also in dealing with problems of censored observations. Finally, estimates of adjustment costs are reported.[1]

2 Model

The dynamic optimization model draws upon the results reported in Cooper and Haltiwanger (2000). The dynamic programming problem, $\forall(A, K)$, is specified as:

$$V(A, K) = \max\{V^i(A, K), V^a(A, K)\} \tag{1}$$

where K represents the beginning of period capital stock and A is the profitability shock. The superscripts refer to active investment "a," where the plant undertakes investment to obtain capital stock K' in the next period, and inactivity "i," where no investment occurs. These options, in turn, are defined by:

$$V^i(A, K) = \Pi(A, K) + \beta E_{A'|A} V(A', K(1-\delta)) \tag{2}$$

and

$$\begin{aligned} V^a(A, K) &= \max_{K'} \Pi(A, K)\lambda - p_b(I > 0)(K' - (1-\delta)K) \\ &+ p_s(I < 0)(K' - (1-\delta)K) - \frac{\nu}{2}\left(\frac{K' - (1-\delta)K}{K}\right)^2 K + \beta E_{A'|A} V(A', K') \end{aligned} \tag{3}$$

So here there are three types of adjustment costs which, as reported in Cooper and Haltiwanger (2000), are the leading types of estimated adjustment costs. The first is a disruption cost parameterized by λ. If $\lambda < 1$, then any level of gross investment implies that a fraction of revenues is lost. The second is the quadratic adjustment cost parameterized by ν. The third is a form of irreversibility in which there is a gap between

[1]This last section is not yet complete.

the buying, p_b, and selling, p_s, prices of capital. These are included in (3) by the use of the indicator function for the buying ($I > 0$) and selling of capital ($I < 0$).

Assume the profit function has the following form

$$\Pi(A, K) = AK^{\alpha}. \tag{4}$$

Cooper and Haltiwanger (2000) discusses the derivation of this profit function from a production function and a demand curve facing the plant.

The first-order condition for the investment decision is

$$p(I) + \nu \left(\frac{K' - (1-\delta)K}{K} \right) = \beta E_{A'|A} V_2(A', K') \tag{5}$$

where $p(I) = p_b$ if $I > 0$ and capital is purchased and $p(I) = p_s$ if $I < 0$ and capital is sold. To evaluate this equation *ex post*, we expand the $E_{A'|A}V_2(A', K')$ term until the plant's next episode of capital adjustment is observed. With non-convex adjustment costs, $\lambda < 1$, adjustment will generally not occur each period. We then replace expectations with realizations to calculate the *ex post* errors from the Euler equation.

To see how this works, suppose the plant adjusts in two consecutive periods, t and $t+1$. Then the *ex post* error, denoted $\varepsilon_{t,t+1}$, from (5) is

$$\varepsilon_{t,t+1} = \nu \frac{I_t}{K_t} + p(I_t) - \beta \left[\lambda \Pi_2(A_{t+1}, K_{t+1}) + p(I_{t+1})(1-\delta) + \nu(1-\delta)\frac{I_{t+1}}{K_{t+1}} + \frac{\nu}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 \right] \tag{6}$$

where $I_t = K_{t+1} - K_t(1-\delta)$. The first two terms here are the period t marginal costs of capital and the remaining terms are the marginal gains for the next period, including the marginal profitability and the marginal effects on adjustment costs next period.

Of course, not all plants adjust every period. It is not appropriate due to selection bias to estimate parameters from the plant's who choose to adjust in two periods.[2] Thus we need a more general condition which allows estimation of the structural parameters.

In general, if the plant adjusts in period t and subsequently in period $t+\tau$, then the *ex post* error, denoted $\varepsilon_{t,t+\tau}$, from the first-order condition is

[2] We later characterize this bias.

$$
\begin{aligned}
\varepsilon_{t,t+\tau} &= \nu\frac{I_t}{K_t} + p(I_t) - \sum_{i=1}^{\tau-1}\beta^i\Pi_2(A_{t+i}, K_{t+i})(1-\delta)^{i-1} - \beta^\tau\lambda\Pi_2(A_{t+\tau}, K_{t+\tau})(1-\delta)^{\tau-1} \\
&\quad -\beta^\tau\left[p(I_{t+\tau})(1-\delta)^\tau + \nu(1-\delta)^\tau\frac{I_{t+\tau}}{K_{t+\tau}} + \frac{\nu}{2}\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right)^2(1-\delta)^{\tau-1}\right].
\end{aligned} \tag{7}
$$

From this general expression, the first term on the right is the marginal cost of adjustment and the second term is the gain in profitability in the period of adjustment. During the periods between adjustment, there is an the effect of capital accumulation on marginal profitability. Finally, in the period of the next adjustment, i.e. when the spell of inactivity ends, there is a final term reflecting the effects of K_{t+1} on the marginal adjustment cost in period $t+\tau$. Note that the non-convex adjustment cost, λ, appears in (7), at the end of the spell of inaction. In addition, both the price of capital in the period of the initial adjustment and in the next adjustment are included as well.

As in the estimation of quadratic adjustment cost models, the *ex post* errors should not be predictable. In the next section we discuss estimation of all parameters, including the non-convex adjustment cost parameter using the orthogonality restrictions generated by optimization.

3 GMM Estimation

Pakes (1994) argues that the logic of Hansen and Singleton (1982) can be applied to the estimation of the structural parameters in a dynamic discrete choice problems. The application in Pakes (1994) is investment coupled with an exit decision. Aguirregabiria (1997) considers a dynamic labor demand model. Here we discuss the estimation of the capital accumulation problem drawing on those contributions.

Starting with (7), we can compute the *ex post* errors between adjustment periods. The optimization condition of the firm, (5), implies that expectations in period t over the *ex post* errors for all completed spells should be zero.

$$
E_t[\varepsilon_{t,t+\tau}] = 0 \tag{8}
$$

for all τ where the expectation is conditional on all variables known in period t. Thus, $\varepsilon_{t,t+\tau}$ ought to be uncorrelated with period t and prior variables. Using a vector of N

variables predetermined in period t, z_t, the following orthogonality condition can be used in a GMM estimation procedure.

$$E_t[z_t \varepsilon_{t,t+\tau}] = 0 \tag{9}$$

The sample analog of this condition is

$$m = \frac{1}{n} Z' \varepsilon(X, \theta) = m(\theta) \tag{10}$$

where Z is the matrix of N variables over T periods and $\varepsilon(X, \theta)$ are the *ex post* errors calculated using the sample data, X, and the parameter vector of interest, θ.

The minimum distance estimator is the $\hat{\theta}$ that minimizes

$$\begin{aligned} s &= m(\hat{\theta})' W^{-1} m(\hat{\theta}) \\ &= \frac{1}{n^2} [\varepsilon(X, \hat{\theta})' Z] W^{-1} [Z' \varepsilon(X, \hat{\theta})]. \end{aligned} \tag{11}$$

Hansen (1982) showed that for this estimator, the optimal choice for W is

$$\begin{aligned} W_{GMM} &= \mathrm{Var}\left(Z' \varepsilon(X, \theta)\right) \\ &= \frac{1}{n^2} Z' \Omega Z \end{aligned} \tag{12}$$

where $\Omega = E[\varepsilon \varepsilon']$. If the errors are uncorrelated, W can be estimated as shown by White (1980) using the following equation.[3]

$$\left(\frac{1}{n}\right) S_0 = \frac{1}{n} \left[\frac{1}{n} \sum_{t=1}^{T} z_t z_t' \varepsilon(x_t, \hat{\theta})^2 \right]. \tag{13}$$

Finally, the estimated asymptotic covariance matrix of the GMM estimator is

$$V(\hat{\theta}) = \left[G(\hat{\theta}) \left(\frac{Z' \hat{\Omega} Z}{n^2} \right)^{-1} G(\hat{\theta})' \right]^{-1} \tag{14}$$

where $G(\hat{\theta}) = \frac{\partial m(\hat{\theta})}{\partial \hat{\theta}}$ is numerically computed.

[3]There is an unresolved issue concerning a correction in the case where the errors are correlated. Because the "observations" in this estimation are spells of different length it is not immediately apparent how to apply a correction similar to that of Newey and West (1987).

4 Monte Carlo

Before estimating this model, we construct a simulation-based exercise. There are a number of goals of this experiment. First, there is the issue of checking the methodology to be sure that we can consistently estimate the parameters of interest.[4] Second, there is the issue of instruments. One can solve (11) for any instruments and, at least in theory, obtain consistent parameter estimates. In practice, it is useful to find instruments that are effective across a broad range of parameterizations. This can be achieved by simulating different models of adjustment costs and evaluating alternative instrument sets.

Third, the estimation strategy outlined above assumes that all investment spells are complete: for all plants adjusting in period t, there is a τ such that adjustment is observed in period $t+\tau$. In practice, spells may not all be complete. In that case, there are two issues. The first concerns the extent of the bias associated with estimation from completed spells only. The second is the development of a correction which is consistent with the dynamic optimization approach.

4.1 Creation of simulated dataset

A data set is simulated in the following steps. First, the structural parameters of the model are chosen and the investment policy functions of the dynamic programming problem are obtained through value function iteration. The parameters of interest in this exercise are those that can be estimated with GMM: $\Theta = \{\alpha, \nu, \lambda, p_s\}$.[5]

We consider three different parameterizations of Θ in order to assess the properties of the estimation procedure. The first case, $\Theta_a = \{0.6, 2, 1, 1\}$, includes only a quadratic cost of adjustment. The second case, $\Theta_b = \{0.6, 2, 0.95, 0.98\}$, adds asymmetry between the buying and selling prices of capital and disruption costs. This parameterization results in a much higher rate of inactivity due to the introduction of the non-convex costs associated with adjustment. The final case, $\Theta_c = \{0.6, 2, 0.8, 0.98\}$, has a much larger disruption cost and therefore leads to more inactivity. This parameterization most closely matches the estimates of Cooper and Haltiwanger.

[4] This is partly a test of our programs and partly an evaluation of the logic associated with this extension of the method of Euler equation estimation.

[5] In this exercise, we normalize $p_b = 1$ and we have chosen not to focus on estimating the discount rate, β, at this point. In the estimation on manufacturing data, we may include interest rate to allow for variation in the discount rate.

The other structural parameters of the model are chosen to be similar to those used by Cooper and Haltiwanger. The frequency of the model is annual, so the discount rate, β, is set at 0.95. The productivity shock, A, consists of an aggregate shock and an idiosyncratic shock. Each of these shocks follows a log-normal autoregressive process. The aggregate shock process has a persistence of 0.75 and the innovation to this process has a standard deviation of 0.05. The idiosyncratic shock process has a persistence of 0.88 and the standard deviation of the innovation is 0.3. The depreciation rate, δ, is 0.07.

The simulated panel data set is created by using the investment policy functions in conjunction with the randomly drawn innovations to the two productivity shock processes. For these exercises, each data set contains 200 plants, the size of an average manufacturing sector. To explore the small sample properties of this estimation method, results are reported for 3 sample lengths: 19 periods, 50 periods, 100 periods.[6]

4.2 Parameter estimation

The parameter vector Θ is estimated by minimizing the weighted sum of squared moments statistic in (11). Two different sets of instruments are used to examine the impact of alternative instruments on the precision of the estimates. The first set of instruments is composed of current and once-lagged values of the state variables of the dynamic programming problem along with a constant; $Z_{1,t} = \{1, A_t, A_{t-1}, K_t, K_{t-1}\}$. The second set of instruments consists of current and once-lagged variables that are observed in the actual data. The variables include the investment rate ($\frac{I}{K}$), the profit rate ($\frac{\pi}{K}$), and the capital stock (K); $Z_{2,t} = \{1, \frac{I_t}{K_t}, \frac{I_{t-1}}{K_{t-1}}, \frac{\pi_t}{K_t}, \frac{\pi_{t-1}}{K_{t-1}}, K_t, K_{t-1}\}$.

The estimates are obtained using a two-stage procedure. In the first stage, an identity matrix is used to weight the moments, and the simplex algorithm is used to obtain the parameter estimates. These first stage estimates are used to estimate a weighting matrix, W, based on the White specification. This weighting matrix is then used in the second stage estimation.

An important issue in the estimation is accounting for incomplete spells in the data set. Non-convex adjustment costs in the form of disruption costs and asymmetry between the prices of buying and selling capital result in periods of inaction for plants. The moment condition underlying this GMM estimation is based upon the expectation taken across ex post errors for all plants that adjusted in a given period. Therefore, if there

[6]The actual data set that is used for estimation in Section 5 contains 19 periods.

is a plant that adjusted in a given period but did not adjust again before the end of the sample, then the ex post error expressed in (7) cannot be computed. We explore different ways of accounting for these incomplete spells.

The first set of results are shown in Tables 1a, 1b, and 1c, corresponding to the three parameterizations being considered. For each estimation exercise, 1000 data sets were simulated. The parameter estimates reported in the tables represent the mean of the estimates from the 1000 data sets. The standard deviation of the estimates is reported in parentheses. For the first set of estimates, we control for incomplete spells by only including observations up to the first period in which there is an observed incomplete spell.

The results in Table 1a show that the GMM estimation procedure performs well in the case with only quadratic adjustment costs, Θ_a, even in the smallest sample exercise. In this case, the true value of the production function parameter, α, is 0.6, and the scalar on the quadratic adjustment cost, ν, is set at 0.2. Using the first instrument set, Z_1, the means of the parameter estimates across the 1000 samples of 200 plants and 19 periods are $\{\bar{\alpha}_{1,19}, \bar{\nu}_{1,19}\} = \{0.600, 1.971\}$. The respective standard deviations across the 1000 parameter estimates are $\{0.025, 0.136\}$. Due to the discrete nature of the value function iteration solution, there are some situations where firms choose to remain inactive. The average number of periods before the first incomplete spell is initiated is 13.6 (listed in the "uncensored periods" column). The average number of complete investment spells (observations) across datasets is 2615. All complete spells starting on or after the date on which the first incomplete spell in initiated are dropped.

Increasing the number of periods in the sample from 19 to 100 leads to a slight improvement in the estimates. The estimate of α is essentially unchanged while the mean estimate of ν is now 1.991. The standard deviations across both parameter estimates decreases by more than two-thirds in comparison to those from the smaller samples. The average number of observations in a sample is 18214.

The lower portion of Table 1a shows estimation results obtained using the second instrument set. The results here are almost identical to those in the upper panel. The mean estimate of ν in the largest data set, 1.978, is slightly further away from the the true value than that reported in the upper panel, but the difference is less than one standard deviation.

Table 1a: GMM Estimates of Θ_a from Monte Carlo Exercise

	α	ν	T	uncensored periods	obs.
Θ_a	0.6	2.0			
GMM estimates using instrument set Z_1	0.600	1.971	19	13.6	2615.2
	(0.025)	(0.136)			
	0.601	1.988	50	44.6	8584.1
	(0.014)	(0.074)			
	0.601	1.991	100	94.6	18214.2
	(0.009)	(0.047)			
GMM estimates using instrument set Z_2	0.600	1.964	19	13.6	2615.2
	(0.025)	(0.129)			
	0.600	1.977	50	44.6	8584.1
	(0.014)	(0.072)			
	0.601	1.978	100	94.6	18214.2
	(0.009)	(0.044)			

Reported estimates are the mean value across estimates from 1000 simulated datasets. Standard deviations of the estimates are reported in parentheses. Each sample contains 200 plants. T denotes the length of the sample period. The last two columns report the mean number of periods in the sample before the first incomplete spell begins and the mean number of observations (completed spells) in the GMM estimation. Denoting $\bar{T}$ as the period in which the first incomplete spell is initiated, all completed spells initiated in periods $t \geq \bar{T}$ have been excluded from the estimation.

Table 1b shows results for the case where the true parameter vector is Θ_b. Here we see evidence that the size of the sample strongly affects the average and the precision of the estimates when disruption costs and capital price asymmetries are present. In the smallest sample exercise, the mean parameter estimates using instrument set Z_1 are $\{\bar{\alpha}_{1,19}, \bar{\nu}_{1,19}, \bar{\lambda}_{1,19}, \bar{p}_{s1,19}\} = \{0.63, 0.24, 0.89, 1.02\}$ with respective standard deviations of $\{0.11, 0.26, 0.32, 0.26\}$. The imprecision of the estimates is due in large part to the length of investment inactivity induced by the nonconvex adjustment costs. After omitting all periods beginning with the earliest observed incomplete spell, the average number of periods used in the estimation is only 4.4 and the average number of observations is 320. In the largest sample exercise, the mean parameter estimates are $\{\bar{\alpha}_{1,100}, \bar{\nu}_{1,100}, \bar{\lambda}_{1,100}, \bar{p}_{s1,100}\} = \{0.602, 0.198, 0.946, 0.978\}$, which are very close to the true values of $\Theta_b = \{0.6, 0.2, 0.95, 0.98\}$. The precision of these estimates is also much improved as the average number of uncensored periods increases to 84 and the average number of observations is not 6207. The respective standard deviations are $\{0.019, 0.050, 0.054, 0.046\}$.

The mean parameter estimates obtained using instrument set Z_2, shown in the lower portion of Table 1b, are very similar to those based on Z_1. The precision of these estimates, however, is much improved. The respective standard deviations of the estimates using the smallest sample are one-third to two-thirds the size of those based on Z_1. In the largest sample, the standard deviations for the four parameter estimates are $\{0.014, 0.019, 0.044, 0.016\}$, representing a reduction of over 50 percent for the estimates of ν and p_s.

Table 1c shows results based on the parameterization that most closely matches the estimates of Cooper and Haltiwanger, Θ_c. This parameterization has a much larger disruption cost, $\lambda = 0.8$, than in the previous case. The larger disruption cost leads to more inactivity and longer observed incomplete spells, which translates into greater imprecision of the estimates due to the number of periods that must be excluded from the estimation. The mean parameter estimates in the smallest sample exercise when using instrument set Z_1 are $\{\bar{\alpha}_{1,19}, \bar{\nu}_{1,19}, \bar{\lambda}_{1,19}, \bar{p}_{s1,19}\} = \{0.61, 0.10, 0.1.17, 0.89\}$. The high degree of impression is reflected in the respective standard deviations of $\{0.28, 0.42, 1.33, 0.57\}$. After controlling for incomplete spells, the average number of periods in the sample is now only 1.8 periods and the average number of observations is 63. In the largest sample exercise, the mean parameter estimates are $\{\bar{\alpha}_{1,100}, \bar{\nu}_{1,100}, \bar{\lambda}_{1,100}, \bar{p}_{s1,100}\} = \{0.595, 0.189, 0.827, 0.967\}$. These mean estimates are not as close to the true values as the comparable estimates in Table 1b, which is a reflection in part of the of greater impression of the estimates. The average number of uncensored periods is 76.6 and the average number of observations is 2841, approximately 30 percent fewer observations than in Table 1b. The respective standard deviations are $\{0.034, 0.059, 0.127, 0.074\}$.

Table 1b: GMM Estimates of Θ_b from Monte Carlo Exercise

	α	ν	λ	p_s	T	uncensored periods	obs.
Θ_b	0.6	0.2	0.95	0.98			
GMM estimates using instrument set Z_1	0.627	0.239	0.894	1.019	19	4.4	320.5
	(0.108)	(0.260)	(0.321)	(0.257)			
	0.603	0.197	0.943	0.979	50	35.2	2583.6
	(0.030)	(0.081)	(0.090)	(0.075)			
	0.602	0.198	0.946	0.978	100	84.5	6207.7
	(0.019)	(0.050)	(0.054)	(0.046)			
GMM estimates using instrument set Z_2	0.621	0.180	0.877	0.968	19	4.4	320.5
	(0.071)	(0.103)	(0.231)	(0.085)			
	0.604	0.194	0.938	0.978	50	35.2	2583.6
	(0.020)	(0.031)	(0.070)	(0.026)			
	0.602	0.197	0.945	0.978	100	84.5	6207.7
	(0.014)	(0.019)	(0.044)	(0.016)			

Reported estimates are the mean value across estimates from 1000 simulated datasets. Standard deviations of the estimates are reported in parentheses. Each sample contains 200 plants. T denotes the length of the sample period. The last two columns report the mean number of periods in the sample before the first incomplete spell begins and the mean number of observations (completed spells) in the GMM estimation. Denoting $\bar{T}$ as the period in which the first incomplete spell is initiated, all completed spells initiated in periods $t \geq \bar{T}$ have been excluded from the estimation.

Table 1c: GMM Estimates of Θ_c from Monte Carlo Exercise

	α	ν	λ	p_s	T	uncensored periods	obs.
Θ_c	0.6	0.2	0.80	0.98			
GMM estimates using instrument set Z_1	0.610	0.099	1.170	0.894	19	1.8	63.2
	(0.275)	(0.422)	(1.332)	(0.572)			
	0.598	0.195	0.823	0.979	50	25.7	953.6
	(0.067)	(0.125)	(0.253)	(0.159)			
	0.595	0.189	0.827	0.967	100	76.6	2841.6
	(0.034)	(0.059)	(0.127)	(0.074)			
GMM estimates using instrument set Z_2	0.622	0.095	0.700	0.904	19	1.8	63.2
	(0.120)	(0.148)	(0.434)	(0.167)			
	0.595	0.185	0.808	0.966	50	25.7	953.6
	(0.029)	(0.044)	(0.125)	(0.047)			
	0.599	0.195	0.808	0.975	100	76.6	2841.6
	(0.017)	(0.027)	(0.072)	(0.030)			

Reported estimates are the mean value across estimates from 1000 simulated datasets. Standard deviations of the estimates are reported in parentheses. Each sample contains 200 plants. T denotes the length of the sample period. The last two columns report the mean number of periods in the sample before the first incomplete spell begins and the mean number of observations (completed spells) in the GMM estimation. Denoting $\bar{T}$ as the period in which the first incomplete spell is initiated, all completed spells initiated in periods $t \geq \bar{T}$ have been excluded from the estimation.

The estimates obtain using instrument set Z_2, shown in the lower portion of Table 1c, are much more precisely estimates and the mean estimate is much closer to the true value in the largest sample. The standard deviations of the four parameter estimates are less that half the size obtained using Z_1 in the smallest sample. In the largest sample exercise, the mean parameter estimates are $\{\bar{\alpha}_{2,100}, \bar{\nu}_{2,100}, \bar{\lambda}_{2,100}, \bar{p}_{s2,100}\} = \{0.600, 0.196, 0.806, 0.976\}$, which are very close to the true values of $\Theta_c = \{0.6, 0.2, 0.8, 0.98\}$. The standard deviations are roughly half the size of those obtained with Z_1. One potential explanation for the improved results using Z_2 is that Z_2 contains two more variables than Z_1. With the additional variables, Z_2 may provide more explanatory power than Z_1. However, instrument set Z_1 contains the current and once-lagged values of the state variables of the dynamic programming problem, which should be the only pieces of information needed to summarize the information set of the plant.

4.3 Incomplete Spells

The previous exercises illustrate that this GMM methodology works well if the sample is of sufficient length. But in cases where the length of the panel is short, the exclusion of periods due to incomplete spells leads to imprecise estimates. An alternative to excluding periods containing incomplete spells is to use all of the available data and attempt to control for the bias created by incomplete spells. In this subsection, we will conduct two experiments to examine the bias introduced by the failure to control for incomplete spells.

The first exercise will be to estimate parameters from the same data sets as used in Tables 1a, 1b, and 1c using a slightly modified criteria in determining which periods to exclude. In the previous exercises, all periods beginning with the earliest observed incomplete spell were omitted from the estimation. This criteria, however, often leads to several periods being omitted in which all investment spells initiated in those period are completed by the end of the sample. So here we will relax the original criteria to include all periods which contain only completed investment spells. The potential for bias is due to the fact that our sample only contains 200 plants. The distribution of spell lengths in a small sample not properly reflect the potential of long periods of inactivity that plants factor into their optimization decisions. Thus, this bias is the similar to that of omitting incomplete spells.

In the second exercise, all periods and all complete spells will be used in the estimation procedure. This exercise will demonstrate the bias that results from not controlling for incomplete spells in the sample. This exercise will set the stage for the following section in which we propose a methodology to control for the presence of incomplete spells.

4.4 Controlling for incomplete spells when A is observed

Here we propose a methodology for controlling for incomplete spells in the estimation procedure. The difficulty introduced by incomplete spells is that the ex post error expressed in (7) cannot be fully evaluated. However, the structure of the dynamic programming problem can potentially be used to approximate the unobserved portion of the incomplete spell.

In the first stage of this methodology, parameter estimates are obtained by including all complete spells as observations in the estimation. We denote these first stage estimates as Θ_1. Assuming that we have obtained all of the other structural parameters of the model from other sources, we then solve the dynamic programming using Θ_1. From this solution, we can compute the expected derivative of the value function that appears in the first-order condition of the investment decision expressed in (5). This expected derivative is a function of the current profitability shock and the capital stock resulting from the investment decision in the current period, conditional on the parameterization Θ_1.

$$\psi(A, K'; \Theta_1) = E_{A'|A} V_2(A', K'; \Theta_1) \tag{15}$$

This function can then be evaluated using observations of A and $K' = (1-\delta)K + I$ from the final period of the sample, and *ex post* errors for all incomplete spells can be computed using the following specification.

$$\varepsilon_{t,incomplete} \;=\; \nu \frac{I_t}{K_t} + p(I_t) - \sum_{i=1}^{T-t} \beta^i \Pi_2(A_{t+i}, K_{t+i})(1-\delta)^{i-1} - \beta^{T-t+1}\psi(A_T, K_{T+1}, \Theta)$$

A second stage estimation including all complete and incomplete spells results in parameter estimates Θ_2. This process can be repeated by computing $\psi(A, K'; \Theta_2)$ and obtaining a third stage estimate, Θ_3. Additional repetitions can be computed until estimates of Θ converge. A Monte Carlo exercise will be conducted to evaluate this proposed methodology and determine the convergence properties of the estimation procedure for various sample sizes.

4.5 Controlling for incomplete spells when A is not observed

The methodology proposed above assumes that the productivity shock is observed in the data. However, in practice it is difficult to obtain a measure of productivity. Therefore, we propose a modification in which we search for a mapping between the derivative of the value function, which is a function of the unobserved productivity shock, and variables that are observed in practice.

The first stage estimates are obtained in the same fashion as described above and the derivative of the value function, $\psi(A, K'; \Theta_1)$, is computed. To determine a mapping between $\psi(.)$ and observable variables, a simulated dataset of the same size as the actual data is simulated. The value function derivative can then be regressed on a polynomial function of the observables $(K', \frac{I}{K}, \frac{\pi}{K})$, estimating the parameter vector Γ.

$$\psi(A, K'; \Theta_1) = f(K', \frac{I}{K}, \frac{\pi}{K}; \Gamma, \Theta_1) \tag{16}$$

The *ex post* errors for incomplete spells can then be computed as

$$\varepsilon_{t,incomplete} = \nu \frac{I_t}{K_t} + p(I_t) - \sum_{i=1}^{T-t} \beta^i \Pi_2(A_{t+i}, K_{t+i})(1-\delta)^{i-1} - \beta^{T-t+1} f(K_{T+1}, \frac{I_T}{K_T}, \frac{\pi_T}{K_T}; \Gamma, \Theta_1)$$

The second stage estimates of Θ are then computed using all complete and incomplete spells. The updating process is repeated until convergence is achieved. A Monte Carlo estimation will be undertaken to determine the best specification of $f(K', \frac{I}{K}, \frac{\pi}{K}; \Gamma, \Theta_1)$ and to examine the properties of this methodology.

5 Estimation

The estimation takes the procedures outlined above to plant-level manufacturing data. The LRD data set is described in some detail in Cooper and Haltiwanger (2000). Some pertinent aspects of the data are summarized in the following table, taken from that paper.

Table 2: Summary Statistics

Variable	LRD
Average Investment Rate	12.2% (0.10)
Inaction Rate: Investment	8.1% (0.08)
Fraction of Observations with Negative Investment	10.4% (0.09)
Spike Rate: Positive Investment	18.6% (0.12)
Spike Rate: Negative Investment	1.8% (0.04)
Serial correlation of Investment Rates	0.058 (0.003)
Correlation of Profit Shocks and Investment	0.143 (0.003)

The approach in Cooper and Haltiwanger (2000) is to use these moments in a minimum distance estimation exercise. In doing so, for each vector of structural parameters, the dynamic programming problem was solved through value function iteration, a data set was simulated and moments were calculated. In addition, a fixed discount factor was assumed through the analysis.

The approach taken here is much faster as it does not require repeated solution of the dynamic programming problem. There is a considerable increase in the speed of the estimation exercise, though, in contrast to the approach of matching the moments in Table 5, the estimation requires access to the actual data rather than summary moments.

The estimation uses (11). As discussed in Cooper and Haltiwanger (2000), the LRD provides enough information to measure all the components of the second instrument set.

[To Be continued as estimation proceeds]

6 Conclusions

This paper had two purposes. The first was to analyze a methodology for using the logic of Euler equation estimation, as in Hansen and Singleton (1982), to settings in which adjustment is infrequent. Our analysis indicates how these procedures can estimate underlying adjustment costs, including those that create the inaction. We have used a simulation environment to identify powerful instruments and to guide us in the analysis of incomplete spells.

The second part of the paper takes this approach to plant-level data for U.S. manufacturers. There we find [to be continued.....]

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