A Likelihood-Based Evaluation of the Segmented Markets Friction in Equilibrium Monetary Models.

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Abstract

This paper estimates and compares the full participation and the segmented markets monetary frameworks. In both models, the real sector and monetary policy determine exogenously the joint process for the aggregate endowment and the short-term nominal interest rate, while the money growth rate and the inflation rate are determined endogenously. Using linearized versions of the models, we use Bayesian methods to compare the two models over the full dimension of the data. This likelihood-based comparison overwhelmingly favors the segmented markets model over the full participation model. The estimate of the fraction of households participating in financial markets is approximately 13%. The segmented markets model generates more persistent and more realistic impulse response functions to monetary policy shocks. Our results strongly suggest that taking the presence of market segmentation into account is important in understanding the short-run dynamics of the monetary sector.

Keywords: limited participation, segmented markets, Bayesian model comparison, monetary policy shocks.
JEL Classification Number: C11, C52, E52.

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1 Introduction

The segmented markets model, has recently been adopted with success as a framework for monetary analysis. Alvarez, Lucas and Weber (2001), Lahiri, Singh and Vegh (2003), and Occhino (2004) are current examples of how the segmented markets framework provides fundamental insights into the nature of the short-run interaction among money, interest rates, exchange rates and inflation.

This paper brings the segmented markets literature one step further, estimating the model and comparing it with the benchmark of full participation. Abstracting from other frictions, we focus on evaluating the impact of introducing market segmentation into an equilibrium model with money. We assume that the real sector and monetary policy determine exogenously the joint process for the aggregate endowment and the short-term nominal interest rate, and then we derive the model predictions on the the money growth rate and the inflation rate. We compare the models with data along two main dimensions. One, statistical, based on the log marginal likelihood, the other, more dependent on economic theory, based on the response to monetary policy shocks.

We use contemporary Bayesian methods to bring non-sample and sample information to bear on this comparison. Linearized versions of each model are calculated and these are used to construct likelihood functions for each model. The information contained in the data, through the likelihood function, and non-sample information, in the form of proper priors, are combined to statistically compare the two models using log marginal likelihoods. As the log marginal likelihood reflects the cumulative out-of-sample prediction performance of a model, we therefore formally compare the two models based on their ability to predict out of sample. We also evaluate the performance of the two models based on their ability to produce impulse responses to monetary shocks that accord with the empirical literature.

The paper is organized as follows. Section 2 describes the economy, defines the competitive equilibrium, and derives the log-linear approximation to the solution. Section 3 details the data, the calibrated parameters, and the estimation procedure. Section 4 describes and comments on the estimation results, the likelihood comparison, and the impulse response analysis. Section 5 concludes.

2 Model

The model is a cash-in-advance endowment economy, with a large number of households and a monetary authority. Time is discrete and is indexed by $t \geq 0$. There are a single non-durable consumption good, money, and one-period nominal bonds, which are claims to one unit of money payable at the end of the period. Households are of two types, traders and non-traders. Let $\omega > 0$ and $\omega^* \geq 0$ be respectively the number of traders and non-traders. We will refer to the case where $\omega^* = 0$ and $\omega^* > 0$ respectively as the full participation model and the segmented markets model.

Households of the same type are identical in all respects. The crucial difference between the two types of households is that non-traders spend all their money purchasing consumption goods, while traders can purchase bonds as well.

Households start each period with cash balances from the previous period. Then, two markets meet in sequence, a bond market and a goods market.

In the bond market, the monetary authority sells one-period nominal bonds to the traders, at the bond price $q_t > 0$. Open market operations are conducted in terms of the short-term nominal interest rate i_t. The monetary authority announces the bond price

$$q_t \equiv \frac{1}{1 + i_t} \tag{1}$$

and stands ready to issue and sell any number of bonds to clear the market at that price. Monetary policy is, then, an exogenous stochastic process for the interest rate, while the bond supply and the money supply are determined endogenously. By assumption, the interest rate is strictly positive, and the bond price is strictly less than one.

After the bond market, all households participate in the goods market. Each trader and each non-trader respectively receive constant fractions $\Lambda > 0$ and $\Lambda^* > 0$ of the exogenous stochastic aggregate endowment $Y_t > 0$, with $\omega\Lambda + \omega^*\Lambda^* = 1$. The endowment cannot be consumed directly, and must be sold in exchange of money at the price $P_t > 0$. Households can only consume goods purchased with money held before the goods market session. The money supply

$$M_t \equiv P_t Y_t \tag{2}$$

is defined as the amount of dollars $P_t Y_t$ spent in the goods market. Bonds are redeemed after the goods market closes.

The aggregate endowment Y_t and the nominal interest rate i_t are the only sources of uncertainty in the economy. Let $\{\overline{Y}_t, \overline{i}_t\}_{t=0}^{\infty}$ be the non-stochastic steady state values of the aggregate endowment and the interest rate, and let us assume that $\overline{Y}_{t+1}/\overline{Y}_t = \alpha$ and $\overline{i}_t = i$ are constant over time. We assume that $\hat{z}_t \equiv [\log(Y_t) - \log(\overline{Y}_t), \log(i_t) - \log(i)]'$ follows the AR(1) process

$$\hat{z}_t = \Pi \hat{z}_{t-1} + \Psi \eta_t \tag{3}$$

where Π and Ψ are two-by-two matrices, Ψ is lower triangular, η_t is a two-by-one vector of independently and identically distributed standard Gaussian shocks.

Each trader chooses consumption C_t, bonds B_t, and next-period cash balances A_{t+1} to solve

$$\begin{aligned}
&\max_{\{C_t>0, B_t, A_{t+1}>0\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right] \\
&\text{subject to:} \\
&q_t B_t + P_t C_t \leq A_t \\
&\qquad A_{t+1} = A_t - q_t B_t - P_t C_t + P_t \Lambda Y_t + B_t,
\end{aligned} \tag{4}$$

given the trader' initial cash balances $A_0 > 0$ in period zero, where E_0 is the expectation conditional on information available after $\hat{z}_0$ has been revealed, the period utility function $u(C) \equiv C^{1-1/\epsilon}/(1-1/\epsilon)$ is constant elasticity of substitution, and the preferences discount factor satisfies $\beta\alpha^{1-1/\epsilon} \in (0, 1)$.

Since the bond price q_t is strictly less than one for all t, holding idle cash balances is never optimal for traders, so the traders' cash-in-advance constraint always holds with equality. Then, the two constraints in the above maximization problem (4) can be substituted with the constraints

$$\begin{aligned} q_t B_t + P_t C_t &= A_t \\ A_{t+1} &= P_t \Lambda Y_t + B_t \end{aligned} \tag{5}$$

Non-traders spend all their initial cash balances purchasing consumption goods. Under this assumption, the behavior of a non-trader is simply described by constraints

$$\begin{aligned} P_t C_t^* &= A_t^* \\ A_{t+1}^* &= P_t \Lambda^* Y_t \end{aligned} \tag{6}$$

given the non-traders' initial cash balances $A_0^* > 0$ in period zero.

The economy is described by the traders' initial assets $A_0 > 0$, the non-traders initial assets $A_0^* > 0$, the initial exogenous state $\hat{z}_0$, and the law of motion (3) for the exogenous state $\hat{z}_t$. An equilibrium is a set of contingent sequences $\{C_t > 0, B_t, A_{t+1} > 0\}_{t=0}^{\infty}$ of consumption demand, bonds demand and cash balances for traders, $\{C_t^* > 0, A_{t+1}^* > 0\}_{t=0}^{\infty}$ of consumption demand and cash balances for non-traders, a contingent sequence $\{D_t\}_{t=0}^{\infty}$ of bonds supplied by the monetary authority, and a contingent sequence $\{P_t > 0\}_{t=0}^{\infty}$ of prices such that, given the prices, the traders' contingent sequence solves the traders' optimization problem (4), the non-traders' contingent sequence satisfies the non-traders constraints (6), and the following bonds and goods market equilibrium condition hold:

$$\begin{aligned} \omega B_t &= D_t \\ \omega C_t + \omega^* C_t^* &= Y_t \end{aligned} \tag{7}$$

The necessary first-order conditions for the traders' optimization problem are

$$\begin{aligned} \beta^t u'(C_t) - \nu_t^1 P_t &= 0 \\ -q_t \nu_t^1 + \nu_t^2 &= 0 \\ -\nu_t^2 + E_t[\nu_{t+1}^1] &= 0 \end{aligned} \tag{8}$$

and the transversality condition is

$$\lim_{t \to \infty} E_0 \left[\nu_t^1 A_t \right] = 0 \tag{9}$$

where ν_t^1 and ν_t^2 are the Lagrange multipliers associated with the two constraints (5). From the first-order conditions, it follows that

$$\begin{aligned} \beta^t u'(C_t) &= \nu_t^1 P_t \\ q_t \nu_t^1 &= E_t[\nu_{t+1}^1] \end{aligned} \tag{10}$$

The system describing the equilibrium is, then, made of the identities (1) and (2), the law of motion 3 for the exogenous state, the traders' first-order conditions 10, the traders' constraints 5, the non-traders' constraints 6, and the equilibrium conditions 7.

For convenience, variables are normalized as follows. As Lucas (1990), nominal variables are normalized by aggregate cash balances available at the beginning of the period. Let $\overline{A}_t \equiv \omega A_t + \omega^* A_t^*$ be the initial aggregate cash balances. Then, $y_t \equiv Y_t/\overline{Y}_t$, $\nu_t \equiv u'(\omega)\nu_t^1 \overline{A}_t / \beta^t u'(\overline{Y}_t)\overline{Y}_t$, $c_t \equiv \omega C_t/\overline{Y}_t$, $b_t \equiv \omega B_t/\overline{A}_t$, $a_t \equiv \omega A_t/\overline{A}_t$, $c_t^* \equiv \omega^* C_t^*/\overline{Y}_t$, $a_t^* \equiv \omega^* A_t^*/\overline{A}_t$, $d_t \equiv D_t/\overline{A}_t$, $\gamma_t \equiv \overline{A}_{t+1}/\overline{A}_t$, $p_t \equiv P_t \overline{Y}_t/\overline{A}_t$, $m_t \equiv M_t//\overline{A}_t$. Also, let us define the traders' share of the aggregate endowment as $\lambda \equiv \omega\Lambda = 1 - \omega^*\Lambda^*$. λ is equal to 1 in the full participation model, and $\lambda \in (0,1)$ in the segmented markets model. It might be helpful to consider the case where the endowment received by a trader is the same as the one received by a non-trader, so $\Lambda = \Lambda^*$. In this case, $\Lambda = 1/(\omega + \omega^*$, and $\lambda = \omega/(\omega + \omega^*)$, so λ is the proportion of traders in the economy.

The system describing the equilibrium can then be written as

$$q_t(1 + i_t) \equiv 1 \tag{11a}$$

$$u'(c_t) = \nu_t p_t \tag{11b}$$

$$q_t \gamma_t \nu_t = \beta E_t[\nu_{t+1}] u'(\alpha)\alpha \tag{11c}$$

$$q_t b_t + p_t c_t = a_t \tag{11d}$$

$$\gamma_t a_{t+1} = p_t \lambda y_t + b_t \tag{11e}$$

$$p_t c_t^* = a_t^* \tag{11f}$$

$$\gamma_t a_{t+1}^* = p_t(1 - \lambda) y_t \tag{11g}$$

$$b_t = d_t \tag{11h}$$

$$c_t + c_t^* = y_t \tag{11i}$$

$$m_t \equiv p_t y_t \tag{11j}$$

$$a_t + a_t^* = 1 \tag{11k}$$

together with the law of motion (3) for the exogenous state. The transversality condition (9) can be written as

$$\lim_{t\to\infty} E_0 \left[\beta^t u'(\overline{Y}_t)\overline{Y}_t \nu_t a_t / u'(\omega)\omega\right] = 0. \tag{12}$$

It is convenient to derive an equivalent system as follows. From the households' budget constraints (11e) and (11g), it follows that

$$\gamma_t a_{t+1} + \gamma_t a^*_{t+1} = p_t \lambda y_t + b_t + p_t(1-\lambda)y_t$$

$$q_t \gamma_t [a_{t+1} + a^*_{t+1}] = q_t p_t y_t + q_t b_t$$

Then, using the households' cash-in-advance constraints (11d) and (11f), the equation (11a), and the goods market equilibrium condition (11i)

$$q_t \gamma_t [a_{t+1} + a^*_{t+1}] = q_t p_t y_t + a_t - p_t c_t + a^*_t - p_t c^*_t$$

$$q_t \gamma_t = q_t p_t y_t + 1 - p_t c_t - p_t c^*_t$$

$$q_t \gamma_t + (1 - q_t) p_t y_t = 1$$

which we use in place of the traders' budget constraint 11e in the previous system 11.

In the non-stochastic steady state, all normalized variables are constant over time, and $y_t = 1$. The non-stochastic steady state can, then, be derived from the system:

$$\begin{aligned}
q(1+i) &\equiv 1 \\
q\gamma &= \beta u'(\alpha)\alpha \\
q\gamma + (1-q)py &= 1 \\
m &\equiv py \\
pc^* &= a^* \\
\gamma a^* &= p(1-\lambda)y \\
c + c^* &= y \\
u'(c) &= \nu p \\
a + a^* &= 1 \\
qb + pc &= a \\
b &= d
\end{aligned} \tag{13}$$

where the variables without the time subscript are the non-stochastic steady state values. Notice that, since $\beta\alpha^{1-1/\epsilon} \in (0,1)$, the transversality condition is satisfied in the non-stochastic steady state.

Log-linearizing the system around the non-stochastic steady state yields

$$
\begin{aligned}
\hat{q}_t + \frac{i}{1+i}\hat{i}_t &\equiv 0 \\
-\frac{1}{\epsilon}\hat{c}_t &= \hat{\nu}_t + \hat{p}_t \\
\hat{q}_t + \hat{\gamma}_t + \hat{\nu}_t &= E_t[\hat{\nu}_{t+1}] \\
qb[\hat{q}_t + \hat{b}_t] + pc[\hat{p}_t + \hat{c}_t] &= a\hat{a}_t \\
q\gamma[\hat{q}_t + \hat{\gamma}_t] + (1-q)py[-\frac{q}{1-q}\hat{q}_t + \hat{p}_t + \hat{y}_t] &= 0 \\
\hat{p}_t + \hat{c}^*_t &= \hat{a}^*_t \\
\hat{\gamma}_t + \hat{a}^*_{t+1} &= \hat{p}_t + \hat{y}_t \\
\hat{b}_t &= \hat{d}_t \\
c\hat{c}_t + c^*\hat{c}^*_t &= y\hat{y}_t \\
\hat{m}_t &\equiv \hat{p}_t + \hat{y}_t \\
a\hat{a}_t + a^*\hat{a}^*_t &= 0
\end{aligned}
\tag{14}
$$

where the variables without the time subscript are the non-stochastic steady state values, while the variables with the hat are the percentage deviations from the steady state values.

The system (14) together with the law of motion (3) for the exogenous state can be reduced to a four equations system in the two exogenous variables $\hat{y}_t$ and $\hat{i}_t$, the endogenous state variable $\hat{a}^*_t$ and the control variable $\hat{\nu}_t$. With standard methods, we, then, derive the linear system describing the equilibrium evolution of the three state variables $\hat{y}_t$, $\hat{i}_t$, and $\hat{a}^*_t$, and linking all the other variables to the three state variables[1]. In particular, we derive the the percentage deviations of normalized money $\hat{m}_t$ and normalized

[1]The solution method is based on the eigenvalue decomposition of the matrix describing the evolution of the state and control variables. Very small imaginary parts of the solution are dropped. As a check, the model has been solved using MATLAB files written by Chris Sims and Paul Klein as well. I thank them for making the files available at the web address http://www.ssc.uwo.ca/economics/faculty/klein/. Their solution method is based on the Schur decomposition of the matrix describing the evolution of the state and control variables. The two methods yield identical solutions.

prices $\hat{p}_t$ as linear functions of the three state variables.

As detailed in the following section, however, the data that we use to estimate the model are the deviations from steady-state values of the money growth rate $\hat{\mu}_t$ and the inflation rate $\hat{\pi}_t$. In the model, $\hat{\mu}_t \equiv \hat{m}_t + \hat{\gamma}_{t-1} - \hat{m}_{t-1}$, and $\hat{\pi}_t \equiv \hat{p}_t + \hat{\gamma}_{t-1} - \hat{p}_{t-1}$, so they both depend on the last-period as well as the current-period state variables. It is then necessary to re-define the state to include the last-period values as well as the current-period values of the original state variables $\hat{y}_t$, $\hat{i}_t$, and $\hat{a}^*_t$. We then define the state as

$$s_t \equiv [\hat{y}_t, \hat{i}_t, \hat{a}^*_t, \hat{y}_{t-1}, \hat{i}_{t-1}, \hat{a}^*_{t-1}]',$$

and we represent the solution of the linearized system (14) and the law of motion (3) as

$$s_t = M_s s_{t-1} + R\eta_t, \tag{15a}$$

where

$$M_s = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & 0 & 0 & 0 \\ \Pi_{21} & \Pi_{22} & 0 & 0 & 0 & 0 \\ m_{31} & m_{32} & m_{33} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad R = \begin{bmatrix} \Psi \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

where Ψ is the 2×2 lower triangular matrix previously defined in the law of motion (3), and $\mathbf{0}$ is a 1×2 vector of zeros. The structure of the model imposes that the exogenous variables $\hat{y}_t$ and $\hat{i}_t$ evolve following a vector AR(1) process, while the endogenous variable $\hat{a}^*_t$ is a function of the last-period values of $\hat{y}_t$, $\hat{i}_t$ and $\hat{a}^*_t$ itself. The shocks η_t are independently and identically distributed standard Gaussian shocks. The model takes as exogenous the dynamics of the percentage deviations of the aggregate endowment and the interest rate from their steady-state values and makes predictions about the endogenous dynamics of the deviations of the money growth rate $\hat{\mu}_t$ and the inflation rate $\hat{\pi}_t$. We then define the data of the model as

$$x_t = [\hat{\mu}_t, \hat{\pi}_t]'$$

and we represent the data equation of the state-space system as

$$x_t = M_x s_t, \tag{15b}$$

where M_x is a 2×6 system matrix.

3 Calculation of Likelihood Function and Estimation of Model

In this paper we use likelihood methods to compare the full participation and the segmented markets models described above. The philosophy behind using likelihood methods is so that all the information contained in the observed data can be brought to bear on the inferential problem at hand. The inferential problem that we face is twofold: First we want to estimate the structural parameters of the models described in Section 2, and second we want to compare various functions of interest, such as the impulse response functions, that are generated by the model.

There have been a number of approaches in the literature that have aimed to answer similar types of inferential problems. The first type of approach is the method of calibration where the structural parameters of laboratory economies are calibrated so that selected functions of the simulated data from the model economy "match" observed values of these functions from data. This approach has been widely used in macroeconomics and has yielded important insights about the way the economy operates in the face of different types of shocks[2].

The method of calibration, however, has not been without criticism, especially in the area of formal model evaluation and comparison[3]. This has led to the use of likelihood methods to estimate and formally evaluate the performance of highly stylized models of the macro-economy. In this paper, we follow authors such as DeJong, Ingram and Whiteman (1996, 2000), Geweke (1999), Schorfheide (2000), Fernandez-Villaverde and Rubio-Ramirez (2004) and Smets and Wouters (2003), and use Bayesian methods to answer the inferential problem described above. To use Bayesian methods we need to be able to calculate the likelihood function of the model we are estimating. For the types of models used in this literature this is not an easy problem. The most common approach is obtain a locally linear approximation to the model and use this to construct a likelihood function. In effect, we treat the linearized version of the model as the "true" model to be estimated.

The linearized version of our model can be written as

$$\begin{aligned} x_t &= M_x s_t \\ s_t &= M_s s_{t-1} + R\eta_t, \end{aligned} \tag{16}$$

where x_t is a $n \times 1$ vector of observed variables, s_t is a $m \times 1$ vector of state variables, and η_t is a $k \times 1$ vector of innovations to the system where $k \leq m$. The system matrices, M_y, M_s, and R are functions of the structural parameters of the model and reflect the restrictions imposed on the observed data by the model.

Suppose that we observe T observations on our data vector, x_t. Let this sample be denoted as $X_T = \{x_t\}_{t=1}^{T}$. Let θ be a $p \times 1$ parameter vector that includes all the parameters that determine the system matrices in (16). Then, using the Kalman Filter

[2] There is a very large literature using calibration methods. Important early papers using these methods are Kydland and Prescott (1982), Hanson (1985), and King, Plosser and Rebelo (1988). A very good summary of calibration and its uses can be found in Kydland and Prescott (1996).

[3] See, for example, papers by Hansen and Heckman (1996), Sims (1996) and Kim and Pagan (1998) for a good discussion of the issues surrounding calibration as an econometric tool.

(see Harvey (1989, page 104)) we can, for any particular value of the parameter vector θ, calculate the value likelihood function for the model given by (16)[4]. Let $p(X_T|\theta, \mathcal{M})$ represent that likelihood function of model $\mathcal{M}$ indexed by parameter vector θ.

Let $p(\theta|\mathcal{M})$ represent the prior density that we, the investigators, place over the parameter vector θ that indexes model $\mathcal{M}$. The prior distribution over the parameters θ represents our beliefs regarding the true values of the parameters of the model, and this acts as a way of imposing non-sample information onto our inferential problem. The information on θ contained in the data is combined with the non-sample information on θ via Bayes' Theorem,

$$p(\theta|X_T, \mathcal{M}) \propto p(\theta|\mathcal{M})p(X_T|\theta, \mathcal{M}). \tag{17}$$

The posterior distribution, $p(\theta|X_T, \mathcal{M})$, contains all information on the value of θ contained in the observed data and all non-sample information on θ supplied by the prior. The inferential problem, described above, boils down to estimating the following expected value:

$$E(g(\theta)|X_T, \mathcal{M}) = \int_{\Theta} g(\theta)p(\theta|X_T, \mathcal{M})d\theta, \tag{18}$$

where $g : \mathrm{R}^p \to \mathrm{R}^q$ is some well-defined (potentially) vector valued function of θ, and Θ is the domain of θ. In all but very special cases the integral defined in (18) cannot be calculated analytically or, because of the "curse of dimensionality", cannot be calculated using numerical integration techniques. In these cases we use Markov chain Monte Carlo (MCMC) methods (see Tierney (1994)) to simulate N serially correlated draws from $p(\theta|X_T, \mathcal{M})$, $\{\theta_1, \ldots, \theta_N\}$. Then as long as $E(g(\theta)|X_T, \mathcal{M}) = \overline{g} < \infty$ and $E(g(\theta) -$

[4]In order to calculate the likelihood function for a state-space model using the Kalman Filter we need to make an assumption about the initial value of the state vector, s_0. In what follows we treat the initial value of the state vector as a parameter of the model rather than using the steady state values of the state vector and the covariance matrix of the state vector as the initial conditions of the filter.

$E(g(\theta))^2|X_T, \mathcal{M}) = \sigma_g^2 < \infty$ then

$$\begin{aligned} \overline{g}(N) &= \frac{1}{N}\sum_{j=1}^{N} g(\theta_j) \xrightarrow{a.s.} \overline{g} \\ \sigma_g^2(N) &= \frac{1}{N}\sum_{j=1}^{N} (g(\theta_j) - \overline{g}(N))^2 \xrightarrow{a.s.} \sigma_g^2. \end{aligned} \tag{19}$$

Examples of functions that we use in this paper are the indicator function that selects one of the elements of θ and the impulse response function of an element of the data vector to a one unit shock to one of the structural shocks of the model.

An important consequence of using Bayesian methods is that we can use Bayesian model selection methods to perform formal model comparison between the two models we study in this paper. Formal model comparison is handled via the Bayes Factor which is the ratio of the marginal likelihoods of the two models. The marginal likelihood of a model, $\mathcal{M}_j$ is defined as

$$p(X_T|\mathcal{M}_j) = \int_{\Theta} p(X_T|\theta, \mathcal{M}_j) p(\theta|\mathcal{M}_j) d\theta, \tag{20}$$

and this represents the probability of observing the data, X_T, under model $\mathcal{M}_j$.

Given a prior probability for model $\mathcal{M}_j$, $p(\mathcal{M}_j)$, the posterior probability of model $\mathcal{M}_j$ conditional on the observed data, X_T, is

$$p(\mathcal{M}_j|X_T) = p(\mathcal{M}_j)p(X_T|\mathcal{M}_j). \tag{21}$$

Model comparison then involves the comparison of competing models that make predictions over the same observable data and / or functions of the data. Suppose for example that we wish to compare two models, $\mathcal{M}_j$ and $\mathcal{M}_k$. Let θ_j be the vector of parameters that index the likelihood function for model $\mathcal{M}_j$ and let θ_k be the vector of parameters that index model $\mathcal{M}_k$. The posterior odds ratio in favor of model $\mathcal{M}_j$ over model $\mathcal{M}_k$ is the ratio of the posterior probability of the respective models given in (21),

$$\frac{p(\mathcal{M}_j|X_T)}{p(\mathcal{M}_k|X_T)} = \frac{p(\mathcal{M}_j)}{p(\mathcal{M}_k)}\frac{p(X_T|\mathcal{M}_j)}{p(X_T|\mathcal{M}_k)}. \tag{22}$$

The first component of the posterior odds ratio is the prior odds ratio and the second component is the ratio of marginal likelihoods, also referred to as the Bayes Factor in favor of model $\mathcal{M}_j$ over model $\mathcal{M}_k$. Thus if we assign equal prior odds to each model the posterior odds ratio is equal to the Bayes Factor. What makes the Bayes Factor a natural model comparison tool is the fact that the marginal likelihood of a model can be shown to reflect the cumulative out-of-sample prediction performance of that model over the observed sample (Geweke (1995)). Model $\mathcal{M}_j$ is favored over model $\mathcal{M}_k$ if it has a higher posterior odds, and in the case of equal prior weights, if it has a higher marginal likelihood. Model $\mathcal{M}_j$, therefore, has a higher marginal likelihood only if it has a superior cumulative out-of-sample prediction performance than model $\mathcal{M}_k$.

The previous paragraphs outline the methods that we employ in this paper to formally compare our two competing models using the full dimension of the observed data. The next subsections now describe the models that we compare in detail and the prior beliefs that we impose.

3.1 Linear State-Space Representation of the Full Participation and Segmented Markets Models

The model that we study is the monetary model described in Section 2. We compare the segmented markets case where the traders' share λ of the aggregate endowment is constrained to be less than 1, and the full participation case where all households are traders ($\lambda = 1$). The full participation model, then, is a limiting case of the segmented markets model.

As we saw in Section 2, the solution to the linearized system describing the equilibrium is represented by the state-space system (15). The parameter vector θ, which indexes the state-space system (15), consists of three sets of parameters.

The first set of parameters are the structural parameters, the traders' share λ of the aggregate endowment, the intertemporal elasticity of substitution ϵ, the preferences discount factor β, and the aggregate endowment growth factor α in the non-stochastic steady state. It is convenient, however, to choose an equivalent set of structural parameters substituting β with the annualized percent real rate of return κ on financial assets in the non-stochastic steady state defined as

$$\kappa \equiv -log(\beta u'(\alpha)) * 400 = -log(\beta) * 400 + (1/\epsilon)\log(\alpha) * 400$$

where the multiplication by 400 converts the quarterly rates into a percent annualized rate. The structural parameters are, then, λ, ϵ and κ, which are estimated, together with α, which is calibrated.

The second set of parameters consists in the matrices Π and Ψ determining the law of motion (3) of the exogenous variables. They are calibrated and fixed in the estimation.

The final set of parameters are the initial values of the state vector, s_0. The estimation procedure that we employ in this paper involve using the Kalman Filter to calculate the likelihood function of the state-space model (15). To do this we need to make some assumptions about how the filter is initialized. We choose to treat the initial value of the state vector as parameters so the the Kalman filter is initialized with the prior mean of s_0 and the prior variance of s_0.

Hence the state-space model is parameterized by

$$\theta = (\lambda, \epsilon, \kappa, s_0', vec(\Pi)', vec(\Psi)', \gamma)' = (\theta_1, \theta_2)'$$

where $\theta_1 = (\lambda, \epsilon, \kappa, s_0')'$ are the "free" parameters of the model and $\theta_2 = (vec(\Pi)', vec(\Psi)', \alpha)'$ are the fixed parameters of the model. In what follows we fix the values of θ_2 and allow

θ_1 to be estimated using the estimation algorithm described in Section 3.2.

3.2 Estimation Algorithm

Given a set of observations on the growth rate of money supply and prices, $X_T = \{x_t\}_{t=1}^T$, and a prior distribution over the unknown parameters of the model, $p(\theta_1)$, the posterior distribution of θ_1 conditional on the observed data, the model and the fixed parameters is given by

$$p(\theta_1|X_T, \theta_2, \mathcal{M}) = p(\theta_1|\theta_2, \mathcal{M})p(X_T|\theta, \mathcal{M}), \tag{23}$$

where the model is either the segmented markets model $(0 < \lambda < 1)$ or the full participation model $(\lambda = 1)$.

Given values for θ and given the data X_T, the prediction error decomposition via the Kalman filter allows us to calculate the value of $p(X_T|\theta, \mathcal{M})$. Thus we are able to calculate the value of the posterior for any value of θ_1 in the domain of θ_1 given by

$$\Theta_1 = [0, 1] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^6.$$

The posterior distribution, given in (23), is in general not a standard distribution. In order to make some draws from this distribution we use a MCMC approach where we construct a Markov chain with $p(\theta_1|X_T, 1\theta_2, \mathcal{M})$ as its limiting distribution. After discarding a number of initial iterations from the algorithm and testing for convergence we use the last N iterations as draws from the posterior. The particular algorithm that we use is the random-walk Hastings-Metropolis algorithm described in Tierney (1994). The i^{th} iteration of this algorithm is as follows:

- given $\theta_1^{(i-1)}$, draw $x \sim N(0, V)$
- let $\tilde{\theta} = f(\theta_1^{(i-1)})$ where f(.) is a function such that $f : \Theta_1 \longrightarrow \mathbb{R}^9$
- let $\tilde{z} = \tilde{\theta}$+x and let $z = f^{-1}(\tilde{z})$

- let $\theta_1^{(i)} = \begin{cases} z & \text{with probability } \max\left[\frac{p(z|X_T,\theta_2,\mathcal{M})}{p(\theta_1^{(i-1)}|X_T,\theta_2,\mathcal{M})}, 1\right] \\ \theta_1^{(i-1)} & \text{else.} \end{cases}$

This algorithm is initialized by drawing $\theta_1^{(1)}$ from the prior for θ_1. The random step is made operational by transforming the parameter space, Θ_1 to $\mathbb{R}^9$ so that the random step is drawn from a Gaussian distribution with mean 0 and variance V. The variance-covariance matrix V is a tuning parameter of the chain and is chosen so that the draws from (23) has good numerical properties.

3.3 Data and Prior Distributions

The data used in this project are obtained from the FRED II database[5]. All series are quarterly data starting in the first quarter of 1959 and ending in the last quarter of 2002. The money supply series is the seasonally adjusted M2 series and the price series is the GDP deflator. Both of these series are detrended using the Hodrick-Prescott (1997) filter with the proportionate deviations from the trend being used in the construction of the likelihood function.

The parameters of the law of motion (3) are calibrated using a VAR(1) model with real output and the federal funds rate. The real output series is the seasonally adjusted real GDP series, and the federal funds rate is constructed from the monthly federal funds rate series by taking the last months value for each quarter. The proportionate deviations from trend of each variable is used to estimate the VAR. Finally, the value of α is calibrated to the long-run growth rate 0.0080 of real output, which translates to an annualized growth rate of 3.2%. A summary of the calibrated values of the fixed parameters can be found in Table 1.

The free parameters of the models are λ (for the case of the segmented markets model only), ϵ, κ, and the initial value of the state vector, s_0. Since all variables in the

[5] http://research.stlouisfed.org/fred2/

Table 1: Calibrated Values of Fixed Parameters: θ_2

Parameter	Value
Π	$\begin{bmatrix} 0.8790 & -0.0028 \\ 3.0946 & 0.9496 \end{bmatrix}$
Ψ	$\begin{bmatrix} 0.0080 & 0 \\ 0.0500 & 0.1272 \end{bmatrix}$
α	0.0080

state vector are measured in proportionate deviations from their long-run trend, the prior mean for each element of s_0 was chosen to be 0. The prior variance for s_0 was chosen to reflect our lack of information regarding the true initial value. Thus

$$s_0 \sim N(0, \mathbf{I}_6). \tag{24a}$$

A crucial parameter is the traders' share λ of the aggregate endowment, for which we choose a uniform prior over the interval $[0, 1]$ for this parameter. Thus,

$$\lambda \sim U(0, 1). \tag{24b}$$

For the intertemporal elasticity of substitution ϵ, we choose a log Gaussian prior with mean equal to 1 and standard deviation set to 0.1. That is,

$$\epsilon \sim logN(-0.005, 0.1). \tag{24c}$$

A value of $\epsilon = 1$ implies log utility. Finally, κ is the annualized percent real rate of return on financial assets in the non-stochastic steady state. We use a log Gaussian prior with

Table 2: Prior Distributions for Free Parameters: θ_1

Parameter	mean	std. dev.	90% HPD Region
λ	0.5	0.2887	—-
ϵ	1	0.1	[0.8181 1.1792]
κ	7.0	1.414	[4.4596 9.4513]

mean equal to 7.0 and variance equal to 1.7. That is,

$$\kappa \sim logN(1.8518, 0.2). \tag{24d}$$

The prior mean of 7 matches the average real rate of return of the S&P500 stock index over the sample period. Using a different financial asset would lead to a different value for κ, so we model the uncertainty surrounding κ with a relatively large prior variance. Table 2 gives the mean, variance and 90% highest prior density regions for these priors. It is assumed that all priors are independent of each other so that the prior for the free parameters is

$$p(\theta_1) = p(\lambda)p(\epsilon)p(\kappa)p(s_0). \tag{25}$$

The prior distributions given in Table 2 and the calibrated values given in Table 1 were then used in the random-walk Hastings-Metropolis algorithm defined above to make a set of drawings from the posterior distribution of θ_1 conditional on the values of θ_2 and the observed data for each model. The results of this are reported below. These results are reported in Table 3 in the second column under *estimation experiment*. In this experiment, the random-walk Hastings-Metropolis chain was used. For both models, because of the serial correlation inherent in the chain, 50,000 draws were made with only the 10^{th} iteration being accepted. As before, the chains were tested for convergence and the first 1000 iterations were excluded.

4 Results

We ran two sets of experiments. The first, which we refer to as the *baseline* experiment, is an experiment where we fixed the intertemporal elasticity of substitution ϵ equal to 1 (logarithmic utility), and the average real rate of return on financial assets κ equal to 7%, the average real rate of return of the S&P500 over the sample period. The second experiment, which we refer to as the *estimation* experiment, is an experiment where we allowed all parameters of the model to be estimated. The same priors for each of the free parameters of both models were used for each experiment. The only difference between the experiments is the fact that in the *baseline* experiment we fixed the intertemporal elasticity of substitution ϵ and the average return on financial assets κ.

In all experiments we used the random-walk Hastings-Metropolis chain to make draws from the posterior distribution of the model at hand. The resulting chain was tested for convergence and after deleting an initial burn-in number of iterations, the rest of the draws were used to compute posterior moments of our functions of interest. In the case of the *estimation* experiment it was necessary, because of increased serial correlation in the chain, to run the chain out for 50,000 iterations rather than the 10,000 iterations used in the *baseline* experiment. However, in all cases the numerical standard error for all moments was significantly less than 10% of the computed standard error for that moment. The iterations that were kept from each chain was used to compute posterior moments for all functions of interest. The first set of functions of interest, the structural parameters of the model, are reported in Table 3. Also reported in this table are the computed log marginal likelihood's for each model. These were calculated using the method proposed by Gelfand and Dey (1994). The standard errors reported for the log marginal likelihoods are numerical standard errors only.

The results for the *baseline* experiment are as follows. For the segmented markets model, the vector of free parameters is $\theta_1^{SM} = (\lambda, s_0')'$ and the vector of fixed parameters is $\theta_2 = (\epsilon, \kappa, vech(\Pi), vech(\Psi))'$. The prior for λ and s_0 is given in Table 2. The

Table 3: Summary of Posterior Estimates and Results

	Baseline Experiment			**Estimation Experiment**		
Segmented Markets Model						
Parameter	Mean	Std. Error.	90% HPD	Mean	Std. Error	90% HPD
λ	0.140	0.008	[0.126 0.153]	0.129	0.028	[0.084 0.175]
ϵ	1	–	–	0.996	0.101	[0.817 1.180]
κ	7	–	–	7.491	0.799	[5.801 9.095]
log ML	-559.398	0.104		-530.771	0.022	
Full Participation Model						
Parameter	Mean	Std. Error.	90% HPD	Mean	Std. Error	90% HPD
λ	1	–	–	1	–	–
ϵ	1	–	–	1.013	0.098	[0.833 1.170]
κ	7	–	–	3.281	0.025	[3.240 3.327]
log ML	-16159.02	0.025		-543.297	0.030	

random-walk Hastings-Metropolis chain was used to draw 10,000 draws from the posterior distribution of the segmented markets model. Various tests were performed to check for convergence, including checking for differences in estimated moments of the estimated parameters using different starting values of the chain. All tests suggested the chain had converged by the 1000^{th} iteration so the last 9000 draws were used to calculate the results given in Table 3. The only free parameter of interest in this experiment is λ, the proportion of households who participate in the bond market. The prior for this parameter was chosen to be uniform over the interval $[0, 1]$ which implies that we place equal prior weight on all possible values of λ. The posterior mean for λ for the segmented markets model was estimated to be 0.14 with a standard deviation of 0.008. The 90% highest posterior density region is [0.126 0.153]. This is significantly different from 1, the value of λ in the full participation model.

In the full participation model, for the *baseline* experiment, the only parameters that are freely estimated are the initial values of the state vector, s_0. It is clear, however, that the full participation model is overwhelmingly outperformed by the segmented markets model as the log marginal likelihood for the segmented markets model is overwhelmingly greater than the log marginal likelihood for the full participation model. Given that the log marginal likelihood reflects a model's out-of-sample prediction performance for the observed sample this suggests that the segmented markets is a better model.

In order to make sure that our results are due to the particular specification we choose for the intertemporal elasticity of substitution, ϵ, and for the average return on financial assets κ, we estimated the two models allowing for all parameters to be freely estimated. These results can be found in the second part of Table 3 under the heading *estimation experiment*. In the segmented markets model the posterior mean of λ is 0.129 with a posterior standard error of 0.028. The 90% highest posterior density region is [0.084 0.175]. In contrast to the results from the *baseline* experiment, the posterior mean for λ is slightly smaller but the posterior standard error is considerable larger. However, as in the previous case the value of λ is substantially smaller than 1, the value of λ in the full participation model.

The posterior mean and standard error for ϵ for each model is very similar and the 90% highest posterior density regions are almost identical to the 90% highest prior density region given in Table 2. This suggests the likelihood function is flat along the ϵ dimension which implies that there is little information regarding the appropriate value of ϵ to be found in the data.

The result for the average return on financial assets is, however, very different for the two models. The posterior mean of κ for the segmented markets model is 7.49% with a standard error of 0.799. The 90% highest posterior density region is [5.801 9.095] which is consistent with observed values of the return to financial assets over the period of our sample. The posterior mean for κ for the full participation model is quite different. The posterior mean is 3.28% with a posterior standard error of 0.025. The 90% highest posterior density region is [3.240 3.327]. This is certainly quite different from the results for the segmented markets model.

The overall performance of both models, as measured by the log marginal likelihood, has increased. The out-of-sample prediction performance of both models is significantly better than in the *baseline* experiment. The most pronounced change is in the performance of the full participation model. The log marginal likelihood for this model has improved from -16159.02 to -543.297. The log marginal likelihood for the segmented markets model has improved from -559.398 to -530.771. While both models' performance improved when we allowed for all the structural parameters to be estimated, it still is the case that the segmented markets model is vastly superior to the full participation model. The log Bayes Factor in favor of the segmented markets model over the full participation model is 12.53 which implies, assuming equal prior model weights, that the posterior probability of the segmented markets model conditional on the observed sample is $e^{12.53} = 2.75 \times 10^5$ higher than the posterior probability of the full participation conditional on the observed sample. That is, allowing for only a fraction of agents to participate in financial markets significantly improves the ability of the model to explain the observed data.

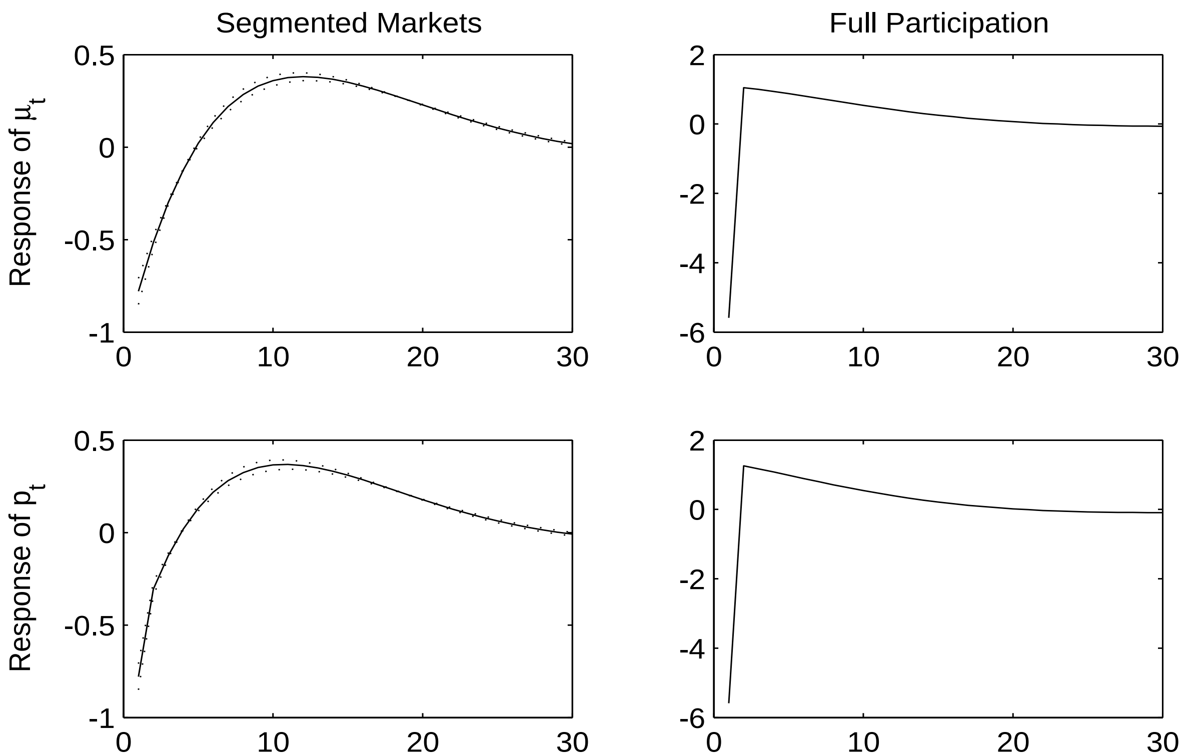
The above results suggest that there is overwhelming statistical evidence that the segmented markets model is a better model at explaining the observed data than the full participation model. An important question, however, is whether the segmented markets model makes better economic, or qualitative, predictions than the full participation model. To investigate this question, we consider the model impulse response function of the growth rate of money, μ_t, and the growth rate of prices, π_t to a monetary policy shock, and we compare it to the empirical impulse responses documented in Christiano, Eichenbaum and Evans (1999). Referring to the empirical impulse responses documented in other VAR studies, like Sims, Leeper and Zha (1996), and Bernanke and Mihov (1998) would not overturn our conclusions.

Following arguments in Bernanke and Blinder (1992) and Bernanke and Mihov (1998), we identify a contractionary monetary policy shock as a one-percent unanticipated increase in the short-term nominal interest rate with no contemporaneous effect on real output. We then identify it ordering real output first and the federal funds rate second and last in the VAR(1) defined in (3), and using a Cholesky decomposition of the covariance matrix. The impulse response function is a function of the structural parameters of the model and so, given our draws of parameters from the posterior distribution of the various versions of the models, we can easily characterize the posterior distribution of these impulse response functions.

The impulse response functions to μ_t and π_t for the *baseline* experiment can be found in Figure 1. The first row of this figure reports the response of the growth rate of money to a contractionary monetary policy shock. The second row reports the response of inflation to a contractionary monetary shock. The first column is the impulse response calculated using the segmented markets model while the second column of the figure reports the impulse response function generated from the full participation model. The solid line in the graphs represent the posterior mean impulse response function and the dotted lines reflect the 90% highest posterior density region for each impulse response function. For the full participation model there are no dotted lines as all structural parameters are fixed.

The impulse response functions generated by the two models are very different qualitatively and quantitatively. In both responses, the initial impact of a contractionary shock is negative. The magnitude of the initial impact, however, varies greatly between models. The initial impact of a 1% increase in the federal funds rate is around -0.8% for both money growth and inflation in the segmented markets model, and around -5.5%

Figure 1: Posterior Estimates of Impulse Response Functions to a Monetary Policy Shock: Baseline Experiment

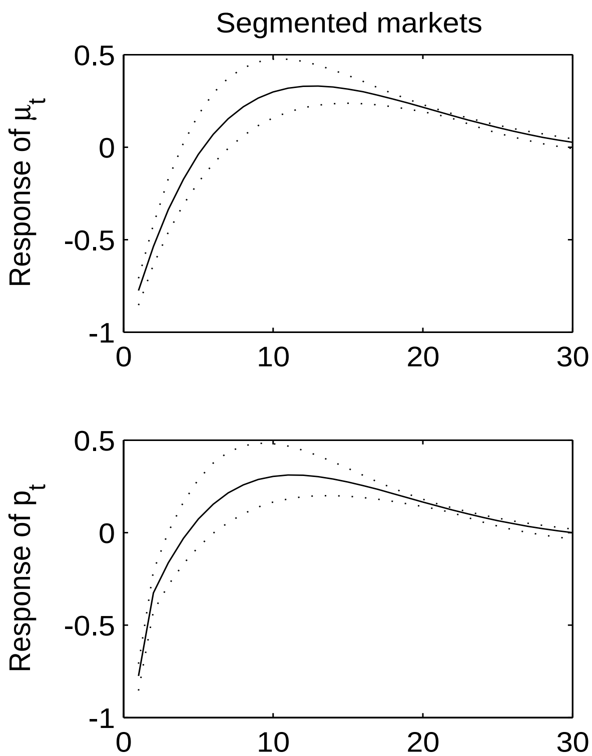


for the full participation model, which we regard as an unrealistic value for the initial response to the monetary policy shock. Consider, for instance, Figure 2 in Christiano, Eichenbaum and Evans (1999), which shows the empirical response to a contractionary monetary policy shock. The shock has an impact effect on the federal funds rate of about 75 basis points. The impact effect on the logarithm of M1 is less than 0.1, and the impact effect on the logarithm of M2 is less than 0.3. There is almost no effect on prices. Equivalently, the impact effect on the annualized M1 and M2 growth rate is respectively less than 0.4% and 1.2%, while the impact effect on the inflation rate is about zero. The segmented markets model does an excellent job in replicating the impact effect on the money growth rate, although it fails to replicate the lack of effect on the inflation rate. The full participation model, however, grossly fails to predict the magnitude of the impact effect on the money growth rate, and counterfactually predicts a much stronger effect on prices.

Furthermore, the impulse response functions of the full participation and the segmented markets models differ importantly with regard to how the shock propagates over time. In the segmented markets model, the responses of the money growth rate and the inflation rate are much smoother and more persistent and remain negative for four and three quarters respectively. This is not the case for the impulse response functions obtained from the full participation model, where the response of money growth and inflation is positive for all periods after the initial period. Figure 2 in Christiano, Eichenbaum and Evans (1999) shows that, empirically, the M1 growth rate is negative for three quarters, the M2 growth rate is negative for five quarters, while the response of prices is not statistically significant. Differently from the full participation model, the segmented markets model succeed in replicating the response of the money growth rate, both qualitatively and quantitatively. It cannot replicate the lack of response of the inflation rate, although the magnitude of the response is much smaller and much closer to data than in the full participation model.

The impulse response functions obtained from the two models under the *estimation* experiment have similar qualitative properties to the ones reported above. These new impulse response functions can be found in Figure 2. The estimation of the structural parameters has affected, not surprisingly, the impulse responses generated by the full participation model the most. Now the initial response to a 1% increase in the federal funds rate leads to a decrease in money growth and inflation of approximately -0.06%. This is much smaller than in the *baseline* experiment but the overall shape of the response has remained the same, so the response is positive for all periods except the first. The responses generated by the segmented markets model, on the other hand, look very similar to the baseline case. The initial response is negative and the response for the first five and four periods are negative for money growth and inflation respectively.

Figure 2: Posterior Estimates of Impulse Response Functions to a Monetary Policy Shock: Estimation Experiment



5 Conclusion

In this paper we used likelihood methods to evaluate the impact of including a market segmentation friction in equilibrium monetary models. To do this we estimated a benchmark full participation monetary model and compared it to a model in which only a fraction of households participate in financial markets. There were two parts to this comparison: First we compared the statistical properties of both models and then we compared their qualitative predictions.

A major finding is that the segmented markets model statistically dominates the full participation model in that the out-of-sample prediction performance of the segmented markets model, as measured by the log marginal likelihood, is vastly superior to the full participation model. Moreover, when we allow the fraction of the households who participate in financial markets to be estimated, we find this fraction to be approximately 13%. This is in contrast to the full participation model, where all agents participate in financial markets.

Christiano, Eichenbaum and Evans (1999) argue that comparing the model response to monetary policy shocks with the empirical response is also an important criterion for selecting a framework for monetary analysis. The segmented markets model dominates the full participation model along this dimension as well. The impulse response functions to monetary policy shocks generated by the segmented markets model are more persistent and more realistic. The response of the money growth rate closely matches the corresponding response documented in the empirical literature. The initial response of the money growth rate to a contractionary monetary policy shock is approximately -0.7%, and remains negative for three quarters, like in the data, whereas in the full participation model the initial response is only -0.06%, and becomes positive immediately after the impact period. The segmented markets model also predicts that the magnitude of the impulse response function of the inflation rate is much smaller and much closer to the empirical response relative to the full participation model. Our results, both statistical and qualitative, therefore strongly suggest that taking the presence of market segmentation into account is important in understanding the short-run dynamics of the monetary sector.

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