The demand for currency at low interest rates: implications for the welfare cost of inflation *

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Abstract

Money holding behavior at low interest rates is central for evaluating the welfare costs of low inflation. We study this issue by focusing on Italian households' demand for currency using a unique panel of micro data that span the 1989-2002 period. The more recent data are extracted from a low inflation sample which offers the possibility to analyze money demand at low interest rates. The results provide information on the "optimal rate of inflation" and offer insights on the properties that should be featured by a data-consistent model of currency holding behavior. A simple modification of the standard currency "inventory" model is proposed which allows us to explain several aspects of the data and to provide an assessment of the welfare costs of inflation.

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Key Words: optimal inflation rate, demand for currency,inventory models.

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1 Introduction

Reducing inflation has been the priority of most central banks during the past 25 years. But while a broad consensus emerged about the "price stability" goal, quantification of this objective remains controversial. This is partly due to the fact that the costs of ("low") inflation are difficult to measure.

Monetary theory offers a measure of the inflation costs. In a monetary economy inflation requires individuals to spend resources to economize on cash balances. Reducing inflation diminishes the resources that each citizen dedicates to such an unproductive activity, and therefore increases welfare. This simple and powerful insight, made explicit by the seminal contributions of Bailey (1956) and Friedman (1969), is still the core of the theory on the costs of inflation.

Measuring the welfare costs of inflation in accordance to this view requires information on the interest elasticity of money demand, to gauge the magnitude of the distortion induced upon the holdings of non-interest bearing assets by a greater rate of inflation. Following this line of reasoning Lucas (2000) estimates that the welfare gain of reducing inflation from about 14 to 3 per cent is comparable to a permanent increase of one per cent of GDP. Moreover, he shows that while this quantification is relatively robust to the money demand specification adopted to fit the historical data, the welfare gain delivered by a further inflation reduction crucially hinges on the interest elasticity of money demand at low interest rates. Depending on the money demand specification, the gain of reducing inflation from 3 per cent to zero ranges from about 1 per cent of GDP to basically nil. As Lucas put it "money holding behavior at low interest rates is central for estimating welfare costs". From a policy perspective, the point is key to gauge whether the current low inflation rates allow citizens to get rid of the bulk of inflation costs or whether an additional inflation reduction would yield more benefits. This paper attempts to provide an answer.

There are two main empirical difficulties, though, in answering this important question. The first one is that, as shown by Attanasio, Guiso and Jappelli (2002), different degrees of financial sophistication across households may bias the inference on money-holdings behavior drawn from aggregate data. This problem requires using micro data, which are not generally available. The second is that most observations available until recently, whether macro or micro, were not drawn from a "low interest rate" sample.

This paper tackles both problems. We use data on currency management at the household level drawn from a unique survey of Italian households conducted by the Bank of Italy approximately every two years between 1989 and 2002. The more recent data include a large number of observations drawn from a low-inflation period; these offer a great natural laboratory to understand money demand at low interest rates.

The next section describes the database and portrays the main developments in the financial variables of interest over the past 15 years. Estimates of a simple inventory-theoretic model of the demand for currency, following Attanasio et al.

(2002), are presented in Section 3. Several results suggest the inadequacy of the basic inventory theoretic model in explaining the data. Section 4 proposes a simple modification of that model which allows us to interpret our results and discuss the welfare costs of inflation. A final section concludes.

2 Currency and financial innovation in the last 15 years: data sources and descriptive statistics

2.1 Sources

The main source of the data is the *Survey of Household Income and Wealth* (SHIW). This is a periodic survey conducted by the Bank of Italy since 1965[1] on a rotating sample of Italian households that includes on top of individual characteristics of the components of the household, information on employment and income, on wealth and on consumption and saving behaviors. We focus here on the surveys from 1989 to 2002 because they include a section that is specifically related to households' use of forms of payment and on cash management. In particular, the first part of this section is focused on the payment technologies available to the household (from bank transfers to periodical payments, from the use of bank checks to that of debit cards), on the number and characteristics of bank accounts (e.g. interest rate received on deposits or the possibility of overdraft) and on the use of ATM and/or credit card. The second part is centered on currency management and includes information on the average monthly number of withdrawals (both through ATM cards and directly from the bank desk) and deposits, to the average amount of cash held by the household, the minimum level of currency holdings that triggers a withdrawal, the amount of consumption expenditures that is done in cash.

Each survey is conducted on a sample of about 8000 households. Samples are not representative, however sampling weights that allow to bring some socio-demographic marginal distributions into line with the corresponding distributions found in Istat population statistics and labor force survey are available.[2] The use of micro data from surveys like the one used in this work poses two important types of problems. The first one is related to intertemporal data comparability that may be affected by changes in the structure of the questions, the second one to well known problems of misreporting. As far as the first problem is concerned, in spite of many changes that have affected SHIW, most of the questions related to the data used in this paper are virtually unchanged. In addition in the few cases where the definition of variables changed over time, proper transformations have been applied to guarantee intertemporal comparability.[3] With respect to the second type of problems,

[1]The frequency of the survey has been annual from 1965 to 1987 and bi-annual since 1989. The only two exception are the fact that it was not conducted in 1985 and that there was a two year interruption from 1995 to 1998.

[2]Such weights have been used to compute the statistics presented in table 1.

[3]See description of the historical database for details.

available studies[4] suggest that they mainly affect data on financial wealth and, to a lesser degree, on real wealth and income; the data used in this work should be only marginally influenced by voluntary misreporting problems.

Other two sources of data are the *Italian Central Credit Register* and the *Supervisory Reports to the Bank of Italy.* The former source[5] includes information on the interest rate paid by banks on checking and saving accounts disaggregated by year and province. The latter is constituted by reports that banks operating on the Italian territory are required to send periodically to the Bank of Italy and include information on balance sheet and income statements, as well as data on financial intermediation and on the diffusion of different forms of payment. From this source we have obtained data on the number of ATM points and POS machines disaggregated at provincial level.

2.2 A first look at the data

The data obtained from the three sources described in the previous section has been merged in a single database. It includes 55643 observations over the sample period 1989 - 2002. All variables reported in Table 1 are expressed in euros of 2002. The figures summarize the main information about households' currency management and other financial variables that are expected to impinge on the demand for currency.

The first line of the table shows that about 15 per cent of the households do not hold a bank (nor a postal) account. This fraction does not vary much from 1989 to 2002. During the same period a steady growth of ATM card possession is recorded; the fraction of households increases from 15 per cent in 1989 to 55 per cent in 2002. This diffusion is accompanied by a marked increase in the number of ATM points, bank branches and POS machines (as a proportion of population).

Looking at currency, the average amount held by the household (measured in real terms) has almost halved during the past 15 years. The reduction, which is common across households with different payment technologies, is largest for those who owns an ATM card. The diffusion of new payment technologies is a likely cause of the increasing number of ATM withdrawals during the nineties.[6] The average size of the ATM withdrawal decreased over the past 15 years. Note, moreover, that the average withdrawal at an ATM is substantially smaller than the typical withdrawal at a bank desk.

Finally notice that the fact that the minimum amount of currency is substantially larger than zero (and greater for agents without the ATM), suggesting that a precautionary motive may play a relevant role in cash holding decisions. A satisfactory model of the demand for currency should integrate this feature.

[4]See for example Battipaglia and D'Alessio (1997).

[5]A detailed description of this database is contained in Miller (2000).

[6]Data for 1991 are not directly comparable with those of the following years since in that year the question on the average number of trips to a bank and to an ATM did not allow for annual frequencies. This is likely to have induced an upward bias in the estimates of the average number of trips and a downward bias in the estimates of the average withdrawal.

Table 1: Currency and Financial Innovation

Variable	1989	1991	1993	1995	1998	2000	2002
Fraction with a bank account	0.88	0.87	0.85	0.85	0.86	0.85	0.85
Fraction using ATMs	0.15	0.29	0.34	0.40	0.49	0.52	0.55
Bank branches[a]	...	341	384	421	470	501	532
ATM points[a]	...	221	315	393	505	570	649
POS machines[a]	...	907	1,547	2,939	6,490	11,218	14,928
Household Holdings of Currency	672	554	396	445	388	374	376
No bank account	660	582	359	441	376	396	494
With bank account	674	549	403	445	390	370	355
No ATM card	665	572	426	482	423	427	405
With ATM card	719	505	368	403	364	335	329
Average withdrawal at a bank	...	487	655	554	520	488	482
No ATM card	...	478	660	547	515	505	484
With ATM card	...	511	645	563	526	474	481
Average withdrawal at an ATM	...	259	243	235	231	232	217
Total number of trips[b]	...	29.0	27.4	30.3	41.9	44.4	43.0
To the bank (no ATM card)	...	18.4	12.5	13.1	19.9	16.6	17.5
To the bank (with ATM card)	...	14.9	12.7	10.7	11.9	13.0	10.5
To the ATM	...	35.5	36.7	39.2	46.8	48.9	46.2
Minimum currency	157	133	141	108	133	137	123
No ATM card	151	131	147	112	157	154	143
With ATM card	184	137	133	103	115	126	113
Nondurable consumption[b]	19,737	19,234	18,563	18,547	17,594	18,214	18,635
Total number of observations	8,262	8,135	7,979	8,134	7,122	8,001	8,010

Note: Averages are computed using sample weights. - Data are deflated (base: 2002) and expressed in euro. - Variables' definitions are available in the appendix. - [a] Per million residents in 1995. - [b] Per year.

Table 2 reports summary statistics on two measures of the interest rates paid on deposits. The first one is obtained from the "Central Credit Register" and is based on interest rates disaggregated by year and province.[7] The second measure, labeled "Individual", is obtained from a specific question available in SHIW since 1995.[8] Two remarks are due. First, interest rates paid on deposits record a substantial reduction since 1989. The decline is particularly significant in the second half of the nineties. Second, the cross-sectional distribution of the interest rate maintains a relatively large coefficient of variation even in the more recent years when the average interest rate is very low. This is an extremely important feature of the data because variability in interest rates is key for the inference on the interest elasticity of money demand.

[7] Italian provinces were 95 until 1995 and became 103 afterwards.

[8] In particular households were asked to report the gross interest rate earned on deposits. Information on individual interest rates is available only for a small subset of our sample. Note also that the smaller sample is likely to be the main reason for the differences in the temporal trend of the different measures of interest rate.

Table 2: Interest rates and inflation

Variable	1989	1991	1993	1995	1998	2000	2002
Interest rate (CCR)[a]	6.96	6.74	6.10	5.23	2.15	1.16	0.77
	(0.49)	(0.52)	(0.42)	(0.32)	(0.23)	(0.22)	(0.15)
Interest rate (Individual)	...	...	...	5.47	2.09	2.07	1.60
	...	...	...	(1.56)	(1.59)	(1.50)	(1.31)
CPI deflator[b]	0.62	0.70	0.77	0.84	0.91	0.95	1.00
Inflation[b]	6.3	6.3	4.6	5.2	2.0	2.5	2.5
Expected inflation current year[c]	6.5	6.4	5.0	5.0	1.9	2.3	2.2
Expected inflation next year[c]	5.8	5.7	4.6	4.4	1.9	1.9	2.0

Note: Standard errors in parenthesis. - [a] Computed as simple mean of individual interest rates disaggregated by province and year. - [b] Consumer price index. All items. - [c] Source: *Consensus Forecast*.

3 The demand for currency: econometric specification and estimation

The descriptive evidence offered in the previous section highlights that currency holding behavior is highly heterogeneous across the population of Italian households, especially concerning the households' payment technology.[9] As argued by Attanasio et al. (2002) (AGJ henceforth) accounting for this heterogeneity, e.g. by controlling for the household payment technology and the possibility of endogenous sample selection, is important to obtain precise measures of the interest elasticity.

Following their lead we present estimates of the demand for currency that integrate their sample, which ended in 1995, with the data made available by the three following surveys (1998, 2000, 2002). As Table 2 shows, the very low interest rates that characterize the more recent data seem appropriate to understand currency demand in a low inflation environment.[10] Let us now turn, therefore, to the estimation of the demand for currency.

As in AGJ, the baseline specification of the demand for currency is derived from the McCallum and Goodfriend (1987) extension of the Baumol-Tobin inventory model. The consumer chooses currency balance, m, to minimize the sum of the cost of transaction time τw (the product of transaction time τ and the time of cost transaction w) and foregone interest rates Rm, subject to a transaction technology:

$$\min_{\tau,m} \tau w + Rm \tag{1}$$

$$\text{subject to} \quad \tau = Ac^{\gamma}\left(\frac{c}{m}\right)^{\beta}$$

[9]Moreover, while part of this heterogeneity has been reduced, e.g. by the increasing diffusion of ATM cards and POS, other important differences persist (e.g. the percentage of households without a bank account is roughly constant).

[10]As noted above, it is important for the estimation that notwithstanding the reduction of the average rate, the cross-sectional variance of the interest rate paid on deposits remains high.

where A measures technology improvements and c is consumption. The *internal* solution of the minimization problem in (1) implies that the average holdings of real balances are equal to:

$$m = \left(\frac{wA\beta}{R}\right)^{\frac{1}{(1+\beta)}} c^{(\beta+\gamma)/(1+\beta)} \tag{2}$$

which yields the benchmark specification for the money demand equation:

$$\log m_{i,t} = \frac{1}{1+\beta}\log\beta + \frac{1}{1+\beta}\log w_{i,t}A_{i,t} - \frac{1}{1+\beta}\log R_{i,t} + \frac{\beta+\gamma}{1+\beta}\log c_{i,t} \tag{3}$$

We begin the analysis by assuming that the parameters of equation (3) are common among individuals who possess at least a bank or postal account. This choice overlooks differences that can arise from the payment technology available to households (e.g. the ATM), but provides us with a large number of observations (about 47,000). Further sample splits to control for the the effect of different payment technologies are considered below. Moreover, we assume that $\log w_{i,t}A_{i,t}$ depends only on calendar time. This is necessary to capture the time-series developments that influence the "cost of time". The main results are reported in the following table, where three specifications for this evolution are considered (quadratic trend, linear trend and year-dummies) .

Table 3: The Demand for Currency (agents with bank or postal account)

	89-02	89-95	98-02	89-02	89-95	98-02	89-02	89-95	98-02
$\log R_{i,t}^{CCR}$	0.025	-0.134	0.043	-0.176	-0.378	0.052	-0.002	-0.159	0.043
	(0.021)	(0.061)	(0.033)	(0.012)	(0.056)	(0.032)	(0.028)	(0.060)	(0.033)
$\log c_{i,t}$	0.299	0.353	0.221	0.302	0.356	0.221	0.297	0.352	0.221
	(0.007)	(0.009)	(0.012)	(0.007)	(0.009)	(0.012)	(0.007)	(0.009)	(0.012)
Trend	-0.112	-0.178	-0.078	-0.082	-0.094	-0.004	...	...	...
	(0.003)	(0.009)	(0.079)	(0.002)	(0.003)	(0.009)	...	...	...
Trend^2	0.004	0.012	0.003	...	...	...	...	...	...
	(0.000)	(0.001)	(0.003)	...	...	...	...	...	...
Constant	3.400	3.260	3.957	3.709	3.590	3.509	3.355	3.125	3.447
	(0.080)	(0.142)	(0.499)	(0.075)	(0.139)	(0.157)	(0.086)	(0.141)	(0.114)
R^2	0.123	0.113	0.019	0.121	0.109	0.019	0.130	0.123	0.019
Total obs.	46,978	27,277	19,701	46,978	27,277	19,701	46,978	27,277	19,701
Year	no	no	no	no	no	no	yes	yes	yes

Note: Standard errors in parenthesis. - The dependent variable in the regression is the logarithm of real currency. - Sample periods at the top of the columns. - Samples are restricted to individual with a bank and/or postal account.

The first column reports the full sample estimates for equation (3), using the quadratic trend specification of AGJ. The magnitude of the consumption elasticity of the demand for currency is comparable to that found by AGJ. But the interest elasticity is much smaller than the one detected by AGJ and statistically not significantly different from zero. This finding suggests that the interest elasticity of the demand for currency is not stable over the whole sample. The regressions in columns 2 and 3 explore this hypothesis. The second column of Table 3 estimates the equation using the AGJ sample. A negative and significant interest elasticity emerges, as found by AGJ. But the regression over the more recent sample indicates a coefficient that is not significantly different from zero (third column). In contrast, the consumption elasticity is positive and highly significant in both subsamples, even though it is greater over the first part of the sample. This could be due to improvements in payment technology (more on this later on). The two alternative specifications for the opportunity cost of time considered (analyzed respectively in columns 4-6 and 7-9) confirm these findings. The interest elasticity of the demand for currency appears clearly over the first part of the sample (until 1995) but the estimates over the more recent sample (1998-02) detect a coefficient that is not significantly different from zero.

The instability of the interest elasticity is robust to more precise estimation of the demand for currency. In particular, OLS estimates of equation (3) could be subject to a "sample selection problem". In essence, the problem arises from the fact that the sample used for the estimation of equation (3) is not randomly selected: the choice of opening a bank account or to have an ATM card is likely to be affected by unobserved individual characteristics that also influence money demand. In this case parameters' estimates obtained through OLS are biased and inconsistent. A procedure developed by Heckman (1979) (the so called "Heckit" estimator) allows us to control for this problem. The methodology consists of two steps. First a probit is estimated over the full sample (including both individual with a bank account (uncensored observations) and those without a bank account (censored observations)) to obtain a measure (the "inverse Mills ratio") of these unobserved characteristics. In a second moment the inclusion of this measure as an auxiliary variable in the baseline OLS regression (that is run on uncensored observations) allows us to correct for the effects of unobserved characteristics and to obtain consistent estimates of the parameters of the money demand equation.

The "Heckit" estimates for households who own an ATM card are presented in Table 4.[11] We focus on this group because the results of AGJ show that the interest elasticity is much greater among these "financially sophisticated" agents than in the rest of the population. Our main question, based on the results of Table 3, concerns whether the value of this elasticity shows any instability in the low-interest rate period (1998-2002) compared to the previous sample (1989-1995).

Indeed, Table 4 highlights a dramatic change in the interest elasticity of the demand for currency. A large, negative and statistically significant interest elasticity (-0.480) is detected over the 1989-1995 sample; the magnitude of the coefficient is close to the one detected by AGJ over the same sample using a slightly different specification (equal to -0.59). The same coefficient becomes statistically insignificant and falls by one order of magnitude (to -0.058) when the equation is estimated over the 1998-2002 sample. No apparent instability emerges about the consumption elasticity of the currency demand. The "Mills ratio" is highly significant, indicating

[11] The results are not sensitive to the choice of the linear trend specification.

Table 4: The Demand for Currency (ATM group) - "Heckit" estimates

	1989-1995	1998-2002
$\log R_{i,t}^{CCR}$	-0.480	-0.058
	(0.109)	(0.039)
$\log c_{i,t}$	0.470	0.410
	(0.023)	(0.023)
Trend	-0.089	-0.023
	(0.008)	(0.011)
Constant	2.305	1.650
	(0.353)	(0.268)
Inverse Mills Ratio	0.184	0.221
	(0.026)	(0.028)
Total obs.	31,457	22,990
Uncensored obs.	9,360	12,150

Note: The equation is estimated using Heckman's two-stage procedure. - The dependent variable in the probit regression for the households with ATM (Account) equals one if the household has an ATM card (Bank or postal account), zero otherwise. The probit regressors also include dummies for education (4 levels), gender and area of residence (city center, semi center, outskirts), as well as a measure of financial wealth and POS and ATM per capita in province. - The dependent variable in the currency equation is the logarithm of real currency.

that the selection problem is important.[12]

As a further check, Table 5 attempts to improve the measurement of the "shopping time technology" and the opportunity cost of money, which influence the demand for currency (through their impact on the variable $w_{i,t}A_{i,t}$). As to the former, we utilize the information on the number of ATM and POS machines, measured at the provincial level (in per capita terms). The idea is that the diffusion of these instruments reduces the time that is necessary to make a certain amount of purchases (for instance because the distance from the nearest withdrawal point diminishes) and/or the cash amount of currency that is necessary to buy a given amount of goods (as purchases can be debited through the POS machine). Both effects tend to reduce $A_{i,t}$, the expected sign is therefore negative. We also include a variable to control for the effects of criminality rate (measured at the provincial level), which is presumed to increase the opportunity cost of currency (a negative sign is thus expected). Given the premises of this specification, estimation is applied only to the households who own an ATM.

This regression confirms the finding that the estimated interest rate elasticity is large and statistically significant in the 1989-95 sample and becomes small and insignificant afterwards. The estimated parameters for the effect of ATM diffusion,

[12]The same qualitative findings emerge if the equation is estimated over the larger sample that contains the households with (at least one) bank or postal account, regardless of whether they possess an ATM.

Table 5: The Demand for Currency (ATM group) - "Heckit" estimates

	1989-1995	1998-2002
$\log R^{CCR}_{i,t}$	-0.632	0.026
	(0.125)	(0.039)
$\log c_{i,t}$	0.367	0.326
	(0.024)	(0.022)
ATM diffusion[a]	-0.505	-0.672
	(0.084)	(0.050)
POS diffusion[a]	-0.043	-0.006
	(0.008)	(0.002)
Crime	-0.046	-0.018
	(0.007)	(0.007)
Trend	-0.071	0.038
	(0.008)	(0.012)
Constant	4.020	2.373
	(0.371)	(0.270)
Mills Ratio	-0.030	-0.016
	(0.030)	(0.029)
Total obs.	31,457	22,990
Uncensored obs.	9,360	12,150

Note: The equation is estimated using Heckman's two-stage procedure. - The dependent variable in the probit regression equals one if the household has an ATM card, zero otherwise. - The probit regressors also include dummies for education (4 levels), gender and area of residence (city center, semi center, outskirts), as well as a measure of financial wealth and POS and ATM per capita in province. - The dependent variable in the currency equation is the logarithm of real currency. - [a] Number of machines per million residents.

POS and the crime variables on cash holdings are significant in both samples and display the expected signs.

Finally, we check the robustness of the previous findings using the households' self-reported information on the interest rate earned on deposits, is available for a small subsample of households since 1995 (see Section 2). This seriously limits the number of observations and, moreover, it excludes high-interest rate years from the sample (the only one left being 1995). The estimates, for the households with an ATM, are reported in Table 6. We present results for three different samples, pairing-up consecutive surveys. The estimates detect a reduction in the interest elasticity of the demand for currency. A small but highly significant elasticity is detected over the 1995-1998 period, approximately of around -0.1. This value decreases tenfold, to 0.01, over the 2000-02 sample and becomes statistically insignificant. Differently from the previous results, the estimated consumption elasticity appears decreasing with time.

Table 6: The Demand for Currency (ATM group) - "Individual" interest rate

	1995-98	1998-00	2000-02
$\log R_{i,t}^{IND}$	-0.078	-0.038	0.009
	(0.027)	(0.019)	(0.021)
$\log c_{i,t}$	0.559	0.470	0.318
	(0.052)	(0.045)	(0.048)
ATM diffusion[a]	-0.315	-0.659	-0.916
	(0.116)	(0.091)	(0.105)
POS diffusion[a]	-0.018	0.004	0.010
	(0.008)	(0.004)	(0.004)
Crime	-0.025	-0.042	-0.032
	(0.015)	(0.012)	(0.014)
Constant	-0.018	-0.005	0.007
	(0.016)	(0.016)	(0.018)
Trend	0.312	1.214	2.769
	(0.614)	(0.519)	(0.556)
Mills Ratio	0.707	0.464	0.107
	(0.104)	(0.092)	(0.095)
Total obs.	4,433	5,614	3,700
Uncensored obs.	3,116	4,108	2,765

[a] See notes in Table 5.

4 Interpreting the data

This section presents a model to interpret the changes in interest elasticity detected over the sample period and discuss its implications for the "shoe leather costs" of inflation. We depart from the McCallum and Goodfriend inventory model, which implies a *constant* interest rate elasticity of the demand for currency. Other aspects of the model, moreover, are at odds with our data. In particular, the average balance before withdrawal is much larger than zero, which suggests a "precautionary component" in the demand for currency that cannot be generated by a deterministic model.

The key feature of the proposed model is to relax the assumption of the inventory model of Section 3 that all cash withdrawals are costly, and to explicitly account for the fact that, in a modern economy, agents often have an opportunity to withdraw at basically no costs, e.g. every time they find a withdrawal point next to the shopping point.

The diffusion of ATMs on the territory provides one example of a financial development which has significantly affected the frequency of low-cost withdrawals. Evidence on the increasing importance of the ATM as withdrawal devices is provided in Table 1. During the nineties both the territorial diffusion of ATMs and the fraction of the population who possesses an ATM card have increased significantly (respectively from 221 to 649 ATM points per million residents and from 15 to 55 per cent).

Altogether, we want to ask whether explicitly accounting for the fact that the cost of withdrawal might change according to specific situations can help in interpreting the evidence presented above.

4.1 A search model of the demand for currency

Consider an agent who, in each period, finances an exogenous stream of consumption expenditures equal to c (time is discrete and the agent has an infinite horizon). The agent needs cash to buy goods (i.e. we rule out credit goods). Shopping takes place in one of several locations of this economy, which are may or may not be endowed with a cash dispenser.

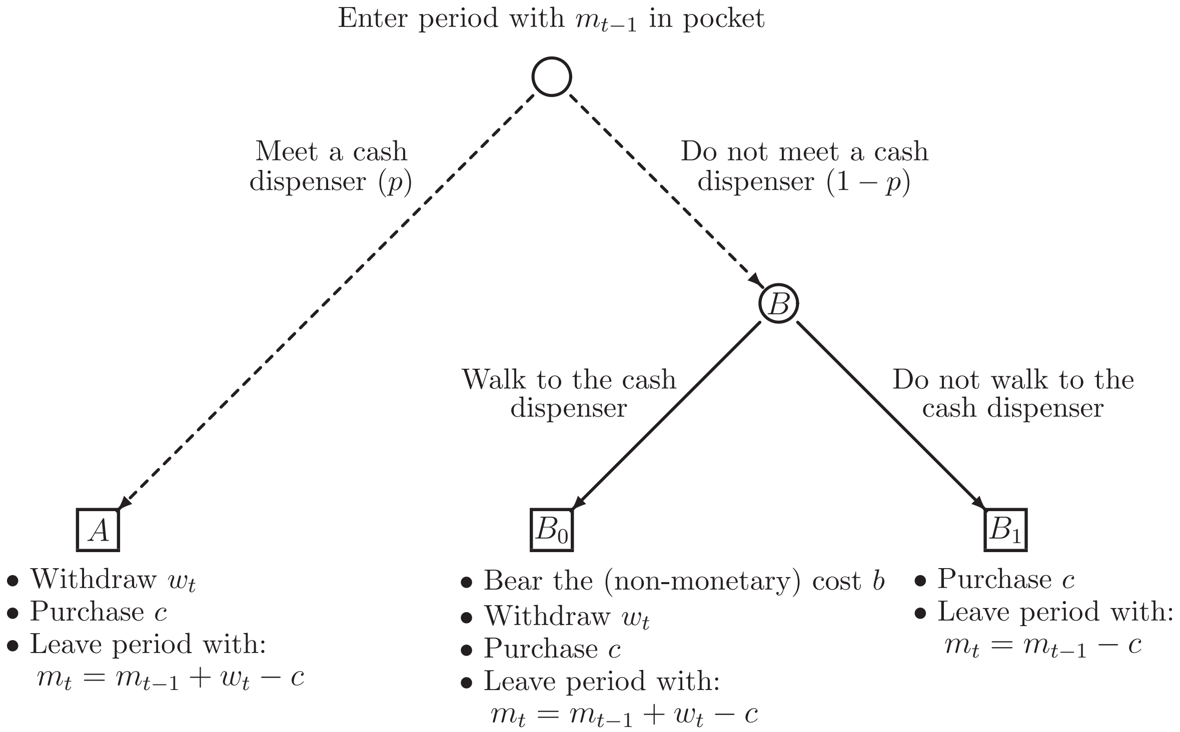
Our model captures the existence of both costly and non-costly withdrawals by assuming that if the agent happens to be shopping at a location that has a cash dispenser (think of it as either an ATM or a bank branch), then a withdrawal can take place at no extra cost. When the agent ends up shopping in a location without cash dispenser, a withdrawal requires her to waste time searching for a cash dispenser, this entails a non monetary cost equal to b (this withdrawal technology is completely analoguos to the one in the inventory model of Section 3). We measure the diffusion of the costly withdrawal technology by the probability $(1-p)$ that the agent ends up shopping in a location without a cash dispenser.

When every shopping point is endowed with a cash dispenser ($p = 1$), the shopper can reduce cash holdings to zero. However, as long as the probability p is less than one, the optimal quantity of currency demanded by the agent will be greater than zero. This quantity weights the costs of forgone interest on deposit (Rm, where R denotes the net interest rate) against the (expected) costs of ending up without cash at a shopping location, in which case the agent must search for a dispenser and bear the non-monetary cost b.

The sequence of events in a typical period of this economy is depicted in Figure 1. The agent goes shopping carrying m_{t-1} currency from the previous period. With probability p, the agent ends up shopping in a location which has a cash dispenser. At this point (node A in the figure), the agent makes a withdrawal w_t, she then consumes c and leaves the period with $m_t = m_{t-1}+w_t-c$ currency. With probability $(1-p)$, the agent ends up in a location without cash dispenser (node B in the figure). She may decide to search for it (bearing the non-monetary cost b) , withdraw w_t and consume c (node B_0 in Figure 1). Or she may decide not to search and pay for current consumption using the available currency holdings. In this case the end of period currency is $m_t = m_{t-1} - c$ (node B_1 in Figure 1).

Formally, the agent's problem can be written recursively as follows. Let $V_A(m)$ be the expected cost of financing the consumption flow for an agent who holds an amount $m \geq 0$ of currency and is matched with a cash dispenser. Let $V_B(m)$ be the corresponding expected cost for an agent who is unmatched. The value function $V_A(m)$ is given by :

Figure 1: Timing of actions within each period



$$V_A(m) = \min_w \{R(m+w) + \delta[pV_A(m+w-c) + (1-p)V_B(m+w-c)]\} \tag{4}$$

subject to $m+w \geq c$. The Bellman equation states that the optimal currency choice weights the current costs of a greater foregone interest with the future (δ denotes the discount factor) expected costs implied by the current withdrawal. For an agent who is not matched with the ATM :

$$V_B(m) = \begin{cases} \min\{b + V_A(m); Rm + \delta[pV_A(m-c) + (1-p)V_B(m-c)]\} & \text{for} \quad m \geq c \\ b + V_A(m) & \text{otherwise} \end{cases} \tag{5}$$

where the second line indicates that the agent must search for a cash dispenser and bear the cost b whenever currency holdings are not sufficient to pay for the current period consumption.

The policy function for this problem $w(s, m)$ is a map that, given the state of the world ($s = A$ or $s = B$) and the amount of money in hand, prescribes the agent whether and how much to withdraw.

Proposition 1. *The optimal policy function is:*

$$w^*(s,m) = \begin{cases} m^* - m & if \quad s = A \\ 0 & if \quad s = B \quad and \quad m \geq c \\ m^* - m & if \quad s = B \quad and \quad m < c \end{cases} \tag{6}$$

Proof. Let $z = m + w$, and define

$$m^* = \arg\min_z \{Rz + \delta(1-p)V_B(z-c)\}$$

Note that:

$$V_A = \frac{Rm^* + \delta(1-p)V_B(m^* - c)}{1 - \delta p}$$

solves the Bellman equation (4). Hence the value function V_A is constant and the optimal withdrawal for the agent matched with a cash dispenser is $w^* = m^* - m$.

Define the function $f(m) \equiv R(m) + \delta[pV_A + (1-p)V_B(m-c)]$, over the domain $m = nc$ with $n = 1, 2, 3, ...$

Note from (4) that $V_A = \min_m f(m) = f(m^*)$, which implies that $V_B \geq V_A$. The integer value $n^* \equiv \frac{m^*}{c}$ corresponds to the number of consumption periods financed by the optimal withdrawal.

Assume (which we verify below) that $V_B(nc) = f(nc)$ for $1 \leq n \leq n^*$. By recursive substitution into 5 we can express the value of V_B as a function of the currency stock nc as follows:

$$V_B(nc) = \frac{1-\varphi^n}{1-\varphi}\delta p V_A + cR\sum_{j=1}^{n}\frac{(1-\varphi^j)}{1-\varphi} + \varphi^n(V_A + b)$$

This shows that $V_B(nc)$ is a convex function of n, hence that it is decreasing for $1 \leq n \leq n^*$, i.e. over the $[c, m^*]$ range. Hence, if it is not optimal to walk to the dispenser when $m = c$, it is never optimal to walk to a dispenser for $m \leq m^*$.

The proof is completed by verifying that it is not optimal to walk to a cash dispenser when $m = c$. Compare the cost of walking to a dispenser $(b + Rm^* + \delta[pV_A + (1-p)V_B(m^* - c)])$ with the cost of not walking: $(b + Rc + \delta[pV_A + (1-p)(V_A + b)])$. Some algebra shows that the difference between these two is always greater than zero. Therefore, an agent at node B only chooses to withdraw when her cash balances are insufficient to finance the current consumption. □

The above result shows that there is an optimal level of currency balances, m^*, which is the same for all agents who decide to withdraw, irrespective of current holdings. We can then define the integer value $n^* \equiv \frac{m^*}{c}$ which corresponds to the number of consumption periods financed by the optimal withdrawal.

The policy rule thus prescribes the following: replenish currency balances till the target level m^* whenever a cash dispenser is available. Otherwise, the rule prescribes not to walk to a dispenser, provided currency holdings are sufficient to finance consumption. The only instance in which the fixed cost b is paid is when the agent is not matched with a dispenser *and* currency holdings are insufficient to pay for consumption.[13]

4.2 The demand for currency and interest elasticity

The model shows that the typical agent in this economy will be holding different amount of currency in different period, depending on whether she was matched with cash a dispenser this period, one period ago, and so on. The model thus generates a whole distribution for currency holdings.

Computing this distribution is necessary to study how the average currency holdings of a representative agent respond to changes in the nominal rate of interest and to the diffusion of cash dispensers (as measured by the probability p). Let the function $\Theta(m)$ denote the density of the end-of-period currency balances m.

Only a fraction π of agents replenish their balances in each period: after consuming they are left with currency holdings equal to $m^* - c$. The fraction π includes p individuals who are matched with a cash-dispenser (node A in Figure 1) and $(1-p)^{n^*}\pi$ agents who pay the cost b and undertake a costly withdrawal. The latter group is given by those agents who remain unmatched with a dispenser for n^* consecutive periods (node B_0 in Figure 1). The value for π thus satisfies: $\pi = p + (1-p)^{n^*}\pi$, which yields $\pi = \frac{p}{1-(1-p)^{n^*}}$.

Table 7: Currency holdings distribution

End of period currency (m_i)	0	c	...	$(n^* - \kappa)\,c$	...	$(n^* - 1)\,c$
Withdrawal (w_i)	m^*	$m^* - c$	...	κc	...	c
Population Fraction $\Theta(m_i)$	$(1-p)^{n^*-1}\,\pi$	$(1-p)^{n^*-2}\,\pi$	...	$(1-p)^{\kappa-1}\,\pi$	...	π

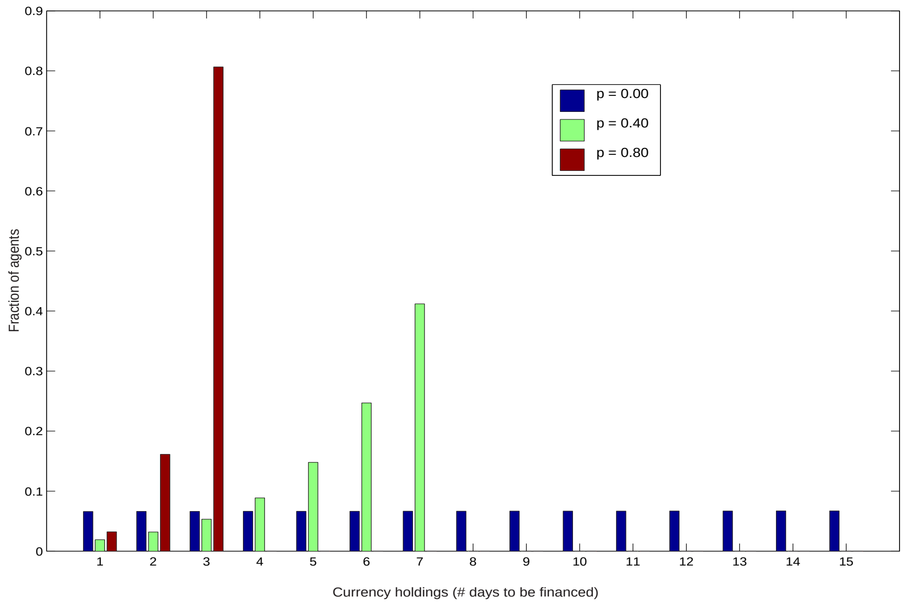
Similarly, one can compute the fraction of agents with end-of-period balances equal to zero, $\Theta(0)$. This is given by those agents who, after a withdrawal, remain un-matched with a cash dispenser for n^*-1 periods (hence $\Theta(0) = (1-p)^{n^*-1}\pi$). Following this line of reasoning, we compute the population fraction of agents with end-of-period balances equal to $m^* - \kappa c$ (where $\kappa = 1, 2, 3, ...n^*$), which is given by $\Theta(m^* - \kappa c) = (1-p)^{\kappa-1}\pi$. Table 7 summarizes the distribution of currency holdings in this economy.

A few numeric examples are used to illustrate the workings of the model for three alternative levels of the probability of a cheap withdrawal p.[14] The basic parametrization assumes the unit length of a period is a day, a daily consumption flow $c = 1$ and that the utility loss of a costly withdrawal is $b = 1/24$, i.e. about the value of one hour of time. We consider values of the nominal interest rate between

[13]Intuitively, for an agent at node B with $m \geq c$ it is not optimal to walk to the dispenser, since this choice entails paying the fixed cost b with certainty while the choice of not walking to the dispenser allows her to postpone paying this cost.

[14]Alvarez and Lippi (2004) provide an analytical characterization of these results.

Figure 2: Currency distribution for different values of p when $R = 0.10$



0.5 and 20 per cent and an intertemporal discount rate $\delta = 0.96$ (both variables are expressed in annual rates).

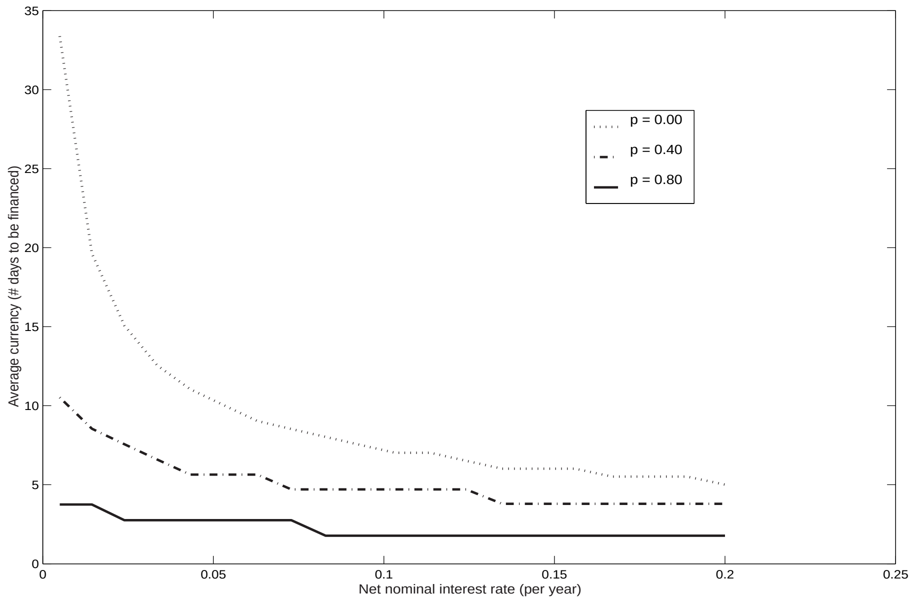
The three distributions of currency holdings originated by this parametrization are shown in Figure 2 for a given net nominal annual interest rate of 10 per cent ($R = 0.1$). When all withdrawals are costly ($p = 0$), the distribution of currency is uniform (the blue bars in the figure), as in the Baumol-Tobin case. As the probability of free withdrawals becomes positive, the distribution becomes unimodal, with the largest mass of agents located at the maximum currency holdings ($m^* - c$), and the remaining mass distributed to the left of this value in a decreasing fashion. It is also apparent that the size of the optimal balance m^* is decreasing in p, as one could intuitively expect. This reflects the fact that the demand for currency in this model has a precautionary component. The reason that motivates an agent to withdraw more currency than what is strictly necessary to pay for today's consumption is to reduce the chances of having to pay the cost b at some point in the future. A greater value of p, by reducing the odds of incurring in a costly withdrawal, reduces the agent's desired currency holdings.

The average currency balance held by each agent in this economy can be readily calculated as:

$$\bar{m} = \Sigma_i \left[m_i \cdot \Theta(m_i) \right] \tag{7}$$

Figure 3 depicts three schedules of the demand for currency corresponding to each of the p values (low, medium, high) described above. Each curve describes the average currency balance of an agent as a function of the nominal interest rate on deposits, R. For low values of p, the curve is downward sloping, with an interest elasticity that is approximately constant at around 0.5. The figure clearly shows that, as the probability of a costly withdrawal decreases, the interest elasticity of the demand for currency gets smaller. The model thus reproduces a qualitative feature of the data, namely that the interest rate elasticity of the demand for currency may decrease with time as the diffusion of ATM machines and bank branches allows the agents to avoid the costly withdrawals implied by the inventory model.

Figure 3: Currency Demand for different values of p



Finally, the model can be used to provide a partial equilibrium assessment of the welfare costs of inflation. The natural measure of welfare costs of our model is, analogously to the Baumol-Tobin approach, the deadweight loss associated to the costly withdrawal: b. The per-period welfare loss can thus be calculated from the proportion of individuals who bear the cost b on average in each period:[15]

$$l = b\,(1-p)^{n^*}\,\pi \tag{8}$$

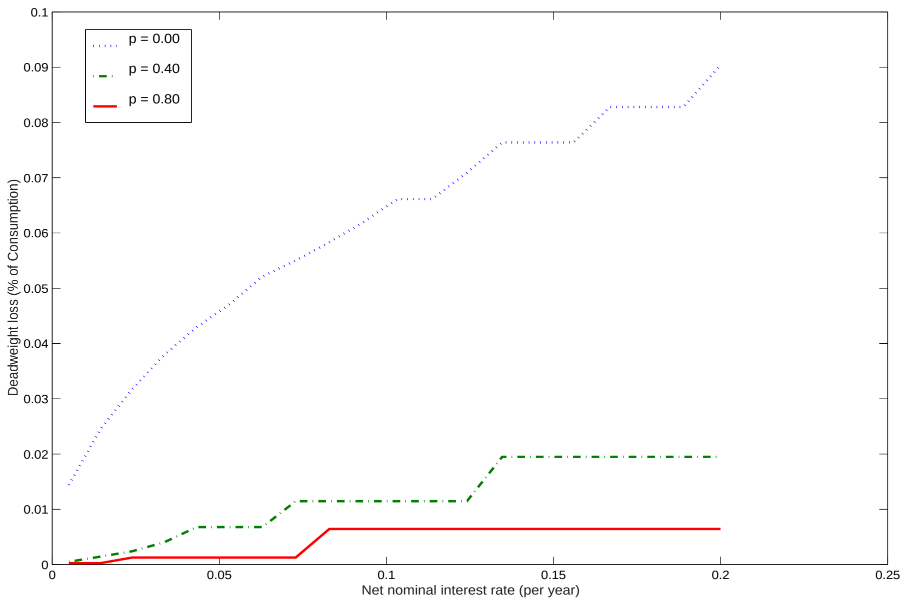
Figure 4 shows three profiles for welfare losses as a function of the net nominal interest rate R corresponding to the three levels of p considered above. When all withdrawals are costly ($p = 0$), the welfare loss is steeply increasing in R. The loss associated with a nominal interest of 5 per cent has a magnitude of about 5 per cent of consumption, quite a large number. The figure shows that the welfare costs of inflation (i.e. nominal interest rate) decrease sharply as the cheap withdrawal

[15]This value can also be interpreted as the expected welfare loss for an individual who lives in this economy.

technology becomes widely available (greater p). The welfare costs of a 5 per cent nominal interest rate when $p = 0.8$ is about 0.1 per cent of consumption.

Finally it is important to notice that in our model the welfare gain achievable from a reduction in inflation crucially depends on the parameter p. In particular, in a regime characterized by a low level of p (the dotted line of Figure 4), the reduction in the welfare loss resulting from a fall in inflation is sistematically larger than that obtained in a regime characterized by an higher level of p (e.g. the solid line of Figure 4).

Figure 4: Welfare Losses for different values of p



5 Concluding remarks

[TBW]

A Data description

This section draws from Attanasio et al. (2002). Information on sample design and response rates of the Survey of Household Income and Wealth can be found in Brandolini and Cannari (1994). In the empirical estimates, all demographic variables are referred to the head of the household. All monetary variables are deflated using the consumer price index, expressed in 1995 lire and then converted to euros.

ATM ownership. In each year, respondents report ownership of an ATM card. The surveys also contain information about the use of ATMs. In practice, virtually all those reporting having an ATM card also report using the ATM card.

Currency. The following question was asked to household heads in each of the surveys: *What is the average amount of currency usually held in your family?*

Minimum amount of currency. The following question was asked to household heads in each of the surveys: *Usually, what is the amount of currency that you have at home before you choose to make a currency withdrawal?*

Number of withdrawals and average withdrawal. The following questions were asked to household heads in each of the surveys: *Think about a normal month. How many currency withdrawals are made by you or members of your household? What is the average currency withdrawn?* These questions are asked separately for withdrawals at a bank or a post office, and at an ATM point.

Consumption. Consumption is the sum of the expenditure on food consumption, entertainment, education, clothes, medical expenses, housing repairs and additions, and imputed rents. Expenditures on durable goods (vehicles, furniture and appliances, or art objects) are therefore not included in the definition of consumption.

Deposits. Include checking accounts, savings accounts, and postal deposits.

Education of the household head. This variable is originally coded as follows: no education (zero), completed elementary school (five years), completed junior high school (eight years), completed high school (13 years), completed college (18 years), and graduate education (more than 20 years). The variables actually used in regression are dummies for each of the classes described above.

Financial wealth. Sum of checking accounts, savings accounts, postal deposits, government paper, corporate bonds, mutual funds and other managed accounts, and stocks. Data have been corrected to guarantee comparability across time.

Interest rate on deposits. We have data on the average nominal interest rate on checking accounts by year (1989, 1991, 1993, 1995, 1998, 2000 and 2002) and provinces (95 up to 1995 and 103 afterwards). The source is the Central Credit Register. This data set is then merged with the 1989-02 SHIW.

Number of ATM points per province. Data on the number of ATM points in each year/province are provided by the Supervisory Reports to the Bank of Italy. This data set is then merged with the 1989-02 SHIW.

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