Import Protection as Export Destruction

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1 Introduction

This paper develops a dynamic, stochastic industry model of heterogeneous firms to examine the effects of trade liberalization on resource reallocation, industry productivity, and welfare in the presence of import and export complementarities. We use the theoretical model to develop an empirical model which we estimate using Chilean plant-level manufacturing data. The estimated model is then used to perform counterfactual experiments regarding different trading regimes to assess the positive and normative effects of barriers to trade in import and export markets.

Empirical evidence suggests that relatively more productive firms are more likely to export (see, for example, Bernard and Jensen(1999), Aw, Chung, and Roberts (2000), and Clerides, Lack and Tybout (1998)). We provide evidence in this paper that *import* status may also be important for explaining differences in plant performance (see also, Kasahara and Rodrigue 2004). Our data also suggests that firms which are both importing and exporting tend to be larger and more productive than firms that are active in either market, but not both.

Other empirical work also indicates that there is a substantial amount of resource reallocations across firms within an industry following trade liberalization and these shifts in resources contribute to productivity growth in the sector. Pavcnik (2002) uses Chilean data and finds that such reallocations contribute to productivity growth after trade liberalization in that country.

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Trefler (2004) estimates these effects in Canadian manufacturing following the U.S.-Canada free trade agreement using plant- and industry-level data and finds significant increases in productivity among both importers and exporters.

Melitz(2003) develops a model of exporters with different productivities which is motivated by the empirical findings regarding exporters described above. To simultaneously address the empirical regularities concerning importers, we begin by extending his model to incorporate imported intermediate goods. In this environment, we allow final goods producers to differ with regard to both their productivity and their fixed cost of importing. We also incorporate complementarities in the fixed costs of importing and exporting. In the analysis, we focus on the role of intermediate imports and their interaction with final goods exports. In the model, the use of foreign intermediates increases firm's productivity but, due to the presence of fixed costs of importing, only inherently high productive firms start importing intermediates. Trade liberalization (e.g., removal of import tariffs) increases aggregate productivity because some inherently productive firms start importing and achieve within-plant productivity gains. This, in turn, leads to a resource reallocation from less productive to more productive importing firms. Furthermore, productivity gains among high productive firms through imported intermediates may allow some of them to start exporting, leading to an additional resource reallocation emphasized by Melitz(2003). Similarly, events that encourage exporting (e.g., liberalization in trading partners or export subsidies) may well have an impact on firm's decision to import since newly exporting firms would have a higher incentive to start importing. Thus, the model identifies an important mechanism whereby import tariff policy affects aggregate exports and whereby export subsidies affect aggregate imports.

To quantitatively examine the impact of trade on aggregate productivity and welfare, we develop and estimate a dynamic, stochastic industry equilibrium model of exports and imports using a panel of Chilean manufacturing plants. Our estimates suggest significant complementarities in both sunk and fixed costs of importing and exporting. Furthermore, the basic observed patterns of productivity across firms with different import and export status is well captured by the estimated model. We end by performing a variety of counterfactual experiments to examine the effects of trade policies. The experiments indicate that the equilibrium price response plays a major role in redistributing resources from less productive firms to more productive firms. In particular, experiments based on a partial equilibrium model that ignores the equilibrium price response provide fairly different estimates of the impact of trade on aggregate productivity. In addition, the welfare gain due to exposure to trade is found to be substantial. Another important finding from those experiments is that because of import and export complementarities, policies which inhibit the importation of foreign intermediates can have a large adverse effect on the exportation of final goods.

The remainder of the paper is organized as follows. Section 2 presents empirical evidence on the static and dynamic distribution of importers and exporters and their performance using Chilean manufacturing plant-level data. Section 3 presents a theoretical model with import and export complementarities. Section 4 presents a dynamic extension of that model. Section 5 provides details of the structural estimation while section 6 presents the data and results. Section 7 concludes.

2 Empirical Motivation

In this section we briefly describe Chilean plant-level data and provide summary statistics to characterize patterns and trends of plants which may or may not participate in international markets.

2.1 Data

We use the Chilean manufacturing census for 1990-1996. In the data set, we observe the number of blue collar workers and white collar workers, the value of total sales, the value of export sales, and the value of imported materials. The export/import status of a firm is identified from the data by checking if the value of export sales and/or the value of imported materials are zero or positive. The value of the revenue from the home market is computed as (the value of total sales)-(the value of export sales). We use the manufacturing output price deflator to convert the nominal value into the real value. The entry/exiting decisions can be identified in the data by looking at the number of workers across years. We use unbalanced panel data of 7234 plants for 1990-1996, including all the plants that have been observed at least one year between 1990 and 1996.

Table 1: Exporters and Importers in Chile for 1990-1996

	1990	1991	1992	1993	1994	1995	1996	1990-96 ave.
A Fraction of Exporters	0.168	0.190	0.199	0.212	0.216	0.222	0.214	0.203
A Fraction of Importers	0.209	0.214	0.237	0.252	0.263	0.245	0.243	0.238
A Fraction of Ex/Importers	0.082	0.096	0.107	0.120	0.131	0.124	0.127	0.112
% of Total Exports by Ex/Importers	0.569	0.571	0.507	0.575	0.660	0.607	0.607	0.585
% of Total Imports by Ex/Importers	0.648	0.683	0.685	0.713	0.788	0.780	0.746	0.720
% of Total Outputs by Exporters	0.567	0.609	0.639	0.630	0.652	0.657	0.659	0.630
% of Total Outputs by Importers	0.555	0.574	0.554	0.594	0.639	0.597	0.618	0.590
% of Total Outputs by Ex/Importers	0.388	0.445	0.405	0.441	0.501	0.454	0.483	0.445
No. of Plants	4722	4628	4938	5084	5040	5123	5455	4999

Note: "Ex/Importers" refer to plants that are both exporting and importing.

2.2 Importers and Exporters Distribution and Performance

Table 1 provides several important basic facts about exporters and importers. A relatively small fraction of plants are exporters and/or importers but their shares are increasing over time as shown in the first three rows of Table 1 and in Figure 1. A fraction of exporters (importers) went from 16.8 (20.9) percent in 1990 to 21.4 (24.3) percent in 1996. A fraction of plants that both export and import also increased from 8.2 percent in 1990 to 12.7 percent in 1996. A majority of exporters are also importers, or vice versa, in 1996. Furthermore, as shown in the fourth and the fifth rows of Table 1, plants that both export and import account for 60.7 percent of total exports and 74.6 percent of total imports in 1996. Plants that engage in both exporting and importing are increasingly common and play an important roles in determining the volume of trade.

While a relatively small fraction of plants are exporters and/or importers, their increasing importance in manufacturing activities is apparent from the sixth to the eighth rows of Table 1. A percentage of total outputs accounted for by exporters (importers) went up from 56.7 (55.5) percent in 1990 to 65.9 (61.8) percent in 1996. Plants that both export and import became increasingly important in accounting for total outputs; they constitute only 12.7 percent of the sample but account for 48.3 percent of total outputs in 1996.

How does the plant performance measures depend on export/import status? While the differences in a variety of plant attributes between exporter and non-exporter are well-known (e.g., Bernard and Jensen, 1999), few previous empirical studies have discussed how the plant performance measures depend on import status. Here we present results on the magnitude of the plant performance gap across different export/import status.

Table 2 presents the estimated premia in the various performance measures associated with export and import. Following Bernard and Jensen (1999), Columns (a)-(c) of Table 2 report the export and import premia estimated from a pooled ordinary least squares regression using the data of 1990-1996.

$$\ln X_{it} = \alpha_0 + \alpha_1 d^x_{it}(1 - d^m_{it}) + \alpha_2 d^m_{it}(1 - d^x_{it}) + \alpha_3 d^x_{it} d^m_{it} + Z_{it}\beta + \epsilon_{it},$$

where X_{it} is a variable of plant attributes (employment, sales, labor productivity, wage, non-production worker ratio, and capital per worker). $d^x_{i,t}$ is a dummy for year t's export status, $d^m_{i,t}$ is a dummy for year t's import status, Z include industry dummies at four-digit ISIC level, year dummies, and total employment to control for size.[1] The export premium α_1 is the average percentage difference between exporters and non-exporters among plants that do not import foreign intermediates. The import premium α_2 is the average percentage difference between importers and non-importers among plants that do not export. Finally, α_3 captures the percentage difference between plants that neither export nor import and plants that both export and import.

The results show that there are substantial differences not only between exporters and non-exporters but also between importers and non-importers. The export premia among non-importers are positive and significant for all characteristics except for the ratio of non-production workers to total workers as shown in column (a). The import premia among non-exporters are positive and significant for all characteristics in column (b), implying the importance of import status in explaining the plant performance even after controlling for export status. Comparing column (a)-(b) with Column (c), plants that are both exporting and importing tend to be larger and be more productive than plants that are engaged in either export or import but not both.[2] The point estimates suggest that the magnitude of the performance gap for various characteristics across different export/import status are substantial.

[1] Regional dummies are available only for a subset of samples and hence we did not include them as controls.

[2] Since export status is positively correlated with import status, the magnitude of the export premia estimated without controlling for import status is likely to be overestimated by capturing the import premia.

Table 2: Premia of Exporter and Importer

	Pooled OLS: 1990-1996			Long-Difference: 1990-1996		
Export/Import Status	(a) Export/ No-Import	(b) Import/ No-Export	(c) Export/ Import	(d) Export/ No-Import	(e) Import/ No-Export	(f) Export/ Import
Total Employment	0.915	0.636	1.463	0.155	0.060	0.214
	(0.019)	(0.016)	(0.018)	(0.040)	(0.026)	(0.034)
Total Sales	0.300	0.551	0.763	0.121	0.106	0.216
	(0.019)	(0.013)	(0.017)	(0.036)	(0.025)	(0.030)
Value Added per Worker	0.313	0.513	0.710	0.122	0.086	0.174
	(0.022)	(0.015)	(0.019)	(0.054)	(0.043)	(0.048)
Average Wage	0.194	0.338	0.435	0.088	0.067	0.092
	(0.011)	(0.009)	(0.010)	(0.024)	(0.019)	(0.023)
Non-Production/Total Workers	0.012	0.229	0.353	0.056	0.025	0.098
	(0.014)	(0.011)	(0.013)	(0.044)	(0.033)	(0.041)
Capital per Worker	0.458	0.489	0.759	0.115	0.130	0.309
	(0.023)	(0.017)	(0.021)	(0.060)	(0.050)	(0.055)
No. of Observations		33721			3248	

Notes: Standard errors are in parentheses. "Total Employment" reports the estimates for exporter/importer premia from regression excluding the logarithm of total employment from a set of regressors.

Columns (d)-(f) of Table 2 report the export and import premia estimated from a long-difference regression to control for plant fixed effects using the data of 1990 and 1996:

$$\begin{aligned}\ln X_{i,96} - \ln X_{i,90} &= \tilde{\alpha}_0 + \alpha_1[d^x_{i,96}(1-d^m_{i,96}) - d^x_{i,90}(1-d^m_{i,90})] + \alpha_2[d^m_{i,96}(1-d^x_{i,96}) - d^m_{i,90}(1-d^x_{i,90})] \\ &\quad +\alpha_3(d^x_{i,96}d^m_{i,96} - d^x_{i,90}d^m_{i,90}) + (Z_{i,96} - Z_{i,90})\beta + \eta_i.\end{aligned}$$

The results based on long-difference regressions in columns (d)-(f) show the similar patterns to those based on the pooled OLS in columns (a)-(c) although the standard errors are now much larger. Notably, all the point estimates for column (f) are larger than those reported in columns (d)-(e), suggesting that plants that are both exporting and importing are larger and more productive than other plants.

2.3 Importers and Exporters Dynamics

Table 3 shows the transition dynamics of export/import status in the sample as well as plant exiting and entry. The first four rows and columns in Table 3 present the empirical transition probability of export/import status conditioned that plants continue in operation. The results show

Table 3: Transition Probability of Export/Import Status and Entry/Exit

	Export/Import Status at $t+1$ conditioned on Staying						
	(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import	Exit at $t+1^a$
No-Export/No-Import at t	0.927	0.024	0.042	0.007	0.031	0.048	0.082
Export/No-Import at t	0.147	0.677	0.013	0.163	0.841	0.176	0.070
No-Export/Import at t	0.188	0.017	0.699	0.096	0.113	0.795	0.035
Export/Import at t	0.025	0.101	0.070	0.804	0.905	0.874	0.022
New Entrants at t^b	0.753	0.096	0.100	0.051	0.147	0.151	0.126
Empirical Dist. in 1990-96[c]	0.686	0.086	0.120	0.108	0.194	0.228	0.068

Note: a). "Exit at $t+1$" is defined as plants that are observed at t but not observed at $t+1$ in the sample. b). "New Entrants t" is defined as plants that are not observed at $t-1$ but observed at t in the sample, of which row represents the empirical distribution of export/import status at t as well as the probability of not being observed (i.e., exit) at $t+1$. c). "Actual Dist. in 1990-1996" is the empirical distribution of Export/Import Status in 1990-1996.

a substantial persistence in plant export/import status. Plants that neither export nor import, categorized as "No-Export/No-Import," is very likely (with 92.7 percent probability) to neither export nor import next period. Plants that both export and import (i.e.,"Export/Import") keep the same status next period with high probability of 80.4 percent. Plants that are either exporting or importing (but not both) keep the same status with probabilities of 67.7 percent and 69.9 percent, respectively, which is also substantial. The existence of persistence in export/import status is suggestive on the presence of sunk cost of export and import.[3]

The probability of exporting next period crucially depends on import status in this period even after controlling for export status. Among non-exporters, in the first and the third rows of the fifth column of Table 3, the probability of exporting next period for importers is 11.3 percent, which is substantially higher than for non-importers, 3.1 percent. Among exporters, the probability of exporting next period for importers is higher by (90.5-84.1=)6.4 percent than that for non-importers. Similarly, the probability of importing next period is higher among exporters than non-exporters even after controlling for importing status. The differences in the probability of importing next period for exporters are higher than for non-exporters by (17.6-4.8=)12.8 percent among importers and by (87.4-79.5=)7.9 percent among non-importers.

[3]Unobserved plant-specific characteristics may also lead to persistence in export/import status.

Plants that are engaged in export activities and/or import activities are more likely to survive as shown in the last column of Table 3. It shows that both export status and import status are important determinants of survival probabilities. While the exiting probability for plants that are both exporting and importing is only 2.2 percent, plants that are either exporting or importing (but not both) are more likely to exit next period with probabilities of 7.0 percent and 3.5 percent, respectively. Plants that are neither exporting nor importing have the highest exiting probability of 8.2 across different export/import status.

The fifth row of Table 3 reports the empirical distribution of export/import status as well as the probability of exiting next period among new entrants. Comparing the empirical distribution of export/import status among all plants reported in the sixth rows, new entrants are less likely to export or import than incumbents; the probabilities of exporting and importing among new entrants are, respectively, 14.7 percents and 15.1 percents, while the (unconditional) probabilities of exporting and importing among all plants are, respectively, 19.4 percent and 22.8 percent. New entrants face the exiting probability of 12.6 percent, which is substantially higher even relative to the exiting probability for plants that are neither exporting nor importing, 8.2 percent.

We now present a static model which is based on Melitz(2003) of heterogeneous firms and import and export cost complementarities to examine the impact of trade liberalization on resource reallocation and productivity. In the remaining sections, we modify this model to introduce interesting dynamic elements and employ structural estimation methods which use this data and the dynamic model.

3 A Model of Import and Export Complementarities

In this section we consider an environment in which final goods producers are heterogeneous with respect to their productivity and their fixed cost of importing intermediates. A firm's productivity is determined after they enter and is constant during the firm's lifetime. A firm's fixed import cost is random and is independent across periods. We compare the autarkic equilibrium to the trading equilibrium and focus on the effects of trade on average productivity and firm's market share and profits. We begin by describing the environment.

3.1 Environment

3.1.1 Consumers

There is a representative consumer who supplies labour inelastically at level L. The consumer's preferences over consumption of a continuum of final goods are given by:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{1/\rho},$$

where ω is an index over varieties, and $0 < \rho < 1$. The elasticity of substitution is given by $\sigma = 1/(1-\rho) > 1$. Letting $p(\omega)$ denote the price of variety ω, we can derive optimal consumption of variety ω to be

$$q(\omega) = Q \left[\frac{p(\omega)}{P} \right]^{-\sigma}, \tag{1}$$

where P is a price index given by

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}, \tag{2}$$

and Q is a consumption index with $Q = U$. We also denote aggregate expenditure as $R = PQ$.

3.1.2 Production

There are two sectors in the economy: a final-good sector and an intermediate-good sector. We first describe the final-good sector with a continuum of firms. As noted above, each firm produces a different variety indexed by ω. Final goods firms sell to domestic consumers and in the trading environment choose whether or not to also export their goods to foreign consumers. In production, final goods producers employ labor, domestically produced intermediates, and in the trading environment, choose whether or not to also use imported intermediates.

There is an unbounded measure of ex ante identical potential entrants. Upon entering, an entrant pays a fixed entry cost, f_e, and draws a productivity parameter, φ, from a continuous cumulative distribution $G(\varphi)$. A firm's productivity remains at this level throughout its operation. After observing φ, a firm decides whether to immediately exit or stay in the market. Final goods producers must pay a fixed production cost, f, each period to continue in operation. In addition, in each period, a firm is forced to exit with probability ξ.

Firms must also pay fixed costs associated with importing and exporting in any period that they choose to be active in those markets. Let $d^x \in \{0, 1\}$ denote a firm's export decision where

$d^x = 0$ implies that a firm does not export their good and let $d^m \in \{0, 1\}$ denote a firm's import decision where $d^m = 0$ implies that a firm does not use imported intermediates. Finally, let $d = (d^x, d^m)$. A firm that is exporting but not importing ($d = (1, 0)$) incurs a non-stochastic cost of $f_x > 0$. A firm that is importing but not exporting ($d = (0, 1)$) incurs a stochastic cost of $f_m + \epsilon$, where f_m is a constant cost common across all firms and ϵ is a firm-specific shock to the fixed cost of importing. The firm draws ϵ from a continuous cumulative distribution $H(\epsilon)$ with density $h(\epsilon)$ defined over $[-f_m, \bar{\epsilon}]$ with zero mean and observes ϵ before making the import/export decision. A firm that is both exporting and importing ($d = (1, 1)$) incurs a fixed cost equal to $\zeta(f_x + f_m + \epsilon)$, where $0 < \zeta \leq 1$ determines the degree of complementarity in fixed costs between exporting and importing. In summary, then, the per-period fixed cost of a firm that chooses d and draws ϵ is given by

$$F(\epsilon, d) = f + \zeta^{d^x d^m}[d^x f_x + d^m(f_m + \epsilon)]. \tag{3}$$

The technology for a firm with productivity level φ and import decision d^m is given by:

$$q(\varphi, d^m) = \varphi l(\varphi)^\alpha \left[\int_{j=0}^{n(d^m)} x(j, \varphi)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}$$

where $l(\varphi)$ is labor input, $x(j, \varphi)$ is the input of intermediate variety j, α is the labor share, and $\gamma > 1$ is an elasticity of substitution between any two intermediate inputs. The variable $n(d^m)$ denotes the range of intermediate inputs which are employed. It is assumed that, if a firm imports foreign intermediate inputs, then it employs a larger variety of intermediates than those who do not import so $n(1) > n(0)$.

The timing of decisions for a final goods producer can be summarized as follows. A new entrant pays the entry cost and then draws a productivity level φ. The firm may then be forced to exit with probability ξ. Surviving firms then choose whether or not to exit based on their φ and their expectations on ϵ. Firms draw an ϵ and then choose whether to import and whether to export. Firms then choose factor inputs and produce. Incumbents follow the same timing except they do not draw a new φ.

In the intermediate goods industry, there is a continuum of firms, each producing a different variety indexed by j. We normalize the measure of firms to 1 which implies that $n(0) = 1$. This industry is competitive and firms have identical linear technologies in labor input: $x(j) = l(j)$. Anyone can access the blueprints of the intermediate production technology for all varieties and there is free entry in this sector. These conditions imply that all intermediates will have the same price and that price will equal the wage, w, which we normalize to one.

We now examine the optimization problems of final goods producers. In the symmetric equilibrium, inputs of all intermediates which are used will be equal so $x(j,\varphi) = x(\varphi)$ for all j. In this case, production is given by

$$q(\varphi, d^m) = a(\varphi, d) l(\varphi)^{\alpha} [n(d^m) x(\varphi)]^{1-\alpha}, \tag{4}$$

where $a(\varphi, d^m) = \varphi n(d^m)^{\frac{1-\alpha}{\gamma-1}}$ is the firm's total factor productivity. Note that the firm's total factor productivity depends on not only inherent productivity, φ, but also the range of varieties of intermediates a firm employs $n(d^m)$, which in turn depends on the firm's import decision. Recalling the normalization that $n(0) = 1$, we can write

$$a(\varphi, d^m) = \varphi \lambda^{d^m} \tag{5}$$

where $\lambda = n(1)^{\frac{1-\alpha}{\gamma-1}} > 1$.

It is well-known that the form of preferences implies that final goods producers will price at a constant markup equal to $1/\rho$ over marginal cost. Hence, using the final goods technology and recalling that all intermediates are priced at the wage, $w = 1$, we have the following pricing rule for final goods sold in the domestic market for a producer with productivity φ and import status d^m:

$$p^h(\varphi, d^m) = \left(\frac{1}{\rho}\right) \left(\frac{1}{Aa(\varphi, d^m)}\right), \tag{6}$$

where $A \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. We also assume that there are iceberg exporting costs so that $\tau > 1$ units of goods has to be shipped abroad for 1 unit to arrive at its destination. The pricing rule for final goods sold in the foreign market then is given by:

$$p^f(\varphi, d^m) = \tau p^h(\varphi, d^m) \tag{7}$$

The total revenue of a final good producer depends on productivity and both export and import status and is given by

$$r(\varphi, d) = r^h(\varphi, d^m) + N r^f(\varphi, d). \tag{8}$$

Using equation (1) and $R = PQ$, we can derive

$$r^h(\varphi, d^m) = p^h(\varphi, d^m) q(\varphi, d^m) = R\left(P\rho A a(\varphi, d^m)\right)^{\sigma-1} \tag{9}$$

and

$$r^f(\varphi, d) = d^x \tau^{1-\sigma} r^h(\varphi, d^m), \tag{10}$$

Thus, we have

$$r(\varphi, d) = (1 + d^x N \tau^{1-\sigma}) r^h(\varphi, d^m), \tag{11}$$

Furthermore, the pricing rule of firms implies that profits of a final good producer with productivity φ, export/import status d, and fixed import cost shock ϵ can be written as

$$\pi(\varphi, \epsilon, d) = \frac{r(\varphi, d)}{\sigma} - F(\epsilon, d) \tag{12}$$

3.2 Exit and Export/Import Decisions

3.2.1 Exit

Under the assumptions of no discounting, that the productivity level for a firm is constant throughout its life, and that the fixed import cost shocks are independent across time, a final goods firm faces a static optimization problem. In particular, a firm will choose to exit if its expected period profits are negative where the expectation is taken over ϵ. Since certainty equivalence holds for firms, a firm will exit if

$$\max_d \pi(\varphi, E(\epsilon), d) = \max_d \pi(\varphi, 0, d) < 0.$$

Since revenue is increasing in φ, there will be some $\tilde{\varphi}$, such that for firms with $\varphi < \tilde{\varphi}$, their expected choice of d equals $(0, 0)$ and for firms with $\varphi \geq \tilde{\varphi}$, their expected choice of d equals $(1, 1)$. Furthermore, the expected profits of firms with $\varphi < \tilde{\varphi}$ must be below firms with $\varphi > \tilde{\varphi}$. Hence, the productivity of the marginal firm which chooses to exit based on expected profits can be determined by setting the expected profits of a firm which expects to neither export nor import equal to zero. Hence, this exit cutoff productivity, φ^*, will satisfy

$$\pi(\varphi^*, 0, 0, 0) = 0$$

or

$$r(\varphi^*, 0, 0) = \sigma f \tag{13}$$

We now relate all other firms' revenues to the revenues of this marginal firm. Using equation (9), we can derive $\forall\ (\varphi, d^m)$:

$$R\,(P\rho A)^{\sigma-1} = \frac{r^h(\varphi^*, 0)}{a(\varphi^*, 0)^{\sigma-1}} = \frac{r^h(\varphi, d^m)}{a(\varphi, d^m)^{\sigma-1}}$$

So

$$r^h(\varphi, d^m) = \left(\frac{a(\varphi, d^m)}{a(\varphi^*, 0)}\right)^{\sigma-1} r^h(\varphi^*, 0) \tag{14}$$

Using equations (5), (11), and (13), we can derive

$$r(\varphi, d) = \left(\frac{\lambda^{d^m}\varphi}{\varphi^*}\right)^{\sigma-1} (1 + d^x N\tau^{1-\sigma})\sigma f \tag{15}$$

or

$$r(\varphi, d) = \left(b_x^{d^x}\right)\left(b_m^{d^m}\right)\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \sigma f, \tag{16}$$

where $b_x = 1 + N\tau^{1-\sigma}$ and $b_m = \lambda^{\sigma-1}$.

3.2.2 Export and Import Decision

We now consider the export and import decisions for firms which choose not to exit. Recall that firms make exit decisions before observing ϵ but make export and import decisions after observing ϵ. Define the following:

$$\Phi(\varphi) \equiv \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} f, \tag{17}$$

For convenience, we can reference firms of different productivity levels by Φ where the dependence on φ is understood. We will refer to this variable as relative productivity. Thus, using equations (44) and (16), we can write profits in terms of Φ:

$$\hat{\pi}(\Phi, \epsilon, d) = \left(b_x^{d^x}\right)\left(b_m^{d^m}\right)\Phi - F(\epsilon, d) \tag{18}$$

To obtain the export and import decision rule for each φ, we define the following variables.

Let $\Phi_x^{d^m}(\epsilon)$ be implicitly defined by $\hat{\pi}(\Phi_x^{d^m}(\epsilon), \epsilon, 1, d^m) = \hat{\pi}(\Phi_x^{d^m}(\epsilon), \epsilon, 0, d^m)$ or

$$\Phi_x^{d^m}(\epsilon) = \frac{\zeta^{d^m} f_x + d^m(\zeta^{d^m} - 1)(f_m + \epsilon)}{b_m^{d^m}(b_x - 1)} \tag{19}$$

So a firm with import status d^m, fixed import cost shock ϵ, and relative productivity $\Phi_x^{d^m}(\epsilon)$ will be indifferent between exporting and not exporting.

Let $\Phi_m^{d^x}(\epsilon)$ be implicitly defined by $\hat{\pi}(\Phi_m^{d^x}(\epsilon), \epsilon, d^x, 1) = \hat{\pi}(\Phi_m^{d^x}(\epsilon), \epsilon, d^x, 0)$ or

$$\Phi_m^{d^x}(\epsilon) = \frac{\zeta^{d^x}(f_m + \epsilon) + d^x(\zeta^{d^x} - 1)f_x}{b_x^{d^x}(b_m - 1)} \tag{20}$$

So a firm with export status d^x, fixed import cost shock ϵ, and relative productivity $\Phi_m^{d^x}(\epsilon)$ will be indifferent between importing and not importing.

Let $\Phi_{xm}(\epsilon)$ be implicitly defined by $\hat{\pi}(\Phi_{xm}(\epsilon), \epsilon, 1, 1) = \hat{\pi}(\Phi_{xm}(\epsilon), \epsilon, 0, 0)$ or

$$\Phi_{xm}(\epsilon) = \frac{\zeta(f_x + f_m + \epsilon)}{(b_x b_m - 1)} \tag{21}$$

So a firm with fixed import cost shock ϵ, and relative productivity $\Phi_{xm}(\epsilon)$ will be indifferent participating in both exporting and importing markets and not participating in either market.

These variables allow us to determine the firms choice of d depending on their Φ and their ϵ. If we let $\theta \equiv f_m + \epsilon$, where $\theta \in (0, fm + \bar{\epsilon})$, then we can graph each of the cutoff variables defined above as a function of θ, and determine firm's export/import choices. We first consider the case with no complementarities in fixed export and import costs, $\zeta = 1$ and Figure 1 graphs cutoff functions for this case. Note that $\Phi(\varphi^*) = f$ so the active firms are those with $\Phi > f$. As the figure demonstrates, the space of (Φ, θ) is partitioned into four areas according to firms' export and import choices. Firms with relatively low productivity and low fixed cost of importing will choose to import but not export ($d = (0, 1)$). Firms with relatively low productivity and higher fixed cost of importing will choose to neither import nor export ($d = (0, 0)$). Firms with relatively high productivity and relatively high fixed cost of importing will choose to export but not import ($d = (1, 0)$). Finally, firms with relatively high productivity and relatively low fixed cost of importing will choose to both import and export ($d = (1, 1)$).

By examining the equations for the different regions, we can also determine the effect of complementarities in the fixed costs of importing and exporting. Recall that a decrease in ζ represents an increase in complementarities. Examination of equations (19)-(21) shows that a decrease in ζ will shift down and decrease the slopes of $\Phi_m^1(\cdot)$, $\Phi_x^1(\cdot)$, and $\Phi_{xm}(\cdot)$. As can be seen from Figure 1, each of these changes would serve to increase the measure of firms choosing to both export and import ($d = (1, 1)$) and decrease the measure of firms in each of the other three areas. This accords with intuition.

3.3 Autarkic and Trading Equilibria

In the previous section, we examined firm's export and import decisions contingent on φ^*. We now seek to characterize the equations which determine φ^* and compare the autarkic equilibrium with the trading equilibrium. We focus on a stationary equilibrium in which the distribution of firms over productivity levels, fixed import costs, and export and import status are constant over time.

3.3.1 Distribution of Firms

We first define the following critical values of θ, which allow us to partition the (Φ, θ) space according to firms' import and export decisions. Let θ_1 satisfy $\Phi_m^0(\theta_1) = f$ or

$$\theta_1 = f(b_m - 1).$$

Let θ_2 satisfy $\Phi_m^0(\theta_2) = \Phi_x^1(\theta_2) = \Phi_{xm}(\theta_2)$ or

$$\theta_2 = \frac{\zeta f_x(b_m - 1)}{b_m(b_x - \zeta) + (\zeta - 1)}.$$

Let θ_3 satisfy $\Phi_m^1(\theta_3) = \Phi_x^0(\theta_3) = \Phi_{xm}(\theta_3)$ or

$$\theta_3 = \frac{f_x(b_x(b_m - \zeta) + (\zeta - 1)}{\zeta(b_x - 1)}.$$

Finally, let $\theta_4 = f_m + \bar{\epsilon}$, which is the maximum value of θ. It can easily be shown that $\theta_1 < \theta_2 < \theta_3 < \theta_4$. Finally, we define $\epsilon_j \equiv \theta_j - f_m$ for $j = 1, \ldots, 4$.

Using the cutoff functions in terms of Φ defined in equations (19)-(21), we can define cutoffs in terms of φ as follows:

$$\tilde{\varphi}_i^j(\epsilon, \varphi^*) \equiv \left(\frac{\Phi_i^j(\epsilon)}{f}\right)^{\frac{1}{\sigma-1}} \varphi^* \quad i \in \{x, m\},\ j \in \{0, 1\} \tag{22}$$

$$\tilde{\varphi}_{xm}(\epsilon, \varphi^*) \equiv \left(\frac{\Phi_{xm}(\epsilon)}{f}\right)^{\frac{1}{\sigma-1}} \varphi^* \tag{23}$$

Using the critical values of ϵ, and these cutoff functions, we can define a weighted average of firms' productivities:

$$\tilde{b}(\varphi^*)^{\sigma-1} = \left[\begin{array}{c} \int_{\epsilon_1}^{\epsilon_2}\left(\int_{\varphi^*}^{\tilde{\varphi}_m^0(\epsilon,\varphi^*)} \frac{\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon + \int_{\epsilon_2}^{\epsilon_3}\left(\int_{\varphi^*}^{\tilde{\varphi}_{xm}(\epsilon,\varphi^*)} \frac{\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon \\ \\ + \int_{\epsilon_3}^{\epsilon_4}\left(\int_{\varphi^*}^{\tilde{\varphi}_x^0(\epsilon,\varphi^*)} \frac{\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon \end{array}\right]$$

$$+\left[\int_{-f_m}^{\epsilon_1}\left(\int_{\varphi^*}^{\tilde{\varphi}_x^1(\epsilon,\varphi^*)} \frac{b_m\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon + \int_{\epsilon_1}^{\epsilon_2}\left(\int_{\tilde{\varphi}_m^0(\epsilon,\varphi^*}^{\tilde{\varphi}_x^1(\epsilon,\varphi^*)} \frac{b_m\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon\right]$$

$$+\left[\int_{\epsilon_3}^{\epsilon_4}\left(\int_{\tilde{\varphi}_x^0(\epsilon,\varphi^*}^{\tilde{\varphi}_m^1(\epsilon,\varphi^*)} \frac{b_x\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon\right]$$

$$+\left[\begin{array}{c} \int_{-f_m}^{\epsilon_2}\left(\int_{\tilde{\varphi}_x^1(\epsilon,\varphi^*)}^{\infty} \frac{b_x b_m\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon + \int_{\epsilon_2}^{\epsilon_3}\left(\int_{\tilde{\varphi}_{xm}(\epsilon,\varphi^*)}^{\infty} \frac{b_x b_m\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon \\ \\ + \int_{\epsilon_3}^{\epsilon_4}\left(\int_{\tilde{\varphi}_m^1(\epsilon,\varphi^*)}^{\infty} \frac{b_x b_m\varphi^{\sigma-1}g(\varphi)}{1-G(\varphi^*)}d\varphi\right)h(\epsilon)d\epsilon \end{array}\right]$$

In this expression, the first bracketed set of terms reflects productivity of firms who neither export nor import, the second set reflects firms who import but not export, the third set reflects firms who export but not import, and the final set reflects firms who both import and export.

Alternatively, we can derive the joint distribution of (φ, d) (*insert these expressions later*) and denote it as $\mu(\varphi, d; \varphi^*)$. We can also define

$$b(\varphi, d) \equiv b_x^{d^x} b_m^{d^m} \varphi^{\sigma-1}$$

Using the joint distribution and this function, then we can write

$$\tilde{b}(\varphi^*) = \left[\sum_d \int \frac{b(\varphi, d)}{1 - G(\varphi^*)} \mu(\varphi, d; \varphi^*) d\varphi \right]^{\frac{1}{\sigma-1}} \tag{24}$$

We can also calculate average fixed costs as

$$\begin{aligned}
\tilde{F}(\varphi^*) = & \left[\begin{array}{c} \int_{\epsilon_1}^{\epsilon_2} \left(\int_{\varphi^*}^{\tilde{\varphi}_m^0(\epsilon,\varphi^*)} g(\varphi) d\varphi \right) F(\epsilon, 0, 0) h(\epsilon) d\epsilon + \int_{\epsilon_2}^{\epsilon_3} \left(\int_{\varphi^*}^{\tilde{\varphi}_{xm}(\epsilon,\varphi^*)} g(\varphi) d\varphi \right) F(\epsilon, 0, 0) h(\epsilon) d\epsilon \\ \\ + \int_{\epsilon_3}^{\epsilon_4} \left(\int_{\varphi^*}^{\tilde{\varphi}_x^0(\epsilon,\varphi^*)} g(\varphi) d\varphi \right) F(\epsilon, 0, 0) h(\epsilon) d\epsilon \end{array} \right] \\
& + \left[\int_{-f_m}^{\epsilon_1} \left(\int_{\varphi^*}^{\tilde{\varphi}_x^1(\epsilon,\varphi^*)} g(\varphi) d\varphi \right) F(\epsilon, 0, 1) h(\epsilon) d\epsilon + \int_{\epsilon_1}^{\epsilon_2} \left(\int_{\tilde{\varphi}_m^0(\epsilon,\varphi^*}^{\tilde{\varphi}_x^1(\epsilon,\varphi^*)} g(\varphi) d\varphi \right) F(\epsilon, 0, 1) h(\epsilon) d\epsilon \right] \\
& + \left[\int_{\epsilon_3}^{\epsilon_4} \left(\int_{\tilde{\varphi}_x^0(\epsilon,\varphi^*}^{\tilde{\varphi}_m^1(\epsilon,\varphi^*)} g(\varphi) d\varphi \right) F(\epsilon, 1, 0) h(\epsilon) d\epsilon \right] \\
& + \left[\begin{array}{c} \int_{-f_m}^{\epsilon_2} \left(\int_{\tilde{\varphi}_x^1(\epsilon,\varphi^*)}^{\infty} g(\varphi) d\varphi \right) F(\epsilon, 1, 1) h(\epsilon) d\epsilon + \int_{\epsilon_2}^{\epsilon_3} \left(\int_{\tilde{\varphi}_{xm}(\epsilon,\varphi^*)}^{\infty} g(\varphi) d\varphi \right) F(\epsilon, 1, 1) h(\epsilon) d\epsilon \\ \\ + \int_{\epsilon_3}^{\epsilon_4} \left(\int_{\tilde{\varphi}_m^1(\epsilon,\varphi^*)}^{\infty} g(\varphi) d\varphi \right) F(\epsilon, 1, 1) h(\epsilon) d\epsilon \end{array} \right]
\end{aligned}$$

Alternatively, we can derive the joint distribution of (ϵ, d) (*insert these expressions later*) and denote it as $\nu(\epsilon, d; \varphi^*)$. Using this joint distribution, then we can write average fixed costs as

$$\tilde{F}(\varphi^*) = \sum_d \int F(\epsilon, d) \nu(\epsilon, d; \varphi^*) d\epsilon \tag{25}$$

3.3.2 Equilibrium Equations

Using equations (12) and (16), we can write average profits in the final goods sector as

$$\bar{\pi} = \left(\frac{\tilde{b}(\varphi^*)}{\varphi^*} \right)^{\sigma-1} \left(\frac{r(\varphi^*, 0, 0)}{\sigma} \right) - \tilde{F}(\varphi^*) \tag{26}$$

Using the zero cutoff profit condition given by (13), we can write our first equilibrium equation in average profits and the cutoff productivity level for exit, φ^*,

$$\bar{\pi} = \left(\frac{\tilde{b}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} f - \tilde{F}(\varphi^*) \tag{27}$$

Our second equilibrium equation is given by the free-entry condition which guarantees that the ex-ante value of an entrant must be equal zero:

$$(1 - G(\varphi^*))\left(\frac{\bar{\pi}}{\xi}\right) - f_e = 0. \tag{28}$$

or

$$\bar{\pi} = \frac{\xi f_e}{(1 - G(\varphi^*))} \tag{29}$$

These two equations ((27),(29)) determine the equilibrium cutoff for exit, φ^*.

Proposition 1

The equilibrium φ^* which satisfies equations ((27),(29)) exists and is unique.

Proof

Omitted.

The equilibrium φ^*, in turn, determines the measure of firms with each type of export/import status as well as average productivity, average revenue, and average profit.

Let the number of domestic firms be given by M. Under symmetry across countries, the variety available in any economy is $(1 + Np_x)M$, where $p_x = \int \sum_{d^m=0}^{1} \mu(\varphi, 1, d^m; \varphi^*) d\varphi$ is the fraction of exporting firms. In such an equilibrium, aggregate price and the aggregate revenue are given by:

$$P = M^{\frac{1}{1-\sigma}} \left[\int [p^h(\varphi, d^m)^{1-\sigma} + d^x N p^f(\varphi, d^m)^{1-\sigma}] d\mu(\varphi, d^x, d^m; \varphi^*)\right]^{\frac{1}{1-\sigma}}, \tag{30}$$

$$R = M \int r(\varphi, d^x, d^m) d\mu(\varphi, d^x, d^m; \varphi^*) = M\bar{r}, \tag{31}$$

where $\bar{r}$ is average revenue. Note that a firm's demand function is determined by the aggregate price and the aggregate revenue as:

$$q = RP^{\sigma-1}p^{-\sigma}. \tag{32}$$

Furthermore, since national product equals national income, we have $L = R = \bar{r}M$. Recall that average profit is given by

$$\pi = \frac{\bar{r}}{\sigma} - \bar{F}$$

so the equilibrium mass of firms must equal

$$M = \frac{L}{\sigma(\bar{\pi} + \bar{F})}.$$

Stationarity also requires that the mass of exits equals the mass of entrants.

3.3.3 Impact of Trade

In the autarkic equilibrium, average productivity as a function of the autarkic cutoff point for exit is given by

$$b_{aut}(\varphi^*_{aut}) = \left(\int_{\varphi^*_{aut}}^{\infty} \frac{\varphi g(\varphi)}{1 - G(\varphi^*_{aut})} d\varphi \right)^{\frac{1}{\sigma - 1}} \tag{33}$$

Using the zero cutoff profit condition, we have

$$\bar{\pi}_{aut} = \left(\frac{b_{aut}(\varphi^*_{aut})}{\varphi^*_{aut}} \right)^{\sigma - 1} f - f \tag{34}$$

The free entry condition given by (29) is unchanged:

$$\bar{\pi}_{aut} = \frac{\sigma f_e}{(1 - G(\varphi^*_{aut}))} \tag{35}$$

Proposition 2

The exit cutoff productivity in trade, φ^*, is above the autarkic exit cutoff productivity in autarky, φ^*_{aut}.

Proof

Incomplete. Follows because the zero cutoff condition (downward sloping) in trade is above that in autarky and the free entry condition (upward sloping) is unchanged. (Verified in numerical simulations).

This proposition implies that opening trade causes firms with lower productivity to exit. This exit, along with the importing of foreign intermediates leads to a resource reallocation from less productive firms to more productive firms and aggregate productivity increases. This result also implies that firms that choose to not import and not export will have lower market shares in the open economy than in autarky. It will also be the case that firms that choose to both import and export will gain market share when the economy opens. Furthermore, among those firms that both import and export, only a subset of firms with sufficiently high productivity will gain profits under the liberalization. These revenue and profit effects are depicted in Figure 2.

(Need to add detailed revenue and profit comparisons and welfare analysis.)

4 The Dynamic Model

4.1 The Environment

In this section, we modify the above environment to introduce interesting dynamics into the model to facilitate structural estimation of the model in the next section. Preferences and production technologies are unchanged but we modify the processes governing productivity. Firms continue to pay a fixed cost of entry and a fixed per-period production cost but we modify the fixed costs associated with exporting and importing. We begin with a description of productivity draws. Upon entering, an entrant draws an initial productivity from a continuous cumulative distribution $G_0(\cdot)$ and the density function $g_0(\cdot)$. After the initial realization, productivity at time t, φ_t, follows a first-order Markov process, where the distribution of φ_{t+1} conditional on φ_t is given by $G(\cdot|\varphi_t)$ with density function $g(\cdot|\varphi_t)$.

There are both per-period fixed costs and one-time sunk costs associated with exporting and importing:

$$F(d_t, d_{t-1}) = f + \zeta_f^{d_t^x d_t^m}(f_x d_t^x + f_m d_t^m) + \zeta_c^{d_t^x d_t^m}[\zeta_{cx}^{d_{t-1}^m} c_x d_t^x (1 - d_{t-1}^x) + \zeta_{cm}^{d_{t-1}^x} c_m d_t^m (1 - d_{t-1}^m)].$$

The second term on the right hand side, $\zeta_f^{d_t^x d_t^m}(f_x d_t^x + f_m d_t^m)$, captures per-period fixed cost while the third term, $\zeta_c^{d_t^x d_t^m}(\zeta_{cx}^{d_{t-1}^m} c_x d_t^x (1 - d_{t-1}^x) + \zeta_{cm}^{d_{t-1}^x} c_m d_t^m (1 - d_{t-1}^m))$, captures one-time sunk cost associated with exporting and importing. The parameter $0 < \zeta_f < 1$ determines the degree of complementarity between current export and import in per-period fixed cost. Similarly, the parameter $0 < \zeta_c < 1$ captures the degree of complementarity between current export and import in one-time sunk cost. While the parameter $0 < \zeta_{cx} < 1$ captures the dynamic effect of past import status on the current sunk cost of exporting, $0 < \zeta_{cm} < 1$ captures the dynamic effect of past export status on the current sunk cost of importing. If $\zeta_{cx} < 1$, then the past importing experience reduces the one-time sunk cost of exporting and hence increases the probability of exporting this period.

A firm's net profit depends on the current productivity φ_t and current and past export/import status (d_t, d_{t-1}):

$$\pi(\varphi_t, d_t, d_{t-1}) = \frac{r(\varphi_t, d_t)}{\sigma} - F(d_t, d_{t-1}).$$

Denote the exiting decision by $\chi \in \{0, 1\}$ where $\chi_t = 1$ indicates that a firm is operating at t and $\chi_t = 0$ implies that a firm exits at the beginning of period t.

There are choice-dependent idiosyncratic cost shocks associated with exiting decisions as well as export/import decisions. If a firm chooses $\chi_t \in \{0,1\}$, then it has to pay the additional cost of $\epsilon_t^{\chi}(\chi_t)$ associated with the exiting choice χ_t, where $\epsilon^{\chi} = (\epsilon^{\chi}(0), \epsilon^{\chi}(1))$ is drawn from the cumulative distribution $H_{\chi}(\cdot)$. Similarly, if a firm chooses $d_t = (d_t^x, d_t^m) \in \{0,1\}^2$ this period, then it has to pay $\epsilon_t^d(d_t)$, where $\epsilon^d = (\epsilon^d(0,0), \epsilon^d(1,0), \epsilon^d(0,1), \epsilon^d(1,1))$ is drawn from the cumulative distribution $H_d(\cdot)$.

The discount factor is given by $\beta \in (0,1)$. The timing of the incumbent's decision with the state (φ_{t-1}, d_{t-1}) within each period is as follows. At the beginning of every period, a firm faces a possibility of a large negative shock that leads it to exit with an exogenous probability ξ. If the firm survives, it draws φ_t from the cumulative distribution $G(\cdot|\varphi_{t-1})$ and the additional choice-dependent idiosyncratic cost shocks associated with exiting decisions, $\epsilon_t^{\chi} = (\epsilon_t^{\chi}(0), \epsilon_t^{\chi}(1))$, from $H_{\chi}(\cdot)$. Given the realizations, the firm decides whether it exits from the market or continues to operate. If the firm decides to exit, it receives the terminal value of $\epsilon_t^{\chi}(0)$. If the firm decides to continue to operate, then it will draw the choice-dependent idiosyncratic cost shocks associated with export and import, $\epsilon_t^d(d)$ for $d \in \{0,1\}^2$. After observing $\epsilon_t^d(d)$,the firm makes export and import decisions. The Bellman's equation is written as:

$$\begin{aligned} V(\varphi_t, d_{t-1}) &= \int \max\{\epsilon^{\chi'}(0), W(\varphi_t, d_{t-1}) + \epsilon^{\chi'}(1)\} dH_{\chi}(\epsilon^{\chi'}) \\ W(\varphi_t, d_{t-1}) &= \int \max_{d_t \in \{0,1\}^2} \left\{ \tilde{W}(\varphi_t, d_t, d_{t-1}) + \epsilon^{d'}(d_t) \right\} dH_d(\epsilon^{d'}), \end{aligned} \tag{36}$$

where

$$\tilde{W}(\varphi_t, d_t, d_{t-1}) = \pi(\varphi_t, d_t, d_{t-1}) + \beta(1-\xi) \int V(\varphi', d_t) G(d\varphi'|\varphi_t).$$

The policy function associated with this Bellman's equation (36), together with the exogenous process of φ_t, determines the transition function of (φ_t, d_t). For a firm with the state (φ_t, d_{t-1}), the exiting probability is:

$$P(\chi_t = 0|\varphi_t, d_{t-1}) = \xi + (1-\xi) \int 1(\epsilon^{\chi'}(0) > W(\varphi_t, d_{t-1}) + \epsilon^{\chi'}(1)) dH_{\chi}(\epsilon^{\chi'}), \tag{37}$$

where $\chi_{it} \in \{0,1\}$ represents the exiting/staying decision. The probability of choosing $d_{it} = (i,j)$ for $(i,j) \in \{0,1\}^2$ conditional on the survival is

$$P(d_t = (i,j)|\varphi_t, d_{t-1}, \chi_t = 1) = \int 1 \left[(i,j) = argmax_{d_t \in \{0,1\}^2} \left\{ \tilde{W}(\varphi_t, d_t, d_{t-1}) + \epsilon^{d'}(d_t) \right\} \right] dH_d(\epsilon^{d'}). \tag{38}$$

4.2 Stationary Equilibrium

As before, we focus on a stationary equilibrium in which the joint distribution of (φ, d) is constant over time and let the stationary distribution of (φ, d) among incumbents is denoted by $\mu(\varphi, d)$. Equations (30)-(32) continue to hold in this dynamic economy.

New entrants are assumed to have no past export/import experience so that $d_{t-1} = (0,0)$. The probability of successful entry among new entrants is $p_{in} = \int P(\chi_t = 1|\varphi', d_{t-1} = (0,0))dG_0(\varphi')$. The expected value of an entering firm is given by $\int V(\varphi', d_t = (0,0))dG_0(\varphi')$, where $V(\cdot)$ is given in (36). Under free entry, this value must be equal to the fixed entry cost f_e:

$$\int V(\varphi', d_t = (0,0))dG_0(\varphi') = f_e. \tag{39}$$

In equilibrium, this free entry condition has to be satisfied.

Given the stationary distribution, the average probability of exiting among incumbent is $\delta = \int P(\chi_t = 0|\varphi', d')d\mu(\varphi', d')$. The stationarity requires that the number of exiting firms is equal to the number of *successful* new entrants:

$$\delta M = p_{in} M_e,$$

where M_e is the mass of new entrants.

The evolution of the probability measure among incumbents has to take account of both the transition of states among survivors and entry/exit processes. Define the probability that a firm with the state (φ_{t-1}, d_{t-1}) continues in operation at t with the state (φ_t, d_t) is

$$P(\varphi_t, d_t, \chi_t = 1|\varphi_{t-1}, d_{t-1}) = P(d_t|\varphi_t, d_{t-1}, \chi_t = 1)P(\chi_t = 1|\varphi_t, d_{t-1})g(\varphi_t|i_{t-1}).$$

The probability that a new successful entrant will operate with the state (φ_t, d_t) is:

$$P_e(\varphi_t, d_t) = P(d_t|\varphi_t, d_{t-1} = (0,0), \chi_t = 1)g_0(\varphi_t). \tag{40}$$

Again, new entrants have no past export/import experience so that $d_{t-1} = (0,0)$ because they have no past export/import experience.

Then, the transition of the measure $\mu(\varphi, d)$ is determined by:

$$\mu_t(\varphi, d) = (T\mu_{t-1})(\varphi, d) \equiv \int P(\varphi, d, \chi = 1|\varphi', d')d\mu_{t-1}(\varphi', d') + \delta P_e(\varphi, d). \tag{41}$$

Here, we impose the equilibrium condition that the mass of entrants is equal to the mass of exits so that $\delta P_e(\varphi, d)$ represents the probability measure of new entrants with the state (φ, d).[4] The stationary distribution, μ, is a fixed point of the operator T: $\mu = T\mu$.

A stationary equilibrium consists of an aggregate price P, an aggregate revenue R, a mass of incumbents M, and a probability distribution of incumbents μ such that

- Given the the aggregates P and R (and hence the demand function (32)), a firm solves the Bellman equation (36).
- Free Entry condition (39) holds.
- The probability distribution μ is a fixed point of the operator T induced by a firm's policy function: $T(\mu) = \mu$.
- The mass of incumbents is a constant over time: $\delta M = p_{in} M_e$.
- Aggregate budget constraint holds: $R = L$.[5]

4.3 Algorithm for Computing a Stationary Equilibrium

Define the demand shifter $K \equiv RP^{\sigma-1}$ (See the demand function (32)).

1. For fixed value of $K = K_j$, we may compute the revenue function r and solve the Bellman equation (36) and obtain the choice probabilities, (37)-(38).
2. Compute $J(K_j) = (\int V(\varphi', (0,0))dG_0(\varphi') - f_e)^2$.
3. Repeat Step 1-2 to find K^* such that $J(K^*) = 0$.
4. Using (37)-(38), compute a stationary distribution μ by computing a fixed point of the operator T.

---

[4]Denote the mass of incumbents at t by M_t. The mass of incumbents with the state (φ_{t-1}, d_{t-1}) at $t-1$ is $M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$. Among them, the mass $P(\chi_t = 0|\varphi_{t-1}, d_{t-1})M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$ exits from the market, and only the mass of $P(\varphi_t, d_t, \chi_t = 1|\varphi_{t-1}, d_{t-1})M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$ will *survive and reach the state* (φ_t, d_t) at t. The mass of incumbents at t with the state (φ_t, d_t) who are not new entrants can be computed by summing up $P(\varphi_t, d_t, \chi_t = 1|\varphi_{t-1}, d_{t-1})M_{t-1}\mu_{t-1}(\varphi_{t-1}, d_{t-1})$ over the distribution $\mu_{t-1}(\varphi_{t-1}, d_{t-1})$: $M_{t-1}\int P(\varphi_t, d_t, \chi_{it} = 1|\varphi', d')d\mu_{t-1}(\varphi', d')$. On the other hand, the mass of successful new entrants with the state (φ_t, d_t) is $p_{in}M_e P_e(\varphi_t, d_t)$. Therefore, the transition of the distribution of incumbent's state is written as $M_t\mu_t(\varphi, d) = M_{t-1}\int P(\varphi, d, \chi = 1|\varphi', d')d\mu_{t-1}(\varphi', d') + p_{in}M_{e,t}P_e(\varphi, d)$. The stationary equilibrium requires that (i) $p_{in}M_{e,t} = \delta M_{t-1}$ and (ii) $M_{t-1} = M_t$. Then, we get $\mu_t(\varphi, d) = \int P(\varphi, d, \chi = 1|\varphi', d')d\mu_{t-1}(\varphi', d') + \delta P_e(\varphi, d)$.

[5]This condition implies labor market clearing.

5. Compute the aggregate price index as $P = (K^*/R)^{\frac{1}{\sigma-1}} = (K^*/L)^{\frac{1}{\sigma-1}}$.

6. Compute M from (31) as $M = \frac{R}{\int r(\varphi',d')d\mu(\varphi',d')} = \frac{L}{\int r(\varphi',d')d\mu(\varphi',d')}$.

5 Structural Estimation

5.1 Empirical Specification

Use the subscript i to represent plant i and the subscript t to represent year t. To develop an estimable structural model, we make the following distributional assumptions:

- We assume that $\ln \varphi_{it}$ follows an AR(1) process: $\ln \varphi_{it} = \psi \ln \varphi_{it-1} + \omega_{it}$, where $\psi \in (0,1)$ and ω_{it} is independently drawn from $N(0, \sigma_\omega^2)$. That is, $g(\varphi_{it}|\varphi_{it-1}) = \frac{1}{\sigma_\omega}\phi((\ln \varphi_{it} - \psi \ln phi_{it-1})/\sigma_\omega)$.

- The logarithm of the initial productivity upon entry is drawn from $N(0, \sigma_0^2)$ so that $g_0(\varphi_{it}) = \frac{1}{\sigma_0}\phi(\ln \varphi_{it}/\sigma_0)$.[6]

- $\epsilon_{it}^\chi(0)$ and $\epsilon_{it}^\chi(1)$ are independently drawn from the identical extreme-value distribution with mean zero and scale parameter ϱ^χ.

- $\epsilon_{it}^d(d)$'s for $d \in \{0,1\}^2$ are independently drawn from the identical extreme-value distribution with mean zero and scale parameter ϱ^d.

We also assume that there is an idiosyncratic shock to export revenue, denoted by η_t, independently drawn from $N(-0.5\sigma_\eta^2, \sigma_\eta)$ and its density function is given by $g_\eta(\eta) = \phi(\eta/\sigma_\eta)/\sigma_\eta$.[7] For simplicity, we assume that this idiosyncratic shock is observed only after the current year's export decision is made so that η_{it} does not affect the firm's export decision. The inclusion of export-revenue specific shocks is necessary to deal with the feature of the data that a substantial variation in export revenue even after controlling for domestic revenue. We also consider labor augmented technological change at the annual rate of α_t.

Modifying the equations in (**??**), we specify the logarithm of the domestic revenue and the export revenue as:

$$\ln r_{it}^h = \alpha_0^h + \alpha_t t + \alpha_m d_{it}^m + \ln \varphi_{it} \quad (42)$$

[6]The mean of initial productivity draws is set to zero in order to achieve the identification. $\ln \varphi_0$ cannot be separately identified from α_0^h and α_0^f.

[7]Assuming that the mean of η is equal to $-0.5\sigma_\eta^2$, we get $E[\exp(\eta)] = 1$.

$$\ln r_{it}^f = \alpha_0^f + \alpha_t t + \alpha_m d_{it}^m + \ln \varphi_{it} + \eta_{it}. \quad (43)$$

Importantly, they are *reduced-form* specifications; we have the following relationships between reduced-form parameters and structural parameters:[8]

$$\begin{aligned} \alpha_0^h &= \ln[\alpha^\alpha(1-\alpha)^{1-\alpha} R P^{\sigma-1}], \\ \alpha_0^f &= \ln[\alpha^\alpha(1-\alpha)^{1-\alpha} R P^{\sigma-1} N \tau^{1-\sigma}], \\ \alpha_m &= (\sigma-1)\ln\lambda. \end{aligned}$$

Note that, since α_0^h and α_0^f are not structural parameters, they will be affected by policy changes as long as policy changes affect R, P, and τ.

Given these specifications for revenues, firm's profit—after detrending and taking an expectation with respect to η_{it}—is

$$\pi(\varphi_{it}, d_{it}, d_{it-1}) = \frac{1}{\sigma}\Big[\exp(\alpha_0^h + \alpha_m d_{it}^m + \ln\varphi_{it}) + d_{it}^x \exp(\alpha_0^f + \alpha_m d_{it}^m + \ln\varphi_{it})\Big] - F(d_{it}, d_{it-1}). \quad (44)$$

Note that, by detrending and redefining the discount factor as $\tilde{\beta} = \beta(1-\xi)e^{\alpha_t}$, the firm's dynamic optimization problem becomes stationary.

Using the properties of the extreme-value distributed random variables (See the Appendix), the Bellman's equation (36) is rewritten as:

$$\begin{aligned} V(\varphi_{it}, d_{it-1}) &= \varrho^\chi \ln(\exp(0) + \exp(W(\varphi_{it}, d_{it-1})/\varrho^\chi)) \\ W(\varphi_{it}, d_{it-1}) &= \varrho^d \ln\left(\sum_{d_{it}} \exp\left([\pi(\varphi_{it}, d_{it}, d_{it-1}) + \tilde{\beta}\int V(\varphi', d_{it}) G(d\varphi'|\varphi_{it})]/\varrho^d\right)\right). \quad (45) \end{aligned}$$

With the solution to the functional equation (45), the conditional choice probabilities of exiting and export/import decisions follow the Nested Logit formula. In particular, the choice probability of exiting and staying conditional on the state (φ_{it}, d_{it-1})—which corresponds to (37)—is given by:

$$P(\chi_{it}|\varphi_{it}, d_{it-1}) = (1-\chi_{it})\xi + \frac{\exp(\chi_{it} W(\varphi_{it}, d_{it-1})/\varrho^\chi)}{\exp(0) + \exp(W(\varphi_{it}, d_{it-1})/\varrho^\chi)}. \quad (46)$$

[8] Also, with abuse of notation, we replace $(\sigma-1)\ln\varphi$ by $\ln\varphi$ since $(\sigma-1)$ cannot be separately identified from the variance of $\ln\varphi$.

Conditional on choosing $\chi_{it} = 1$ (i.e., continuously operating), the choice probabilities of d_{it}, corresponding to (38), are given by the multinomial logit formula:

$$P(d_{it}|\varphi_{it}, d_{it-1}, \chi_{it} = 1) = \frac{\exp([\pi(\varphi_{it}, d_{it}, d_{it-1}) + \tilde{\beta} \int V(\varphi_{it+1}, d_{it}) G(d\varphi_{it+1}|\varphi_{it})]/\varrho^d)}{\sum_{d' \in \{0,1\}^2} \exp([\pi(\varphi_{it}, d', d_{it-1}) + \tilde{\beta} \int V(\varphi_{it+1}, d') G(d\varphi_{it+1}|\varphi_{it})]/\varrho^d)}, \quad (47)$$

To achieve the identification, we normalize the profit function by $1/\sigma$; the various components of the fixed cost as well as the scale parameter ϱ^d are estimated up to the scale of σ.

The probability that a new successful entrant will operate with the state (φ_t, d_t), corresponding to (40), is:

$$P_e(\varphi_{it}, d_{it}) = P(d_{it}|\varphi_{it}, d_{it-1} = (0,0), \chi_{it} = 1) P(\varphi_{it}|\chi_{it} = 1), \quad (48)$$

where $P(d_{it}|\varphi_{it}, d_{it-1} = (0,0), \chi_{it} = 1)$ is given in (47) and $P(\varphi_{it}|\chi_{it} = 1)$ can be computed, using (46), as:

$$P(\varphi_{it}|\chi_{it} = 1) = \frac{P(\chi_{it} = 1|\varphi_{it}, d_{it-1} = (0,0)) g_0(\varphi_{it})}{\int P(\chi_{it} = 1|\varphi', d_{it-1} = (0,0)) g_0(\varphi') d\varphi'}.$$

To obtain the transition function of the state, define the probability that a firm with the state $(\varphi_{it-1}, d_{it-1})$ continues in operation at t with the state (φ_{it}, d_{it}) as:

$$P(\varphi_{it}, d_{it}, \chi_{it} = 1|\varphi_{it-1}, d_{it-1}) = P(d_{it}|\varphi_{it}, d_{it-1}, \chi_{it} = 1) P(\chi_{it} = 1|\varphi_{it}, d_{it-1}) g(\varphi_{it}|\varphi_{it-1}).$$

Then, the transition of the measure $\mu(\varphi, d)$ is determined by (41).

5.2 Econometric Approach

The parameter vector to be estimated is $\theta = (\alpha_t, \alpha_0^h, \alpha_m^h, \alpha_0^f, \alpha_m^f, \sigma_\eta^2, \sigma_\omega^2, igma_0^2, f, f_x, f_m, \zeta_f, c_x, c_m, \zeta_c, \varrho^\chi, \varrho^d, \xi)$ The discount factor $\tilde{\beta}$ is not estimated and is set to 0.96. The parameters are estimated by the method of Maximum Likelihood.

Given the data and the parameter vector θ, we may compute the estimates of $\ln \varphi_{it}$ and η_{it} as:

$$\begin{aligned} \ln \tilde{\varphi}_{it}(\theta) &\equiv \ln r_{it}^h - \alpha_0^h - \alpha_t t - \alpha_m d_{it}^m \\ \tilde{\eta}_{it}(\theta) &\equiv \ln r_{it}^f - \alpha_0^f - \alpha_t t - \alpha_m d_{it}^m - \ln \tilde{\varphi}_{it}(\theta). \end{aligned}$$

Denote the first year and the last year in which firm i appears in the data by $T_{i,0}$ and $T_{i,1}$, respectively. The unbalanced plant-level data spans from 1990 to 1996. Thus, $T_{i,0}$ is either 1990 or the year in which firm i entered between 1991 and 1996. $T_{i,1}$ is either 1996 or the year in which firm i exited between 1991 and 1995.

For $t > T_{i,0}$, we can observe the past state variables $(d_{it-1},\ \tilde{\varphi}_{it-1})$; we may compute the likelihood contribution of the current observation $(d_{it}, \tilde{\varphi}_{it}, \tilde{\eta}_{it})$ conditioned on the observable past state variable $(d_{it-1},\ \tilde{\varphi}_{it-1})$ as

$$L_{it}(\theta) = \begin{cases} P(\chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1})P(d_{it}|\tilde{\varphi}_{it}, d_{it-1}, \chi_{it} = 1)g(\tilde{\varphi}_{it}|\tilde{\varphi}_{it-1})g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}}, & \text{for } \chi_{it} = 1 \\ \int P(\chi_{it} = 0|\varphi', d_{it-1})g(\varphi'|\tilde{\varphi}_{it-1})d\varphi', & \text{for } \chi_{it} = 0 \end{cases} \tag{49}$$

Each of the terms is explained as follows. First, $P(d_{it}, \chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1}) = P(\chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1})$ $P(d_{it}|\tilde{\varphi}_{it}, d_{it-1}, \chi_{it} = 1)$ is the likelihood of observing $(d_{it}, \chi_{it} = 1)$ conditioned on the current productivity $\tilde{\varphi}_{it}$ and the past export/import decision d_{it-1}. $g(\tilde{\varphi}_{it}|\tilde{\varphi}_{it-1})$ is the likelihood of observing $\tilde{\varphi}_{it}$ conditioning on the past state variable. If a firm is exporting (i.e., $d^x_{it} = 1$), then the likelihood of observing $\tilde{\eta}_{it}$ is also computed as $g_{\eta}(\tilde{\eta}_{it})$. The likelihood $L_{it}(\theta)$ for $\chi_{it} = 1$ is, therefore, the probability of observing $(d_{it}, \tilde{\varphi}_{it}, \tilde{\eta}_{it})$ conditioned on the past state variables $(d_{it-1},$ $\tilde{\varphi}_{it-1})$. In the case of $\chi_{it} = 0$ (i.e., firm i exists at t), we do not observe the current productivity shock $\tilde{\varphi}_{it}$. So, in order to compute the probability of exiting conditioned on $(d_{it-1},\ \tilde{\varphi}_{it-1})$, we integrate out $P(\chi_{it} = 0|\varphi_{it}, d_{it-1})$ over unobserved current shock φ_{it} using the conditional density function $g(\varphi_{it}|\varphi_{it-1})$.

For $t = T_{i,0}$, there are two cases. The first case is that firm i enters into the market after 1991 (i.e., $T_{i,0} > 1990$). The second case is that firm i has operated in 1990.

Consider the first case of $T_{i,0} > 1990$. The likelihood of observing $(d_{it}, \tilde{\varphi}_{it}, \tilde{\eta}_{it})$ for $t = T_{i,0}$ is written as

$$L_{it}(\theta) = P_e(\tilde{\varphi}_{it}, d_{it})g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}}, \tag{50}$$

where $P_e(\varphi, d)$ is given in equation (48). For a new successful entrant, productivity shock φ_{it} is distributed according to $P_e(\tilde{\varphi}_{it}|\chi_{it} = 1)$, which is the distribution of initial draws conditioned on the successful entry.

Next, consider the case of $T_{i,0} = 1990$. While the evaluation of the choice-probabilities (46)-(47) as well as the probability of observing φ_{it} requires the past state variables $(d_{it-1},$ $\varphi_{it-1})$, we do not observe these past state variables at $t = 1990$ in this case. Note, however, that the theory suggests that the past state variables $(d_{it-1},\ \varphi_{it-1})$ is distributed according to a stationary distribution of $(d_{it-1},\ \varphi_{it-1})$, implied by the policy function associated with the parameter vector θ. Thus, we may use the stationary distribution to integrate out the unobserved past state variables to compute the likelihood of the initial observations. The likelihood is

$$L_{it}(\theta) = \int P(\chi_{it} = 1|\tilde{\varphi}_{it}, d')P(d_{it}|\tilde{\varphi}_{it}, d', \chi_{it} = 1)g(\tilde{\varphi}_{it}|\varphi')g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}} d\mu(\varphi', d'). \tag{51}$$

In sum, the likelihood contribution from the observation of firm i at t is computed as

$$L_{it}(\theta) = \begin{cases} P_e(\tilde{\varphi}_{it}, d_{it})g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}} & \text{for } t = T_{i,0} > 1990, \\ \int P(\chi_{it} = 1|\tilde{\varphi}_{it}, d')P(d_{it}|\tilde{\varphi}_{it}, d', \chi_{it} = 1)g(\tilde{\varphi}_{it}|\varphi')g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}} d\mu(\varphi', d') & \text{for } t = T_{i,0} = 1990, \\ P(\chi_{it} = 1|\tilde{\varphi}_{it}, d_{it-1})P(d_{it}|\tilde{\varphi}_{it}, d_{it-1}, \chi_{it} = 1)g(\tilde{\varphi}_{it}|\tilde{\varphi}_{it-1})g_{\eta}(\tilde{\eta}_{it})^{d^x_{it}}, & \text{for } \chi_{it} = 1 \text{ and } t > T_{i,0}, \\ \int P(\chi_{it} = 0|\varphi', d_{it-1})g(\varphi'|\tilde{\varphi}_{it-1})d\varphi' & \text{for } \chi_{it} = 0 \text{ and } t > T_{i,0}. \end{cases}$$

The parameter vector θ can be estimated by maximizing the logarithm of likelihood function

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \sum_{t=T_{i,0}}^{T_{i,1}} \ln L_{it}(\theta). \tag{52}$$

Evaluation of the log-likelihood involves solving computationally intensive dynamic programming problem that approximates the Bellman equation (45) by discretization of state space. For each candidate parameter vector θ, we solve the discretized version of (45) and then obtain the choice probabilities, (46) and (47), as well as the stationary distribution from the associated policy function. Once the choice probabilities and the stationary distribution are obtained for a particular candidate parameter vector θ, then we may evaluate the log-likelihood function (52). Repeating this process, we can maximize (52) over the parameter vector space of θ to find the estimate.

In practice, we use the Gauss-Hermit quadrature grids to discretize the space of φ so that the integral in (45) can be approximately evaluated using the Hermit quadrature formula (c.f., Tauchen and Hussey, 1991; Stinebricker, 2000). Another important issue is that, while the conditional choice probabilities, (46) and (47), can be evaluated only on the set of finite quadrature grids, we need to evaluate the conditional choice probabilities on the *realized* value of φ_{it} which is not necessarily on the set of grid points. We evaluate the conditional choice probabilities (46) and (47) at φ_{it} that is not on the quadrature grids by using cubic spline interpolation. The appendix discusses the approximation method in greater detail. To maximize the log-likelihood function (52), we first use the simplex method of Nelder and Mead to reach the neighborhood of the optimum and then use the BFGS quasi-Newton method.

6 Data and Results

6.1 Data

We use the Chilean manufacturing census for 1990-1996. We focus on the following five observable variables: d^x_{it}, d^m_{it}, χ_{it}, r^h_{it}, and r^f_{it}, where i represents plant's identification and t represents the year t. In the data set, we observe the number of blue workers and white workers, the value of total sales, the value of export sales, and the value of imported materials. The export/import status, (d^m_{it}, d^x_{it}), is identified from the data by checking if the value of export sales and/or the value of imported materials are zero or positive. The value of the revenue from the home market, r^h_{it}, is computed as (the value of total sales)-(the value of export sales). We use the manufacturing output price deflator to convert the nominal value into the real value. The entry/exiting decisions, χ_{it}, can be identified in the data by looking at the number of workers across years. We use unbalanced panel data of 7234 plants for 1990-1996, including all the plants that has been observed at least one year between 1990 and 1996: $\{\{d^x_{it}, d^m_{it}, \chi_{it}, r^h_{it}, r^f_{it}\}_{t=T_{i,0}}^{T_{i,1}}\}_{i=1}^{7234}$. Here, $T_{i,0}$ is the first year in which firm i appears in the data, which is either 1990 or the year in which firm i entered between 1991 and 1996. $T_{i,1}$ is the last year in which firm i appears in the data, which is either 1996 or the year in which firm i exited between 1991 and 1995.

6.2 Parameter Estimates

Table 4 presents the maximum likelihood estimates of the empirical models and their asymptotic standard errors, which are computed using the outer product of gradients estimator. The parameters are evaluated in the unit of million US dollars in 1990. The standard errors are generally small.

The estimate of α_t implies that the revenue is growing at the annual rate of 5.2 percent. The estimate of α_m is 0.04, indicating that importing materials from abroad has a substantial impact, a 4.0 percent increase, on the total revenues. The standard error for export revenue is estimated as 2.40, which seems to be high. This high estimate might be due to our specification for export revenue function.[9]

The estimated per-period fixed cost of operating in the market is $\hat{f} = 2.03/\sigma$ million dollars.

[9]The current specification might not be rich enough to capture the process of export revenue shocks. One possibility is to consider the vector autoregressive process for the shocks in domestic revenue and export revenue.

Table 4: Maximum Likelihood Estimates

Parameters	Estimates	S.E.
α_t	0.052	(0.003)
α_0^h	-1.046	(0.002)
α_m	0.040	(0.002)
α_0^f	-3.317	(0.011)
σ_η	2.565	(0.014)
σ_ω	0.417	(0.000)
σ_0	1.223	(0.013)
σf	2.032	(0.127)
σf_x	2.342	(0.117)
σf_m	1.811	(0.093)
ζ_f	0.217	(0.020)
σc_x	42.622	(0.974)
σc_m	38.330	(0.840)
ζ_c	0.873	(0.009)
ζ_{cx}	0.961	(0.015)
ζ_{cm}	0.938	(0.016)
$\sigma \rho_d$	9.093	(0.182)
$\sigma \rho_\chi$	8.944	(0.927)
ψ	0.990	(0.000)
ξ	0.037	(0.001)
f_e	57.944	
$\exp(\alpha_0^h + \ln \bar{\varphi})$	0.800	
$\sigma c_x + \sigma E(\epsilon_d \mid d_t^x = 1, d_{t-1}^x = 0)$	12.962	
$\sigma c_m + \sigma E(\epsilon_d \mid d_t^m = 1, d_{t-1}^m = 0)$	12.000	
log-likelihood	70687.22	
No. of Plants	7234	

Notes: Standard errors are in parentheses. The parameters are evaluated in the unit of million US dollars in 1990.

Thus, if, say, $\sigma = 4$, the estimated per-period fixed cost is approximately equal to 51 thousand US dollars. On the other hand, this per-period fixed cost is more than twice of the profit from domestic sales for the "average" incumbent with the average productivity $\ln\varphi = 0.82$, $(\exp(\alpha_0^h + 0.82)/\sigma =)0.80/\sigma$. Taken for face value, the result indicates that many firms put up with the profit loss for a potential big success in the future—a possible realization of a high value of φ. The estimated per-period fixed costs for export and import are also substantial: $\hat{f}_x = 2.34/\sigma$ and $\hat{f}_m = 1.81/\sigma$. The parameter determining the degree of complementarity in the per-period fixed cost associated with export and import, ζ_f, is estimated as 0.217, indicating that a firm can save more than a half of per-period fixed cost associated with trade by engaging in both export and import activities.

The estimated one-time sunk costs associated with export and import are also large: $\hat{c}_x = 42.6/\sigma$ and $\hat{c}_m = 38.3/\sigma$. We have to be careful, however, in interpreting these numbers since a firm is also paying a random cost $\epsilon(d)$. Since a firm tends to start exporting when a firm draws a lower random cost $\epsilon_d(d^x = 1)$, the average value of ϵ_d a firm is paying is low and tends to be negative. If we take into account the random cost, the average sunk cost paid among the firms that start exporting (or importing) is computed as $\hat{c}_x + \hat{E}(\epsilon_d(d_t^x = 1)|d_t^x = 1, d_{t-1}^x = 0) = 13.0/\sigma$ (or $\hat{c}_m + \hat{E}(\epsilon_d(d_t^m = 1)|d_t^m = 1, d_{t-1}^m = 0) = 12.0/\sigma$), which is substantially lower than $\hat{c}_x = 42.6/\sigma$ (or $\hat{c}_m 38.3/\sigma$). This means that, on average, a firm pays the sunk cost of exporting (importing) that is about 16 (15) times as large as the average incumbent's annual profit.

The parameter estimate $\hat{\zeta}_c = 0.873$ indicates that a firm can save more than 12 percent of one-time sunk cost for export and import if it simultaneously exports and imports. On the other hand, $\hat{\zeta}_{cx} = 0.961$ and $\hat{\zeta}_{cm} = 0.938$ imply that the past import experience reduces the sunk cost for exporting by 3.9 percent while the past export experience reduces the sunk cost for importing by 6.2 percent.

The estimated magnitudes of the shocks associated with the exiting decision and the export/import decisions are large relative to the per-period profit. While the estimate of $\rho_d = 9.09/\sigma$, implying the standard error of $\frac{\pi}{\sqrt{6}} \times 9.09/\sigma = 11.66/\sigma$ in export/import cost shocks, which is more than fourteen times as large as the "average" incumbent's profit. This indicates that a firm faces a large uncertainty in the magnitude of the one-time fixed cost for exporting and importing.

The estimate of the AR(1) coefficient for the φ process, ψ, is 0.990 so that φ follows a highly

Table 5: The Mean of Productivity

Mean of φ at Entry Trial	1.00
Mean of φ at Successful Entry	1.04
Mean of φ at Steady State	5.84
Mean of $\varphi \exp(d^m \alpha_m)$ at Steady State	35.40

Notes: The reported numbers are relative to the productivity level at entry. In particular, the original numbers are divided by the mean of φ at entry (i.e., $\int \varphi g_0(\varphi) d\varphi$).

persistent process. The exogenous exiting probability is estimated as 3.7 percent, implying that even a large and highly productive firm faces a non-negligible probability of exiting due to a negative shock. The fixed entry cost, which is estimated from the free entry condition $\hat{f}_e = \int \hat{V}(\varphi, (0,0)) \hat{g}_0(\varphi') d\varphi'$, is $57.94/\sigma$, which is 72 times as large as the domestic profit for the "average" incumbent.

In the model, the higher productivity firms are more likely to survive than the lower productivity firms. To examine how important such a selection mechanism to determine aggregate productivity, the first to the fourth rows of Table 5 compare the mean of productivity across different groups of firms. The average productivity level among successful new entrants is 4 percent higher than the average productivity level of the initial draws from $g_0(\varphi)$, indicating that those who initially drew the relatively higher productivity are more likely to succeed in entering into the market. Over time, the selection leads to a larger impact on the average productivity. The average productivity at the steady state is more than five times higher than the average productivity across initial draws. When we also include the effect of import, as shown in the fourth row, the average productivity at the steady state is more than 35 times as high as the average of initial draws.

Table 6 compares the actual and the predicted transition probability of export/import status and entry/exit together. The transition pattern of export and import are replicated by the estimated model pretty well. The predicted invariant distribution is also very close to the empirical distribution for 1990-1996. On the other hand, the estimated models do not predict export/import decisions among new entrants well.[10]

Table 7 compares the actual and the predicted average productivity and market shares across

[10] One possible reason is the presence of unobserved heterogeneity. In the future, we will incorporate unobserved heterogeneity into the model.

Table 6: Transition Probability of Export/Import Status and Entry/Exit (Actual vs. Predicted)

	Export/Import Status at $t+1$ conditioned on Staying						
Actual	(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import	Exit at $t+1^a$
No-Export/No-Import at t	0.927	0.024	0.042	0.007	0.031	0.048	0.082
Export/No-Import at t	0.147	0.677	0.013	0.163	0.841	0.176	0.070
No-Export/Import at t	0.188	0.017	0.699	0.096	0.113	0.795	0.035
Export/Import at t	0.025	0.101	0.070	0.804	0.905	0.874	0.022
New Entrants at t^b	0.753	0.096	0.100	0.051	0.147	0.151	0.126
Predicted							
No-Export/No-Import at t	0.916	0.030	0.046	0.009	0.038	0.054	0.073
Export/No-Import at t	0.175	0.669	0.012	0.144	0.813	0.156	0.044
No-Export/Import at t	0.202	0.008	0.695	0.095	0.103	0.790	0.045
Export/Import at t	0.026	0.104	0.092	0.778	0.882	0.870	0.038
New Entrants at t	0.922	0.027	0.043	0.007	0.034	0.051	0.082

Note: a). "Exit at $t+1$" is defined as plants that are observed at t but not observed at $t+1$ in the sample. b). "New Entrants t" is defined as plants that are not observed at $t-1$ but observed at t in the sample, of which row represents the empirical distribution of export/import status at t as well as the probability of not being observed (i.e., exit) at $t+1$.

different export/import states. In the actual data, while a 68.6 percent of the firms are neither exporting nor importing, their market shares account only for a 22.8 percent of total outputs. On the other hand, only a 10.8 percent of the firms are both exporting and importing but they account for 44.1 percent of total output. As shown in Table 7, this pattern in the market share is well replicated by the estimated models. As the actual data suggests (in the third row), relatively small number of exporters and importers account for large market shares because they tend to be more productive and hence employ more workers relative to non-exporters and non-importers. This basic observed pattern on the productivity across different export and import status is also captured by the estimated model as shown in the last row. The estimated model indicates that the exporting and the importing firms are more productive on average than the non-exporting and the non-importing firms since the more productive firms, expecting the higher returns from exporting and importing, are more willing to pay the one-time fixed costs of exporting and importing and hence are more likely to become exporters and importers.

6.3 Counterfactual Experiments

While some of the structural parameters are not identified from the empirical model (crucially, we cannot identify σ), we may solve for the change in the equilibrium aggregate price as a result

Table 7: Productivity and Market Shares by Export/Import Status (Actual vs. Predicted)

		Export/Import Status					
		(1) No-Export /No-Import	(2) Export /No-Import	(3) No-Export /Import	(4) Export /Import	(2)+(4) Export	(3)+(4) Import
Actual	Dist. of Ex/Im Status	0.686	0.086	0.120	0.108	0.194	0.228
	Market Shares	0.228	0.187	0.145	0.441	0.627	0.586
	Average of $\ln \varphi$	0.484	1.337	1.474	2.381	1.911	1.898
Predicted	Dist. of Ex/Im Status	0.662	0.092	0.126	0.120	0.212	0.246
	Market Shares	0.221	0.108	0.076	0.595	0.703	0.671
	Average of $\ln \varphi$	0.520	1.249	0.921	2.072	1.714	1.482

of counterfactual experiments as follows.

Denote the equilibrium aggregate price under the parameter θ by $P(\theta)$. Under the estimated parameter $\hat{\theta}$, we may compute the estimate of fixed entry cost as: $\hat{f}_e = \int V(\varphi', (0,0); \hat{\theta}) g_0(\varphi'; \hat{\theta}) d\varphi'$, where $V(\varphi, d; \theta)$ is the fixed point of the Bellman's equation (45) under the parameter θ and $g_0(\varphi; \theta)$ is the probability density function of the initial productivity under θ.

Suppose that we are interested in a counterfactual experiment characterized by a counterfactual parameter $\tilde{\theta}$ that is different than the estimated parameter $\hat{\theta}$. Note that the following relationships hold between α_0^h and α_0^f, on the one hand, and the aggregate price P, on the other,

$$\begin{aligned} \hat{\alpha}_0^h &= \ln(\alpha^\alpha (1-\alpha)^{1-\alpha} R P(\hat{\theta})^{\sigma-1}), \\ \hat{\alpha}_0^f &= \ln(\alpha^\alpha (1-\alpha)^{1-\alpha} R P(\hat{\theta})^{\sigma-1} N \tau^{1-\sigma}). \end{aligned}$$

Then, we may write the estimated profit function, (44), evaluated at the counterfactual aggregate price $P(\tilde{\theta})$ as:

$$\begin{aligned} &\hat{\pi}(\varphi_t, d_t, d_{t-1}; P(\tilde{\theta})) \\ &= \frac{1}{\sigma} \exp[(\sigma-1)\ln(P(\tilde{\theta})/P(\hat{\theta}))][\exp(\hat{\alpha}_0^h + \hat{\alpha}_m d_t^m + \ln \varphi_t) + d_t^x \exp(\hat{\alpha}_0^f + \hat{\alpha}_m d_t^m + \ln \varphi_t)] - \hat{F}(d_t, d_{t-1}), \\ &= \frac{1}{\sigma} \exp[\ln(K(\tilde{\theta})/K(\hat{\theta}))][\exp(\hat{\alpha}_0^h + \hat{\alpha}_m d_t^m + \ln \varphi_t) + d_t^x \exp(\hat{\alpha}_0^f + \hat{\alpha}_m d_t^m + \ln \varphi_t)] - \hat{F}(d_t, d_{t-1}), \end{aligned}$$

where $K(\theta) = RP(\theta)^{\sigma-1}$ is the demand shifter under the parameter θ.

Then, we may compute the equilibrium changes in the demand shifters, $\ln(K(\tilde{\theta})/K(\hat{\theta})) = (\sigma-1)\ln(P(\tilde{\theta})/P(\hat{\theta}))$. Specifically, we may use the same algorithm for computing a stationary equilibrium using the free entry condition under the counterfactual parameter $\tilde{\theta}$:

$$\hat{f}_e = \int V(\varphi', (0,0); \tilde{\theta}) g_0(\varphi'; \tilde{\theta}) d\varphi'.$$

Table 8: Counterfactual Experiments

		Counterfactual Experiments			
		(1)	(2)	(3)	(4)
	Trade	No Export	No Import	Autarky	No Comp.
With Equilibrium Price Effect					
Welfare measured by $-(\sigma-1)\ln P$	0.000	-0.100	-0.049	-0.140	-0.012
Exiting Rates at Entry	0.086	0.075	0.063	0.068	0.069
ln(Average φ)	0.000	-0.030	-0.103	-0.064	-0.065
ln(Average Revenue)	0.000	-0.006	-0.094	-0.019	-0.075
A Fraction of Exporters	0.212	0.000	0.082	0.000	0.087
A Fraction of Importers	0.246	0.125	0.000	0.000	0.124
Without Equilibrium Price Effect					
Exiting Rates at Entry	0.086	0.106	0.112	0.124	0.094
ln(Average φ)	0.000	0.092	0.114	0.158	0.046
ln(Average Revenue)	0.000	0.016	0.074	0.064	0.025
A Fraction of Exporters	0.212	0.000	0.094	0.000	0.093
A Fraction of Importers	0.246	0.130	0.000	0.000	0.132

We may quantify the impact of counterfactual experiments on the welfare level by examining how much the equilibrium aggregate price level P changes as a result of counterfactual experiments since the aggregate price level P is inversely related to the welfare level W.[11]

To quantitatively investigate the impact of international trade, we conduct the four counterfactual experiments with the following counterfactual parameters:

(1) No Export: $f_x, c_x \to \infty$.

(2) No Import: $f_m, c_m \to \infty$.

(3) Autarky: $f_x, c_x, f_m, c_m \to \infty$.

(4) No Complementarity: $\zeta_f = \zeta_c = \zeta_{cx} = \zeta_{cm} = 1$.

Table 8 presents the results of counterfactual experiments using the estimated model. To examine the importance of equilibrium response to quantify the impact of counterfactual policies, Table 8 reports the results both with and without equilibrium aggregate price response. According to the experiment, moving from autarky to trade may decrease the equilibrium aggregate price by a $14.0/(\sigma-1)$ percent. This implies that, say if $\sigma = 4$, exposure to trade increases the real income by a (14.0/4=)3.7 percent, leading to a substantial increase in welfare. Intuitively,

[11]To see this, note that the income is constant at the level of L. From the budget constraint $PQ = L$ and the definition of aggregate product $W = Q = \left[\int_{\omega\in\Omega} q(\omega)^\rho d\omega\right]^{1/\rho}$, the utility level is equal to $W = P^{-1}L$.

when a country opens up its economy, more productive firms start exporting and importing, which in turn increases the aggregate labor demand and hence leads to an increase in the real wage.

As shown in the second row of Table 8, the exiting rates at entry trial increase from 6.8 percent to 8.6 percent by moving from autarky to trade. The higher real wage in open economy leads to higher exiting rates among new entrants who tend to be less productive. Furthermore, the higher exiting rates among less productive firms shift the productivity distribution to the right, allocating resources from higher productivity firms to lower productivity firms; as a result, exposure to trade increases the average productivity—defined as $\int \varphi d\mu(\varphi, d)$—by 6.4 percent as shown in the third row. By comparing the impact of trade on exiting rates and average productivity between with and without the equilibrium price responses, we notice that it is the equilibrium price response that is responsible for the redistribution of resources from lower productive firms to higher productive firms. Without equilibrium price response, moving from autarky to trade may lead to lower exiting rates among less productive firms and higher average productivity.[12]

The counterfactual experiments under no export or no import (but not both) highlight the interaction between aggregate export and aggregate import in the presence of heterogenous firms. According to the estimated model, when the economy moves from trade to no export, a fraction of *importers* declines from a 24.6 percent to a 12.5 percent; when the economy moves from trade to no import, a fraction of *exporters* declines from a 21.2 percent to a 8.2 percent. Thus, policies that prohibits the import of foreign materials could have a large negative impact on the export of final consumption goods, or vice-versa. The similar results hold even without equilibrium price effect and thus the equilibrium price response is little to do with these results; rather, it is due to the complementarity between export and import within both revenue function $r(\cdot)$ and sunk-cost function $F(\cdot)$.

[12]The following two assumptions are important to understand the result of the experiment without equilibrium price response. First, there are choice-dependent cost shocks so that even least productive firms may start exporting and importing. Second, a firm-specific productivity, φ, follows an AR(1) process so that even the currently less productive firms may become highly productive in the future. When the variance of cost shocks, ϵ_d, is large, a substantial fraction of less productivity firms are exporting and importing in open economy. Given the possibility of becoming productive in the future and having already paid a large one-time sunk cost for exporting and importing, the low productive firms that are exporting and/or importing are willing to stay in the market. As a result, the average exiting probability among less productivity firms could be lower in trade than in autarky.

To examine the role of complementarity between export and import in the sunk-cost function—relative to the role played by the complementarity in the revenue function—we conducted what would happen to a fraction of importers and/or a fraction of exporters had there been no complementarity between export and import in the sunk cost function. The results are striking. Eliminating the complementarity between export and import in the sunk cost function has essentially the same effect on export and import, respectively, as restricting to no import and no export. A fraction of exporters is 8.2 percent under no import while eliminating the complementarity in sunk-cost function decreases a fraction of exporters from 21.2 percent to 8.7 percent. Similarly, eliminating complementarity decreases a fraction of importers from 24.6 percent to 12.4 percent, which is almost identical to a fraction of importers under no export. Thus, it is the complementarity in the sunk cost function, rather than the complementarity in the revenue function, that determines the impact of exporting policies (e.g., export subsidies) on intermediate imports or the impact of importing policies (e.g., import tariffs) on exports.

7 Conclusions

We have developed and estimated a dynamic, stochastic, industry model of import and export with heterogeneous firms. The analysis highlights interactions between imports of intermediate goods and exports of final goods. In doing so, we have identified a potential mechanism whereby import policy can affect exports and export policy can affect imports.

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8 Appendix I

Here, we discuss the properties of the Type I extreme-value distributed random variables. Assume that $\epsilon(0)$ and $\epsilon(1)$ are independently drawn from the identical extreme-value distribution with mean zero and variance normalized to $\frac{\pi^2}{6}$.[13] Let $V(0)$ and $V(1)$ be some real numbers. All we have to know about the properties of the extreme-value distributed random variables is the following two properties.

The first property is:

$$E[\max(V(0)+\epsilon(0), V(1)+\epsilon(1)] = \ln[exp(V(0)) + \exp(V(1))],$$

where the expectation is taken with respect to the distribution of $\epsilon(0)$ and $\epsilon(1)$.

The second property is:

$$P(V(0)+\epsilon(0) > V(1)+\epsilon(1)) = \frac{\exp(V(0))}{\exp(V(0)) + \exp(V(1))}.$$

In multivariate case, when we have $\epsilon(d)$ for $d = 0, 1, 2, ..., J$, the first property is

$$E[\max_{j=0,1,\ldots,J} V(j) + \epsilon(j)] = \ln[\sum_{j=0}^{J} \exp(V(j))].$$

The second property is

$$P[V(d)+\epsilon(d) > V(j)+\epsilon(j) \text{ for all } j \neq d] = \frac{\exp(V(d))}{\sum_{j'=0}^{J} \exp(V(j'))}.$$

One implication is

$$\begin{aligned} E[\max(V(0)+\varrho\epsilon(0), V(1)+\varrho\epsilon(1)] &= \varrho E[\max(V(0)/\varrho+\epsilon(0), V(1)/\varrho+\epsilon(1)] \\ &= \varrho \ln[\exp(V(0)/\varrho) + \exp(V(1)/\varrho)], \end{aligned}$$

which I have used to derive the first equation in (45). Note that by letting $\varrho \to 0$, we have $E[\max(V(0)+\varrho\epsilon(0), V(1)+\varrho\epsilon(1)] \to \max(V(0), V(1))$ so that, if we do not like adding extreme-value distributed random variables into the model, then we may manually set ϱ to be a very small number so that we get a computational advantage of this specification while getting the almost identical result as the case of no extreme-value distributed random variables.

[13] The cumulative distribution function of $\epsilon(d)$ for $d = 0, 1$ is $\exp(-\exp(-(\epsilon(d) - \gamma)))$, where γ is Euler's constant.

Assume that $\epsilon(0)$ and $\epsilon(1)$ are independently drawn from the identical extreme-value distribution with mean zero and variance normalized to $\frac{\pi^2}{6}$. To derive labor demand in the presence of extreme value distributed shocks, we need to know $P(\epsilon(0)|\epsilon(0)+V(0) \geq \epsilon(1)+V(1))$. Below, we prove that $\epsilon(0)$ conditional on $d=0$ is chosen is extreme value distributed with the mean $-\ln P(0)$.

First,

$$P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1)|\epsilon(0)) = \exp(-e^{-(\epsilon(0)-\gamma+V(0)-V(1))}).$$

Then,

$$\begin{aligned} P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1),\epsilon(0)) &= P(\epsilon(0))P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1)|\epsilon(0)) \\ &= -e^{-(\epsilon(0)-\gamma)}\exp(-e^{-(\epsilon(0)-\gamma)})\exp(-e^{-(\epsilon(0)-\gamma+V(0)-V(1))}). \end{aligned}$$

Using $P(V(0)+\epsilon(0) > V(1)+\epsilon(1)) = \frac{\exp(V(0))}{\exp(V(0))+\exp(V(1))}$,

$$\begin{aligned} P(\epsilon(0)|\epsilon(0)+V(0) \geq \epsilon(1)+V(1),\epsilon(0)) &= \frac{P(\epsilon(0)+V(0) \geq \epsilon(1)+V(1),\epsilon(0))}{P(V(0)+\epsilon(0) > V(1)+\epsilon(1))} \\ &= -e^{-(\epsilon(0)-\gamma+\ln P(0))}\exp(-e^{-(\epsilon(0)-\gamma+\ln P(0))}). \end{aligned}$$

9 Appendix II

This section discusses the numerical methods we use in detail.

Since there is no closed-form solution to the functional equation (36), we discretize the state space of $\ln\varphi$ using the quadrature grids and solve the approximated decision problem numerically by backward induction. In particular, the integral on the left-hand-side of (36) is approximately evaluated using the Gauss-Hermit quadrature formula as follows.

As explained in Section 7.2 of Judd (1998), if $\ln\varphi$ is distributed $N(\mu,\sigma^2)$, then the expected value of $f(\ln\varphi)$ for some function $f(\cdot)$ can be approximately evaluated as

$$\begin{aligned} E[f(\ln\varphi)] &= (2\pi\sigma^2)^{-1/2}\int_{-\infty}^{\infty} f(\ln\varphi)e^{-(\ln\varphi-\mu)^2/2\sigma^2}d\ln\varphi \\ &= \pi^{-1/2}\int_{-\infty}^{\infty} f(\sqrt{2}\sigma x+\mu)e^{-x^2}dx \\ &\approx \pi^{-1/2}\sum_{i=1}^{n}\omega_i f(\sqrt{2}\sigma x_i+\mu) \end{aligned}$$

where the second equality uses the linear change of variables $x = (\ln\varphi - \mu)/\sqrt{2}\sigma$ and the third approximation uses the Gauss-Hermite quadrature rule with the Gauss-Hermite quadrature weights ω_i and nodes x_i.

Note that this formula only applies to the normally distributed random variable without any serially correlation. On the other hand, we have AR(1) process for $\ln\varphi$, $\ln\varphi_{t+1} = \psi\ln\varphi_t + \epsilon_t$ with ϵ_t distributed as $N(0,\sigma^2)$, and what we would like to evaluate is the conditional expectation of some function of $\ln\varphi_{t+1}$ given $\ln\varphi_t$: $E[V(\ln\varphi_{t+1})|\ln\varphi_t] = \int_{-\infty}^{\infty} V(\ln\varphi')G(d\ln\varphi'|\ln\varphi_t)$. To apply the Gauss-Hermite quadrature method in this context, we use the following trick:

$$\begin{aligned}
E[V(\ln\varphi_{t+1})|\ln\varphi_t] &= (2\pi\sigma^2)^{-1/2}\int_{-\infty}^{\infty} V(\ln\varphi')e^{-(\ln\varphi'-\psi\ln\varphi_t)^2/2\sigma^2}d\ln\varphi' \\
&= (2\pi\sigma^2)^{-1/2}\int_{-\infty}^{\infty}\left[V(\ln\varphi')\frac{e^{-(\ln\varphi'-\psi\ln\varphi_t)^2/2\sigma^2}}{e^{-(\ln\varphi'-\mu)^2/2\sigma^2}}\right]e^{-(\ln\varphi'-\mu)^2/2\sigma^2}d\ln\varphi' \\
&= (2\pi\sigma^2)^{-1/2}\int_{-\infty}^{\infty}\left[\tilde{V}(\ln\varphi)\right]e^{-(\ln\varphi'-\mu)^2/2\sigma^2}d\ln\varphi' \\
&= \pi^{-1/2}\int_{-\infty}^{\infty}\tilde{V}(\sqrt{2}\sigma x+\mu)e^{-x^2}dx \\
&\approx \pi^{-1/2}\sum_{i=1}^{n}\omega_i\tilde{V}(\sqrt{2}\sigma x_i+\mu),
\end{aligned}$$

where

$$\tilde{V}(\ln\varphi) \equiv V(\ln\varphi')\frac{e^{-(\ln\varphi'-\psi\ln\varphi_t)^2/2\sigma^2}}{e^{-(\ln\varphi'-\mu)^2/2\sigma^2}},$$

and ω_i and x_i are the Gauss-Hermite quadrature weights and grids.