

Privatization, Unemployment and Subsidy for Low-skilled Labor*

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December 21, 2004

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* We express our thanks to Zsombor Cseres-Gergely and Barna Garay for their useful comments.

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Abstract

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In their seminal paper, Aghion and Blanchard (1993) (for short: A–B) have modeled the dynamic interconnection between privatization and unemployment. A key feature of the transition (not analyzed by A–B) is the huge difference between the unemployment rates of skilled (H) and unskilled (L) labor. Generalizing the A–B-model, in this paper we prove that by introducing an appropriate subsidy, the employment of L-labor can be significantly increased without essentially diminishing that of H-labor. Since the subsidy increases the long-term net wage of L and decreases that of H, there is a limit on subsidies.

Keywords: transition, privatization, unemployment, subsidies

JEL numbers: E62, H23 and P30

1. Introduction

In their seminal paper, Aghion and Blanchard [1993] (A–B for short) have modeled the dynamic interactions between privatization, wages, taxes and employment. Several papers of the ‘optimal speed of transition’ literature that grew out of the A–B model have relaxed its assumptions. including Brixiova and Kiyotaki (1997), Jurajda and Terrell (2003) and Boeri (2001). The model closest to ours was built by Commander and Tolstopyatenko (1997). Commander and Tolstopyatenko (2001) distinguished between the formal and informal sectors, with workers choosing between formal and informal employment in response to taxes and the punishment of tax evasion, while the present authors distinguished between the skilled and unskilled (or high and low productivity workers).

Thus we follow them in assuming heterogeneity (of workers and sectors) while adhering to the logics of the original benchmark A–B model, as far as possible. Under state socialism, all workers are employed in the state sector and are paid a uniform wage irrespective of differences in their (potential) productivity. Following an initial shock, privatization begins and the state (government) sector workers gradually lose their jobs. Hiring decisions in the emerging private sector depend on profit per worker that is affected by workers’ productivity (skill endowments), wages, and taxes. Wages vary with unemployment benefits and the probability of being hired in the private sector, while taxes have to be sufficiently high to cover the costs of unemployment benefits and possible other transfers. Heterogeneity is taken into account in a most simple way by distinguishing between two types of labor (skilled versus unskilled) and two sectors with different skills composition. In the first part of the paper, a model is analyzed assuming a fully segmented economy wherein skilled and unskilled workers are employed in two separate sectors denoted by M and S. Later on we show that the qualitative conclusions do not change if we allow Leontief technologies, with one sector employing predominantly skilled and another employing mostly unskilled labor.

The model has three policy variables: the speed of closing the state sector, the level of the unemployment benefit, and a possible transfer intended to reduce the relative costs of low-skilled labor. (The per-worker subsidy and an endogeneously determined poll tax determine the total tax burden on employment). The parameters are chosen at the onset of transition and left unchanged thereafter. By ‘subsidy’ we mean any kind of assistance, which is financed from tax revenues and reduces the user cost of unskilled labor.

Similarly to A–B we find that there is a maximum speed of closure compatible with successful transition. However, our main point of interest is how employment and wages of the two groups are affected by benefits and net taxes. We identify two regimes conducive to high aggregate employment and relatively small differences in employment probabilities: one with low benefits and another with high benefits combined with subsidy for the low-productivity workers (sector). Generous benefits combined with no transfer lead to lower aggregate employment and severe inequalities.

Why is it important to introduce heterogeneity to the modeling of transition, and how can we justify the idealizations that we make? Distinguishing between skilled and unskilled labor (and the sectors, which employ them) is motivated by the observation of an exceptionally strong bias against less-educated labor in Central and East-European (CEE) labor markets. Recently, the employment ratio of the population having no

upper secondary education ranges between 30 per cent in Slovakia and 47 per cent in the Czech Republic compared to a 57 per cent average in the OECD as a whole. (Table 1). While in the last decade low-skilled employment was on the rise in the OECD it fell with two-digit percentages in the CEEs. This decline continued in most countries until recently. Micro-data available for us in Hungary suggest that the gap between unskilled and skilled employment is indeed skill-specific rather than a statistical artifact reflecting age-specific differentials.

Table 1.

Employment–population ratios in the age range 25-64 by educational attainment, 2001

	Below upper secondary	Upper secondary	Tertiary	Below upper sec. 1995=100
OECD average	57	75	84	101.7
(standard dev.)	(10)	(6)	(7)	
Czech Republic	47	76	88	83.9
Hungary	37 ¹	72	83	100.0 ²
Poland	41	65	84	82.0
Slovakia	30	70	87	76.9
Slovenia ³	40	74	87	...

Source: OECD (2003). Notes: 1) Hungary 1995: authors' calculation from the Labor Force Survey. 2) 86.2 in 1992-95. 3) European Commission, 2003, p. 56, population aged 15-64

Furthermore, the exceptionally low employment ratio of the less-educated accounts for the bulk of the gap that exists between the aggregate employment ratios of the OECD and the CEEs. This is shown in Table 2 where the gaps are decomposed using data on the population's educational composition and education-specific employment ratios. In Hungary, Slovenia and Slovakia almost the entire gap is accounted for by the exceptionally low employment ratio of the less educated; in the Czech Republic this is the only component having a negative contribution; while in Poland other factors (such as the poor employment prospects of workers with vocational secondary school background) also play a role.

Table 2. *Deviation of the employment-population ratio from the OECD average – Decomposition, 2002, per cent*

	Aggregate employment, deviation from the OECD mean per cent	Weighted with the educational composition of the OECD		Weighted with the educational composition of the country	
		Parameter effect of L-employment	All other effect	Parameter effect of L-employment	All other effect
Hungary	– 7.8	–6.4	–1.2	–5.8	–2.0
Poland	–10.7	–6.1	–4.6	–3.4	–7.3
Slovakia	– 5.0	–9.3	4.3	–4.1	–0.9
Slovenia	– 6.0	–5.4	–0.6	–5.8	–0.2
Czech Rep.	2.6	–3.8	6.4	–1.4	4.0

Data source: OECD (2004). Low-skilled stands for less than upper secondary degree

We admittedly make at least three strong simplifying assumptions in modeling the transition with heterogeneous labor. First, we disregard within-sector substitution between skilled and unskilled labor. This assumption can be supported by translog cost function estimates available for Hungary (Halpern et al., 2004) suggesting that the demand for skilled and unskilled labor are nearly independent. Second, we consider two separate segments of the economy with no capital mobility between them, and therefore no tendency to eliminate the profit differentials that occasionally arise. The second idealization is more difficult to justify. It applies as far as we talk of segments with rather different skills requirements and different sources of finance, such as rural SMEs versus large industrial organizations, street-corner shops versus huge shopping centers, traditional micro-businesses versus modern service networks, and so on. We believe that there is indeed little capital mobility between these segments and that their ‘struggle for life’ had crucial impact on the fate of low-skilled labor in transforming economies. Unlike in Southern Europe – a region with similar skill endowments – agriculture, traditional handicraft, retail trade, services and low-tech manufacturing proved incapable of creating jobs for low-skilled wage labor at a large scale.

Third, we do not close our model by showing explicitly what happens to the profits earned by one sector or another – the model simply establishes a link between expected profits and subsequent job creation. Assuming a feedback from higher ex post profits to more job creation would magnify the tendencies that we identify, and would therefore be redundant. A formal account of the linkages between profits, price formation, product market competition and investment decisions extends beyond the capacity of this model, which proves quite complicated even under its restrictive assumptions. We hope that it can even so provide useful insights to the policy options of the new democratic governments and the implications of their choices.

In Sections 2, 3 and 4 we examine the simple model, where the skilled and unskilled labor work exclusively in the M- and S-sectors, respectively. In Section 2 we derive the dynamic equations: the closing of the government sector, the emergence of unemployment and the creation of the private sector.

Section 3 analyzes the simple model. First we show that under moderate subsidies the employment as well as the net wage of the skilled labor is higher than those of the unskilled labor, respectively. Then the post-transition period is analyzed and the stability conditions of full employment are determined.

In Section 4, using simulation, it is demonstrated that for a relatively large period of initial unemployment rates, the system is viable and stable. By choosing an appropriate subsidy, the difference between the two types of employment can be significantly diminished. More precisely, the subsidization of the unskilled labor hardly changes the employment of skilled labor (the burden of subsidies is put on the skilled workers' wage), while the employment of the unskilled labor is radically raised (and the subsidy increases their wages as well). This comparison is repeated for several settings, changing the parameter values of the unemployment benefit and the speed of closing of the government sector.

In Section 5 we relax the counterfactual assumption of the simple model and we only assume that the M-sector employs skilled labor in a much higher proportion than the S-sector does. In the complex model the impact of subsidies is smaller, because the employment of the skilled labor requires the employment of the unskilled, too.

At the end of the introduction, we summarize the essence of our simulation results. For the sake of transparency, it is assumed that the share of the skilled and unskilled in the total population is 0.5-0.5. In contrast to the simple model, in the M-sector 80 per cent of the employees are skilled, and in the S-sector, 80 per cent of the employees are unskilled. We compare the case of no subsidies to that of subsidies (1/4 of the difference between the productivities) and the case of high versus low unemployment benefit (30 per cent vs. 20 per cent of the high productivity). The transition last for 24 years. Then the employment data at the end of transition are as follows.

Table 3. *End-of-transition employment structures*

Model	Unemployment benefit	Subsidy	Skilled Employment	Unskilled Employment
Simple	high	no	0.380	0.199
	high	yes	0.384	0.274
	low	no	0.431	0.338
	low	yes	0.428	0.366
Complex	high	no	0.369	0.245
	high	yes	0.375	0.291

In all the six cases, the employment (rate) of the skilled is much higher than that of the unskilled; the largest difference arises in the simple model with high unemployment benefits and no subsidies (the ratio is close to 2). For high unemployment benefit and given subsidies, the employment is much higher in the complex model than in the simple one: e.g. 24.5 vs. 19.9 for the high benefit–no subsidies case. (In terms of employment rates, the corresponding numbers are twice as high: 49 and 39.8 per cent.) On the other hand, in the simple model with high unemployment benefit, the introduction of realistic subsidies dramatically increases the employment of unskilled, while slightly increasing the employment of the skilled, too. In the simple model with low unemployment benefit and in the complex model, the impact of subsidies is smaller.

Section 6 concludes. The more sophisticated proofs are relegated to a separate Appendix.

2. The simple model

In this Section we present a simple generalization of the A–B-model, where the skilled and unskilled workers are exclusively employed in the M- and the S-sectors, respectively. There is no free flow between the two sectors, therefore the profit differences need not be eliminated. The ingredients of this simple model are the following.

The beginning

For decades, there was a socialist economy with full employment and without any significant difference between skilled and unskilled labor. Due to the sudden collapse of the Soviet-type system, full employment was replaced by a partial employment with an initial unemployment rate u^0 .

Dual labor market

We distinguish two types of labor: skilled (H-type) and unskilled (L-type). The time-invariant productivity of i -type is equal to y_i , $y_H > y_L$.

The i -type's *earning*, w_i changes endogenously: (4) below. For simplicity, let us assume that the government charges a *poll tax* z for every worker, whose value is determined by the macrobudget equation (5). The private sector only employs an i -type worker if the per capita profit π_i is positive. Since the unemployment benefit is independent of type, it is impossible to pay a sufficiently low wage to the unskilled workers which would make their employment profitable. To alleviate unemployment, the government pays the firms *transfer* (k) for every employed of L-type. One part of the transfer (k_1) is simply a *compensation*, without which the unskilled would pay higher tax rates than the skilled ones. (Like in the A–B-model, the government employees do not get any compensation!) The remaining part (k_2) serves as *employment subsidies*: $k = k_1 + k_2$. The net profits per worker are given by $\pi_H = y_H - w_H - z$, $\pi_L = y_L - w_L - z + k$, respectively.

The introduction of a compensation transfer ensures the equality of taxes rather than of tax rates in the two sectors. Indeed, given the other parameters of the model, one can determine a transfer $k = k_1$, so that the tax rates $\tau_H = z/w_H$ and $\tau_L = (z - k)/w_L$ be equal at period $T/2$ and $3T/2$, where T is the length of transition.

Thus the (differential) equations of the A–B-model are modified and doubled as follows.

Closing of government sector

We must break down the government jobs as well as their closing to H- and L-types: let E_H and E_L be the number of H- and L-type workers, respectively, and $E = E_H + E_L$ be their sum. We shall assume that the *initial drop in employment* from $E^* = 1$ to $E^0 < 1$ was followed by a continuous drop in the *stock of government jobs*. Breaking down this process: the initial drop was from (E_H^*, E_L^*) to $(E_H^0, E_L^0) = E^0(E_H^*, E_L^*)$. It will be useful to introduce government employment rates $e_i = E_i/E_i^*$, $e^0 = E^0$. Let $s > 0$ be rate of closing government jobs. Then

$$\dot{e}_H = -s, \quad e_H^0 = e^0 \quad \text{given} \quad (1 - H)$$

and

$$\dot{e}_L = -s, \quad e_L^0 = e^0. \quad (1 - L)$$

Obviously, $e_H = e_L = e = e^0 - st$. The elimination of the government sector is completed at date $T = E^0/s$.

Creation of new jobs

We turn now to the description of the creation of private jobs. Let N_i be the number of i -type workers employed in the private sector. Denote $n_i = N_i/E_i^*$ the private employment rate of i -type workers. We assume that the employment rate increases in proportion to the firm's profit per worker:

$$\dot{n}_H = a(y_H - w_H - z), \quad \text{where} \quad n_H = \frac{N_H}{E_H^*} \quad (2 - H)$$

and

$$\dot{n}_L = a(y_L - w_L - z + k), \quad \text{where} \quad n_L = \frac{N_L}{E_L^*}. \quad (2 - L)$$

Unemployment rate

It is easy to see that the creation of private jobs cannot keep up with the building down of the government jobs, therefore unemployment emerges. The two types of the unemployment rates are respectively

$$u_H = 1 - e - n_H = \Delta e - n_H, \quad U_H = u_H E_H^* \quad (3 - H)$$

and

$$u_L = 1 - e - n_L = \Delta e - n_L, \quad U_L = u_L E_L^*, \quad (3 - L)$$

where Δe is the absolute value of the change in the government employment rate (both H- and L-types). We also need the total unemployment $U = U_H + U_L$ and the total change in the government jobs $\Delta E = \Delta E_H + \Delta E_L$. It is assumed that every unemployed receives a benefit $b > 0$.

Wage equations

We retain the wage equation of A–B, but we break it down for the skilled and the unskilled labor, respectively. Let r be the rate of interest and c the surplus value of working over being unemployed. Then

$$w_H = b + c \left(r + \frac{\dot{n}_H}{u_H} \right) \quad (4 - H)$$

and

$$w_L = b + c \left(r + \frac{\dot{n}_L}{u_L} \right). \quad (4 - L)$$

Taxes and transfers

We have a macro budget equation for the taxes, the benefits and the transfers: $Ub + N_L = (1 - U)z$, i.e.

$$(E_H^*u_H + E_L^*u_L)b + E_L^*n_Lk = (1 - E_H^*u_H - E_L^*u_L)z. \quad (5)$$

System of differential equations

Note that we have a simultaneous system of equations: the wage depends on the employment, the employment in turn (via the profits) depends on the wage. As we will show in the Appendix, substituting (4) into (2) and using (5) yields

$$\dot{n}_H = a \frac{u_H}{u_H + ca} \left(y_H - cr - \frac{b + kE_L^*n_L}{1 - E_H^*u_H - E_L^*u_L} \right), \quad n_H^0 = 0 \quad (6 - H)$$

and

$$\dot{n}_L = a \frac{u_L}{u_L + ca} \left(y_L - cr - \frac{b - k(e + E_H^*n_H)}{1 - E_H^*u_H - E_L^*u_L} \right), \quad n_L^0 = 0. \quad (6 - L)$$

Although we have six equations, two of them [(1)] can be solved directly, another two [(3)] can be eliminated. In fact, we have only two independent equations: nonlinear differential equations. Introducing the notation $\bar{y}_i = y_i - cr$ for the *reduced productivity* of type i , we have

$$\dot{n}_H = a \frac{\Delta e - n_H}{\Delta e - n_H + ca} \left(\bar{y}_H - \frac{b + kE_L^*n_L}{e + E_H^*n_H + E_L^*n_L} \right), \quad n_H^0 = 0 \quad (7 - H)$$

and

$$\dot{n}_L = a \frac{\Delta e - n_L}{\Delta e - n_L + ca} \left(\bar{y}_L - \frac{b - k(e + E_H^*n_H)}{e + E_H^*n_H + E_L^*n_L} \right), \quad n_L^0 = 0. \quad (7 - L)$$

We shall call a system *viable* if all its variables are nonnegative: $0 \leq n_H \leq \Delta e$ and $0 \leq n_L \leq \Delta e$.

3. Analysis

In the section we shall analyze and simulate the simple model with special regard on the impact of subsidies on the employment of the unskilled workers. Following the order of logic, we analyze the post-transition and the full periods intermittently.

Preparations

We shall start the analysis with preparations. We shall make three natural assumptions.

A1. The product of the reduced unskilled productivity and the initial employment rate is greater than the unemployment benefit: $\bar{y}_L(1 - u^0) > b$.

A2. The difference between the two productivities is larger than the transfer: $0 \leq k < y_H - y_L$.

A3. The ratio of skilled to unskilled workers is not very far from 1, say $1/2 < E_H^*/E_L^* < 2$.

Due to its utmost simplicity, it is worth starting the preparations with the beginning of the transition process. Let us recall that before the private sector takes off, full employment in government sector ceases to exist: $E^0 < E^*$, and unemployment suddenly arises: $u^0 = 1 - e^0 > 0$.

Theorem 1. a) *The initial rises in the private employment of the two types are given by*

$$\dot{n}_H(0) = \frac{au^0}{ca + u^0} \left(\bar{y}_H - \frac{b}{1 - u^0} \right) \quad \text{and} \quad \dot{n}_L(0) = \frac{au^0}{ca + u^0} \left(\bar{y}_L - \frac{b - k(1 - u^0)}{1 - u^0} \right).$$

b) *The initial earnings are as follows:*

$$w_H(0) = b + c \left(r + \frac{\dot{n}_H(0)}{u^0} \right) \quad \text{and} \quad w_L(0) = b + c \left(r + \frac{\dot{n}_L(0)}{u^0} \right).$$

Remarks. 1. Due to A2–A1, $\dot{n}_H(0) > \dot{n}_L(0) > 0$, i.e. with common u^0 , $w_H(0) > w_L(0)$.

2. While the initial change in H-employment is independent of the value of the transfer, that in L-employment is a strongly increasing function of k .

Proof. (5) implies

$$z(0) = \frac{u^0}{1 - u^0} b.$$

(6) implies $\dot{n}_i(0)$ and (4) implies $\dot{w}_i(0)$. ■

We have seen in Theorem 1 that at the start the employment as well as the wage is respectively greater in the H-sector than in the L-sector. We shall show now that this is true for the entire period. Because of simultaneity, the direct comparisons of (2–H) and (2–L) or (4–H) and (4–L) is not enough. Furthermore, it would be economically unacceptable if an L-worker earned more than an H-worker.

Theorem 2. *Under A2, (and apart from the start) the employment as well as the wage is respectively greater in the H-sector than in the L-sector: $n_H > n_L$ and $w_H > w_L$.*

Remark. The essence of the proof of $n_H > n_L$ is as follows: it is the last factor what is decisive in (7), and by $\bar{y}_H > \bar{y}_L + k$, it is larger for H than for L (see the proof of Theorem 3 below). In turn, $w_H > w_L$ is a relatively easy consequence of the wage equations and $n_H > n_L$.

Proof. Appendix.

Theorem 1 states that the increase in the subsidies increases the unskilled employment without (much) changing the skilled employment. Because of continuity, this favorable result holds for a while. This results is extended for the whole period by

Conjecture 1. *With a well-chosen subsidy, the unemployment of the unskilled can significantly be reduced without much affecting that of the skilled.*

Remark. Without assumption A3, if the stock of unskilled workers were much higher than that of the skilled workers, the conjecture may not be true.

In the next Example, we shall show that in the excluded limit case $\bar{k} = y_H - y_L$, the differences between the employment rates and the wages of the two types disappear, respectively.

Example 1. In the limit case $\bar{k} = y_H - y_L$, the employment rate and the wage of the skilled are identical with those of the unskilled: $n_H \equiv n_L$ and $w_H \equiv w_L$. In fact, in that case both factors of the RHS of (7–H) are equal to those of the RHS of (7–L). By (4), the earning paths are also identical.

Post-transition period

The *post-transition period* is characterized by the lack of government sector: $[T, \infty)$. Here (1) drops out, thus (7) simplifies to a time-invariant system:

$$\dot{n}_H = a \frac{1 - n_H}{1 - n_H + ca} \left(\bar{y}_H - \frac{b + kE_L^* n_L}{E_H^* n_H + E_L^* n_L} \right), \quad n_H(T) = n_H^T \quad (8 - H)$$

and

$$\dot{n}_L = a \frac{1 - n_L}{1 - n_L + ca} \left(\bar{y}_L - \frac{b - kE_H^* n_H}{E_H^* n_H + E_L^* n_L} \right), \quad n_L(T) = n_L^T \leq n_H(T). \quad (8 - L)$$

Full employment is a steady state: $n_H^o = 1$ and $n_L^o = 1$, because the first factors of (8) are zero. It is of certain interest that in addition to full employment, there may exist other viable steady states if the expressions below are positive:

$$n_H = 1 \quad \text{and} \quad n_L = \frac{b - (\bar{y}_L + k)E_H^*}{\bar{y}_L E_L^*} > 0,$$

or

$$n_L = 1 \quad \text{and} \quad n_H = \frac{b - (\bar{y}_H - k)E_L^*}{\bar{y}_H E_H^*} > 0.$$

In fact, if the unemployment benefit b is sufficiently high, then both additional steady states can be positive.

We turn to the next question: is full employment a locally asymptotical stable steady state of the post-transition system?

Theorem 3. a) Under A1–A2, full employment is the only locally asymptotical stable steady state.

b) The employment of one type goes monotonically to zero if the pair of end-of-transition employment rates $(n_H(T), n_L(T))$ satisfy the condition

$$(\bar{y}_L + k)E_H^* n_H(T) + \bar{y}_L E_L^* n_L(T) < b. \quad (9 - 1)$$

c) Both types' employment rates rise monotonically to 1 if the pair of end-of-transition employment rates $(n_H(T), n_L(T))$ satisfy condition

$$\bar{y}_H E_H^* n_H(T) + (\bar{y}_H - k)E_L^* n_L(T) > b. \quad (9 - 2)$$

Remark. 1. Our assumptions are only sufficient but not necessary, nevertheless, they are not very restrictive. The transfer is reasonably assumed to be much less than the difference between the two productivities, and the unemployment benefit in terms of productivity is also quite small. We shall distinguish two cases: high(er) and low(er) unemployment benefit.

2. The condition (9–2) is not empty, since full employment satisfies it: $(\bar{y}_L + k)E_H^* + \bar{y}_L E_L^* \geq \bar{y}_L E_H^* + \bar{y}_L E_L^* = \bar{y}_L > b$.

3. According to A2, there exists such a zone between the domains of stability and of instability, the points of which generate nonmonotonic paths. We should apply sophisticated tools to decide whether any of these points generates a stable or an unstable path. According to the folklore, two saddle-point unstable steady states are connected by a separatrix path; points above the separatrix generate stable paths, while points below the separatrix generate unstable and unviable ones. In the limit case $\bar{k} = y_H - y_L$, this zone reduces to a single straight line $\bar{y}_H E_H^* n_H(T) + \bar{y}_L E_L^* n_L(T) = b$.

Proof. Using the traditional method, first we shall examine in which states the one or the other variable of the system remains constant: By (8–H), $n_H = \text{const.}$ if

$$n_H = 1 \quad \text{or} \quad \bar{y}_H E_H^* n_H + (\bar{y}_H - k) E_L^* n_L = b;$$

By (8–L), $n_L = \text{const.}$ if

$$n_L = 1 \quad \text{or} \quad (\bar{y}_L + k) E_H^* n_H + \bar{y}_L E_L^* n_L = b.$$

Four possible pairs would give the steady states: (i) full employment; (ii)–(iii) the additional pairs mentioned before presenting Theorem 3; and (iv) the intersection of the two straight lines. A simple computation, however, reveals that in state (iv), (8) is not defined.

Next we turn to the issue, in which states the one or the other variable of the system increase: By (8–H), $\dot{n}_H > 0$ if and only if

$$\bar{y}_H E_H^* n_H + (\bar{y}_H - k) E_L^* n_L > b; \quad (10-H)$$

By (8–L), $\dot{n}_L > 0$ if and only if

$$(\bar{y}_L + k) E_H^* n_H + \bar{y}_L E_L^* n_L > b. \quad (10-L)$$

By A2—in view of $\bar{y}_H - \bar{y}_L = y_H - y_L$ —(10–L) implies (10–H). (In the opposite case, it would be also reversed.) Because the variables are strictly increasing, (10–H) always holds. Since in this region, only full employment is a steady state, the paths starting from this region converge to full employment. The instability can similarly be proved. ■

Theorem 4. *Full-employment tax and wages are respectively simple functions of the model's parameters:*

$$z(\infty) = E_L^* k, \quad w_H(\infty) = y_H - E_H^* k, \quad w_L(\infty) = y_L + E_L^* k.$$

Remark. Under A2, $w_H(\infty) > w_L(\infty)$ holds: a completion of Theorem 2. It is remarkable that in the long run the skilled workers bear the burden of subsidies, because their net wages diminish by $E_L^* k$, while the net wages of the unskilled increase by $E_H^* k$, both from their subsidy-free values.

Proof. Asymptotically, $\dot{n}_i(\infty) = 0$, (5) implies $z(\infty) = E_L^* k$, i.e. (2) implies $\pi_i(\infty) = 0$, etc.. ■

In the following part we shall analyze the difference between the two types' employment rates. To do this, we need the following notations. Let $\gamma(x) = x - ca \ln(1 - x)$ be a function, mapping $[0, 1] \rightarrow [0, \infty]$. By simple derivation, it can be shown that

$$\gamma'(x) = 1 + \frac{ca}{1 - x} > 0,$$

therefore γ has an inverse, which is strictly increasing. Let $\Gamma_i(t) = \gamma(n_i(t))$, $i = H, L$. Then we have

Theorem 5. *In the post-transition period, the two employment rates satisfy the equation*

$$\Gamma_H(t) - \Gamma_L(t) = a(y_H - y_L - k)t + \Gamma_H(T) - \Gamma_L(T), \quad t \geq T. \quad (11)$$

Proof. Appendix.

We shall see from the simulations to be presented below that $n_H(t, k)$ hardly depends on k . Then (11) “implies” that $n_L(t, k)$ is a strictly increasing function of k (to be also seen in the simulation).

What is the impact of the transfer on the employment rates? To answer this question, let us introduce the following sensitivity indices:

$$m_H(t) = \frac{\partial n_H}{\partial k} \quad \text{and} \quad m_L(t) = \frac{\partial n_L}{\partial k}.$$

These indices approximately show the ratios of the change in the employment rates to a small change in the transfer. With their help, one can formulate

Theorem 6. *In the post-transition period, the two employment rates and sensitivity rates satisfy*

$$m_L = \frac{\gamma'(n_H)m_H + at - c_k}{\gamma'(n_L)} \quad (12)$$

where $c_k = \gamma'(n_H(T))m_H(T) - \gamma'(n_L(T))m_L(T)$ is a constant.

Remark. Similarly to Theorem 5, (12) only provides a relative information: if $m_H \geq 0$ is known, then $n_H > n_L$ (Theorem 2) and (12) imply $m_L > 0$, moreover, $m_L > m_H$. However, if $m_H \leq 0$, then we are lost.

Proof. Appendix

Analysis of transition

To guarantee a successful transition, we need even stricter conditions; but they are difficult to obtain analytically. In our 2-sector model, we can only conjecture and simulate the analytical observation of the 1-sector model of A–B.

Conjecture 2. *To any closing rate $s > 0$ there belongs a maximal unemployment rate $u^0(s)$, for which the system is still viable. Higher closing rate determines a lower maximal unemployment rate.*

4. Simulation

Having reached the limits of our analytical power, we turn now to simulation.

Simulating the post-transition period

Since in our model full employment is only reached in the long run (at the end of the post-transition period) and only for sufficiently high end-of-transition employment, we must discuss the transient paths of the post-transition period. Because of their complexity, we must use simulation.

Since we only touch the government sector's closing, we deviate from A–B at several points. We shall work with the following parameter set: $E_H^* = 0.5$; $E_L^* = 0.5$; $u^0 = 0.04$. $y_H = 1$. $y_L = 0.7$; $b = 0.3$; $a = 0.24$, $c = 1.5$; $r = 0.1$. Then $T = 24$ years. For the time being, we exclude subsidies, i.e. $k_2 = 0$ and $k_1 = 0.03$.

First we shall display the stability and instability domains of Theorem 3.

Figure 1

It can be seen in the figure that for “initial” states $N_H(T) = 0.26$ and $N_L(T) = 0.26$ (where the unemployment rate is 48 per cent), the system still converges. Diminishing the end-of-transition employment rates and stepping over the instability line (9–1), the path starting from $N_H(T) = 0.2$ and $N_L(T) = 0.16$ is unstable.

Simulating the full period

Turning to the simulation of the full period, we retain the zero subsidy assumption: $k_2 = 0$. Using the same data as in Figure 1, the system converges to full employment, although the employment rate is very low at the closure of the government sector: $N_H(T) = 0.38$ and $N_L(T) = 0.2$; the unemployment is tragically high and extremely disproportionate: $U_H(T) = 0.12$ and $U_L(T) = 0.3$ (Figure 2a). The wages first decrease, then increase (Figure 2b). The tax rate first increases, then decreases. The two types' tax rates do not diverge too much (Figure 2c) and they are equal in average.

Figure 2

On the basis of Figure 1, one might suppose that for parameters not so well-chosen the transition path ends out of the stability domain, therefore the post-transition system explodes.

As was already mentioned, we are working with time-invariant transfers, namely $k_1 = 0.03$. Our calculation would be more precise if the transfers were at least cleared ex-post. Let us denote the length of an elementary interval by h , then $k(t + h)$ be determined such a way as to ensure the equality of the tax rates at t :

$$\frac{z(t)}{w_H(t)} = \frac{z(t) - k(t + h)}{w_L(t)}.$$

Numerical simulations attest that using an averaging constant transfer is conducive to almost the same.

In Figure 3, we discuss the path with transfer $k = 0.1$ of which $k_2 = 0.07$ can be considered as subsidies. As we hoped for, the introduction of subsidies pushed up the employment of the unskilled, while hardly affecting that of the skilled. Again, we cite

the end-of-transition data: $N_H(T) = 0.384$; $N_L(T) = 0.271$. The employment of the unskilled jumped by 7 per cent points, and even the employment of the skilled rose by 0,4 per cent points.

Figure 3

We arrived now to the centerpiece of our analysis: the role of unemployment benefit. Until now the unemployment benefit was quite high: 0.3. Now we investigate what happens if the benefit is reduced to 0.2 (cf. rows 3-4 of Table 3 above). We shall discuss both cases of no subsidies and subsidies, but now the compensation k_1 drops from 0.03 to 0.015. With no subsidies: $N_H(T) = 0.431$; $N_L(T) = 0.338$ (Figure 4); with subsidies: $N_H(T) = 0.428$; $N_L(T) = 0.366$ (Figure 5). The creation of new jobs becomes quicker and full employment is reached earlier. At the same time, for a lower unemployment benefit, the introduction of subsidies diminishes the skilled employment by 0.3 per cent and raises that of the unskilled by only 2.8 per cent.

Figure 4

Figure 5

Until now we have displayed various scenarios (high benefit–no subsidy, high benefit–subsidy, low benefit–no subsidy, etc.) separately. It is worthwhile to compare them, too. Let us start the comparison with the key variable, namely the employment of low-skilled (Figure 6a). The introduction of subsidy improves the situation, but lowering the unemployment benefit (without the help of subsidy) is even more effective. This ordering is even more visible with the employment dynamics of high-skilled (Figure 6b), where only the lowering of the unemployment benefit has a favorable impact. Concerning the total employment, the impact of the subsidy is also important (Figure 6c).

Turning to the wages, the emerging picture is similar to the previous one. The introduction of subsidy with high unemployment benefit raises the wage of the low-skilled workers, but much less than its replacement with a lower unemployment benefit (Figure 7a.) Concerning the wage of high-skilled workers, the favorable impact of subsidy is negligible in comparison with that of the lowering of the benefit (Figure 7b). Finally, turning to the wage inequalities, represented by $\ln(w_H/w_L)$ is more complex (Figure 7c): The introduction of a subsidy obviously diminishes the inequality, but the lowering of the benefit also reduces it. In the first 30 years, the reduction is stronger with the subsidy scenario, but after that subsidy proves stronger.

Before finishing the simulation of the simple model, we must simulate Conjecture 2 concerning the connection between the closing rate of government sector (Column 1) and the maximal unemployment rate (Column 2), yielding a viable path. The compensation rate is uniformly 0.035.

Table 4. *Minimal viable initial employment as a function the rate of building-down*

Rate of building-down s	Minimal viable initial employment e_0
0.01	0.53
0.02	0.56
0.03	0.66
0.04	0.80

It is visible that the faster the closing, the lower initial unemployment rate is feasible. Indeed, for $s = 0.01$, $u^0 = 0.47$; while for $s = 0.04$, “only” $u^0 = 0.2$.

5. The complex model

For the sake of simplicity, up to now we have pretended that every skilled worker is employed in the M-sector and every unskilled worker is employed in the S-sector. It is high time to give up this simplification. We shall only assume that the M-sector employs skilled workers in high proportions and the S-sector does the same with unskilled workers. This makes our model more realistic, and justifies the persistent relation: the profit rate is higher in the M-sector than in the S-sector. Here we return from the employment rates to the original variables.

For simplicity, we assume that due to complementarities, both types of sectors employ workers in fixed proportions: let β_{fi} be the proportion of i -type workers employed by f -type sector: $\beta_{fH} + \beta_{fL} = 1$, $f = M, S$. By assumptions, $0 \leq \beta_{SH} < \beta_{MH} \leq 1$. In the simple model, $\beta_{SH} = 0$ and $\beta_{MH} = 1$.

New equations

We present the new, further desaggregated equations.

Closing of government sector

$$\dot{e}_i = -s, \quad e_i^0 \text{ given.} \quad (1)$$

Jobs in the private sector

$$n_{fi} = \beta_{fi}N_f, \quad N_f = N_{fH} + N_{fU}, \quad N_i = N_{Mi} + N_{Si}.$$

Per worker profit for firm f

$$\pi_f = \sum_i \beta_{fi}(y_i - w_i - z + k_i). \quad (13)$$

New jobs

$$\dot{N}_{fi} = a_{fi}\pi_f, \quad \text{where } a_{fi} = \beta_{fi}aE_i^*. \quad (2^*)$$

Unemployment

$$U_i = E_i^* - E_i - N_i = \Delta E_i - N_i, \quad U = U_H + U_L. \quad (3)$$

Wages

$$w_i = b + c \left(r + \frac{\dot{N}_i}{U_i} \right). \quad (4)$$

Taxes and subsidies

$$Ub + N_L k = (1 - U)z. \quad (5)$$

After substitutions, as described in the Appendix, one obtains the explicit basic differential equation system:

$$\dot{N}_H = \frac{(1 + \alpha_{LL})C_H - \alpha_{HL}C_L}{\delta} \quad \text{and} \quad \dot{N}_L = \frac{(1 + \alpha_{HH})C_L - \alpha_{LH}C_H}{\delta}, \quad (6^*)$$

where

$$\delta = (1 + \alpha_{HH})(1 + \alpha_{LL}) - \alpha_{HL}\alpha_{LH},$$

$$C_i = \sum_f a_{fi} \sum_{i'} \beta_{fi'} (y_{i'} - b - cr - z(N_H, N_L) + k_{i'}), \quad (14)$$

$$\alpha_{ii'} = c \frac{\sum_f a_{fi} \beta_{fi'}}{U_{i'}}, \quad (15)$$

$$z(N_H, N_L) = \frac{Ub + N_L k_L}{1 - U}. \quad (5^*)$$

New results

The following question arises: will the earlier result survive in the new model? It seems to be obvious that the sufficient condition for stability remains valid if the two sectors are different enough in terms of skill composition. To obtain a precise stability condition, one must analyze the 2×2 derivative matrix $f'_N(\mathbf{N})$ taken at $\mathbf{N} = (E_H^*, E_L^*)$: if both characteristic roots are less than 1 in absolute value, then the system is asymptotically locally stable.

The statements on the initial and the end values remain valid.

New simulations

Let us suppose that the M-sector employs skilled workers in 80 per cent, while the S-sector employs unskilled workers on the same scale: $\beta_{MH} = 0.8$ and $\beta_{SL} = 0.8$. In case of no subsidies ($k_2 = 0$), the employment of the skilled workers grows slower than in the simple model, but the employment of the unskilled workers significantly rises: $N_H(T) = 0.369$ and $N_L(T) = 0.245$ – there is an intra-firm subsidy (Figure 8a). At the same time, the wage of skilled workers may temporarily surpass even their productivity (Figure 8b). Full employment is a stable steady state, it is reached within 80 years.

Figure 8

We could display the impacts of introducing subsidies or of diminishing the unemployment benefit on employment rates. Here we only refer to Table 3 given at the end of the Introduction. The change with respect to the simple model is favorable but moderate: $N_H(T) = 0.375$ and $N_L(T) = 0.291$.

6. Conclusions

The A–B-paper has clearly modeled the dynamic interaction between privatization and creation of new jobs, but by necessity, neglected the heterogeneity of the workers and the sector. In the present paper, we only modified the A–B-model by distinguishing skilled and unskilled workers, employed by the M- and S-sectors with different proportions, respectively. The employment of the unskilled is subsidized. First we examined the simple model, where M-sector employs only skilled workers and S-sector employs only unskilled workers. Here a suitable choice of subsidies raises the employment of

the unskilled labor without affecting that of the skilled labor. In the complex model, the impact of subsidization is more limited because through the fixed employment proportions, the employment of the skilled labor necessitates that of the unskilled labor. Further analysis is needed for a better understanding of the two models.

Appendix

Derivation of (6)

Let us introduce $k_H = 0$ and $k_L = k$. Substituting (4) into (2), yields for $i = H, L$

$$\dot{n}_i = a \left(y_i - b - c \left(r + \frac{\dot{n}_i}{u_i} \right) - z + k_i \right).$$

Expressing \dot{n}_i :

$$\dot{n}_i = \frac{au_i}{u_i + ca} (\bar{y}_i - b - z + k_i).$$

Inserting

$$z = \frac{Ub + N_L k_L}{1 - U}$$

and rearranging, yields (6-H) and (6-L). ■

Proof of Theorem 2

a) First we prove the employment inequality. We shall formulate system (7) in a more general form:

$$\dot{n}_H = g(t, n_H) h_H(t, n_H, n_L), \quad n_H^0 = 0 \quad (\text{A1-H})$$

and

$$\dot{n}_L = g(t, n_L) h_L(t, n_H, n_L), \quad n_L^0 = 0. \quad (\text{A1-L})$$

According to Theorem 1, H-employment increases faster than L-employment, both starting from 0. We shall prove the inequality for an arbitrary $t > 0$, assuming the contrary. Let us assume that it is at date $t^o > 0$, where the inequality is first upset: $n_H(t)$ intersects $n_L(t)$ from above. Substituting $n_H(t^o) = n_L(t^o) = n^o$ into (A1), the first factors are again equal, and for the second factors, $h_H(t^o, n^o, n^o) \geq h_L(t^o, n^o, n^o)$ holds, i.e. by (A1), $\dot{n}_H(t^o) \geq \dot{n}_L(t^o)$, contradicting the intersection condition.

b) Assume the contrary: there exists an instant t^o such at which $w_H(t^o) \leq w_L(t^o)$. Since $w_H(0) > w_L(0)$, and the wage-time functions are continuous, there exists a date $\bar{t} \leq t^o$ at which $w_H(\bar{t}) = w_L(\bar{t})$. Consider the difference between (2-H) and (2-L) at this instant:

$$\dot{n}_H(\bar{t}) - \dot{n}_L(\bar{t}) = a(y_H - y_L - k).$$

By A2, $\dot{n}_H(\bar{t}) > \dot{n}_L(\bar{t})$. Due to the employment inequality, $u_H(\bar{t}) < u_L(\bar{t})$. Comparing (4-H) and (4-L): $w_H(\bar{t}) > w_L(\bar{t})$, a contradiction. ■

Proof of Theorem 5

Rearrange (8) so that only the expression in () remain at the RHS:

$$\frac{1 - n_H + ca}{a(1 - n_H)} \dot{n}_H = \bar{y}_H - \frac{b + kE_L^* n_L}{E_H^* n_H + E_L^* n_L}, \quad n_H^T = n_H(T) \quad (8' - H)$$

and

$$\frac{1 - n_L + ca}{a(1 - n_L)} \dot{n}_L = \bar{y}_L - \frac{b - kE_H^* n_H}{E_H^* n_H + E_L^* n_L}, \quad n_L^T = n_L(T) \leq n_H(T). \quad (8' - L)$$

Taking into account

$$\frac{1 - n_i + ca}{1 - n_i} = \gamma'_i(n_i),$$

and deducing (8'-L) from (8'-H), yields

$$\dot{\Gamma}_H - \dot{\Gamma}_L = a(y_H - y_L - k). \quad (A.2)$$

Integration yields (11). ■

Proof of Theorem 6

Take the derivative of both sides of (11) with respect to k :

$$\gamma'(n_H)m_H - \gamma'(n_L)m_L = -at + c_k. \quad (A.3)$$

This already implies (12). ■

Derivation of (6*)

Inserting (4) into (13), and then the newly obtained equation into (2*):

$$\dot{N}_{fi} = a_{fi} \sum_{i'} \beta_{fi'} \left[y_{i'} - b - c \left(r + \frac{\dot{N}_{i'}}{U_{i'}} \right) - z + k_{i'} \right].$$

Fixing i and summarizing for the two types of firms:

$$\dot{N}_i = \sum_f a_{fi} \sum_{i'} \beta_{fi'} (y_{i'} - b - cr - z + k_{i'}) - c \sum_f a_{fi} \sum_{i'} \beta_{fi'} U_{i'}^{-1} \dot{N}_{i'}.$$

Introducing notation (14) for the first double sum and interchanging the order of summation in the second, yields

$$\dot{N}_i = C_i - c \sum_{i'} \left(\sum_f a_{fi} \beta_{fi'} \right) U_{i'}^{-1} \dot{N}_{i'}.$$

Introducing notation (15), one obtains a two-variable two-dimensional algebraic linear equation system for \dot{N}_H and \dot{N}_L . Warning: the coefficients appearing in (14), (15) and (5) are *not* constants! ■

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