Search Frictions and Asset Price Volatility

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1 Introduction

LeRoy and Porter (1981) and Shiller (1981) calculated the time series for asset prices using the simple present value formula – the current price of an asset is equal to the expected discounted present value of its future dividends. Using a constant interest rate to discount the future, they showed that the variance of the observed prices for U.S. equity exceeds the variance implied by the present value formula (see figure 1). This is the *excess volatility puzzle*. Equilibrium models of asset pricing deliver a generalized version of the present value formula. In Lucas (1978), for instance, the discount factor is stochastic and depends on the intertemporal marginal rate of substitution (IMRS) of the representative consumer. There have been several attempts to explain the excess volatility puzzle. LeRoy and LaCivita (1981) and Michener (1982) examine the role of risk aversion. Flavin (1983) and Kleidon (1986) examine whether small sample bias can statistically account for violations of the variance bound. Marsh and Merton (1986) try to resolve the puzzle with different statistical assumptions on the dividend process.[1] Shiller (1984) and Ingram (1990) explore whether the existence of rule-of-thumb traders can account for the excess volatility.

In related work, Mehra and Prescott (1985) showed that for reasonable values of risk aversion the Lucas asset pricing model cannot reproduce the observed equity premium. This is the *equity premium puzzle*. Resolutions of the equity premium puzzle have followed two distinct paths. One approach was to retain the complete frictionless markets framework of Lucas, but abandon the separability assumptions in the preferences. Prominent examples of this approach are Weil (1989) and Epstein and Zin (1991), who use state non-separable preferences, and Constantinides (1990), who uses time non-separable preferences. The second approach to resolving the equity

[1] West (1988) develops a volatility test that circumvents the above small sample bias and dividend process criticisms and shows that the observed stock prices are indeed too volatile.

premium puzzle abandons the complete frictionless markets framework.[2] Aiyagari (1993), Lucas (1994) and Heaton and Lucas (1996) are examples of this approach. The frictions in these models include uninsured idiosyncratic risks and transaction costs.

This paper follows the frictions approach. Our purpose is to examine the quantitative effect of search frictions in product markets on asset prices. We combine several features from Shi (1997) and Lagos and Wright (2002) in a model without money. Households prefer special goods and general goods. Special goods can be obtained only via trade in decentralized markets. This trading process involves search and bargaining. Similar to Shi and Lagos-Wright, the search frictions make intertemporal trade infeasible in our model. General goods can be obtained via trade in centralized competitive markets and via ownership of an asset. There is only one asset in our model and that is similar to a Lucas tree that yields fruits that can be consumed directly. The asset is also used as a medium of exchange in the decentralized market to obtain the special goods. The value of the asset in facilitating transactions in the decentralized market is determined endogenously.[3] If we shut down the decentralized trading process (i.e., special goods and search frictions), then our model is identical to that of Lucas (1978).

With only one asset, our model cannot address the equity premium puzzle, but we provide parameters for which the model delivers the average rate of return on equity and the volatility of equity price. It turns that the Lucas model can deliver the average rate of return on equity for reasonable values of risk aversion, but it cannot simultaneously deliver the volatility. The price-dividend ratio implied by the model is high relative to the data while for the same risk aversion the Lucas model underpredicts the price-dividend ratio. When we calibrate the model to deliver the

[2]Mehra and Prescott (1985) suggest this approach in their concluding remarks – "Perhaps introducing some features that make certain types of intertemporal trades among agents infeasible will resolve the puzzle."

[3]See Bansal and Coleman (1996) for a reduced form model of the transaction role of assets and its implications for asset returns.

observed price-dividend ratio, the implied value for the medium of exchange role of the asset is on average 14.3% above the Lucas model.

The rest of paper is organized as follows. In the next section we set up the economic environment and derive the equilibrium asset pricing equation. In section 3, we study the quantitative implications of the model.

2 The Environment

Consider a discrete-time non-monetary economy with special goods and general goods, decentralized day markets and centralized night markets, and aggregate uncertainty. The special and general goods and the day and night markets are similar to Lagos and Wright (2002). There are $H \geq 3$ types of households and there is a continuum of households in each type. The type size is normalized to one. A type h household consumes only good h but produces only good $h+1$. The utility from consuming c units of the special good is $u(c)$. The utility function is increasing and strictly concave, and satisfies $u(0) = 0$, $u'(0) = \infty$ and $u'(\infty) = 0$. To produce q units of the special good, households incur q units of disutility. The special goods are non-storable between periods.

There is an infinitely lived asset (Lucas tree) in this economy that yields dividends (fruits) each period. Fruits are general goods and they follow an exogenous stationary stochastic process. The utility from consuming d units of fruits is $U(d)$, where $U(\cdot)$ is increasing and strictly concave. Note that there is no cost to producing the fruits. The fruits are also perishable. Each household is initially endowed with one (divisible) tree.

Special goods are exchanged in a decentralized market in daytime where agents meet in pairs randomly, as in standard search theory. The random matching technology combined with the household preferences rules out barter in pairwise meetings. Furthermore, there is no public record of transactions to support any credit arrangements. Thus, in pairwise meetings special goods are exchanged for trees. General goods are available for trade only in the centralized market at night. The night market is frictionless and trees are exchanged for general goods at the competitive equilibrium price p.

Time is indexed by $t = 0, 1, ...$ The discount factor between periods is β. There is no discounting between day and night.

Random matching during the day will typically result in non-degenerate distributions of asset holdings. In order to maintain tractability, we use the device of large households along the lines of Shi (1997). Each household consists of a continuum of worker-shopper (or, seller-buyer) pairs. Buyers cannot produce the special good, only sellers are capable of production. We assume the fraction of buyers = fraction of sellers = $\frac{1}{2}$. Let $\alpha = \frac{1}{H}$. Then, the probability of single coincidence meetings during the day is $\frac{1}{4}\alpha$. Each household sends its buyers to the decentralized day market with *take-it-or-leave-it* instructions (q, s) – accept q units of special goods in exchange for s trees. Each household also sends its sellers with "accept" or "reject" instructions. There is no communication between buyers and sellers of the same household during the day. After the buyers and sellers finish trading in the day, the household pools the trees and shares the special goods across its members each period. By the law of large numbers, the distribution of trees and special goods are degenerate across households. This allows us to focus on the representative household. The representative household consumption of the special good is $\frac{\alpha}{4}q$.

2.1 Timing of events in each period

- The representative household starts the period with a trees.
- It observes the aggregate state d (fruits per tree), but the fruits are not available for trade during the day.
- The household determines the take-it-or-leave-it offer (q, s). It allocates s trees

to each buyer in the household and provides trading instructions to its sellers and buyers.

- The sellers and buyers from households of *all* types are randomly matched in the decentralized market. In single coincidence meetings, the sellers produce the special good in exchange for trees from the buyers.
- Each household then pools its purchases and consumes the special goods.
- Next, each household enters the centralized market at night with its new asset balance and fruits. Households trade fruits and trees in the centralized competitive asset market (much like the standard consumption based asset pricing model) at price p.
- Then, they consume the fruits and end the period with a' trees.

2.2 Optimization

We begin with the representative household's instructions to its buyers and sellers. Clearly, if a member of the household is not in a single coincidence meeting, the instruction is not to trade. The instruction to the buyers in single coincidence meetings is a the take-it-or-leave-it offer (q, s). For another household's seller to be indifferent between accepting and rejecting the buyer's offer in the random match, (q, s) has to satisfy the seller's participation constraint:

$$\Omega s - q = 0, \tag{1}$$

where Ω is the other household's valuation of the asset. The first term on the left hand side is the gain to the seller from obtaining s trees in the trade. The second term is the disutility from q units of the special good. The take-it-or-leave-it offer will leave no surplus for the seller, so the right hand side is 0 (since $u(0) = 0$). We will assume that the seller will accept the offer whenever he is indifferent. An additional restriction on the offer is that the total number of trees allocated to the buyers by the representative household cannot exceed the number of trees that the household started the period with:

$$\frac{1}{2}s \leq a. \tag{2}$$

This is because (i) the decentralized market does not support credit arrangements, so the buyer cannot short-sell the asset and (ii) the buyer is temporarily separated from other members of the household, so he cannot borrow from the other members of the household. We can eliminate s by combining the two constraints (1) and (2):

$$\frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a.$$

The representative household's instruction to its sellers in single coincidence meetings are straightforward. Suppose that the buyer from the other household offers (Q, S). The instruction is, if the surplus from (Q, S) is non-negative, accept the offer and produce Q units of the special good; otherwise, reject the offer and do not trade.

The representative household's problem then is described by the following dynamic program:

$$v(a, d) = \max_{q,x,a'} u\left(\frac{\alpha}{4}q\right) - \frac{\alpha}{4}Q + U(x) + \beta E_{d'|d}v(a', d') \tag{3}$$

$$\text{s. t. } \frac{1}{2}\left(\frac{q}{\Omega}\right) \leq a \tag{4}$$

$$x + pa' = \left\{a + \frac{\alpha}{4}S - \frac{\alpha}{4}\left(\frac{q}{\Omega}\right)\right\}(p + d), \tag{5}$$

where Q is the amount of the special good obtained by the buyers from other households and S is the number of trees obtained by the sellers from other households. The second constraint is the wealth constraint for the household. Note that p is the relative price a tree in terms of the fruits in the centralized night market.

Remark 1 *If $\alpha = 0$ (i.e., no search frictions or special goods), then our model is identical to that of Lucas (1978). In this case, the asset has positive value since it yields dividends. The presence of search frictions ($\alpha > 0$) implies an additional "liquidity" value to the asset.*

Uniqueness, concavity and differentiability of $v(\cdot)$ follows from theorems 9.6, 9.7, and 9.8 in Stokey, Lucas and Prescott (1989).

2.3 Equilibrium

Definition 2 *An equilibrium consists of a sequence* $\{q_t, x_t, s_t, a_{t+1}\}_{t=0}^{\infty}$, *given initial asset holdings, such that*

1. *Given other households' offers and valuations, each household's choices solve the dynamic program (3);*

2. *The choices and the asset valuations are the same across households;*

3. *The centralized markets clear for all t: $x_t = d_t$, $a_{t+1} = 1$.*

Let $\frac{\alpha}{2}\lambda$ be the multiplier on the constraint (4). The first order conditions for the representative household with respect to q and a' are as follows.

$$u'(\frac{\alpha}{4}q) = \frac{1}{\Omega}\left\{(p+d)\,U'(x) + \lambda\right\} \tag{6}$$

$$pU'(x) = \beta E_{d'|d}\frac{\partial v(a', d')}{\partial a'} \tag{7}$$

In these conditions, we have used the wealth constraint (5) to substitute for x. Note that if the no-short-sales constraint (4) does not bind, then $\lambda = 0$. The envelope condition for a implies that

$$\frac{\partial v(a, d)}{\partial a} = (p+d)\,U'\left(x\right) + \frac{\alpha}{2}\lambda \tag{8}$$

Using (6) to substitute for λ in (8), we get

$$\frac{\partial v(a, d)}{\partial a} = \left(1 - \frac{\alpha}{2}\right)(p+d)\,U'\left(x\right) + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q\right)\Omega.$$

We can rewrite (7) using the above expression for $\frac{\partial v}{\partial a}$:

$$pU'(x) = \beta E_{d'|d}\left\{\left(1 - \frac{\alpha}{2}\right)(p'+d')\,U'\left(x'\right) + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q'\right)\Omega'\right\}. \tag{9}$$

We have to now impose the equilibrium conditions on (9). The valuation of the asset, Ω, by other households in the decentralized market during the day, has to equal the valuation, ω, by the representative household, in equilibrium. We can determine ω as follows. An additional unit of asset obtained in the decentralized market yields d fruits at night; the asset can also be sold for p fruits in the centralized market at night. On the margin these additional fruits are valued at $U'(x)$. In equilibrium, the general goods market clearing at night implies $x = d$. Hence,

$$\omega = \Omega = (p + d)\, U'(d)\,.$$

Using the equilibrium values for Ω and x, we can write (9) as

$$pU'(d) = \beta E_{d'|d} \left\{ (p' + d')\, U'(d') \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} u' \left(\frac{\alpha}{4} q' \right) \right] \right\}.$$

Hence, the equilibrium sequence of asset prices satisfy

$$p_t U'(d_t) = \beta E_t \left\{ (p_{t+1} + d_{t+1})\, U'(d_{t+1}) \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} u' \left(\frac{\alpha}{4} q_{t+1} \right) \right] \right\}. \qquad (10)$$

Again, note that if $\alpha = 0$, then the above asset pricing equation is identical to that of Lucas (1978). In the presence of search frictions, the price in the competitive asset market accounts for the future liquidity value of the asset as well.[4]

To solve for the equilibrium sequence $\{q_t\}$, we have to account for two possible scenarios. If the constraint (4) does not bind in period t, then λ_t equals zero and $u'(\frac{\alpha}{4} q_t) = 1$. Denote the solution to this equation as q^*. Note that the solution does not depend on the aggregate state and, hence, is time-invariant. Furthermore, if $q_t = q^*$ for all t, then the search frictions are irrelevant for the asset pricing implications and the price sequence in our model is the same as in Lucas (1978). If the constraint (4) binds in period t, then

$$q_t = 2\,(p_t + d_t)\, U'(d_t)\,. \qquad (11)$$

[4] Vayanos and Wang (2002), Duffie, Garleanu and Pedersen (2003) and Weill (2003) consider search frictions in the asset market and present models of liquidity premium.

3 Quantitative Implications

To examine the quantitative implications of our model, we restrict the utility functions to be of the CRRA class,

$$
\begin{aligned}
u(c) &= \frac{c^{1-\sigma}}{1-\sigma} \\
U(x) &= \frac{x^{1-\delta}}{1-\delta}
\end{aligned}
$$

where $0 < \sigma < 1$ and $0 < \delta < \infty$ are the coefficients of relative risk aversion. Hence, q^* is the unique solution to $\left(\frac{\alpha}{4}q\right)^{-\sigma} = 1$.

When the no-short-sales constraint (4) binds, $q = 2(p+d)d^{-\delta}$. In our computation later we will assume that the constraint (4) binds for all t. Thus, we combine (10) and (11) and write the asset pricing equation for these functional forms as

$$
p_t d_t^{-\delta} = \beta E_t \left\{ \left(p_{t+1} + d_{t+1}\right) d_{t+1}^{-\delta} \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} \left\{ \frac{\alpha}{2} \left(p_{t+1} + d_{t+1}\right) d_{t+1}^{-\delta} \right\}^{-\sigma} \right] \right\}. \tag{12}
$$

With the equilibrium values for the price sequence we can compute the quantities $\{q_t\}$ and we will verify ex-post whether the constraint (4) is indeed binding for all periods.

3.1 Numerical method

To compute the price sequence, we follow the Monte Carlo simulation method proposed by Judd (1998). Since we need to generate the sequence of prices over sample period, we compute the asset price in each period given the realized data up to that period instead of the pricing function.

First rewrite the pricing equation (12) as

$$
p_t = \beta E_t \left\{ \begin{array}{c} \left(1 - \frac{\alpha}{2}\right) \left(\frac{d_{t+1}}{d_t}\right)^{-\delta} \left(p_{t+1} + d_{t+1}\right) \\ + \left(\frac{\alpha}{2}\right)^{1-\sigma} d_{t+1}^{\sigma\delta} \left(\frac{d_{t+1}}{d_t}\right)^{-\delta} \left(p_{t+1} + d_{t+1}\right)^{1-\sigma} \end{array} \right\}. \tag{13}
$$

Since the current price is a non-linear function of future prices for $\sigma \in (0, 1)$, it is difficult to write the current price as a function of expected future dividend streams.

We overcome this problem by approximating part of the pricing equation. The term $(p_{t+1}+d_{t+1})^{1-\sigma}$ can be written as $(p_{t+1}/d_{t+1}+1)^{1-\sigma}\, d_{t+1}^{1-\sigma}$, and we linearize $(p_{t+1}/d_{t+1}+1)^{1-\sigma}$ around its mean $\overline{w}+1$. The first order Taylor expansion of $(p_{t+1}/d_{t+1}+1)^{1-\sigma}$ is:

$$\left(\frac{p_{t+1}}{d_{t+1}}+1\right)^{1-\sigma} \approx (1-\sigma)\left(\overline{w}+1\right)^{-\sigma}\left(\frac{p_{t+1}}{d_{t+1}}+1\right)+\sigma\left(\overline{w}+1\right)^{1-\sigma}. \tag{14}$$

(The mean price-dividend ratio, $\overline{w}$, is 22.75 in our sample.) Plug (14) into (13) to obtain

$$p_t=\beta E_t\left\{\begin{array}{c}\left(p_{t+1}+d_{t+1}\right)\left(1-\frac{\alpha}{2}+(1-\sigma)\left(\frac{\alpha}{2}\right)^{1-\sigma}\left(\overline{w}+1\right)^{-\sigma} d_{t+1}^{\sigma\delta-\sigma}\right)\left(\frac{d_{t+1}}{d_t}\right)^{-\delta}\\ +\sigma\left(\frac{\alpha}{2}\right)^{1-\sigma}\left(\overline{w}+1\right)^{1-\sigma} d_{t+1}^{1+\sigma\delta-\sigma}\left(\frac{d_{t+1}}{d_t}\right)^{-\delta}\end{array}\right\}. \tag{15}$$

Let

$$\begin{aligned}F_{t+1} &= 1-\frac{\alpha}{2}+(1-\sigma)\left(\frac{\alpha}{2}\right)^{1-\sigma}\left(\overline{w}+1\right)^{-\sigma} d_{t+1}^{\sigma\delta-\sigma}\\ G_{t+1} &= \sigma\left(\frac{\alpha}{2}\right)^{1-\sigma}\left(\overline{w}+1\right)^{1-\sigma} d_{t+1}^{1+\sigma\delta-\sigma}\end{aligned}$$

so (15) becomes

$$p_t=\beta E_t\left\{\left[\left(p_{t+1}+d_{t+1}\right)F_{t+1}+G_{t+1}\right]\left(\frac{d_{t+1}}{d_t}\right)^{-\delta}\right\}. \tag{16}$$

The no-bubbles solution can be obtained by repeated substitution of prices using (16) i.e.,

$$p_t=E_t\sum_{j=1}^{\infty}\beta^j\left\{\left(\frac{d_{t+j}}{d_t}\right)^{-\delta}\left[\left(\prod_{i=1}^{j}F_{t+i}\right)d_{t+j}+\left(\prod_{i=1}^{j}F_{t+i-1}\right)G_{t+j}\right]\right\}. \tag{17}$$

where F_t is defined to be 1.

Given the price sequence $\{p_t\}$, we can calculate the asset returns by

$$R_{t+1}=\frac{p_{t+1}+d_{t+1}}{p_t}.$$

The key problem now is to calculate the expectation in (17). This can be done by estimating the stochastic process for dividend assuming the dividend is trend stationary and simulating the sequence of dividends.[5] (See Figure 2.)

1. Assume that the log of dividends follows a trend stationary process:

$$\ln d_{t+1} = b_0 + b_1 \ln d_t + b_2 t + \eta_{t+1} \tag{18}$$

   where η_{t+1} is the disturbance with mean 0.

2. For each period t, use data in period t as the initial value to simulate a time series for dividends of length 200. That is, generate a sequence d_{t+j}, $j = 1, ..., 200$ using the coefficients in (18) and drawing the disturbances η_t from its empirical distribution. (An alternative is to draw these disturbances under the assumption that η is normally distributed.) Plug the appropriate values into (17) to calculate

$$\sum_{j=1}^{200} \beta^j \left\{ \left(\frac{d_{t+j}}{d_t} \right)^{1-\delta} \left[\left(\prod_{i=1}^{j} F_{t+i} \right) d_{t+j} + \left(\prod_{i=1}^{j} F_{t+i-1} \right) G_{t+j} \right] \right\}.$$

3. Repeat step 2 many times. The number of replications we use is 1000. The average value of these 1000 calculations is p_t.

4. Repeat step 2 and 3 for periods $t+1$, $t+2$, ..., until the end of sample period.

Using the time series of p calculated from above steps, we can compute the rate of return sequence $\{R_{t+1}\}$ for the whole sample period. This will allow us to calculate the unconditional moments of prices and returns. Two issues about this calculation are worth noting. The first is whether the bubble term will indeed converge to zero. The parameters we use in the following sections will satisfy this requirement. The product $\beta^{200} \left(\frac{d_{t+200}}{d_t} \right)^{-\delta} \left(\prod_{j=1}^{200} F_{t+j} \right)$ is very close to zero and, hence, a time series of length 200 provides a good approximation for the infinite sum in (17). The second issue is

[5] See DeJong and Whiteman (1991) for evidence on trend stationarity.

the number of replications used to calculate the expectation. When we quadruple the number of replications to 4000, our results are unchanged.

3.2 Data and Parameters

The data are all in real terms and obtained from Shiller's website. The sample period is 1871-1995. We measure the asset prices and dividends by the S&P 500 prices and per capita dividends. We measure the volatility of a variable by the standard deviation of the detrended time series of the variable. The average rate of return on equity in this sample is 8% and the standard deviation of the equity price is 81. The mean growth rate of dividend is 1.91% and the standard deviation of detrended dividend is 1.61.

Other than the coefficients in the trend stationary process, we have three preference parameters, σ, δ, and β, and one parameter α that describes the extent of departure from the standard asset pricing model. The estimates of the coefficients are

$$\begin{aligned} b_0 &= 0.308 \\ b_1 &= 0.802 \\ b_2 &= 0.002 \end{aligned}$$

and the variance of η is 0.0136.

We set $\beta = 0.96$. We searched for α, σ and δ to match the observed average rate of return on equity and standard deviation of the asset price. There are several restrictions on these parameters. Recall that we have assumed $u(0) = 0$, so σ must be less than 1. The number of types of special goods in our model is assumed to be 3 or more ($H \geq 3$), so $\alpha \leq \frac{1}{3}$. Finally, we have assumed that the no-short-sales constraint binds, so we have to verify that our equilibrium quantities and prices satisfy (11).

3.3 Results

For the benchmark parameters in the table below the average rate of return on the asset is 8% and the standard deviation of the asset price is 84.

Table 1. Benchmark Parameters

β	α	σ	δ
0.96	0.03	0.11	3.0

Recall that the mean return in the data is 8% and the standard deviation of the asset price is 81. In figure 3, we illustrate the equilibrium price sequence implied by the model. In figure 4 we illustrate the price-dividend ratio. The mean price-dividend ratio in the data is 22.75 while the model implies a mean of 26.

In figures 5 and 6, we plot the price sequence implied by the model as we vary the parameters σ and α. (The other parameters β and δ are fixed at their benchmark values.) Changes in σ affect the curvature of the utility function associated with the special consumption good. As σ increases, the asset price volatility increases. As we move farther away from the standard frictionless asset pricing model (increase in α), the asset price volatility increases. The price-dividend ratio exhibits a similar pattern. Figures 7, 8 and 9 illustrate the effects of σ and α on the average rate of return, the volatility and the price-dividend ratio. The table below present a summary of the comparative dynamics associated with changes in σ and α.

Table 2. Comparative dynamics ($\beta = 0.96$ and $\delta = 3$)

Average rate of return (%)				
	σ			
α	*0.05*	*0.10*	*0.13*	*0.15*
0.01	8.96	8.71	8.53	8.39
0.03	8.70	8.19	7.85	7.63
0.05	8.51	7.82	7.45	7.31
0.10	8.13	7.33	7.75	8.99

Std. deviation of the asset price				
	σ			
α	*0.05*	*0.10*	*0.13*	*0.15*
0.01	62.2	66.8	70.9	74.5
0.03	66.9	80.2	93.7	108
0.05	71.4	95.3	126	173
0.10	82.8	158	662	6000

Mean Price-dividend ratio				
	σ			
α	*0.05*	*0.10*	*0.13*	*0.15*
0.01	20.8	22.1	23.3	24.3
0.03	22.2	26.0	29.8	33.6
0.05	23.5	30.2	38.5	49.9
0.10	26.8	46.6	127	639

The standard asset pricing model ($\alpha = 0$) delivers the observed average rate of return on equity for risk aversion $\delta = 2.65$. In figure 10, we plot the price sequence for this case. The standard deviation of the asset price, however, is 53 while the observed volatility is 81. In figure 11, we illustrate the price-dividend ratio. The mean price-dividend ratio implied by the standard asset pricing model is 20.7 while the observed mean is 22.75. Changes in δ affect the price sequence as shown in figure 12. As α approaches zero in our model, the average rate of return and the volatility in our model approach the values in the Lucas model.

It is clear from comparing the price-dividend ratio in figure 4 to that in figure 11 that the asset in our model has a significant value as the medium of exchange. To compute the "liquidity premium" of the asset, we calibrate the model to match the observed mean price-dividend ratio. Holding β, α and δ at their benchmark values, when we decrease σ to 0.06, the mean price-divided ratio implied by the model is the same as in the data. These new parameters imply an average equity return of 8.6% and a price volatility of 69. We then calculate the asset prices for a model with $\alpha = 0$ and β and δ set at their benchmark values. This is, of course, the standard asset pricing model. (Note from (12) that the value of σ is irrelevant for this calculation.) Since the standard model does not assign any medium of exchange role to the asset, the difference between the prices implied by the two models would be the premium paid for liquidity. The mean price-dividend ratio in the standard model is 20. In figure 13, we illustrate the liquidity premium as a fraction of the price implied by the standard model i.e., liquidity premium $= \frac{P_{\text{model}} - P_{\text{Lucas}}}{P_{\text{Lucas}}}$. The mean liquidity premium implied by the model with search frictions is 14.3%.

3.3.1 The Hansen-Jagannathan Bound

In this section we examine whether the IMRS in our model satisfies the Hansen and Jagannathan (1991) bound. Hansen and Jagannathan proposed a test that generalizes the variance bounds developed by LeRoy and Porter (1981) and Shiller (1981). They used asset return data to derive a lower bound on the volatility of a representative household's IMRS. An asset pricing model is said to be consistent with the data if the volatility of the IMRS implied by the model is greater than the HJ bound. To derive the bound, Hansen and Jagannathan projected the model IMRS onto a space of contemporaneous asset returns and utilized only a necessary condition associated with dynamic models, namely the intertemporal Euler equation. For instance, in the Lucas model, the unconditional version of the Euler equation can be written as

$$ER_{t+1}m_{t+1} = 1,$$

$$\text{where } R_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} \text{ and } m_{t+1} = \beta \left(\frac{d_{t+1}}{d_t} \right)^{-\delta}.$$

To compute the HJ bound for the case of 1 risky asset, consider the least squares projection of the IMRS onto the linear space spanned by a constant and contemporaneous returns. The projection is of the form

$$m = Em + (R - ER)\theta + \nu,$$

where Em is the mean of the model IMRS and ER is the mean asset return. The projection error ν is orthogonal to the constant as well as contemporaneous returns, so $ER\nu = 0$, and $E\nu = 0$. Hence,

$$\begin{aligned}
\text{var}(m) &= \theta^2 var(R) + \text{var}(\nu) \\
&\geq \theta^2 \text{var}(R).
\end{aligned}$$

(The notation var(x) refers to variance of x.) The projection coefficient $\theta = \frac{Cov(R,m)}{var(R)}$, where the numerator is the contemporaneous covariance between R and m. We can rewrite $\theta = \frac{ERm - EmER}{var(R)}$. The Euler equation then implies $\theta = \frac{1-EmER}{\text{var}(R)}$. Satisfying the HJ bound amounts to verifying whether

$$\begin{aligned}
\text{var}(m) &\geq \frac{(1 - EmER)^2}{\text{var}(R)}, \text{ or} \\
\text{std}(m) &\geq \frac{1 - EmER}{\text{std}(R)}
\end{aligned}$$

for the chosen preference parameters and observed dividend data.

He and Modest (1995) and Luttmer (1996) showed that the presence of frictions alters the HJ bound. The unconditional version of the Euler equation could be, for instance,

$$ER_{t+1}m_{t+1} = \psi < 1.$$

In this case, the lower bound on the volatility of the IMRS is $\frac{\psi - EmER}{\text{std}(R)}$. They then choose the value of ψ that minimizes the volatility bound. Clearly, such a strategy assumes that ψ does not depend on the model parameters. The environment described in section 2 suggests a different approach. Suppose that we can measure the medium of exchange transactions q. The asset pricing equation (10) can be written as

$$E\left\{\left(\frac{p_{t+1} + d_{t+1}}{p_t}\right)\beta\frac{U'(d_{t+1})}{U'(d_t)}\left[1 - \frac{\alpha}{2} + \frac{\alpha}{2}u'\left(\frac{\alpha}{4}q_{t+1}\right)\right]\right\} = 1.$$

Rewrite this equation in the familiar form

$$ER_{t+1}m_{t+1} = 1,$$

$$\begin{aligned} \text{where } R_{t+1} &= \frac{p_{t+1} + d_{t+1}}{p_t} \text{ and} \\ m_{t+1} &= \beta \frac{U'(d_{t+1})}{U'(d_t)} \left[1 - \frac{\alpha}{2} + \frac{\alpha}{2} u' \left(\frac{\alpha}{4} q_{t+1} \right) \right]. \end{aligned}$$

The HJ bound then is $\frac{1-EmER}{\mathrm{std}(R)}$, exactly the same as in the case without frictions. However, the IMRS is very different.

4 Conclusion

In this paper, we consider an environment with search frictions in the goods market. The asset in our model is used to facilitate trading in the goods market. This transaction role makes the asset pricing implications of our model different from those in the standard asset pricing model. We show that a "small" departure from the standard asset pricing model can simultaneously deliver the observed average rate of return on equity and the volatility of the asset price.

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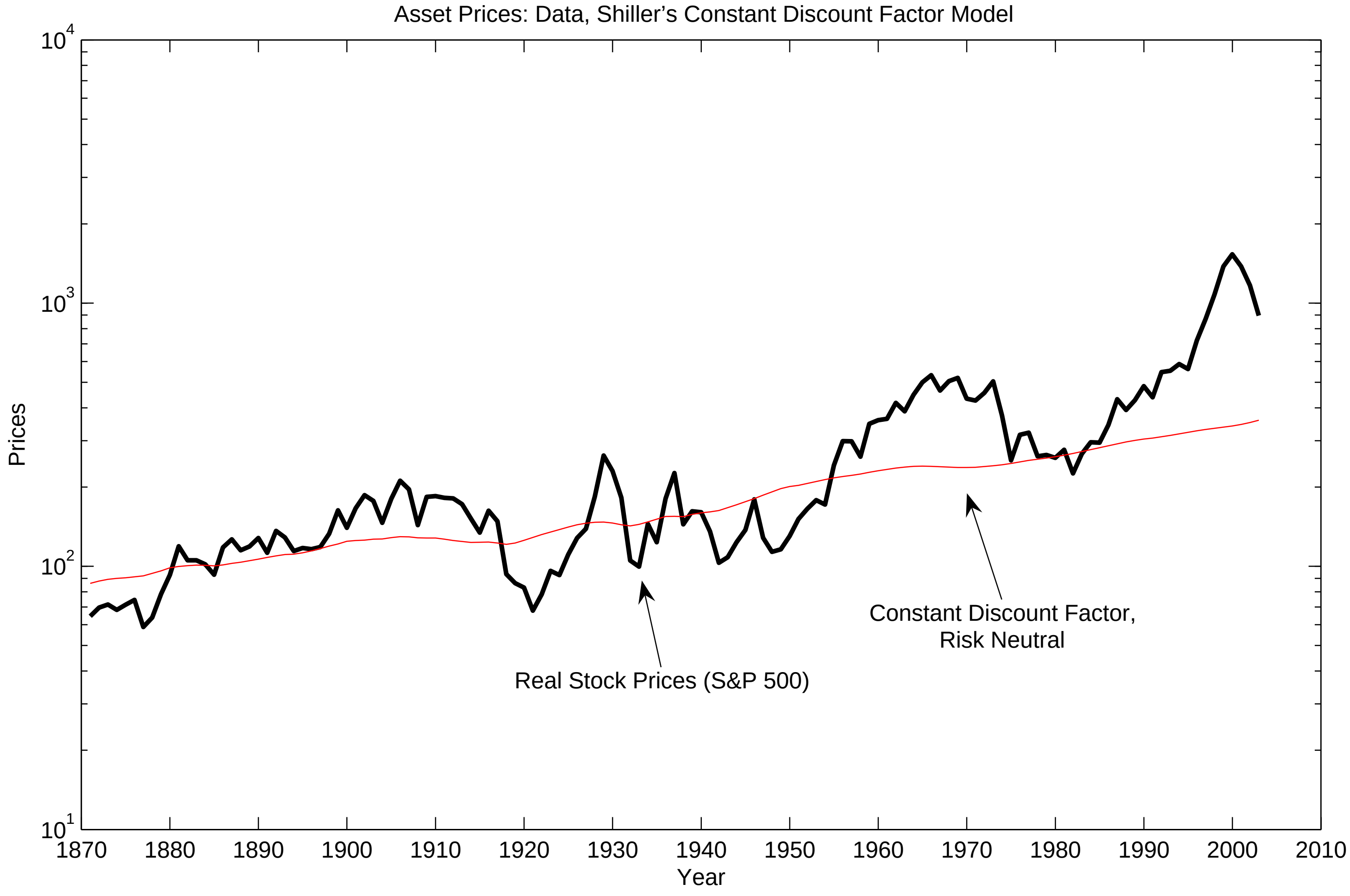

Figure 1: Data and Shiller's Constant Discount Factor Model

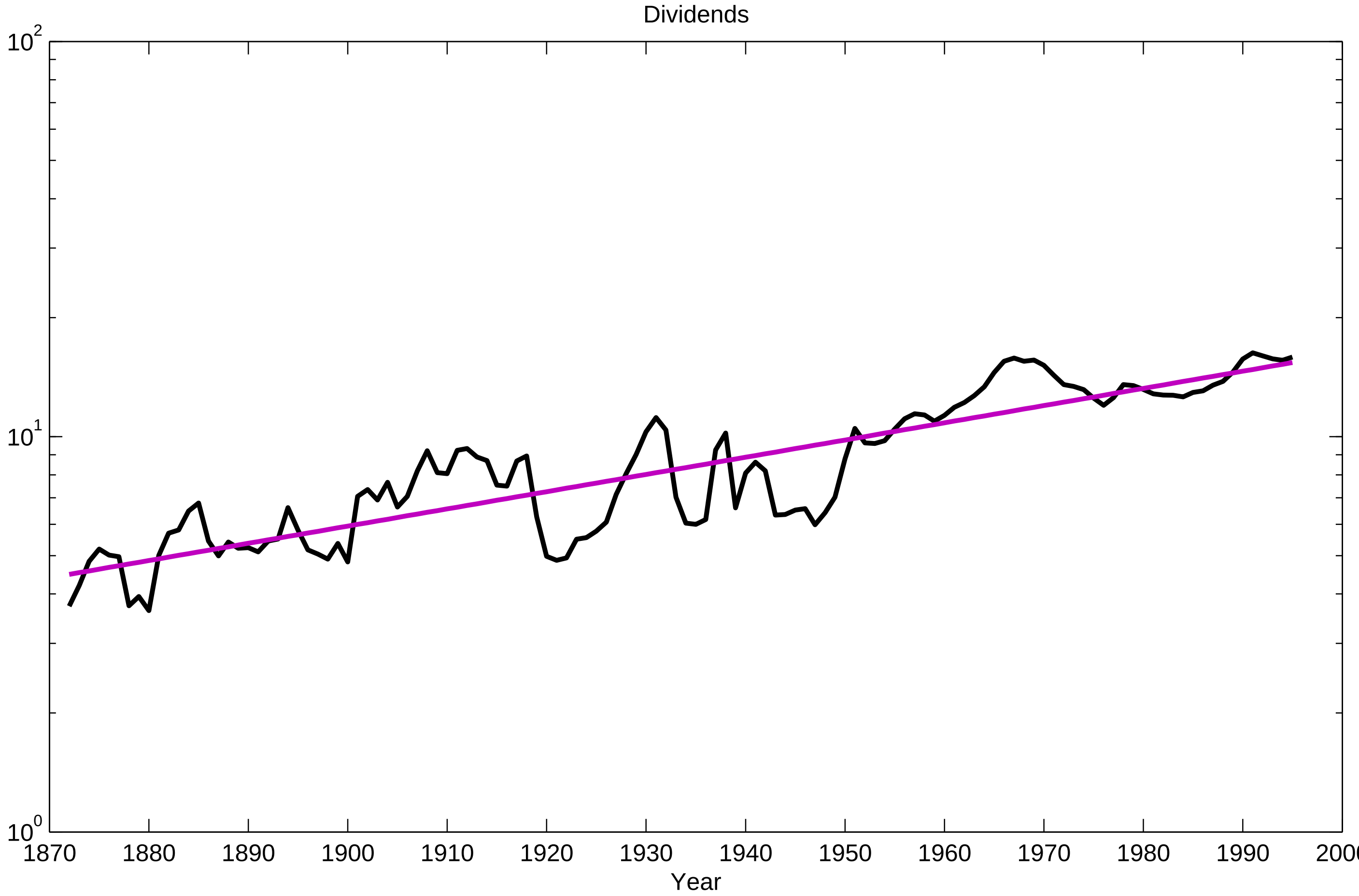

Figure 2: Dividends

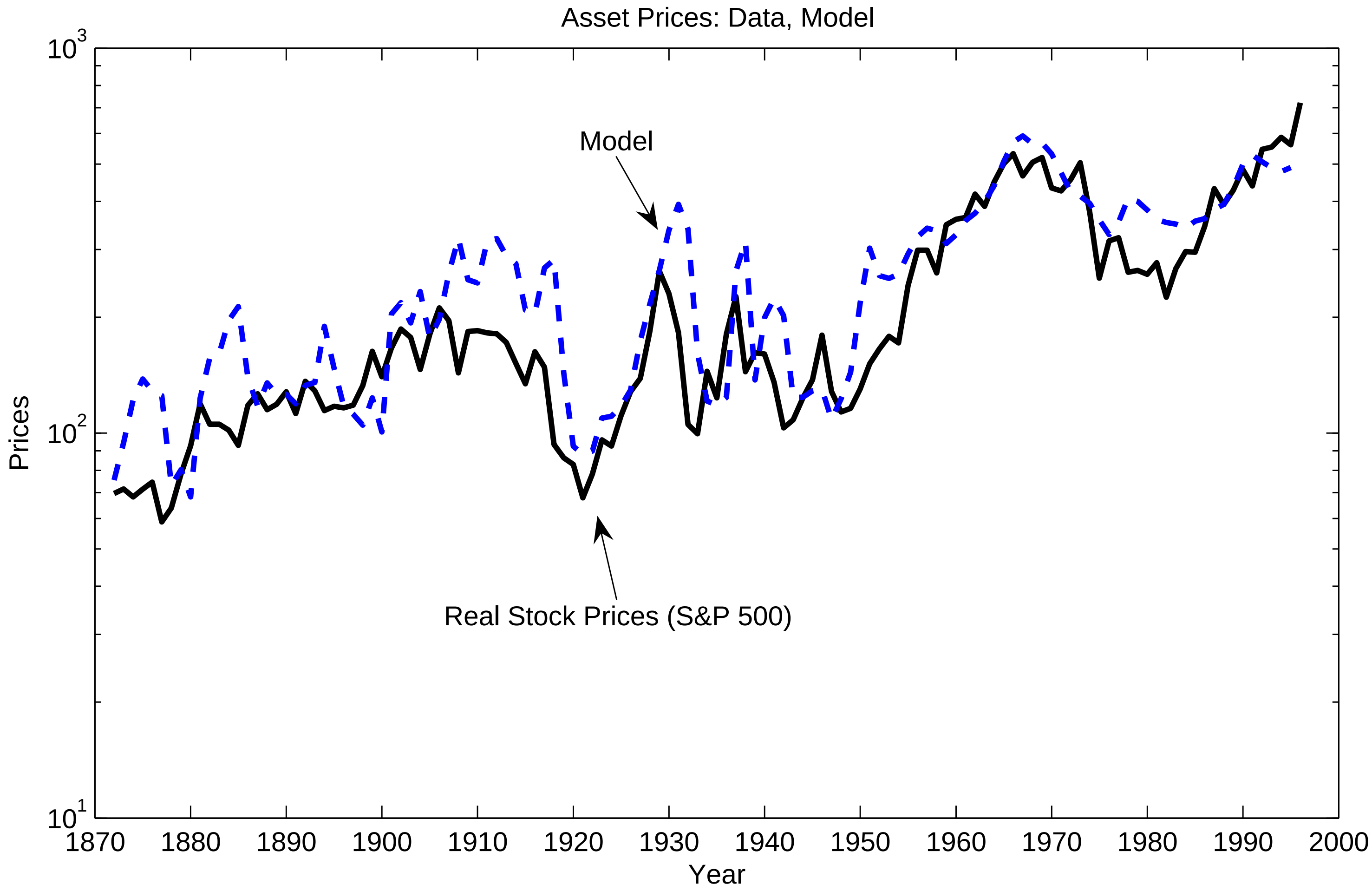

Figure 3: Data and Model

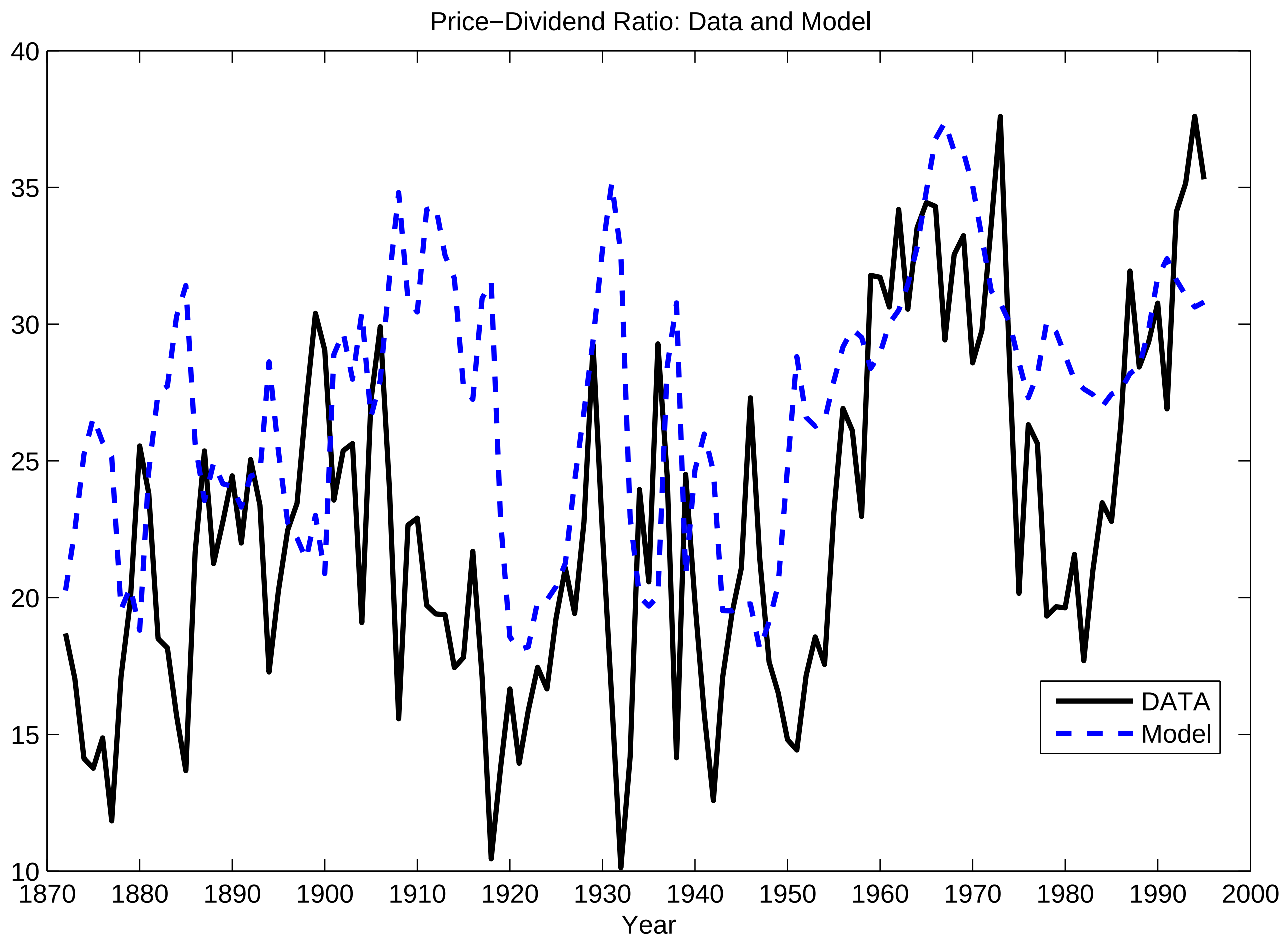

Figure 4: Price-Dividend Ratio - Data and Model

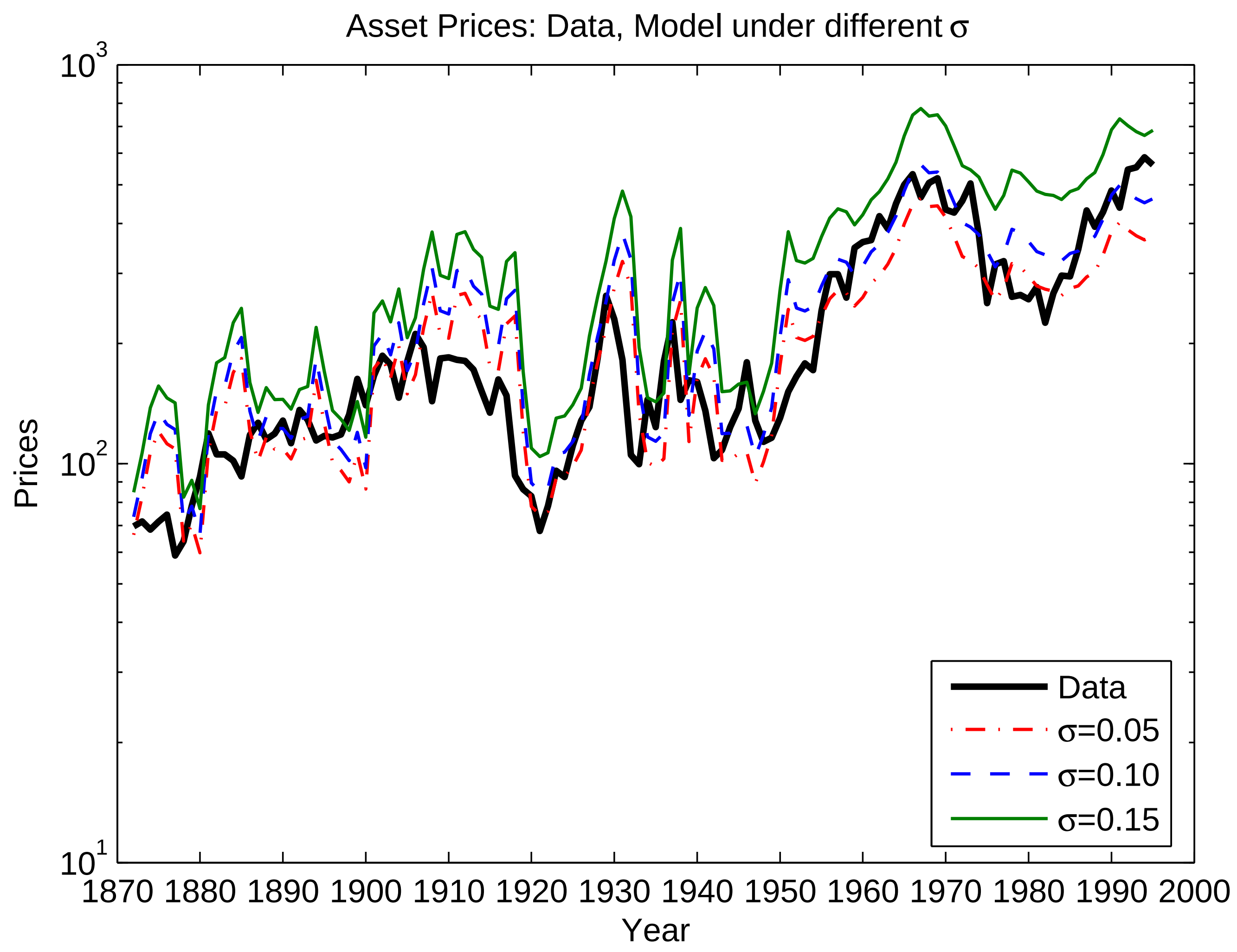

Figure 5: Model under different σ

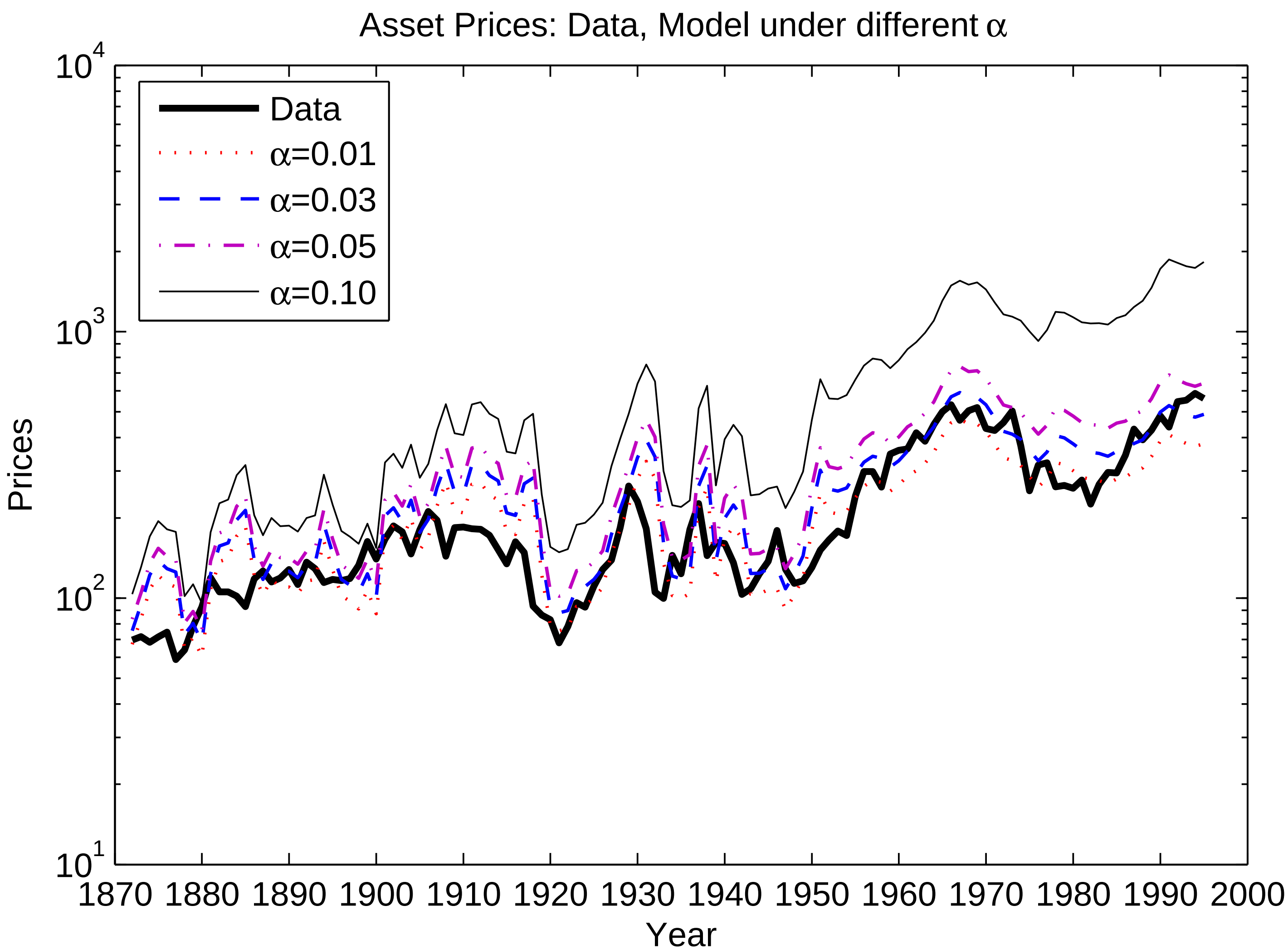

Figure 6: Model under different α

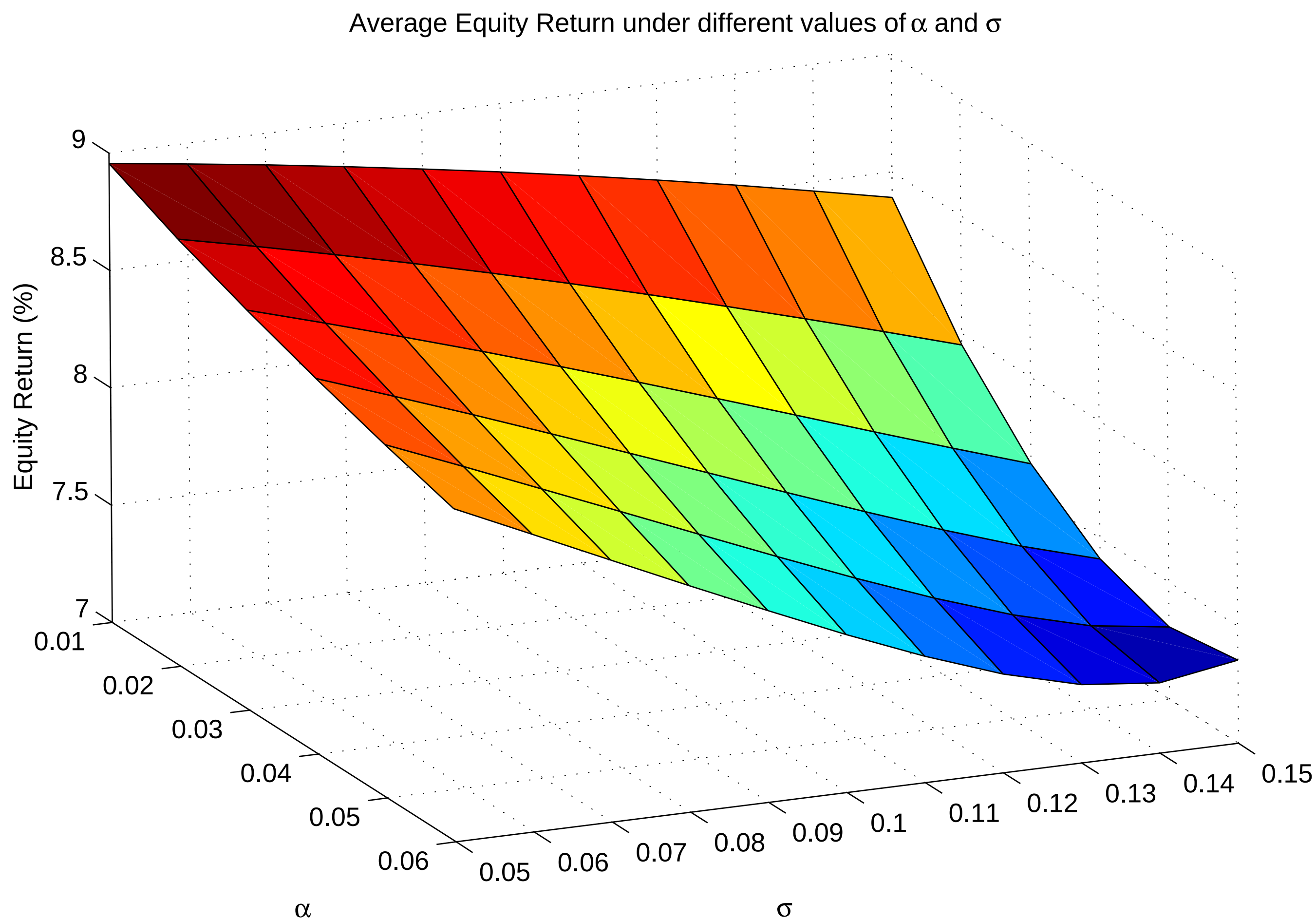

Figure 7: Average Equity Return under different values of α and σ

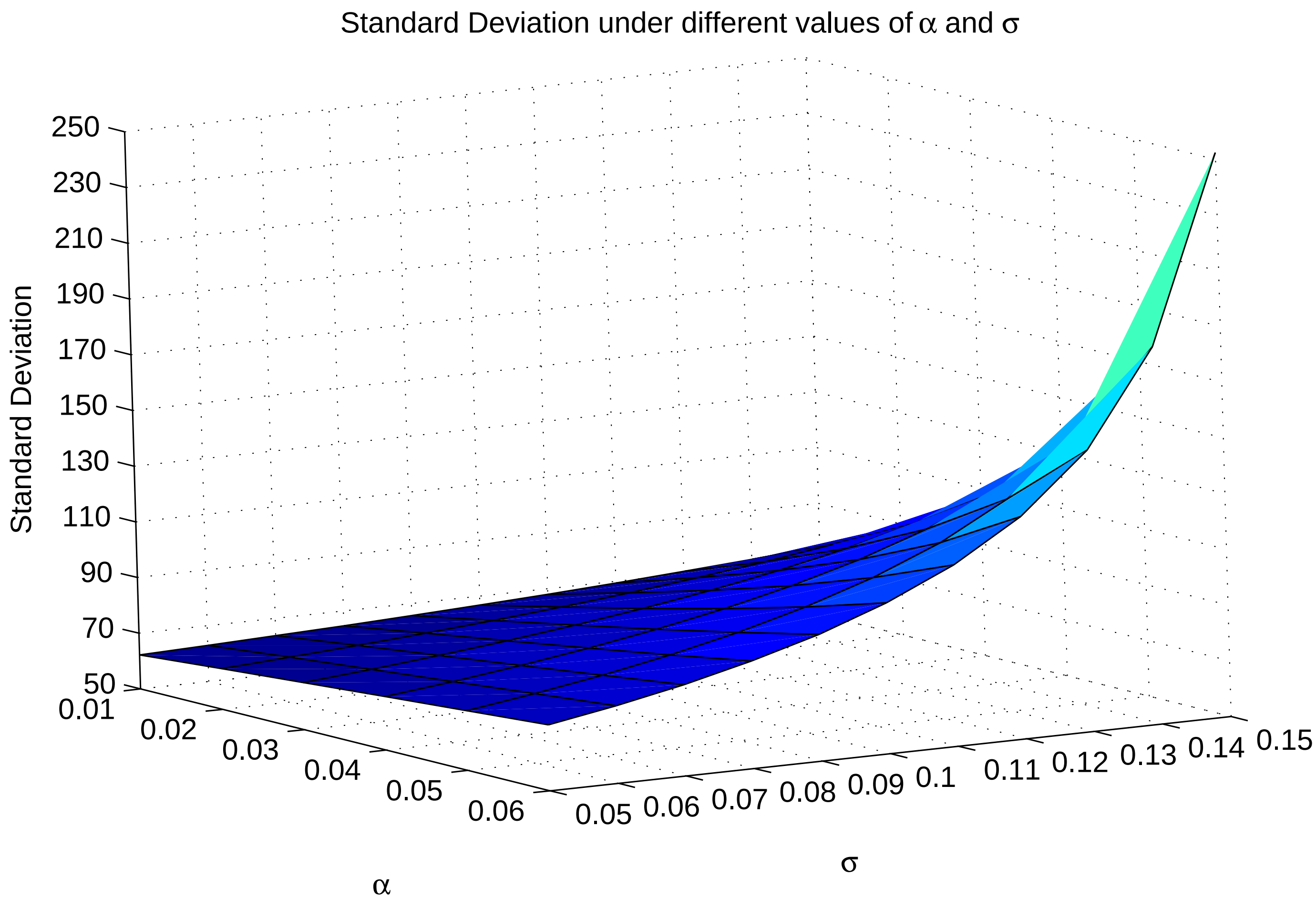

Figure 8: Standard Deviation of Asset Price under different values of α and σ

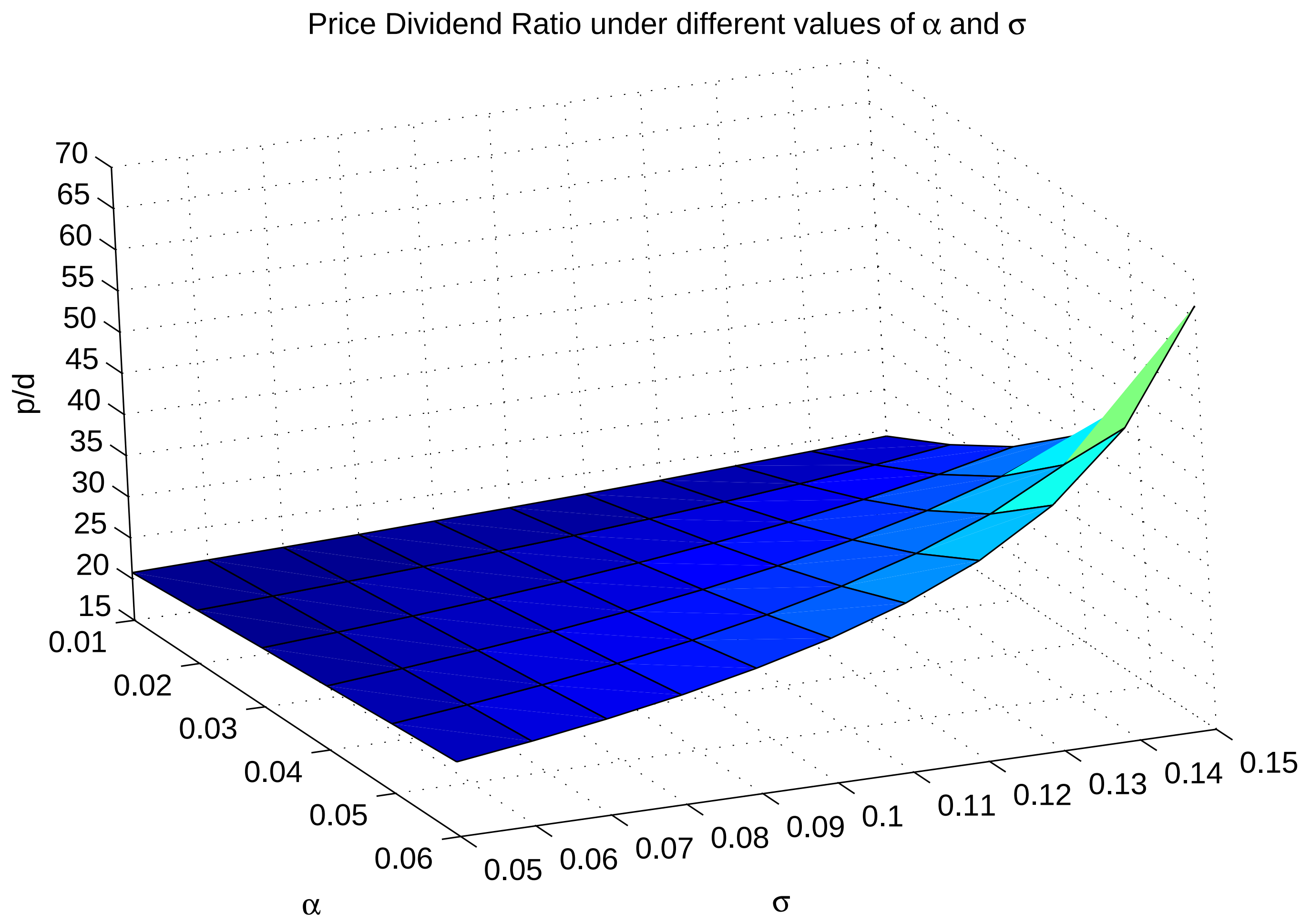

Figure 9: Mean Price-Dividend Ratio under different values of α and σ

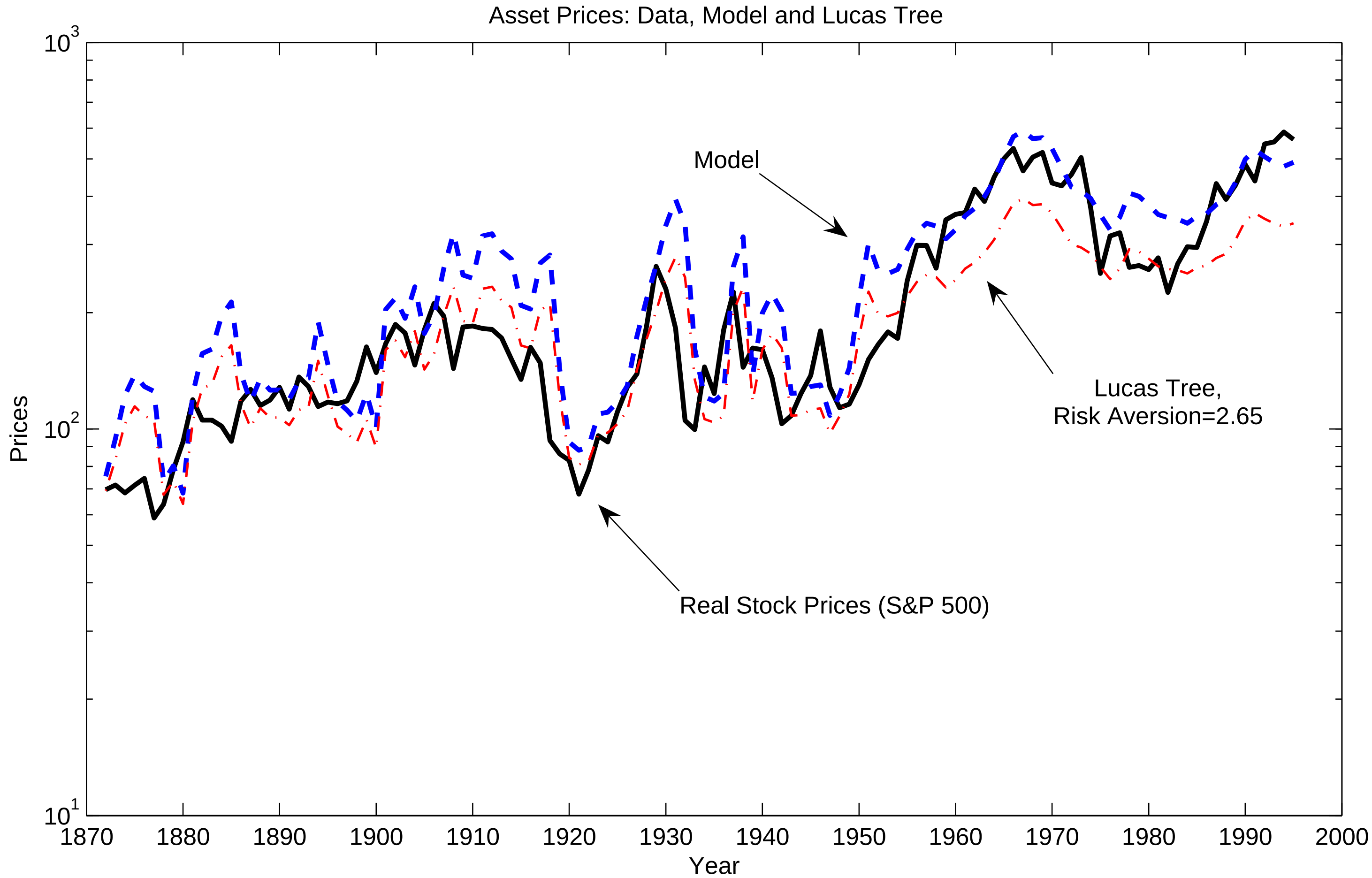

Figure 10: Asset Prices in the Lucas Tree Model that matches the Average Equity Return

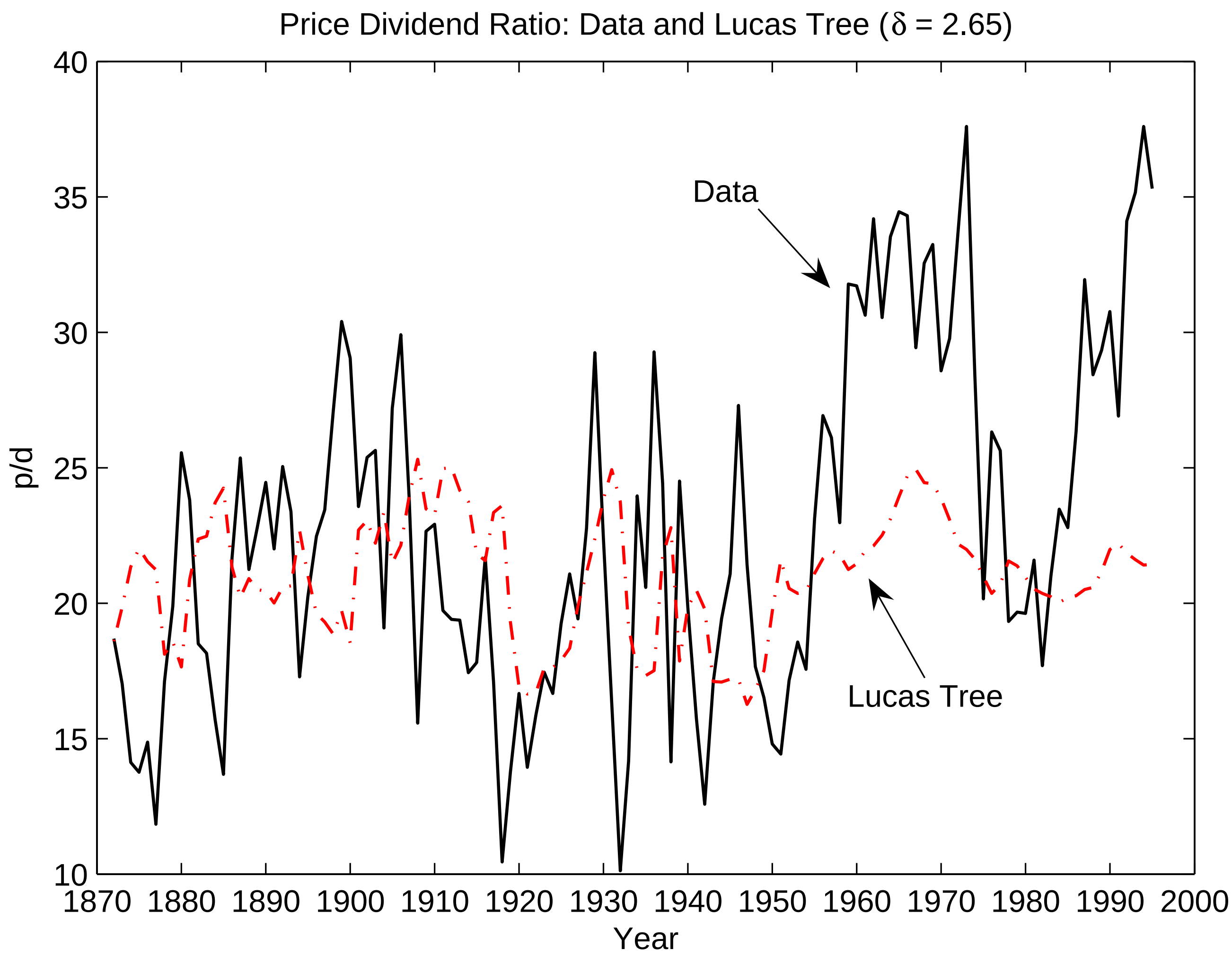

Figure 11: P/D Ratio in the Lucas Tree Model that matches the Average Equity Return

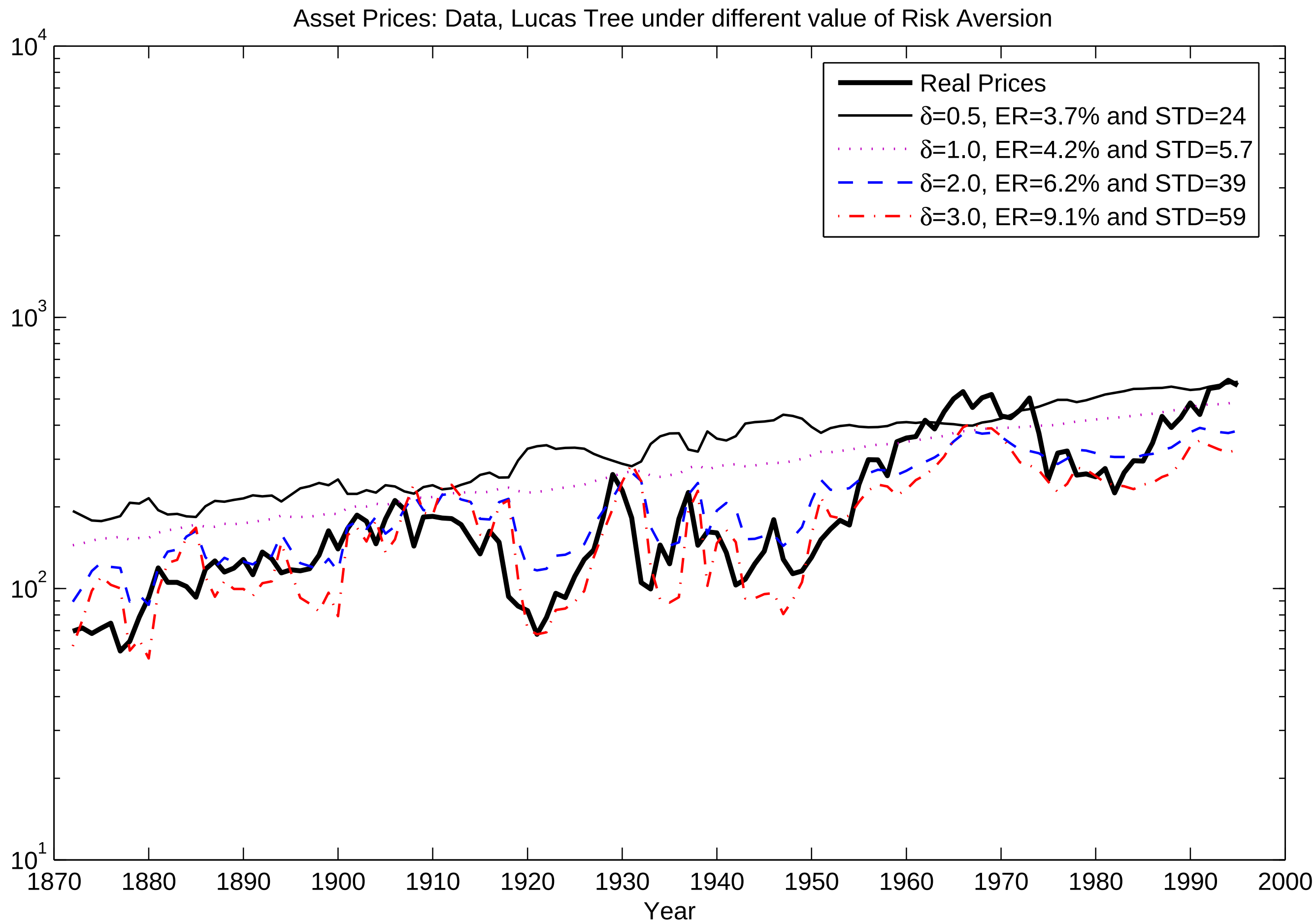

Figure 12: Asset Prices in the Lucas Tree Model under different values of risk aversion

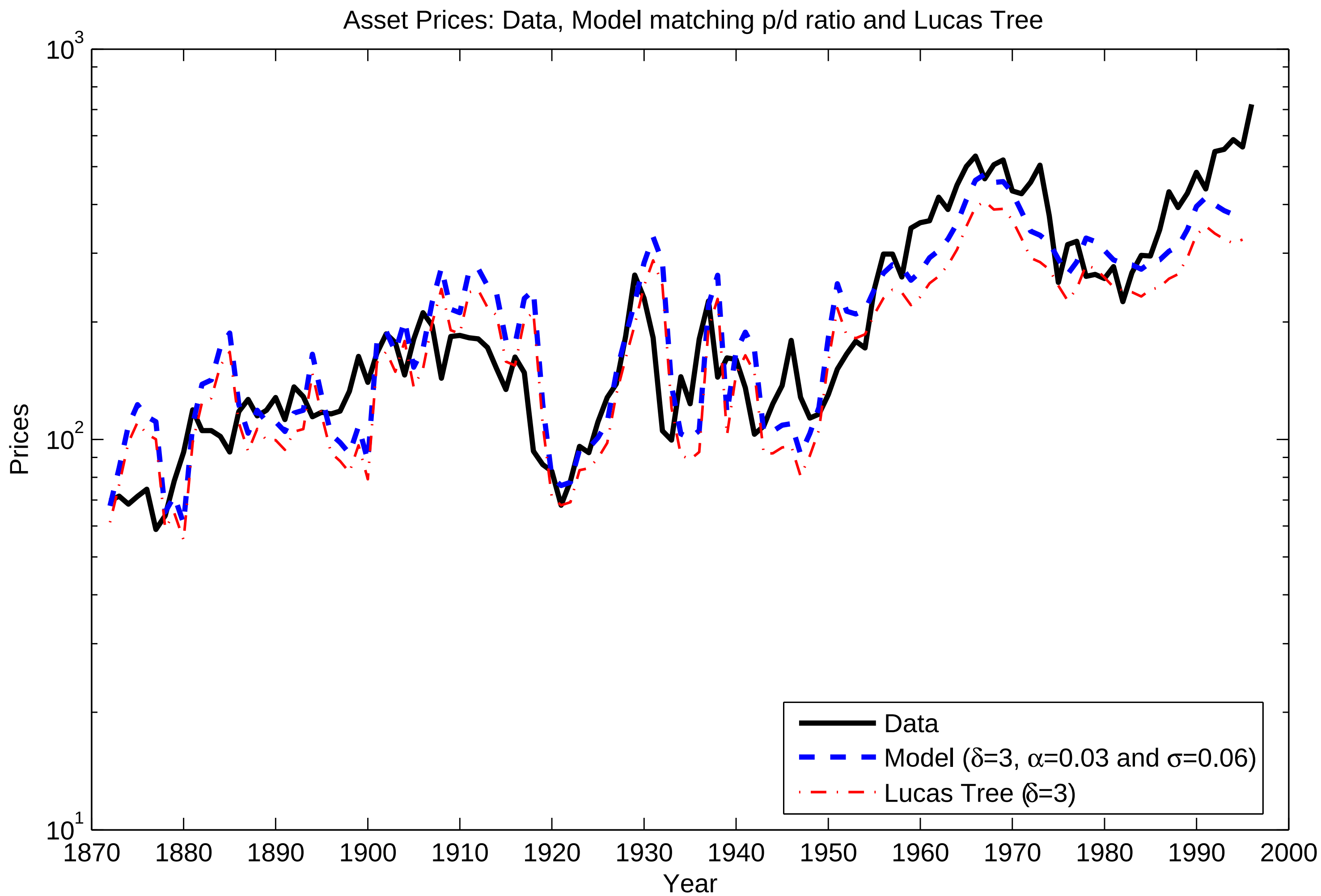

Figure 13: Model matching mean p/d ratio and Lucas Tree Model

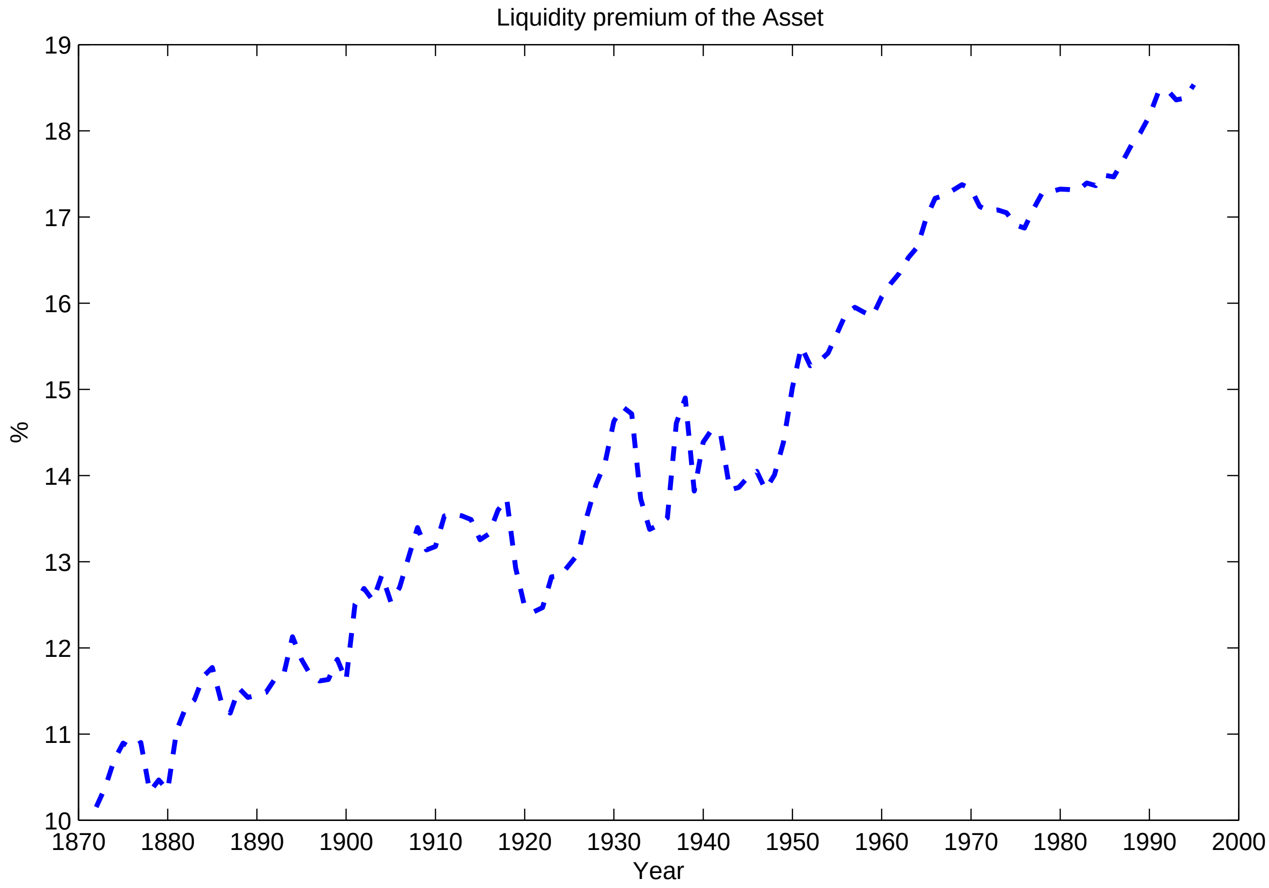

Figure 14: Liquidity Premium of the Asset