

The Value of Information in Credit Markets

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Abstract

We study firm and industry dynamics in economies where there is asymmetric information about the firms' survival probability. Young firms grow as they acquire financial reputation in credit markets through two channels: selection and screening. Our economy exhibits several features of firm and industry dynamics documented for the U.S. Importantly, while information problems undermine production of young firms, general equilibrium forces induce old firms to produce more, thereby largely offsetting the adverse effect of asymmetric information on aggregate output.

1 *Introduction*

We tend to believe that information problems in credit markets could create sizable distortions in production. Common wisdom argues that imaginative entrepreneurs come across productive ventures but, typically, have no means to fund them. When they step into credit markets, lenders simply refuse to take their word for the goodness of a project foreseen to have fuzzy cash flows. In response, small and usually expensive loans are granted, at least until cash flows prove lenders wrong. With financial reputation, funding becomes available and firms expand. But it takes time to put credit problems behind. In the meantime,

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struggling firms start small, confirming the wisdom that the lack of information undermines production. In this paper we study an environment that justifies the existence of this wisdom: Because of information problems in credit markets, younger firms produce much less than older firms. But we also show that the wisdom is wrong: General equilibrium forces considerably weaken the role of information problems in determining aggregate production. In other words, information problems are relevant for firm and industry dynamics, but seem unimportant for determining how much is produced in aggregate.

The role of information is studied in a stationary environment. Every period entrepreneurs start up firms. They choose the scale of production, and operate a decreasing returns to scale technology using labor and capital. The probability of survival, which is the type or quality of the project, remains unchanged along the firms' lifetime. There is a continuum types, which are private information. This generates a repeated adverse selection problem in credit markets because lenders do not observe the firms' survival (and default) probabilities. Credit markets observe the age and the wealth invested by entrepreneurs, which help markets learn about the entrepreneurs' types. The firms' age is important for credit markets to account for the "selection" that takes place in the economy, as lower types die with higher probability (in the spirit of Jovanovic (1982)). The net worth invested in the firm matters because it helps "screen" types, as only those with high survival probability would be willing to risk a large amount of wealth.

We set up a mechanism design problem to solve for the optimal allocations of economies with asymmetric information and competitive lending. We find that when entrepreneurs have all the wealth they need, lenders can screen firms right from the beginning. In this case, firms adopt the efficient scale from startup, and the asymmetry of information creates no distortions in production (as with full information). But when entrepreneurs do not own enough wealth to invest in their firms, distortions arise in production because the highest types get pooled. Indeed, the equilibrium is such that there is a threshold for every cohort of firms, below which types take separating contracts, and above which all types take the same pooling contract. Pooling is inefficient because projects with different productivities (survival probabilities) undertake the same scale of production.

The amount of wealth of the best entrepreneurs determines the position of the threshold in each cohort. With an initial low level of wealth, many types get pooled in one financial contract, as high types invest all they have and lower types mimic them. Imitating a high type has benefits because they borrow more, and at lower rates. But it is not free of cost, since a larger amount of wealth must be gambled (and lower types know they are more likely to fail). As surviving entrepreneurs build net worth and re-invest, lower types gradually drop out of the pool. Both because of natural selection and screening, the average type in the pool improves. The best types acquire financial reputation, pay lower interest rates, borrow more, and expand the scale of production. Eventually all the asymmetry of information among members of the cohort is solved, but it takes time. In the meantime, new firms arise every period, keeping the information problem alive.

The economy exhibits firm and industry dynamics that resemble several of the features documented for the US. On the financial side, smaller and younger firms pay fewer dividends, face higher interest rates and borrow less, while growing and younger firms are more leveraged, and exhibit a greater sensitivity of investment to cash flows (Fazzari, Hubbard and Petersen (1988), Petersen and Rajan (1994), and Gilchrist and Himmelberg (1995)).¹ Moreover, it rationalizes the role of financial reputation for firm and industry dynamics, an issue absent in the literature. On the real side, the model exhibits decreasing mean and volatility of growth rates, job creation, job destruction, net job creation and exit rates, both by age and by size (Hall (1987), Evans (1987), Dunne, Roberts and Samuelson (1989a, 1989b), and Davis, Haltiwanger and Schuh (1996)). Furthermore, the model also generates these firm dynamics by age when we condition on firm size, as reported by Evans (1987).

Qualitatively, more information is shown to be better for production. Under asymmetric information, the entrepreneurs' wealth affects production because it is used as a screening device in credit markets. We analytically show that total production, employment, capital, wages and measured total factor productivity increase when more types are revealed. When wealth is large, all types get screened and total production is at the maximum (full information). We also show that under asymmetric information, production is always greater than

¹Also see Gomez (2001) for a critical view of this evidence.

under uncertain information, where neither entrepreneurs nor lenders know the type of the project. In this case learning only happens through selection, as in Jovanovic (1982).

Quantitatively though, the information problem in credit markets is shown to have a minor effect on aggregate production. We implement a numerical exercise by parameterizing our model to match relevant features of the US economy. We analyze the aggregate output performance under full, asymmetric, and uncertain information in partial and general equilibrium. At the same wages, production under asymmetric information is shown to be about 14.1 percent lower than under full information. This is because younger-constrained firms produce much more under full information. Consequently, wages must rise. But then, the higher cost of labor forces older cohorts, which had overcome the asymmetry of information, to cut back employment. Surprisingly, this general equilibrium effect almost offsets the gains from information. This is so because production at the firm level is more sensitive to the labor cost when firms are larger, as it is more often the case with older firms. Moreover, our sensitivity analysis shows that our results are robust to alternative parameterizations of the returns to scale and the labor supply elasticities.

In summary, this paper argues that the fact that younger firms struggle for credit because of information problems does not imply that these problems are important in the aggregate. Future research should investigate whether the interaction of information problems with other distortions have a more significant impact on production.

Our work relates to a growing literature to which we will not do justice. Part of the firm dynamics is due to selection as in Jovanovic (1982), while the rest is due to screening. The feature of screening makes this paper the first one to analyze information problems in credit markets *and* firm dynamics. Reputation in credit markets have been studied before by Diamond (1989), although we depart in several important aspects. First, equity can be accumulated, making the reputation effects (imitation) only a temporary feature of the cohort's life (as eventually all types are screened). Second, the scale of production is a choice variable, which allows us to study the interactions between reputation and firm dynamics. Third, we study all these issues under general equilibrium, which turns out to be important. In this sense, this paper also departs from Jovanovic (1982), as well as other contributions on

financial imperfections and firm dynamics such as Clementi and Hopenhayn (2002), Quadrini (2004) and Albuquerque and Hopenhayn (2004). Cooley and Quadrini (2001) study firm dynamics –in a model *a la* Hopenhayn (1992)– with costly monitoring (instead of information), but do not focus on aggregate performance. Cooley, Marimon and Quadrini (2004) study the aggregate cyclical consequences of limited contract enforceability, but they do not characterize the allocation efficiency as we do in this paper. In a more stylized model, Carranza, Fernandez-Villaverde and Galdon Sanchez (2003) report output losses of up to 30 percent. The list continues but we stop here. Next, we present the environment before we lay down a roadmap for this paper.

In the next section we present the environment. Next, we solve the allocation problem and analytically characterize the main properties of the model under asymmetric information. Finally, we numerically study our economy under different informational environments, to quantify the implications of information for aggregate production.

2 Environment

The economy is populated with two classes of agents: workers and entrepreneurs. There is a mass μ of infinitely lived workers. They maximize their welfare by choosing the level of consumption, savings, leisure and the time spent at work at every point in time over the following utility function:

$$U_t^W = \sum_{j=t}^{\infty} \left(\frac{1}{1+r} \right)^{j-t} u \left(c_j^W - \varphi(l_j^s) \right) \quad (1)$$

where c_t^W and l_t^s represent consumption and labor supplied respectively at time t and $u(\cdot)$ and $\varphi(\cdot)$ have the usual properties. Superscript W stands for worker. Workers satisfy the budget constraint at every date, given by

$$c_t^W + a_{t+1} \leq w l_t^s + (1+r) a_t \quad (2)$$

where a_{t+1} are the units of assets held by consumers between period t and $t+1$. These assets promise a predetermined return r as usual (equal to the subjective discount rate). Notice that

wages and interest rates are not indexed by time since there is no macroeconomic uncertainty and we will only study the properties of this economy in a stationary environment.

Entrepreneurs are infinitely lived, risk neutral agents, with preferences given by

$$U_t^E = E_t \sum_{j \geq t} \gamma^{j-t} c_j \quad (3)$$

where superscript E stands for entrepreneur and γ is the discount factor. Furthermore, we assume that entrepreneurs are impatient relative to workers, or $\gamma(1+r) < 1$. Nonetheless, they will save in equilibrium because they have access to very profitable investment opportunities for which it will be expensive to borrow.

Entrepreneurs are endowed with one unit of labor at every period, which they can transform into ω units of the final good. Not all entrepreneurs get the same labor income. There is a distribution function $g(\omega)$ of endowments, with cumulative distribution $G(\omega)$. This heterogeneity helps generate features of industry dynamics similar to those of the US.

A fraction of the entrepreneurs produces while the rest look for ideas to start a firm. There is a unit mass of projects that entrepreneurs can start per unit of time and projects are randomly allocated among those entrepreneurs looking for ideas. Furthermore, the probability of getting a project is small so that, in equilibrium, entrepreneurs always prefer to consume rather than to save their income while searching for ideas.² In this way, the distribution of wealth for those entrepreneurs starting projects is also given by $g(\omega)$, since no accumulation takes place during the searching period. This assumption allows us to feed the model with different wealth distributions of newborn entrepreneurs to study the role of information in credit markets. It is also assumed that entrepreneurs cannot borrow against future labor endowment.

When entrepreneurs find a project or idea, they make consumption, production and financing decisions. When a project is started, managers hire workers and rent capital at current prices, w and r_k (capital depreciates at a rate δ every period). Entrepreneurs produce using the following technology

²Entrepreneurs would only save if the probability of getting the project is high, since they are relatively impatient.

$$y_{t+1} = \theta_t(k_t)^\alpha(l_t)^\beta, \quad \theta_t = \begin{cases} A & \text{with prob } p \\ 0 & \text{o.w.} \end{cases} \quad (4)$$

where $\alpha, \beta > 0$ and $\alpha + \beta < 1$, implying decreasing returns in production over labor and capital. Furthermore, managerial talent is indivisible and it takes a manager to run a firm.

After inputs are allocated in the firm, θ_t realizes as either $A > 0$ or 0 (see Figure 1). This variable reveals to the entrepreneur whether he will remain in business or exit. With probability p entrepreneurs keep their market and produce successfully for the period. With probability $1 - p$, they lose their market, fail to generate revenues in the current period and exit the industry. Entrepreneurs keep the same probability p over the firm lifetime, making this parameter the type of the manager or, equivalently, the quality of the project undertaken.

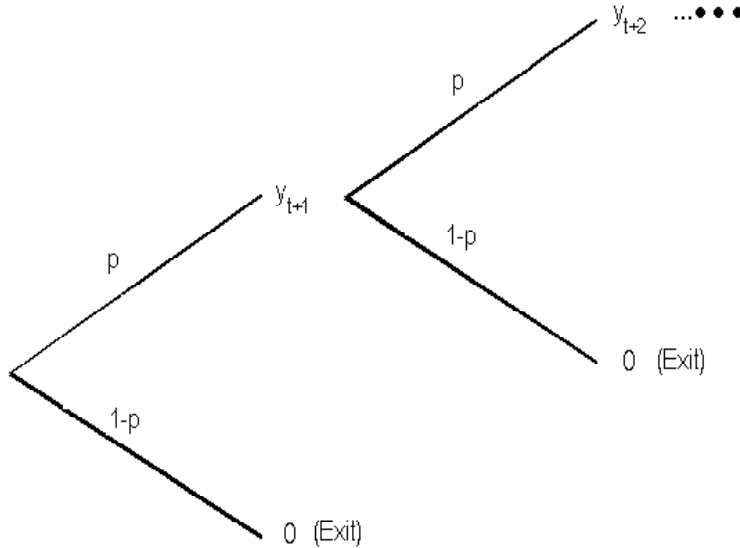


Figure 1: Firm Life Expectancy

Not all the projects are of equal quality in this economy. There is a continuum of types drawn from a density function $f(p)$ defined in the support $[0, \bar{p}]$ with $\bar{p} \leq 1$, differentiable and such that $f(p) > 0 \forall p$. This distribution of types is assumed to be public information.

In this paper we study the role of information in credit markets. We proceed by focusing on three alternative environments: full, asymmetric, and uncertain information. Information is uncertain when types are unknown to both entrepreneurs and lenders, in the spirit of Jo-

vanovic (1982). Information is asymmetric, or private, when entrepreneurs, and nobody else, observes the quality of the project. There is full (or complete) information when all agents observe the types. We make further simplifying assumptions about credit markets, which are only relevant for the case of asymmetric information, to embed all these environments into one model.

We assume that competitive financial intermediaries borrow from households (or depositors) and lend to entrepreneurs (or firms) at no cost. Furthermore, entrepreneurs are unable to commit to future actions to obtain better financial contracts in the present and banks cannot commit to remain in long-term financial relations. This implies that the equilibrium features contracts that break even at every period, keeping the model analytically tractable to a great extent. Furthermore, while age and the entrepreneur's net worth are publicly observable, details such as the past lending rates are not.³ For this reason, financial contracts cannot depend on those details.

Alternative assumptions about the contract environment do not affect the main message. We show that the production level of an economy under asymmetric information is in between those under uncertain and full information. Furthermore, we find that the level of production under private information is remarkably close to that under complete information. This implies that, while environments that allow for richer contracts can better overcome informational problems in credit markets, and hence affect the firm dynamics, the main result of this paper will remain. That is, the value of information in credit markets is much less than we think!

2.1 *Allocations in credit markets*

The value function of entrepreneurs searching for projects is given by

$$V^S(\omega) = \omega + \gamma q E_p [V(p, 1, \omega)] + \gamma(1 - q)V^S(\omega) \quad (5)$$

³This would be the case, for example, if banks anticipate that managers and loan officers can forge the firms' credit history.

where $V(p, 1, \omega)$ is the entrepreneurs' utility function with endowment ω , when they find a new project of quality p . Notice that they produce and consume ω . This occurs because, on one hand, they prefer not to save since $(1+r)\gamma < 1$ and the probability of finding a project q is (assumed to be) small. On the other, entrepreneurs cannot collateralize borrowing with future values of ω and hence consumption equals current income during the searching periods.

When a project is found, competitive lenders offer contracts to entrepreneurs based on all observable information. Then entrepreneurs make consumption, production and financing decisions. They do not hold deposits at the interest rate r in equilibrium since $\gamma(1+r) < 1$ and they are risk neutral. For this reason, and without loss of generality, we simplify the allocation problem by eliminating the possibility of holding deposits altogether.

Our problem is to find a competitive allocation for this economy. Our strategy is to solve the constrained efficient allocation of the planner's problem, who maximizes the total welfare of all entrepreneurs that belong to a cohort of age $n \in N$, with wealth $W \in \mathbb{R}_+$.

We set up the allocation problem as a mechanism design problem. The planner asks members of each cohort of age n and wealth W (observable) to report their types (unobservable) given that the planner commits to assign the allocations according to the announcements (\hat{p}) .⁴ Broadly speaking, the problem is to find the allocations of this game that maximize the total welfare of the cohort, subject to various constraints.

To that end, we apply the Revelation Principle to our problem. This allow us to look for the optimal allocations in a smaller, truth telling, set of allocations without loss of generality. We define an allocation as follows

Definition 1 *An allocation $\tau : [0, \bar{p}] \times N \times \mathbb{R}_+ \rightarrow \mathbb{R}_+^7$ is a collection of mappings from announcements, ages and wealth to consumption $c(\hat{p}, n, W)$, production $\{k(\hat{p}, n, W), l(\hat{p}, n, W), y(\hat{p}, n, W)\}$, and financial contracts $\{i(\hat{p}, n, W), L(\hat{p}, n, W), e(\hat{p}, n, W)\}$, including a lending rate, a loan size and a level of entrepreneur's investment.*

⁴See that we do not mix cohorts of different ages and wealth because both variables are observables and hence competitive markets would also differentiate them in this way.

Again, notice that the allocation functions depend on age and wealth. No other relevant information such as the true type or past announcements are available to lenders and, for this reason, to the planner. The allocation problem for each cohort solves

$$\max_{\tau} \int V(p, n, W) dH(p) \quad (6)$$

where

$$V(p, n, W) = \max_{\hat{p}} \left[c(\hat{p}, n, W) + \gamma p V(p, n', W'(\hat{p}, n, W)) + \gamma (1 - p) V^S(\omega) \right]$$

subject to

$$c(\hat{p}, n, W) + e(\hat{p}, n, W) \leq W \quad \forall \hat{p} \quad (7)$$

$$W'(\hat{p}, n, W) = y(\hat{p}, n, W) - [1 + i(\hat{p}, n, W)] L(\hat{p}, n, W) + \omega \quad \forall \hat{p} \quad (8)$$

$$y(\hat{p}, n, W) = Ak(\hat{p}, n, W)^\alpha l(\hat{p}, n, W)^\beta \quad \forall \hat{p} \quad (9)$$

$$r_k k(\hat{p}, n, W) + w l(\hat{p}, n, W) \leq e(\hat{p}, n, W) + L(\hat{p}, n, W) \quad \forall \hat{p} \quad (10)$$

$$(1 + r)L(\hat{p}, n, W) = \hat{p}[1 + i(\hat{p}, n, W)] L(\hat{p}, n, W) \quad \forall \hat{p} \quad (11)$$

$$c(p, n, W) + \gamma p V(p, n', W'(p, n, W)) \geq c(\hat{p}, n, W) + \gamma p V(p, n', W'(\hat{p}, n, W)) \quad \forall p, \hat{p} \quad (12)$$

$$V(p, n, W) \geq V^S(\omega) \quad (13)$$

$$n' = n + 1 \quad (14)$$

$$c(\hat{p}, n, W) \geq 0 \quad \forall \hat{p} \quad (15)$$

$$\{r_k, w, r\} \quad \text{given}$$

First, notice that the planner weights the entrepreneurs' utility functions according to some arbitrary function $H(p)$. This weighting function can be the same or different than the actual distribution of types. Think of this problem as one where the planner offers allocations $\tau(\hat{p}, n, W)$ to entrepreneurs in a cohort of age n and wealth W , and they sequentially choose an announcement \hat{p} . The planner's problem is to find the allocation τ that maximizes the cohort's welfare, subject to various constraints that we now explain.

Equation (7) states that consumption plus the entrepreneur's investment in the firm (e) cannot exceed the entrepreneur's wealth, a state variable in our problem. Equation (8) defines the law of motion for wealth contingent on continuation for each type. If the manager succeeds to produce and remains in business, the next period wealth is determined by the firm's cash flow, defined as the total output minus principal plus interests (i) on the loan (L), and next period's endowment ω . Expression (9) is the output produced contingent on success. Equation (10) is the firm's resource constraint, stating that the total cost of inputs must be financed with equity and/or debt (loans). Again note that all prices are constant because there is no macroeconomic uncertainty and we assume stationarity.

Equation (11) is the banks' participation constraint. The expected revenues for intermediaries (in the RHS), should equal the expected costs (in the LHS) at every period and for *every* contract. This follows from our assumptions that the credit markets are competitive, and given that lenders are unable to commit to stay in a long-term financial relationship.

Equation (12) is the incentive compatibility constraint for entrepreneurs to report their true type. Both the LHS and the RHS of this expression are the part of the entrepreneurs' value function inside the max operator that depends on the announcement or report. By the Revelation Principle we can narrow the choice set of allocations by focusing only on truth telling allocations, without loss of generality. Because this constraint is strictly binding in equilibrium, we know that the solution to the problem with asymmetric information is dominated, from the welfare standpoint, by allocations of the problem with full information. Condition (13) is the participation constraint for all entrepreneurs. Equation (14) is the updating rule for age, another state variable. Finally Equation (15) is the non-negativity constraint for consumption.

The assignment here is to find the optimal allocations τ that solves the planner's problem. We split the analysis into two cases.

2.1.1 When wealth is not a constraint

We first solve the allocations under the assumption that Condition (15) is not binding. This is equivalent to assuming that the wealth level of all entrepreneurs is high enough. The

case where all entrepreneurs have enough wealth is a benchmark that helps understand the allocation for the general case.

For any given arbitrary level of investment in the firm (e), optimality requires that inputs are chosen to maximize the firm's profits subject to the lenders' participation constraint and the resource constraint. From Condition (11) we obtain that in a separating equilibrium, the lending rate is simply

$$i(\hat{p}, n, W) = \frac{1+r}{\hat{p}} \quad (16)$$

Note that the lending rate is determined by the lenders' perception about the entrepreneurs type (or announcement). The higher the belief about the probability of survival, the lower is the lending rate charged to entrepreneurs. Maximizing the firm's cash flow implies that the optimal input choices are

$$k(\hat{p}, n, W) = \left[\frac{\hat{p}A\alpha^{1-\beta}\beta^\beta}{w^\beta (1+r) r_k^{1-\beta}} \right]^{\frac{1}{1-\alpha-\beta}} \quad (17)$$

$$l(\hat{p}, n, W) = \left[\frac{\hat{p}A\alpha^\alpha\beta^{1-\alpha}}{w^{1-\alpha} (1+r) r_k^\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \quad (18)$$

Inputs depend negatively on their prices and positively on the productivity parameter A . More meaningfully, both inputs depend positively on the banks' perception about their project quality. The higher the market's belief about the firm probability of survival, the lower the interest rate on loans and the greater the demand for both inputs. This feature follows from our assumption on decreasing returns in capital and labor. It is interesting to notice that inputs are not determined by the entrepreneur's true type p , but only by the perception about the firm's quality \hat{p} . This occurs because in equilibrium banks are indeed the marginal suppliers of funds in the economy. From this result we obtain our first proposition

Proposition 2 (*Modigliani and Miller's Neutrality Theorem*). *Under complete information, the scale of production is independent of the entrepreneurs wealth.*

Proof. See that under perfect information $\hat{p} = p$. ■

When entrepreneurs and credit markets share the same information regarding the success probability of the firm, agency problems in financial contracts disappear. Then, from Modigliani and Miller (1958) we know that there is a financial contract that can implement the efficient scale of production at every period. In an environment with full information, the economy is unable to generate firm dynamics as the type of the project (and hence the productivity of the firm) remains constant over the life cycle of the project, and firms start right away from the efficient scale of production.

Plugging Expressions (17) and (18) into (10) and (9), and them together with (16) into (8), allows us to simplify the law of motion for wealth to

$$W' = (1 - \alpha - \beta) \left(\frac{\hat{p}^{\alpha+\beta} A \alpha^\alpha \beta^\beta}{w^\beta (1+r)^{\alpha+\beta} r_k^\alpha} \right)^{\frac{1}{1-\alpha-\beta}} + \frac{1+r}{\hat{p}} e + \omega \quad (19)$$

where the first term represent the profits of the firm while the second, the opportunity cost of equity adjusted for risk, $(1+r)/\hat{p}$.

From the mechanism design standpoint we can reformulate our problem as one solving for an announcement of the entrepreneur's type \hat{p} and the financial contract $i(\hat{p}, n, W)$, $L(\hat{p}, n, W)$, and $e(\hat{p}, n, W)$, thus reducing the dimensionality of the problem. For convenience, we re-name $W'(\hat{p}, n, W)$ as $W^+(\hat{p}, e(\hat{p}, n, W))$ (see Expression (19)). Now our allocation problem is

$$\max_{\tau} \int V(p, n, W) dH(p) \quad (20)$$

$$V(p, n, W) = \max_{\hat{p}} \left[c(\hat{p}, n, W) + \gamma p V(p, n', W^+(\hat{p}, e(\hat{p}, n, W))) + \gamma (1-p) V^S(\omega) \right]$$

subject to

$$c(\hat{p}, n, W) + e(\hat{p}, n, W) \leq W \quad \forall \hat{p} \quad (21)$$

$$W^+(\hat{p}, e(\hat{p}, n, W)) = (1 - \alpha - \beta) \left(\frac{\hat{p}^{\alpha+\beta} A \alpha^\alpha \beta^\beta}{w^\beta (1+r)^{\alpha+\beta} r_k^\alpha} \right)^{\frac{1}{1-\alpha-\beta}} + \frac{(1+r)}{\hat{p}} e(\hat{p}, n, W) + \omega \quad \forall \hat{p} \quad (22)$$

$$c(p, n, W) - c(\hat{p}, n, W) \geq \gamma p \left\{ V \left[p, n', W^+(\hat{p}, e(\hat{p}, n, W)) \right] - V \left[p, n', W^+(p, e(p, n, W)) \right] \right\} \quad \forall p, \hat{p} \quad (23)$$

$$V(p, n, W) \geq V^S(\omega) \quad (24)$$

$$n' = n + 1 \quad (25)$$

$$\begin{aligned}
c(\hat{p}, n, W) &\geq 0 && \forall \hat{p} \\
\{r_k, w, r\} &&& \text{given}
\end{aligned} \tag{26}$$

Notice that for any allocation proposed by the planner, each entrepreneur will announce a type that maximizes his utility. In other words, the announcement constitutes the entrepreneurs' best response to the mechanism he is facing. In order to find the entrepreneurs' best response, we compute each entrepreneur's first order condition with respect to their announcements \hat{p}

$$V(p, n, W) = \max_{\hat{p}} \left\{ W - e(\hat{p}, n, W) + \gamma p V \left[p, n', W^+(\hat{p}, e(\hat{p}, n, W)) \right] + \gamma (1 - p) V^S(\omega) \right\} \tag{27}$$

Subject to (22), the non-negativity constraint for consumption, and given a mechanism $\{i(\hat{p}, n, W), L(\hat{p}, n, W), e(\hat{p}, n, W)\}$, which is first assumed, and later confirmed, to be differentiable with respect to \hat{p} .

The entrepreneurs' best response for announcements satisfies

$$\frac{\partial e(\hat{p}, n, W)}{\partial \hat{p}} = \gamma p \frac{\partial V(p, n', W^+)}{\partial W^+} \frac{\partial W^+(\hat{p}, e(\hat{p}, n, W))}{\partial \hat{p}} \quad \forall p \tag{28}$$

As long as W is big enough an interior solution exists for the entrepreneurs' problem. The interior solution exists not only at time t but also at every future period, because wealth never decreases along the firm life. This result follows from the fact that $W^+(\hat{p}, e(\hat{p}, n, W)) > e(\hat{p}, n, W)$ as shown by expression (19). When the entrepreneurs' wealth is large enough, the first order condition holds with equality and we can apply the Envelope Theorem to prove that $\frac{\partial V(p, n', W^+)}{\partial W^+} = 1$. We rearrange terms in our first order condition and impose truth telling by letting $\hat{p} = p$ (this guarantees that Condition (23) is satisfied for all types). This expression finally results in the following condition

$$\begin{aligned}
(\alpha + \beta)\gamma \left(\frac{A\alpha^\alpha \beta^\beta}{w^\beta (1+r)^{\alpha+\beta} r_k^\alpha} \right)^{\frac{1}{1-\alpha-\beta}} \hat{p}^{\frac{\alpha+\beta}{1-\alpha-\beta}} &= \frac{(1+r)\gamma}{\hat{p}} e(\hat{p}, n, W) + \\
&[1 - (1+r)\gamma] \frac{\partial e(\hat{p}, n, W)}{\partial \hat{p}} \quad \forall p. \tag{29}
\end{aligned}$$

Expression (29) imposes conditions on $e(\hat{p}, n, W)$ such that our mechanism satisfies the incentive compatibility constraint of the planner's problem, when entrepreneurs have enough wealth. In principle we should use this expression as a constraint in the planner's problem to find the optimal allocation τ . But that is not necessary. Equation (29) implicitly defines the function $e(\hat{p}, n, W)$ up to a constant, and by making the participation constraint of the lowest type bind, we can pin down this part of the allocation without having to go back to the planner's problem. A nice feature of our setup is that our allocation has a closed form solution.

Proposition 3 *When the entrepreneurs' wealth is large enough, the optimal financial allocation is given by*

$$\begin{aligned} e(\hat{p}, n, W) &= \frac{(1 - \alpha - \beta)\gamma(1 + r)(\alpha + \beta)}{[1 - \gamma(1 + r)(\alpha + \beta)]} \left(\frac{A\alpha^\alpha\beta^\beta}{w^\beta(1 + r)r_k^\alpha} \right)^{\frac{1}{1 - \alpha - \beta}} \hat{p}^{\frac{1}{1 - \alpha - \beta}} \quad \forall p \quad (30) \\ i(\hat{p}, n, W) &= \frac{1 + r}{\hat{p}} \quad \forall p \\ L(\hat{p}, n, W) &= r_k k(\hat{p}, n, W) + wl(\hat{p}, n, W) - e(\hat{p}, n, W) \quad \forall p \end{aligned}$$

Where $k(\hat{p}, n, W)$ and $l(\hat{p}, n, W)$ are given by Expressions (17) and (18).

Proof. See Appendix. ■

From Proposition 3 we can solve for the rest of the allocations in this economy (consumption and production). Notice that when entrepreneurs own enough wealth to truthfully reveal their types ($W > e(p, n, W) \forall p$), a separating equilibrium arises. We highlight some features of this allocation.

Corollary 4 *Contingent on wealth being enough, the financial contracts $\{i(\hat{p}, n, W), L(\hat{p}, n, W), e(\hat{p}, n, W)\}$ only depend on the announcement \hat{p} , but not on age or wealth.*

Proposition 3 presents a system of equations that uniquely defines $e(p, n, W)$, $i(p, n, W)$, $k(p, n, W)$, $l(p, n, W)$ and $L(p, n, W)$. But age and wealth are not present in these equations, and hence the solutions cannot depend on these variables. Because the firm's scale of

production only depends on the firms' type and not on age or wealth, there cannot be any firm dynamics in this case.

The optimal contract suggested by Proposition 3 is fairly intuitive. The investment required by entrepreneurs increases with their announcements \hat{p} (see Figure 2). In other words, they are asked to risk more of their own wealth in the investment project in order for credit markets to believe they are actually running a good project.

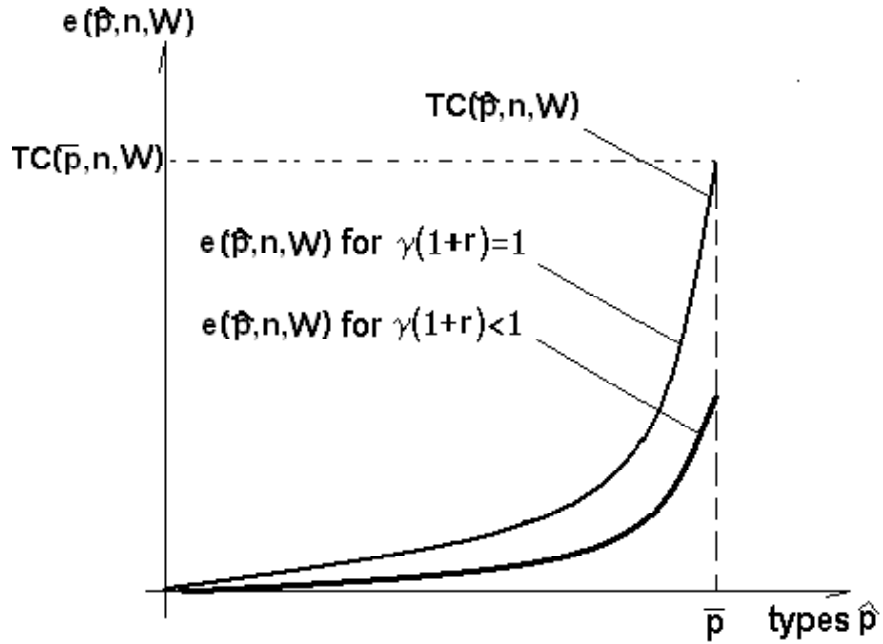


Figure 2: Contract under Full Separation

Notice that the schedule of equity depends on the entrepreneurs' discount factor γ as well. To see how this works, assume for a moment that $\gamma(1+r) = 1$. From Expression (30) we see that the investment asked of entrepreneurs of type p is then

$$e(p, n, W) = (\alpha + \beta) \left(\frac{A\alpha^\alpha \beta^\beta}{w^\beta (1+r) r_k^\alpha} \right)^{\frac{1}{1-\alpha-\beta}} p^{\frac{1}{1-\alpha-\beta}} \quad (31)$$

But a careful inspection at this expression shows that this is the total cost of production for those types, or $TC(p, n, W) = r_k k(p, n, W) + wl(p, n, W) = e(p, n, W)$. A truth telling contract is one where the owner finances the whole cost of investment. In other words,

entrepreneurs find it incentive compatible to report the true type only when there is no borrowing!

Figure 2 also shows how the schedule shifts when entrepreneurs are relatively impatient in this economy. While when $\gamma(1+r) = 1$ there is no borrowing under full revelation, if $\gamma(1+r) < 1$, managers finance a fraction of the costs of production by borrowing from financial intermediaries even under full revelation. This feature allows firms to be leveraged even when they operate at the efficient scale of production. The interpretation is that as managers become more impatient and the cost of postponing consumption increases, the contracts can screen the types by requiring a lower level of investment.

Lastly, from Equations (17) and (18) we observe that under complete revelation, the economy achieves efficiency in production. That is, when wealth is not a constraint, the economy will exhibit no firm dynamics and the asymmetry of information in credit markets will have no consequences for aggregate production.

2.1.2 When wealth is a constraint

We now focus on the case where entrepreneurs are not wealthy enough to truthfully reveal their types (or $W < e(p, n, W)$ for some $p < \bar{p}$). For now, assume that every entrepreneur has the same endowment ω , which implies that they start with the same level of wealth.

With limited wealth, the allocation function $e(\hat{p}, n, W)$ will not coincide with that found before for every type. Because it does coincide for some types, it is convenient to re-name with $e_s(\hat{p}) : [0, \bar{p}] \rightarrow \mathbb{R}_+$ the mapping described by Expression (30). Subscript s stands for separating contract (see definition below). In the same way, let $k_s(\hat{p}) : [0, \bar{p}] \rightarrow \mathbb{R}_+$ and $l_s(\hat{p}) : [0, \bar{p}] \rightarrow \mathbb{R}_+$, and $y_s(\hat{p}) : [0, \bar{p}] \rightarrow \mathbb{R}_+$ be the mappings described by (17), (18) and (9) respectively.

Equilibrium contracts can be of two possible kinds,

Definition 5 *A separating financial contract is a lending rate, a loan size and a required level of the firm's net worth, that is taken by only one type of entrepreneur in equilibrium.*

A pooling financial contract is a lending rate, a loan size and a required level of the firm's net worth that is taken by more than one type of entrepreneur in equilibrium.

Figure 3 shows the net worth schedule required of firms under full screening. When the level of wealth of an entrepreneur of type p is large enough ($W \geq e_s(\bar{p})$), the equilibrium is fully revealing and each type takes a *separating contract*. When the entrepreneur's wealth is such that $W < e_s(\bar{p})$, as in Figure 3, a fraction of the entrepreneurs in the cohort will be unable to afford a separating contract.⁵

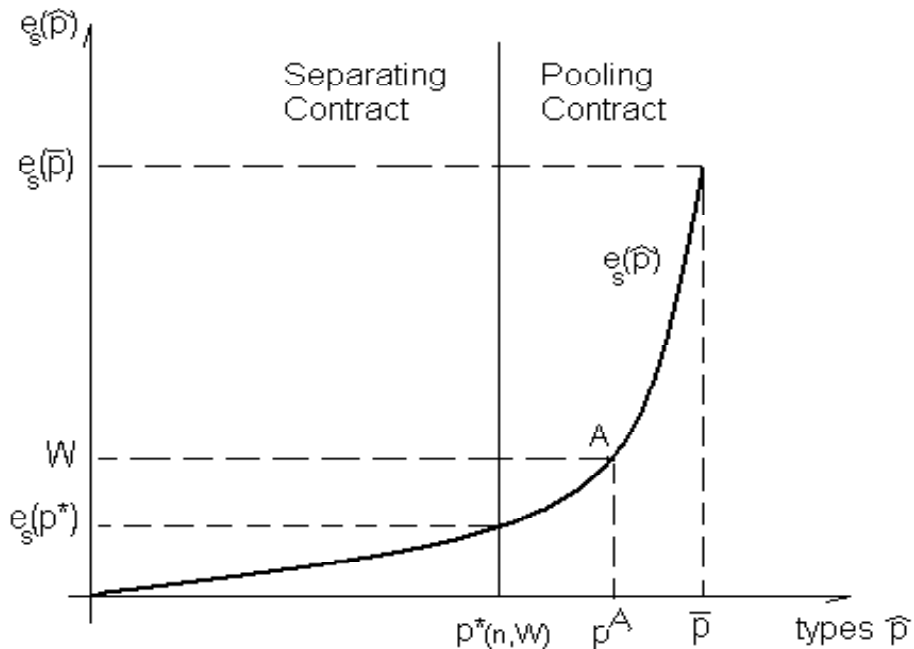


Figure 3: Intuition for the Emergence of Pooling

We now show that pooling arises in equilibrium. Start by conjecturing that, because wealth is limited to W , all types $p > p^A$ will take a pooling contract. The markets could then compute the conditional mean of all types in the pooling contract given that the distribution of types is public information. Let $\tilde{p}(n, p^A) \equiv E(p/p > p^A, n)$ be the conditional average of types. Age is important to compute the conditional average type participating in the pool since the distribution of types for a cohort of age n is given by $f(p) p^{n-1}$. If the average

⁵Remember that the type's participation constraint implies that $e_s(0) \geq 0$.

type taking this contract is $\tilde{p}(n, p^A)$, the interest rate in a competitive market should be $1 + i(n, p^A) = \frac{1+r}{\tilde{p}(n, p^A)}$. Furthermore, if this was an equilibrium outcome, then the net worth level required in the contract would be $e = W$, for those $p > p^A$, and $e_s(\hat{p})$ for the rest.

But this is not an equilibrium outcome. A type p^A is strictly better off by taking the pooling contract than by taking a separating one. Notice that the net worth required by both contracts are the same while the lending rate paid under the pooling contract is significantly lower, as $p^A < \tilde{p}(n, p^A)$. Following this reasoning some types below p^A will find it profitable to free ride the pooling contract.

In equilibrium there should be a threshold type $p^*(n, W)$ for each cohort of age n , with wealth W , such that this type is indifferent between taking the pooling contract or a separating one ($p^* : N \times \mathbb{R}_+ \rightarrow [0, \bar{p}]$). Also notice that $p^*(n, W)$ will be strictly positive as long as entrepreneurs have some wealth, as a type sufficiently close to zero will not risk $W > 0$ knowing that he faces a large probability of failure (we formalize this below). To compute $p^*(n, W)$, we go back to our incentive compatibility constraint as it was written in Expression (12) and equate the utility that a type $p^*(n, W)$ gets under the pooling and separating (truth telling) contracts. This gives

$$W - e_s(p^*) + \gamma p^* V(p^*, n', W^+(p^*, e_s(p^*))) = \gamma p^* V(p^*, n', W^+(\tilde{p}(n, p^*), W)) \quad (32)$$

where

$$\tilde{p}(n, p^*) = E(p/p > p^*, n) = \frac{\int_{p^*}^{\bar{p}} f(p) p^n dp}{\int_{p^*}^{\bar{p}} f(p) p^{n-1} dp} \quad (33)$$

Notice that expressions (32) and (33) implicitly define $p^*(n, W)$. The LHS of (32) is the utility when a type p^* takes a separating contract, while the RHS when he takes a pooling contract. In the latter case, there is no present consumption since all the entrepreneurs' wealth is invested in the firm.

The analysis simplifies when we assume conditions such that if a type p^* is indifferent between separating and pooling at date t , he will strictly prefer to separate at time $t + 1$ and

onwards. In this way, the payoffs for a type p^* under the pooling and the separating strategies only differ in the first two periods since, after that, he would receive the same separating allocations regardless of his decision today. This simplifies the computation of the incentive compatibility constraint.⁶ Under this condition, Expression (32) can be re-written by only taking into account the consumption streams of the first two periods as follows.

$$[W - e_s(p^*)] + \gamma p^* [W^+(p^*, e_s(p^*)) - e_s(p^*)] = \gamma p^* [W^+(\hat{p}(n, p^*), W) - e_s(p^*)] \quad (34)$$

The incentive compatibility constraint illustrates the trade off faced by type p^* . If he joins the pooling contract, he would risk more in the project than if he took a separating contract because $e_s(p^*) < W$, as suggested by Figure 3. If he separates, he would consume $W - e_s(p^*)$ right away, while if he joins the pool, he would be risking this additional amount to a project that he knows has a low probability of surviving. See that when $\gamma(1 + r) < 1$, the benefits of separating increase due to the relative impatience.

On the other hand, by joining the pooling contract, the entrepreneur would benefit from: a) lower lending rates, since the average type in the pool has a greater probability of surviving than himself, and b) the return to a larger internal capital ($W > e_s(p^*)$).⁷ These incentives

⁶To get the intuition about what is required, think of a type p^* that is indifferent between pooling or separating today. If he pools, he receives the same wealth as the best type in his cohort, contingent on success. Because the best type wants to signal he is a high type, he will re-invest all his wealth in the firm in the following period. At that point, p^* must decide whether to take a pooling contract again or drop off from the pool to take a separating one. On one hand, if he remains in the pool, he will be facing smaller lending rates compared to that paid in the current contract, as larger types tend to survive more often than lower types, thus improving the average of the pool (“selection”). On the other hand, if he remains in the pool, he will have to risk all his wealth, which will be larger than what it is in the current period as Expression (19) shows. As long as the second effect dominates, a type that is indifferent between separating and pooling at date t , will strictly prefer to separate at date $t + 1$. It turns out that the assumptions needed over the distribution of types for the second effect to dominate are quite general. It also turns out that while these assumptions simplify the exposition and computations, in principle the equilibrium can be computed for any arbitrary distribution of types.

⁷Computing the derivative of $W^+(\hat{p}, W)$ with respect to the first argument and then rearranging terms we observed that,

$$\frac{\partial W^+(\hat{p}, W)}{\partial \hat{p}} = \frac{1 + r}{\hat{p}^2} [TC(\hat{p}) - W] > 0$$

drive the adverse selection problem in the economy.

The characterization of $e(\hat{p}, n, W)$, for the case where wealth is a constraint, is depicted in Figure 4. All types above p^* take the same pooling contract while the rest take separating contracts. Somewhere in between p^* and \bar{p} we find the average type in the pool $\tilde{p}(n, p^*)$.

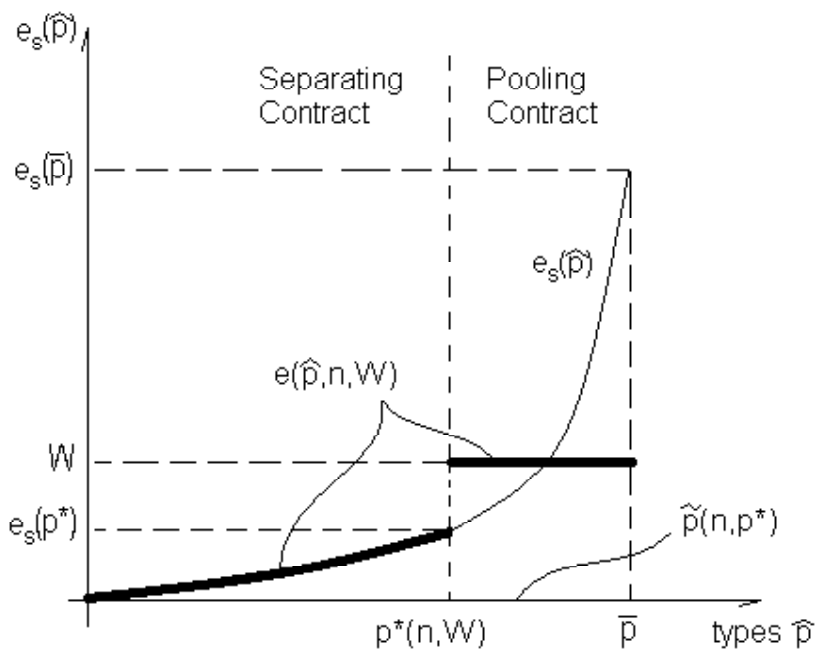


Figure 4: Contract under Pooling

We now show that $e(\hat{p}, n, W)$ is an equilibrium allocation. No type above the average \tilde{p} can get a better contract because they are unable to afford it, since they are already investing everything they have ($e(\hat{p}, n, W) = W$). A better contract would violate the non-negativity constraint for consumption. At the contracts offered in the market, summarized by $e(\hat{p}, n, W)$, those below the average have two options, to take a pooling contract or to separate. Condition (32) shows that pooling is the best response for all types above $p^*(n, W)$. The same condition proves that separation is optimal for those below the threshold. Also see that all the separating and the pooling contracts yield zero profits, implying that these allocations are a competitive equilibrium. Furthermore, the allocation is truth telling since

since there is always some borrowing in equilibrium.

types above p^* get the same contract (and hence have no incentives to lie), while those below p^* find it incentive compatible to report their true types (expression (30) is satisfied).

These allocations solve the planner's problem for any weighting function $H(p)$. We know that a solution to the planner's problem is such that incentive compatible constraints for all types and individually rational constraints for banks and the worst type bind. It turns out that there is only one allocation for which this is true, and hence the planner's weighting function $H(p)$ is irrelevant.

Replacing $e_s(p^*)$ and the law of motion for wealth both under separating and pooling ($W^+(p^*, e_s(p^*))$ and $W^+(\tilde{p}(n, p^*), W)$) in condition (34), allows us to find the threshold p^* . Furthermore, we can study its relation with the wealth invested by entrepreneurs, to understand how net worth affects the degree of adverse selection in a cohort.

Proposition 6 *The lowest and average type participating in a pooling financial contract, $p^*(n, W)$ and $\tilde{p}(n, p^*(n, W))$ are increasing functions of the entrepreneurs net worth W .*

Proof. See appendix. ■

The intuition for this result is that only high enough types would be willing to risk more of their own wealth in the firm to benefit from the lower lending rate offered in the pooling contract. To check our result, it is worth noticing that if the level of net worth W in equation (34) was equal to the level required in a separating contract for the highest type \bar{p} , that is $e_s(\bar{p})$ given by Proposition 3, then Condition (34) would only hold for $p^* = \tilde{p}(n, p^*) = \bar{p}$. That is, no type will be willing to participate in the pooling contract other than the best type itself! This shows that our results for pooling and separating contracts are consistent with each other. On the other hand if $W = 0$, all types would be willing to participate in the pooling contract simply because they would benefit from a lower lending rate without risking net worth.

The main features of the equilibrium allocations are characterized in the following proposition.

Proposition 7 *In a stationary environment, for every cohort of age n and with wealth W , there will be a threshold $p^*(n, W)$ such that all types $p < p^*(n, W)$ will take separating*

contracts while all types $p \geq p^*(n, W)$ will take a pooling contract. Furthermore, the optimal allocations of the planner's problem are

$$\begin{aligned}
e(\hat{p}, n, W) &= \begin{cases} e_s(\hat{p}) & \forall \hat{p} \leq p^*(n, W) \\ W & o.w. \end{cases} \\
i(\hat{p}, n, W) &= \begin{cases} \frac{1+r}{p} & \forall \hat{p} \leq p^*(n, W) \\ \frac{1+r}{p(n, p^*(n, W))} & o.w. \end{cases} \\
k(\hat{p}, n, W) &= \begin{cases} k_s(\hat{p}) & \forall \hat{p} \leq p^*(n, W) \\ k_s[\tilde{p}(n, p^*(n, W))] & o.w. \end{cases} \\
l(\hat{p}, n, W) &= \begin{cases} l_s(\hat{p}) & \forall \hat{p} \leq p^*(n, W) \\ l_s[\tilde{p}(n, p^*(n, W))] & o.w. \end{cases} \\
L(\hat{p}, n, W) &= wl_s(\hat{p}, n, W) + r_k k_s(\hat{p}, n, W) \\
c(\hat{p}, n, W) &= W - e_s(\hat{p}, n, W) \\
W'(\hat{p}, n, W) &= \begin{cases} W^+(\hat{p}, e_s(\hat{p})) & \forall \hat{p} \leq p^*(n, W) \\ W^+[\tilde{p}(n, p^*(n, W)), W] & o.w. \end{cases}
\end{aligned} \tag{35}$$

These allocations show that consumption is equal to zero for all types $p > p^*(n, W)$, since the best type invests all his wealth while those $p > p^*(n, W)$ will mimic him. As wealth increases, $p^*(n, W)$ does too. Eventually, the best entrepreneur in the cohort should have built up enough net worth to signal that he is the best ($W > e(\bar{p})$). Then, the adverse selection problem in the cohort will be completely resolved.

As an example, the mass of firms for cohorts of age 1, 2 and 3 with endowment ω are plotted in Figure 5.⁸ Let $f_n(p) = f(p)p^{n-1}$ be the distribution of types for surviving firms of age n (for a newborn cohort, assume that $f_1(p) = f(p) = 6p(\bar{p} - p)$). We assume that full separation is not possible from the start. Hence some pooling arises in equilibrium, and revelation occurs gradually.

⁸To generalize the environment to one where cohorts have different endowments we must split the newborn cohort according to endowments and proceed with the same exercise, assuming that ω is public information.

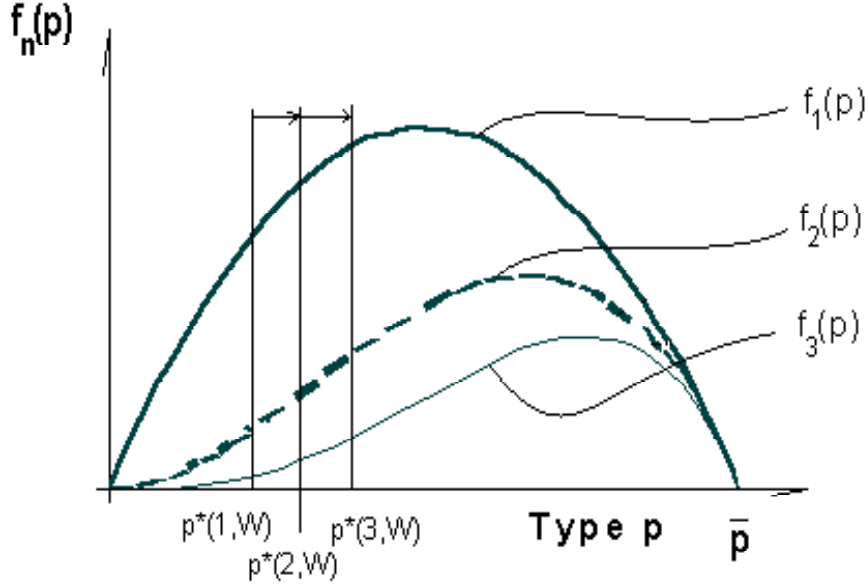


Figure 5: Distribution of Types by Age

We know from Condition (34) that there exists a type $p^*(1, W)$ such that he is indifferent between pooling and separating. If $W > 0$, $p^*(1, W) > 0$. Also from Proposition 7, those types to the right of this threshold will take the same pooling contract while those types to the left will take separating contracts.

The next period wealth of all types in the pool will be greater than it was in the previous period since $W^+(\tilde{p}, W) > W$. When the second period arrives, all surviving firms in the pool will have the same level of wealth, as they implement the same production plan in the first round of investments. The best type will invest as much as possible and some types will mimic him. We know that as long as the distribution of types is not extreme, as we assumed without loss of generality, then $p^*(2, W^+(\tilde{p}(1, p^*), W)) > p^*(1, W)$, given that the best type in the pool will raise the stake by re-investing everything he obtained from the first round. The same occurs in successive periods until eventually the best type in the cohort has accumulated enough wealth to take a separating contract, or $W \geq e_s(\bar{p})$. Because entrepreneurs start with different wealth levels, not all best types in a cohort will get to truthfully signal their types in the same number of periods. But once all of them start taking separating contracts, the asymmetry of information in the cohort will be completely resolved.

We can characterize the firm dynamics in our environment. First notice that when a firm reveals its type, it keeps implementing the same production plan from then on and it stops growing. This production plan is determined by the input scale of production, as given by (35). Firm growth occurs for the best types in each cohort because $\tilde{p}(n, p^*)$ increases with age n due to two reasons. First, because of the selection process, the distribution of types improves over time as lower types exit with higher probabilities ($\partial \tilde{p}(n, p^*) / \partial n > 0$, $\forall p^* < \bar{p}$). Second, screening makes low types drop off the pooling contract as members of the pool are risking more wealth in their firms, making it costly for the lower types to follow. Because the perception of credit markets about the entrepreneurs in the pool improves over time, lending interest rates decrease and firms expand the scale of production. These firm dynamics follow from the allocations described by (35). Notice that when a firm of type p drops off the pooling contract, the manager pays a higher interest rate and scales down production since it must be that $p \leq p^*(n, W) < \tilde{p}(n, p^*(n, W))$. We summarize all these results in the following corollary.

Corollary 8 *Conditional on survival, the dynamics for a firm of type p is such that firms*

- face decreasing lending rates and implement increasing scales of production as long as $p^*(n, W) \leq p$,*
- pay a higher interest rate and scale down production when taking the first separating contract, and*
- pay the same lending rate and maintain the same production scale when $p^*(n, W) > p$.*

Firm dynamics do not occur as a consequence of learning or technological improvements, since entrepreneurs operate the same technology throughout. On the contrary, in the presence of asymmetric information in credit markets, firms dynamics occur because of financial reasons, since it takes time for good firms to build up financial reputation and convince banks of their quality. Pooling is inefficient because entrepreneurs with different success rates, and hence different expected productivity, get to invest the same amount (since different types take the same pooling contract). As credit markets perceive firms to have greater chances of

surviving (reputation), interest rates on loans decrease and firms grow in scale by employing additional labor and capital.

2.2 The worker's problem

Workers maximize utility subject their budget constraint and taking prices as given. The solution to the workers' problem, at steady state, follows from

$$\varphi'(l_t^s) = w \quad \forall t > 0 \quad (36)$$

$$\sum_{t=0}^{\infty} \frac{c^W}{(1+r)^t} \leq \sum_{t=0}^{\infty} \frac{wl^s}{(1+r)^t} + a \quad (37)$$

Expression (36) is the workers' labor supply. Notice that the labor supply has no income effect (since it only depends on wages). Also the Euler equation of the workers problem implies that, in a stationary environment, the interest rate is the same as the worker's subjective discount.

3 Equilibrium

We now present the definition of equilibrium. Let $\Omega[\mu, f(p), g(\omega)]$ be the economy described above.

Definition 9 *A competitive equilibrium allocation for economy $\Omega[\mu, f(p), g(\omega)]$ is a set of prices $\{r_k, w, r\}$; productive allocations for capital, labor and output $\{k(p, n, W), l(p, n, W), y(p, n, W)\}$; financial contracts $\{i(p, n, W), L(p, n, W), e(p, n, W)\}$; allocations for entrepreneurs' consumption $c(p, n, W)$ and wealth $W^+(p, e(p, n, W))$; and consumption and saving allocations for workers $\{c^W, a\}$ such that entrepreneurs and workers maximize their utility, banks are willing to offer every contract, and labor and capital markets clear.*

Because there is competition in the rental market for capital, and inputs are paid in advance, the equilibrium rental price of capital is $r_k = (r + \delta)/(1 + r)$. Let us now concentrate on the equilibrium conditions in the labor and capital markets.

Before we studied the allocations for a cohort with the same endowment ω . Since entrepreneurs have different endowments ω , we must now aggregate the demand for labor and capital inputs across all firms.

Notice that $p^*(1, W) = p^*(1, \omega)$, while $p^*(2, W^+) = p^*(2, W^+(\tilde{p}(1, p^*(1, \omega)), \omega))$. In other words, all thresholds are only a function of age and the endowment ω , as the second threshold depends on the average type in the first pooling contract, $\tilde{p}(1, p^*(1, \omega))$, and on the endowment, ω . Using the law of motion $W^+(\tilde{p}(n, p^*), W)$ recursively, we can demonstrate that this result extends to all successive thresholds. Given that the age and the endowment are enough to define all thresholds, it is convenient for what follows to let $p_\omega^*(n) \equiv p^*(n, W^+)$, where W^+ is a function of ω as we showed. For the same reason, let $W_\omega(n) \equiv W^+(\tilde{p}(n, p_\omega^*(n)), W)$ be the wealth level of those types of age n with endowment ω that *participate in a pooling contract*.

Finally, also let $\eta_\omega(n)$ be the mass of firms of age n and endowment ω , taking a pooling contract at time t , or

$$\eta_\omega(n) = \int_{p_\omega^*(n)}^{\bar{p}} f(p)p^{n-1}dp$$

Having said that, equilibrium in the labor market holds if and only if⁹

$$\sum_{n=1}^{\infty} \left\{ \int \left[\int_0^{p_\omega^*(n)} l_s(p)p^{n-1}f(p)dp + l_s(\tilde{p}(n, p_\omega^*(n))) \eta_\omega(n) \right] dG(\omega) \right\} = \mu l^s \quad \forall t. \quad (38)$$

Likewise, equilibrium in the capital market implies

$$\mu a + \sum_{n=1}^{\infty} \left\{ \int \left[\int_0^{p_\omega^*(n)} e_s(p)p^{n-1}f(p)dp + W_\omega(n)\eta_\omega(n) \right] dG(\omega) \right\} = K \quad \forall t \quad (39)$$

where

$$K = \sum_{n=1}^{\infty} \left\{ \int \left[\int_0^{p_n^{\omega*}} k_s(p, n, W)p^{n-1}f(p)dp + k_s(\tilde{p}(n, p_\omega^*(n))) \eta_\omega(n) \right] dG(\omega) \right\}.$$

⁹Notice that the cross section of a variable is equal to the time series since the economy is in a stationary environment.

3.1 Contracts

We now discuss the role played by each of the assumptions that affect the set of contracts used in equilibrium.

Our assumption that banks can walk out of the financial relationship at any time is important for firm dynamics. Note that every type's discount factor is effectively given by γp . Higher types are relatively patient compared to lower types since the latter survive less often. Because of this feature, a multi-period financial arrangement could, in principle, induce separation even when the types have a limited amount of initial wealth. For example, banks could offer contracts where they retain all entrepreneurs' earnings during a certain number of periods to then rebate these earnings back at some date in the future, contingent on the survival of the firm. Because lower types discount these payments more heavily, as γp is lower, they would be less willing to take that contract.

With the same purpose, we have assumed that firms cannot commit to future actions.¹⁰ If they could, high types would commit to re-invest all earnings for a certain number of periods, and hence reveal as a high type. In essence, the possibility of committing to future investment plans would replicate the multi-period contract suggested above, even when banks lack the commitment to stay in the financial relationship.

Finally, the history of previous announcements are assumed to be unobserved to banks (only entrepreneurs and credit officers know them). If they were observed, once an entrepreneur takes a separating contract, his type would be revealed to the market. If contracts depended on past announcements, and if $\gamma(1+r) < 1$, no net worth would be required after the type is revealed, and entrepreneurs would benefit from consuming rather than investing $e_s(p)$. In other words, the contract we have characterized would not be optimal. Nonetheless, if $\gamma(1+r) = 1$, the optimal financial contract would be independent of past announcements, because there is no benefit in consuming rather than investing the amount $e_s(p)$. We prefer to work under the assumption that past announcements are not public information and let $\gamma(1+r) < 1$ because, then, entrepreneurs that have overcome the financial constraint will

¹⁰This assumption is typically used in most models of financial imperfections.

keep borrowing in equilibrium.

As was mentioned before, while these assumptions might affect the firm dynamics of our model, alternative environments do not affect the main results regarding the role of information in credit markets for aggregate production.

4 Information and aggregate performance

From the model we can rank the output performance of economies under alternative informational environments. Let $G_\Omega(\omega)$ and $G_\Delta(\omega)$ be the distribution of entrepreneurs' endowment of economies Ω and Δ , respectively. Our ranking is stated in the following proposition.

Proposition 10 *Under asymmetric information, if $G_\Omega(\omega)$ first order stochastically dominates $G_\Delta(\omega)$, economy Ω will exhibit more employment, aggregate output, capital stock, higher wages and higher measured total factor productivity than economy Δ . Moreover, economies under full information outperform those under asymmetric information, which outperform those under uncertain information.*

Proof. See Appendix. ■

First think about economies under asymmetric information. Proposition 6 shows that the thresholds $p^*(n, W)$ are increasing in wealth and hence in endowments as shown by iterating over Expression (19). Under asymmetric information, if the same cohort had more endowment ω , there would be less pooling since $p_\omega^*(n) < p_{\omega'}^*(n) \forall \omega' > \omega$. We show in the proof of Proposition 10 that if there is more pooling in a cohort, total employment, capital and production in that cohort will be lower.¹¹ If entrepreneurs' endowments are larger (in the

¹¹This result contradicts previous findings by De Meza and Web (1990) who claim that “in a pooling equilibrium there is always too much investment”. The discrepancy arises because they study adverse selection with a fixed size of investment projects. In an equilibrium with production, pooling implies that both the good and bad types will invest, and this leads to too much investment. Instead, in our case pooling leads to higher lending rates, inducing the best types to scale down production (and investment). Proposition 10 shows that the second effect dominates and, hence, pooling generates less investment when compared to the first best.

first order stochastic sense), all cohorts will produce more and there will be excess demand for labor at initial wages. The equilibrium is only restored at higher wages and employment (since labor supply has no income effect).

Second, economies under full information produce generally more than those under asymmetric information. Note that production allocations under asymmetric information converge to those under full information, for the limiting case where no entrepreneur is financially constraint or, equivalently, all types are screened ($p_{\omega}^*(n) \geq 1 \forall \omega, n$).

Third, compare economies under asymmetric and uncertain information. In our model with asymmetric information, banks learn over time the average type participating in each contract via two ways: “selection” (in the spirit of Jovanovic (1982)) and “screening”. Banks update their beliefs based on the age of the firm because those types that survived a round of investment have a higher average probability of survival than those that have exited (selection). This effect is the only one present under uncertain information. In particular, note that the expected probability of survival under uncertain information is given by $\tilde{p}(n, 0) = E(p/p > 0, n)$, where $\tilde{p}(n, 0)$ is increasing in age and $\lim_{n \rightarrow \infty} \tilde{p}(n, 0) = \bar{p}$.¹² But under asymmetric information, banks can additionally learn from the revelation of lower types, that are unwilling to keep up with the path of investment set up by the best types in each cohort (screening). Banks can estimate the thresholds $p^*(n, W)$, which we know are generally greater than $p^*(n, 0)$. As shown in Proposition 10, production falls when more types are pooled, and all types are pooled under uncertain information. Thus, this logic shows that there is more production under asymmetric versus uncertain information.

5 Quantifying the role of information

Next, we turn to numerical analysis to study some properties of industry dynamics displayed by the economy under asymmetric information, and to measure the loss of output caused by informational imperfections.

¹²Because the scale of production increases with $\tilde{p}(n, 0)$, as shown by (17) and (18), firms will always grow under uncertain information (as long as they survive).

5.1 Parameterization

We assume every round of investment matures in a year, and hence the model is calibrated annually. The workers' discount rate (the interest rate in the economy) is set at two percent. The disutility from working and the mass of workers are calibrated so that $l = 1$ and wages are equal to .9 in the initial steady state. Furthermore, the labor supply is assumed to be $\kappa l^{\frac{1+\varepsilon}{\varepsilon}}$ where κ is a normalization parameter and ε , the labor supply elasticity is set at one as in previous studies of firm dynamics and business cycles. The entrepreneur's discount factor is set so that the leverage of unconstrained firms, defined as the debt-to-equity ratio, is $1/2$ ($\gamma = 0.968$).¹³

Basu and Fernald (1997) estimate the returns to scale to be close to one or slightly decreasing in manufacturing. We follow Cooley and Quadrini (2001) and utilize a return to scale of .975. Nonetheless, we run a sensitivity analysis on the range between .85, as suggested by Quadrini (2004), and .99. The labor share is calibrated to match the US labor income share, $\beta = .67$. Hence $\alpha = .305$. Depreciation is assumed to be five percent, as usual in quantitative studies. The plant productivity parameter A is calibrated so that the best firm operating at the efficient scale employs 600 workers, to be able to match the US distribution of firms, for which we have data for plants with over 500 workers.

The main conclusions of this paper, regarding the role of information for aggregate production, are not sensitive to alternative distributions of types $f(p)$ or endowments $g(\omega)$. Nonetheless we try to roughly match a few features of industry characteristics. The distribution of entrepreneurial wealth $g(\omega)$ was discretized. The support chosen is $\{2, 9, 15\}$ with probabilities $\{.6, .3, .1\}$, so that the mean labor income generated by an entrepreneur is six times larger than that of workers. We also present simulations with higher means (although same other moments) for the distribution of the entrepreneurs' endowments. We use a simple function of the form $f(p) = (1 - p)^a p^b$ with $p \in [\underline{p}, \bar{p}]$ for the distribution of talents. We set $\bar{p} = .99$ so that all firms have at least a one percent probability of default (to match the prime rate in the US). Parameters a , b , and \underline{p} , together with $g(\omega)$, were chosen to roughly approximate the size distribution of firms in manufacturing for 1987 as reported

¹³Gama is computed by setting $e/[TC - e] = 1/2$.

by Van Ark and Monnikhof (1996).¹⁴ Also, we generate a right-skewed size distribution of newborn firms as reported by Cabral and Mata (2003). An exhaustive procedure to search for the parameter values to approximate the unconditional size distribution of firms could not be implemented since the mapping from parameters to the simulated distribution is highly non-linear. Through trial and error, we were able to find parameters that seem to do a good job in this regard. The parameter values adopted are $a = 1.2$, $b = 1$ and $\underline{p} = .8$ for the distribution of types. Figure 6 shows the US size distribution of firms for 1987 and the simulated size distribution of firms of our calibrated economy.

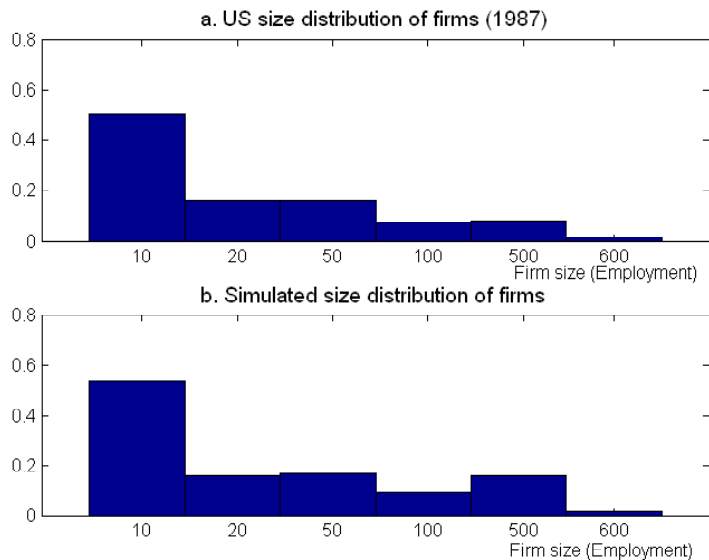


Figure 6: Invariant Size Distribution of Firms

Our parameterization of the distribution of types gives average entry and exit rates of around 10 percent per annum, measures slightly higher than the (annualized) rates reported by Evans (1987) (which we believe under-estimate the true exit rates given that very small firms are under-represented in his sample).

5.2 Information and firm dynamics

Firm dynamics: The firm dynamics characterized in Proposition 7 can also be followed in Figure 7. Panel *a* shows the labor employment dynamics of a firm of type \bar{p} , contingent

¹⁴Cabral and Mata (2003) report similar distributions for other OECD economies.

on survival, for our three different levels of initial wealth $\omega \in \{2, 9, 15\}$. Notice that the maximum employment size reached by a type \bar{p} is 600, as calibrated. The figure shows that a firm of type \bar{p} that starts with the highest net worth, reaches the efficient scale by year 15. The same type starting with the lowest net worth matures in 32 years. They also start at different scale (five versus around thirty employees). The slopes of these curves show that firms with looser financial constraints grow much faster. See that this panel also exhibits the evolution of types other than the highest. We know from our characterization of the equilibrium that those types below \bar{p} in a pooling contract share the same firm dynamics with the highest type. Once they separate, they scale down production and remain producing at their efficient scale (at some employment level below 600 employees).

Panel *b* shows the dynamics of net worth (solid line) and debt (dotted), for the highest type under the three different endowment levels, again contingent on survival. Panel *c* shows that leverage (debt-to-equity ratio) is decreasing in the firm's age. The debt-to-equity ratio remains at 1/2 once firms have reached the efficient scale of production. This shows that growing firms are also more levered. Additionally, note that wealthier entrepreneurs start with a lower leverage level and reach adulthood in a shorter period.

Panel *d* shows that lending rates decrease with age. As firms grow, they acquire financial reputation and, hence, they have access to lower lending rates. Moreover, entrepreneurs with higher endowments ω can signal that they are better types and then pay lower lending rates from startup (11 versus 16 percent, in our simulation).

We can follow the amount of adverse selection that takes place in equilibrium from Panel *e*. Remember that thresholds $p_n^*(\omega)$ delimit the types that take a pooling contract versus those that take separating ones, for each cohort of age n and endowment ω .

Two observations follow. First, $p_n^*(\omega)$ increases with age, since surviving firms accumulate net worth and then the best types are better able to undertake more aggressive investment projects (and hence separate from lower types). Second, $p_n^*(\omega)$ is increasing in the endowment ω . These thresholds reach their maximum at $\bar{p} = .99$, when the best type in the cohort has enough wealth to take a separating contract (when they reach adulthood). For this reason, these thresholds reach the maximum level \bar{p} in 15 years for cohorts with the highest initial

endowment, and in 32 years for cohorts with the lowest endowment. In other words, full revelation occurs sooner for wealthier cohorts. Panel *f* presents the fraction of constrained firms in a cohort with different endowments and ages. This fraction is the relative mass of firms under pooling contracts. It follows that the mass of constrained firms decreases with age and endowment until, eventually, no firm is constrained. At that point, the asymmetry of information vanishes and financial markets are able to screen all types within a cohort. Nonetheless, it takes a long time for this to happen (up to 32 years in our simulation).

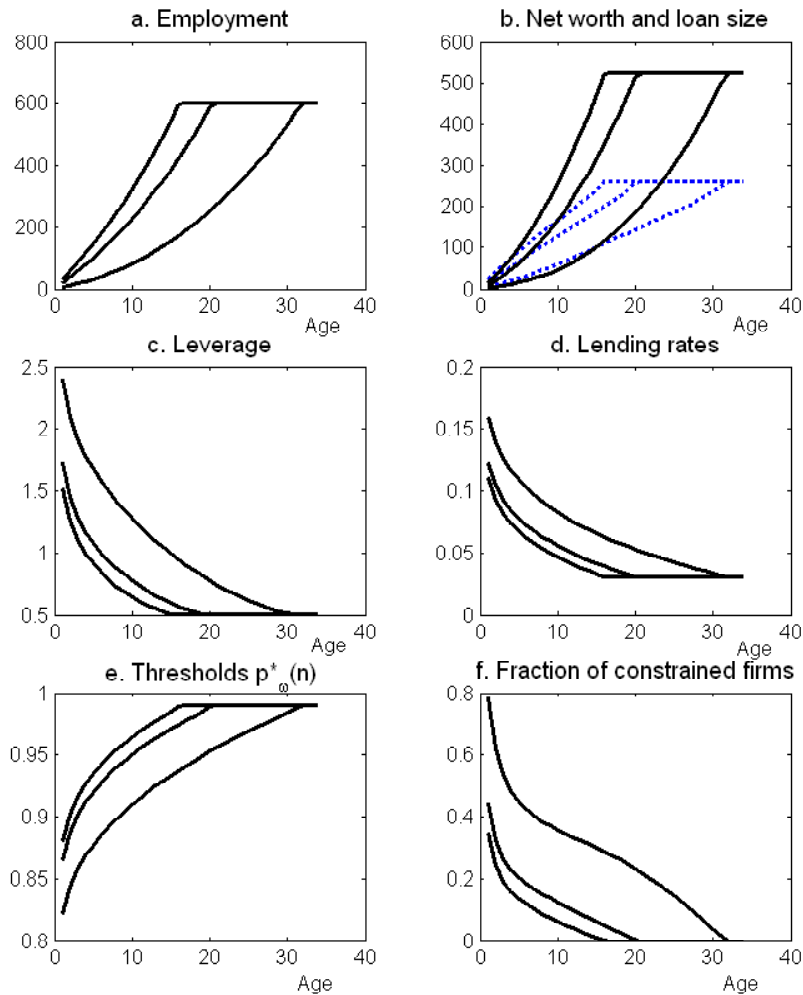


Figure 7: Firm Dynamics

Industry dynamics: Figure 8 presents relevant measures of industry dynamics gener-

ated by our simulation. Panel *a* shows that the exit rate is a decreasing function of age. The age dependence of the exit rate is important because models with exogenous exit rates overestimate the amount produced by constrained firms.¹⁵ Indeed, under exogenous exit rates, too many large-unconstrained firms exit in equilibrium. Panel *b* displays the fraction of financially constrained firms by age. A little over 60 percent of firms in a newborn cohort are financially constrained (although in different degrees), while by age 32 no firm is producing below their efficient scale. Consistent with previous findings by Hall (1987) and Evans (1987), Panels *c* and *d* show that both growth rates and standard deviation of growth rates are inversely related to the firm's age and size.¹⁶ In our model, firms grow only when they are financially constrained, and while reputation is being acquired (since there is no other source of growth). Firms shrink in size when they leave a pooling contract to take the first separating contract. All other firms remain at their efficient scale (where they are financially unconstrained). Again, by age 32 no firm is growing or slowing down since all types have been revealed. Exiting firms have not been considered in the computations of means and volatilities of growth rates.

Panels *e* and *f* show job reallocation (creation and destruction) in rates, by age and size. Creation of jobs due to entry has not been considered as part of the simulated rates. Two features of job reallocation rates stand out. First, the simulated job creation and destruction rates are generally decreasing in age and size. The only exception is the job creation rates for the smaller firms (which increase with size), a consequence of not including the job creation by newborn firms, most of which belong to this size category. Second, creation rates are higher than destruction rates for younger and smaller firms, and lower for older and bigger firms. Both facts are nicely aligned with empirical findings reported for the US by Davis, Haltiwanger and Schuh (1996).¹⁷ Furthermore, conditional on size, younger firms grow faster and have a higher volatility of growth rates in the model, as documented by Evans (1987).¹⁸

¹⁵Some examples are, Cooley and Quadrini (2001), Cooley, Marimon and Quadrini (2004) and Cagetti and De Nardi (2003).

¹⁶Growth rates are computed based on employment as in the cited empirical literature.

¹⁷Also, net job creation rates become negative around age (10) and size (50+), similar to findings for the US.

¹⁸Cooley and Quadrini (2001) show that they need entrants to have relatively high productivity for the

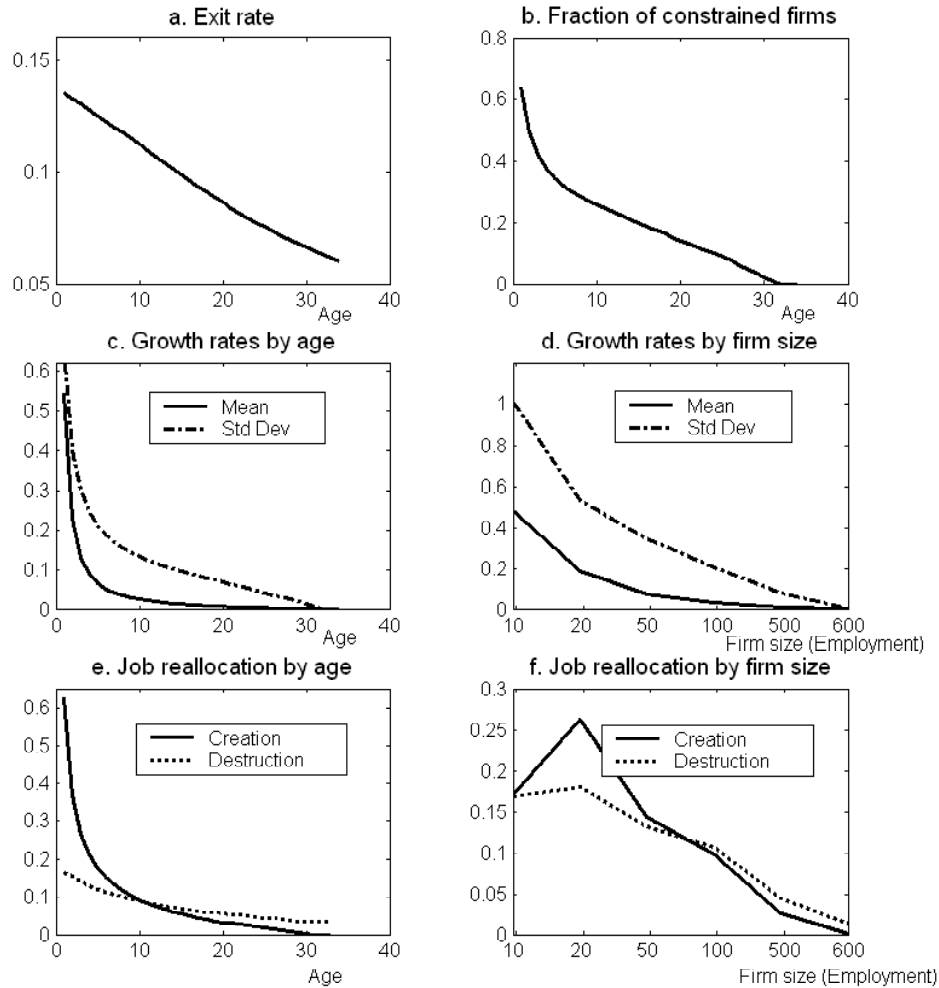


Figure 8: Industry Dynamics

Although the model does well in several dimensions, it fails to match the size dependence of firm dynamics (see Evans (1987)). Conditional on age, the dynamics of firms, regarding growth rates, volatility of growth rates, and job reallocation should decrease with firm size. But in the model, only the larger firms in a cohort actually get to grow, because they are financially constrained. Smaller firms do not create jobs, although they destroy jobs due to exit. A model with heterogeneity in technology, both between newborns and generations age dependence to have the right sign (older firms exhibit lower growth rates, volatility of growth rates, and job reallocation). In our model entrants have, on average, a lower productivity than incumbents (and financial imperfections make the productivity on entrants even lower).

of firms (vintage), could be better equipped to match the conditional size dependence fact, although we leave this issue for the future.

5.3 The value of Information

Proposition 10 states that economies where more information is revealed produce more at the aggregate level. In this section we quantify the role played by information for aggregate production within our model. We do so by exploring the consequences for production of increasing the entrepreneur's endowments ω (by changing $G(\omega)$ in the first order stochastic sense).

Partial equilibrium: First we study the role of information when wages remain unchanged in response to changes in the distribution of endowments or assumptions on the information structure. Wages are kept constant at the equilibrium wages arising from our calibrated, asymmetric information economy.

That the value of information in credit markets could be large for individual types follows from Figure 7a. Note that older firms produce several times more than younger firms, even for the same type (same survival probability and technology).

From a different perspective, Figure 9a presents the total level of employment by cohort of age n , under three environments:

- full information, where $p_\omega^*(n) = 1 \forall \omega, n$,
- asymmetric information, where $p_\omega^*(n)$ increases with ω and n , and
- uncertain information, where $p_\omega^*(n) = 0 \forall \omega, n$.

Output by cohort is proportional to employment under partial equilibrium, since $wM = \beta/(1+r)Y$, as show in the proof of Proposition 10. Under full information, employment and output by cohort falls with age because the mass of firms in a cohort shrinks due to exit. Under asymmetric information, employment and output by cohort are hump-shaped. Notice that a newborn cohort produces around 1/4 of its potential under asymmetric information. Initially, the scale of production is expanded due to the revelation of information, despite the

exit of some firms. By age 9, cohorts start producing less as the exit effect dominates the gains from information revelation. By age 25, the gains from revelations are almost exhausted, although we know they fully disappear by age 32. A better distribution of endowments $G(\omega)$ (in the first order sense), would move the asymmetric information allocations closer to the full information one. The tension between information revelation and exit is also present under uncertain information, although the former is weaker since $p_\omega^*(n) = 0$ for all ages (neither credit markets nor entrepreneurs get to know the exact survival probability in Jovanovic's world). Because learning also takes place under uncertain information, eventually all cohorts end up producing at first best levels (although convergence occurs in the very long run in this case).

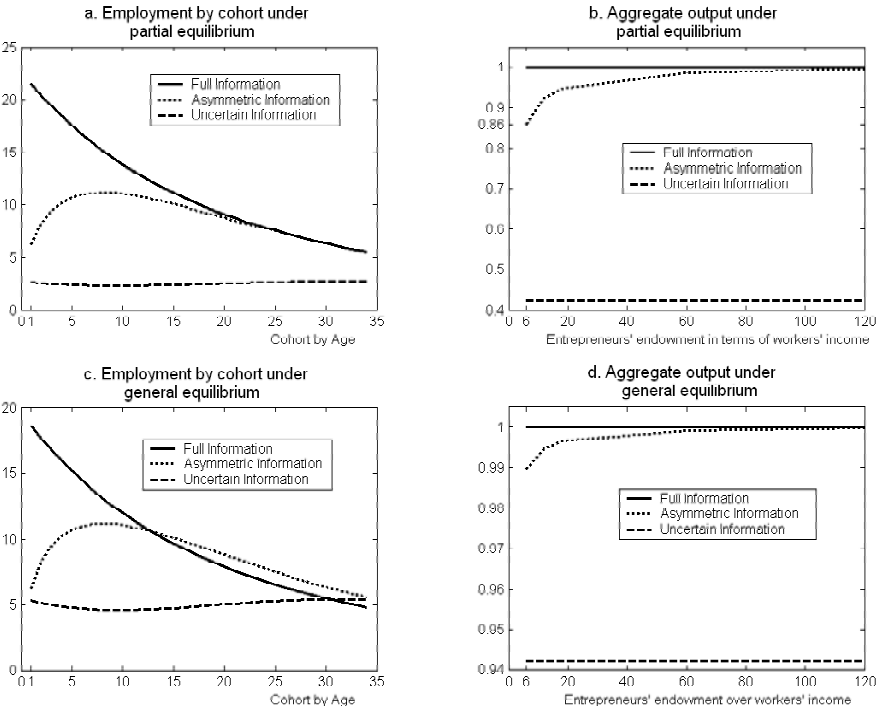


Figure 9: Partial vs General Equilibrium

The area between these curves reflect the loss of employment and output caused by the lack of information in credit markets under partial equilibrium. Panel *b* plots the total employment under asymmetric and uncertain information, relative to the first best alloca-

tions, for different levels of entrepreneurs' endowment.¹⁹ Because wages remain unchanged, this graph plots both the employment and output (in percents terms). In our simulations, output under asymmetric and uncertain information are 14.1 and 57.5 percent lower than output under full information, when wages do not adjust in response to a larger demand for employment, and the average mean of endowments is around 6 times the workers' yearly income (as calibrated). Only the mean, and no other moment, of the distribution $G(\omega)$ was increased for computing this graph. Furthermore, the entrepreneurs' wealth only matters under asymmetric information since, otherwise, Proposition 2 holds.

General equilibrium: Panels c and d display the same variables as Panels a and b , but under general equilibrium, that is, when wages are allowed to change in response to the amount of information in the economy. Total employment by cohort under asymmetric information is the same as that presented in Panel a . We have also computed the full and uncertain information allocations. Note that total employment with full information under general equilibrium lies below the one corresponding to the partial equilibrium (Panel a), since under full information wages are higher in response to a more efficient allocation of resources. Panel c shows that under more information and higher wages, older firms produce less (since it is more costly to produce) while younger firms produce more (because there is more information in credit markets).²⁰ This feature is also true when comparing the uncertain information allocations to any of the alternatives. In other words, this graph shows that the general equilibrium effect works in opposition to the informational effect.

How strong is the general equilibrium effect? Panel d shows that this effect is actually quite strong. While under partial equilibrium, total output loss under asymmetric information was 14.1 percent, the *equilibrium* output loss under asymmetric information is only 1.1 percent. This is important because we are saying that, for our calibration, the general equilibrium effects wash out most of the adverse effects coming from our credit market imperfections!

Employment loss is about half of the output loss. We work under the assumption that a

¹⁹Aggregate employment under full information is normalized to one.

²⁰Allocations under full and asymmetric information coincide when wealth is large (all types are screened).

labor supply elasticity is one, and then wages and employment move proportionally. Since $wM = \frac{\beta}{(1+r)}Y$, and output loss under asymmetric versus full information is around one percent, wages and employment losses must be around half a percent.

Sensitivity analysis: Figure 10 shows that our results are robust to alternative parameterizations of the technology and labor supply elasticity. Panel *a* shows the output losses under asymmetric and uncertain information, relative to the full information case, for different returns to scale in production. Profits equal to x percent correspond to a technology exhibiting $1 - x$ returns to scale. The simulations in Figure 10 were run by re-calibrating our productivity parameter (A) so that the best firm employs 600 workers. Parameters α and β were adjusted proportionally to our original calibration ($\alpha/\beta = .305/.67$). Under general equilibrium, output loss under asymmetric information reaches a maximum of only two percent around a return to scale of .95. Panel *b* shows that even by stretching the labor supply elasticity to a value of two, the output loss due to the lack of information in credit markets would be less than three percent. When labor supply elasticity is .5, the most commonly used value, the maximum output loss under asymmetric information is less than 1.5 percent. This confirms our results that general equilibrium effects wash out most of the adverse effects of information.²¹

There are a few relevant insights about Figure 10*a*. First, as the return to scale approaches 1, the output loss disappears both under asymmetric and uncertain information. The intuition is the following. Under almost constant returns to scale, those firms perceived to be the best will get to produce while the rest will produce an extremely small amount. If we take this argument to the extreme, only the highest types would produce, both under full and uncertain information. In this case, information cannot have a large impact on aggregate production since it helps to avoid allocating production in low-type firms but these

²¹Restuccia and Rogerson (2003) evaluate a model with heterogenous plants where distortions are correlated (high and low productivity plants are taxed and subsidized, respectively). This model also shares this feature since information makes more productive firms (the best types) produce less while less productive ones (free riders) more. Despite this, our model shows that information is unable to undermine aggregate production.

firms would not produce in the first place.²²

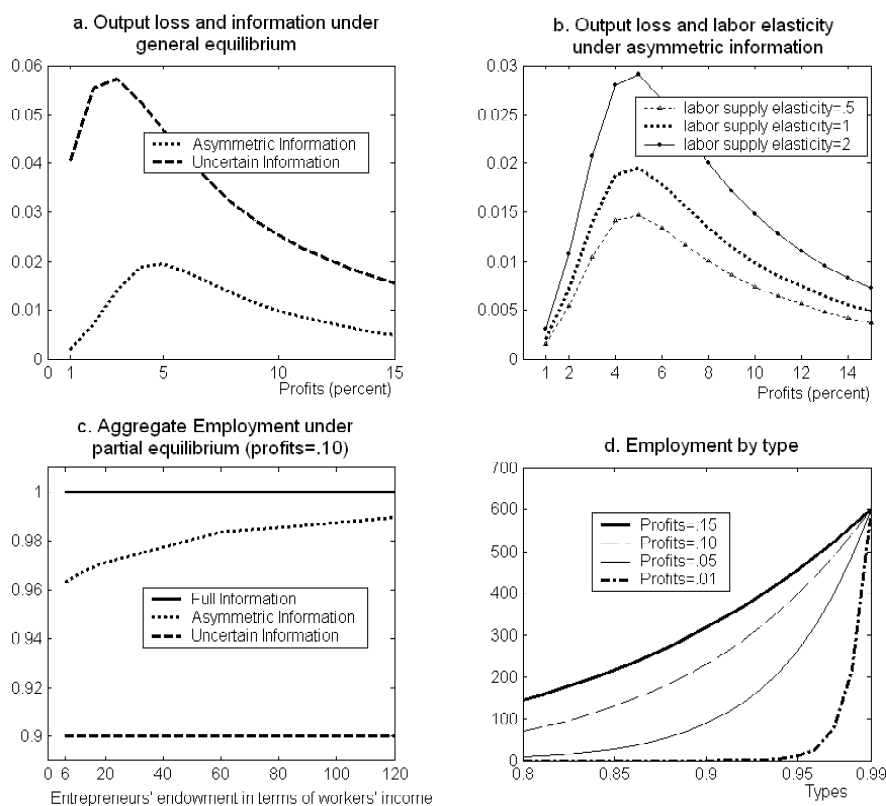


Figure 10: Sensitivity

Why are the output loss functions in Figure 10a non-monotonic? First note that the explanation cannot be due to financial imperfections since the output loss under uncertain information is also non-monotonic. Second, the non-monotonicity observed in Figure 10a is not due to general equilibrium arguments. Panel c shows the total employment (and output) under partial equilibrium, when returns to scale are .9. In this case, output loss under uncertain information is much lower (10 percent, versus 57.5 percent for profits of 2.5 percent in Figure 9b). In other words, smaller returns to scale reduce the value of information about the types, even under partial equilibrium.

²²Atkeson, Khan and Ohanian (1996) use a version of Hopenhayn and Rogerson (1993) to study the implications of labor market distortions and to argue that returns to scale are far from constant. As we show below, even under lower returns to scale we find that information is not relevant at the aggregate level.

Panel *d* plots the optimal employment level for every firms' type (Equation (18)), for different returns to scale. Again we re-calibrate A so that the highest type employs 600 workers. As the return to scale falls, the labor schedule loses its convexity and shifts up for all types but the highest. Less convexity *eventually* implies that the gains from full versus uncertain information shrink in absolute terms.²³ When returns to scale are .85, most of the convexity of $l_s(p)$ disappears, and the output loss under uncertain information shrinks. This explains the non-monotonic relation.

Lastly, output loss under asymmetric information also falls with profits. In addition to the previous argument, higher profits imply a faster wealth accumulation, and a sooner revelation within cohorts.

6 Conclusions

We study a general equilibrium environment where entrepreneurs know more about the firm's survival probability than credit markets (asymmetric information). Entrepreneurs build up financial reputation through selection, as the worse firms die with higher probability, and screening, as they are asked to risk larger amounts of wealth if they claim to be a higher type. In this environment, the best entrepreneurs in each cohort face decreasing lending rates, borrow more, and expand production. The firm and industry dynamics of the model exhibit several of the properties documented for the US. Smaller and younger firms pay fewer dividends, face higher interest rates and borrow less, while growing and younger firms are more levered, and exhibit a greater sensitivity of investment to cash flows. The mean and volatility of growth rates, job creation, job destruction, net job creation and exit rates, decrease with size and age (even when we condition by firm size). Moreover, the model rationalizes the role of financial reputation for firm and industry dynamics, an issue absent in the literature.

We investigate the aggregate implications that information problems in credit markets

²³This occurs because $l_s \left(\int_{\underline{p}}^{\bar{p}} pf(p)/(1-p)dp \right) \rightarrow \int_{\underline{p}}^{\bar{p}} l(p)f(p)/(1-p)dp$ as $l_s(p)$ gets flatter. (The unconditional distribution of types is $f(p)(1+p+p^2+\dots) = f(p)/(1-p)$.)

have on output and employment in these economies. We show that younger firms produce much less than older firms only because of the asymmetry of information in credit markets (technology is always the same). Then we ask how much more output could be produced, and labor employed, in the absence of information problems. Surprisingly, the answer is not much. When the distortions in credit markets are removed, younger firms employ more labor. Wages increase however, and older (and unconstrained) firms respond by cutting down employment. Our quantitative exercise shows that these equilibrium forces almost offset informational problems in credit markets. That is, information affects mainly the organization of production, but only slightly how much is produced. Furthermore, these results seem robust to alternative parameterizations of the return to scale and the labor supply elasticity.

We leave for the future investigating alternative environments where information matters. Indeed, the model has several limitations. Information is studied in a stationary economy, and hence, we ignore the role of information in environments with technological growth, macroeconomic uncertainty, or capital accumulation. Moreover, the asymmetry of information is the only distortion in our environment, and hence we ignore how would it interact with other ones. Also in our model, the number of startups is independent of the financial imperfections, but this view has been contradicted by Evans and Jovanovic (1989). Lastly, we think that more sophisticated contractual arrangements would mitigate informational problems, but this should not be taken for granted since little is known about these environments.

Appendix

Proof. of Proposition 3. Equation (37) is a differential equation that fits into the following general form of linear differential equations

$$w(\hat{p}) = u(\hat{p})e(\hat{p}, n, W) + \frac{\partial e(\hat{p}, n, W)}{\partial \hat{p}}$$

and its closed form solution is given by

$$e(\hat{p}, n, W) = \exp\left(-\int u(\hat{p})d\hat{p}\right) \left(A + \int w(\hat{p}) \exp\left(\int u(\hat{p}) d\hat{p}\right)d\hat{p}\right)$$

Where the constant A is chosen such that the worst type $p = 0$ participates (Equation (13) is satisfied for all types), and given that $e(p, n, W) \geq 0$ (entrepreneurs do not borrow for consumption against future income and every contract must break even in expectation). The solution is Equation (30).

Now we verify that if this is the schedule of investments required of entrepreneurs, reporting their true type is indeed a maximum of Problem (27). The second order condition with respect to the announcement \hat{p} is given by

$$-\frac{\partial^2 e(\hat{p}, n, W)}{\hat{p}^2} + \gamma p \frac{\partial^2 W^+(\hat{p}, e(\hat{p}, n, W))}{\hat{p}^2}$$

Plugging Equation (30) into (19) gives

$$W^+(\hat{p}, e(\hat{p}, n, W)) = \frac{(1 - \alpha - \beta)(1 + r)\hat{p}^{\frac{\alpha+\beta}{1-\alpha-\beta}}}{[1 - \gamma(1 + r)(\alpha + \beta)]} + \omega$$

With this, our second order condition under truth telling becomes

$$-\frac{\partial^2 e(p, n, W)}{\hat{p}^2} + \gamma p \frac{\partial^2 W^+(\hat{p}, e(\hat{p}, n, W))}{\hat{p}^2} = -\frac{\gamma(1 + r)(1 - \alpha - \beta)^2 p^{\frac{\alpha+\beta}{1-\alpha-\beta}-1}}{[1 - \gamma(1 + r)(\alpha + \beta)]} < 0 \blacksquare$$

Proof. of Proposition 6. Replacing $e_s(p^*)$ and the law of motion for wealth both under separating and pooling ($W^+(p^*, e_s(p^*))$ and $W^+(\tilde{p}(n, p^*), W)$) in condition (34), allows us to find the type p^* . This condition is

$$\left(\frac{A\alpha^\alpha\beta^\beta}{w^\beta(1+r)r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} \left[\tilde{p}(n, p^*)^{\frac{\alpha+\beta}{1-\alpha-\beta}} - \frac{(1 - \alpha - \beta)}{[1 - (1 + r)\gamma(\alpha + \beta)]} p^{*\frac{\alpha+\beta}{1-\alpha-\beta}}\right] = W \frac{[\tilde{p}(n, p^*) - (1 + r)\gamma p^*]}{(1 - \alpha - \beta)(1 + r)\gamma p^* \tilde{p}(n, p^*)} \quad (40)$$

for all n , where \tilde{p} is defined by (33).

Because the distribution of types $f(p)$ is assumed to be differentiable, from expression (33) we obtain

$$\frac{\partial \tilde{p}(n, p^*)}{\partial p^*} = \frac{f(p^*)p^{*n}}{\int_{p^*}^{\tilde{p}} f(x)x^{n-1}dx} \left(\frac{\tilde{p}(n, p^*)}{p^*} - 1 \right) > 0 \quad \forall p^* \in [0, \tilde{p}] \quad (41)$$

The sign of this derivative is expected, given that p^* is the lower bound of the conditional average of types. From Condition (40) it follows that

$$W = C \frac{p^* \tilde{p}(n, p^*) \left[\tilde{p}(n, p^*)^{\frac{\alpha+\beta}{1-\alpha-\beta}} - \frac{(1-\alpha-\beta)}{[1-(1+r)\gamma(\alpha+\beta)]} p^{*\frac{\alpha+\beta}{1-\alpha-\beta}} \right]}{[\tilde{p}(n, p^*) - (1+r)\gamma p^*]}$$

where C is a constant that depends on parameters and prices. Call [1] the expression between brackets in the numerator. Differentiating the participation constraint with respect to p^* , and simplifying gives

$$\begin{aligned} \frac{dW}{dp^*} &= \frac{\tilde{p}}{[\tilde{p} - (1+r)\gamma p^*]} \left[\tilde{p}[1] - [\tilde{p} - (1+r)\gamma p^*] \frac{\alpha + \beta}{[1 - (1+r)\gamma(\alpha + \beta)]} p^{*\frac{\alpha+\beta}{1-\alpha-\beta}} \right] + \\ &\frac{\partial \tilde{p}}{\partial p^*} \frac{p^*}{[\tilde{p} - (1+r)\gamma p^*]^2} \left[-(1+r)\gamma p^*[1] + [\tilde{p} - (1+r)\gamma p^*] \frac{\alpha + \beta}{(1 - \alpha - \beta)} \tilde{p}^{\frac{\alpha+\beta}{1-\alpha-\beta}} \right] \end{aligned}$$

Where $(1+r)\gamma \leq 1$ by assumption. Now, let [2] and [3] be the first and second expressions between brackets in this derivative. The proof follows by showing that these two expressions are positive for all possible values of p^* . Since $\partial \tilde{p}(n, p^*)/\partial p^*$ is always positive, then $dW/dp^* > 0$ for all values of p^* .

Rearranging terms, [2] becomes

$$\begin{aligned} [2] &= \frac{\tilde{p}^{\frac{1}{1-\alpha-\beta}}}{[1 - (1+r)\gamma(\alpha + \beta)]} \left[[1 - (1+r)\gamma(\alpha + \beta)] - \left(\frac{p^*}{\tilde{p}} \right)^{\frac{\alpha+\beta}{1-\alpha-\beta}} + (1+r)\gamma(\alpha + \beta) \left(\frac{p^*}{\tilde{p}} \right)^{\frac{1}{1-\alpha-\beta}} \right] \\ &= \frac{\tilde{p}^{\frac{1}{1-\alpha-\beta}}}{[1 - (1+r)\gamma(\alpha + \beta)]} \Lambda \left(\frac{p^*}{\tilde{p}} \right) \end{aligned}$$

where $p^*/\tilde{p} \in [0, 1]$. It is easy to show that $\Lambda(0) > 0$, $\Lambda(1) = 0$, and $\Lambda'(p^*/\tilde{p}) < 0 \quad \forall p^*/\tilde{p} \in [0, 1]$. This implies that [2] > 0 . Similarly,

$$\begin{aligned}
[3] &= \frac{\tilde{p}^{\frac{1}{1-\alpha-\beta}}}{(1-\alpha-\beta)} \left[\frac{(1+r)\gamma(1-\alpha-\beta)^2}{[1-(1+r)\gamma(\alpha+\beta)]} \left(\frac{p^*}{\tilde{p}}\right)^{\frac{1}{1-\alpha-\beta}} - (1+r)\gamma\frac{p^*}{\tilde{p}} + (\alpha+\beta) \right] \\
&= \frac{\tilde{p}^{\frac{1}{1-\alpha-\beta}}}{(1-\alpha-\beta)} \Theta\left(\frac{p^*}{\tilde{p}}\right)
\end{aligned}$$

Now, $\Theta(0) = \alpha + \beta > 0$, $\Theta(1) = (\alpha + \beta)[1 - (1+r)\gamma]^2 > 0$, and $\Theta'(x) < 0 \forall p^*/\tilde{p} \in [0, 1]$.

This implies that $[3] > 0$. ■

Proof. of Proposition 10. We prove the first statement of the proposition since the second follows from this proof and the text. The total amount of labor demanded and output produced within a particular cohort of age n and endowment ω , is

$$Y(n, p_\omega^*(n)) = \int_0^{p_\omega^*(n)} py_s(p)f(p)p^{n-1}dp + y_s(\tilde{p}(n, p_\omega^*(n))) \int_{p_\omega^*(n)}^{\bar{p}} pf(p)p^{n-1}dp \quad (42)$$

$$M(n, p_\omega^*(n)) = \int_0^{p_\omega^*(n)} l_s(p)f(p)p^{n-1}dp + l_s(\tilde{p}(n, p_\omega^*(n))) \int_{p_\omega^*(n)}^{\bar{p}} f(p)p^{n-1}dp \quad (43)$$

where py is the average output produced by a type p , and $l_s(p) = C(w)p^{\frac{1}{1-\alpha-\beta}}$ is given by Equation (18). The proof consists in first showing that $Y(n, p^*)$ and $M(n, p^*)$ are increasing in p^* for a given wage rate, and then to show that wealthier economies produce more, employ more labor and capital, etc. Thus,

$$\frac{\partial M(n, p^*)}{\partial p^*} = \frac{\partial l_s(\tilde{p})}{\partial \tilde{p}} \frac{\partial \tilde{p}}{\partial p^*} \int_{p^*}^{\bar{p}} f(p)p^n dp - [l_s(\tilde{p}) - l_s(p^*)] f(p)p^n$$

From Expression (41) we know that

$$\frac{\partial \tilde{p}}{\partial p^*} \int_{p^*}^{\bar{p}} f(p)p^{n-1} dx = f(p^*)p^{*n-1} [\tilde{p} - p^*]$$

Together with $l_s(p) = C(w)p^{\frac{1}{1-\alpha-\beta}}$ we get

$$\begin{aligned}
\frac{\partial M(n, p^*)}{\partial p^*} &= \frac{C(w)\tilde{p}^{\frac{1}{1-\alpha-\beta}} f(p^*)p^{*n-1}}{1-\alpha-\beta} \left[\alpha + \beta - \frac{p^*}{\tilde{p}} + (1-\alpha-\beta) \left(\frac{p^*}{\tilde{p}}\right)^{\frac{1}{1-\alpha-\beta}} \right] \\
&= \frac{C(w)\tilde{p}^{\frac{1}{1-\alpha-\beta}} f(p^*)p^{*n-1}}{1-\alpha-\beta} \Xi\left(\frac{p^*}{\tilde{p}}\right) > 0
\end{aligned}$$

where $p^*/\tilde{p} \in [0, 1]$. See that $\Xi(0) > 0$, $\Xi(1) = 0$ and $\Xi'(p^*/\tilde{p}) < 0$ for all ratios $p^*/\tilde{p} \in [0, 1]$, proving that $\Xi(p^*/\tilde{p}) > 0$ for any possible ratio p^*/\tilde{p} . Similarly, we can prove that, at given prices, $\partial K(n, p^*)/\partial p^* > 0$ and $\partial Y(n, p^*)/\partial p^* > 0$.

Holding wages fixed, if the distribution of initial wealth improves in the first order stochastic sense, the aggregate demand for labor ($M(w) = \int \sum_n M(n, p_\omega^*(n)) dG(\omega)$) will rise. Then, there will be excess demand for labor at the original wage rate. Because the labor supply schedule has no income effect, as shown by Expression (36), wages and employment must increase.

Since $k = \frac{\alpha w}{\beta r_k} l$ for every firm, then $K = \frac{\alpha w}{\beta r_k} M$ must increase even more than M since w rises. From (18) and (9) we see that $wl = \frac{\beta}{(1+r)} py$ for every firm, as expected from the fact that the production function is Cobb Douglas. Hence, $wM = \frac{\beta}{(1+r)} Y$, where Y is the aggregate production of $\int \sum_n Y(n, p_\omega^*(n)) dG(\omega)$. Because both w and M increase if the initial distribution of wealth improves in the first order sense, then aggregate output rises even more than total employment.

Finally we prove that if the initial distribution of wealth improves in the first order stochastic sense, TFP increases. Define the empirical measure of aggregate TFP as

$$A = \frac{Y}{K^\chi M^{1-\chi}}$$

where $1 - \chi$ is the empirical measure of labor income share. Since $K = \frac{\alpha w}{\beta r_k} M$ and $Y = \frac{(1+r)w}{\beta} M$, we obtain

$$A = \frac{(1+r)}{\alpha^\chi} \left(\frac{w}{\beta} \right)^{1-\chi}$$

Since wages increase with larger endowments (in the first order sense), TFP increases with endowments as well. ■

References

- [1] Atkeson, A., Khan, A. and Ohanian, L. “Are Data on Industry Evolution and Gross Job Turnover relevant for Macroeconomics?” *Carnegie-Rochester Conference Series on Public Policy*, 44(1), pp. 216-250, 1996.
- [2] Albuquerque, R. and Hugo Hopenhayn. “Optimal Lending Contracts and Firm Dynamics.” *Review of Economics Studies*, 71 (2), pp. 285-315. April 2004.
- [3] Basu, S. and J. G. Fernald. “Returns to Scale in U.S. Production: Estimates and Implications.” *JPE*, 1997, 105(2), pp. 249-283.
- [4] Bernanke, B. and M. Gertler: “Financial Fragility and Economic Performance,” *QJE*, February 1990.
- [5] Bernanke, B., M. Gertler and S. Gilchrist: “The Financial Accelerator in a Quantitative Business Cycle Framework”, NBER working paper 6455, 1998.
- [6] Cabral, Luis M. B. and Jose Mata. “On the Evolution of the Firm Size Distribution: Facts and Theory.” *American Economic Review*, 93 (4), September 2003.
- [7] Cagetti, Marco and Cristina De Nardi. “Entrepreneurship, Frictions, and Wealth”, Minneapolis Fed, Research Department Staff Report 322.
- [8] Carranza, Luis, Jesus Fernandez-Villaverde and Jose E. Galdon Sanchez. “Entrepreneurship, Financial Intermediation and Aggregate Activity.” Working paper, University of Pennsylvania, 2003.
- [9] Clementi, G. L. and Hugo Hopenhayn: “A theory of Financing Constraints and Firm Dynamics”, Mimeo, May 2002.
- [10] Cooley, T.F, R. Marimon, and V. Quadrini: “Aggregate Consequences of limited contract enforceability”, December 2003, forthcoming *JPE*.
- [11] Cooley, T.F. and V. Quadrini: “Financial Markets and Firm Dynamics”, *AER* 91, December 2001.

- [12] Davis, S. J., J. C. Haltiwanger and S. Schuh: Job Creation and Destruction. MIT Press, 1996.
- [13] De Meza, D. and D. Webb: "Risk, Adverse selection and Capital Market Failure," The Economic Journal 100, March 1990.
- [14] Diamond, Douglas: "Reputation Acquisition in Debt Markets," JPE 97(4), 1989.
- [15] Dunne, Timothy, Mark Roberts, and Larry Samuelson. "The Growth and Failure of U.S. Manufacturing Plants." Quarterly Journal of Economics, 104, pp. 671-98, 1989.
- [16] Dunne, Timothy, Mark Roberts, and Larry Samuelson. "Plant Turnover and Gross Employment Flows in the U.S. Manufacturing Plants." Journal of Labor Economics, 7, pp. 48-71, 1989.
- [17] Engers, Maxim and Luis Fernandez: "On the Existence and Uniqueness of Signalling Equilibria", Econometrica 55, 1987.
- [18] Evans, David S.: "Tests of Alternative Theories of Firm Growth," JPE 95(4), 1987.
- [19] Evans, David S. and Boyan Jovanovic: "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints." Journal of Political Economy 97(4), 1989.
- [20] Fazzari, S. M, R. G. Hubbard and Bruce C. Petersen: "Financing Constraint and Corporate Investment", Brookings Papers on Economic Activity 1, 1988.
- [21] Gertler, M. and S, Gilchrist: "Monetary Policy, Business Cycles, and the Behavior of small manufacturing Firms." QJE CIX, May 1994.
- [22] Gilchrist, Simon and Himmelberg, Charles P. "Evidence on the Role of Cash Flows for Investment." Journal of Monetary Economics, 36 (3), pp. 541-72.
- [23] Gomes, Joao F. "Financing Investment." American Economic Review, 91 (5), pp.1263-1285, 2001.

- [24] Hall, Bronwyn H. “The Relationship Between Firm Size and Firm Growth in the U.S. Manufacturing Sector.” *Journal of Industrial Economics*, 35(4), pp. 583-606, June 1987.
- [25] Hopenhayn, Hugo: “Entry, Exit, and Firm Dynamics in Long Run Equilibrium”, *Econometrica* 60 (5), September 1992.
- [26] Hopenhayn, Hugo and Richard Rogerson. “Job Turnover and Policy Evaluation: A General Equilibrium Analysis.” *Journal of Political Economy*, 101 (5), 1993.
- [27] Jovanovic, Boyan: “Selection and the Evolution of Industry,” *Econometrica* 50 (3), May 1982.
- [28] Modigliani F.F. and M.H. Miller: “The cost of capital corporation finance, and the theory of investment”, *AER* 48, 1958.
- [29] Petersen, M. A. and Raghuram G. Rajan: “The Benefits of Lending Relationships: Evidence from the Small Business Data”, *The Journal of Finance*, March 1994.
- [30] Quadrini, Vincenzo. “Investment and Liquidation in Renegotiation-proof Contracts with Moral Hazard”, *Journal of Monetary Economics*, 51, pp. 713-751, 2004.
- [31] Restuccia, Diego and Richard Rogerson. “Policy Distortions and Aggregate Productivity with Heterogeneous Plants”, Working Paper, July 2003.
- [32] Van Ark, Bart and Erik Monnikhof. “Size Distribution of Output and Employment: A Data Set for Manufacturing Industries in Five OECD countries, 1960s-1990.” *Organization of Economic Co-Operation and Development*, Working Paper No. 166, 1996.