GROWTH EXPECTATIONS AND BUSINESS CYCLES

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ABSTRACT. We examine the role played by rational expectations about future productivity in explaining economic fluctuations within standard business cycle models. We show that, contrary to what one might intuitively expect, within standard RBC models a revision of agents' rational growth expectations does not induce business cycle-like fluctuations, that is positive co-movements of key macroeconomic variables. Moreover, optimism about future growth leads to a reduction of current real activity. We point out the general mechanism that can be incorporated into RBC models in order to enable them to exhibit business cycles induced by rational growth expectations (E-RBCs). In this paper the focus is on slack in the labor market as one incarnation of the general mechanism. In particular, we show that standard labor market matching models can exhibit E-RBCs.

Key Words: Growth expectations, Real Business Cycle, Labor market matching

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1. Introduction

Economists have long recognized the importance of expectations in explaining economic fluctuations. As early as 1927, Pigou postulated that "the varying expectations of business men [...] constitute the immediate cause and direct causes or antecedents of industrial fluctuations." A recent episode where many academic and non-academic observers attribute a key role to expectations is the economic expansion of the 1990s. During the 1990s, economic agents observed an increase in current productivity levels but also became more and more optimistic regarding future growth rates of productivity. In fact, there was a strong sense of moving towards a new era—the "new economy"—of higher average productivity growth rates for the foreseeable future. With the benefit of hindsight it is easy to characterize the optimism about future growth rates as "unrealistic" but it should not be forgotten that at the time the observed increases in current-period productivity were in fact remarkable and that the view that the economy had entered a new era was shared by many experts, including economic policy makers such as Alan Greenspan.[1] Many commentators perceive those high expectations about future growth rates to have at least magnified, if not caused, the economic expansion of the 1990s. The Economist, to take an example, writes: "Firms overborrowed and overinvested on unrealistic expectations about future profits and the belief that the business cycle was dead. [...] The boom became self-reinforcing as rising profit expectations pushed up share prices, which increased investment and consumer spending. Higher investment and a strong dollar helped to hold down inflation and hence interest rates,

[1] In a speech on April 7, 2000: "[...] there can be little doubt that not only has productivity growth picked up from its rather tepid pace during the preceding quarter-century but that the growth rate has continued to rise, with scant evidence that it is about to crest. In sum, indications [...] support a distinct possibility that total productivity growth rates will remain high or even increase further."

fuelling faster growth and higher share prices. That virtuous circle has now turned vicious."[2]

The example of the 1990s makes us wonder, what role expectations by themselves play in explaining economic fluctuations. That is, what is the effect of a change in expectations about future productivity at a given level of current productivity? Based on "anecdotal" evidence from episodes such as the boom of the late 1990s, but also at an intuitive level, we would maybe conjecture that rising optimism should have a positive impact on current economic conditions. And in fact, Beaudry and Portier (2004b) provide us with more formal empirical evidence that expectations are indeed important in explaining post-war business cycle fluctuations:

> "Hence, our empirical results suggests that an important fraction of business cycle fluctuations may be driven by changes in expectations."

Virtually all modern business cycle models are dynamic and stochastic. Moreover, since agents are forward-looking, current-period decisions are affected by agents' expectations about the future. For example, in real business cycle (RBC) models, expectations about future productivity and the resulting expected consequences for future decisions affect both the current consumption and investment decision as well as current labor supply and demand. However, productivity is typically modelled as a simple AR process, implying that changes in the expected values of future productivity levels and changes in the current-period level are perfectly (positively) correlated.[3] As a result, changes in current-period decisions induced by changes in current productivity capture both a response to the different level of

[2] The Economist. "The Unfinished Recession." September 26, 2002.

[3] Most popular are AR(1) processes with an autoregressive coefficient either close to unity or equal to unity.

current productivity as well as a response to different expected future productivity levels. However, to understand the role expectations play within those models, it becomes crucial to disentangle those two responses, and to analyze the effect of a change in expected future productivity levels on current decisions for a fixed level of current productivity.

An important, and to the best of our knowledge only, theoretical contribution is the startling result in another paper by Beaudry and Portier (2004a) where they show that in a wide class of business cycle models changes in growth expectations cannot generate business cycle-like fluctuations. To obtain this result, Beaudry and Portier rely on a definition whereby a change in expectations is considered as generating a business cycle if and only if immediately after the revision of expectations consumption and investment move in the same direction. For this definition it is actually quite intuitive that in the standard neoclassical growth model expectations cannot generate business cycles. To see this, note that in the neoclassical growth model output depends on current-period productivity and a capital stock that is predetermined. It follows that a change in expectations without a change in current productivity leaves the aggregate amount of available resources unchanged. Thus, either consumption or investment could increase, but directly following an upward revision of expectations, consumption and investment cannot both increase at the same time.[4] The authors show that several extensions such as flexible labor or variable capacity utilization

[4]Note that the intuition given here does not depend on the change in expectations being a change in expectations about future productivity levels. The same negative result holds if productivity levels are constant and the model is allowed to have increasing returns to scale so that a sunspot solution exists. Again, the fact that aggregate resources are fixed means that consumption and investment cannot both move in the same direction following a shock to the sunspot variable.

do not affect this result.[5] In this paper we show that for standard parameter values the result still holds even if we employ a broader definition of a business cycle by taking into account the behavior of consumption and investment for several periods after the change in expectations has occurred.

We consider as problematic the finding that standard RBC models predict that in response to an increase in the expected values of future productivity levels (keeping current productivity fixed) consumption increases but investment and total hours worked decline. This means that optimism about the future actually leads to a *reduction* of current real activity, thereby standing in sharp contrast to both the view discussed in the beginning of this introduction that optimism about the future was an important driver of the boom of the 1990s as well as to the more formal empirical evidence provided by Beaudry and Portier (2004b). This paper therefore develops a model in which increased optimism about future productivity growth not only increases current real activity (keeping current productivity levels fixed) but in which the increase in real activity is a standard business cycle boom in that output, consumption, investment, and employment all move in the same direction. Agents are fully rational and productivity is an exogenous process. As a consequence, even though increases in expected values of future productivity levels may very well not materialize, on average they do. We will refer to business cycles induced by changes in expected future productivity levels, keeping current productivity fixed, as E-RBCs.[6]

As discussed above, what lies at the heart of the inability of standard RBC models to

[5]In standard business cycle models, leisure is a normal commodity. Increased optimism about the future induces agents to want more of all commodities, including leisure. Agents thus reduce the amount of hours worked as a response to higher growth expectations.

[6]We provide a definition at the beginning of section 2.

exhibit business cycle fluctuations whenever a change in growth expectations occurs is the fact that the amount of aggregate resources remains unchanged. This suggests that models in which expectations do induce business cycle fluctuations must allow for "slack" or "idle resources" in the aggregate economy, so that the economy is not at its full capacity when the change in growth expectations occurs. If the amount of slack can be reduced by an increase in growth expectations, then consumption and investment can increase at the same time.

In this paper we focus on one of the many dimensions along which we could incorporate aggregate idle resources into our model economies: We draw upon slack in the labor market as a mechanism to relax the aggregate budget constraint. In particular, we show that in standard labor market matching models, changes in expectations about future productivity can cause business cycle fluctuations even if they are not accompanied by changes in the current level of productivity. The intuition for this result is fairly straightforward: In these models, slack is represented by the pool of unemployed. Because the amount of vacancies firms post and, therefore, the amount of hiring that takes place in any given period depends on firms' expectations about future profits, it follows that whenever profit expectations are suddenly revised upwards, more vacancies are posted, more jobs are generated, and employment rises. This will cause production to increase even though current productivity has remained unchanged. Moreover, both consumption and investment go up so that the increase in expectations results in a standard business cycle.

The remaining sections of the paper are structured as follows: In Section 2 we discuss the difficulties standard RBC models experience in generating business cycles induced by growth expectations. In Section 3 we develop the model and in Section 4 we document that it does constitute an E-RBC model. Section 5 concludes.

2. Business Cycles Induced by Rational Growth Expectations: E-RBCs

We define a business cycle induced by rational growth expectations (E-RBC) as a positive co-movement in key macroeconomic variables, namely output, consumption, investment, and employment, induced by a change of agents' rational expectations regarding future productivity levels, that is, without an alteration in current productivity levels.

2.1. A Framework for Changes in Rational Expectations about Future Economic Growth. In this section we construct a stochastic process for productivity that allows us to analyze the effect of changes in rational expectations about future productivity levels while keeping current-period productivity fixed.

To that end, we assume that in each period t, the economy can be in one of three possible regimes, with regime-specific values for the level of productivity $\theta_t \in \left\{\overline{\theta}^1, \overline{\theta}^2, \overline{\theta}^3\right\}$, where $\overline{\theta}^1$ is productivity in regime 1, $\overline{\theta}^2$ is productivity in regime 2, etc. The law of motion for the productivity process is modelled as a first-order Markov process with the transition matrix $\Omega = (\Omega_{ij})$, where Ω_{ij} denotes the probability of a switch to regime j conditional on being in regime i.

In order to model a change in rational expectations about future productivity, we set:

$$\overline{\theta}^1 = \overline{\theta}^2 < \overline{\theta}^3, \tag{1}$$

and

$$\Omega_{12} > 0, \Omega_{13} = 0, \Omega_{23} > 0. \tag{2}$$

Let us assume that a switch from regime 1 to regime 2 occurs. We note that productivity has not changed, since $\overline{\theta}^1 = \overline{\theta}^2$, while the transition probabilities have changed. As a result,[7]

$$E_t\left[\theta_{t+j} \mid \theta_t = \overline{\theta}^2\right] > E_t\left[\theta_{t+j} \mid \theta_t = \overline{\theta}^1\right]. \tag{3}$$

Throughout the paper we will work with the following parameterization of the productivity process:[8]

$$\theta_t = \{1.000, 1.000, 1.005\}, \tag{4}$$

$$\Omega = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.20 & 0.00 & 0.80 \end{pmatrix}. \tag{5}$$

To operationalize a "positive co-movement in key macroeconomic variables" from our definition of E-RBCs, we simply check if the correlations conditional on being in regime 2 of the key macroeconomic variables output, consumption, investment, and employment are jointly pairwise positive as a measure of the ability of a given model to exhibit E-RBCs.

We would like to highlight that the process as laid out above is not calibrated to any dimensions of empirical productivity data and is, obviously, stationary. The process as it stands serves merely as an analytical tool to simulate a rational change in productivity growth expectations. We are currently working on a more realistic process that disentangles shocks to the current level of productivity from shocks to the growth rate of productivity. However, a regime switching process with three regimes as laid out above is probably the simplest starting point to separate changes in the current level of productivity from changes in expected future levels of productivity without actually incorporating productivity growth

[7] As can be seen in figure 1, $E_t\left[\theta_{t+j} \mid \theta_t = \overline{\theta}^2\right] > E_t\left[\theta_{t+j} \mid \theta_t = \overline{\theta}^1\right]$ holds for appr. $1 \leq j \leq 18$.

[8] Under this parameterization, the average time the economy remains in any one regime before experiencing a switch is $\frac{1}{1-0.80} = 5$ periods (quarters).

in the model. For now, the regime switching process is sufficient to establish whether a given RBC model constitutes an E-RBC model. The quantitative assessment of E-RBCs within E-RBC models is more difficult, as we would need a calibrated productivity-driving process for that purpose. We therefore only offer preliminary conclusions regarding the quantitative impact of E-RBCs and postpone a more thorough quantitative assessment to a later version of our work.

2.2. Standard Business Cycle models and E-RBCs. In this section we analyze the role expectations play in standard RBC models by examining if those models can exhibit E-RBCs. First we discuss Beaudry and Portier (2004a) who provide some formal results regarding the ability of a wide class of business cycle models to generate E-RBCs. Then we go on to investigate the role of E-RBCs in standard RBC models with both fixed and flexible labor, and with variable capacity utilization.

Beaudry and Portier (2004a). Beaudry and Portier (2004a), henceforth BP, formally show that for a fairly large class of business cycle models it is not possible to generate "expectations driven business cycles". They conclude: "[...] most commonly used macro models restrict the production possibility set in a manner that precisely rules out the possibility of expectations driven business cycles in the presence of market clearing. The main technological features we identify as being necessary for expectations driven business cycles is that of a multi-sector setting where firms experience economies of scope."

In order to be able to provide analytical proof of the impossibility of expectations driven business cycles within standard business cycle models, BP operationalize their definition of E-RBCs in the following way: By examining the signs of the partial derivatives of key macroeconomic variables with respect to each other, BP focus exclusively on the instantaneous and marginal co-movement of those variables.[9] In other words, the BP proof is based on a definition of E-RBCs that considers only the immediate impact of very small shocks on output, consumption, investment, and employment. For example, a case where as an instantaneous response to a change in expectations $\Delta C = -\varepsilon < 0$ and $\Delta I > 0$, and immediately after that both $\Delta C > 0$ and $\Delta I > 0$ would not constitute an E-RBC according to BP. However, in the next section and in section 4.3 we show that cases like the one just described are indeed possible. Our approach, which operationalizes our broad definition of E-RBCs by examining the signs of the correlations of key macroeconomic variables conditional on being in the regime where growth expectations are high, avoids this shortcoming, as it takes into account the behavior of output, consumption, investment and employment several periods after the change in expectations.

For the BP approach to E-RBCs, the inability of standard RBC models to exhibit E-RBCs is fairly intuitive: If expectations change without a change in current economic conditions, agents are faced with an unchanged current budget constraint. As a result, agents can either increase consumption by decreasing their savings and thus the capital stock, or increase their savings, thus increasing investment and the capital stock, by means of reducing their current consumption. It follows that directly after a revision of growth expectations, consumption and investment can not move in the same direction, rendering E-RBCs in the first period impossible.

[9] Beaudry and Portier (2004a), Lemma 1: "Expectations driven business cycles can arise in a Walrasian equilibrium only if [...] $\frac{\partial C}{\partial I} > 0$ and $\frac{\partial L}{\partial I} > 0$."

Standard RBC Models with Fixed Labor. Consider a standard economy in which aggregate consumption is chosen with the aim to maximize the expected utility of a representative agent:

$$\max_{\{c_{t+j}\}_{j=0}^{\infty}} E_t \left[\sum_{j=0}^{\infty} \beta^j u(c_{t+j}) \right] \qquad u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}. \tag{6}$$

Subject to the constraint:

$$k_{t+1} = f(k_t) + (1-\delta)k_t - c_t, \tag{7}$$

$$f(k_t) = \theta_t k_t^{\alpha}, \tag{8}$$

where θ_t is driven by the regime switching process as laid out in section 2.1. The first order condition and budget constraint follow as:

$$c_t^{-\gamma} = \beta E_t \left[c_{t+1}^{-\gamma} \left(\alpha \theta_{t+1} k_{t+1}^{\alpha-1} + (1-\delta) \right) \right], \tag{9}$$

$$c_t = \theta_t k_t^{\alpha} + (1-\delta)k_t - k_{t+1}. \tag{10}$$

When expectations about future productivity increase (increase in $E_t \left[\theta_{t+1} \mid \theta_t \right]$), there are two well-known basic effects:

1. Substitution effect: Increase in the expected marginal productivity of capital ($E_t \left[\alpha \theta_{t+1} k_{t+1}^{\alpha-1} \right]$). This increase in expected productivity renders saving (investment) more attractive, as the expected return on invested capital is now higher.

2. Income effect: Due to the higher expected productivity of the existing capital stock, agents expect their consumption to increase in the future, resulting in a lower expected marginal utility of consumption (decrease in $E_t \left[c_{t+1}^{-\gamma} \right]$). This motivates agents to smooth consumption over time by increasing current consumption.

In the remainder of this section we investigate the effects of a rational change in expectations about future productivity, i.e. the effects of a switch from regime 1 to regime 2 of our driving process under different parameterizations of the standard RBC model.

Parameterization 1: The Brock-Mirman Model.

$$\alpha = 0.33, \qquad \gamma = 1.00,$$
$$\beta = 0.99, \qquad \delta = 1.00.$$

The Brock-Mirman parameterization of the standard RBC model has the following well-known analytical solution:

$$c_t = (1-\alpha\beta)\theta_t k_t^{\alpha}, \tag{11}$$
$$k_{t+1} = \alpha\beta\theta_t k_t^{\alpha}. \tag{12}$$

The Brock-Mirman policy functions thus depend only on current productivity, not on expectations about future productivity levels. Note that this result holds for any specification of θ_t. It follows trivially that there are no E-RBCs in this model. To put it differently, a switch from regime 1 to regime 2 has no effect on agents' consumption-savings decision, precisely because a switch from regime 1 to regime 2 has no effect on current productivity (θ_t).

Note that the fact that the expectations operator "cancels out" of the policy functions of the Brock-Mirman parameterization lies at the heart of the reason why we can solve this model in closed form. In the Brock-Mirman model, higher expectations about future productivity ($E(\theta_{t+1})$) raise the expected productivity of capital (substitution effect) by the same amount as they lower the expected marginal utility of consumption (income effect). Substitution effect and income effect thus cancel out, and the change in expectations about future productivity has no effect on the current consumption-savings decision.

Parameterization 2: Risk Neutrality - Only Substitution Effect, no Income Effect.

$$\alpha = 0.33, \qquad \gamma = 0.00,$$

$$\beta = 0.99, \qquad \delta = 0.025.$$

Risk neutral agents are indifferent with respect to the lower expected marginal utility of consumption (income effect) induced by higher expectations about future productivity ($E(\theta_{t+1})$). Instead, their focus lies entirely on the higher expected productivity of capital (substitution effect). With $\gamma = 0$, the first order condition becomes:

$$1 = \beta E_t \left[\alpha \theta_{t+1} k_{t+1}^{\alpha-1} + (1-\delta) \right]. \tag{13}$$

Thus:

$$k_{t+1} = \left[\frac{1-\beta(1-\delta)}{\alpha \beta E_t(\theta_{t+1})} \right]^{\frac{1}{\alpha-1}}. \tag{14}$$

Consumption is derived from the budget constraint:

$$c_t = \theta_t k_t^{\alpha} + (1-\delta)k_t - k_{t+1}. \tag{15}$$

It follows that the agents will react to higher expected productivity ($E_t(\theta_{t+1})$) with an increase in current savings (investment) at the expense of current consumption. Consequently, E-RBCs are impossible in the first period. However, the higher capital stock allows the agents to increase consumption again in subsequent periods. In figure 2 we show the effect of a switch from regime 1 to regime 2 on key macroeconomic variables. And indeed, in terms of steady state values, both consumption and capital (and thus output) increase relative to their steady state values before the change in expectations. We can show this result analytically from (14) and (15). $E_t(\theta_{t+1})$ is in regime 1 and regime 2 respectively:[10]

$$E\left[\theta_{t+1} \mid \theta_t = \overline{\theta}^1\right] = 0.2 \times 1.0 + 0.8 \times 1.0 = 1.0, \tag{16}$$

$$E\left[\theta_{t+1} \mid \theta_t = \overline{\theta}^2\right] = 0.9 \times 1.0 + 0.1 \times 1.005 = 1.0005. \tag{17}$$

Steady state values for capital stock and consumption in regime 1 and regime 2 follow:

$$k^{R1} = \left[\frac{1-\beta(1-\delta)}{\alpha\beta E\left[\theta_{t+1} \mid \theta_t = \overline{\theta}^1\right]}\right]^{\frac{1}{\alpha-1}} \approx 28.3484, \tag{18}$$

$$k^{R2} = \left[\frac{1-\beta(1-\delta)}{\alpha\beta E\left[\theta_{t+1} \mid \theta_t = \overline{\theta}^2\right]}\right]^{\frac{1}{\alpha-1}} \approx 28.3696, \tag{19}$$

$$c^{R1} = \overline{\theta}^1 \left(k^{R1}\right)^{\alpha} - \delta k^{R1} \approx 2.3066, \tag{20}$$

$$c^{R2} = \overline{\theta}^2 \left(k^{R2}\right)^{\alpha} - \delta k^{R2} \approx 2.3068. \tag{21}$$

In spite of the initial negative co-movement of investment and consumption, in the second period after a switch from regime 1 to regime 2 both consumption and the capital stock (and thus investment and output) are eventually higher in regime 2 compared to regime 1 ($k^{R2} > k^{R1}$ and $c^{R2} > c^{R1}$).[11] We compute the conditional correlations (conditional on being in regime 2) of the key macroeconomic variables consumption (c), investment (i), and output (y):

	c	i	y
c	1.00	-1.00	1.00
i	-1.00	1.00	-1.00
y	1.00	-1.00	1.00

[10] For parameterization of $\theta_t = \left\{\overline{\theta}^1, \overline{\theta}^2, \overline{\theta}^3\right\}$ and $\Omega = (\Omega_{ij})$, see section 2.1.

[11] Note that from $E\left[\theta_{t+1} \mid \theta_t = \overline{\theta}^2\right] > E\left[\theta_{t+1} \mid \theta_t = \overline{\theta}^1\right]$, $k^{R2} > k^{R1}$ follows in general. $c^{R2} > c^{R1}$ depends on the parameter choice for α and δ.

Thus, both according to Beaudry and Portier (2004a) as well as according to our definition, the fluctuations in the macroeconomic variables induced by a change in growth expectations are not considered E-RBCs (because the conditional correlations are not jointly pairwise positive). BP rule E-RBCs out because of the very fact of the initial negative co-movement, we do so due to the magnitude of the initial negative co-movement, as reflected in the perfect negative correlation between both consumption and investment, and output and investment.

Parameterization 3: Log-Utility - Both Substitution Effect and Income Effect.

$$\alpha = 0.33, \qquad \gamma = 1.00,$$
$$\beta = 0.99, \qquad \delta = 0.025.$$

Now we use standard values in the literature for RBC models with quarterly periods and log utility. We solve the model numerically and show the effect of a change in expectations about future productivity (switch from regime 1 to regime 2) on key macroeconomic variables in figure 3. Clearly, consumption and investment move in opposite directions after a switch from regime 1 to regime 2. Our observation is confirmed when we compute the conditional correlations:

	c	i	y
c	1.00	-1.00	1.00
i	-1.00	1.00	-1.00
y	1.00	-1.00	1.00

The basic RBC model therefore does not constitute an E-RBC model, applying either the Beaudry and Portier (2004a) definition or our definition. The intuition for this result is as follows: With log-utility, the income effect outweighs the substitution effect in the standard RBC model, and agents immediately consume more and save less in the period when the upward revision of expectations regarding future productivity occurs. This will lead to a decrease in the capital stock and therefore a tighter budget constraint in the subsequent period. It follows that the agents in the second period, endowed with even less resources than in the first period, but with the same expectations about future productivity, are again not able to increase both the capital stock and consumption relative to the first period, and thus also not relative to the situation before the change in expectations has occurred. E-RBCs are therefore not possible in standard RBC models whenever the income effect is stronger than the substitution effect. When is this the case? We have already demonstrated that for risk neutral agents ($\gamma = 0$) the substitution effect outweighs the income effect. In fact, it turns out that in the standard RBC model the substitution effect dominates the income effect only for relatively low coefficients of risk aversion (approximately $\gamma < 0.35$), potentially resulting in E-RBCs. We present results for the case of $\gamma = 0.20$ below.

Parameterization 4: Low CRRA - The Substitution Effect outweighs the Income Effect.

$$\alpha = 0.33, \qquad \gamma = 0.20,$$

$$\beta = 0.99, \qquad \delta = 0.025.$$

We demonstrate the effect of a change in growth expectations in figure 4. Conditional correlations are:

	c	i	y
c	1.00	-1.00	1.00
i	-1.00	1.00	-1.00
y	1.00	-1.00	1.00

As discussed, the substitution effect outweighs the income effect, and agents increase investment at the expense of consumption as an immediate response to higher expectations about future productivity. However, as can be seen in the graphs, it takes the economy a very long time to reach a situation where both consumption and output experience a net increase.[12] Our measure of conditional correlations reflects the strong initial negative co-movement of consumption and investment and the subsequent slow adjustment process by a negative correlation between consumption and output with investment.

Standard RBC Models with Flexible Labor. Intuitively, one might think that making the labor-leisure decision flexible would allow standard RBC models to exhibit E-RBCs, because agents could increase their labor supply as an immediate response to a positive revision of future productivity growth expectations, that way increasing output in the current period and relaxing the budget constraint. However, as also pointed out by Beaudry and Portier (2004a), if leisure is a normal good, just like consumption, agents will want to enjoy more leisure whenever permanent income increases, that is whenever they expect economic conditions to improve in the future. When agents expect future productivity to increase, they will thus not generate additional resources in the current period by increasing their supply of labor. In fact, we would conjecture the opposite to be true.

To demonstrate, we consider the standard Hansen (1985) model. Aggregate consumption (c_t) and leisure (l_t) are chosen with the aim to maximize the expected utility of a

[12] In our case with $\gamma = 0.20$ it takes the economy about 20 quarters to reach a situation where consumption is higher relative to its level before a switch from regime 1 to regime 2.

representative agent:[13]

$$\max_{\{c_{t+j},l_{t+j}\}_{j=0}^{\infty}} E_t\left[\sum_{j=0}^{\infty}\beta^j u(c_{t+j},l_{t+j})\right] \qquad u(c_t,l_t)=\log(c_t)+Bl_t. \tag{22}$$

Subject to the budget constraint:

$$k_{t+1} = f(k_t)+(1-\delta)k_t - c_t, \tag{23}$$

$$f(k_t) = \theta_t k_t^{\alpha} h_t^{1-\alpha}, \tag{24}$$

$$h_t = 1-l_t. \tag{25}$$

We use the same value for B as in Hansen (1985):

$$\alpha = 0.33, \qquad B = 2.85,$$

$$\beta = 0.99, \qquad \delta = 0.025.$$

We solve the model numerically and show the effect of a change in expectations about future productivity (switch from regime 1 to regime 2) on key macroeconomic variables in figure 5. Conditional correlations are:

	c	i	y	h
c	1.00	-1.00	-1.00	-1.00
i	-1.00	1.00	1.00	1.00
y	-1.00	1.00	1.00	1.00
h	-1.00	1.00	1.00	1.00

We conclude that making the labor-leisure decision flexible does not support standard RBC models in their ability to exhibit E-RBCs.

[13]Hansen (1985) derives the utility function for the representative agent $u(c_t,l_t)=\log(c_t)+Bl_t$ (which is linear in leisure) from the utility function of the individual households $u(c_t,l_t)=\log(c_t)+A\log(l_t)$ by introducing an "employment lottery" and complete unemployment insurance, where households provide labour services h_0 with probability α_t. Hansen shows $B=\frac{-A\log(1-h_0)}{h_0}$.

Standard RBC Models with Variable Capacity Utilization. [To come]

3. The Model

3.1. Employment Relationships. Every employment relationship consists of one worker and one firm. Relationships are productive through discrete time until they are severed exogenously at the beginning of period, where ρ^x denotes the probability of an exogenous break-up of the employment relationship. We abstract from endogenous destruction decisions. We also abstract from the labor force participation decision of workers, and assume instead that workers are either productive within an active employment relationship or are part of the unemployment pool, searching for new employment. Those abstractions make a thorough understanding of the model possible, while we can safely assume that they do not have a fundamental impact on the intuition of any of our results. We conjecture that endogenous destruction would quantitatively enforce our results, and are therefore planning to incorporate endogenous destruction into a later version of our model.

At the beginning of each period t, firms observe current-period productivity θ_t. Matched with a worker, a firm rents capital k_t at the market clearing interest rate r_t and pays a fixed wage $\overline{w}$ to its worker.[14] A firm, matched with a worker, produces output with the technology $\theta_t k_t^\alpha$, where θ_t is aggregate productivity, driven by the stochastic process as described in section 2.1. The firm's profits π_t follow as:

$$\pi_t = \theta_t k_t^\alpha - r_t k_t - \overline{w}. \tag{26}$$

Firms are taken as maximizing profits on behalf of their owners. The amount of capital a

[14] As discussed in section 4.3, Hall (2004) proposes this version of the model with sticky wages.

firm rents, taking the interest rate r_t as given, is thus:

$$\theta_t \alpha k_t^{\alpha-1} - r_t = 0, \tag{27}$$

$$k_t = \left(\frac{r_t}{\alpha\theta_t}\right)^{\frac{1}{\alpha-1}}. \tag{28}$$

3.2. Matching Market. Employment relationships are formed in a matching market. In the economy there is a continuum of employees with unit mass and a continuum of firms with potentially infinite mass. The mass of unmatched workers seeking employment in period t is denoted by U_t, the mass of firms posting vacancies in period t is denoted by V_t. The matching process within a period takes place after observation of aggregate productivity for that period, but before actual production takes place.

After exogenous destruction of employment relationships at the beginning of period t has occurred, but before the matching process in period t takes place, the number of (still) active employment relationships ready to produce is $(1-\rho^x) N_{t-1}$, where N_{t-1} is the number of active and producing employment relationships in period $(t-1)$. The number of unemployed workers who enter the matching market in period t follows as:

$$U_t = 1 - (1-\rho^x) N_{t-1}. \tag{29}$$

Firms who are not in active employment relationships can choose freely whether or not to post a vacancy in a particular period at a fixed cost ψ, thereby entering the matching market in that period.

The number of successful matches in any given period is determined by a standard Cobb-Douglas specification:[15]

$$m_t = \min\left\{\mu\left(U_t^{\nu} V_t^{1-\nu}\right), U_t, V_t\right\}. \tag{30}$$

[15]Note that for small enough values for U_t or V_t, the Cobb-Douglas specification $m_t = \mu\left(U_t^{\nu} V_t^{1-\nu}\right)$ by itself can lead to values for m_t such that $m_t > \min(U_t, V_t)$. As the number of matches must not exceed

The unmatched workers of the current period re-enter next period's matching market together with the workers whose employment relationships are severed at the beginning of next period.

3.3. Household behavior. The capital stock and all firms are owned by a representative household.[16] All workers are members of this household and at the end of the period the household receives all wages earned by the workers. Aggregate consumption is chosen with the aim to maximize the expected utility of the representative household:[17]

$$\max_{\{C_{t+j}\}_{j=0}^{\infty}} E_t\left[\sum_{j=0}^{\infty}\beta^j u(C_{t+j})\right] \qquad u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{31}$$

The aggregate income of the representative household is composed out of the following components: Aggregate labor income ($N_t\overline{w}$), the proceeds from the rental of capital to firms ($r_tK_t = r_tN_tk_t$), aggregate profits ($\Pi_t = N_t\pi_t$), net of the aggregate posting costs of vacancies (ψV_t). The budget constraint of the representative household follows as:

$$K_{t+1} = N_t\overline{w} + r_tK_t + \Pi_t + (1-\delta)K_t - C_t - \psi V_t, \tag{32}$$

where δ denotes the depreciation rate.

K_{t+1} and C_t are determined by maximization of (31) subject to (32), for which the

the pool of either unemployed workers or posted vacancies, we specify $m_t = \min\left\{\mu\left(U_t^{\nu}V_t^{1-\nu}\right), U_t, V_t\right\}$ to rule out those cases, as is standard in the literature. In the numerical solution of our benchmark model $m_t < \min(U_t, V_t)$ always holds.

[16] That is, we implicitly assume perfect risk sharing between the agents in the economy.

[17] We assume that at the beginning of each period the representative household splits up into a continuum of workers with unit mass. After production, at the end of each period, workers pool their income and aggregate into the representative household once again.

following is a sufficient condition:

$$C_t^{-\gamma} = \beta E_t \left[C_{t+1}^{-\gamma} \left(r_{t+1} + (1-\delta) \right) \right]. \tag{33}$$

3.4. Equilibrium. At the beginning of period t, firms observe the productivity of capital, θ_t, and decide how many vacancies, V_t, to post at cost ψ. The expected net present value of all future profits that can be made within an active employment relationship is:

$$G_t = E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} (1-\rho^x)^j \left(\theta_{t+j} k_{t+j}^{\alpha} - \overline{w} - r_{t+j} k_{t+j} \right) \right]. \tag{34}$$

In equilibrium, a firm will post a vacancy at time t only if the expected benefit of posting a vacancy, which is the probability to get matched once a vacancy has been posted, λ_t^f, times the expected value of an active employment relationship conditional on being matched, G_t, are equal to the cost of posting a vacancy, ψ:

$$\psi = \lambda_t^f \times G_t, \tag{35}$$

where the probability to get matched out of the perspective of a firm, λ_t^f, is the number of successful matches in any given period divided by the number of posted vacancies in that period:[18]

$$\lambda_t^f = \frac{m_t}{V_t} = \mu \left(\frac{U_t}{V_t} \right)^{\nu}. \tag{36}$$

From the free entry condition (35) together with (34) and (36) the equilibrium number of vacancies posted, V_t, follows, determining the number of active employment relationships

[18] For the sake of expositional clarity we abstain here from using the full specification for the matching technology ($m_t = \min \left\{ \mu \left(U_t^{\nu} V_t^{1-\nu} \right), U_t, V_t \right\}$). Instead, we show equilibrium for $m_t = \mu \left(U_t^{\nu} V_t^{1-\nu} \right)$, implicitly assuming $m_t > \min(U_t, V_t)$ always to hold, as is the case for our benchmark model.

in period t, N_t:

$$\psi = \mu \left(\frac{U_t}{V_t}\right)^{\nu} \times G_t, \tag{37}$$

$$V_t = U_t \left(\frac{\mu G_t}{\psi}\right)^{\frac{1}{\nu}}, \tag{38}$$

$$N_t = (1-\rho^x)N_{t-1} + \lambda_t^f V_t. \tag{39}$$

In period t, N_t firms enter the capital market to rent capital. The total supply of capital is fixed in period t, K_t, as it is determined by the savings decision of the representative household in period $(t-1)$, *before* observation of the random shock θ_t. The capital market clears when capital demand is equal to the capital supply in period t:

$$N_t k_t = K_t. \tag{40}$$

The market clearing interest rate, r_t, follows from the equilibrium condition (40) together with the single firm's optimal choice of capital (28) as:

$$r_t = \theta_t \alpha \left(\frac{K_t}{N_t}\right)^{\alpha-1}. \tag{41}$$

We assume that θ^i is driven by a first-order Markov process so that the state variables follow as θ^i, N_{-1}, and K.[19] The recursive equilibrium consists of functions $G(\theta^i, N_{-1}, K)$, $N(\theta^i, N_{-1}, K)$, $C(\theta^i, N_{-1}, K)$, $K'(\theta^i, N_{-1}, K)$ such that (26), (29), (32), (33), (34), (35), (36), (39), (40), (41) hold simultaneously.

[19] The current regime of the economy is denoted by θ^i, where $i \in \{1, 2, 3\}$. For details see section 2.1.

4. Results and Interpretation

In this section we present results for our calibration of the labor market matching model. We show that the model exhibits E-RBCs, and we offer a preliminary assessment of the quantitative impact of E-RBCs within the model. We also provide an extensive interpretation of the results, whereby our aim is to "unravel the black box" and provide the reader with a clear understanding of the mechanisms within the model that render E-RBCs possible.

We proceed as follows: In section 4.1 we briefly discuss our calibration. In sections 4.2 and 4.3 we present the numerical results and interpret. In section 4.4 we assess the quantitative impact, and in section 4.5 we check the robustness of our results.

4.1. Calibration. We choose the following calibration of the model:[20]

Parameter	Value	Parameter	Value
α	0.33	μ	0.60
β	0.99	ν	0.70
γ	1.00	ψ	0.10
δ	0.025	ρ^x	0.08
		$\overline{w}$	2.00

Parameters for preferences and production technology, α, β, γ, δ are standard values for quarterly parameterization. We calibrate the parameters for the matching technology, following Blanchard and Diamond (1990), as is standard in the literature, by setting the curvature parameter ν in line with estimates by Blanchard and Diamond (1990), and adjusting μ and ψ so as to match statistics from simulated data to empirical measures of the worker and firm matching probabilities (λ^w and λ^f respectively), which we manage to some

[20] We refer to this particular calibration as our "benchmark model" in the text.

extent:[21]

	U.S. data	Model
λ^w	0.45	0.69
λ^f	0.71	0.44

In setting the quarterly rate of separation ρ^x to 8%, we follow Den Haan et al. (2000) who rely on standard values from Hall (1995) and Davis et al. (1996) to justify a range for ρ^x between 8% and 10%. Finally, fixing the wage level at $\overline{w}$=2.00 corresponds to a steady state value of the labor share of 69%, in line with its widely known empirical value. In section 4.5 we demonstrate robustness of our results to different choices of the parameters ν and ρ^x. We provide an extensive discussion of the role of the fixed wage $\overline{w}$ and its level in sections 4.3 and 4.5.

4.2. Business Cycles Induced by Rational Growth Expectations: E-RBCs. We solve our model numerically and show the effect on key macroeconomic variables of a change in expectations about future productivity (a switch from regime 1 to regime 2) in figure 6. We can clearly see that output, consumption, investment, and employment jointly increase in the first period after an upward revision of growth expectations, and continue to increase. This fact is reflected in the matrix of conditional correlations, our usual measure to evaluate

[21] The Blanchard and Diamond (1990) point estimate is $\nu = 0.80$. As will become clear in section 4.3, the choice of $\nu = 0.70$ leads to a higher sensitivity of employment to expected productivity changes. We will show in section 4.5 that our results are robust to the choice of $\nu = 0.80$.

the success of a model to generate E-RBCs:

	C	I	Y	N
C	1.00	0.98	1.00	1.00
I	0.98	1.00	0.98	0.99
Y	1.00	0.98	1.00	1.00
N	1.00	0.99	1.00	1.00

Contrary to all standard RBC models considered in section 2, the matrix now contains only positive entries, clearly confirming that our model constitutes an E-RBC model. Another observation we make in figure 6 is that a change in growth expectations has immense propagation-effects, that is the expectations "shock" seems to be extremely persistent. We will account for this fact in the next section, where we explain in detail the mechanisms at work within the model that allow for E-RBCs.

4.3. Unraveling the Black Box.

The Effect of a Change in Rational Growth Expectations. The model relies on slack in the labor market to generate the necessary additional resources for the representative household to increase both consumption and investment. The effect of a change in expectations on the labor market, translating into a relaxation of the budget constraint, is depicted in figure 7. We observe that the change in expectations drives the NPV of future profits to the firm (G) up. As a result, the number of vacancies (V) that are posted in equilibrium shoot up, with the immediate consequence of an increase of employment (N) in the current period. Note that the matching market together with the matching friction are crucial for this result: If firms would have the guaranty that posting a vacancy in any given period would result in a successful match with a worker and thus production in that same period, firms would always wait to post a vacancy until they observe actual current productivity to go up, as opposed to increasing the number of vacancies because of the sheer anticipation of future profit opportunities. However, due to the free entry condition combined with the matching technology, the probability of a firm to get matched (λ^f) is smaller than unity, thus inducing firms to increase the number of vacancies for fear of foregone profit opportunities whenever economic prospects improve.

Finally, the higher employment translates into higher aggregate output (Y) in the current period for the following reasons: First of all, employment has gone up, resulting in more firms producing. Second of all, capital is employed more productively as it is "spread more thinly" across more firms, which in the presence of diminishing returns to scale increases output.[22]

The Labor Market. The model relies on the labor market as the creator of additional resources in response to a change in rational expectations. The following equations are the key to understand how higher expected productivity translates into higher employment:

1. The equation for the expected net present value of all future profits that can be made within an active employment relationship:

$$G_t = E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} (1-\rho^x)^j \left(\theta_{t+j} k_{t+j}^{\alpha} - \overline{w} - r_{t+j} k_{t+j} \right) \right]. \tag{42}$$

[22] Our timing assumes that the capital stock K_t is fixed at the beginning of period t, as it is determined by the savings decision of the representative household at the end of period $(t-1)$. This assumption somewhat supports the model to generate E-RBCs already in the first period, as the effect that takes advantage of the curvature of the production function (diminishing returns to scale) is strengthened during the first period. In any case, our definition of E-RBCs does not rely on first-period effects.

2. The free entry condition:

$$\psi = \lambda_t^f \times G_t. \tag{43}$$

3. The law of motion for employment:

$$N_t = (1-\rho^x)N_{t-1} + \lambda_t^f V_t. \tag{44}$$

The amount of movement in the labor market induced by a change in expected productivity ($E_t[\theta_{t+j}]$) critically depends on the sensitivity of profits G_t to changes in $E_t[\theta_{t+j}]$. As can be seen from equation (42), there is a direct positive effect of $E_t[\theta_{t+j}]$ on expected profits via the production technology $E_t\left[\theta_{t+j}k_{t+j}^{\alpha}\right]$. There are two indirect negative effects, firstly via the interest rate $E_t[r_{t+j}]$, which is expected to go up due to an expected increase in the number of firms competing to rent capital, and secondly via the relative marginal utility of consumption $E_t\left[\frac{u'(C_{t+j})}{u'(C_t)}\right]$ which is expected to decrease due to higher expected future levels of consumption.

The free entry condition (43) implies that λ_t^f changes by the same relative amount as G_t (of course in the opposite direction). From (43) and (44) together with

$$V_t = U_t \mu^{\frac{1}{\nu}} \left(\frac{G_t}{\psi}\right)^{\frac{1}{\nu}}, \tag{45}$$

(see equation (38)) we can express N_t as a function of G_t:

$$\begin{aligned} N_t &= (1-\rho^x)N_{t-1} + \frac{\psi}{G_t} \times U_t \mu^{\frac{1}{\nu}} \left(\frac{G_t}{\psi}\right)^{\frac{1}{\nu}}, && (46) \\ &= (1-\rho^x)N_{t-1} + U_t \mu^{\frac{1}{\nu}} \left(\frac{G_t}{\psi}\right)^{\frac{1-\nu}{\nu}}. && (47) \end{aligned}$$

From the law of motion for employment as a function of G we recognize that N_t depends on $E_t[\theta_{t+j}]$ only through G_t. How strongly N_t responds to changes in $E_t[\theta_{t+j}]$ thus depends first of all on the sensitivity of G_t to $E_t[\theta_{t+j}]$, and second of all on the curvature parameter ν of the matching technology: The lower the value of ν, the more sensitive the response of N_t to changes in G_t and thus to $E_t[\theta_{t+j}]$.[23]

As discussed, our results rely on sufficient movement in the labor market induced by a change in expectations in order to generate the necessary additional resources for E-RBCs. However, the standard Diamond (1982), Mortensen (1982), and Pissarides (1986) (DMP) setting with Nash-bargaining has a well-known and crucial shortcoming, namely too little fluctuation in posted vacancies. In this version of our paper we therefore follow Hall (2004) in using a sticky wage rule to generate sufficient sensitivity of G_t to changes in $E_t[\theta_{t+j}]$. Sufficient movement in N_t as a response to changes in $E_t[\theta_{t+j}]$ then follows as laid out above.

Now, why do sticky wages increase the sensitivity of profits (G_t) to changes in expected productivity ($E_t[\theta_{t+j}]$)? Intuitively, in a model with Nash-bargaining the work force will absorb a (large) part of the expected additional profits arising from higher expected productivity. With sticky wages, a change in (expected) productivity is not shared with the labor force and thus hits the "bottom line" to a stronger extent. The higher we set the level of the fixed wage, the higher the relative sensitivity of G_t to changes in $E_t[\theta_{t+j}]$. Consider a simple example to illustrate. We let:

$$G_t^N = (1-\pi)\, E_t[\theta_{t+j}], \tag{48}$$

$$G_t^{SW} = E_t[\theta_{t+j}] - \overline{w}, \tag{49}$$

where G_t^N and G_t^{SW} are the expected profits to the firm under Nash-bargaining and with sticky wages respectively. Let there be a change of $E_t[\theta_{t+j}]$ from, say, 100 to 110. First, we

[23] In section 4.5 we show that our results are robust to the choice of ν.

demonstrate the effect on G_t^N (under Nash-bargaining), where the worker receives a share π of profits:

$$\%\Delta G_t^N = \frac{(1-\pi)\times 110}{(1-\pi)\times 100} = 10\%. \tag{50}$$

Then, we fix the wage at $\overline{w} = 50$:

$$\%\Delta G_t^{SW} = \frac{110-50}{100-50} = 20\%. \tag{51}$$

Finally, we increase the fixed wage to $\overline{w} = 80$:

$$\%\Delta G_t^{SW} = \frac{110-80}{100-80} = 50\%. \tag{52}$$

As also demonstrated in Hall (2004), replacing Nash-bargaining with sticky wages in the DMP setting improves the magnification properties of the model. We report standard magnification ratios for our benchmark model:[24]

σ_Y/σ_θ	2.17	σ_C/σ_Y	0.60
σ_C/σ_θ	1.31	σ_I/σ_Y	2.49
σ_N/σ_θ	1.40		

The Capital Market. In our model shocks are very persistent, leading to strong propagation-effects, as can be seen in figure 6. We share this feature with Den Haan et al. (2000). They show that the propagation-effects are due to feedback effects from the capital market on the labor market: A look at figure 8 reveals that about three periods after a switch from regime 1 to regime 2 the economy experiences a fall in the interest rate and an increasing capital stock during the same period. In a given next period, the rising capital stock causes profit expectations (G) to go up; as higher capital supply implies lower expected interest

[24]We simulate a long series for Y, C, N, θ, take logs, and compute standard deviations.

rates and thus higher profits. This increase in G in turn causes employment to rise during the same period. If the net effect on interest rates of the higher capital stock (higher supply of capital) and the higher employment (higher demand for capital) is such that higher supply outweighs higher demand, then interest rates decline further, as we observe in our model (see figure 8). Note that this justifies the increase in G, as it confirms agents' anticipation of lower interest rates. At the same time, higher employment results in higher output, and thus higher investment and an increase in the capital stock. The economy, driven by falling interest rates and higher levels of capital from the previous period, experiences another period with higher levels of capital and yet lower interest rates.

To further establish that the propagation in our model is caused by the same interaction between capital- and labor markets as described in Den Haan et al. (2000), we solve a model in which we set the interest rate as constant. We observe from figure 9 that there is only minimal propagation left in the model with a constant interest rate, as variables rapidly move to new steady states.

4.4. Quantitative Assessment of E-RBCs within our E-RBC model. Because at this stage we do not work with a calibrated process driving productivity, as discussed in section 2.1, the quantitative assessment of E-RBCs is difficult. Nonetheless, we would like to offer conclusions for the driving process we consider in this paper, even though this process mainly serves analytical purposes. We postpone a thorough and more solid quantitative assessment of the impact of E-RBCs for a calibrated productivity process to a later version of the paper.

The first "back-of-the-envelope" approach we take to get a handle on the quantitative impact of E-RBCs is to take a look at "steady-state" values of key macroeconomic variables in different regimes.[25] In figure 10 we basically focus on the difference in steady state values between regime 1 and regime 2.[26] Judging from these differences, the role of E-RBCs clearly seems present, but somewhat limited.

However, we would like to caution from interpreting too much into the quantitative assessment at this point. For example, a relatively minor change to the parameters of our model, generating additional movement in the labor market, results in a considerable increase in the quantitative impact of E-RBCs, as judged by steady state values: We keep the same parameter values as in our benchmark model, but increase the wage level from $\overline{w} = 2.00$ to $\overline{w} = 2.01$, in order to generate additional movement in the labor market. To keep the steady state value of employment in regime 1 in line, we adjust the posting costs from $\psi = 0.10$ to $\psi = 0.05$ and μ from $\mu = 0.60$ to $\mu = 0.70$. In addition, we increase productivity in regime 3 from $\overline{\theta}^3 = 1.005$ to $\overline{\theta}^3 = 1.010$. This pushes the economy into full employment in regime 3, with the result that almost all slack in the labor market is already reduced in regime 2 in anticipation of the high productivity in regime 3. We can see in figure 11 that now E-RBCs seem to play a prominent role.

The second approach we take to judge the quantitative impact of E-RBCs is to compare standard magnification ratios of our benchmark model with a model that is based on a slightly different transition matrix Ω, with all other parameters the same. We manipulate the transition matrix in such a way as to minimize the role expectations play at a switch from regime 1 to regime 2 by making regime 2 more similar to regime 1 in terms of expected

[25] We define steady-state values for a given regime as the constant values the variables take after the economy has been in that regime for a sufficient number of periods so that the variables do not change any longer from one period to the next.

[26] Graphs are taken from our benchmark calibration as described in sections 4.1 and 4.2.

productivity. By increasing the probability of a switch to regime 1 at the expense of a switch to regime 3, we can reduce expected productivity in regime 2. Compared to our standard specification as described in section 2.1 we adjust the elements Ω_{21} and Ω_{23}:

$$\Omega^{QA} = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ \mathbf{0.19} & 0.80 & \mathbf{0.01} \\ 0.20 & 0.00 & 0.80 \end{pmatrix}.$$

The resulting magnification ratios are:[27]

	σ_Y/σ_θ	σ_C/σ_θ	σ_N/σ_θ
Benchmark Model	2.17	1.31	1.40
Model with Ω^{QA}	2.06	1.11	1.35

For the model with $\overline{w} = 2.01$ the ratios are:

	σ_Y/σ_θ	σ_C/σ_θ	σ_N/σ_θ
Model with $\overline{w} = 2.01$	2.46	1.67	1.82
Model with Ω^{QA}	1.48	0.70	0.68

Both from the inspection of steady state levels, as well as from the comparison of magnification ratios we conclude that it is not possible for us to assess the quantitative impact of E-RBCs at this point. In our benchmark model the quantitative impact is clearly limited. However, as we demonstrate, relatively minor changes to our model and targeted changes to the driving process strongly increase the quantitative impact of E-RBCs. We decided to include the discussion of the quantitative impact of E-RBCs in this version of our paper in order to motivate the need to move our research project towards a calibrated process for productivity that disentangles shocks to the level of productivity from shocks to the growth rate.

[27] We simulate a long series for Y, C, N, θ, take logs, and compute standard deviations.

4.5. Robustness. In this section we discuss the robustness of our results to modifications of the important parameters of the model. We would like to highlight up-front that it is relatively easy to get the model to generate E-RBCs according to our definition. It is somewhat more difficult to calibrate the model in such a way that it can generate E-RBCs according to the stricter Beaudry and Portier (2004a) definition of E-RBCs, that is to observe positive co-movement of all key macroeconomic variables already during the first period. Also, as discussed in section 4.4, the quantitative role played by E-RBCs in the model depends on both the calibration of the model as well as the parameterization of the productivity driving process.

In what follows, we discuss robustness to the matching market parameters ν and ρ^x, as well as to the level of the fixed wage rate $\overline{w}$. We also examine critical changes to the transition matrix and the productivity level in regime 3 on the ability of the model to generate E-RBCs.

Robustness to the Parameters ν and ρ^x. As shown in section 4.3, decreasing ν, the curvature parameter of the matching function, leads to additional movement in the labor market. To check robustness we therefore increase ν from $\nu = 0.70$ to $\nu = 0.80$, keeping all other parameters the same as in our benchmark model. Note that $\nu = 0.80$ corresponds to the point estimate of Blanchard and Diamond (1990), a standard value in the literature. We find that our results are robust to this change, as we still observe E-RBCs from the first period on, just like for our benchmark model (see figure 12). We also observe that a higher value for ν has the effect of making the labor market somewhat less responsive to changes in productivity, leading to a reduction of the quantitative impact of E-RBCs.

We change ρ^x, the rate of exogenous destruction, from $\rho^x = 0.08$ to $\rho^x = 0.10$. Again, we keep all other parameters the same. Note that in our choice of $\rho^x = 0.08$ for our benchmark calibration we follow Den Haan et al. (2000), who rely on Hall (1995) and Davis et al. (1996) to justify a range for ρ^x between 0.08 and 0.10. We find that our results are robust to this change. This time we observe a slight increase in the responsiveness of the labor market to changes in productivity (see figure 13).

Robustness to the Driving Process. We keep all parameters of the model the same as in our benchmark model and only adjust either the transition matrix Ω or the levels of productivity θ, as described below.[28]

As in a controlled experiment, we take four transition matrices that differ in crucial aspects from the transition matrix of our benchmark model, and hope to convince the reader this way that our results do not rely on the choice of a particular transition matrix:

$$\Omega^1 = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ \mathbf{0.19} & 0.80 & \mathbf{0.01} \\ 0.20 & 0.00 & 0.80 \end{pmatrix}, \quad \Omega^2 = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ \mathbf{0.01} & 0.80 & \mathbf{0.19} \\ 0.20 & 0.00 & 0.80 \end{pmatrix},$$

$$\Omega^3 = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ \mathbf{0.01} & 0.00 & \mathbf{0.99} \end{pmatrix}, \quad \Omega^4 = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ \mathbf{0.99} & 0.00 & \mathbf{0.01} \end{pmatrix}.$$

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[28] For the reader's convenience, the parameterization of the productivity level and the transition matrix in our benchmark model:

$$\theta = \{1.000, 1.000, 1.005\}$$

$$\Omega = \begin{pmatrix} 0.80 & 0.20 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.20 & 0.00 & 0.80 \end{pmatrix}$$

- Ω^1: A switch from regime 2 to regime 3 is much less likely compared to the transition matrix of the benchmark model:

  Our results are robust to Ω^1 (we already observe positive co-movement of all variables during the first period). As we would expect, the quantitative impact of E-RBCs seems smaller compared to our benchmark model.

- Ω^2: Opposite "experiment" as Ω^1. Now we make a switch to regime 3 much more likely, at the expense of the probability of a switch back to regime 1:

  Our results are robust to Ω^2. Again as expected, the quantitative impact of E-RBCs seems stronger now compared to the benchmark model.

- Ω^3: Regime 3 is much more persistent compared to the transition matrix of the benchmark model:

  This is the most interesting case, as it turns out that our results are only robust by our own definition of E-RBCs, not according to Beaudry and Portier (2004a). We observe a positive (net) co-movement of all relevant macroeconomic variables only from the second period onwards (see figure 14). The reason is that the income effect is now very strong relative to the higher expected return on savings (substitution effect). This is due to the fact that induced by regime 3's high persistence, agents will increase their consumption more in regime 3 compared to the benchmark model.

  The quantitative impact of E-RBCs appears to be stronger here than in the benchmark model.

- Ω^4: Regime 3 is much less persistent:

  Our results are robust to Ω^4 in terms of generating first-period-E-RBCs, while the quantitative impact of E-RBCs now seems weaker. Compared to our benchmark model, under Ω^4 the relation of higher consumption to higher productivity, i.e. the strength of the income effect relative to the substitution effect, is more conducive to E-RBCs, because agents now build up less consumption in regime 3 due to the high likelihood of a drop back to regime 1. On the other hand, the fact that regime 3 is now less attractive in general apparently dampens the quantitative impact of E-RBCs.

We also change the productivity levels in regime 3 (drastically) to demonstrate robustness of our results along that dimension:

$$\begin{aligned}\theta^L &= \{1.000, 1.000, 1.001\}\,,\\ \theta^H &= \{1.000, 1.000, 1.025\}\,.\end{aligned}$$

Under θ^L, regime 3 is much less productive compared to regime 3 under our benchmark parameterization ($\theta = \{1.000, 1.000, 1.005\}$). Under θ^H, regime 3 is characterized by a much higher productivity level.

Both under θ^L, as well as under θ^H, we observe a positive co-movement of all key macroeconomic variables already during the first period after a change in rational expectations. As we would expect, the quantitative impact of E-RBCs gets stronger the higher we set productivity in regime 3.

Robustness to the Level of the Fixed Wage Rate. As demonstrated in section 4.3, the level of the fixed wage rate is crucial for generating sufficient movement in the labor market. It becomes interesting to see by how much we can lower the wage rate before our model is no longer able to generate E-RBCs. We start by decreasing the wage rate from

$\overline{w} = 2.00$ to $\overline{w} = 1.975$, adjusting ψ so that steady state values roughly stay in line.[29] We solve the model and report results in figure 15. We observe that our model still exhibits E-RBCs, however only from the second period after a change in growth expectations onwards. Our usual measure, the conditional correlations, reflect our observation:

	C	I	Y	N
C	1.00	0.34	1.00	1.00
I	0.34	1.00	0.36	0.40
Y	1.00	0.36	1.00	1.00
N	1.00	0.40	1.00	1.00

The conditional correlation between investment on the one hand, and consumption, output and employment on the other hand is still positive, but substantially less than unity. Beaudry and Portier (2004a) would exclude this case from the class of E-RBC models in accordance with their narrow definition. Now, arguably the model with $\overline{w} = 1.975$ represents as much an E-RBC model in principle as the model with $\overline{w} = 2.00$. The difference between the two models is a difference in degree rather than category, illustratively summarized by the lower but still positive conditional correlations of the model with $\overline{w} = 1.975$ compared to our benchmark model with $\overline{w} = 2.00$.

We continue to decrease the fixed wage rate further from $\overline{w} = 1.975$ to $\overline{w} = 1.90$.[30] Please see figure 16 for results. We observe that our model is not able to generate E-RBCs at this wage level. The amount of movement in the labor market, induced by a change in growth expectations, is not sufficient any more to generate enough resources so as to increase both

[29] We adjust the vacancy posting cost from $\psi = 0.10$ to $\psi = 0.25$ and keep all other parameters the same as in our benchmark model.

[30] We accordingly adjust ψ from $\psi = 0.25$ to $\psi = 0.60$, keeping all other parameters the same.

consumption and investment. Once again, the conditional correlations confirm:

	C	I	Y	N
C	1.00	-0.96	1.00	1.00
I	-0.96	1.00	-0.95	-0.93
Y	1.00	-0.95	1.00	1.00
N	1.00	-0.93	1.00	1.00

5. Conclusion

We have investigated the role rational growth expectations play in explaining economic fluctuations within standard business cycle models. In particular, we have addressed the question of whether a change in rational expectations about future productivity levels at a given level of current productivity can generate business cycle-like fluctuations within RBC models, that is positive co-movements between output, consumption, investment, and employment. In order to model changes in expectations we have used a regime switching process for productivity, which in turn enabled us to employ the conditional correlations of key macroeconomic variables in the regime with high growth expectations as a measure of the occurrence of E-RBCs in a given RBC model.

We have demonstrated that within standard RBC models, optimism about the future leads to a reduction of current real activity. The unchanged aggregate resources whenever expectations about future productivity change (keeping current productivity fixed) have been identified as the main culprit for this fact. As a consequence, RBC models that aspire to exhibit E-RBCs must allow for slack or idle resources in the aggregate economy, so that the economy is not at its full potential when a change in growth expectations takes place.

We have drawn upon a class of standard labor market matching models that incorporate slack in the labor market as a possibility to release aggregate resources whenever growth expectations change. And indeed, these models can constitute E-RBC models. To that end they have to generate sufficient movement in the labor market in response to fluctuations in expected productivity. In the current version of our model we rely on a sticky wage rule proposed by Hall (2004) to render the labor market sufficiently responsive. We are planning to incorporate endogenous destruction into a later version of the model as an additional mechanism to generate realistic magnitudes of movement in the labor market.

Another extension of our work we want to undertake is to include financial markets (financial frictions) as a second major dimension along which mechanisms to relax the aggregate budget constraint can be deployed.

There seems to be a strong intuitive case for the notion of business cycle-like fluctuations induced by a revision of growth expectations. The economic expansion of the 1990s may serve as the most recent example where many observers would agree that growth expectations at least contributed to the boom. There is also empirical evidence by Beaudry and Portier (2004b) that suggests a strong role for growth expectations in the explanation of business cycle fluctuations. This renders the result that within standard RBC models higher growth expectations do not induce business cycle-like fluctuations problematic. Therefore, we consider as relevant both our general results regarding the mechanisms that allow a transformation of standard RBC models into E-RBC models, as well as our demonstration that the standard labor market matching model can generate E-RBC fluctuations.

So far, our results are based on an un-calibrated process for productivity. For the theoretical assessment of whether any given RBC model constitutes an E-RBC model, this shortcoming is irrelevant. However, it does bar us from drawing reliable conclusions with respect to the quantitative relevance of E-RBCs within our E-RBC models. For that purpose we are currently working on a realistic process that disentangles shocks to current levels of productivity from shocks to the growth rate of productivity. If for a calibrated driving process it would turn out that E-RBCs play a strong quantitative role within our models, then this might suggest that our calibrated driving process in combination with our version of the standard labor market matching model, or another type of model that incorporates slack or idle resources and is sufficiently responsive to growth expectations, constitutes an important apparatus for the design of E-RBC models.

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