Housing, House Prices, and the Equity Premium Revisited*

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Abstract

We show that explicitly including housing services as a seperate argument from other consumption in a standard utility function does not resolve the equity premium puzzle. We estimate the feasible set of parameter values for a Lucas-tree model with CES preferences defined over housing services and other consumption using a parsimonious set of moment restrictions implied by the model and aggregate quarterly data starting from 1970. We show that the sets of parameters that match our housing moments yield an intra-temporal elasticity of substitution of housing services and other consumption at least as large as 2.0, much more substituable than has previously been assumed. Any estimate below 2.0 (and with an intertemporal elasticity above 0.05) produces model house price errors that are negatively correlated with time implying that house prices have not grown fast enough on average over the past 35 years. In addition, we find that we cannot simultaneously price Treasury bills and either gross equity returns or

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housing. The set of parameter values that are consistent with the gross equity and housing moments imply an annual real risk-free rate of no less than 11 percent.

Keywords: House Prices, Housing, Equity Premium

1 *Introduction*

[Introduction and conclusion are not finalized.]

We wish to understand whether housing, considered as a distinct good in the utility function, can be useful in explaining economic activity. In particular, we wish to understand whether housing in the utility function is sufficient to explain the equity premium puzzle. There is a growing literature (Lustig and Van Nieuwerburgh (2004) and Piazzesi et al. (2004)) that attempts to resolve the equity premium puzzle by explicitly including housing in the utility specification. There is also a growing and related literature which uses housing in this fashion to explain wealth and expenditure puzzles (Gruber and Martin (2004) and Krueger and Fernandez-Villaverde (2001) for example). However, despite the importance of housing to their results, none of these papers takes seriously the estimation of the parameters of the utility function[1].

We show that housing services and consumption are much more substitutable than had previously been assumed in the literature[2]. This is of first order importance if we wish to use housing to explain empirical phenomena. For example, this fact will imply that in no way does housing help to resolve the equity premium puzzle. With housing estimated as fairly strong substitutes, pricing of equity returns implies very low intertemporal elasticities of substitution. For these values, the risk free rate is near 11 percent.

[1] Lack of estimation is not a fault of these papers. Identification of the parameters is elusive. We will show that we can achieve identification only over feasible sets of parameters. However, and importantly, these feasible sets will not intersect with the parameters which have been used in previous studies.

[2] Empirical estimates of the substantiality between consumption of housing services and other goods have often found the two to be substitutable. The estimates range from slightly more complementary than Cobb-Douglas (Ruppert et al. (1995) or Ogaki and Reinhart (1998)) to a an elasticity of substitution near 2 (McGratten et al. (1997)). However, macro papers have all (to our knowledge) chosen preferences at least as complementary as Cobb-Douglas (Krueger and Villaverde (2001) and Piazessi et al (2004) choose Cobb-Douglas while Lustig and Van Nieuwerburgh (2004) and Gruber and Martin (2004) choose elasticity of substitution well below 1 (Cobb-Douglas).

Implying, that housing alone can not resolve the equity premium puzzle. Given that our estimates differ so greatly from that used in the macro literature, it seems questionable whether its use to date to resolve puzzles will hold up.

Explicit incorporation of housing seems to be a fruitful line of research, as measured expenditures on housing services are a large portion of total consumption expenditures and the statistical properties of housing services are distinct from those of other consumption goods and services[3]. Table 1 shows the unconditional correlations between the growth rate of the housing stock, the growth rate of consumption, and a set of Fama-French portfolios. As can be seen, the growth rate of the housing stock covaries negatively with asset returns whereas consumption covaries positively[4].

To understand how housing may help resolve the equity premium puzzle, recall the standard consumption asset pricing formula: $1 = E\left[m_{t+1}R_{t+1}\right]$ with $m_{t+1} = \beta\left(\frac{c^*_{t+1}}{c^*_t}\right)^{-\sigma}$. c^* is total consumption expenditure and all goods and services are assumed to be perfect substitutes. σ represents the coefficient of risk. Hansen and Cochrane (1992), for example, have shown that using this definition of c^* and using aggregate consumption data the model requires $\sigma > 30$ (and intertemporal elasticity of substitution of less than 0.03, which in turn implies a risk free rate in the vicinity of 17 percent per quarter. The difficulty lies in the fact that using c^* and reasonable estimates of σ, $1 < E\left[m_{t+1}R_{t+1}\right]$. Consumption does not covary sufficiently with R to support low risk free rates and price historical equity returns.

Now consider a utility function that explicitly separates housing services from other consumption, that is

$$U\left(c_t, h_t\right) = \frac{\left(\gamma c_t^{\alpha} + (1-\gamma)\, h_t^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma}$$

where c is consumption of nondurable goods, h is consumption of housing, α governs intratemporal elasticity

[3] Throughout, we will use the convention of referring to consumption of goods and services excluding housing services simply as consumption.

[4] Notice, the covariances are both very small and of hte same sign for both equities and T-Bills. Therefore, explaining the difference in rates of return between the two assets is going to be difficult in any model. In a nutshell, that is the equity premium puzzle.

of substitution between the two goods, and σ governs the intertemporal elasticity of substitution, $\alpha = 1$ implies perfect substitutes and $\alpha = -\infty$ is Leontief utility. With this utility function the asset pricing equation becomes

$$1 = E\left[m_{t+1}\left\{\left(\frac{\gamma + (1-\gamma)\left(\frac{h_{t+1}}{c_{t+1}}\right)^{\alpha}}{\gamma + (1-\gamma)\left(\frac{h_t}{c_t}\right)^{\alpha}}\right)^{\frac{1-\sigma-\alpha}{\alpha}}\right\}R_{t+1}\right]$$

where the extra term in brackets captures the effect of a change in the relative consumption of housing services versus other consumption[5]. LVN and Piazzesi et al. find that when $\alpha \leq 0$, that is consumption and housing services at least as complementary as Cobb-Douglas, they are able to price excess returns assuming empirically plausible coefficient of risk values between 3 and 10. Thus, it appears, that housing has the ability to resolve the equity premium puzzle[6].

Our basic insight into this literature is that if one wishes to add an asset to the utility function in order to price other assets, this specification should also do well in pricing the added asset (i.e. we must be able to price houses in this framework). To this end, we will estimate the parameters of the utility function by only considering parameters which are also consistent with the sequences of house price data observed in the data. We will do this by estimating a pricing equation for housing.[7]

In this paper, we show – using quarterly data from 1970 to 2004Q2 on house prices, the housing stock, a portfolio of Fama-French portfolios, and consumption – that housing services and consumption are actually fairly strong substitutes: In order to price housing in a way that does not imply trending house price errors, either α must be larger than around 0.2 or σ must be greater than 10. While negative values of α are permissible, these are associated with values of σ greater than 20. Values of α below -2 are not compatible with any σ below 200. However, more importantly, for the parameters to also be consistent with equity

[5] Piazzesi et al. refer to this term as composition risk.

[6] We should note there is also a macro literature which uses a similar set up in an effort to explain both the implications of durable goods on the households risk exposure (Chetty et al. (2004), Gyourko and Tracy (2004), Franantoni (2001)) and the inequality of the wealth distribution (Gruber and Martin (2003) and Diaz and Luengo-Prado (2004)).

[7] The pricing equation for housing will be a generalization of the above pricing equation. The difference will lie in the fact that housing enters the utility function directly and hence we must also consider the user cost of housing explicitly.

moments α is always greater than 0.5. That is, if we wish to pick parameters that are consistent with both house prices and equity returns, α must be greater than 0.5 and σ must be less than 10. Further, for this feasible set of parameters, the average real risk free rate is in excess of 11 percent per year.

The simple intuition for this result is that given the measured historical average growth rates of consumption and the stock of housing, house prices have not grown fast enough to support strong complementarities between the two types of goods. Surprisingly, we also find that there is no combination of parameters that can successfully price average real one quarter T-bill returns; our best estimates suggest that the T-bill returns are underpriced by an average of 3 percent per year.

In addition, the presence of large transaction costs in housing markets also would imply very high welfare costs if housing is not sufficiently substitutable. If housing services and other consumption is relatively substitutable, the transaction cost minimizing strategy of the household which faces a rising income stream of buying a very large house and temporarily forgoing other consumption is relatively inexpensive. If, on the other hand, the two consumption streams are complementary, the welfare costs are large and one might expect a market to exist to minimize such frictions.

Our results lead us to infer that the incorporation of housing into the period utility function is in itself insufficient to alleviate the equity premium puzzle. The parameters needed to price equities and housing simultaneously are completely inconsistent with real T-Bill returns. We find that over the feasible set the lowest possible real-risk-free interest rate is in excess of 11 percent. While this is better, than a model without housing services, we do not consider it to be a sucess.

2 Model

Our model is an endowment economy with a single representative agent. The representative agent has per-period utility over a CES-composite consumption good:

$$U\left(c_t, h_t\right) = \frac{\left(\gamma c_t^{\alpha} + (1-\gamma)\, h_t^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma}, \tag{1}$$

where h_t denotes housing services and c_t denotes consumption exclusive of housing services, hereafter called simply "consumption." The parameter $\gamma \in (0,1)$ weights housing services and consumption in utility, $\sigma \in (1,\infty)$ measures the degree of relative risk aversion, and $\alpha \in (-\infty, 1)$ captures the intra-temporal elasticity of substitution between housing services and consumption. Preferences are Cobb-Douglas in the limit as $\alpha \to 0$, $lim_{\alpha\to 0} U\left(c_t, h_t\right) \approx \frac{\left(c_t^{\gamma} h_t^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$, and separable when $\alpha = 1-\sigma$, $U\left(c_t, h_t\right)_{\alpha=1-\sigma} = \frac{\gamma c_t^{1-\sigma}}{1-\sigma} + \frac{(1-\gamma) h_t^{1-\sigma}}{1-\sigma}$.

The representative agent maximizes his discounted expected remaining lifetime utility,

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right), \tag{2}$$

β the discount factor, by choosing sequences of c_t, h_t, A_t (consumption of nondurable goods, consumption of housing services, and vectors[8] of financial assets one of which may be risk free) subject to the following per-period budget constraint:

$$c_t + p_t h_t + \mathbf{A}_{t+1} = \mathbf{R}_t \mathbf{A}_t + p_t h_{t-1}. \tag{3}$$

p_t denotes the price of one unit of housing in units of consumption and R_t is a vector of total returns on the financial assets, including both capital gains and dividends. The expectation in (2) is taken over the future sequences of $\{c_t, h_t, R_t\}_{t=0}^{\infty}$.

Denoting λ_t as the Lagrange multiplier on the budget constraint at period t, and assuming interior

[8] We will follow convention and denote vectors in bold.

solutions, this model implies the following set of first order conditions,

$$c_t \quad : \quad U_c\left(c_t, h_t\right) - \lambda_t = 0 \tag{4}$$

$$h_t \quad : \quad U_h\left(c_t, h_t\right) - p_t \lambda_t + \beta E_t p_{t+1} \lambda_{t+1} = 0 \tag{5}$$

$$A_t \quad : \quad -\lambda_t + \beta E_t \mathbf{R}_{t+1} \lambda_{t+1} = 0 \tag{6}$$

Rearranging these first order conditions yields Euler equations that we directly use in our empirical work to price financial and housing assets:

$$\begin{aligned} 1 &= \frac{1}{p_t} \frac{U_h\left(c_t, h_t\right)}{U_c\left(c_t, h_t\right)} + E_t\left[m_{t+1} \frac{p_{t+1}}{p_t}\right] \\ 1 &= E_t\left[m_{t+1} R_{t+1}^i\right], \end{aligned}$$

where m_{t+1} (the pricing kernel) is $\beta\left(\frac{c_{t+1}}{c_t}\right)^{\alpha-1}\left[\frac{\gamma c_{t+1}^\alpha+(1-\gamma) h_{t+1}^\alpha}{\gamma c_t^\alpha+(1-\gamma) h_t^\alpha}\right]^{\frac{1-\sigma-\alpha}{\alpha}}$ given our utility function. In equation (**??**), the various financial assets are indexed by i. We assume throughout that the representative agent is not constrained at the optimum and that there are no transaction costs in the sale and purchase of housing.

In the housing Euler equation, we assume the dividend, $\frac{U_h(c_t,h_t)}{U_c(c_t,h_t)} = \frac{1-\gamma}{\gamma}\left(h_t/c_t\right)^{\alpha-1}$, is paid in the current period and thus separate the dividend from its capital gain, unlike our treatment of financial assets. Note that the current period dividend for housing is the user cost of housing, and it is the sequence of this ratio that the BLS is attempting to measure with its rental cost of owner-occupied housing series. Of course, once parameters are chosen for the utility function, the user cost of housing is completely determined the model given housing services and consumption. User cost, or owner-occupied rent, and the kernel used to price other assets are not independent entities, implying that BLS series for rents can not be taken as independent data that is exogenous from any other predictions of the model.

Finally, we should note that the pricing equation for housing is a difference equation in p. One solution to this equation, and the only non-bubble solution to the equation is

$$p_t = E_t \sum_{s=0}^{\infty} \left(\prod_{j=1}^{s} m_{j+t} \right) \frac{U_h\left(c_{s+t}, h_{s+t}\right)}{U_c\left(c_{s+t}, h_{s+t}\right)} \tag{7}$$

The current price is equal to the present discounted value of user costs, exactly analogous to the common result for security pricing.

3 Data

Our consumption and housing data are expressed as real per-capita quantities. Our estimates of the population for the entire United States are taken from the Regional Economic Accounts, produced by the Bureau of Economic Analysis (BEA).[9] From 1970 to 2004, the US population has grown at approximately 1 percent per year.

Consumption. To measure consumption, we subtract real consumption of housing services (as reported in line 14 of Table 2.3.6 of the National Income and Product Accounts) from line 1 of that same table, total real consumption inclusive of expenditures on consumer durable goods.[10] Our price index for consumption used to measure inflation in the calculation of real returns, denoted p_t^{c*}, is consistent with this definition.[11] Note that in the process of purging housing services from overall consumption, we may introduce a tiny amount of measurement error into our consumption series, increasing the volatility of consumption and inducing negative serial correlation in the growth rates. To understand the latter effect, denote observed consumption in period t as c_t^o, true consumption as c_t, and an i.i.d. measurement error draw as e_t. If $\log\left(c_t^o\right) = \log\left(c\right) + e_t$ then $\Delta \log\left(c_t^o\right) = \Delta \log\left(c\right) + \Delta e_t$; obviously, Δe_t is negatively serially correlated. We

[9] The Regional Accounts publish annual population estimates, approximately for July of each year. We convert these to a quarterly frequency by assuming constant growth between years.

[10] The data nuts out there should rest assured that we appropriately use chain-weight aggregation.

[11] We set our price indexes and housing stock data to be beginning-of-quarter estimates; T-Bill returns and inflation for any period t are measured from the beginning-of-quarter t to the beginning-of-quarter t+1; and, consumption is measured as a flow throughout the quarter. Also, a * superscript denotes a nominal price index.

assume that the true process for $\log(c_t)$ is a random walk, that is $\Delta \log(c) = u_t$ with innovations u_t that are uncorrelated with e_s for any s and t, and use the Kalman Filter to uncover an underlying series for $\log(c_t)$ that we use in our data analysis. The top panel of Figure 1 compares the level of Kalman-Filter predicted per-capita real consumption against the original unsmoothed level; the bottom panel plots the growth rates of the two series. The growth rates of the filtered series appear less negatively autocorrelated than the unfiltered growth rates but the level of the two series are basically identical.

Stocks and Treasury Bills. We use nominal beginning-of-quarter 3-month Treasury yields to proxy for nominal risk-free bond returns. For nominal quarterly equity returns, we study the six Fama-French portfolios that are available on Kenneth French's web site, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.[12] Quickly summarizing, the Fama-French portfolios classify stocks as either "small" or "big," where the split-point is the median NYSE market equity. For both small and big stocks, the portfolios are split again into three groups based on the ratio of book-to-market equity; stocks with the highest book-to-market equity ratios are called "value" stocks, the lowest are the "growth" stocks, and those in between "neutral." The split points are the 30th and 70th NYSE percentiles. The average returns across these portfolios differ quite a bit: Over the 1970-2004 period, an index constructed based on real returns of the small value stocks has grown by 13 percent per year on average whereas an index constructed on real returns of big growth stocks has only grown by 8 percent per year on average. For our bond and stock portfolios, we convert nominal to real returns by appropriately accounting for realized consumer-price inflation $p^{c^*}_{t+1}/p^{c^*}_t$.

Price and Quantity of Housing Services. Our data on house prices and housing services are sufficiently different from those used in previous studies that a detailed discussion seems appropriate. We assume in our empirical work that real housing services are proportional to the real stock of housing, that is

$$h_t = \kappa H_t, \tag{8}$$

[12] We assume that these returns are consistent with our timing convention.

where H_t is the real stock of housing and κ maps the stock of housing into housing services. Obviously, this assumption is not without loss of generality, and in our concluding section we discuss some alternatives.

Equation (8), however, implies that we can use new data from Davis and Heathcote (2004) on house prices and the stock of houses directly in our empirical work. For house prices, Davis and Heathcote use the Freddie Mac USA Conventional Mortgage Home Price Index (CMHPI), a price index of house values for the entire United States. The CMHPI is a repeat-sales price index that approximately holds the quality of houses constant between any two consecutive periods but allows the quality of the stock to change over time as the quality of the aggregate stock changes. The CMHPI starts in 1970:Q1, explaining our sample period in estimation. Davis and Heathcote (2004) apply a Kalman filter to the log level of the inflation-adjusted CMHPI to correct for measurement error that if left unadjusted induces negative serial correlation in the growth rates of house prices (as with consumption). We calculate the relative price of housing p_t^h as the nominal price index for housing $p_t^{h^*}$ divided by $p_t^{c^*}$.

Davis and Heathcote calculate the nominal stock of housing (owned and rented) from 1970:Q1 to 2004:Q2 using a perpetual accounting method that adjusts the aggregate nominal value of housing in period t, $p_t^{h*}h_t$, for capital gains as measured by the growth rate in the (modified) CMHPI, $\frac{p_{t+1}^{h*}}{p_t^{h*}}$, and then adds to that net-new aggregate investment in housing, $p_{t+1}^{h*}\Delta h_{t+1}$, that is derived from BEA data on investment in residential structures,

$$p_{t+1}^{h*}h_{t+1} = \left(\frac{p_{t+1}^{h*}}{p_t^{h*}}\right)p_t^{h*}h_t + p_{t+1}^{h*}\Delta h_{t+1}. \tag{9}$$

This system is benchmarked to an estimate of the aggregate nominal value of housing in 2000 that is derived from micro data reported in the 2000 Decennial Census of Housing. Davis and Heathcote find that this perpetual inventory system, benchmarked only to the 2000 Decennial Census of Housing, matches the nominal value of the housing stock within 5 percent to estimates derived from the 1980 Decennial Census of Housing.[13]

[13] See Davis and Heathcote (2004) for more details.

We define real stock of housing in constant \$2000 as the nominal stock $p_{t+1}^{h*}h_t$ divided by the price index p_t^{h*} and set the real value such that it equals the nominal in 2000. In some ways our measure of the the real housing stock is similar to the real consumption of housing services series that is published by the BEA in the NIPA and has been used in previous studies such as Piazzesi et. al. For example, the growth rates of the two series in per-capita terms are highly correlated (a correlation of 0.62) and both are weakly correlated (0.11) with our filtered per-capita consumption estimate. The two series are different in a very important way, however: Since 1970, our measure of the real per-capita stock of housing has grown at just 0.4 percent per year whereas the real per-capita consumption of housing services as published by the BEA in the NIPA has grown at 1.7 percent per year.

We have no idea why the BEA estimate has grown so much faster than that of Davis and Heathcote, but we suspect that Davis and Heathcote estimate is more accurate, for thee reasons. First, the Davis and Heathcote data are consistent with Decennial Census of Housing data and repeat-sales measures of house prices, suggesting that at face value this data may be correct. Second, the Personal Consumption Expenditure handbook that is published by the BEA (U.S. Department of Commerce, 1990) describes the growth in the estimate of owner-occupied nominal rent (the hypothetical amount owner-occupiers would pay to rent their housing units from a landlord) as being affected by year-by-year "judgemental" (p. 61) adjustments for changes to the quality of the housing stock. Finally, the BEA deflates the nominal estimate of "space rent" in the NIPA by the CPI for either owners' equivalent rent or tenant rent (depending on the specific line item) to produce a real estimate. A vast literature[14] suggests that the CPI estimate of changes to rental prices has a large downward bias, which would induce an upward bias to growth of real housing services as published in the NIPA. The fact that our estimate of the housing stock is growing more slowly than the BEA estimate has important implications for our parameter estimates, and can potentially explain some of the differences between our findings and that of previous studies.

[14] For a recent paper, see Gordon and vanGoethem 2003.

All of our data are available for download at http://morris.marginalq.com/.

4 Estimation and Analysis

Our goal is to find model parameters which are consistent with the observed time series of consumption of goods and housing services, and the observed prices for both types of goods and a given set of financial assets. We use our pricing equations derived above directly in estimation by writing

$$E_t\left[m_{t+1}\frac{p_{t+1}}{p_t}\right] = m_{t+1}\frac{p_{t+1}}{p_t}+\epsilon^h_{t+1} \tag{10}$$

$$E_t\left[m_{t+1}R^i_{t+1}\right] = m_{t+1}R^i_{t+1}+\epsilon^i_{t+1} \tag{11}$$

and assuming, as is typical, that the ex-post housing errors ϵ^h_{t+1} and ex-post vector of financial errors indexed by i, ϵ^i_{t+1} are not forecastable using any information dated at time t or before.

Hence, we will estimate the following equations:

$$\begin{aligned} 1 &= \frac{1}{p_t}\frac{U_h\left(c_t,h_t\right)}{U_c\left(c_t,h_t\right)}+m_{t+1}\frac{p_{t+1}}{p_t}+\epsilon^h_{t+1} \\ 1 &= m_{t+1}R^i_{t+1}+\epsilon^i_{t+1} \end{aligned}$$

In what follows, we wish to identify a feasible range for the parameters of the above equation using as few moment restriction as possible. To foreshadow the results, each of our moment restrictions are going to imply strong identification of γ and α pairs. For equities and housing, we will be able to identify a non-empty intersection of these sets[15]. However, the intersection between either house prices and bonds or equities and

[15] For the equity moment, we will attempt to identify parameters using an equally weighted portfolio of the six Fama-French portfolios identified in the data section. We require only that this moment have zero average error. It turns out, and we discuss this at length in the next subsection, that we will always be able to set the average error in the housing equation equal to zero. Therefore, we will require that the house price errors not trend over time. That is, we require the correlation between house price errors and time to be zero.

bonds is empty. .

4.1 Housing Moment

As our view is that any model that includes housing as an argument to explain financial returns should first explain historical housing returns, we will start by using the housing Euler equation to inform us as much as possible as to the set of feasible parameters. Only then will we add the equity moment in an attempt to refine this feasible set. But first we must deal with the identification issue alluded to above.

4.1.1 Identification

First, note that we cannot, in general, estimate the value of the parameter κ that links the real housing stock to real housing services. To see this, suppose we rescale c and h by k_c and k_h respectively. This produces

$$\frac{\left(\gamma\left(k_c c_t\right)^{\alpha}+(1-\gamma)\left(k_h h_t\right)^{a}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma} = \tag{12}$$

$$\left(\gamma k_c^{\alpha}+(1-\gamma) k_h^{\alpha}\right)^{\frac{1-\sigma}{\alpha}} \frac{\left(\frac{\gamma k_c^{\alpha}}{\gamma k_c^{\alpha}+(1-\gamma)k_h^{\alpha}}\left(c_t\right)^{\alpha}+\frac{(1-\gamma)k_h^{\alpha}}{\gamma k_c^{\alpha}+(1-\gamma)k_h^{\alpha}}\left(h_t\right)^{a}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma} = \tag{13}$$

$$A\frac{\left(\hat{\gamma}\left(c_t\right)^{\alpha}+(1-\hat{\gamma})\left(h_t\right)^{a}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma} \tag{14}$$

Since the utility function is scale invariant, our rescaling of c and h has no effect on the solution except insofar as it affects our estimate of γ. For numerical reasons, it is convenient if c and h have the same average value. Therefore, we rescale both consumption and housing by dividing by their average values. The parameter γ must then rescale the h and c to make them consistent with the data. An important note is that we will not be able to directly compare our results for γ across estimations. Since, our various estimations will imply different values for α, they will automatically imply different values for γ which as can be seen above is a function of α.

Next, note that because we do not directly observe a price of houses, but rather a price index for housing, we can always set the average value of ϵ_t^h to zero. Define the price of houses as δp_t^h, where p_t^h is our observed price index for housing and δ is a parameter to be estimated that links our price index to the true price level. The value of δ that sets the average house price error to zero given the other parameters of the model satisfies

$$\frac{1}{\delta} = \frac{1 - \frac{1}{T-1} \sum_{t=1}^{T-1} \left(m_{t+1} \frac{p_{t+1}}{p_t} \right)}{\frac{1}{T-1} \sum_{t=1}^{T-1} \frac{1-\gamma}{\gamma} \left(h_t / c_t \right)^{\alpha - 1} (1/p_t)} \tag{15}$$

where T denotes the number of observations and m_{t+1} is defined as before. In the parameter estimates that we subsequently report or show, δ is always set such that (15) is satisfied.

4.2 Estimation

To make headway on uncovering the other parameters of the model, we impose what seems to us the next most basic moment restriction for housing (given that the average error is identically zero): that the housing errors do not trend over time (i.e. we minimize the correlation of ϵ_t^h and t). First, we discuss identification of α and γ given a σ, β.

Given, a σ and a β, the set of feasible $\alpha' s$ is actually quite large. For example, with $\sigma = 5$ and $\beta = .995$ any α above 0.4 appears to be permissible. For this (σ, β), Figure 1 gives the absolute value of the correlation between the house price error and time. From this graph it can be seen immediately that any good numerical routine should be able to find its way to the valley which runs from the pair$(\gamma = .38,\ \alpha = .999)$ to the pair $(\gamma = .999,\ \alpha = .38)$. Therefore, one need only search along the floor of the valley to find the absolute minimum correlation. However, while there does appear to be a true minimum in the valley (it appears to reach a minimum around $\alpha = .7$), numerical error is relatively large percent of the difference between points within the valley. While the scale of the ridges is easy to see in the graph, they are of order 0.1, the scale of points within the valley are between order 1x10^{-12} and 1x10^{-11} close to the order of magnitude of our numerical error. In other words, after accounting for numerical error (of course, even very minor errors in the data or small changes in our sample would also play a role here), we will consider all points in the valley to be potential candidates for a solution. Notice, if we had some a priori knowledge of either α or γ, we would have exact identification. Unfortunately, we do not know α and γ is not the true weighting between consumption and housing in the utility function because of the scaling issues discussed above. All we can say for γ is that as it increases more and more weight is being placed on the growth rate housing.

We now wish to determine the feasible set for α. We define this set as the union of all valleys as we vary both σ and β. It turns out that the value, $\alpha = .999$ always lies within the feasible set. As the valley is also always continuous, it is sufficient to identify the lowest α which is permissible for each β and each σ. Figure 2 plots this line.

Figure 2 plots, for several different βs, the lowest α as a function of σ. To demonstrate this relationship, we allow for values of σ between 1.1 and 80, a range which is considerably large than what is normally considered to be admissible. We plot a line for β = .95, .99, .995, and .999. We consider .95 to be well below the permissible range for β when using quarterly data as it implies an annual risk free rate in excess of 20 percent. Notice, that, while negative α's are permissible, they are only permissible for $\sigma > 20$. In fact if we believe (and we do) that σ lies well below 10, then α must be greater than 0.2 (significantly more substitutable than Cobb-Douglas). Importantly, values of α used in either Gruber and Martin (2004) (α near -4) or LVN (2004) ($\alpha = -5$) are not permissible for any $\sigma < 200$.

This result on its own is quite useful. The result implies that consumption of housing services and consumption of other goods are actually quite substitutable relative to what has previously been assumed in the literature. As a result, the user cost of housing must be less volatile than was once thought and the welfare costs of large transaction costs in the purchase of owner occupied housing must be smaller than has been assumed (as utility is less affected by composition changes, the agent is free to follow policies which minimize lifetime costs of transaction costs).

We have now found our feasible set for α. In order to refine the feasible set, we will attempt to jointly estimate the equity moment with the housing moment. In other words, we will try to find a set of parameters which simultaneously prices equities and housing. But first we take a short detour in order to explain the basic intuition behind the results for α.

4.2.1 Intuition for α

Lets step back and consider the basic intuition for the finding that, in general, housing services and consumption of other goods are substitutes in the period utility function. In the model section above, we showed the following solution to the difference equation in house prices

$$p_t = E_t \sum_{s=0}^{\infty} \left(\prod_{j=1}^{s} m_{j+t} \right) \frac{U_h \left(c_{s+t}, h_{s+t} \right)}{U_c \left(c_{s+t}, h_{s+t} \right)}$$

Lets work for the moment in a world where fluctuations are small enough we can ignore the expectation and where m is well approximated by a constant. In this world, the growth rate of prices should exactly equal the growth rate of user cost. We realize that these assumptions are extreme but they will serve us well for understanding the basic intuition for our house price moments. With our utility function we have the following

$$\Delta \log p = (1 - \alpha) \left(\Delta \log c - \Delta \log h \right)$$

The estimate of α implied by the above equation is sensitive to the end point of the sample. For example, using data from 1970 through 1996, the equation implies an $\alpha \approx 0.5$;while, using data from 1970 through 2004Q2, the equation implies an $\alpha \approx 0.25$. The difference owes almost entirely to differences in the growth rate of house prices in the latter part of the sample and occurs during the time period when the expected capital gain portion of prices may have been abnormally large. In any case, the estimates of α implied by the equation are positive. Given the growth rates of consumption and housing, if α were a negative number, take $\alpha = -1$ as an estimate, the growth rate of house prices would have been more than double the average growth rate observed even if we use the entire sample. In other words, house prices have not grown fast enough over the period in order to support complements in housing.

4.3 Houses and Equity Together

We will conduct a similar exercise in the equity moment as we did on the housing moment. First, as with the housing moment, we will be able to identify only a feasible set of values for α. Figure 3 plots the average error in the equity pricing equation, fixing $\sigma = 5$ and $\beta = 995$. As before, any good numerical algorithm should be able to find the valley and, as with the housing moment, difference between points in the valley are distinguished primarily by numerical error. However, unlike the housing moment here one might be able to identify a definite slope to the valley. While not precise, values of α close to 1 are on average one to two orders of magnitude larger than the errors for αs below zero. In this sense, we feel that the housing moments are not quite compatible with the equity moments. However, we will stick with our identification scheme and admit any solution which falls in the valley of the equity moment. Our task then is quite simple; we must simply identify the intersection of valleys between the house moment and the equity moment for all values of σ and β.

Figure 4 shows, for several values of β, the σ and α pairs such that we satisfy the stock and housing moments simultaneously. Notice, the lowest line in $(\sigma,\ \alpha)$ space is for $\beta = .999$. Therefore, the lowest admissible α which satisfies both the equity and the housing moment is 0.55. As β is reduced, the set of feasible $(\sigma,\ \alpha)$ pairs shrinks towards the point $(\sigma = 1,\ \alpha = 1)$. The set of feasible points becomes empty for β between 0.98 and 0.985. The set of feasible σ and α is defined by the set contained above the curve for $\beta = .999$ and above a line with slope around -.12 and intercept near 1. Hence, larger σ values are associated with higher $\alpha' s$. The minimum elasticity of substitution permissible is 2.2.

One might think that we have come so far simply pick a point within this space with the minimum error (defined as some weighted sum of the error for house prices and the error for equities) and call that point the solution. Unfortunately, the numerical error within these points completely swamps any slope which might exist. Further refinement of the feasible set would require the addition of an additional moment. The natural moment, given that we wish to discuss the equity premium puzzle would be a risk free rate moment.

5 Conclusion

[Introduction and conclusion are not finalized.]

Using the same methodology as above for housing and equity, we examine the moment conditions for the quarterly T-Bill. Following the same procedure once again, we find that we can price T-Bills so long as both α and γ are near 1. The intersection of this set and the set which prices both housing and equity is empty. We find the lowest risk free rate which falls into the feasible set for housing and equities is 11 percent per year. While this is smaller than has been found in the past for the risk free rate, we do not consider it to be a success.

At the end of the day, we are also left with a housing puzzle for several reasons. There is significant autocorrelation in the errors for housing. Perhaps the autocorrelation can be attributed to transaction costs which prevent rapid housing adjustment but as these are unmodelled we do not know. Also, and perhaps most importantly, and not discussed in depth in the literature to date, is that, as housing services grow on average at a different rate than consumption, real interest rates are not stationary without additional assumptions on the utility function. This can be seen immediately from the pricing kernel

$$\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{\gamma+(1-\gamma)\left(h_{t+1}/c_{t+1}\right)^{\alpha}}{\gamma+(1-\gamma)\left(h_t/c_t\right)^{\alpha}}\right)^{\frac{1-\sigma-\alpha}{\alpha}}$$

With this kernel, interest rates will trend over time unless: (A) h and c grow at the same average rate or; (B) $\alpha \in \{1-\sigma,\ 0,\ 1\}$. Under case A, the second term is a constant on average. However, this also implies that, on average, the growth rate of house prices should roughly match the growth rate of the price of consumption. A quick look at 30+ years of data shows that this is absolutely rejected.

Turning to case B. The first case is separable utility. In this case, h is not an argument in the kernel. The second case is Cobb-Douglas. The relative growth rates do not matter, the kernel is stationary so long as both the growth rate of consumption and housing services are each stationary. Finally, with $\alpha = 1$ consumption and housing are perfect substitutes and only the average of their combined growth rate maters.

An alternative possibility, which we are currently exploring, is that the assumption that housing services are linearly proportional to the housing stock where the proportionality factor is constant is a poor choice. If instead housing services are produced as say $zf(H)$, where H is the stock of housing (as before) and z is an unobservable home "productivity shock", then we have much more freedom of choosing the other parameters of the utility function. Indeed, there exists a large class of processes for z such that

$$1 = E\left[m_{t+1} \left(\frac{\gamma + (1-\gamma)\left(\frac{z_{t+1} f(h_{t+1})}{c_{t+1}}\right)^{\alpha}}{\gamma + (1-\gamma)\left(\frac{z_t f(h_t)}{c_t}\right)^{\alpha}} \right)^{\frac{1-\sigma-\alpha}{\alpha}} R_{t+1} \right] = E\left[m_{t+1} g_{t+1}(z) R_{t+1}\right]$$

interest rates are stationary and (we suspect) the moment condition for housing is also satisfied. The fact that z is fundamentally unobservable gives us broad freedom in matching moment conditions. Although housing services are produced and z is viewed as a productivity parameter some discipline may be found in the home production literature itself (see for example Greenwood, Rogerson, and Wright (1995)). But this remains to be seen. Everything lies in the interpretation of $g(z)$, for example, this would also correspond to the liquidity constraint multiplier used by LVN.

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Figure 1: Housing Correlation Moment Sigma=5, Beta=.995

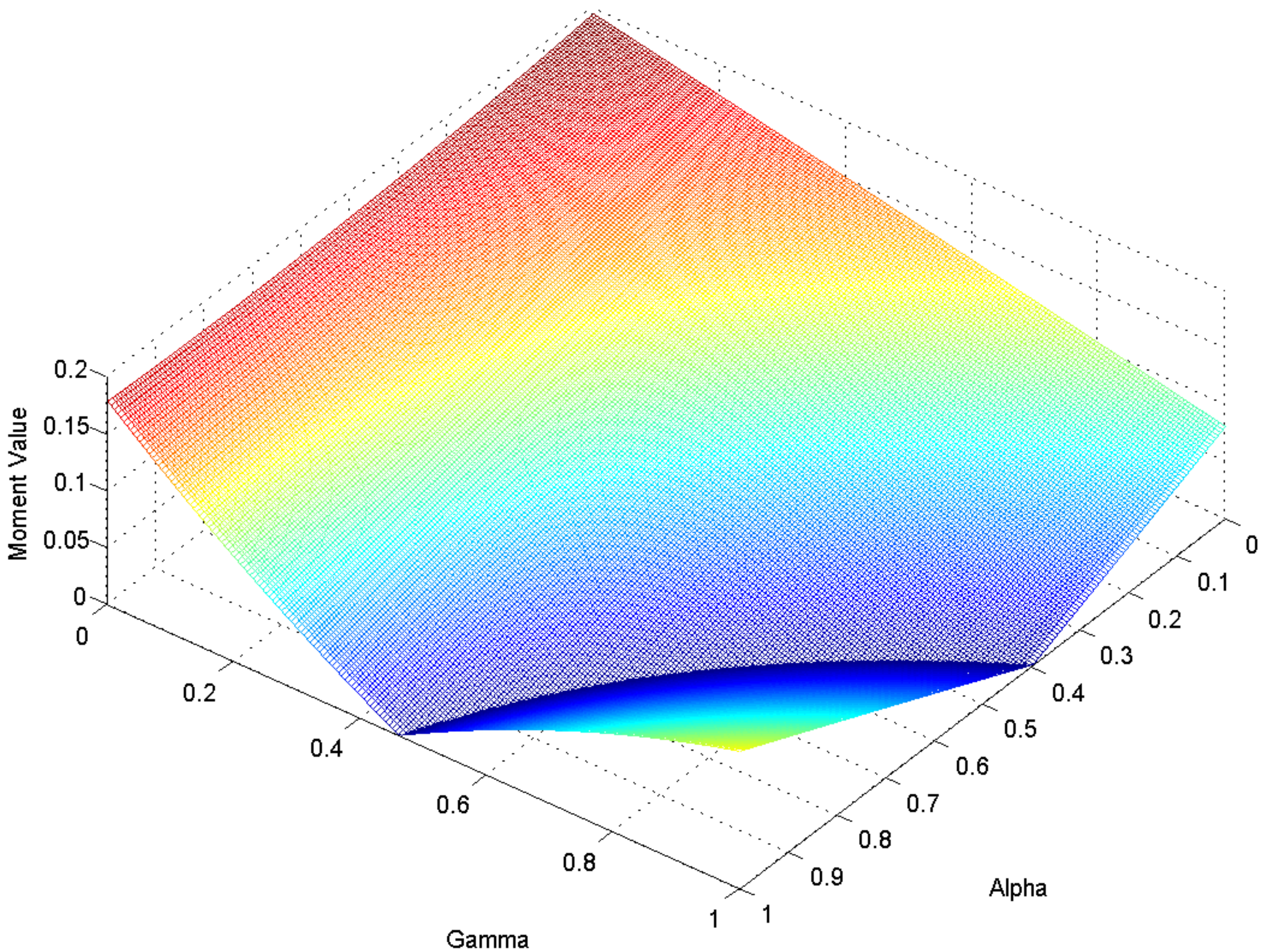

Figure 2: Minimum Alpha Consistent with Housing Moment
Beta=.95 to .999

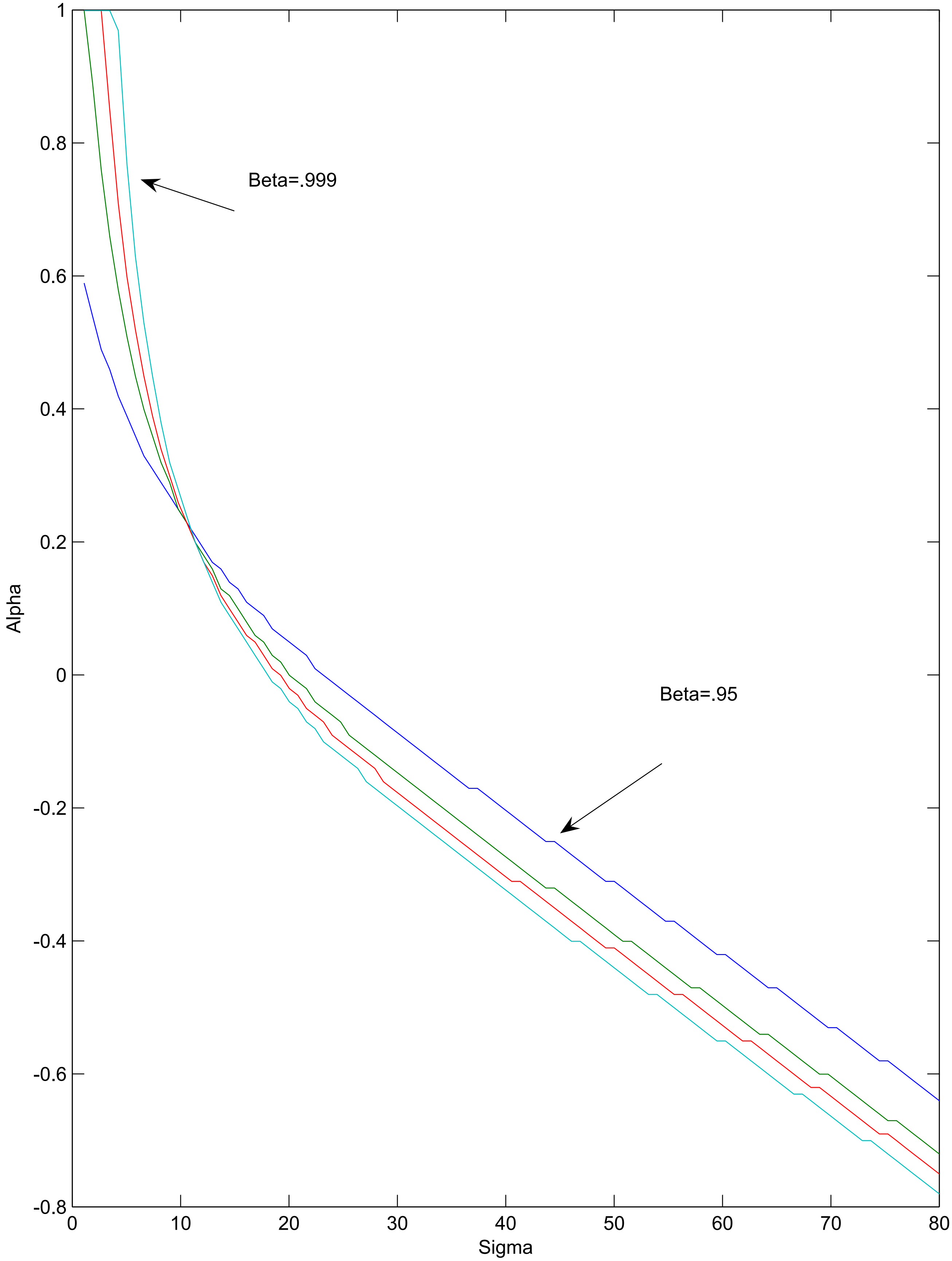

Figure 3: Equity Moment Sigma=5, Beta=.995

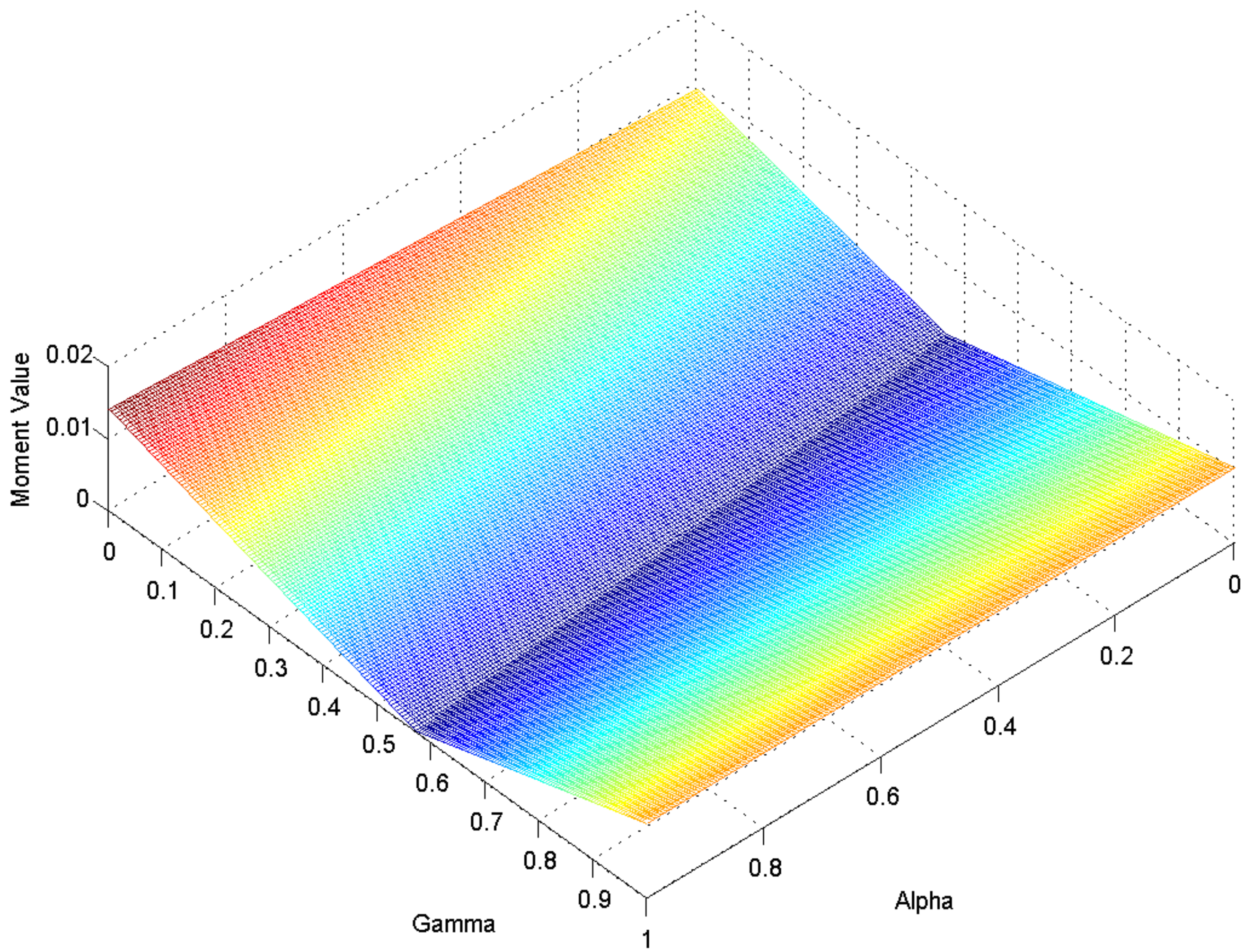

Figure 4: Feasible Parameter Set: Equities and Housing