# Private Debt and Idiosyncratic Volatility: A Business Cycle Analysis

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[ VERY PRELIMINARY ]

**Abstract**

I develop and estimate a heterogeneous agents business cycle model featuring aggregate and idiosyncratic shocks on the one hand, collateralized and uncollateralized debt on the other. I use the model to ask two sets of questions: (1) what has caused the large increase in private debt over GDP in the US in the last 30 years? (2) can the model account for some features of the distribution of assets and expenditure across the population?

In answer to (1), I find that most of the increase in private debt can be explained by a rise in the volatility of idiosyncratic shocks (coupled with the unit root behavior of idiosyncratic debt itself). In response to (2), I find that the model developed can explain not only the trend but also the cyclical behavior of financial assets over time, as well as the dynamics in earnings and consumption inequality which are found in the data. (JEL E31, E32, E44, E52, R21)

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Debts of households and businesses have since the 1980s jumped out of proportion with real activity. This phenomenon has occurred alongside two separate and striking changes in economic volatility. On the one hand, aggregate volatility has fallen: in the US, the standard deviation of GDP growth has roughly halved between the period 1960-1983 and the period 1983-2002. On the other, cross-sectional risk has risen, as well as earnings inequality between and within income groups.

This paper constructs a dynamic general equilibrium model to understand the interaction between the interaction between income volatility, private sector financial balances, and the distribution of consumption. In detail, I construct a three-agents general equilibrium model with idiosyncratic income shocks, aggregate income shocks and financial shocks and ask whether the model helps replicating the historical behavior of financial assets in the US in the last three decades. In particular, I address the following questions:

1. What has caused the rise in debt? Does the increase in borrowing reflect the response of agents to easing liquidity constraints, the reduction in aggregate uncertainty or the rise in individual income volatility?
2. Does the causality running from the primitive shocks to the quantity of debt feeds back into the observed macroeconomic volatility?

The key model features are heterogeneity in discount rates, market incompleteness (in the form of liquidity constraints for some of the agents), idiosyncratic income shocks and financial shocks. Three agents populate the economy: they issue bonds, produce a final good using capital and labor and consume durables and non-durables. Agents 1 and 2 only differ in their idiosyncratic productivity. Idiosyncratic shocks give rise to a desire to borrow and lend. Agent 3 also differs in preferences: being more impatient, agent 3 would like to borrow at the equilibrium interest rate. This agent faces borrowing constraints tied to durable values. Changes in the tightness of borrowing constraints affect how much agent 3 borrows from 1 and 2.

Using annual data on between and within-group income inequality, I estimate stochastic process for the idiosyncratic income shocks; using data on loan-to-value ratios and productivity, I estimate processes for "financial" shocks and aggregate productivity shocks. Once the shocks are estimated, I simulate the model to test whether the timing of the shocks can explain the patterns in the data, in particular the trend and the cyclical behavior of private debt, and the distribution of consumption and income across the population.

The key finding of the paper lies in the ability of a heterogeneous agents model to explain at the same time two salient feature of the data:

1. On the one hand, the model can explain the timing and the magnitude of rise in private debt over GDP. Debt rises when the cross-sectional risk increases and when borrowing constraints become

looser. Of the total variation in debt, a large fraction is due to increased idiosyncratic uncertainty, and a smaller one to financial liberalization (time variation in the tightness of the borrowing constraints).

2. On the other, the model can reconcile the sharp increase in income inequality over the last 30 years with a modest rise in consumption inequality over the same time period.

# 1 The model

## 1.1 Overview

There are three agents. Each agent works, produces and accumulates assets over time. The mass of each set of agents is normalized to 1. Absent shocks, patient agents only differ in their total productivity: hence *within-group* differences between them can be ascribed to some exogenous, unobservable fixed effect. Impatient agents differ from patient agents along two key dimensions: they discount the future more heavily, and they face borrowing constraints which limit the amount of assets they can trade in order to smooth consumption. I refer to differences between impatient and patient agents as *between-group* differences. I now describe the behavior of each agent.

## 1.2 Patient Agent 1

Each agent in the economy is subject to an aggregate productivity shock and to an idiosyncratic shock to its own productivity. For each agent, the production function used to produce output $y$ combines capital $k$ and labor $l$ through a standard constant returns to scale technology. The resulting output can be either consumed, invested in business capital $k$ (which depreciates at rate $\delta_k$) or in durables / housing capital $h$ (which depreciates at rate $\delta_h$ and provides utility services).

The patient agent 1 chooses consumption $c$, durables $h$, capital $k$ and labor supply $l$ in order to maximize:[1]

$$\max E_0 \sum_{t=0}^{\infty} \beta'^t \left( \log c'_t + j \log h'_t - \tau' \frac{(l'_t)^{1+\eta'}}{1+\eta'} \right).$$

The budget constraint and production function are respectively:

$$c'_t + \left(h'_t - (1-\delta_h)\, h'_{t-1}\right) + \left(k'_t - (1-\delta_k)\, k'_{t-1}\right) + b'_t + \phi_t^{b'} - d_t + \phi_t^d = y'_t + R'_{t-1} b'_{t-1} - R_{t-1} d'_{t-1} \quad (1)$$

$$y'_t = \frac{A_t}{W_t Z_t^{\zeta}} k'^{\mu}_{t-1} \left(X_t l'_t\right)^{1-\mu}. \quad (2)$$

where $b'_t$ are loans made by agent 1 to the impatient agent 3, whose behavior will be described below, and $d$ is borrowing of agent 1 from agent 2.

[1] The preferences here are consistent with balanced growth. See King and Rebelo (1999).

In the production function, $A_t$ reflects aggregate total factor productivity (TFP), which is assumed to follow an $AR(1)$ process in logs. Shocks to TFP are perfectly correlated across all agents. To capture secular growth in output, I assume that there is a deterministic component to productivity $X_t$ which is assumed to expand at a constant rate over time. That is:

$$X_t = (1+g)\, X_{t-1}.$$

Finally, $W_t$ and $Z_t$ reflect instead idiosyncratic shocks (which are assumed to follow $AR(1)$ processes in logs). Shocks to $W_t$ only affect the productivity of patient agents and are negatively correlated across agents: given that patient agents are identical except that for the scale $A$ of their production function, shocks to $W_t$ can be interpreted as a source of within-group income volatility. Conversely, shocks to $Z_t$ affect all agents: when these shocks hit positively the patient agents, they also hit negatively the impatient agents: such shocks can be interpreted as a source of between-group income inequality.

In addition, for the generic bond $i$, I also assume a trading cost of the form:

$$\phi_t^i = \phi \frac{\left(i_t - (1+g)\, i_{t-1}\right)^2}{2X_t}$$

The first order conditions for this problem are:

$$\frac{1}{c_t'}\left(1 + \frac{\phi}{X_t}\left(b_t' - (1+g)\, b_{t-1}'\right)\right) = E_t\left(\frac{\beta'}{c_{t+1}'}\left(R_t' + \frac{\phi}{X_t}\left(b_{t+1}' - (1+g)\, b_t'\right)\right)\right) \tag{3}$$

$$\frac{1}{c_t'}\left(1 - \frac{\phi}{X}\left(d_t' - (1+g)\, d_{t-1}'\right)\right) = E_t\left(\frac{\beta'}{c_{t+1}'}\left(R_t - \frac{\phi}{X_t}\left(d_{t+1}' - (1+g)\, d_t'\right)\right)\right) \tag{4}$$

$$\frac{1}{c_t'} = \frac{j'}{h_t'} + \beta' E_t\left(\frac{1-\delta_h}{c_{t+1}'}\right) \tag{5}$$

$$\frac{1}{c_t'} = \frac{\beta}{c_{t+1}'}\left(\frac{\mu y_{t+1}'}{k_t'} + 1 - \delta_k\right) \tag{6}$$

$$\frac{1}{c_t'}(1-\mu)\frac{y_t'}{l_t'} = \tau'\left(l_t'\right)^{\eta'}. \tag{7}$$

I assume that these agents' asset position is such that the borrowing constraints that they face are never binding. This assumption is safe if their wealth is large enough of if their maximum borrowing limit is large enough relative to their wealth.

### 1.3 Patient agent 2

Patient agens 2 are indexed by a double prime (″). They are identical to agent 1, except for the scale of their production function (in steady state, this is chosen so as to match observed within-income inequality in the data). These agents can lend (borrow) $d_t$ $(-d_t)$ to patient agent 1 and can lend $b_t''$ to impatient agent 3. Their production function is:

$$Y_t'' = \frac{A_t W_t^{\theta}}{Z_t^{\gamma}} k_{t-1}''^{\mu}\left(X_t l_t''\right)^{1-\mu}.$$

The parameters $\theta$ and $\gamma$ are chosen in a way to guarantee that, keeping capital and labor input fixed, the aggregate effect of given idiosyncratic shocks $Z_t$ and $W_t$ are zero (the restrictions on $\theta$ and $\gamma$ will be derived in the next section).

### 1.4 Impatient agent 3

Impatient agents are assumed to discount the future more heavily than agents 1 and 2 and to face a liquidity constraint that limits the amount of borrowing to their durable assets. With this very simple assumption, I want to capture the idea that durables can be used as a form of collateral.

The problem the impatient agent solves is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_t + j \log h_t - \tau \frac{(l_t)^{1+\eta}}{1+\eta} \right)$$

subject to the following budget constraint, where $b'$ and $b''$ denote respectively borrowing from patient agent 1 and patient agent 2:

$$c_t + h_t - (1-\delta_h) h_{t-1} + R''_{t-1} b''_{t-1} + R'_{t-1} b'_{t-1} + k_t - (1-\delta_k) k_{t-1} + \phi_t^{b'} + \phi_t^{b''} = Y_t + b'_t + b''_t \tag{8}$$

where

$$y_t = A_t Z_t k_{t-1}^{\mu} (X_t l_t)^{1-\mu}$$

and the borrowing constraints are respectively:

$$b'_t < \alpha m_t h_t / R_t \tag{9}$$

$$b''_t < (1-\alpha) m_t h_t / R''_t. \tag{10}$$

In the borrowing constraint, $\alpha$ is the fraction of collateral which is pledged respectively to each of the two agents. Since any value of $\alpha$ maximizes the borrowing capacity of the agent, I set $\alpha$ to be equal to the relative income of each patient agent: that is, $\alpha = y' / (y' + y'')$. The first order conditions are:

$$\frac{1}{c_t}\left(1 - \frac{\phi}{X_t}\left(b'_t - (1+g) b'_{t-1}\right)\right) = E_t \left( \frac{\beta R'_t}{c_{t+1}} \left(1 - \frac{\phi}{X_{t+1}} \left(b'_{t+1} - (1+g) b'_t\right)\right)\right) + \lambda'_t \tag{11}$$

$$\frac{1}{c_t}\left(1 - \frac{\phi}{X_t}\left(b''_t - (1+g) b''_{t-1}\right)\right) = E_t \left( \frac{\beta R''_t}{c_{t+1}} \left(1 - \frac{\phi}{X_{t+1}} \left(b''_{t+1} - (1+g) b''_t\right)\right)\right) + \lambda''_t \tag{12}$$

$$\frac{1}{c_t} = \frac{j}{h_t} + \beta E_t \left( \frac{1-\delta_h}{c_{t+1}} \right) + m_t \left( \frac{\alpha \lambda'_t}{R'_t} + \frac{(1-\alpha)\lambda''_t}{R''_t} \right) \tag{13}$$

$$\frac{1}{c_t} = E_t \frac{\beta}{c_{t+1}} \left( \mu \frac{y_{t+1}}{k_t} + 1 - \delta_k \right) \tag{14}$$

$$\frac{1}{c_t}(1-\mu)\frac{y_t}{l_t} = \tau (l_t)^{\eta}. \tag{15}$$

It is straightforward to show that, in a neighborhood of the steady state, agent 3 will be borrowing constrained and the multiplier $\lambda$ on the borrowing constraint will be stricly positive.[2]

[2] See Iacoviello (forthcoming) for a related application.

### 1.5 Markets and equilibrium

The definition of equilibrium is standard. In equilibrium, all the markets clear and the interest rates work to equate demand and supply in the goods market and in the market for bonds.

### 1.6 The shocks, and how to measure them

The economy is hit by four shocks: an aggregate productivity shock, a financial shock, and two types of idiosyncratic shocks hitting differently the different agents in the economy.

It is instructive to think of the two patient/unconstrained agents as otherwise identical agents. Shocks to the income of one relative to the other are therefore a source of *within-group* earnings inequality in the population. Instead, shocks to the income of constrained agents relative to the unconstrained agents are a source of *between-group* earnings inequality.

I extract measures of the shocks from the data as follows: I take a total measure of income inequality over time (broken down by within-group and between-group) and recover from such measure the time series for the idiosyncratic shocks. The next section explain this procedure in more detail.

## 2 Recovering idiosyncratic shocks from earnings inequality

### 2.1 The data

Several papers have documented upward trends in income inequality in the US in the last 30 years (see for instance Kats and Autor, 1999, and Moffitt and Gottschalk, 2002). I follow common practice and decompose the overall income distribution[3] into differences in wages between groups (defined by skill, demographic or observable categories) and within group wage dispersion (residual wage inequality, measuring the effect of unobservables).

Krueger and Perri (2003), among the others, document the levels and the trends of the above variables. Disposable earnings variance was 36% in 1970, and had risen to about 53% in 2000. Between and within variances were respectively 4% and 32% in 1970, and 8% and 45% in 2000. They also document another important piece of information: consumption inequality only rises by a small amount throughout the same period, mostly driven by an increase in between-group consumption inequality.

[3]Since labor income is 2/3 of total income earned, I refer for now to earnings-wage-income inequality interchangeably. Future work should relax this assumption.

### 2.2 The methodology: within $(w)$ and between-group $(z)$ variance

Setting aside time variations in labor and capital input, total factor and deterministic productivity,[4] I can write the agents' log income processes as

$$\begin{aligned}\log y_t &= \log y_0 + \log z_t \\ \log y_t' &= \log y_1 - \log w_t - \zeta \log z_t \\ \log y_t'' &= \log y_2 + \theta \log w_t - \gamma \log z_t\end{aligned}$$

$\log(z)$ and $\log(w)$ are shocks which are normalized to zero in period 0. By definition, I want the shocks to satisfy the following conditions:

1. Idiosyncratic shocks must be such that total income would be unchanged keeping hours and capital input constant. It is easy to show that this implies the following restrictions on $\theta$, $\gamma$ and $\zeta$:

$$\begin{aligned}\theta &= y_1/y_2 \\ y_0 &= \zeta y_1 + \gamma y_2\end{aligned}$$

2. Idiosyncratic shocks must be such that between-group $z_t$ shocks do not affect *within* group log income inequality. From the formula of within income inequality:

$$V_{wt} = \frac{2}{3}\left(\frac{1}{2}(\log y_1 - \log y_2) - \frac{(1+\theta)}{2}\log w_t + \frac{(\gamma-\zeta)}{2}\log z_t\right)^2$$

this requires $\zeta = \gamma$. Hence:

$$\gamma = \frac{y_0}{y_1 + y_2}$$

3. Given these conditions, once within group shocks are calculated, between group shocks will be extracted as the residual consistent with conditions (1) and (2).

### 2.3 Within-group log variance

Within-group variance measures inequality between patient agent 1 and agent 2, who otherwise have same preferences. I am interested in recovering a time series for their income shock $\log w_t$ which is consistent with known observations about time variations in the within group log-income variance $V_w$, which is observable from the data. At each point in time, within group variance is given by:

$$V_{wt} = \frac{(\log y_1 - \log w_t - E_H)^2 + (\log y_2 + \theta \log w_t - E_H)^2}{3}$$

[4] By this, I mean that these terms are all subsumed into the constants $Y_0, Y_1$ and $Y_2$.

where $E_H = \frac{1}{2}\left(\log y_1 + \log y_2 - (1-\theta)\log w_t\right)$. At time zero, all shocks are normalized to zero, so that (using the normalization $\log y_2 > \log y_1$)

$$V_{w0} = \frac{1}{6}\left(\log y_2 - \log y_1\right)^2$$

Simple algebra can be shown to derive vector of shock processes which is consistent with given observations on log income inequality, starting from an arbitrary level $V_{w0}$:

$$\log \mathbf{w}_t = \frac{1}{1+\theta}\left(\sqrt{6V_{wt}} - \sqrt{6V_{w0}}\right)$$

### 2.4 Between-group variance

Between-group variance measures inequality between log income of the impatient agent and the mean between log incomes of agents 1 and 2, that is:

$$V_{bt} = \frac{1}{3}\left(\left(\log y_0 + \log z_t - E\right)^2 + 2\left(\frac{1}{2}\left(\log y_1 - \log w_t + \log y_2 + \theta \log w_t\right) - E\right)^2\right)$$

where $E = \frac{1}{3}\left(\log y + \log y' + \log y''\right)$ is the mean of log incomes across all agents. Plugging in the formula for $E$ gives:

$$V_{bt} = \frac{1}{18}\left(2\log y_0 - \log y_1 - \log y_2 + 2\left(1+\gamma\right)\log \mathbf{z} + \left(1-\theta\right)\log \mathbf{w}\right)^2.$$

At time 0, shocks equal zero, so that

$$V_{b0} = \frac{1}{18}\left(2\log y_0 - \log y_1 - \log y_2\right)^2.$$

Thus:

$$\log \mathbf{z} = \frac{1}{2\left(1+\gamma\right)}\left(\sqrt{18V_{b0}} - \sqrt{18V_b} - \left(1-\theta\right)\log \mathbf{w}\right)$$

where $\gamma$ depends on the initial steady state.

### 2.5 Initial steady state

In the initial steady state of the model, $\mathbf{w}_0 = 0$ and $\mathbf{z}_0 = 0$. Hence I set:

$$\log y_2 = \log y_1 + \sqrt{6V_{w0}}$$

so that within group variance replicates the value found in the data. Analogously, in order to replicate between group inequality, I set:

$$\log y_1 = \log y_0 + \frac{1}{2}\left(\sqrt{18V_{b0}} - \sqrt{6V_{w0}}\right).$$

## 3 Calibration and simulation

To check whether the model can account for the main stylized facts, I use the following procedure:

1. I calibrate the structural parameters of the model, so that the initial steady state matches key observations of the US economy in the year 1969. In detail, I set the parameters describing preferences and technology so that in the initial steady state the debt to GDP ratio, the ratios of the components of spending to output and the distribution of income replicate the data.

2. I estimate from the data the sequence of technology shocks, financial shocks and between and within-group income shocks.

3. I feed the estimated shocks into the model decision rules (calculated under the assumption that the shocks that hit the economy are drawn by the same distribution from which the decision rules are calculated) starting from the year 1969, and check whether the time series generated from the model can replicate the cyclical and trend behavior of financial assets, income inequality and consumption inequality.

### 3.1 Calibration

The time period is set equal to one year. This reflect the lack of higher frequency measures of income inequality over time, which are needed to recover the processes for the idiosyncratic shocks.

Table 3.1 summarizes the calibrated parameters. As explained above, these parameters are meant to capture the initial (year 1969) steady state distribution of income and financial assets, as well as the consumption, business fixed and housing investment to output ratios. Given that patient agents do not face borrowing constraints, their discount factor (together with the deterministic growth rate of the economy $g$, which is estimated to be 1.6 percent per annum) pins down the steady state real interest rate at 3% per year.

In this version, the cost of bond transactions is assumed to be small for all agents. Labor supply elasticity is assumed to be $1/\eta = 1/3$, a number which is in the range of microeconometric estimates of labor supply elasticity.[5] The capital share $\mu$ and the housing preferences parameter $j$ are chosen so as the match the steady state stocks and flow of structures and investment which are found in the data. It is assumed that housing depreciates more slowly than business capital. The discount factor for impatient agents is assumed to be 0.9 (see Iacoviello (forthcoming) for a discussion).

In the aggregate, these choices result in 29% fixed investment over output ratio, 4% housing investment output ratio, 67% non-durable consumption output ratio. These data are roughly consistent with US post-world war II experience.

[5] See for instance Browning, Hansen and Heckman (1999).

$$
\begin{aligned}
\beta &= 0.9 \\
\beta' &= \beta'' = 0.985 \\
\mu &= 0.33 \\
j &= j' = j'' = 0.1 \\
\delta_k &= 0.1 \\
\delta_h &= 0.01 \\
\eta &= \eta' = \eta'' = 3 \\
V_{b0} &= \sqrt{0.04} \\
V_{w0} &= \sqrt{0.30} \\
m &= 0.75 \\
\phi &= 0.05
\end{aligned}
$$

Table 3.1: Calibrated Parameter Values

In order to match income variability in the data, I pick $(A, A', A'')$ so as to match cross-sectional income volatility at the beginning of the sample. In particular, the values of $A$'s are chosen so that in the initial steady state $y = 1$, $y' = 0.77$ and $y'' = 3.03$. These values imply that moving from the initial steady state, a relative increase in $y''$ (coming from positive shocks to $W_t$) versus $y'$ leads to an increase in within-group income inequality, whereas a relative increase in $y$ (coming from positive shocks to $Z_t$) versus $y'$ and $y''$ leads to a decrease in between-group income inequality.

In addition, at the beginning of the sample period, total debt over GDP is 91%. I pick a typical loan-to-value ratio $m = 75\%$ at the beginning of sample period. This results in a ratio of collateralized debt to output of 25%. The unconstrained debt cannot be pinned uniquely down in the steady state of the model, however we can choose it so that total debt matches its beginning of period value: that is, unconstrained debt is chosen to be $91\% - 25\% = 66\%$. A final decision has to be made as to whether patient agent 1 or patient agent 2 is a borrower or a lender. Since in the data I observe an increase in debt over time alongside an increase in within-group income inequality over time, it is self-evidence that the model can explain a rise in debt only if the poorer agent is a borrower at the beginning of the sample period. This is also consistent with the basic stylized facts that show how rich agents have, in general, higher saving rates (see for instance Dynan, Skinner and Zeldes, 2004).

### 3.2 Stochastic processes for income and financial liberalization

The stochastic processes for the shocks are calculated as follows:

1. I measure the technology shock from the BLS Manufacturing Multifactor Productivity Series. To decompose this series into deterministic trend and stochastic cycle, I fit a linear trend to this series to extract $\gamma$, the deterministic productivity component. The residual from this regression is then assumed to follow an $AR(1)$ process and used to construct $\log(A_t)$. That is,

$$\log A_t = \rho_A \log A_{t-1} + e_{At}$$

where $e_{At}$ is verified to be iid over time.[6]

2. The financial shock / financial liberalization measure is perhaps the hardest to construct. Likely candidates would include:

    1. observed measures of loan-to-value ratios: these are available from the Federal Reserve Federal Housing Board (see http://www.fhfb.gov/mirs/mirs_downloads.htm). These are also readily identifiable from the data;
    2. measures of housing affordability (see www.realtor.org);
    3. measures of home ownership rates (see www.census.gov);
    4. measures of the ability of banks to recover loans.

    In reality, financial liberalization in the United States has been a combination of (1) increased credit market access and (2) increase in loan-to-value ratios. All these elements have occurred through a variety of channels and it is impossible to expect that the time variation in $m_t$ can capture much of this.

    Therefore, any measure entirely based on $(a)$ is likely to underestimate the growth in credit due to financial liberalization. For now, I simply take loan-to-price ratios as a measure of financial shocks and assume that they vary stochastically over time according to an $AR(1)$ process. This way, I can construct a measure of time-varying liquidity constraints, which gives me the process for $\log(m_t)$.

3. The idiosyncratic shock processes are calculated using the methods described above.

Figure 1 shows the time series for the shock processes constructed normalized to 0 in the 1969, which is taken to be the base year. The increase in $W_t$ over time reflects an increase in within group income inequality, since income of patient agent 2 (who starts richer) rises over time at the expense of income of agent 1. The decrease in $Z_t$ over time reflect instead a decline over time of the income share of the impatient agents, which feeds back into a widening between group income inequality. For $\log(m)$, $\log(w)$,

[6] See King and Rebelo (1999) for details. I use the Manufacturing Multifactor Productivity (MFP). This is the most widely reported measure of technical change, also referred to in other contexts as total factor productivity or the Solow residual.

$\log(z)$ and $\log(A)$ the autocorrelations are found to be respectively $\rho_m = 0.78$, $\rho_w = 0.95$, $\rho_z = 0.98$, $\rho_A = 0.82$. It is interesting to notice how idiosyncratic shocks are very estimated to be very persistent: trivially, the high persistence of these shocks reflects the extremely high persistence in the movements in income inequality.[7]

# 4 Evidence on the behavior of the model

## 4.1 Impulse Responses

Figure 2 shows the basic workings of the model, by showing the impulse responses of the variables to each of the shocks constructed above.

The key message of the impulse responses of Figure 2 is that even purely idiosyncratic shocks can have aggregate effects by causing heterogeneous responses of investment and labor supply on part of the different agents.

## 4.2 Model simulations

I start with the findings which are "too dependent" on the properties of the estimated shocks to be taken as a serious measure of the success of the model in explaining the data.

### 4.2.1 You get what you put into it

- *The model captures the behavior of GDP over the last 30 years*

  Remember that GDP (as well as all other trending variables) has been detrended with the linear trend extracted from the multifactor productivity processes. As shown from Figure 3, the remaining cyclical component moves together with the model counterpant. This is not surprising, since GDP behavior is mostly driven by the exogenous technology shock, something which is well known in the real business cycle literature (see for instance King and Rebelo, 1999).

- *The model captures the evolution of income inequality over the last 30 years.*

  Income inequality is mainly driven by the idiosyncratic shocks and by the heterogeneous responses in capital and hours worked from the agents. So long as labor supply is not too elastic and given the small share of fixed capital in production, it is not surprising that the model replicates the behavior of between and within income inequality extremely well. See the top two panels of Figure 4.

[7] Krueger and Perri (2004) retrieve the persistence of the idiosyncratic income processes by estimating income processes for classes of individuals in the Consumer Expenditure Survey. Their measure of persistence (see Table 1) is the second largest eigenvalue of a transition matrix and cannot be directly compared to ours.

### 4.2.2 The success stories of the model

1. *The model captures the trend behavior of debt over GDP.*

   In the data, debt over GDP rises in the data from 90% to 165%. In the model, it rises from 90% to a very similar magnitude. See top-left panel of Figure 5.

2. *The model captures the cyclical behavior of debt.*

   See the bottom panels of Figure 5. I do so by comparing the first difference of log debt in the model and in the data. The correlation coefficient between the two is positive and different from zero.

3. *The model predicts a modest rise of consumption inequality, as found in the data, despite the strong increase in income inequality.*

   As in the data, I find that the model predicts a modest rise in within-group consumption inequality,[8] while it also predicts a sharper increase in between-group inequality (see the bottom-right panel of Figure 4). However, unlike in Krueger and Perri (2003) model, the result needs not to rely on endogenous developments in the credit markets.

4. *The model attributes most of the increase in debt to idiosyncratic volatility, in particular to within group shocks.*

   This is shown in the top-right panel of Figure 5.

   Of the increase in debt over GDP, a huge fraction is due to increased idiosyncratic volatility. This might appear at first sight surprising, since one is led to believe that all the debt claims issued by agent 1 and 2 refer to unsecured debt. In fact, this debt might well be backed by collateral: the model specification simply assumes that for these agents wealth holdings are large enough that the collateral constraint is never binding.

   Thus point raises in fact one thorny issue for the model. At the beginning of the sample period, agents balance sheets and financial positions (relative to their incomes) are as follows (notice that positive values of $d$ or $b$ indicate a debt):

| | $y$ | $c/y$ | $h/y$ | $k/y$ | $d/y$ | $b/y$ | $(d+b)/y$ |
|---|---|---|---|---|---|---|---|
| Impatient agent | 1 | 0.77 | 1.40 | 1.55 | 0 | 1.09 | 1.09 |
| Patient agent 1 | 0.77 | 0.59 | 1.52 | 2.74 | 4.33 | −0.29 | 4.03 |
| Patient agent 2 | 3.03 | 0.66 | 1.70 | 2.74 | −1.09 | −0.29 | −1.38 |
| Aggregate economy | | 0.67 | 1.61 | 2.49 | 0 | 0 | 0 |

[8] Attanasio, Battistin and Ichimura (2004) argue against the evidence shown by Krueger and Perri (2003), suggesting that within-group consumption inequality has risen in the last 30 years. See also Attanasio and Davis (1996).

Hence, at the beginning of the sample period, even the most indebted agents (patient 1) have enough assets, in the form of housing and business capital, to make up for most of the debt. In a sense, therefore, their net worth is still positive. Over the simulation period, as they pile up debt faster than net worth, their net worth becomes slightly negative, so one can imagine that they are forced to rely on "unsecured" debt to finance their negative idiosyncratic shocks, something which seems to be entirely consistent with the recent US experience.[9]

## 5 Conclusions

The paper which is most closely related to mine is Krueger and Perri (2003).[10] They argue that, in the data, consumption inequality has risen much less than income inequality. They present a model of endogenous market incompleteness in which the incentive to trade assets is directly related to the uncertainty faced at the individual level. They show that only such a model is able to predict a modest *decrease* in within-group consumption inequality alongside an increase in between-group consumption inequality.

In my model, the increase in standard deviation of log consumption within groups is very small. My overall results of a modest rise consumption inequality are also in line with the findings of, among the others, Attanasio, Battistin and Ichimura (2004).

From the methodological point of view, the main element of novelty of my paper lies in its ability to show how a tractable dynamic general equilibrium model with heterogeneous agents can provide a careful and sensitive description of trends not only of income over time, but also of the distribution of consumption and holdings of financial assets across the population. In addition, I focus not only on consumption, but also on durables and on the behavior of financial assets. Moreover, I retrieve the actual shocks and feed them in the model, in order to see whether the model can explain not only the trend but also the cyclical behavior of consumption, income and financial assets. Understanding the cyclical behavior of consumption and income inequality can provide better evidence of the success and the relative merits of different business cycle theories in explaining the evolution of financial assets and macroeconomic variables over time.

[9] See discussion in Sullivan (2004).

[10] See also the work by Campbell and Hercowitz (2004), although their main focus is on collateralized debt.

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## Appendix A: The steady state

Define variables with a bar on them the variables normalized by $X$. That is, $c/X = \overline{c}, y/X = \overline{y}$ and so on...

Let $\zeta_{\#}$ be constants which depend on the structural parameters, and let $\varphi_{\#}$'s be constants which are functions of $\zeta_{\#}$'s and structural parameters. Normalize the $\tau$ so that $l = l' = l'' = 1$. Let $d' = d > 0$ (double prime agent lends to single prime agent if $d > 0$). Set lending shares to constrained are a fraction of relative income $\alpha = Y_1/\left(Y_1 + Y_2\right)$. Let $r = R - 1$.

Let $\frac{\delta_h+g}{1+g} = \delta_{h0}$, $\frac{\delta_k+g}{1+g} = \delta_{k0}$, $r_0 = \frac{r-g}{1+g}$. The steady state can be described as follows:

$$R = (1+g)/\beta' = (1+g)/\beta''$$

$$\overline{\lambda} = \left(1 - \beta/\beta'\right)/\overline{c}$$

$$\overline{k} = \frac{\beta\mu(1+g)}{1+g-\beta(1-\delta_k)}\overline{y} = \zeta_0\overline{y}$$

$$\overline{k'} = \frac{\beta'\mu(1+g)}{1+g-\beta'(1-\delta_k)}\overline{y'} = \zeta_1\overline{y'}$$

$$\overline{k''} = \frac{\beta''\mu(1+g)}{1+g-\beta''(1-\delta_k)}\overline{y''} = \zeta_2\overline{y''}$$

$$\overline{h} = \frac{j(1+g)}{1+g-\beta(1-\delta_h)-(\beta'-\beta)m}\overline{c} = \zeta_3\overline{c}$$

$$\overline{h'} = \frac{j'(1+g)}{1+g-\beta'(1-\delta_h)}c' = \zeta_4\overline{c'}$$

$$\overline{h''} = \frac{j''(1+g)}{1+g-\beta''(1-\delta_h)}c'' = \zeta_5\overline{c''}$$

$$\overline{b} = m\overline{h}/R = (m\zeta_3/R)\,\overline{c} = \varphi_5\overline{c}$$

$$\overline{b'} = \alpha\overline{b}$$

$$\overline{b''} = (1-\alpha)\,\overline{b'}$$

$$\overline{c} + \delta_{h0}\overline{h} + \delta_{k0}\overline{k} = \overline{y} - r_0\overline{b}$$

$$\overline{c} = \frac{1-\delta_{k0}\zeta_0}{1+\delta_{h0}\zeta_3 + r_0\varphi_5}\overline{y} = \varphi_0\overline{y}$$

$$\overline{c'} + \delta_{h0}\overline{h'} + \delta_{k0}\overline{k'} = \overline{y'} + \alpha r_0\overline{b} - r_0\overline{d}$$

$$\overline{c'} = \frac{1-\delta_{k0}\zeta_1}{1+\delta_{h0}\zeta_4}\overline{y'} + \frac{\alpha r_0\varphi_5\varphi_0}{1+\delta_h\zeta_4}\overline{y} - \frac{r_0}{1+\delta_h\zeta_4}\overline{d} = \varphi_1\overline{y'} + \varphi_2\overline{y} - \varphi_{d1}\overline{d}$$

$$\overline{c''} + \delta_{h0}\overline{h''} + \delta_{k0}\overline{k''} = \overline{y''} + (1-\alpha)\,r_0\overline{b} + r_0\overline{d}$$

$$\overline{c''} = \frac{1-\delta_{k0}\zeta_2}{1+\delta_{h0}\zeta_5}\overline{y''} + \frac{(1-\alpha)\,r_0\varphi_5\varphi_0}{1+\delta_{h0}\zeta_5}\overline{y} - \frac{r_0}{1+\delta_h\zeta_5}\overline{d} = \varphi_3 y'' + \varphi_4 y + \varphi_{d2} d$$

This set of equations offers a complete characterization of the steady state.

## Appendix B: The linearized model

In the initial steady state only labor supply and interest rates are constant, whereas all the other variables grow at a constant rate given by $g$. The model below is written in terms of the variables scaled by $X$, the deterministic productivity component. The shocks to $m$, $w$, $z$ and $A$ follow the $AR(1)$ processes described in the text.

Endogenous states: $h \quad h' \quad h'' \quad k \quad k' \quad k'' \quad b' \quad b'' \quad d \quad l \quad l' \quad l'' \quad \lambda' \quad \lambda'' \quad R' \quad R'' \quad R \quad Y \quad C \quad H \quad K$

Endogenous jump variables: $y \quad y' \quad y'' \quad c \quad c' \quad c''$ - Shocks $m \quad w \quad z \quad A$

The state equations (let $\omega = m(\beta' - \beta)$, $w_h = \frac{1-\delta_h}{1+g}$, $w_k = \frac{1-\delta_k}{1+g}$, $\widetilde{\Delta} h_t = h_t - w_h h_{t-1}$)

$$y_t = A_t + z_t + \mu k_{t-1} + (1-\mu) l_t \tag{1}$$

$$y'_t = A_t + w_t - \gamma_0 z_t + \mu k'_{t-1} + (1-\mu) l'_t \tag{2}$$

$$y''_t = A_t - \alpha_0 w_t - \gamma_0 z_t + \mu k''_{t-1} + (1-\mu) l''_t \tag{3}$$

$$y' y'_t - b' b'_t + d d_t = c' c'_t + h' \widetilde{\Delta} h'_t - \left(b'/\beta'\right)\left(R'_{t-1} + b'_{t-1}\right) + \left(d/\beta'\right)\left(R_{t-1} + d_{t-1}\right) + k' \widetilde{\Delta} k'_t \tag{4}$$

$$y'' y''_t - b'' b''_t - d d_t = c'' c''_t + h'' \widetilde{\Delta} h''_t - \left(b''/\beta''\right)\left(R''_{t-1} + b''_{t-1}\right) - \left(d/\beta'\right)\left(R_{t-1} + d_{t-1}\right) + k'' \widetilde{\Delta} k''_t \tag{5}$$

$$Y Y_t = C C_t + K\left(K_t - w_k K_{t-1}\right) + H\left(H_t - w_h H_{t-1}\right) \tag{6}$$

The expectational equations: Impatient 1

$$c_t = \left(\beta/\beta'\right) c_{t+1} - \left(\beta/\beta'\right) R'_t - \left(1 - \beta/\beta'\right) \lambda'_t - \phi b' \left(\left(1 + \beta/\beta'\right) b'_t - b'_{t-1} - \left(\beta/\beta'\right) b'_{t+1}\right) \tag{7}$$

$$c_t = \left(\beta/\beta'\right) c_{t+1} - \left(\beta/\beta'\right) R''_t - \left(1 - \beta/\beta'\right) \lambda''_t - \phi b'' \left(\left(1 + \beta/\beta'\right) b''_t - b''_{t-1} - \left(\beta/\beta'\right) b''_{t+1}\right) \tag{8}$$

$$c_t = \left(1 - \beta w_h - \omega\right) h_t + \beta\left(1 - \delta_h\right) c_{t+1} - \omega\left(m_t + \alpha\left(\lambda'_t - R'_t\right) + (1-\alpha)\left(\lambda''_t - R''_t\right)\right) \tag{9}$$

$$c_t = -\left(1 - \beta w_k\right)\left(y_{t+1} - k_t\right) + c_{t+1} \tag{10}$$

$$y_t = c_t + (1+\eta) l_t \tag{11}$$

$$b'_t = m_t + h_t - R'_t \tag{12}$$

$$b''_t = m_t + h_t - R''_t \tag{13}$$

The patient 1

$$c'_t = c'_{t+1} - R'_t + \phi b' \left((1 + 1/R) b'_t - b'_{t-1} - (1/R) b'_{t+1}\right) \tag{14}$$

$$c'_t = c'_{t+1} - R_t - \phi d \left((1 + 1/R) d_t - d_{t-1} - (1/R) d_{t+1}\right) \tag{15}$$

$$c'_t = \left(1 - \beta' w_h\right) h'_t + \beta' w_h c'_{t+1} \tag{16}$$

$$c'_t = -\left(1 - \beta' w_k\right)\left(y'_{t+1} - k'_t\right) + c'_{t+1} \tag{17}$$

$$y'_t = c'_t + (1+\eta) l'_t \tag{18}$$

The patient 2

$$c''_t = c''_{t+1} - R''_t + \phi b'' \left((1 + 1/R) b''_t - b''_{t-1} - (1/R) b''_{t+1}\right) \tag{19}$$

$$c''_t = c''_{t+1} - R_t + \phi d \left((1 + 1/R) d_t - d_{t-1} - (1/R) d_{t+1}\right) \tag{20}$$

$$c''_t = \left(1 - \beta' w_h\right) h''_t + \beta' w_h c''_{t+1} \tag{21}$$

$$c''_t = -\left(1 - \beta' w_k\right)\left(y''_{t+1} - k''_t\right) + c''_{t+1} \tag{22}$$

$$y''_t = c''_t + (1+\eta) l''_t \tag{23}$$

The definitions

$$YY_t = yy_t + y'y'_t + y''y''_t \tag{24}$$

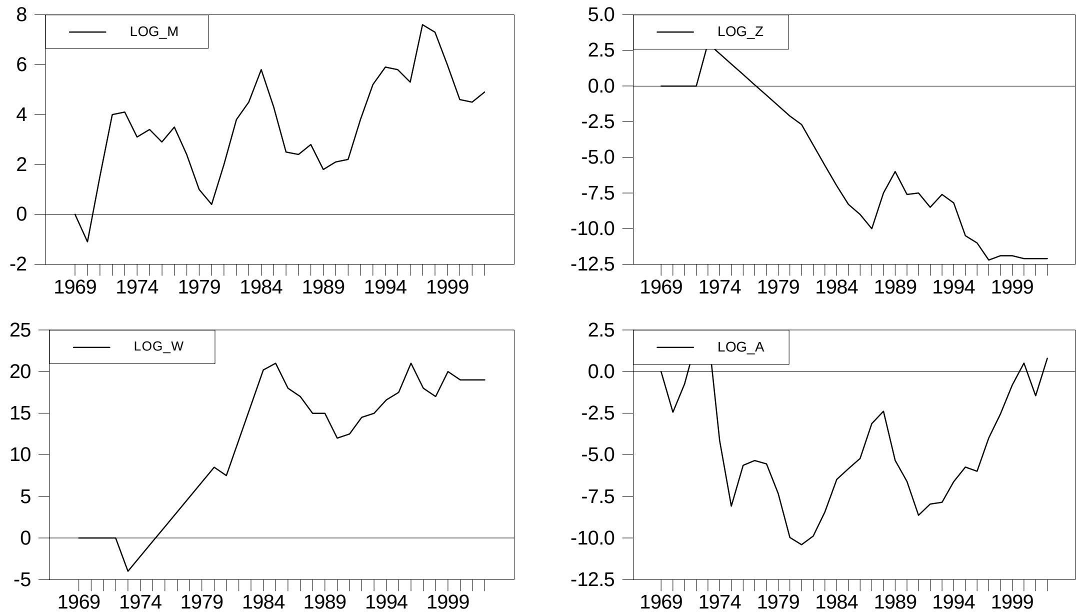
$$CC_t = cc_t + c'c'_t + c''c''_t \tag{25}$$

$$KK_t = kk_t + k'k'_t + k''k''_t \tag{26}$$

$$HH_t = hh_t + h'h'_t + h''h''_t \tag{27}$$

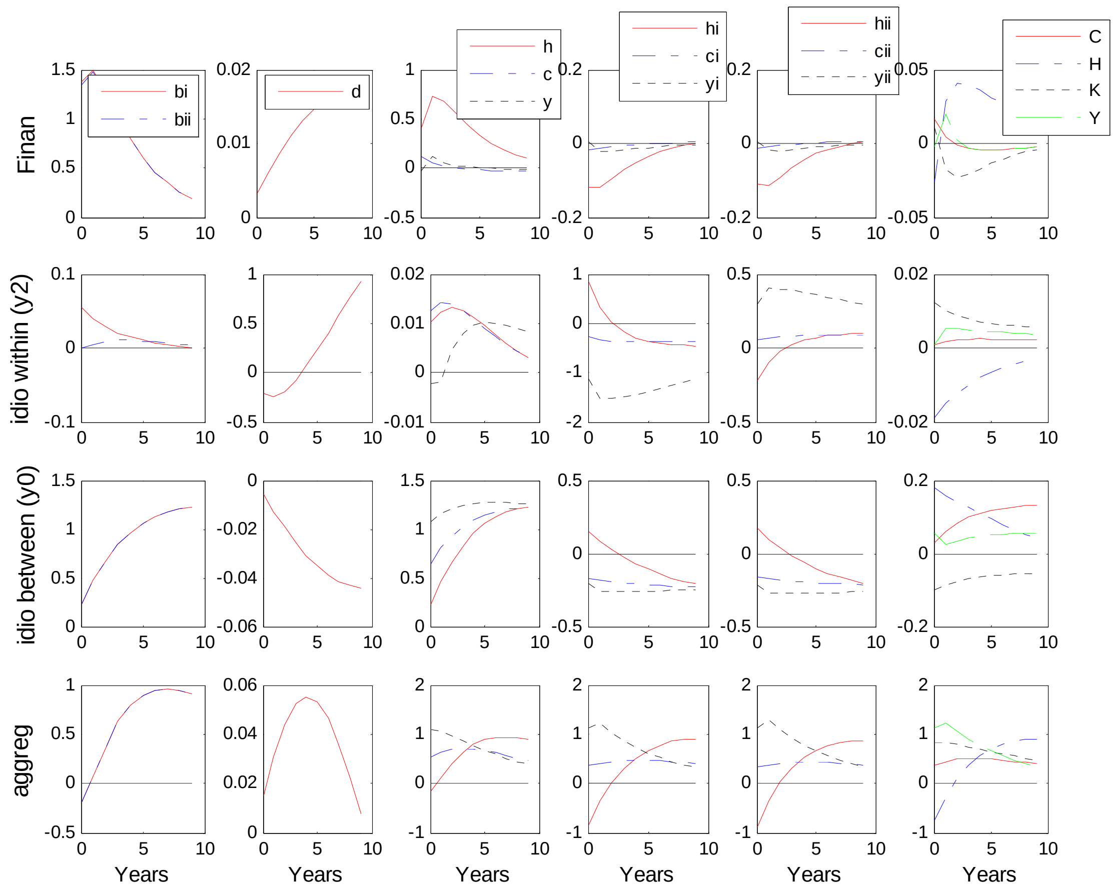
# Figures

FIGURE 1: The estimated stochastic processes




*Notes:* The variables are expressed in percentage deviations from the steady state, which is taken to be the year 1969.

FIGURE 2: Impulse Responses of the model




*Notes:* Vertical axis measures percent deviations from steady state.

FIGURE 3: Comparison between model and data: simulated income

FIGURE 4: Simulated profiles of consumption and income inequality

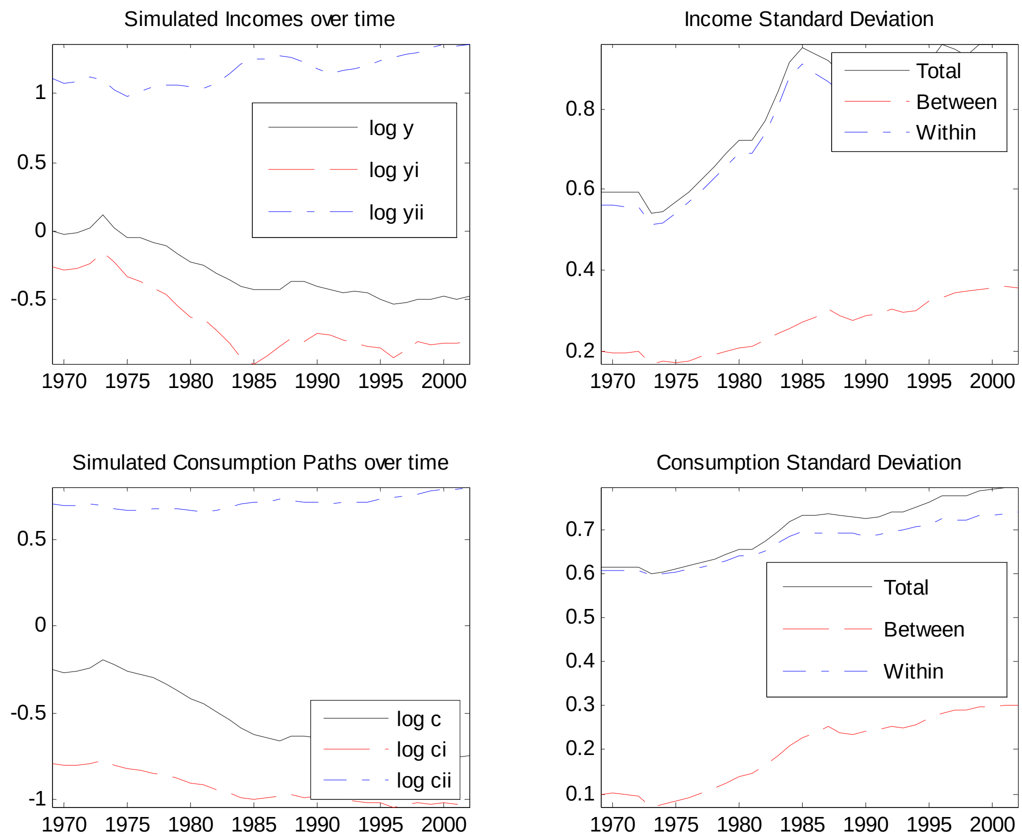


FIGURE 5: Simulated profiles of debt