# Forbearance and Prompt Corrective Action

Narayana Kocherlakota<br/>' and Ilhyock  $\operatorname{Shim}^{\scriptscriptstyle\ddagger}$ 

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<sup>†</sup> Narayana Kocherlakota: Professor, Department of Economics, Stanford University. Also, Federal Reserve Bank of Minneapolis, and NBER.

<sup>‡</sup> Ilhyock Shim: Economist, Monetary and Economic Department, Bank for International Settlements.

#### Abstract

This paper investigates whether a bank regulator should terminate problem banks promptly or exercise forbearance. We construct a dynamic model economy in which entrepreneurs pledge collateral, borrow from banks, and invest in long-term projects. We assume that collateral value has aggregate risk over time, that entrepreneurs can in any period abscond with the project but losing the collateral, and that depositors can withdraw deposits in any period. We show that optimal banking regulation exhibits forbearance if the ex-ante probability of collapse in collateral value is sufficiently low, but exhibits prompt termination of problem banks if this probability is sufficiently high.

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Narayana Kocherlakota: Professor, Department of Economics, Stanford University. Also, Federal Reserve Bank of Minneapolis, and NBER. Ilhyock Shim: Economist, Monetary and Economic Department, Bank for International Settlements.

# 1 Introduction

This paper is about a key question in banking regulation. A regulator becomes aware that a bank is failing. Should the regulator immediately intervene and liquidate the bank's assets to provide partial payment to the bank's owners and depositors? Or should the regulator let the bank continue operating? The first type of regulatory response is called prompt corrective action (PCA). It was embedded into United States banking regulation by the Federal Deposit Insurance Corporation Improvement Act of 1991. The second type of regulatory response is called forbearance and is generally regarded to have been practiced during the 1980's by the Federal Home Loan Bank Board and the Federal Savings and Loan Insurance Corporation (the United States regulators of the savings and loan associations).<sup>1</sup> In this paper, we ask which of these two regulatory responses is superior.

This question can only be answered by an explicitly dynamic model of banking. We build such a model around the following two assumptions. First, bank loans are enforced primarily through the threat that the borrower will lose his collateral. The motivation for this assumption is that it is typically difficult for banks to seize a borrower's human wealth by garnishing wages. Second, we assume that the value of collateral is subject to aggregate risk that is independent of the risk that affects the borrower's project. Suppose for example that a borrower takes out a loan to finance a restaurant. The loan is backed by the borrower's house. The value of land may move in ways that have nothing to do with the value of restaurant services.

Under these two assumptions, the question of the optimality of PCA can be re-posed as follows. Suppose the value of collateral falls precipitously and is expected to remain low. Such a fall reduces the ability of all banks to collect from their borrowers, and therefore reduces the value of the claims held on the bank by depositors. Should a banking regulator liquidate the assets of the banking system or not? We find that whether or not a regulator

<sup>&</sup>lt;sup>1</sup>White (1991) documents that the Federal Home Loan Bank Board allowed hundreds of insolvent thrifts to remain in operation throughout the 1980's.

should use PCA or forbearance depends not just on the realized history of shocks to the value of collateral, but also on the ex-ante probability of occurrence of that history. If the fall in the value of collateral was regarded as relatively likely, then the regulator should use PCA. If the fall in the value of collateral was extremely surprising, then the regulator should exercise forbearance.

The details of our analysis are as follows. Our model is a dynamic generalization of Kocherlakota (2001)'s static setup.<sup>2</sup> There are three types of agents: borrowers, lenders, and outsiders. All agents are risk-neutral and receive utility in the final period from consumption goods and collateral goods. Borrowers have a long-term project and are endowed with collateral goods. Lenders are endowed with durable investment goods. Outsiders are endowed with consumption goods in the final period. The value of collateral, both to borrowers and to others, follows a stochastic process.

There are two key enforcement frictions in the environment. First, borrowers may choose to leave the society in any period. If they do, they lose their collateral goods. However, they keep the project and any goods already invested in the project. Second, lenders are free to leave the society in any period. They take with them any uninvested goods, and also a fraction of the goods that have already been invested.

In the model, we consider the properties of socially optimal contracts. We view the borrowers as being entrepreneurs borrowing from banks, and lenders as depositors investing in banks, and finally outsiders as taxpayers who cannot walk away. Then, the optimal social contracts among firms, depositors and taxpayers can be regarded as the optimal bank regulation that taxpayers and depositors agree to ex ante. We compare PCA versus forbearance by looking at whether an optimal contract mandates liquidation in any date and state in which it becomes known that with probability one, the banking sector will require funds from taxpayers. The main result in the paper is that, given sufficiently low costs of liquidation,

 $<sup>^{2}</sup>$ Kocherlakota's framework is closely related to that of Holmstrom and Tirole (1998). The main difference is that in Kocherlakota's model, the government has to respect the same enforcement constraints as the private sector does.

the optimal contract exhibits forbearance if the ex-ante probability of reaching such a state is low, but exhibits PCA if the ex-ante probability of reaching such a state is high.

The intuition behind our result is simple. First, as in Kocherlakota (2001), deposit insurance may be efficient in this setting. A fall in the value of collateral does not affect whether a given project is socially beneficial or not. However, such a fall does reduce the ability of taxpayers and depositors to share in these social benefits. Since depositors may then want to (inefficiently) liquidate the project, they should be insured against falls in the value of collateral.

From an ex-ante perspective, though, taxpayers will not be willing to provide this insurance against collateral value fluctuations unless they receive sufficiently high deposit insurance premia when collateral values are high. It is this ex-ante participation constraint that generates the possibility of PCA. If the probability of a fall in the value of collateral is sufficiently low, then the deposit insurance premia can be made sufficiently large that taxpayers can fully insure depositors against all fluctuations in the value of collateral. In this case, regulators engage in forbearance: projects are never liquidated, even though taxpayers may learn that they will definitely be bailing out depositors.

However, if the probability of a fall in collateral value is sufficiently high, then deposit insurance premia will be paid with a low probability. Taxpayers will not be willing to fully insure depositors against all fluctuations in collateral value. Instead, it is necessary to liquidate the project in some states of the world. We show that, at least in some cases, it is optimal for this liquidation to take place as soon as the taxpayers know that they will be bailing out the depositors.

As we stated earlier, to analyze the welfare properties of prompt corrective action, one needs to use a dynamic model. However, there are few dynamic models of banking regulation. Sleet and Smith (2000) and Shim (2004) are two exceptions. Sleet and Smith (2000) consider the appropriate design of a safety net for the banking system in a two-period model when a government runs a deposit insurance program and a discount window. They show that, for some economies, the case for closing troubled banks "promptly" is not strong in the presence of social costs of closure. Shim (2004) considers banking regulation as a mechanism implementing the optimal incentive-feasible allocation in an infinite-horizon model of interactions between a banker and a regulator. He shows that an optimal regulatory plan features stochastic termination/bailout of an undercapitalized bank. In contrast, current United States regulatory practice requires deterministic termination with no possibility of bailout.

The paper proceeds as follows: Section 2 presents the model environment and defines social contracts. Section 3 shows optimal social contracts featuring deposit insurance and corrective actions under different settings. In section 4, we discuss the implications of the model on the recent banking crisis in Japan. Finally, section 5 concludes.

# 2 Model Specification

In this section, we describe the basic environment. We define what we mean by incentivefeasibility, and social optimality. Finally, we discuss how to interpret various real-world aspects of banking regulation in the context of the setup. Our notations and definitions follow Kocherlakota (2001).

#### 2.1 Environment

The economy lasts T + 1 periods, indexed by t = 0, 1, 2, ..., T. There are three types of agents: borrowers, lenders, and outsiders. There are unit measures of each type.

There are three types of goods: investment goods, consumption goods, and collateral goods. In period 0, each borrower is endowed with one unit of perfectly durable collateral that is specific to him (although others get utility from it), so there are actually a continuum of different types of collateral goods. In period 0, each lender is endowed with one unit of perfectly durable and divisible investment goods. In period T, each outsider is endowed with one unit of divisible consumption goods.

In terms of technology, each lender has a technology that converts investment goods one for one into consumption goods in period T. Each borrower is endowed with a technology that converts x units of investment goods in period t into  $R^{T-t}x$  units of consumption goods in period T. We assume R > 1, which means that the borrowers' projects expand the amount of social resources available. At any date s, it is possible to generate  $\Psi y_s$  units of durable consumption goods in period s by reducing the amount of investment goods in the technology by  $y_s$ . This reduces the period T payoff from the technology by  $R^{T-t}y_s$  units of consumption goods. We assume  $\Psi \leq 1$ , which means that (partial) liquidation of the project is socially costly.

All agents are risk-neutral over final-period consumption goods and collateral goods. The borrowers' utility function is given by:

$$c_T^b + \lambda \delta_T^b V_T$$

where  $c_T^b$  represents his period T consumption and  $\delta_T^b$  represents his period T collateral goods. Here,  $V_T$  is the value of his collateral to others. Lenders and outsiders have the same utility functions, given by:

$$c_T + \delta_T V_T$$

where  $c_T$  is the consumption of period T consumption goods and  $\delta_T$  is period T consumption of others ' collateral goods. Here, we assume  $\lambda > 1$ , which means that lenders and outsiders are less willing to substitute consumption for collateral goods than are borrowers. Thus,  $(\lambda - 1)V_T$  is borrower-specific utility from consuming the collateral.

The timing of events is as follows. At the very beginning of the initial period (period 0), the planner determines an ex-ante social contract which specifies the consumption of all

agents at the final period as well as the investment/withdrawal at every period. There are two stages in every period  $0 \le t \le T$ .

In the first stage of period  $0 \le t \le T$ , the realization of  $V_t$  is determined, which is publicly observable. We assume that  $\{V_t\}_{t=1}^T$  is a stochastic process with finite and nonnegative support. Investment  $x_t$  and withdrawal  $y_t$  take place at the end of the stage.

In the second stage of period t, lenders and borrowers have the option to walk away from the environment. A lender who walks away retains his uninvested investment goods and can liquidate any previously invested goods. A borrower who walks away retains his project and any investment goods that are still invested in it, although he loses the collateral. At the end of period T, lenders, borrowers and outsiders consume the consumption goods and the collateral.

We interpret the borrowers in this economy as being entrepreneurs who have a positive net present value project, the lenders as being bank depositors, and the outsiders as being taxpayers. We can then think of the two key enforcement constraints as follows. First, an entrepreneur can choose to seize his project at the cost of abandoning his collateral to the bank depositors. The motivation for this constraint is that it is often easy for entrepreneurs to divert the returns from a project into perks or wages that are difficult for a bank to seize. In contrast, it is generally easy for the bank to seize collateral goods. Second, a bank, representing its depositors, can always choose to recall its loans. The entrepreneur responds to the recall by liquidating his projects and returning the proceeds to the bank.

#### 2.2 Social Contracts

A social contract in this environment specifies non-negative  $V^T$ -measurable random variables  $\{c_T^b, c_T^o, c_T^l, \delta_T^b, \delta_T^o, \delta_T^l\}$  and two non-negative stochastic processes  $\{x_t, y_t\}_{t=0}^T$ .

A contract  $(\{c_T^b, c_T^o, c_T^l, \delta_T^b, \delta_T^o, \delta_T^l\}, \{x_t, y_t\}_{t=0}^T)$  is feasible if

$$c_T^b + c_T^l + c_T^o \le 1 + \left(1 - \sum_{t=0}^T x_t\right) + \Psi \sum_{t=0}^T y_t + \sum_{t=0}^T R^{T-t} (x_t - y_t), \ \forall V^T$$
  

$$\delta_T^b + \delta_T^l + \delta_T^o \le 1, \ \forall V^T$$
  

$$1 \ge \sum_{s=0}^t x_s \ge \sum_{s=0}^t y_s, \ \forall V^t, \forall t$$
  

$$c_T^b, c_T^o, c_T^l, \delta_T^b, \delta_T^o, \delta_T^l \ge 0, \ \forall V^T$$
  

$$x_t, y_t \ge 0, \ \forall V^t, \forall t.$$
  
(1)

The first constraint in (1) shows that the sum of consumption by borrowers, lenders and outsiders is equal to or less than the sum of four types of available resources at the final period: (i) outsiders' endowment of consumption goods, 1; (ii) the remaining investment goods at period T converted to consumption goods,  $\left(1 - \sum_{t=0}^{T} x_t\right)$ ; (iii) the sum of all consumption goods available at period T, generated by reducing investment goods by  $y_t$ ,  $\Psi \sum_{t=0}^{T} y_t$ ; and (iv) the sum of all consumption goods available at period T produced by the borrowers' net investment at  $t \leq T$ ,  $\sum_{t=0}^{T} R^{T-t}(x_t - y_t)$ . The second constraint shows that the collateral is consumed by borrowers, lenders, and/or outsiders. The third constraint is to make sure that the total amount of disinvestment does not exceed the accumulated investment at any period.

Beyond the physical restrictions, there are two enforcement limitations. First, at every period, lenders can walk away from the contract. Second, at every period, borrowers can opt to walk away with the project and the invested goods, leaving the collateral behind. Thus, society cannot prevent borrowers and lenders from walking away. We define an *incentivecompatible* contract to be one such that for all  $V^t$ , t = 0, 1, ..., T - 1, it is weakly optimal for borrowers not to walk away, and weakly optimal for lenders not to walk away.

**Definition 1** A social contract  $(\{c_T^b, c_T^o, c_T^l, \delta_T^b, \delta_T^o, \delta_T^l\}, \{x_t, y_t\}_{t=0}^T)$  is incentive-compatible if

$$E(c_T^l + \delta_T^l V_T \mid V^t) \ge \left(1 - \sum_{s=0}^t x_s\right) + \Psi \sum_{s=0}^t x_s, \quad \forall V^t, \forall t$$
(2)

$$E(c_T^b + \lambda \delta_T^b V_T \mid V^t) \ge \sum_{s=0}^t R^{T-s} (x_s - y_s), \ \forall V^t, \forall t$$
(3)

The condition (2) requires that the expected utility of a lender at T conditional on the realized history of collateral value up to t is greater than the utility of the lender at T arising from the uninvested investment goods as of t and the liquidation of all previously invested goods at t. The condition (3) requires that the expected utility of a borrower at T conditional on the realized history of collateral value up to t is greater than the utility of the borrower at T arising from consuming all investment goods still invested as of t.

A contract that is both incentive-compatible and feasible is *incentive-feasible*.

A main goal of this paper is to characterize *optimal* social contracts. By "optimal", we mean social contracts that solve the following social planner's problem.

$$\max E(c_T^b + \lambda \delta_T^b V_T) \tag{4}$$

such that

$$E(c_T^o + \delta_T^o V_T) \ge 1 \tag{5}$$

$$E(c_T^l + \delta_T^l V_T) \ge 1 \tag{6}$$

and the contract satisfies (1), (2) and (3).

In this problem, condition (5) guarantees that outsiders are no worse off than autarky in the ex-ante sense; condition (6) expresses the same restriction for lenders.

#### 2.3 Properties of Social Contracts

Just like in the real world, a social contract in this economy is an elaborate web of interactions between private banks (borrowers and lenders in the model) and taxpayers (outsiders). In what follows, we identify real-world aspects of banking regulation with properties of social contracts. We introduce the following definitions to investigate the implications of the model in the context of banking regulation.

We begin with a definition of deposit insurance. In the real world, deposit insurance explicitly refers to a payment from taxpayers to depositors. Hence, following Kocherlakota (2001), we use the following definition of deposit insurance.

#### **Definition 2** A contract involves deposit insurance if $c_T^o < 1$ with positive probability.

If  $c_T^o < 1$  with positive probability, the outsiders or taxpayers are making a payment to the banking system. Under definition 2, we are interpreting any such payment as being "deposit insurance." Definition 2 implies that we view the insurance between lenders and outsiders as deposit insurance, since we can view lenders as depositors and outsiders as taxpayers who cannot walk away.

We next turn to the concepts of *forbearance* and *corrective action*. Both of these terms refer to regulatory behavior in the context of a crisis in the banking sector. In our model setting, we think of such a crisis as being a situation in which it is known with probability one that the banking sector will require funds from taxpayers. Hence, in the real world, *forbearance* refers to a situation in which bank loans are not liquidated, even though it is known that taxpayers will have to compensate bank depositors. We translate this notion into our model as follows.

**Definition 3** A contract exhibits forbearance in history  $\overline{V}^t$  if  $y_t(\overline{V}^t) = 0$  and  $\Pr(c_T^o < 1 | V^t = \overline{V}^t) = 1$ .

In the real world, corrective action is the opposite of forbearance: it refers to a situation in which the regulator liquidates loans. We translate this notion into our setting as follows.

**Definition 4** A contract exhibits corrective action in history  $\overline{V}^t$  if  $y_t(\overline{V}^t) > 0$ .

Finally, we turn to *prompt corrective action* (PCA). A contract is said to exhibit PCA if the projects are liquidated as soon as it is known that the total net profits of outsiders are negative with probability one.

**Definition 5** A contract exhibits PCA if  $y_t(\overline{V}^t) > 0$  in any history  $\overline{V}^t$  such that  $\Pr(c_T^o < 1 | V^t = \overline{V}^t) = 1$  and  $\Pr(c_T^o < 1 | V^s = \overline{V}^s) < 1$  for s < t.

In what follows, we explore under what circumstances the optimal contract displays these attributes.

# **3** Optimal Social Contracts

In this section, we investigate the properties of optimal contracts under different subsets of parameters. The crucial problem confronting the planner in this setting with limited enforcement is that the *shareable* societal pie that can be consumed by all agents in the economy is different from the total societal pie. To be concrete, suppose that all of the investment goods are invested in period 0 and there is no liquidation. Then, the total societal pie is given by  $R^T + \lambda V_T + 1$ . However, not all of this pie is shareable with outsiders and lenders, because borrowers can threaten to walk away. If the value of the collateral is  $V_T$ , then outsiders and lenders cannot receive more than min  $(\lambda V_T, R^T)$  from the borrowers. The shareable pie is thus only  $1 + \min(\lambda V_T, R^T)$ . The problem for the planner is to split the *shareable* pie among the participants in such a way so as to make the *total* pie as large as possible.

#### 3.1 First-Best Investment

In this subsection, we describe when it is optimal for the planner to prescribe the first-best level of investment. We shall see that, as in Kocherlakota (2001), attaining the first-best level of investment may involve the use of deposit insurance.

The first proposition demonstrates that if  $\lambda V_T > 1$  for all  $V_T$ , all optimal contracts involve the first-best investment. Moreover, in this case, the first-best level of investment can be achieved without deposit insurance.

**Proposition 1** If  $\lambda V_T > 1$  with probability one, then in any optimal contract,  $x_0 = 1$ ,  $x_t = 0, \forall V^t, \forall t \ge 1, y_t = 0, \forall V^t, \forall t \ge 0, \delta_T^b(V^T) = 1, \forall V^T$ . There exists an optimal contract in which  $c_T^o = 1$  with probability one.

**Proof.** For any optimal contract, the ex-ante participation constraints of lenders and outsiders ((5) and (6)) and the resource constraints (the first and second constraints in (1)) must be satisfied with equality. Substitute these constraints into the borrowers' ex-ante utility function. Then, it is clear that for any contract with investment/withdrawal process  $\{x_t, y_t\}_{t=0}^T$ , the borrower's ex-ante utility is

$$B(\{x_t, y_t\}_{t=0}^T, \delta_T^b, \delta_T^o, \delta_T^l) = E\left[\sum_{t=0}^T \left(R^{T-t} - 1\right) x_t\right] - E\left[\sum_{t=0}^T \left(R^{T-t} - \Psi\right) y_t\right] + \lambda E[\delta_T^b V_T] + E\left[\left(\delta_T^o + \delta_T^l\right) V_T\right].$$

This is maximized by setting  $x_0 = 1$ ,  $x_t = 0$ ,  $\forall V^t$ ,  $\forall t \ge 1$ ,  $y_t = 0$ ,  $\forall V^t$ ,  $\forall t \ge 0$ ,  $\delta_T^b = 1$ ,  $\delta_T^o = \delta_T^l = 0$ ,  $\forall V^T$ . Thus, no contract can attain a value for the planner's objective higher than  $B^* = (R^T - 1) + \lambda E[V_T]$ .

Now consider the following contract:

$$\begin{split} x_0 &= 1, \ x_t(V^t) = 0, \ \forall V^t, \forall t \ge 1 \\ y_t(V^t) &= 0, \ \forall V^t, \forall t. \\ \delta^b_T(V^T) &= 1, \ \delta^o_T(V^T) = \delta^l_T(V^T) = 0, \ \forall V^T \\ c^o_T(V^T) &= 1, \ c^b_T(V^T) = R^{T-1} - 1, \ c^l_T(V^T) = 1, \ \forall V^T \end{split}$$

This contract satisfies the social planner's constraints and maximizes the borrower's objective. Hence, this contract is optimal. Moreover, any contract with  $x_0 < 1$  achieves a borrower's utility lower than that with  $x_0 = 1$ .

The point of Proposition 1 is that, when  $\lambda$  is sufficiently high and the lowest possible value of  $V_T$  is not too small so that  $\lambda V_T > 1$ ,  $\forall V_T$ , then the shareable pie under the first-best investment is always greater than the sum of the endowment of outsiders and the maximum autarkic consumption by lenders, which is 2. Now lenders are willing to make the first-best investment and outsiders also want to participate in the contract, because borrowers can recompense them sufficiently whatever collateral value is realized.

The next proposition considers the case in which the *expected* shareable pie is greater than 2, but with some positive probability, the shareable pie is less than the sum of the outsiders' endowment and the lenders' outsider option under the first-best investment, which is  $1 + \Psi$ . In this case, the optimal contract features deposit insurance.

**Proposition 2** If  $\min_{V_T} \lambda V_T < \Psi$  and  $E\left[\min\left(\lambda V_T, R^T\right)\right] \ge 1$ , then in any optimal contract,  $x_0 = 1, x_t = 0, \forall V^t, \forall t \ge 1, y_t = 0, \forall V^t, \forall t \ge 0, and \delta_T^b = 1, \forall V^T$ . There exists no optimal contract in which  $c_T^o \ge 1$  with probability one.

**Proof.** Note that the expected shareable pie,  $1 + E\left[\min\left(\lambda V_T, R^T\right)\right]$ , is greater than 2. As in the proof of Proposition 1, any contract with investment/withdrawal process  $\{x_t, y_t\}_{t=0}^T$  achieves the following borrower's ex-ante utility:

$$B(\{x_t, y_t\}_{t=0}^T, \delta_T^b, \delta_T^o, \delta_T^l) = E\left[\sum_{t=0}^T \left(R^{T-t} - 1\right) x_t\right] - E\left[\sum_{t=0}^T \left(R^{T-t} - \Psi\right) y_t\right] + \lambda E[\delta_T^b V_T] + E\left[\left(\delta_T^o + \delta_T^l\right) V_T\right].$$

The unique maximizer of this function is obtained by setting  $x_0 = 1$ ,  $x_t = 0$ ,  $\forall V^t, \forall t \ge 1$ ,  $y_t = 0, \forall V^t, \forall t \ge 0$ , and  $\delta_T^b = 1$ ,  $\delta_T^o = \delta_T^l = 0, \forall V^T$ .

Let  $\zeta_T = \min(R^T, \lambda V_T)$ . Consider the following contract:

$$\begin{split} c^b_T &= R^T - \zeta_T + E(\zeta_T) - 1\\ c^o_T &= \zeta_T + 1 - E(\zeta_T)\\ c^l_T &= 1, \ \forall V^T\\ x_0 &= 1, \ x_t = 0, \ \forall V^t, \forall t \geq 1\\ y_t &= 0, \ \forall V^t, \forall t \geq 0\\ \delta^b_T &= 1, \ \delta^o_T &= \delta^l_T = 0, \ \forall V^T \end{split}$$

Note that:

$$E(c_T^b + \lambda V_T | V^t) = R^T + E(\zeta_T) - 1 + E(-\zeta_T + \lambda V_T | V^t)$$
$$= R^T + E(\zeta_T) - 1 + E(-\min(R^T, \lambda V_T) + \lambda V_T | V^t)$$
$$\geq R^T + E(\zeta_T) - 1$$
$$\geq R^T$$

so that borrowers will not walk away in any period. The lenders' ex-post participation constraints are satisfied, as is the outsiders' ex-ante participation constraint. Thus, this contract is optimal.

We now need to prove that there is no optimal contract such that  $c_T^b(V^T) \ge 1$  for all  $V^T$ . In any optimal contract,  $x_0 = 1$ . The resource constraint implies that in any optimal

contract,  $c_T^o(V^T) + c_T^l(V^T) \leq 1 + R^T - c_T^b(V^T)$ ,  $\forall V^T$ . From the borrowers' participation constraints and the nonnegativity constraint on  $c_T^b$ ,  $c_T^b(V^T) \geq R^T - \min(R^T, \lambda V^T)$ .

Now,  $c_T^o(V^T) + c_T^l(V^T) \leq 1 + \min(R^T, \lambda V_T)$ ,  $\forall V^T$ . Since  $R^T > 1$  and for some  $V_T^*$ ,  $\lambda V_T^* < \Psi$ , we get  $\min(R^T, \lambda V_T^*) < \Psi$ . The lenders' participation constraint at time T,  $c_T^l(V^T) \geq \Psi$ ,  $\forall V^T = (V^{T-1}, V_T^*)$ , implies that  $c_T^o(V^{T-1}, V_T^*) < 1$ ,  $\forall V^{T-1}$ .

The result in Proposition 2 shows that deposit insurance plays an essential role in the optimal allocation of resources. Outsiders make a transfer to lenders when collateral value is low, and receive a fraction of the lenders' loan proceeds when collateral value is high. Since we can view lenders as bank depositors and outsiders as the government representing taxpayers, these payments from taxpayers to depositors can be viewed as deposit insurance against fluctuations in the value of collateral.

Under the conditions of Proposition 1, there are optimal contracts in which  $\Pr(c_T^o < 1) = 0$ ; there are no banking crises in this world. In contrast, under the assumptions of Proposition 2, in any optimal contract, there is a state  $V^T$  such that  $c_T^o(V^T) < 1$ . However, liquidation never takes place. Under the conditions of Proposition 2, forbearance is always optimal.

#### **3.2** Corrective Action

In the previous subsection, we showed that even in the presence of enforcement limitations, the optimal contracts can support the first-best investment even without any form of corrective action. We now consider parameter settings in which corrective action is optimal. The next proposition shows that if  $E[\min(R^T, \lambda V_T)] < 1$ , any contract with full initial investment and no repayment via collateral features corrective action.

**Proposition 3** Suppose  $E[\min(R^T, \lambda V_T)] < 1$ . Consider any contract in which  $x_0 = 1$  and  $\delta_T^b = 1$  with probability one. In that contract, there exists  $(t, V^t)$  such that  $y_t(V^t) > 0$ .

**Proof.** Let  $x_0 = 1$ . For a general  $\{y_t\}_{t=0}^T$ , the borrower's participation constraint at T is

$$c_T^b(V^T) \ge R^T - \sum_{t=0}^T R^{T-t} y_t - \lambda V_T.$$

Adding the nonnegativity constraint, this constraint becomes

$$c_T^b(V^T) \ge \max\left[0, R^T - \sum_{t=0}^T R^{T-t} y_t - \lambda V_T\right].$$

To satisfy the ex-ante participation constraints of lenders and outsiders, we need:

$$E\left[1 + \Psi \sum_{t=0}^{T} y_t + R^T - \sum_{t=0}^{T} R^{T-t} y_t - c_T^b\right] \ge 2,$$

or equivalently,

$$E\left[\Psi\sum_{t=0}^{T} y_t + \min\left(R^T - \sum_{t=0}^{T} R^{T-t} y_t, \ \lambda V_T\right)\right] \ge 1.$$

Suppose all  $y_t$ 's are zero. Then, we get  $E[\min(R^T, \lambda V_T)] \ge 1$ , which contradicts the assumption  $E[\min(R^T, \lambda V_T)] < 1$ . Therefore, we need at least one  $y_t$  to be positive.

If  $E[\min(R^T, \lambda V_T)] < 1$ , then the expected shareable pie is insufficient to compensate outsiders and lenders for participating in the optimal contract. This means that it is not possible to simply set  $x_0 = 1$  and never liquidate. Instead, the regulator must liquidate projects in states of the world in which he expects  $\lambda V_T$  to be low, in order to provide outsiders and lenders with repayment  $\Psi y_t$  in those states.

Proposition 3 applies to contracts with full initial investment and no repayment via collateral. Note that if  $\Psi$  is equal to 1, then it is at least weakly optimal to set  $x_0 = 1$ , because the planner can costlessly withdraw investment if it ever becomes necessary to do so. Also, if  $\lambda$  is sufficiently large, then repayment via collateral will be suboptimal (that is,  $\delta_T^b = 1$ ). Hence, Proposition 3 applies to optimal contracts as long as  $\Psi$  is sufficiently close to 1 and  $\lambda$  is sufficiently large.

We now turn to assessing when this corrective action is prompt. So far, we have assumed that  $V_t$  is a general stochastic process. For the next proposition, we consider the special case in which  $V_{t+1} = V_t/p$  with probability p,  $V_{t+1} = 0$  with probability (1 - p), and  $V_0 > 0$ . Under this restriction, a social contract exhibits prompt correction action in a history  $V^t$  if  $y_t(V^t) > 0$ ,  $V_s > 0$  for all s < t, and  $V_t = 0$ . We show that, depending on the value of p, PCA may take place in some or all histories.

**Proposition 4** Suppose  $V_t = V_{t-1}/p$  with probability p and  $V_t = 0$  with probability (1 - p), where pR < 1. Suppose too that  $\Psi = 1$  and  $\lambda$  is sufficiently large that  $\lambda V_0 > R^T$  and  $\delta_T^b = 1$ with probability one in any optimal contract. Then, the following statements are both true.

1. If p is sufficiently large that

$$(1 - p^T R^T) \le p^{T-1}(1 - p)$$

then in any optimal contract,  $y_T(V^{T-1}, 0) > 0$  if  $V_t > 0$  for all  $t \leq T - 1$ , and  $y_t(V^t) = 0$  for all other  $(t, V^t)$ .

2. If p is sufficiently small that

$$(1-p) > p^T (R^T - 1)$$

then in any optimal contract,  $y_{t+1}(V^t, 0) > 0$  if and only if  $V_s > 0$  for all  $s \leq t$ .

**Proof.** In Appendix.

The whole goal of corrective action is to be able to satisfy the ex-ante participation constraints. Proposition 3 shows that if  $E(\min(\lambda V_T, R^T)) < 1$ , then there has to be corrective action. If p is large enough, then this correction needs only take place infrequently. It is best to wait as much as possible to correct to give borrowers longer to run their long-term beneficial project. However, if p is small, then the intervention must take place immediately. Also, note that when we assume  $\Psi = 1$ , the optimal contract under corrective action again features a form of deposit insurance. When collateral value does not collapse by T, outsiders consume  $R^T > 1$ . On the other hand, when collateral value collapses at  $t \leq T - 1$ , outsiders can only consume the proceeds from liquidation or corrective action,  $y_t(V^{t-1}, 0)$ . Note that  $0 \leq y_t(V^{t-1}, 0) \leq 1$ , for all  $1 \leq t \leq T$ , where equalities hold depending on the value of p.

### 4 Understanding the Japanese Banking Crisis

After real estate prices fell and the macroeconomy slowed in Japan in the early 1990s,<sup>3</sup> the banking sector has become very weak and many banks are undercapitalized. During this banking crisis, Japanese bank regulators provided implicit (and later explicit<sup>4</sup>) guarantees of bank deposits and were extremely averse to liquidation of problem banks.<sup>5</sup> They are often criticized for both practices.

This paper argues that both their insurance of deposits and their forbearance may have been optimal. In particular, we show that when it is inevitable to liquidate the project partially in order to guarantee the ex-ante participation of depositors and taxpayers in the system, prompt liquidation of the projects facing the collapse of collateral value is not always socially optimal. The more general lesson is that in the face of collateral shocks, deciding when to liquidate apparently insolvent banks depends not just on current conditions, but on the a priori probability of that failure taking place.

 $<sup>^{3}</sup>$ See Figure 2 in Ueda (1999) for the time series of the rate of change in Japanese land prices.

<sup>&</sup>lt;sup>4</sup>Formal deposit insurance existed in Japan since 1971 through JDIC (Japan Deposit Insurance Corporation), explicitly guaranteeing deposits up to 10 million yen, as in US. Since deposits larger than 10 million yen were not guaranteed, facing greater turmoil in the financial sector, the Japanese government introduced in early 1998 an "explicit" blanket guarantee of all bank deposits without limits. This blanket guarantee was supposed to expire in April 2003, but extended indefinitely. See Dekle and Kletzer (2003) for details.

<sup>&</sup>lt;sup>5</sup>Japan actually introduced its own version of Prompt Corrective Action in April 1998, which resembles the general structure of the US Prompt Corrective Action with the threshold level of capital ratios lower than that of the US. However, Japanese bank regulators were still reluctant to close problem banks even after the introduction of Prompt Corrective Action. For details on financial reform in Japan during the 1990, see Hall (1998), and Dekle and Kletzer (2003).

# 5 Conclusion

This paper considers the question of the optimality of prompt corrective action in a model akin to that of Kocherlakota (2001). It shows that the decision to use PCA or forbearance depends crucially on properties of the stochastic process that determines the value of collateral. In particular, when a fall in the value of collateral is a low-probability event, it is best to use forbearance if collateral actually does fall in value. However, when a fall in the value of collateral is a high-probability event, it is best to use prompt corrective action if collateral falls in value.

This paper focuses on collateral risk and abstracts from moral hazard. It is interesting to contrast the results about PCA in this framework to those obtained by Shim (2004), who did not consider collateral shocks but stochastic returns and moral hazard. Shim (2004) models repeated interactions between a risk-neutral banker and the risk-neutral FDIC under the following informational frictions: every period the banker can privately choose the level of costly effort which affects the distribution of returns in that period; every period the banker can give up the bank and enjoy outside option whenever he wants. Shim (2004) shows that stochastic termination/bailout of a problem bank is socially preferable to full/deterministic termination can be an alternative to stochastic termination. The main difference between this paper and Shim (2004) in terms of termination timing is that the optimal termination policy in Shim (2004) is always "prompt" in the ex-ante sense, be it stochastic or partial, whereas this paper shows that PCA is not always optimal.

How can we best reconcile the results in Shim (2004) and in this paper? The appropriate regulatory response depends crucially on the nature of the shock affecting the banking system. Shim considers shocks to project returns, and argues for a stochastic version of prompt corrective action in response to such shocks. This paper considers shocks to the value of collateral, and argues that forbearance may well be optimal. The lesson from these papers is that the optimal banking regulation cannot be couched solely in terms of the current value of a bank's portfolio. The optimal response to banking insolvency depends on the nature of the shock that led to the insolvency.

# 6 Appendix: Proof of Proposition 4

The expected shareable pie  $1 + E[\min(R^T, \lambda V_T)]$  is less than 2. As in the proof of Proposition 1 and by assumption  $\delta_T^b = 1$  with probability one, for any optimal contracts,

$$B(\{x_t, y_t\}_{t=0}^T) = E\left[\sum_{t=0}^T \left(R^{T-t} - 1\right)(x_t - y_t)\right] + \lambda V_0$$

Let  $\widetilde{V}^t = (V_0/p, ..., V_0/p^t), t \ge 1$ . Because  $\lambda V_0 > R^T$ , the ex-post participation constraint of borrowers associated with any  $\widetilde{V}^t$  does not bind. Then, together with the non-negativity constraint on  $c_T^b$ , we need to consider the following participation constraints for borrowers:

$$\begin{split} c_T^b(\widetilde{V}^T) &\geq 0 \\ c_T^b(\widetilde{V}^{T-1}, 0) &\geq \sum_{t=0}^{T-1} R^{T-t} (x_t(\widetilde{V}^{T-1}) - y_t(\widetilde{V}^{T-1})) + x_T(\widetilde{V}^{T-1}, 0) - y_T(\widetilde{V}^{T-1}, 0) \\ c_T^b(\widetilde{V}^{T-2}, 0, 0) &\geq \sum_{t=0}^{T-2} R^{T-t} (x_t(\widetilde{V}^{T-2}) - y_t(\widetilde{V}^{T-2})) + R[x_{T-1}(\widetilde{V}^{T-2}, 0) - y_{T-1}(\widetilde{V}^{T-2}, 0)] \\ &\quad + \max\left\{0, \ x_T(\widetilde{V}^{T-2}, 0, 0) - y_T(\widetilde{V}^{T-2}, 0, 0)\right\} \\ &\vdots \\ c_T^b(0, ..., 0) &\geq R^T(x_0 - y_0) + R^{T-1}(x_1(0) - y_1(0)) + \max\{0, \ R^{T-2}(x_2(0, 0) - y_2(0, 0)), \\ R^{T-2}(x_2(0, 0) - y_2(0, 0)) + R^{T-3}(x_3(0, 0, 0) - y_3(0, 0, 0)), \\ &\qquad R^{T-2}(x_2(0, 0) - y_2(0, 0)) + \cdots + R(x_{T-1}(0, ..., 0) - y_{T-1}(0, ..., 0)) \end{split}$$

$$+ (x_T(0,...,0) - y_T(0,...,0)) \}$$

Note that since  $\Psi = 1$  and  $\delta_T^l = 0$  for all  $V^T$ , in order to satisfy the ex-ante participation constraint for lenders with equality, we need  $c_T^l(V^T) = 1, \forall V^T$ .

Using the above inequalities together with the resource constraints, state-independent consumption by lenders (i.e.  $c_T^l(V^T) = 1, \forall V^T$ ) and the ex-ante participation constraints for lenders and outsiders, we get the following condition:

$$E\left[\sum_{t=0}^{T} (R^{T-t} - 1)(x_t - y_t)\right] - p^{T-1}(1-p)\left[\sum_{t=0}^{T-1} R^{T-t}(x_t(\widetilde{V}^{T-1}) - y_t(\widetilde{V}^{T-1})) + x_T(\widetilde{V}^{T-1}, 0) - y_T(\widetilde{V}^{T-1}, 0)\right] - p^{T-2}(1-p)\left[\sum_{t=0}^{T-2} R^{T-t}(x_t(\widetilde{V}^{T-2}) - y_t(\widetilde{V}^{T-2})) + R\left[x_{T-1}(\widetilde{V}^{T-2}, 0) - y_{T-1}(\widetilde{V}^{T-2}, 0)\right] + \max\left\{0, x_T(\widetilde{V}^{T-2}, 0, 0) - y_T(\widetilde{V}^{T-2}, 0, 0)\right\}$$

. . .

$$-(1-p) \left[ \begin{array}{c} R^{T}(x_{0}-y_{0})+R^{T-1}(x_{1}(0)-y_{1}(0)) \\ 0, \ R^{T-2}(x_{2}(0,0)-y_{2}(0,0)), \\ R^{T-2}(x_{2}(0,0)-y_{2}(0,0))+R^{T-3}(x_{3}(0,0,0)-y_{3}(0,0,0)), \cdots, \\ R^{T-2}(x_{2}(0,0)-y_{2}(0,0))+\cdots+R(x_{T-1}(0,...,0)-y_{T-1}(0,...,0)) \\ +(x_{T}(0,...,0)-y_{T}(0,...,0)) \end{array} \right\} \right]$$

$$\geq 0$$

Denote the left-hand side of this inequality as  $\Omega(x, y)$ . Now define a relaxed problem P as follows:

$$\max_{\{x_t, y_t\}_{t=0}^T} E\left[\sum_{t=0}^T (R^{T-t} - 1)(x_t - y_t)\right]$$
  
s.t.  $\Omega(x, y) \ge 0$   
 $1 \ge \sum_{s=0}^t x_s(V^t), \ \forall V^t, \forall t$   
 $\sum_{s=0}^t x_s(V^t) \ge \sum_{s=0}^t y_s(V^t), \ \forall V^t, \forall t$   
 $y_t(V^t) \ge 0, \ \forall V^t, \forall t$ 

We use the Lagrangian multiplier  $\mu$  for the first constraint,  $m_1(V^t)$  for the second set of constraints,  $m_2(V^t)$  for the third set of constraints, and  $m_3(V^t)$  for the last set of constraints.

Then, we first show  $\mu > 0$ . Suppose  $\mu = 0$ . The FOCs with respect to  $x_t(V^t)$  is as follows:

$$pr(V^{t}) \cdot [(R^{T-t} - 1)] + \mu \frac{\partial \Omega}{\partial x_{t}(V^{t})} - \sum_{\tau \ge t} \sum_{(V^{\tau} \ge V^{t})} m_{1}(V^{\tau}) + \sum_{\tau \ge t} \sum_{(V^{\tau} \ge V^{t})} m_{2}(V^{\tau}) = 0.$$

In particular, the FOCs with respect to  $x_0$ ,  $x_1(V_0/p)$  and  $x_1(0)$ , respectively, are as follows:

$$(R^{T} - 1) + \mu(-1 + p^{T}R^{T}) - \sum_{t \ge 0} \sum_{V^{t}} m_{1}(V^{t}) + \sum_{t \ge 0} \sum_{V^{t}} m_{2}(V^{t}) = 0$$
$$p(R^{T-1} - 1) + \mu(-p + p^{T}R^{T-1}) - \sum_{t \ge 1} \sum_{(V^{t} \ge V_{1} = V_{0}/p)} m_{1}(V^{t}) + \sum_{t \ge 1} \sum_{(V^{t} \ge V_{1} = V_{0}/p)} m_{2}(V^{t}) = 0$$
$$(1 - p)(R^{T-1} - 1) + \mu(-1 + p) - \sum_{t \ge 1} \sum_{(V^{t} \ge V_{1} = 0)} m_{1}(V^{t}) + \sum_{t \ge 1} \sum_{(V^{t} \ge V_{1} = 0)} m_{2}(V^{t}) = 0$$

where  $V^t \ge V_1 = V_0/p$  represents a history  $V^t$  with  $V_1 = V_0/p$ , and  $V^t \ge V_1 = 0$  a history  $V^t$  with  $V_1 = 0$ . From these conditions,  $m_1(V_0) > 0$  holds, which implies  $x_0 = 1$ . Also, the FOC with respect to  $y_t(V^t)$  is as follows:

$$pr(V^{t}) \cdot \left[ -(R^{T-t} - 1) \right] + \mu \frac{\partial \Omega}{\partial y_t(V^{t})} - \sum_{\tau \ge t} \sum_{(V^{\tau} \ge V^{t})} m_2(V^{\tau}) + m_3(V^{t}) = 0$$

From these conditions, all  $m_3$ 's are positive, which means all y's are zero. However, this violates the first constraint because we assumed pR < 1. Therefore,  $\mu > 0$  and  $\Omega(x, y) = 0$ .

When we substitute  $\Omega(x, y) = 0$  into the objective,  $x_0 = 1$  maximizes the objective. Note that since  $\Omega(x, y) = 0$ , we can determine  $c_T^b$  directly from the optimal x's and y's. Also, note that given  $x_0 = 1$ , from the nonnegativity constraints on y's, the terms with max operator in  $\Omega(x, y)$  disappear. Now we consider a simplified problem P' as follows:

$$\begin{split} \max_{\{y_t\}_{t=0}^T} (R^T - 1) - E \left[ \sum_{t=0}^T (R^{T-t} - 1) y_t(V^T) \right] \\ s.t. \quad \Omega(x_0 = 1, y) \equiv (R^T - 1) - E \left[ \sum_{t=0}^T (R^{T-t} - 1) y_t(V^T) \right] \\ \quad - p^{T-1} (1 - p) \left[ R^T - \sum_{t=0}^{T-1} R^{T-t} y_t(\widetilde{V}^{T-1}) - y_T(\widetilde{V}^{T-1}, 0) \right] \\ \quad - p^{T-2} (1 - p) \left[ R^T - \sum_{t=0}^{T-2} R^{T-t} y_t(\widetilde{V}^{T-2}) - R y_{T-1}(\widetilde{V}^{T-2}, 0) \right] \\ \quad - \cdots - (1 - p) \left[ R^T (1 - y_0) - R^{T-1} y_1(0) \right] = 0, \\ 1 \ge \sum_{s=0}^t y_s(V^t), \ \forall V^t, \forall t, \\ y_t(V^t) \ge 0, \ \forall V^t, \forall t. \end{split}$$

We need to find out which y should be positive to satisfy the first constraint with equality. First, note that for all  $V^T$  except for  $(\tilde{V}^{T-1}, 0)$ , a change of  $y_T(V^T)$  does not affect the objective nor the first constraint. Thus, without loss of generality, we set  $y_T(V^T) = 0$  for all  $V^T$  except for  $(\tilde{V}^{T-1}, 0)$ . An increase in  $y_T(\tilde{V}^{T-1}, 0)$  does not change the objective while increasing  $\Omega(x_0 = 1, y)$ . Thus, when all y's are zero and  $\Omega(x_0 = 1, y) < 0$ , an increase in  $y_T(\tilde{V}^{T-1}, 0)$  is least costly to the objective while increasing the value of  $\Omega(x_0 = 1, y)$ .

Also,  $\frac{\partial\Omega}{\partial y_t(V^t)} < 0$  for all histories  $V^t$  other than  $\widetilde{V}^t$  and  $(\widetilde{V}^{t-1}, 0)$  for all  $1 \le t \le T - 1$ . Thus, from the above FOCs with respect to y's,  $m_3(V^t) > 0$  and  $y_t(V^t) = 0$  for all  $V^t$  other than  $\widetilde{V}^t$  and  $(\widetilde{V}^{t-1}, 0)$  for all  $1 \le t \le T - 1$ .

Finally, we need to determine which y to use first to satisfy the first constraint. We already showed that  $y_T(\tilde{V}^{T-1}, 0)$  is not costly to the objective while increasing  $\Omega(x_0 = 1, y)$ . Note that an increase in any other  $y_t(V^t)$  for all  $V^t$  for all  $t \leq T - 1$  strictly reduces the objective. Thus, it is optimal to set  $y_T(\tilde{V}^{T-1}, 0) > 0$  first, in order to make  $\Omega(x_0 = 1, y) = 0$ . Also note that  $y_T(\tilde{V}^{T-1}, 0)$  cannot be greater than 1, and that when  $y_T(\tilde{V}^{T-1}, 0) = 1$ ,  $y_t(\tilde{V}^{T-1}, 0)$ for all  $t \leq T - 1$  should be zero from the second constraint in problem P'. That is, when  $y_T(\tilde{V}^{T-1}, 0) = 1$ ,  $y_0 = y_1(V_0/p) = y_2(V_0/p, V_0/p^2) = \cdots y_{T-1}(\tilde{V}^{T-1}) = 0$ . Therefore, only y's associated with histories  $V^t = (\tilde{V}^{t-1}, 0)$  for all  $t \geq 2$  can be positive. One unit increase in  $y_t(\tilde{V}^{t-1}, 0)$  for  $1 \leq t \leq T - 1$  decreases the objective by  $p^{t-1}(1-p)(R^{T-t}-1)$  units, while increasing  $\Omega(x_0 = 1, y)$  by  $p^{t-1}(1-p)$ . Note that switching from a unit increase in  $y_t(\tilde{V}^{t-1}, 0)$ to a unit increase in  $y_{t-1}(\tilde{V}^{t-2}, 0)$  decreases the objective by the factor of  $\frac{1}{p} \frac{(R^{T-t+1}-1)}{(R^{T-t}-1)}$  and increases  $\Omega(x_0 = 1, y)$  by the factor of  $\frac{1}{p}$ , where  $\frac{(R^{T-t+1}-1)}{(R^{T-t-1})} > 1$ . Thus, after  $y_T(\tilde{V}^{T-1}, 0) = 1$ , the second least costly way to satisfy  $\Omega(x_0 = 1, y) = 0$  is to set  $y_{T-1}(\tilde{V}^{T-2}, 0) > 0$ , and after  $y_{T-1}(\tilde{V}^{T-2}, 0) = 1$ , the third least costly way is to set  $y_{T-2}(\tilde{V}^{T-3}, 0) > 0$ , and so on. In particular, the optimal choice of y's depends on the relative value of p and R.

If 
$$\frac{1-p^T R^T}{p^{T-1}(1-p)} \leq 1$$
, then  $y_T(\widetilde{V}^{T-1}, 0) = \frac{1-p^T R^T}{p^{T-1}(1-p)} \leq 1$  and all the other y's are zero.  
If  $\frac{1-p^T R^T}{p^{T-1}(1-p)} > 1$  and  $\frac{1-p^T R^T}{p^{T-2}(1-p)} - p \leq 1$ , then  $y_T(\widetilde{V}^{T-1}, 0) = 1$ ,  $y_{T-1}(\widetilde{V}^{T-2}, 0) = \frac{1-p^T R^T}{p^{T-2}(1-p)} - p$ .

and all the other y's are zero.

If 
$$\frac{1-p^T R^T}{p^{T-2}(1-p)} - p > 1$$
 and  $\frac{1-p^T R^T}{p^{T-3}(1-p)} - p(1+p) \le 1$ , then  $y_T(\tilde{V}^{T-1}, 0) = y_{T-1}(\tilde{V}^{T-2}, 0) = 1$ ,  
 $y_{T-2}(\tilde{V}^{T-3}, 0) = \frac{1-p^T R^T}{p^{T-3}(1-p)} - p(1+p)$ , and all the other y's are zero.  
Finally, if  $\frac{1-p^T R^T}{p(1-p)} - p(1+p+\dots+p^{T-3}) > 1$ , then  $y_T(\tilde{V}^{T-1}, 0) = \dots = y_2(\tilde{V}^1, 0) = 1$ ,  
 $y_1(0) = \frac{1-p^T R^T}{(1-p)} - p(1+p+\dots+p^{T-2}) \le 1$ , and all the other y's are zero.

Once we determine the optimal value of y's, the value of  $c_T^b$ 's and  $c_T^o$ 's are determined as follows:

$$\begin{split} c_T^b(\widetilde{V}^T) &= 0 & c_T^o(\widetilde{V}^T) = R^T \\ c_T^b(\widetilde{V}^{T-1}, 0) &= R^T - y_T(\widetilde{V}^{T-1}, 0) & c_T^o(\widetilde{V}^{T-1}, 0) = y_T(\widetilde{V}^{T-1}, 0) \\ c_T^b(\widetilde{V}^{T-2}, 0, 0) &= R^T - Ry_{T-1}(\widetilde{V}^{T-2}, 0) & c_T^o(\widetilde{V}^{T-2}, 0, 0) = y_{T-1}(\widetilde{V}^{T-2}, 0) \\ \vdots & \vdots \\ c_T^b(0, ..., 0) &= R^T - R^{T-1}y_1(0) & c_T^o(0, ..., 0) = y_1(0) \end{split}$$

Finally, we can easily see that the solution to the problem P' also satisfies all constraints in the main problem.  $\hline$ 

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