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# Endogenous Borrowing Constraints in the Presence of Shipping Costs*

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**Abstract**

The goal of this paper is to analyze the extent of international risk sharing in the presence of trade costs and credit constraints, focusing on three issues : how the frictions play, how they interact and whether they can be disentangled empirically. I build a simple model of a two-country endowment economy featuring both shipping costs and non-enforceable financial contracts. In a recursive contract framework, I show how these two features jointly determine countries external imbalances. An important result is that the credit constraints generated by the non-enforceability of contracts become looser as trade costs decrease. This creates a multiplier for the impact of trade integration on risk-sharing. In the absence of aggregate uncertainty, I find that one could not tell whether risk-sharing is limited due to the segmentation of goods markets or to limited debt capacity on financial markets. This observational equivalence no longer holds when aggregate uncertainty is allowed for, suggesting a way to disentangle the two sources of low international risk-sharing.

**Keywords:** Financial and Trade Integration, Enforcement Frictions, Recursive Contracts
**JEL Codes:** F34, F36, F15

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# 1 Introduction

By the logic of the balance of payments, there would be no scope for international borrowing and lending in a world with no tradable goods. Conversely, absent international capital flows, current accounts would always have to be balanced. These limit cases, though extreme, illustrate a crucial point : trade flows can be restricted due to constraints on international borrowing and capital flows can be hindered by frictions on international trade. In this paper, I draw on this insight to analyze the causes of low international risk sharing, by which I refer to a set of well-established stylized facts : low current account imbalances (Feldstein & Horioka [1980]), home bias in portfolio (French & Poterba [1991]) and high intra-country correlation between output and aggregate consumption (Backus, Kehoe & Kydland [1992]).

I build a simple model of a two-country endowment economy in which trade in goods and assets can possibly be limited both on the real side by trade costs, and on the financial side by borrowing constraints. Within this framework, I analyze the determination of countries external imbalances and tackle the following three questions : 1) is the presence of trade costs on *goods markets* of first order importance to account for the segmentation of international *asset markets*, once imperfections specific to capital markets have been taken into account? 2) how do each constraints interact with each other? 3) is it possible to discriminate empirically which constraint is binding, so that a precise cause could be assigned to low international risk sharing? These questions have both a theoretical interest and far-reaching implications for policy recommendations on the issues of economic and financial integration.

This work is related to the vast amount of literature dealing with international portfolio diversification, international borrowing and lending, and international risk sharing. In this literature, a majority of papers typically invokes enforceability problems or asymetric information on international financial markets to match the puzzling stylized facts mentionned above[1] . But in

[1] See the survey by Lewis [1999].

a thought-provoking paper, Obstfeld & Rogoff [2000] offer a challenging view : they argue that the so-called "puzzles" vanish once the existence of trade costs on international goods markets is taken into account, with no need to invoke financial market imperfections[2] . My paper goes beyond these alternative views by nesting them into a same theoretical framework. On the real side, I follow Obstfeld & Rogoff [2000] and model trade costs as *iceberg costs*, though my results could be extended to an economy featuring the traditional "tradable vs. non-tradable goods" dichotomy[3] . On the financial side, I have complete markets and full information but allow for the possibility of default. Like most of the literature on international debt, I only consider "national default risk", taking each country as a single entity that makes all lending, borrowing and default decisions[4] . I take the view that international contracts are not enforceable, which means that in case of default creditors have no way to recoup anything from the defaulter, either by force or by recourse to a third party. Hence in my model, international lending is not supported by any collateral but "reputation". To make things tractable I assume that after a default, a country is eternally plagued by bad reputation and loses access to capital markets forever – which is meant to capture the fact that defaulter countries often experience an external credit crunch and face higher interest rates. Endogenous borrowing constraints arise from this lack of enforceability and from the lack of commitment to repay, in such a way that at equilibrium all contracts are self-enforceable and default never happens[5] . To incorporate this feature into the model, I extend the literature on risk-sharing with imperfect commitment and endogenous borrowing constraints[6] .

[2] The "spill-over" mechanism from frictions on goods markets to asset market segmentation goes through inflation and real interest rates: trade costs imply deviations from purchasing power parity and a link between a country's external position and its price level. For the empirical relevance of trade costs, see Anderson & van Wincoop [2004].

[3] For an analysis of the implications of trade costs on current account and real exchange rate, see Dumas (1992). For their implication for portfolio choice, see Sercu, Uppal & Van Hulle (2000).

[4] An analogous setting featuring "resident default risk" on private international debt contracts has been analyzed in Jeske [2001] and Wright [2004]. The consequences of the separation of *private* borrowing decision and *government* decision to repudiate or take other steps detrimental to foreign creditors is the topic of Tirole [2003].

[5] This is made possible by the assumption of a complete menu of contingent assets. In an incomplete market setting, Chatterjee, Corbae, Nakajima and Rios-Rull [2002] have developed a similar model in which defaults do occur in equilibrium (in bad states of the world, which is a nice feature).

[6] See Kocherlakota [1996], Ligon, Thomas & Worral [2002], Alvarez & Jermann [2000] and Kehoe & Perri [2002].

Technically, my paper is part of a broad literature that uses "recursive contracts" to design optimal insurance mechanism under various incentives problems[7] . Within this literature, my contribution consists in solving for constrained optimal allocations with both limited enforcement and a proportional cost on transfers. Throughout the paper, I rely on "not too tight borrowing constraints" *à la* Alvarez & Jermann [2000] to implement this constrained optimal allocation in a decentralized setting[8] .

The model yields the following results. *First,* it clearly shows how imperfections on goods and capital markets jointly determine the overall degree of international risk sharing. For a simple version of the model, I can define four risk sharing regimes according to the role played by each friction in the determination of countries external imbalances and I state conditions on a set of key parameters for the economy to be in each regime. *Second,* I show that credit constraints endogenously relax as trade costs decrease[9] . The intuition for this impact of trade costs on credit constraints is that the value of a "good reputation" is an increasing function of the gains from trade. When a country loses access to capital markets, it also loses its ability to finance trade deficits and enjoy the gains from trade (in the one-good setting I adopt, there is only intertemporal trade and the gains from trade amount to the gains from intertemporal consumption smoothing). As trade costs diminish, more risk-sharing possibilities would be lost by returning to autarky. Because the sanction in case of default becomes more painful, the threshold level of debt beyond which a debtor would be tempted to default increases. Therefore, a decrease in trade costs causes an increase in international flows not only directly, as it costs less to ship, but also by increasing the value of reputation, relaxing the borrowing constraints and allowing higher current account imbalances. This extensive margin beyond the intensive margin creates a multiplier for the impact

[7] Recursive contracts have been applied in various fields : growth (Marcet & Marimon (1992)), development economics (Ligon, Thomas & Worral (2002)), labor economics (Hopenhayn & Nicolini (1997)), corporate finance (Clementi & Hopenhayn (2002)), public economics (Attanasio & Rioss-Rull (2000), Golozov & Tsyvinski (2003)).

[8] A formal treatment of the implementation issue is available upon request.

[9] Lane (2001) gets this relationship more mechanically through *ad hoc* assumptions.

of a decrease in trade costs on the extent of risk-sharing: I find this amplification mechanism to be quantitively significant. The *third* result bears on the empirical identifiability of the quantitative impact of each friction. In the absence of aggregate uncertainty, I point at an observational equivalence: without information on the underlying parameters of the economy, one could not tell whether international risk sharing is due to the segmentation of goods markets or to imperfections on financial markets. When aggregate uncertainty is allowed for, this observational equivalence no longer holds : financial market imperfections induce history dependence (which materializes in real exchange rate persistence) and the model suggests a way to identify the impact of trade costs on risk-sharing.

The remaining of the paper is organized as follows. Section 2 presents the setup of the model in a simple two-state case without aggregate uncertainty. This simple version of the model is instrumental in illustrating the logic of the paper. I solve it in section 3 and comment the results in section 4. In section 5, I extend the results to more general assumptions on countries'endowment processes, allowing for aggregate uncertainty. Section 6 concludes.

# 2 The model

I consider a pure exchange economy, with two countries $i = 1, 2$, each composed of a large number of infinite-lived identical agents. The countries are endowed in a same kind of non-storable good. I refer to country $i$'s endowment by $y_i$. To start with, I assume that the joint process for $(y_1, y_2)$ takes a very simple form : there are two states of nature $s = 1, 2$ and $p \in [0, 1]$ denotes the probability of remaining in the same state from one period to the next (alternatively, I write $\pi_{ss'} = p$ if $s = s'$ and $\pi_{ss'} = 1 - p$ if $s \neq s'$). If $s = 1$, $y_1 = y_H$ and $y_2 = y_L$, with $y_L < y_H$. If $s = 2$, $y_1 = y_L$ and $y_2 = y_H$. Hence, I assume no aggregate uncertainty and a perfect symmetry in countries'endowments[10] .

[10] The markovian assumption makes the model irrelevant to study questions of development : the endowment process is not altered by any kind of action. The logic is not to borrow to invest and foster growth. For the same reason, introducing longer debt contracts would be redundant.

The good can be exchanged on a competitive international market but subject to a trading cost which takes the form of an "iceberg cost". I call $\tau \in [0,1]$ the percentage lost in shipping. The presence of these costs allows deviations from purchasing power parity but by arbitrage these deviations are bounded.

On the financial side, I assume complete markets, with two contingent one-period securities traded each period. I assume countries can default on their debt when this makes them better off. In case of default, creditors have no way to recoup anything : contracts are not *enforceable*. Trade in financial assets is possible though because when a country defaults it is excluded from asset markets forever[11] . *Ex post*, a country takes the decision to default when the short-run gain of doing so (increasing in the amount it has to repay) is higher than the long-run loss due to its return to autarky. Hence, *ex ante*, creditors do not lend above a certain amount, above which they know their debtor would default : the non-enforceability of contracts generates endogenous credit contraints.

I assume that the two countries share the same preferences. I consider two representative agents who maximise

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau)$$

with an iso-elastic period utility function :

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

The parameter $\beta$ denotes the time discount factor ($0 < \beta < 1$), and $\rho$ is the relative risk aversion parameter.

[11] Three remarks are in order here, having to do with realism, renegociation-proofness and the Bulow & Rogoff's critique. Realism is not an issue : even if markets do not have a long memory, defaults are typically followed by a temporary credit crunch and high interest rates – this is what this assumption captures and relaxing it to make it more realistic would not alter the main points of our analysis. The other two issues are addressed by Kletzler & Wright (2000). They show that our equilibrium allocation can be sustained by renegociation-proof strategies, without resorting to the threat of autarky and they also show that with symmetric inability to commit and with "cheat the cheater strategies" the Bulow & Rogoff's critique no longer holds.

# 3 Solution

## 3.1 Stationary constrained efficient allocation

In this simple symmetric case with two states of nature and no aggregate uncertainty (like in the example considered in Alvarez & Jermann [2001]), one can focus on a stationary constrained efficient allocation, as argued in the following proposition.

**Proposition 1 :** *whatever the initial distribution of wealth, countries consumptions end up in an ergodic set* $\{c_H, c_L\}$ *and then are stationary, in the sense that they only depend on the current state :*

$$\begin{array}{ccc} & s=1 & s=2 \\ c_1 & c_H & c_L \\ c_2 & c_L & c_H \end{array}$$

$(c_H, c_L)$ *is the solution of the following problem :*

$$\max_{c_H, c_L} \quad u(c_H) + u(c_L)$$

$$s.t. \quad c_L - y_L = (1-\tau)(y_H - c_H)$$

$$\begin{aligned} V_H &\geq U_H^{aut} \\ V_L &\geq U_L^{aut} \end{aligned}$$

*where the values* $V_H$, $V_L$, $U_H^{aut}$ *and* $U_L^{aut}$ *are defined by :*

$$\begin{cases} V_H = u(c_H) + \beta\left[pV_H + (1-p)V_L\right] \\ V_L = u(c_L) + \beta\left[pV_L + (1-p)V_H\right] \end{cases}$$

$$\begin{cases} U_H^{aut} = u(y_H) + \beta\left[pU_H^{aut} + (1-p)U_L^{aut}\right] \\ U_L^{aut} = u(y_L) + \beta\left[pU_L^{aut} + (1-p)U_H^{aut}\right] \end{cases}$$

The inequality constraints are *participation constraints.* Their meaning is that at any time and in any state, a country should be at least as well by cooperating with the other country as by returning to autarky forever. Theses constraints are the exact counterpart in the planner problem of borrowing constraints in the decentralized setting.

Further on, I will concentrate on the quantities shipped and consumed in the stationary allocation. Before to start with the algebra, it should be clear that depending on the severity of each friction, the economy can find itself in one of the following four situations : 1) complete autarky due to trade costs, 2) complete autarky due to participation constraints, 3) some integration limited by credit constraints, possibly interacting with trade costs if any, and 4) some integration limited only by transportation costs (with the special case of full integration when $\tau = 0$).

## 3.2 Impact of trade costs

The impact of frictions on "real flows" is the place to start with. Indeed, if trade costs are prohibitive, countries will never import from abroad and credit constraints is not an issue.

**Proposition 2 :** *Transfers occur only when the utility gain brought by smoothing outweighs the iceberg cost of transfer. That is the case if* $(1-\tau)u'(y_L) > u'(y_H)$.

Using the assumption of isoelastic utility, this condition can rewrite alternatively :

$$\tau < 1 - \left(\frac{y_L}{y_H}\right)^{\rho}$$

$$\Leftrightarrow \frac{y_H}{y_L} > \left(\frac{1}{1-\tau}\right)^{1/\rho}$$

These conditions are easily interpeted : for a certain size of country specific shocks (captured by $\frac{y_L}{y_H}$), trade costs are prohibitive if they are above a certain threshold : $1 - \left(\frac{y_L}{y_H}\right)^{\rho}$. Alternatively, in terms of decentralized equilibrium allocation, transfers only occur when the shadow real exchange rate in autarky is above $(1-\tau)^{-1}$. For given trade costs $\tau$, endowment shocks have to be large enough ($y_H/y_L > (1-\tau)^{-1/\rho}$) to bring deviations from purchasing power parity large enough to compensate for trade costs. The size of the no-shipping region is decreasing in $\rho$, which captures the willingness to smooth over time.

### 3.3 Impact of participation constraints

I am now considering the impact of participation constraints when trade costs are not prohibitive. I start by defining the consumption allocation that would prevail absent participation constraints (full-enforceability case). It is such that : $(1-\tau)u'(c_L^*) = u'(c_H^*)$.

**Proposition 3 :** *when participation constraints are not binding, the level of transfer $T^*$ (before the losses due to "melting") from the country having a good shock to the other is given by :*

$$\frac{y_H - T^*}{y_L + (1-\tau)T^*} = \left(\frac{1}{1-\tau}\right)^{1/\rho}$$

Now, the presence of participation constraints can have different implications, according to their severity, which is itself a function of some key parameters (concerning both preferences, like $\beta$ and $\rho$, and endowments, like $p$ and $y_H/y_L$). Proposition 4 states conditions on the parameters of the economy such that participation constraints never bind (the proofs for propositions 4 and 5 are in the appendix).

**Proposition 4 :** *Define $V_H^*$ and $V_L^*$ as follows*

$$V_H^* = u(c_H^*) + \beta\left[pV_H^* + (1-p)V_L^*\right]$$
$$V_L^* = u(c_L^*) + \beta\left[pV_L + (1-p)V_H^*\right]$$

*If $V_H^* > U_H^{aut}$, then $(c_H^*, c_L^*)$ is the stationary equilibrium allocation.*

Condition $V_H^* > U_H^{aut}$ is equivalent to a condition on $\tau$ being below a certain threshold.

When participation constraints do play, the remaining question is how much risk sharing they allow, i.e. how far they push the economy away from $(c_H^*, c_L^*)$. Proposition 5 characterizes the allocation resulting from the interplay between small trade costs and binding participation constraints. In particular, it states conditions under which participation constraints prevent any risk sharing among the two countries.

**Proposition 5 :** *when* $1 - \frac{1-\beta p}{\beta(1-p)}\left(\frac{y_L}{y_H}\right)^\rho < \tau < 1 - \left(\frac{y_L}{y_H}\right)^\rho$*, countries remain in autarky because of the self-enforceability constraint. When* $\tau < 1 - \frac{1-\beta p}{\beta(1-p)}\left(\frac{y_L}{y_H}\right)^\rho$ *but* $\tau$ *is still too high for condition* $V_H^* > U_H^{aut}$ *to hold, the equilibrium allocation is implicitely defined in the following equation*

$$u(c_H) = \frac{(1-\beta)(1-2\beta p+\beta)}{1-\beta p}U_H^{aut} - \frac{\beta(1-p)}{1-\beta p}u[y_L + (1-\tau)(y_H - c_H)].$$

The presence of costs on transfers does not alter the usual comparative statics results of the literature on risk sharing without commitment: there is all the more risk sharing as $\beta$, $\rho$ and $y_H/y_L$ increase and as $p$ decreases.

# 4 Findings

The severity of the non-enforceability of contracts is not captured by one single parameter. The appeal of Alvarez & Jermann's "not too tight borrowing constraints" is precisely that they do not depend on an exogenously and arbitrarily fixed parameter, supposed to capture the level of imperfections on credit market, but on a whole set of fundamental parameters[12] . To illustrate the results of the model, I will use $p$, the probability of remaining in a same state from one period

[12] A few papers in the literature on limited risk sharing show that models with endogenous borrowing limits perform better than models with arbitrarily fixed borrowing constraints. See for example Kehoe & Perri [2002].

to the other, as a measure of the severity of the credit constraints brought about by contract unenforceability. When $p = 1$, one country is constantly hit by luck and the other is constantly unlucky. The rich never lends to the poor in such a world. When $p = 0$, countries alternate deterministically from good to bad state : this is the most favorable scenario for risk sharing to take place. In between, the higher $p$ the more limited the extent of risk-sharing.

### 4.1 Risk sharing regimes

Figure 1 shows in the space $(p, \tau)$ the frontiers delineating the four possible regimes for our two-country economy[13] . For $\tau$ above $1 - (y_L/y_H)^\rho$, trade costs are so high that they prohibit any current account imbalance (I label this region "autarky due to transportation costs"). For $\tau$ below this threshold, there are three more regions, depending on the tightness of borrowing constraints, which itself depends both on $p$ and $\tau$.

For a given $\tau$, as the persistence parameter $p$ goes from 1 to 0, the economy typically goes from no-risk sharing to a full risk-sharing regime, going through a zone of limited risk-sharing. For $\tau = 0$, the three segments correspond exactly to the three risk-sharing regimes in Alvarez & Jermann [2001]. Compared to their paper, I add the vertical dimension.

For a given $p$, these zones correspond (in the same ordering) to a decreasing $\tau$. For a given level of endowment persistence, when trade costs decrease, credit constraints become looser – so that the economy can reach a new regime more intensive in risk sharing. The mechanism is the following : as trade costs decrease, transfers are easier to make, so that expected gains from trade are higher – and hence the sanction incurred in case of default would be more painful. Lower trade costs thus increase the incentive not too default and therefore relax the credit constraints. The frontier separating the no risk-sharing zone from the limited risk-sharing corresponds to the locus $\tau = 1 - \frac{1-\beta p}{\beta(1-p)} \left(\frac{y_L}{y_H}\right)^\rho$. The other frontier has no closed-form expression : it corresponds to

[13] The figures are drawn for the following calibration : $y_H = 0.6$, $y_L = 0.4$, $\beta = 0.8$ and $\rho = 3$.

the $V_H^* = U_H^{aut}$ condition in proposition 4. The concave shape of the frontiers follows from the interaction between credit constraints and trade costs : the lower $p$, the lower $\tau$ has to be to make a certain level of risk-sharing sustainable.

The message delivered in figure 1 is that fostering trade integration (i.e. lowering $\tau$) is a pre-requisite for financial risk-sharing as it increases countries debt capacity[14] . This figure also enlightens the pains and benefits associated with the process of globalization : as $\tau$ decreases, the typical trajectory is to go from complete autarky due to trade costs to integration with risk sharing – but in-between, there is an ungrateful regime to cross where the gains from integration are not reaped because of imperfections on international financial markets.

## 4.2 Accounting for low external imbalances

Figure 2b shows $T(p, \tau)$ the level of transfer (all at once net and gross) flowing from the lucky country to the unlucky. In the model, due to the absence of aggregate uncertainty and because of stationarity, this level of transfer is constant in each period. The level of $T$ for a given $(p, \tau)$ should be compared with $(y_H + y_L)/2$, which is the first-best level of transfers, guaranteeing a constant consumption level. The computed example is for $y_L = 0.4$ and $y_H = 0.6$, so that the first-best situation would imply transfers of 0.1 from the lucky to the unlucky and constant consumption equal to 0.5. By projection on the base of this three-dimensional picture, one easily recognizes the shape of the regions exhibited in figure 1. Obviously, autarky due to trade costs looks exactly like autarky due to credit constraints: $T = 0$ (more on this in section 4.5).

Figures 3 and 5 give a cross view of the surface. Figure 3 is drawn for $\tau = 0$ : it exhibits the pure impact of credit constraints (the tightness of which increases with $p$). Figure 5 shows the impact of trade costs for a given persistence parameter $p = 0.8$. This is to compare with figure 4,

[14] It should be clear that my model, at least in this totally symmetric specification, is not relevant to understand the North-South integration but rather the integration of similarly rich countries.

which corresponds to the model with trade costs and perfectly enforceable contracts, then showing the pure impact of trade costs.

What is the quantitative impact of the existence trade costs on the extent of risk-sharing? To assess the impact of trade costs in a world featuring imperfections on credit markets, I start by measuring the distance to the first best level of transfers implied by enforcement frictions : $T_{FB} - T_{BC}$. I then consider the level of transfer when both frictions are present and compute $T_{FB} - T_{TC\&BC}$. The percentage of the distance to the first best that can be claimed to be due to the presence of trade costs in addition to enforcement frictions is :

$$1 - \frac{T_{FB} - T_{BC}}{T_{FB} - T_{TC\&BC}}.$$

For $y_H = 0.6$, $y_L = 0.4$, $\beta = 0.8$ and $\rho = 3$, the calculations give that if $\tau = 20\%$ the presence of trade costs accounts for 50% of the distance to first best, while trade costs of 10% would still account for 33% of the distance to first best.

## 4.3 Multiplier for the impact of trade integration

In the intermediate region defined in proposition 5, the interaction between trade costs and credit constraints amplifies the overall impact of a decrease in trade costs on risk-sharing. This shows up in the fact that under full contract enforceability a decrease in $\tau$ from 20 to 10% would cause a 5% increase in transfers, whereas when contracts are not enforceable (and for $p = 0.8$), the same change in trade costs implies that the amount of transfers increases by 55.4%.

## 4.4 Welfare impact of frictions

It might be more telling to assess the impact of frictions in welfare terms. Figure 2a shows the welfare impact of frictions. Based on this, I can use the standard measure "*à la* Lucas", computing the permanent consumption decrease which would cause a welfare loss equivalent to that due to the introduction of frictions. For $\tau = 0.2$ and $p = 0.8$, the overall welfare loss corresponds to a

3.56% permanent consumption decrease. The presence of trade costs accounts for 35.5% of it. For $\tau = 0.1$, the corresponding figures are 2.06% and 28.7%.

### 4.5 Observational equivalence

A question different from that of "risk-sharing accounting" is that of the "empirical identifiability" of the causes of limited integration. The Obstfeld & Rogoff [2000] paper immediately raises this question : say we have no idea on $\tau$, can we deduce from consumption, output and current account data alone the importance of trade costs in limiting international risk-sharing? In my simple model, the answer is no : *any possible joint process for endowments and consumption allocations can result from different vectors of parameters, each exhibiting a different level for* $\tau$. For instance, for $p = 0.8$, $\rho = 3$, $y_H = 0.6$ and $y_L = 0.4$, there is a perfect observational equivalence between a world with $\tau = 0$ and $\beta = 0.742$ and a world with $\tau = 20\%$ and $\beta = 0.8$ : high impatience or high trade costs constitute alternative explanations for a given low level of risk-sharing. In section 5, I show that this result must be qualified as soon as more general assumptions are made on the endowment process.

### 4.6 Approximate portfolio bias

In the section of their paper dealing with the impact of trade costs on portfolio bias, Obstfeld & Rogoff [2000] adopt a model with a complete asset market. They easily compute the consumption allocation, which due to their assumption of completeness is optimal. Then they argue that though this allocation can not be supported as an equilibrium by trading only shares on countries outputs[15] , they can still approximate the composition of country portfolios that would give an allocation close to the allocation they get assuming trading in a complete menu of assets.

I can do the same kind of exercise and possibly carry out the same kind of accounting than

[15] For isoelastic utility, it is the case that, absent trade costs, the complete market allocation can be exactly replicated by trading only output shares. This is because in this case, at optimum, each country's consumption is a constant share of global output. This no longer holds with trade costs.

in section 4.2. This means computing $\theta_H^{dom}$, $\theta_H^{foreign}$, $\theta_L^{dom}$ and $\theta_L^{foreign}$, with $\theta_H^{dom} + \theta_L^{foreign} = \theta_L^{dom} + \theta_H^{foreign} = 1$, such that :

$$\begin{aligned} c_H &= \theta_H^{dom} y_H + \theta_H^{foreign}(1-\tau) y_L \\ c_L &= \theta_L^{dom} y_L + \theta_L^{foreign}(1-\tau) y_H \end{aligned}$$

Then the approximate share of foreign assets in country portfolio is $\theta^{foreign} = (\theta_H^{foreign} + \theta_L^{foreign})/2$. The value of $\theta^{foreign}$ for given parameters of the model would have to be compared with the first-best level of diversification: $\theta^{foreign} = 50\%$.

# 5 Extension to S states and aggregate uncertainty

I now show how the simple model of the previous section extends to more general assumptions on the joint endowment process. There are now $S$ possible states of the world at each period, the transition probabilities being given by $\pi_{ij} = prob(s_{t+1} = i \,|s_t = j\,)$. The $\Pi(s^{t+i}\,|s_t)$ are the conditional probabilities induced by the matrix $[\pi_{ij}]$. These assumptions nest iid and markov distributions. They allow for aggregate uncertainty and for any correlation between endowments (and for any relationship between idiosyncratic and aggregate risk). I will denote a history of state realizations $(s_0, s_1, ..., s_t)$ by $s^t$, where $s_t$ is the state realized in period $t$.

I now turn to the general planning problem that yields the constrained optimal allocation. In what follows, I describe the transfers by two policy functions $b(.)$ and $l(.)$ : $b$ denotes the transferred quantity received by country 1, $l$ denotes the quantity taken from 1 and transferred to 2. The constrained planning problem is :

$$\max_{l(s^t),b(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \Pi(s^t\,|s_0) \left\{\lambda_1 u[c_1(s^t)] + \lambda_2 u[c_2(s^t)]\right\}$$

$$s.t. \qquad \forall i, \forall t, \forall s^t, \quad u(c_i(s^t)) + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} u(c_i(s^\tau)) \geq V_i^{aut}(s_t)$$

$$
\begin{aligned}
c_1(s^t) &= y_1(s_t) + b(s^t) - l(s^t) \\
c_2(s^t) &= y_2(s_t) - \frac{1}{1-\tau} b(s^t) + (1-\tau) l(s^t)
\end{aligned}
$$

$$\forall s^t, \quad b(s^t), l(s^t) \geq 0$$

$E_t(.)$ denotes the conditional expectation operator associated with probabilities $\Pi(s^\tau \,|s_t)$, $\tau \geq t$. $\lambda_1$ and $\lambda_2$ are the *initial* Pareto-Negishi weights.

This problem consists in finding optimal history-contingent allocations that are both *resource-feasible* and *sustainable*, in the sense that they respect participation constraints. These participation constraints capture the fact that at any time and for any given history, the parties involved in the risk-sharing contract have the possibility to "default", i.e. to renege on the risk-sharing contract and return to autarky.

In this general formulation, the problem is much more complex than the simple case I considered up to now. The trick to deal with this kind of problem consists in finding an appropriately defined state space in which the problem is stationary. I follow Marcet & Marimon [1994] and take the *modified relative weight* as a state variable[16] .

## 5.1 From history dependence to stationarity

In what follows, $\alpha_i(s^t)$ denotes the Lagrange multipliers, normalized by $\Pi(s^t \,|s_0)\, \beta^t$, for country i's participation constraint at time t after history $s^t$. Writing the Lagrangian and rearranging terms using Abel's partial summation formula (see appendix for details), the optimization problem can be rewritten equivalently as the following saddle-point problem :

[16] This method originated in papers by Kydland & Prescott [1980] and Hansen, Hepple and Roberds [1985]. For a textbook presentation, see Ljungqvist & Sargent [2004].

$$\min_{\{\alpha_t^i(s^t)\}} \max_{l(s^t), b(s^t)} \quad \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \Pi(s^t \,|s_0\,) \left\{ \begin{array}{c} M_1(s^t).u[c_1(s^t)] + M_2(s^t).u[c_2(s^t)] \\ -\alpha_1(s^t) V_1^{aut}(s_t) \\ -\alpha_2(s^t) V_2^{aut}(s_t) \end{array} \right\}$$

$$\begin{aligned} s.t. \quad c_1(s^t) &= y_1(s_t) + b(s^t) - l(s^t) \\ c_2(s^t) &= y_2(s_t) - \frac{1}{1-\tau} b(s^t) + (1-\tau) l(s^t) \end{aligned}$$

$$\begin{aligned} M_i(s^t) &= M_i(s^{t-1}) + \alpha_i(s^t) \\ M_i(s^0) &= \lambda_i + \alpha_i(s_0) \end{aligned}$$

$$\alpha_t^i(s^t) \geq 0$$

$$\forall s^t, \quad b(s^t), l(s^t) \geq 0.$$

The first-order condition with respect to consumption yields the following expression :

$$(1-\tau)\,\frac{\lambda_2 + \alpha_2(s_0) + ... + \alpha_2(s^t)}{\lambda_1 + \alpha_1(s_0) + ... + \alpha_1(s^t)} \leq \frac{u'(c_1(s^t))}{u'(c_2(s^t))} \leq \frac{1}{1-\tau}\,\frac{\lambda_2 + \alpha_2(s_0) + ... + \alpha_2(s^t)}{\lambda_1 + \alpha_1(s_0) + ... + \alpha_1(s^t)}.$$

This expression captures all at once the logic of optimal allocations with transportation costs and the logic of optimal risk sharing contracts with participation constraints. The former is easy to understand : it is about sparing iceberg costs when transfers result in too little utility gain. The latter is less obvious : it has to do with backloading as an incentive device. It implies that a country's "effective" weight in the planner's objective is not constant across dates and states : the better its history of shocks, the higher the country's relative weight. More precisely, a country's relative weight is increased when its participation constraint is binding (i.e. when $\alpha_i(s^t) > 0$). This plays as an incentive mechanism : when a country is tempted to default, it is induced to respect the contract by being credibly promised a higher continuation value[17] .

[17] The formulation in terms of "promised value" rather than in terms of modified weights follows from Abreu, Pearce & Stacchetti [1990].

Introducing $z(s^t)$ the modified relative weight of country 2, one can rewrite more succintly :

$$(1-\tau)z(s^t) \leq \frac{u'(c_1(s^t))}{u'(c_2(s^t))} \leq \frac{1}{1-\tau} z(s^t).$$

This modified relative weight is a key variable : from the first order conditions, it is immediate that *when $z$ is added to the current state of nature as a state variable the problem is stationary.*

## 5.2 Optimal policy functions

Solving for constrained-optimal allocations amounts to solving for policy functions $l, b, C_1, C_2, \mu_1, \mu_2, Z, V_1, V_2$ which satisfy the following *system of equations* for any $x = (z, s)$ in $[0, +\infty] \times \{1, ..., S\}$:

– FOC with respect to consumption

$$(1-\tau)Z(z,s) \leq \frac{u'[C_1(z,s)]}{u'[C_2(z,s)]} \leq \frac{1}{1-\tau} Z(z,s)$$

– transition law

$$Z(z,s) = \frac{1-\mu_1(z,s)}{1-\mu_2(z,s)}.z$$

– positivity and coherence of transfers

$$b(z,s), l(z,s) \geq 0$$

$$b(z,s).l(z,s) = 0$$

$$(1-\tau)z \leq \frac{u'(y_1(s))}{u'(y_2(s))} \leq \frac{1}{1-\tau} z \quad \Rightarrow \quad b(z,s) = l(z,s) = 0$$

– resource constraint

$$\begin{aligned} C_1(z,s) &= y_1(s) + b(z,s) - l(z,s) \\ C_2(z,s) &= y_2(s) - \frac{1}{1-\tau} b(z,s) + (1-\tau) l(z,s) \end{aligned}$$

– value of the contract for agent i

$$V_i(z,s) = u[C_i(z,s)] + \beta \sum_{s'} \pi_{ss'} V_i(Z(z,s); s')$$

– participation constraints and complementary slackness conditions

$$\mu_i(z, s) \geq 0$$

$$V_i(z, s) - V_i^{aut}(s) \geq 0$$

$$\mu_i(z, s).[V_i(z, s) - V_i^{aut}(s)] = 0$$

The optimal policy functions can be found numerically. I use an algorithm iterating directly on policy functions, starting from the optimal policy functions for the problem featuring trade costs but full-enforceability[18] . Once the optimal policy functions are obtained, getting results on average level of transfers or on macro correlations and volatility just involves computing unconditional expectations by using the ergodic distribution induced by the markov chain.

### 5.3 Some insights

For given relative weight $z$ (inherited from the past and fully summarizing it), the logic is exactly the same as in section 3. If the state $s$ is such that $(1-\tau)\,z \leq \frac{u'(y_1(s))}{u'(y_2(s))} \leq \frac{1}{1-\tau}z$ (i.e. the ratio of autarkic marginal utilities is within the band), there is no shipping. The question then is : for "extreme" states, do participation constraints prevent the planner from implementing the "optimal" allocation (i.e. optimal given $\tau$)? Without loss of generality, I consider a situation in which the state vector $(z, s)$ is such that :

$$\frac{u'(y_1(s))}{u'(y_2(s))} > \frac{1}{1-\tau}z$$

Absent participation constraints (the constraint that could bind here is that of country 2), the planner would like to make a transfer $b^* > 0$ from country 2 to country 1 such that :

$$\frac{u'(c_1)}{u'(c_2)} = \frac{1}{1-\tau}z$$

[18] In the problem without trade costs, the starting point is also crucial for convergence of the algorithm (see Kehoe & Perri [2002]). This is due to the fact that the underlying functional operator is not a contraction mapping.

If no participation constraint is violated for such a transfer,

$$
\begin{aligned}
b(z,s) &= b^*, \\
l(z,s) &= 0, \\
Z(z,s) &= z.
\end{aligned}
$$

Otherwise, the planner has to find a $Z(z,s)$ and a transfer $b(z,s)$ such that :

$$
Z(z,s) > z,
$$

$$
\frac{u'[C_1(z,s)]}{u'[C_2(z,s)]} = \frac{1}{1-\tau} Z(z,s),
$$

$$
u[C_2(z,s)] + \beta \sum_{s'} \pi_{ss'} V_2(Z(z,s); s') = V_2^{aut}(s).
$$

Overall, it is clear that, for a given history, depending on the state that realizes, the economy finds itself in one of the four situations I identified as possible stationary outcomes in the simple version of the model.

## 5.4 A 4-state example with aggregate uncertainty

To illustrate the properties of the model, I take $S = 4$ and I assume that country endowments in each state are as follows :

| | $s=1$ | $s=2$ | $s=3$ | $s=4$ |
|---|---|---|---|---|
| $y_1$ | $y_H$ | $y_H$ | $y_L$ | $y_L$ |
| $y_2$ | $y_H$ | $y_L$ | $y_H$ | $y_L$ |

The optimal policy and value functions all take $z$ and $s$ as arguments and also depend on the whole range of parameters : $\beta$, $\rho$, $p$, $y_H$, $y_L$ and $\tau$. The baseline calibration is for $\beta = 0.85$, $\rho = 3$, $yH = 1.694$ and $yL = 0.590$. The matrix of transition probabilities follows from the individual probability of remaining in the same state : $p = 0.9$.

*Impact of trade costs on transfers and welfare* – Figure 6 shows the transfer taking place in state 1 as a function of $z$, for different values of $\tau$ (net transfer from country 2 to country 1 before trade costs). In this state, both countries have a high endowment. There is a region around $z = 1$ for which no transfer takes place. For $z$ low enough, transfers go towards country 1. For high z, transfers go to country 2. The impact of $\tau$ on transfers shows up both in the size of the no-shipping region and in the size of transfers (see more details below).

To give an idea of the resulting impact of trade costs on welfare, I show in figure 7 how constrained Pareto frontiers depart from first-best frontiers. In each state, going from the upper-right to the lower-left, the figure displays the first-best welfare frontier, the frontier with capital market imperfections and the constrained Pareto frontiers with both credit constraints and trade costs of 20%. The welfare impact of trade costs beyond credit constraints is large.

Figure 8 shows the admissible values for z in each state for different values of $\tau$, for $\tau$ going from 0 to 30%. In state 2 for instance, a state favorable to country 1, for given $\tau$, there is a value of z, $\bar{z}_2 < 1$, above which the expected value of country 1 is higher when defaulting and returning to autarky than when transferring to the other country and keeping access to risk-sharing. Similarly, one can define a lower bound $\underline{z}_3 > 1$ on the corresponding intervals for state 3. The intervals $[\underline{z}_s, \bar{z}_s]$ directly determine the policy function $Z(z, s)$. In this particular example, it must be the case (at least after a finite history of shocks) that $Z(z, 2) = \bar{z}_2$ and $Z(z, 3) = \underline{z}_3$.

*Multiplier for the impact of trade integration* – Figure 8 illustrates that $\bar{z}_2(\tau)$ is decreasing with $\tau$ while $\underline{z}_3$ is increasing in $\tau$. This is exactly what lies behind the interaction between trade costs and credit constraints and the amplification effect that I emphasized in the preceding section. The optimal transfer in state 2 for instance, $T_2$, is given by

$$\left( \frac{y_H - T_2}{y_L + (1 - \tau)T_2} \right)^{-\rho} = (1 - \tau)\bar{z}_2(\tau)$$

By differentiating this expression, it can be easily shown that the term $d\bar{z}_2(\tau)/d\tau < 0$ makes $|dT_2/d\tau|$ larger. This term captures the extensive margin in risk-sharing.

*Limited risk sharing and past dependence* – Figures 9 to 11 illustrate the allocation resulting from a given simulation for the markov chain (this is for $\tau = 20\%$ and starting with $z = 1$, the sequence of shocks is listed at the bottom of figure 9). The first figure clearly shows that for this calibration choice, trade costs and credit contraints leave room for some but imperfect risk-sharing. It also illustrates the role of past dependence : a country's consumption does not only depend on the current state but also on the preceding states.

Figure 10 shows the behavior of the Pareto-Negishi weight : switching to state 2 (resp. state 3) causes $z$ to jump to $\bar{z}_2$ (resp. $\underline{z}_3$). Otherwise, the relative weight remains constant.

Eventually, figure 11 shows the behavior of the marginal utility ratio (circles). On this figure, one can see that there are four possible values for $u'(c_1)/u'(c_2)$. The two extreme values, $(1-\tau)\bar{z}_2$ and $\underline{z}_3/(1-\tau)$, each correspond to state 2 and 3, in which endowments are asymetric. The marginal utility ratios in state 1 and 4 take intermediate values, $\bar{z}_2/(1-\tau)$ or $(1-\tau)\underline{z}_3$, depending on whether there was a state 2 or 3 in the close history of shocks. If $\tau$ was equal to zero, the band around $z$ would collapse.

## 5.5 Disentangling the frictions

The key difference between the general setting of this section and that of the rest of the paper has to do with the role of the history of shocks. In the general case, "history matters" : it is a feature specific to the "two-state and no aggregate uncertainty" model that the resulting allocation is stationary[19] .

When the question of "empirical identifiability" is revisited now, the message is modified. The presence of imperfections on financial markets induces a specific kind of "persistence" in countries marginal utility ratio. The presence of trade costs induces no-shipping bands which affect transfers "at the margin" even when they do not prevent them. As a result, *the behavior of the marginal*

[19] The stationarity of the 2-state problem without uncertainty is easy to understand now. It comes from the fact that, in this degenerate case, the marginal utility ratio bounds between $(1-\tau)\bar{z}_1(\tau)$ and $\underline{z}_2(\tau)/(1-\tau)$.

*utility ratio is substantially different in the four cases of a) a model without frictions, b) a model with trade costs only, c) a model with enforcement frictions only and d) a model with both capital markets imperfections and trade costs.* It is interesting to notice that when net transfers occur in each period (as is the case in the example above), the process followed by the real exchange rate corresponds exactly to the process followed by the marginal utility ratio $u'(c_1)/u'(c_2)$. The model therefore suggests that enforcement frictions could be a possible explanation for the observed persistence in real exchange rates.

In the model, the ratio of marginal utilities $(u'(c_1)/u'(c_2))$ in state 3 divided by the ratio of marginal utilities in a subsequent symmetric state directly gives $(1-\tau)^2$ (while the marginal utility ratio in state 2 divided by the ratio of marginal utilities in a subsequent symmetric state gives $1/(1-\tau)^2$). Since the process followed by the marginal utility ratio $u'(c_1)/u'(c_2)$ is related to the real exchange rate process, this suggests a way to recover the size of $\tau$ from real exchange rate data. Another strategy would consist in calibrating the parameters $p$, $y_H$ and $y_L$ to approximate output processes for a pair of countries via a markov chain (Tauchen's method). Then, for standard values of $\beta$ and $\rho$, the ratio of $sd(C)/sd(Y)$ would only be a function of $\tau$. Once properly calibrated, the model could be used to evaluate the role of trade costs (beyond that of capital market imperfections) in limiting international risk-sharing and to quantify their impact on quantities, prices and welfare.

# 6 Conclusion

The goal of this paper was to analyze how frictions on international trade and on international capital markets jointly determine countries external imbalances. I carried out this analysis for the case of a symmetric two-country economy and sketched how each constraint plays and possibly interacts with the other. The model exhibits a strong interaction between real and financial frictions, pointing at a form of complementarity between real integration (to be understood as a decrease in trade costs) and financial integration, going through credit constraints. It yields a clear message concerning globalization : trade integration has to come first and it has to be fostered beyond a certain level for countries to reap the welfare gains associated with more international risk-sharing. Once this threshold is reached, the impact of a marginal decrease in trade costs is strongly amplified via the credit market channel.

On an empirical ground, recent studies (see Rose [2002], Aviat & Coeurdacier [2004]) have documented a robust positive relationship between trade in goods and bilateral capital flows. My model is instrumental in understanding this relationship as it captures the very essence of the standard informal argument that debtors repay for fear of the loss in gains from trade following default[20] .

Possible extensions of the model could consist in considering more than one good[21] . The first direction is to have non-tradable goods in each country. The second direction is to have two differentiated (imperfect substitute) tradable goods, each good being produced in only one country. In such a model, there would be "real" trade in goods (as opposed to intertemporal trade) and we could possibly distinguish gross and net flows. There would be at least two key differences : in the default scenario, trade would still be possible after reverting to financial autarky provided a trade balance constraint is respected. Thus the autarky value would also increase when $\tau$ decreases. Second, the terms of trade movements would create an alternative risk-sharing mechanism, as emphasized by Cole and Obstfeld [1991]. I leave the analysis of the impact of the degree of goods susbtitutability on the extent of risk-sharing for future research.

[20] Some empirical evidence in favor of this argument can be found in Rose & Spiegel [2002], who show that past default has long lasting negative impact on subsequent trade. Another informal argument emphasizes *information*: trade creates a relationship and involves information flows that reduce information asymetries. This channel is certainly relevant too.

[21] Bodenstein [2004] has a model analogous to mine with richer assumptions for commodity markets (borrowed from Corsetti, Dedola and Leduc [2003]), though he has no shipping costs. His richer structure allows him to address convincingly both the exchange rate volatility puzzle and the Backus-Smith puzzle.

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# 7 Appendix

## 7.1 Proofs for section 3.3

Throughout this appendix, $\tau < 1 - \left(\frac{y_L}{y_H}\right)^{\rho}$. The problem is to characterize transfers in regime 3 and find the frontiers delineating this zone (as shown in figure 1).

Under full-enforceability, the level of transfer $T^*(\tau)$ would be such that[22]

$$\frac{u'(y_L + (1-\tau)T^*)}{u'(y_H - T^*)} = \frac{1}{1-\tau} \tag{1}$$

For given $T$, define the values

$$\begin{aligned} V_H &= u(y_H - T) + \beta\left[pV_H + (1-p)V_L\right] \\ V_L &= u(y_L + (1-\tau)T) + \beta\left[pV_L + (1-p)V_H\right] \end{aligned}$$

Solving for $V_H$ and $V_L$, these recursive definitions imply, for $\gamma \equiv \frac{1-\beta p}{1+\beta-2\beta p}$,

$$V_H(T) = \frac{1}{1-\beta}\left[\gamma u(y_H - T) + (1-\gamma)u(y_L + (1-\tau)T)\right]$$

$$V_L(T) = \frac{1}{1-\beta}\left[(1-\gamma)u(y_H - T) + \gamma u(y_L + (1-\tau)T)\right]$$

- **Lemma :** $V_H(T) \geq V_H(0) \equiv V_H^{aut} \Rightarrow V_L(T) \geq V_L(0) \equiv V_L^{aut}$

The proof goes as follows.

$V_H(T) - V_H^{aut} \geq 0 \Leftrightarrow \frac{u(y_L+(1-\tau)T)-u(y_L)}{u(y_H)-u(y_H-T)} \geq \frac{\gamma}{1-\gamma}$

$V_L(T) - V_L^{aut} \geq 0 \Leftrightarrow \frac{u(y_L+(1-\tau)T)-u(y_L)}{u(y_H)-u(y_H-T)} \geq \frac{1-\gamma}{\gamma}$

It is immediate that $\gamma > 1/2$, and for $\gamma > 1/2$, $\frac{\gamma}{1-\gamma} > \frac{1-\gamma}{\gamma}$, which ends the proof.

[22] This is the level of transfer that prevails in regime 4. Under mild parameter condition, $T^*$ is decreasing in $\tau$. The possible ambiguity for the sign of the derivative comes from the definition of transfer as "pre-loss". Looking at the "post-loss" transfer would yield an unambiguous negative relationship.

Then, for given parameter values, there are 2 possibilities:

– if $V_H(T^*) \geq V_H(0) \equiv V_H^{aut}$, $T^*(\tau)$ is sustainable (regime 4),

– else risk-sharing is limited (regime 3) or completely impossible (regime 2) due to the possibility of default.

- **Level of transfer in regime 3**

In regime 3, it must be the case that $V_H(T) = V_H^{aut}$.

Indeed, $V_H(T) < V_H^{aut}$ would not be sustainable and $V_H(T) > V_H^{aut}$ would be suboptimal : an increase in transfers would decrease consumption volatility.

The problem then can be restated as

$$V_L = u(y_L + (1-\tau)T) + \beta\left[pV_L + (1-p)V_H^{aut}\right]$$

$$V_H^{aut} = u(y_H - T) + \beta\left[pV_H^{aut} + (1-p)V_L\right]$$

There are two equations and two unknowns : $T$ and $V_L$. These equations can be rewritten

$$V_L = \frac{1}{1-\beta p}\left[u(y_L + (1-\tau)T) + \beta(1-p)V_H^{aut}\right] \tag{2}$$

$$u(y_H - T) = (1-\beta p)V_H^{aut} - \beta(1-p)V_L \tag{3}$$

Substituting (2) into (3), I get

$$\begin{aligned} u(y_H - T) &= (1-\beta p)V_H^{aut} - \frac{\beta(1-p)}{1-\beta p}\left[u(y_L + (1-\tau)T) + \beta(1-p)V_H^{aut}\right] \\ &= \frac{(1-\beta)(1+\beta-2\beta p)}{1-\beta p}V_H^{aut} - \frac{\beta(1-p)}{1-\beta p}u(y_L + (1-\tau)T) \end{aligned}$$

This equation implicitely defines $T(\tau)$ in regime 3.

- **Frontier 2 − 3**

When do participation constraints start preventing any transfer at all?

Define $Q \equiv \frac{(1-\beta)(1+\beta-2\beta p)}{1-\beta p}V_H^{aut}$ and $\alpha \equiv \frac{\beta(1-p)}{1-\beta p}$.

The equation defining $T$ rewrites

$$u(y_H - T) + \alpha u(y_L + (1-\tau)T) = Q$$

It is easy to check that this equation admits at least one solution : $T = 0$. The problem is to find a condition on $\tau$ such that this equation admits a positive solution.

One needs to check the derivative of $u(y_H - T) + \alpha u(y_L + (1-\tau)T)$ with respect to $T$ at $T = 0$. Indeed, the second derivative is negative. The strict concavity implies that a positive level of transfer is feasible if and only if the first derivative is strictly positive at $T = 0$.

the first derivative is

$-u'(y_H - T) + (1-\tau)\alpha u'(y_L + (1-\tau)T)$

the second derivative is

$u''(y_H - T) + (1-\tau)^2 \alpha u''(y_L + (1-\tau)T) < 0$

the value of the first derivative at $T = 0$ is

$-u'(y_H) + (1-\tau)\alpha u'(y_L)$

Therefore, a positive transfer is feasible when

$-u'(y_H) + (1-\tau)\alpha u'(y_L) > 0$

$\Leftrightarrow (1-\tau)\frac{\beta(1-p)}{1-\beta p}u'(y_L) > u'(y_H)$

$\Leftrightarrow 1-\tau > \frac{1-\beta p}{\beta(1-p)}\frac{u'(y_H)}{u'(y_L)}$

$$\Leftrightarrow \tau < 1 - \frac{1-\beta p}{\beta(1-p)}\frac{u'(y_H)}{u'(y_L)}$$

- **Frontier $3-4$**

This is the locus where $V_H(T^*) = V_H^{aut}$. Equivalently, given the expression for $V_H(T)$, this is the locus where

$$\beta\,(1-p)\,[u(y_L + (1-\tau)T^*(\tau)) - u(y_L)] = (1-\beta p)\,[u(y_H) - u(y_H - T^*(\tau))]$$

This equation can not be solved for $\tau$ in closed-form. It must be solved numerically.

What can be said is that the condition $V_H(T^*(\tau)) \geq V_H^{aut}$ amounts to $\tau$ being below a certain threshold. Indeed, $V_H \circ T^*$ should be decreasing in $\tau$.

The fact that region 4 is indeed below region 3 comes from the fact that for $\tau$ located on the frontier $2-3$, for any given positive transfer $T$, $V_H(T) < V_H^{aut}$, in particular $V_H(T^*(\tau)) < V_H^{aut}$.

- **Variation of $T$ with $\tau$ in regime 3**

The equation defining $T$ rewrites

$$u(y_H - T) + \alpha u(y_L + (1-\tau)T) = Q$$

Suppose $\tau < 1 - \frac{1-\beta p}{\beta(1-p)} \frac{u'(y_H)}{u'(y_L)}$ but $\tau$ is still too high for $T^*(\tau)$ to be sustainable. The problem is to look at the impact of $d\tau < 0$ on transfer.

Differentiating :

$-u'(y_H - T)dT + \alpha(1-\tau)u'(y_L + (1-\tau)T)dT - \alpha T u'(y_L + (1-\tau)T)d\tau = 0$

$\Rightarrow \frac{dT}{d\tau} = \frac{\alpha T u'(y_L + (1-\tau)T)}{\alpha(1-\tau)u'(y_L+(1-\tau)T) - u'(y_H - T)}$

The numerator is positive. The sign of the derivative then is the sign of

$$\alpha(1-\tau)u'(y_L + (1-\tau)T) - u'(y_H - T)$$

This is typically negative because of $\alpha(1-\tau)$.[23]

[23] If we look at the "post loss" transfer, the condition becomes

- **Multiplier on the impact of a decrease in $\tau$ in regime 3**

In regime 3

$$\frac{dT}{d\tau} = \frac{\alpha T u'(y_L + (1-\tau)T)}{\alpha(1-\tau)u'(y_L + (1-\tau)T) - u'(y_H - T)}$$

whereas absent enforceability problems,

$$\frac{dT}{d\tau} = \frac{u'(y_L + (1-\tau)T) + T(1-\tau)u''(y_L + (1-\tau)T)}{u''(y_H - T) + (1-\tau)^2 u''(y_L + (1-\tau)T)}$$

Suppose parameters are such that

$$\alpha(1-\tau)u'(y_L + (1-\tau)T) < u'(y_H - T)$$

$$u'(y_L + (1-\tau)T) + T(1-\tau)u''(y_L + (1-\tau)T) > 0$$

so that both derivatives above are negative.

The *mutiplier effect* shows up for given $\tau$ if

$$\left|\frac{dT}{d\tau}\right|_{regime3} > \left|\frac{dT}{d\tau}\right|_{full-enforceability}$$

i.e.

$$\frac{\alpha T u'(y_L + (1-\tau)T)}{u'(y_H - T) - \alpha(1-\tau)u'(y_L + (1-\tau)T)} > -\frac{u'(y_L + (1-\tau)T) + T(1-\tau)u''(y_L + (1-\tau)T)}{u''(y_H - T) + (1-\tau)^2 u''(y_L + (1-\tau)T)}$$

$$\frac{\alpha T u'(c_L)}{u'(c_H) - \alpha(1-\tau)u'(c_L)} > -\frac{u'(c_L) + T(1-\tau)u''(c_L)}{u''(c_H) + (1-\tau)^2 u''(c_L)}$$

We know that $\tau$ is between two bounds, perhaps this can help show that the inequality holds.

The computed example of section 4.2 illustrates the size of the multiplier.

---

$$u(y_H - \frac{1}{1-\tau}t) + \alpha u(y_L + t) = Q$$

Differentiating yields:
$-\frac{1}{1-\tau}u'(y_H - \frac{1}{1-\tau}t)dt - \frac{t}{(1-\tau)^2}u'(y_H - \frac{1}{1-\tau}t)d\tau + \alpha u'(y_L + t)dt = 0$
$\Rightarrow \frac{dt}{d\tau} = \frac{\frac{t}{(1-\tau)^2}u'(y_H - \frac{1}{1-\tau}t)}{\alpha u'(y_L+t) - \frac{1}{1-\tau}u'(y_H - \frac{1}{1-\tau}t)}$
The sign of the derivative then is the sign of $\alpha u'(y_L + t) - \frac{1}{1-\tau}u'(y_H - \frac{1}{1-\tau}t)$.
This is typically negative too, but not unambiguously. Perhaps using the fact that $\tau$ is between two bounds would lead to conclude that the derivative is always negative.

## 7.2 Appendix for section 5.1

By the Lagrange theorem, if $\{c_i^*(s^t)\}$ is solution of the planning problem displayed at the start of section 5, there is some $\{\alpha_i^*(s^t)\}$ such that $\{c_i^*(s^t), \alpha_i^*(s^t)\}$ is solution of the following problem :

$$\min_{\{\alpha_i(s^t)\}} \max_{\{c_i(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \Pi(s^t) \left\{\lambda_1 u[c_1(s^t)] + \lambda_2 u[c_2(s^t)]\right\}$$

$$+ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left\{ \begin{array}{c} \alpha_1(s^t) \left[u(c_1(s^t)) + \sum_{i=1}^{\infty} \beta^i \sum\limits_{s^{t+i}} \Pi(s^{t+i} \,| s_t)\, u(c_1(s^{t+i})) - V_1^{aut}(s_t)\right] \\ +\alpha_2(s^t) \left[u(c_2(s^t)) + \sum_{i=1}^{\infty} \beta^i \sum\limits_{s^{t+i}} \Pi(s^{t+i} \,| s_t)\, u(c_2(s^{t+i})) - V_2^{aut}(s_t)\right] \end{array} \right\}$$

$$s.t. \quad if\ c_1(s^t) > y_1(s_t), \quad c_1(s^t) = y_1(s_t) + \underbrace{(1-\tau)[y_2(s_t) - c_2(s^t)]}_{b(s^t)}$$

$$if\ c_2(s^t) > y_2(s_t), \quad c_2(s^t) = y_2(s_t) + (1-\tau)\underbrace{[y_1(s_t) - c_1(s^t)]}_{l(s^t)}$$

$$\alpha_i(s^t) \geq 0.$$

In order to get the formulation of this saddle-point problem that I show in the text, just rewrite the sum of all participation constraints for each agent as follows :

$$\begin{aligned}
& \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \alpha_1(s^t) \left[u(c_1(s^t)) + \sum_{i=1}^{\infty} \beta^i \sum_{s^{t+i}} \Pi(s^{t+i} \,| s_t)\, u(c_1(s^{t+i})) - V_1^{aut}(s_t)\right] \\
= & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \alpha_1(s^t) \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau} \Pi(s^\tau \,| s_t)\, u(c_1(s^\tau)) - V_1^{aut}(s_t)\right] \\
= & \sum_{t=0}^{\infty} \sum_{s^t} \sum_{\tau \geq t} \sum_{s^\tau \geq s^t} \beta^\tau \Pi(s^\tau) \alpha_1(s^t) u(c_1(s^\tau)) - \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \alpha_1(s^t) V_1^{aut}(s_t) \\
= & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left[\left(\sum_{r=0}^{t} \alpha_1(s^r_{s.t.\ s^r \leq s^t})\right) u(c_1(s^t)) - \alpha_1(s^t) V_1^{aut}(s_t)\right].
\end{aligned}$$

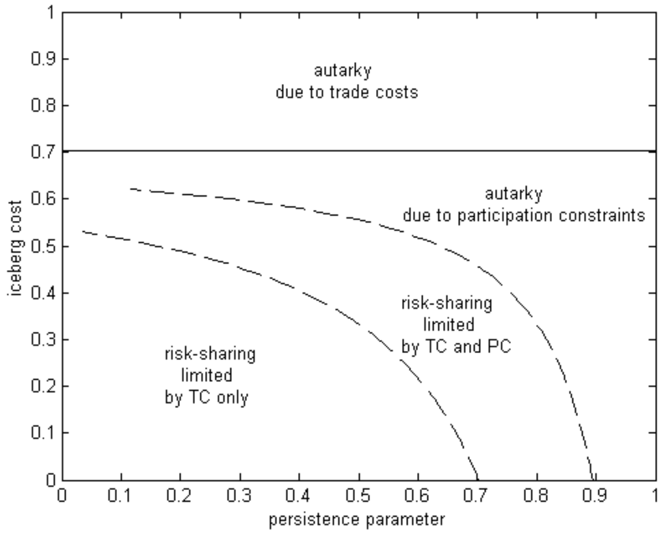


Fig. 1 : risk sharing regions

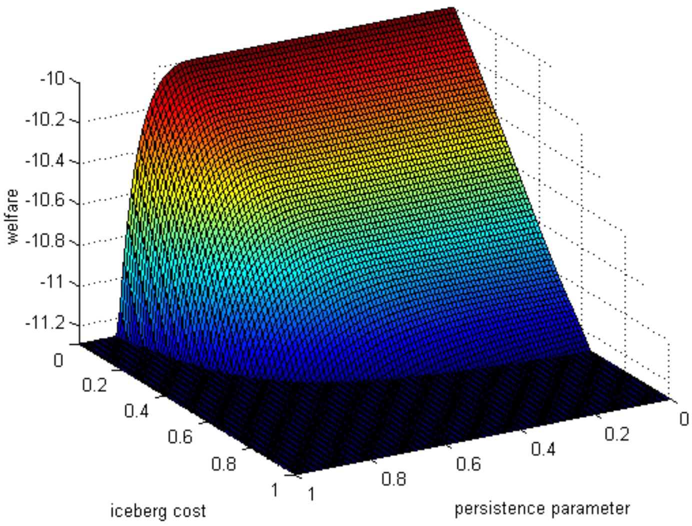


Fig. 2a : welfare

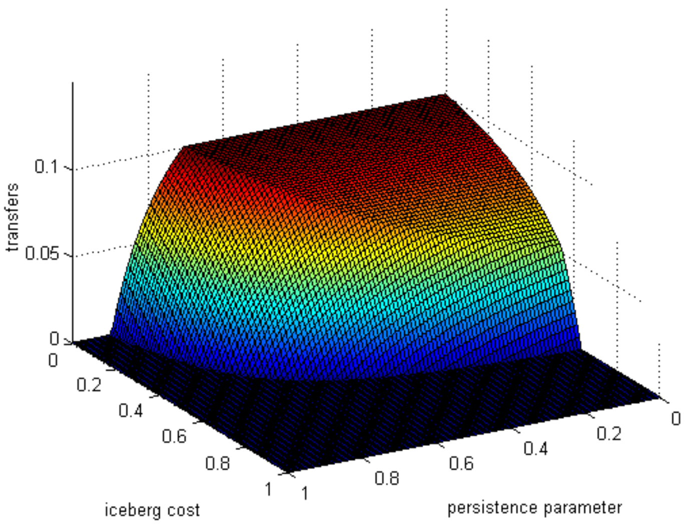


Fig. 2b : transfers

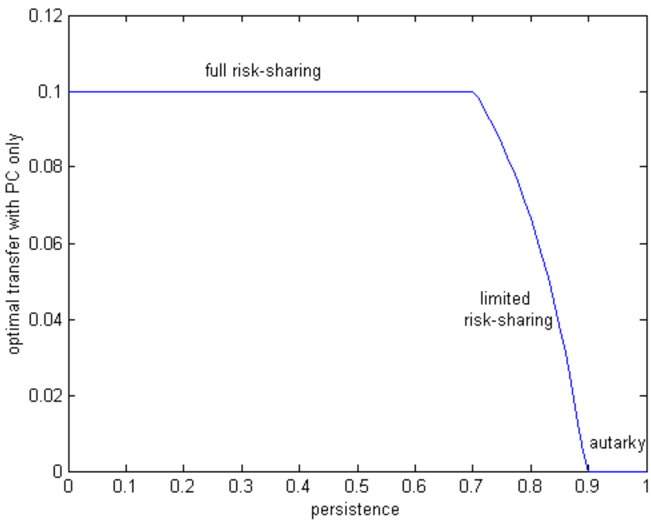


Fig. 3 : pure impact of participation constraints

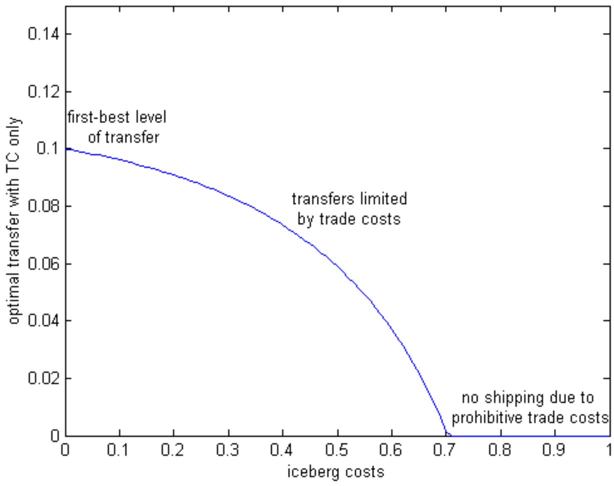


Fig. 4 : pure impact of trade costs

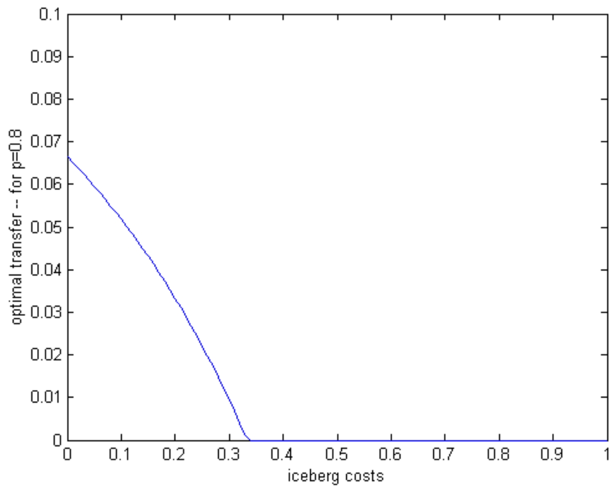


Fig. 5 : impact of trade costs with participation constraints

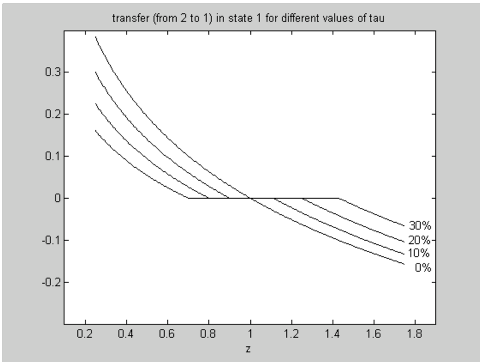


Fig. 6 : impact of trade costs on transfers

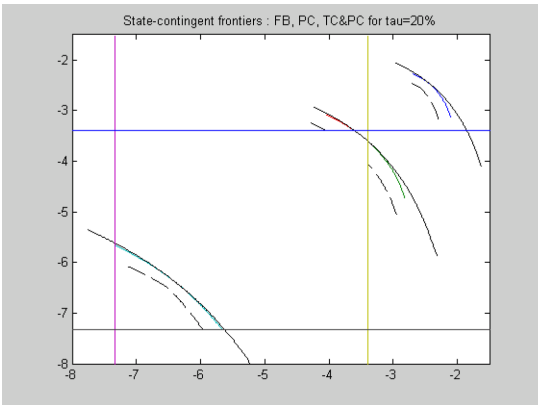


Fig. 7 : effect of imperfections on Pareto frontiers

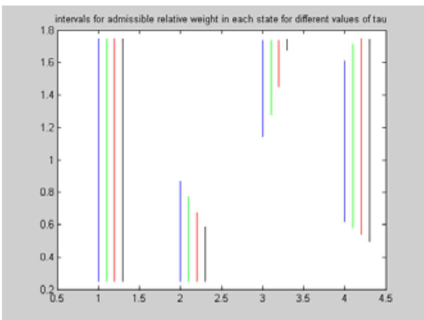


Fig. 8 : admissible values for z for $\tau$ from 0 to 30% (the states 1 to 4 read on the horizontal axis)

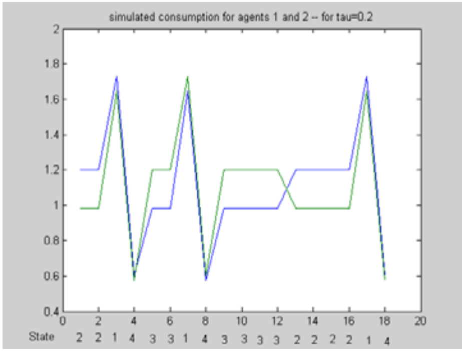


Fig. 9 : simulated consumption

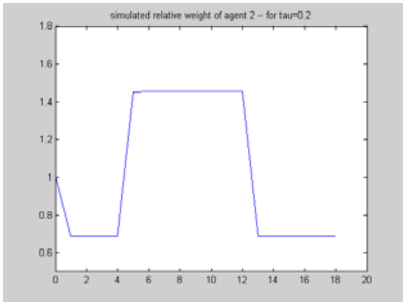


Fig. 10 : simulated Pareto-Negishi weight

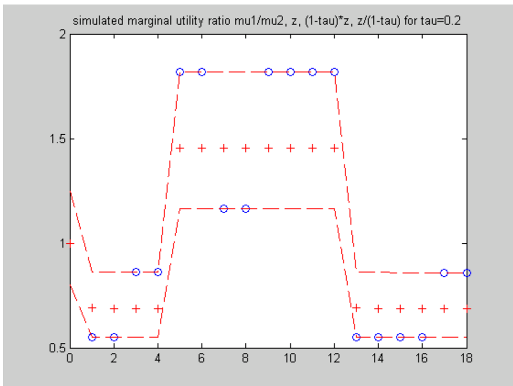


Fig. 11 : simulated marginal utility ratio