Shooting the Auctioneer

Roger E. A. Farmer
UCLA, Dept. of Economics
8283 Bunche Hall
Box 951477
Los Angeles, CA 90095-1477
rfarmer@econ.ucla.edu

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Abstract

Most dynamic stochastic general equilibrium models (DSGE) of the macroeconomy assume that labor is traded in a spot market. Two exceptions (Andolfatto [3], Merz [9]) combine the two-sided search model of Mortenson and Pissarides, [14], [13], [15] with a one-sector real business cycle model. These hybrid models are successful, in some dimensions, but they cannot account for observed volatility in unemployment and vacancies. Following a suggestion by Hall, [4] [5], building on work by Shimer [18], this paper shows that a relatively standard DSGE model with sticky wages *can* account for these facts. Using a second-order approximation to the policy function I simulate moments of an artificial economy with and without sticky wages. I compute the welfare costs of the sticky wage equilibrium and find them to be small.

1 Introduction

Most dynamic stochastic general equilibrium models (DSGE) of the macroeconomy are built around a spot market for labor. Two exceptions (Andolfatto [3], Merz [9]) combine the two-sided search model of Mortenson and Pissarides [14], [13], [15] with a one-sector real business cycle model. These hybrid models are successful, in some dimensions, at explaining how unemployment and vacancies move over the business cycle but they cannot account for observed volatility in unemployment and vacancies (Shimer [18]). This paper shows that a DSGE model with rigid wages *can* account for these facts.

Shimer [18] suggests that the problem with search theoretic models is that they are typically closed with a Nash bargaining solution. Nash bargaining, as a wage-setting mechanism, allows too much wage flexibility relative to the data. Hall [4], [5], [6] has explored Shimer's suggestion that models with rigid or partially adjusting wages may be more successful than flexible wage economies at explaining the facts. This paper builds on the Hall-Shimer approach by constructing a fully specified dynamic general equilibrium model and studying the properties of alternative wage determination mechanisms.

I construct a version of a real business cycle model in which I add a two-sided matching technology similar to that studied by Andolfatto [3] and Merz [9]. I use this artificial economy to study the properties of three alternative equilibria. In the first, the wage is chosen to mimic the social planning solution; I call this a flexible wage economy. In the second equilibrium the real wage grows at the rate of underlying technological progress but is unresponsive to current productivity shocks. I call this the rigid wage solution. Finally, I study an economy in which the real wage adjusts 15% of the way towards the efficient solution in every quarter. I call this a sticky wage economy.

I find two main results. The first is that the sticky wage economy does a good job of explaining the time series properties of unemployment and vacancies in the U.S.. The second is that the welfare costs of the sticky wage equilibrium are small. Both of these properties have been shown to hold in risk neutral economies (the first by Hall [5] and the second by Shimer [19]); this paper shows that the main features of the Hall-Shimer sticky wage equilibrium continue to hold in a standard production economy with risk averse consumers and capital accumulation whilst preserving the ability of the standard RBC model to explain other features of the data.

2 The Social Planning Problem

In this section I describe an artificial economy that adapts the standard one-sector real-business-cycle model by adding a search technology for moving labor between leisure and productive activities. I solve for the social planning optimum and I show how the model with unemployment and vacancies is related to a standard environment with a spot market for labor.

2.1 Setting up the problem

The social planner maximizes the discounted present value of the function

$$J_t = \max \sum_{t=s}^{\infty} \beta^{t-s} E_s \left[\log (C_t) - \chi \frac{L_t^{1+\gamma}}{1+\gamma} - b (U_t + V_t) \right].$$

The first term in the square bracket represents the utility of consumption which I take to be logarithmic. The second term represents the disutility of working in market activity and the third is the utility cost of searching for a job. The cost of search has two components; U_t is time spent searching by a worker for a job, and V_t is time spent by the representative family in its role as an employer searching for workers.

The stock of employment evolves according to the expression

$$L_t = (1 - \delta_L) L_{t-1} + M_t,$$

where I assume that matches separate exogenously at rate δ_L. The term

$$M_t = B (U_t)^{\theta} (V_t)^{1-\theta} \tag{1}$$

is the matching function which I take to be Cobb-Douglas with weight θ.

The problem is constrained by a sequence of capital accumulation constraints,

$$K_{t+1} = K_t (1 - \delta_K) + Y_t - C_t, \qquad t = 1...,$$

and by a production function,

$$Y_t = (K_t)^{\alpha} \left((1+g)^t A_t L_t \right)^{(1-\alpha)}. \tag{2}$$

Output, Y_t is produced using labor L_t and capital K_t which depreciates at rate δ_K. The term $(1+g)^t$ measures exogenous technological progress and A_t is an autocorrelated productivity shock which follows the stationary process

$$A_t = A_{t-1}^{\rho} \exp(\varepsilon_t), \qquad 0 < \rho < 1,$$

$$E_{t-1}(\varepsilon_t) = 0.$$

I assume that ε_t is a described by a Markov process with bounded support and I let ε^t be the history of shocks, defined recursively as;

$$\begin{aligned} \varepsilon^t &= \varepsilon^{t-1} \times \varepsilon_t, \\ \varepsilon^1 &= \varepsilon_1. \end{aligned}$$

The assumption of bounded support is required in Section 5 in which I compute a second order approximation to the policy function.

2.2 Solving the Social Planning Problem

The social planner can alter the stock of workers in productive activities by varying the search intensity of workers or firms. Since the stock of workers can only be increased by hiring, one might think that the inclusion of employment as a state variable would provide an additional propagation mechanism for shocks. However, in practice the separation rate from firms is so high that the labor market operates as if the entire workforce were fired and rehired every period.[1] In effect, movements in employment at business cycle frequencies are caused by variations in search intensity either by firms or by workers.

To model the movements in unemployment and vacancies that would be observed in an efficient allocation I solve the social planning problem. To move labor into and out of productive activity the planner chooses contingent sequences $\{U_t(\varepsilon_t), V_t(\varepsilon_t)\}$. The first-order conditions for the choice of these variables are given by Equations (3) and (4);

$$U_t = \frac{\theta M_t}{bC_t}\left((1-\alpha)\frac{Y_t}{L_t} - C_t \chi L_t^{\gamma}\right), \tag{3}$$

$$V_t = \frac{(1-\theta) M_t}{bC_t}\left((1-\alpha)\frac{Y_t}{L_t} - C_t \chi L_t^{\gamma}\right). \tag{4}$$

Notice the symmetry in these expressions. From a planning perspective, the most efficient way to move labor from leisure to employment is to increase

[1] Shimer [18] cites data from Abowd and Zellner [1] and from the Job Openings and Labor Turnover Survey, to argue that separations occur at a rate of approximately 10% per quarter in the U.S. data. This is a big number - it implies that 40% of the labor force separates from employment in a year.

unemployment and vacancies together; hence percentage movements in U_t should be perfectly correlated with percentage movements in V_t.

By combining Equations (3) and (4) with the definition of the matching function (1) and the first order condition for the choice of capital, I arrive at the following expressions,

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(1 - \delta_K + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \right], \tag{5}$$

$$(1-\alpha) \frac{Y_t}{C_t L_t} - \chi L_t^{\gamma} = \kappa, \qquad \kappa = \frac{1}{B} \left[\frac{b}{\theta^{\theta} (1-\theta)^{1-\theta}} \right]. \tag{6}$$

Equation (5) is the consumption Euler equation and Equation (6) is closely related to the static optimizing condition from a standard model. The term $(1-\alpha) Y_t/L_t$ is the marginal product of labor and $C_t \chi L_t^{\gamma}$ is the ratio of the marginal disutility of effort, χL_t^{γ}, to the marginal utility of consumption, $1/C_t$. The constant κ measures the costs of search; as $\kappa \to 0$, Equation (6) converges to the familiar first order condition for labor in the one-sector RBC model and, for small values of κ, the time paths of capital, gdp, consumption and hours are close to the solutions obtained from an RBC economy with a spot market for labor.

2.3 A Comparison with Previous Literature

The model so far is almost the same as that of Andolfatto [3] and Merz [9]; there are some differences in the way I have modeled search costs but these are not important. My reason for reconsidering models of this kind is based on the findings of Shimer [18] who points out that when search models are closed with a Nash bargaining solution, the models deliver counterfactual labor market predictions.

Unemployment and vacancies are two ways of moving labor from non-market to market activities. In the Andolfatto and Merz models, these two variables enter differently into utility since vacancies use units of commodities but unemployment uses labor as an input. In the current model I have made them symmetric (unemployment and vacancies both impose a time cost) to emphasize a stark implication of the RBC model. In the social planning solution both vacancies and unemployment should be procyclical.[2]

[2] In order to generate negatively correlated unemployment and vacancies, both Mertz

3 Competitive Search Equilibrium

To decentralize the equilibrium in the usual way, one would require that all of the objects that enter utility functions and production functions can be traded, including time allocated to search by households and firms. For example, one might conceive of competition between employment agencies. Each agency would purchase, from households, the exclusive right to match unemployed workers with available vacancies. Similarly the employment agency would purchase, from firms, the exclusive right to match vacant jobs with unemployed workers. The employment agency would use the available constant-returns matching technology and produce matches which represent a joint product valued both by firms and households. The match would be sold to the worker-firm-pair at a competitive price that ensured zero profits for the employment agency.

The employment market does not operate in this way, perhaps because of moral hazard problems. In the decentralized solution envisaged by competitive theory, unemployed workers receive compensation from employment agencies for the right of exclusive representation. In practice it would be difficult to enforce a contract in which the worker commits to sign-up with a single agency. One might conceive of unemployed workers signing with multiple agencies. Further, once matched, a household could refuse to take a job and continue to search by selling its employment rights to a different firm. This points to a problem with a decentralized solution to the matching problem and it implies that there is inherently a missing market.

As an alternative to the standard decentralization, a number of authors have studied decentralized search equilibria.[3] In a search equilibrium, households and firms choose search intensities, U and V, taking the matching probabilities as given. The problem of the missing market shows up as an equilibrium concept with one fewer equations than unknowns. Typically, this problem is solved by positing an ex-post Nash bargain in which the firm and the worker split the surplus of a match according to an exogenous but arbitrary bargaining weight.

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and Andolfatto study versions of their respective models in which search intensity by workers is fixed. Merz studies a version of her model with variable search intensity in which she finds (Merz [9] Table 3 page 282) that unemployment and vacancies are positively correlated.

[3] See the survey by Rogerson, Shimer and Wright [16] for a detailed description of works in this literature.

Espen Moen has proposed an alternative concept that he calls *competitive search equilibrium*. In Moen's work [10] the wage is determined by ex-ante competition between market-makers, instead of the ex-post determination of the wage with a Nash bargain. Moen conceives of firms and workers who choose to search in one of many locations, each of which is run by a market-maker. Market makers charge an entry fee to firms and workers for the right to search at their location. In equilibrium, free-entry ensures that this fee will be zero. The market maker posts the wage at which bargains will be consummated if search takes place at his location. In a competitive search equilibrium, entry is free and the wage is chosen to maximize the expected utility of the worker; i.e. competitive search equilibrium decentralizes the solution to the social planning problem.

4 A Decentralized Model

In this section I study a decentralized version of the model. I assume that a representative worker/firm takes the real wage as given and chooses capital, unemployment and vacancies to maximize expected utility. To close the model I introduce three alternative solution concepts to determine the real wage.

In the first solution concept I adopt the idea that competition between market makers forces the wage to maximize the expected utility of potential workers, that is, the wage is chosen to implement the social planning optimum. I compare this solution with an alternative, suggested by Hall ([4]), in which the real wage is unresponsive to current market conditions. I implement this solution by assuming that the real wage is that which would prevail along the non-stochastic balanced growth path. Since the fixed wage solution leads to fluctuations in unemployment and vacancies that are too volatile relative to the data, I also consider a third equilibrium concept in which the real wage adjusts partially each period towards its optimal value.

4.1 Setting up the Problem

In a decentralized economy the representative agent acts both as a household and as a firm. In its role as a household, the agent supplies labor L_t^S. His period utility function is

$$U = \log\left(C_t\right) - \frac{\chi\left(L_t^S\right)^{1+\gamma}}{1+\gamma} - b\left(U_t + V_t\right),$$

and labor supply in period t is related to search effort U_t and lagged labor supply by the expression

$$L_t^S = \left(1-\delta_L\right) L_{t-1}^S + U_t \frac{\bar{M}_t}{\bar{U}_t}. \tag{7}$$

$\bar{M}_t/\bar{U}_t$ is the increase in employment when the household increases its search intensity, U_t, by one unit. This probability is parametric to the household, but is determined in equilibrium as the ratio of aggregate matches $\bar{M}_t$ to aggregate search intensity $\bar{U}_t$.

The representative worker/firm faces the following sequence of budget constraints;[4]

$$K_{t+1} = K_t\left(1-\delta_K\right) + A_t K_t^{\alpha}\left(\left(1+g\right)^t L_t^D\right) + W_t L_t^S - W_t L_t^D - C_t. \qquad t = 1, \ldots \tag{8}$$

The household can increase its stock of workers, L_t^D by incurring a utility cost $-bV_t$ of search. Every additional unit increase in V_t leads to an increase in the stock of employed workers of $\bar{M}_t/\bar{V}_t$ where $\bar{V}_t$ is aggregate search intensity by all other firms and $\bar{M}_t$ is the aggregate number of matches. This leads to the following expression

$$L_t^D = \left(1-\delta_L\right) L_{t-1}^D + V_t \frac{\bar{M}_t}{\bar{V}_t}, \tag{9}$$

which is the accumulation equation faced by the household in its role as a labor demander.

[4] Although I have modeled an economy with a single asset, storable capital, nothing of substance would be added by including a complete set of contingent claims markets. Since this is a representative agent economy, additional markets would serve only to determine the prices for additional assets at which the representative agent would choose not to trade.

4.2 Solving the problem

The representative agent chooses state contingent sequences

$$\left\{K_{t+1}\left(\varepsilon^t\right), U_t\left(\varepsilon^t\right), V_t\left(\varepsilon^t\right)\right\}_{t=1}^{\infty},$$

taking as given the production function (2) and the accumulation constraints (7), (8) and (9). The first order condition for the choice of capital leads to the Euler equation;

$$\frac{1}{C_t} = \beta E_t\left[\frac{1}{C_{t+1}}\left(1 - \delta_K + \alpha\frac{Y_{t+1}}{K_{t+1}}\right)\right].$$

The first order conditions for the choice of time spent searching in his capacity as a worker and a firm leads to the following two first-order conditions;

$$b = \frac{\bar{M}_t}{\bar{U}_t}\left(\frac{W_t}{C_t} - \chi\left(L_t^S\right)^{\gamma}\right), \tag{10}$$

$$b = \frac{\bar{M}_t}{\bar{V}_t}\left(\frac{(1-\alpha)Y_t}{C_t L_t^D} - \frac{W_t}{C_t}\right). \tag{11}$$

In a competitive equilibrium, the model is closed by the market equilibrium conditions

$$L_t \equiv L_t^S = L_t^D, \quad \bar{U}_t = U_t, \quad \bar{V}_t = V_t,$$

and by the definition of the aggregate matching function

$$M_t = BU_t^{\theta}V_t^{1-\theta}.$$

Since there are two ways of moving labor between leisure and employment, but only one price, the model as it stands is missing an equilibrium condition. Typically, a model of this kind would be closed by positing a Nash bargaining solution to fix the real wage, W_t. The Nash bargaining solution, for appropriate choice of bargaining weights, can be shown to implement the social planning solution.[5] Alternatively one might appeal to Moen's idea of

[5] The generalized Nash bargaining solution divides the surplus of a match in proportion to an exogenous bargaining weight. For the case of a Cobb-Douglas matching function, this solution implements the social planning optimum when the bargaining weight is equal to the elasticity parameters θ of the matching function. This result is a generalization of the Hosios condition [8] to a model with more general utility functions.

competitive market makers to argue that the wage will be chosen to maximize the expected utility of the representative worker. In either case, the imposition of the efficient solution leads to a wage equation of the form

$$W_t = \frac{(1-\alpha)\,\theta Y_t}{L_t} + b\,(1-\theta)\,C_t. \tag{12}$$

Combining (12) with the equilibrium conditions and the first-order conditions of the competitive model, Equations (10) and (11), leads to a pair of equations for unemployment and vacancies that are identical to the first-order conditions of the Social Planner, Equations (3) and (4).

In the context of a search model with linear preferences, Hall [4] and Shimer [18] have shown that the Nash solution leads to fluctuations of unemployment and vacancies that are an order of magnitude too small compared with the observed fluctuations in the data. Hall has suggested instead, that one should look at a different equilibrium concept in the which the real wage is fixed in the face of shocks. Unlike standard sticky-wage models, Hall's model does not present an incentive for an incumbent firm to offer a different wage as long as the range of variation of shocks to the model leaves the fixed wage within the range of variation permitted by the bargaining set.

Within the context of a standard search model with linear preferences, Hall has shown that a sticky wage equilibrium is associated with fluctuations of unemployment and vacancies of the same magnitude that one observes in the data. The question remains however, as to whether this solution can be combined with other features of a standard macroeconomic model and in particular, whether existing features of a real business cycle model will be preserved once the labor market is modeled as a search equilibrium with wages that are insensitive to productivity shocks. The following section addresses this issue by studying the properties of equilibria under three alternative wage determination mechanisms in the artificial economy described above.

5 Computational Issues

This section describes the procedure that I used to compute the properties of equilibria in the artificial economy. I begin by describing the solution algorithm that I used to compute the properties of artificial time series generated by the model. I then describe the alternative wage determination mechanisms that I used to close the model.

5.1 The Solution Algorithm

To compute solutions to the model I used a second order approximation to the policy function due to Schmitt-Grohé and Uribe [17]. Their procedure requires that the variables of the model be separated into a set of non predetermined variables p_t and a set of predetermined variables q_t. To implement this procedure, one must first find a representation of the model in which all of the variable are stationary.

To compute a stationary transformation of the model, I defined the following variables

$$\begin{aligned}
k_t &= \log\left(\frac{K_t}{(1+g)^t}\right), \; a_t = \log\left(\frac{A_t}{(1+g)^t}\right), \\
y_t &= \log\left(\frac{Y_t}{(1+g)^t}\right) \quad c_t = \log\left(\frac{C_t}{(1+g)^t}\right), \quad l_t = \log(L_t), \\
u_t &= \log(U_t), \qquad v_t = \log(V_t), \qquad m_t = \log(M_t), \\
z_t &= \log\left(\frac{Y_t}{L_t}\right), \quad w_t = \log\left(\frac{W_t}{(1+g)^t}\right), \quad j_t = \left(\frac{J_t}{(1+g)^{\frac{t}{1-\beta}+\frac{\beta}{(1-\beta)^2}}}\right).
\end{aligned}$$

The vector of predetermined variables q_t consists of the variables

$$q_t = \{a_t, k_t, w_{t-1}, l_{t-1}\} \in R^4,$$

and the vector of endogenous variables, p_t is given by

$$p_t = \{y_t, c_t, l_t, u_t, v_t, m_t, z_t, w_t, j_t\} \in R^9.$$

I assume that all uncertainty arises from stochastic productivity shocks that take the form

$$a_{t+1} = \rho a_t + \sigma \varepsilon_{t+1}$$

where ε_{t+1} is an i.i.d. random variable with bounded support and unit variance and σ is the standard deviation of the innovation to the productivity shock. The model is a set of equations

$$E_t f(p_{t+1}, p_t, q_{t+1}, q_t) = 0, \tag{13}$$

where the function f consists of a set of identities, model definitions and first-order conditions. These equations are defined in Appendix A.

5.2 Alternative wage determination mechanisms

To compute the decentralized solution under alternative wage determination mechanisms I solved for the time path of w_t in the social planning optimum. This is given by the expression;

$$w_t^{SP} = \log\left((1-\alpha)\,\theta e^{y_t - l_t} + b\,(1-\theta)\,e^{c_t}\right).$$

Next I computed the steady state value $\bar{w}$;

$$\bar{w} = \log\left((1-\alpha)\,\theta e^{y-l} + b\,(1-\theta)\,e^{c}\right).$$

To compute alternative equilibria I simulated sequences for a set of equations in which the wage is given by the expression

$$w_t = (1-\lambda)\,w_{t-1} + \lambda w_t^{SP}.$$

By setting $\lambda = 1$ this solution implements the social planning optimum. Alternatively, setting $\lambda = 0$ fixes the wage equal to its unconditional mean along the balanced growth path. Choosing any other value of λ in the interval $(0,1)$ implements a partial adjustment mechanism in which the logarithm of the real wage adjusts a fraction λ of the way towards the social planning optimum in any given period.

The solution to the model, when it exists and is unique, is of the form

$$\begin{aligned} \underset{9\times 1}{p_t} &= g\,(q_t,\sigma)\,, \\ \underset{4\times 1}{q_{t+1}} &= h\,(q_t,\sigma) + \eta\sigma\varepsilon_{t+1}, \end{aligned}$$

where σ is the standard deviation of the shock ε_t and η is the column vector

$$\eta = [1,0,0,0]'\,.$$

Schmitt-Grohé and Uribe provide code that generates analytic first and second derivatives of the function f in Equation (13).[6] Evaluating these derivatives at the point

$$\bar{p} = g\,(\bar{q},0)\,,\ \bar{q} = h\,(\bar{q},0)$$

[6] The exact relationship between these expressions and the Schmitt-Grohé-Uribe code is explained in Appendix B.

leads to the second order approximation

$$\underset{9\times1}{\tilde{p}_t} = \underset{9\times1}{\mu_p} + \underset{9\times9}{g_q}\underset{9\times1}{\tilde{q}_t} + \frac{1}{2}\underset{9\times16}{G_{qq}}\underset{16\times1}{\tilde{m}_t}, \tag{14}$$

$$\underset{4\times1}{\tilde{q}_{t+1}} = \underset{4\times1}{\mu_q} + \underset{4\times4}{h_q}\underset{4\times1}{\tilde{q}_t} + \frac{1}{2}\underset{4\times16}{H_{qq}}\underset{16\times1}{\tilde{m}_t} + \underset{4\times1}{\eta}\sigma\varepsilon_{t+1}. \tag{15}$$

The terms

$$\tilde{q}_t = (q_t - \bar{q}), \ \tilde{p}_t = (p_t - \bar{p})$$

are deviations of p_t and q_t from their non-stochastic steady states and

$$\mu_p = \frac{1}{2}g_{\sigma\sigma}\sigma^2, \quad \mu_q = \frac{1}{2}h_{\sigma\sigma}\sigma^2$$

are bias terms that cause $\tilde{p}_t$ and $\tilde{q}_t$ to differ from zero when the model is nonlinear. The variable

$$\tilde{m}_t = \tilde{q}_t \otimes \tilde{q}_t \equiv vec(\tilde{q}_t\tilde{q}_t')$$

is a vector of cross product terms.

The Schmitt-Grohé-Uribe solution has a number of advantages over alternative algorithms. First, it uses the symbolic math feature of Matlab to compute analytic derivatives of a user specified set of functions. This feature mechanizes the process of solving for derivatives by hand and removes a potential source of error. Second, the program computes a second order approximation to the policy function which is essential if one is interested in a welfare comparison of alternative wage determination mechanisms.

6 Taking the model to the data

Table 1a lists the values of four key moments that I used to calibrate parameters.

Table 1a:	Product Market: Moments from the U.S. data	
Moment	Description	Value in baseline calibration
g	Average quarterly growth rate of per capita gdp	0.045
r	Average quarterly real interest rate	0.162
c_y	Average ratio of consumption to gdp	0.75
l_s	Labor's share of gdp	0.66
ρ	Autocorrelation of TFP	0.99
σ_ε	Standard deviation of TFP	0.007

I used these observed moments to calibrate the parameters g, β, δ_K and α. Since the artificial economy is based on a Solow growth model, the parameter g which represents the quarterly growth rate of technological progress is equal to the quarterly per capita growth rate of gdp. This was set at 0.045 which implies an annual per capita growth rate of 1.8%, equal to the U.S. average for the past century. To calibrate the elasticity of capital in production, α, I used the assumptions of competitive labor markets and constant returns-to-scale which imply that α is equal to $1 - l_s$, where l_s is labor's share of gdp.

To compute the time series properties of the productivity shock I computed a time series for total factor productivity in the data using the expression

$$TFP = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}.$$

I regressed the log of TFP on its own lagged value and computed the first order autocorrelation coefficient and the standard deviation of the residual. This led to values of $\rho = 0.99$ and σ_ε of 0.007.

To compute the discount factor and the quarterly depreciation rate I solved the steady state equations,

$$(1 + r) = 1 - \delta_K + \frac{\alpha}{k_y}, \tag{16}$$

$$(g + \delta_K)\, k_y = 1 - c_y, \tag{17}$$

for k_y, the steady state capital to gdp ratio and δ_K, the depreciation rate as functions of r, α, g and c_y. Equation (16) is a no-arbitrage relationship in the asset markets and (17) is the steady state gdp accounting identity.

The unknowns r and c_y were set equal to their historical averages in the data; I set $r = 0.162$ which represents an annual rate of 6.5% (computed as the average annual yield on the S&P 500) and $c_y = 0.75$, which is the average ratio of consumption to gdp when government consumption is included as part of consumption.

The discount factor, β was set to 0.99 by solving the the steady state Euler equation which implies that

$$\frac{\beta\,(1+r)}{(1+g)} = 1.$$

Table 1b lists the values for the parameters g, β, δ_K and α, implied by this calibration exercise.

Table 1b: Parameter values implied by Moments from Table 1a

Parameter	Description	Value in baseline calibration
g	Average quarterly growth rate of productivity	0.045
β	Quarterly discount factor	0.99
δ_K	Quarterly depreciation rate for physical capital	0.165
α	Elasticity of capital in production	0.34

Table 2a lists the moments from data that were used to calibrate key labor market facts. I set the separation rate, δ_L, at 10% per quarter based on Shimer's [18] interpretation of the JOLT data, the unemployment rate at 5.8%, and the participation rate to equal 70%.

Table 2a:	Labor Market: Parameters chosen to match moments from the U.S. data	
Moment	Description	Value in baseline calibration
u	Average unemployment rate	0.058
p	Average participation rate	0.7
κ	Search wedge as a fraction of consumption	0.1
γ	Inverse labor supply elasticity	0
θ	Elasticity of the matching function	0.5
δ_L	Average quarterly separation rate	0.1

The steady state values of U and L are related to the unemployment rate u, the participation rate p, and population size N by the definitions

$$p = \frac{L + U}{N}, \tag{18}$$

$$u = \frac{U}{U + L}. \tag{19}$$

Since the model contains a single representative agent, I normalized the population size to 1 and computed L and U from Equations (18) and (19). This led to a value of $L = 0.66$, and $U = .041$ which implies that the representative agent spends 66% of his time in paid employment and 4.1% searching for a job.

Table 2b:	Labor market parameter values implied by Tables 1a and 2a	
Moment	Description	Value in baseline calibration
U	Fraction of time unemployed	0.041
V	Fraction of time searching for workers	0.041
L	Fraction of time working	0.66
m	Match parameter	0.066
b	Marginal disutility of search	0.081
χ	Marginal disutility of effort (when $\gamma = 0$)	1.18
B	Constant of the matching function	1.62

The data on time spent searching by firms is based on an index of help wanted. Since this is an index number, one cannot directly infer the relationship of observed help-wanted to time spent searching by firms. To identify the steady state value of V, I exploited the fact that U and V enter symmetrically to the social planning problem and I set $V = [(1-\theta)/\theta]\,U$.

There are two parameters to the matching function, θ and B. Blanchard and Diamond [2] estimate θ to equal 0.4. In my calibration I set $\theta = 0.5$ and I set $m = .066$ to match the steady state accumulation equation

$$m = \delta_L L.$$

The definition of the matching function

$$B = \frac{m}{U^{\theta} V^{1-\theta}},$$

gives a value for $B = 1.62$.

To pick the parameter γ I used the fact that real business cycle models require high labor elasticity to generate sufficient volatility of hours. In the base-line calibration I picked $\gamma = 0$ which has become standard following Hansen's work [11] on indivisibilities. The parameter κ indexes the costs of search. It represents the wedge between the marginal product of labor and the disutility of effort measured as a fraction of consumption,

$$\kappa C_t = \left((1-\alpha) \frac{Y_t}{L_t} - \chi L_t^{\gamma} \right).$$

To pick κ I experimented with simulations and found that the volatility of unemployment and vacancies is highly sensitive to this parameter. I chose a value of 0.1 to match the volatility of unemployment in the U.S. data when the wage is chosen to be sticky with respect to current market conditions. Having chosen κ, the parameters b and χ are implied by the steady state values of the other variable and by the remaining parameterization. They are given by the expressions

$$\begin{aligned} b &= \kappa B \theta^{\theta} (1-\theta)^{1-\theta}, \\ \chi &= \frac{\left((1-\alpha)\frac{Y}{CL} - \kappa\right)}{L^{\gamma}}. \end{aligned}$$

For my baseline calibration I find a value of $b = 0.081$ and $\chi = 1.18$ which implies that an extra hour spent searching for a job generates about 7% of the disutility associated with an extra hour working.

7 Matching the Data

Table 3 reports the volatilities and correlations with gdp of gdp, consumption, investment, hours worked, labor productivity, the real wage, unemployment and vacancies. The first column reports the moments of the quarterly data from 1955 first quarter, to 2002, fourth quarter. Consumption and investment are both defined as the sum of private plus government components and all variables are in 1996 U.S. dollars and deflated by U.S. resident population. Hours is defined as employment per person multiplied by average hours where employment is total non farm employment from the establishment survey. Productivity is Gdp deflated by hours and the real wage is computed as compensation to employees divided by hours and deflated by the 1996 Gdp price index. Unemployment is the U.S. unemployment rate for persons over 16 years old and vacancies is an index of help wanted from the St. Louis Federal Reserve data base. All variables have been passed through the HP filter with a smoothing parameter of 1600.

Table 3 — Standard deviations in percent (a) and correlations with Gdp (b) for U.S. and artificial economies

	Quarterly U.S. time series (1955.1-2002.4)		Flexible wage economy ($\lambda = 1$)		Sticky wage economy ($\lambda = 0.15$)	
Series	(a)	(b)	(a)	(b)	(a)	(b)
Gdp	1.59	1.00	1.44	1.00	1.45	1.00
	NA	NA	(0.15)	(0.00)	(0.18)	(0.00)
Consumption	1.04	0.85	0.64	0.96	0.65	0.96
	NA	NA	(0.07)	(0.006)	(0.09)	(0.007)
Investment	5.45	0.93	3.92	0.99	3.97	0.99
	NA	NA	(0.39)	(0.002)	(0.50)	(0.002)
Hours	1.80	0.92	0.84	0.98	1.05	0.97
	NA	NA	(0.08)	(0.005)	(0.11)	(0.005)
Productivity	0.74	−0.07	0.64	0.96	0.65	0.96
	NA	NA	(0.07)	(0.006)	(0.09)	(0.007)
Real wage	0.74	−0.24	0.64	0.96	0.33	0.36
	NA	NA	(0.07)	(0.006)	(0.06)	(0.019)
Unemployment	**11.57**	**−0.86**	**6.26**	**0.43**	**11.02**	**−0.88**
	NA	NA	(0.03)	(0.034)	(1.54)	(0.023)
Vacancies	**13.00**	**0.90**	**6.26**	**0.43**	**16.68**	**0.90**
	NA	NA	(0.03)	(0.034)	(1.59)	0.005

The second two columns of Table 3 report the same moments for two artificial economies. The middle panel is an economy in which the social planner chooses unemployment and vacancies to move labor between employment and leisure; it was implemented by choosing the parameter λ to equal 1 which allows the wage to adjust each period to a value that causes the decentralized solution to mimic the social planning optimum. The third panel reports data for an economy in which the log of the real wage adjusts by a fraction 0.15 towards the social planning optimum in any given period. I chose a value of 0.15 by experimenting with different values of λ between 0 and 1. When $\lambda = 0$, the real wage follows the balanced growth path and is completely unresponsive to market conditions. For equilibria of this kind, the standard deviation of gdp in 100 simulations of the model was equal 0.38 compared with a value of 1.69 in the data. The standard deviations of unemployment and vacancies, in contrast were equal to 32 and 50, compared to 11.57 and 13 in the data. As I increased λ from 0 to 1 I found that, for modest values of λ, gdp, consumption and investment are very similar in the sticky wage and flexible wage economies. However, the standard deviations of unemployment and vacancies are very sensitive to variations in λ. I chose a value of $\lambda = 0.15$ to capture the observed standard deviation of unemployment.

Columns 2 and 3 of Table 3 are generated by simulating 100 runs of the model for the baseline parameters setting $\lambda = 1$ for column 2 and $\lambda = 0.15$ for column 3. I refer to the former as a flexible wage economy and to the latter as a sticky wage economy. The numbers in parentheses are standard deviations of the reported parameter over 100 simulations. In each case, the column labeled (a) reports the standard deviation of a variable and the column labeled (b) is its correlation with gdp. All artificial data has been passed through the HP filter with a smoothing parameter of 1600 in the same way as the artificial data.

There are two features of Table 3 that are important. Notice first, that gdp, consumption investment and hours have the same statistical properties in the fixed wage and the flexible wage economies. In each case the correlations with gdp and the standard deviations of these series are within one standard deviation of each other. The reasons for the differences of these statistics from the U.S. data are, by now, well understood and the model does not add much that is new in this dimension. The fixed and flexible wage economies differ substantially, however, in their predictions for the behavior of unemployment and vacancies.

In the data the standard deviations of unemployment and vacancies are equal to 11.57 and 13.00. Vacancies are procyclical with a correlation coefficient with gdp of 0.9 whereas unemployment is countercyclical with a correlation coefficient of -0.86. In the social planning optimum, in contrast, unemployment and vacancies each have a standard deviation of 6.26, they are perfectly correlated with each other and correlated with gdp with a coefficient of 0.43.

Contrast this with the sticky-wage artificial economy. Here, unemployment has a standard deviation of 11.02 and vacancies has a standard deviation of 16.68. As in the U.S. data, these variables are negatively correlated with each other. Vacancies is procyclical with a correlation coefficient with gdp of 0.9 and unemployment is countercyclical with a correlation coefficient with gdp of -0.88.

8 Welfare Costs of Sticky Wages

If the sticky wage economy does a good job of replicating the real world data, one might ask the question; what is the difference in welfare between the flexible wage equilibrium and the equilibrium with sticky prices? To answer this question, I computed a second order approximation to expected utility for values of $\lambda = 1$, $\lambda = 0.15$ and $\lambda = 0$. When $\lambda = 1$, the simulated data mimics the social planning solution and in this case the wage is equal, each period, to a linear combination of the marginal product of labor and the marginal disutility of work. When $\lambda = 0$, the wage grows each period at the rate g, but is unresponsive to innovations in the technology shock. For a value $\lambda = 0.15$, the wage adjusts each period by 15% of the difference between its previous value and the optimal wage for the period.

To compute the welfare loss I specified utility as one element of the vector of endogenous variables and obtained a second order approximation to the time path of utility around its steady state value. Recall that the solution to the model is represented by the following difference equation;

$$\begin{aligned} \underset{9\times 1}{\tilde{p}_t} &= \underset{9\times 1}{\mu_p} + \underset{9\times 9}{g_q}\underset{9\times 1}{\tilde{q}_t} + \frac{1}{2}\underset{9\times 16}{G_{qq}}\underset{16\times 1}{\tilde{m}_t}, & (20) \\ \underset{4\times 1}{\tilde{q}_{t+1}} &= \underset{4\times 1}{\mu_q} + \underset{4\times 4}{h_q}\underset{4\times 1}{\tilde{q}_t} + \frac{1}{2}\underset{4\times 16}{H_{qq}}\underset{16\times 1}{\tilde{m}_t} + \underset{4\times 1}{\eta}\,\sigma\varepsilon_{t+1}, & (21) \\ \tilde{m}_t &= \tilde{q}_t \otimes \tilde{q}_t. & (22) \end{aligned}$$

Appendix C derives the following expressions for the unconditional expectations of the moment vector $\tilde{m}$

$$\tilde{m} \simeq \left[I - H_{qq} \otimes \mu_q - h_q \otimes h_q - \mu_q \otimes H_{qq}\right]^{-1} \bar{\mu}, \quad (23)$$

and the unconditional expected value of deviations of the endogenous variables from their non-stochastic means $E[\tilde{p}]$

$$E[\tilde{p}] = \mu_p + G_{qq}\tilde{m}. \quad (24)$$

By taking the component of $E[\tilde{p}]$ that corresponds to utility I can compute the deviation of expected utility from its non-stochastic mean for alternative parameterizations of the artificial economy. To compute the equivalent compensating variation in consumption (in the sense of Lucas ([12])) I computed the percentage variation in permanent consumption that yields the same variation in expected utility.

Table 4	Welfare cost of alternative Equilibria (Percentage of consumption)		
	Flexible wage economy ($\lambda = 1$)	Sticky wage economy ($\lambda = 0.15$)	Rigid wage economy ($\lambda = 0$)
$\rho = 0$	0.0079	0.0072	0.0049
$\rho = 0.5$	0.0097	0.0074	-0.001
$\rho = 0.99$	0.0351	-0.0089	-3.48

Table 4 tabulates these percentage variations for alternative values of ρ, the autocorrelation of the shock process, and λ the degree of real wage inertia. The first surprising feature of these results (at least to me) is that in the flexible wage economy the representative agent likes uncertainty. This is not a computational error, it follows from the fact that the technology is convex in inputs plus uncertainty.[7] As the parameter λ moves further away from 1, its efficient value, the expected utility gain from uncertainty decreases and for some values of ρ and λ it becomes negative.

An interesting feature of these results is that, all but one of the entries in this table is tiny. In an economy with no serial correlation the representative agent would be prepared to pay less than one percent of permanent consumption to live in a world of uncertainty. This is a welfare *gain*. The welfare *loss* from sticky wages is even smaller. Holding constant the calibrated value of the innovation to the productivity shock, the welfare loss of living in a sticky wage economy ($\lambda = 0.15$) is 0.0007 percent of average consumption which is about 4 cents per person per quarter.[8]

Higher serial correlation raises welfare costs of sticky wages because, when the wage cannot adjust to market conditions, the representative agent's consumption and labor allocations may deviate from the first best for a very long time. This is reflected in the third row of the third column of Table 4 which

[7] Consider the one-shot economy in which $U = \frac{C^{1-\rho}}{1-\rho} - \frac{L^{1+\gamma}}{1+\gamma}$, and $C = AL$. A is a random variable, L is labor supply, C is consumption and ρ and γ are non-negative parameters. If the representative agent chooses L to maximize expected utility his indirect utility, as a function of A, is given by $U = \frac{A^{\varepsilon}}{\varepsilon}$ where $\varepsilon = \left(\frac{(1-\rho)(1+\gamma)}{(\gamma+\rho)}\right)$. For $\rho < 0.5$ there are values of γ for which this is a convex function of A. When there are opportunities for storage this result holds for a much larger class of utility functions as the representative agent can smooth consumption.

[8] This is computed as $[(0.0097 - .0072)/100] \times 6000$ where \$6,000 is a rough estimate of per capita quarterly consumption in 1996 dollars.

reports the welfare loss of a wage that is completely insensitive to market conditions. This number is approximately 3.5% of permanent consumption. This is roughly $210 per quarter which is of a magnitude that begins to make countercyclical policy seem attractive. However, numbers of this magnitude persist *only* for the economy in which wages do not adapt at all to market conditions.

The most relevant welfare number in this table, is the entry for a sticky wage economy in which the real wage adjusts 15% of the way towards its efficient value in any given quarter. For this economy, there is still a welfare loss in the sticky wage economy, but it is much lower than in the rigid wage economy. The representative agent would be willing to pay 0.062 per cent of consumption in order to live in the flexible wage economy. This is roughly $1.57 per person per quarter which is a relatively small number. If there are unmodeled costs of more rapid wage adjustment, menu costs for example, then welfare may well be higher in the sticky wage economy than in the flexible wage solution.

9 Conclusion

I have shown that a relatively simple modification to a standard real business cycle model, of the kind initially studied by Andolfatto and Merz, does a very bad job of explaining the properties of unemployment and vacancies in the U.S. data. The problem with this model is the one identified by Shimer: unemployment and vacancies are not volatile enough and they have the wrong correlation with gdp. I modified the model using Hall's suggestion that a model with rigid wages may provide a better representation of the data. As pointed out by Hall, the rigid wage model does not leave firms or workers with an incentive to change their behavior, in effect, because the search model has a missing market.

Although the rigid wage model does better in some dimensions than the flexible wage economy, it overshoots on unemployment volatility and leads to gdp fluctuations that are too small. An intermediate model in which the real wage adjusts by 15% of the way each quarter towards the flexible wage solution, does a much better job. This model performs as well as the standard RBC model at capturing the volatility of hours, gdp, investment and consumption. In addition it captures the observed volatility of unemployment and has close to the correct volatility for vacancies. More important, I find that unemployment is countercyclical and vacancies are procyclical, just as they are in the U.S. data.

I compared the welfare properties of alternative equilibria and found that the rigid wage solution is associated with a welfare cost of roughly \$210 per person, a relatively large number. The partial adjustment equilibrium, on the other hand, is associated with a welfare cost of only \$1.57 per quarter. This suggest that there may some small unmodeled cost of wage adjustment that is missing from the model, but which causes the sticky wage equilibrium to dominate, at least for fluctuations of the magnitude that we have observed in the post-war period.

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Appendix A: The function f

Production Function

$$f_1 = y_t - a_t - \alpha k_t - (1-\alpha) l_t. \tag{A1}$$

Euler equation

$$f_2 = e^{-c_t} - \frac{\beta}{1+g} e^{-c_{t+1}} (1 - \delta_K + \alpha e^{y_{t+1} - k_{t+1}}). \tag{A2}$$

Gdp accounting identity

$$f_3 = (1+g) e^{k_{t+1}} - (1-\delta_K) e^{k_t} - e^{y_t} + e^{c_t}. \tag{A3}$$

Technology shock process

$$f_4 = a_t - \rho a_{t-1}. \tag{A4}$$

Utility function definition

$$f_5 = c_t - b(e^{u_t} + e^{v_t}) - \frac{\chi}{1+\gamma} (e^{l_t})^{1+\gamma} + \beta e^{j_{t+1}} - e^{j_t}. \tag{A5}$$

Wage equation

$$f_6 = w_t - (1-\lambda) w l_t - \lambda \log \left(\theta (1-\alpha) e^{y_t - l_t} + (1-\theta) \chi (e^{l_t})^{\gamma} e^{c_t} \right). \tag{A6}$$

Vacancy first order condition

$$f_7 = b - (1-\alpha)\, e^{m_t + y_t - l_t - v_t - c_t} + e^{w_t + m_t - c_t - v_t}. \tag{A7}$$

Unemployment first order condition

$$f_8 = b - e^{w_t + m_t - c_t - u_t} + \chi (e^{l_t})^{\gamma} e^{m_t - u_t}. \tag{A8}$$

Labor accumulation equation

$$f_9 = e^{l_t} - e^{m_t} - (1-\delta_L) e^{ll_t}. \tag{A9}$$

Definition of productivity

$$f_{10} = z_t - y_t + l_t. \tag{A10}$$

Housekeeping equations

$$f_{11} = l_t - ll_{t+1}, \tag{A11}$$

$$f_{12} = w_t - wl_{t+1}. \tag{A12}$$

Equations (A11) and (A12) occur because labor and the wage occur both as endogenous and exogenous variables. As endogenous variables they are represented by l_t and w_t and as lagged exogenous variables thay are represented by ll_t and wl_t.

Appendix B

The Schmitt-Grohé-Uribe code generates arrays $h_x, h_{xx}, h_{\sigma\sigma}, g_x, g_{xx}$ and $g_{\sigma\sigma}$. The arrays g_{xx} and h_{xx} are three dimensional and may be unpacked into nine 9×4 and four 4×4 matrices respectively. The second order approximation in matrix form can then be written as follows

$$\tilde{p}_t = g_{\sigma\sigma}\sigma^2 + g_q\tilde{q}_t + \begin{bmatrix} \tilde{q}_t' g_{qq}^1 \tilde{q}_t \\ \vdots \\ \tilde{q}_t' g_{qq}^9 \tilde{q}_t \end{bmatrix}, \tag{B1}$$

$$\tilde{q}_{t+1} = h_{\sigma\sigma}\sigma^2 + h_q\tilde{q}_t + \begin{bmatrix} \tilde{q}_t' h_{qq}^1 \tilde{q}_t \\ \vdots \\ \tilde{q}_t' h_{qq}^4 \tilde{q}_t \end{bmatrix}, \tag{B2}$$

where $\tilde{q}_t$ is 4×1 and $\tilde{p}_t$ is 9×1. Using Kronecker product notation and the fact (see Hamilton [7] page 265) that

$$vec\,(ABC) = (C' \otimes A)\,vec(B), \tag{B3}$$

it follows that

$$vec(\tilde{q}_t' h_{qq}^i \tilde{q}_t) = \tilde{q}_t' \otimes \tilde{q}_t' vec(h_{qq}^i) = vec\left(h_{qq}^i\right)' \tilde{m}_t,\ i = 1, ..4 \tag{B4}$$

$$vec(\tilde{q}_t' g_{qq}^i \tilde{q}_t) = \tilde{q}_t' \otimes \tilde{q}_t' vec(g_{qq}^i) = vec\left(g_{qq}^i\right)' \tilde{m}_t,\ i = 1, ..9 \tag{B5}$$

where

$$\underset{16\times1}{\tilde{m}_t} = \underset{4\times1}{\tilde{q}_t} \otimes \underset{4\times1}{\tilde{q}_t}, \tag{B6}$$

is a 16×1 column vector. Defining

$$G_{qq} = \begin{bmatrix} vec\left(g_{qq}^1\right)' \\ \vdots \\ vec\left(g_{qq}^9\right)' \end{bmatrix}, \tag{B7}$$

$$H_{qq} = \begin{bmatrix} vec\left(h_{qq}^1\right)' \\ \vdots \\ vec\left(h_{qq}^4\right)' \end{bmatrix}, \tag{B8}$$

and $\mu_q = h_{\sigma\sigma}\sigma^2,\ \mu_p = g_{\sigma\sigma}\sigma^2$ leads to the expressions

$$\tilde{p}_t = \mu_p + g_q\tilde{q}_t + G_{qq}\tilde{m}_t, \tag{B9}$$

$$\tilde{q}_{t+1} = \mu_q + h_q\tilde{q}_t + H_{qq}\tilde{m}_t, \tag{B10}$$

which corrspond to equations (14) and (15) in the text.

Appendix C

Using Equations (25) we can write the unconditional expectation of the moment matrix $\tilde{x}\tilde{x}'$ as follows

$$\tilde{M} = E\left(\tilde{q}\tilde{q}'\right) = E\left(\mu_q + h_q\tilde{q} + H_{qq}\tilde{m}\right)\left(\mu_q + h_q\tilde{q} + H_{qq}\tilde{m}\right)'. \tag{C1}$$

Using the fact that $E\left(\tilde{x}\right) = 0$, this expression can be expanded as

$$\tilde{M} = \mu_q\mu_q' + \mu_q\tilde{m}'H_q' + h_q\tilde{M}h_q' + H_{qq}m\mu_q' + o\left(\sigma^3\right) \tag{C2}$$

where $o\left(\sigma_3\right)$ denotes terms of order σ^3 and higher. Using the vec operator and the algebra of Kronecker products this expression can be written as

$$\begin{aligned} vec\left(\tilde{M}\right) &= \mu_q \otimes \mu_q' + H_{qq} \otimes \mu_q\tilde{m} + h_q \otimes h_q\tilde{m} \\ &\qquad + h_q \otimes h_q\tilde{m} + \mu_q \otimes H_{qq}\tilde{m} + o\left(\sigma^3\right). \end{aligned} \tag{C3}$$

Now define $\bar{\mu} = \mu_q \otimes \mu_q'$ and note that $vec\left(\tilde{M}\right) = \tilde{m}$ and drop terms in σ^3 and higher to give the following second order approximation to the unconditional expectation of the vectorized matrix of cross-product terms.

$$\tilde{m} \simeq \left[I - H_{qq} \otimes \mu_q - h_q \otimes h_q - \mu_q \otimes H_{qq}\right]^{-1}\bar{\mu}. \tag{C4}$$

Using this expression for $\tilde{m}$ we obtain the following approximation for the unconditional expected value of $\tilde{p}$, which is a vector of deviations of the endogenous variables from their non-stochastic means.

$$E\left[\tilde{p}\right] = \mu_p + G_{qq}\tilde{m}. \tag{C5}$$

Appendix D: The equations of the model in levels

Social planning problem

$$U = \log\left(C_t\right) - \chi \frac{L_t^{1+\gamma}}{1+\gamma} - b\left(U_t + V_t\right), \tag{D1}$$

$$K_{t+1} = K_t\left(1 - \delta_K\right) + A_t\left(\left(1+g\right)^t K_t\right)^{\alpha} L_t^{1-\alpha} - C_t \tag{D2}$$

$$L_t = L_{t-1}\left(1 - \delta_L\right) + M_t, \tag{D3}$$

$$M_t = B\left(U_t\right)^{\theta}\left(V_t\right)^{1-\theta}, \tag{D4}$$

Labor market FOC

$$b = \frac{\theta M_t}{U_t}\left(\left(1-\alpha\right)\frac{Y_t}{C_t L_t} - \chi L_t^{\gamma}\right) \tag{D5}$$

$$b = \frac{\left(1-\theta\right) M_t}{V_t}\left(\left(1-\alpha\right)\frac{Y_t}{C_t L_t} - \chi L_t^{\gamma}\right) \tag{D6}$$

$$\left(\left(1-\alpha\right)\frac{Y_t}{C_t L_t} - \chi L_t^{\gamma}\right) = \kappa \tag{D7}$$

$$\kappa = \frac{b}{B\left(\theta\right)^{\theta}\left(1-\theta\right)^{1-\theta}} \tag{D8}$$

Competitive Equilibrium

$$U = \log\left(C_t\right) - \chi \frac{\left(L_t^S\right)^{1+\gamma}}{1+\gamma} - b\left(U_t + V_t\right), \tag{D9}$$

$$K_{t+1} = K_t\left(1 - \delta_K\right) + A_t\left(\left(1+g\right)^t K_t\right)^{\alpha}\left(L_t^D\right)^{1-\alpha} + W_t L_t^S - W_t L_t^D - C_t \tag{D10}$$

$$L_t^D = L_{t-1}^D\left(1 - \delta_L\right) + V_t \frac{\bar{M}_t}{\bar{V}_t}, \tag{D11}$$

$$L_t^S = L_{t-1}^S (1-\delta_L) + U_t \frac{\bar{M}_t}{\bar{U}_t}, \tag{D12}$$

$$\bar{M}_t = B \left(\bar{U}_t\right)^{\theta} \left(\bar{V}_t\right)^{1-\theta}, \tag{D13}$$

Labor market FOC

$$b = \frac{W_t}{C_t} \frac{\bar{M}_t}{\bar{U}_t} - \chi \left(L_t^S\right)^{\gamma} \frac{\bar{M}_t}{\bar{U}_t} \tag{D14}$$

$$b = \frac{1}{C_t} \frac{\bar{M}_t}{\bar{V}_t} \frac{(1-\alpha) Y_t}{L_t} - \frac{W_t}{C_t} \frac{\bar{M}_t}{\bar{V}_t} \tag{D15}$$

First best wage

$$W_t = \left(\frac{(1-\alpha)\theta Y_t}{L_t} + (1-\theta) \chi L_t^{\gamma} C_t \right) \tag{D16}$$