MONETARY POLICY IN AN EQUILIBRIUM PORTFOLIO BALANCE MODEL

Michael Kumhof, International Monetary Fund

Stijn Van Nieuwerburgh, New York University

January 30, 2005

Abstract

This paper analyzes the general equilibrium effects of monetary policy choices on portfolio shares of domestic and foreign currency denominated securities. Concentrating on the small open economy case, it relates the optimal choice of portfolio shares to the domestic-foreign interest rate differential. The first contribution of the paper is to show that there are indeed conditions under which a portfolio balance relationship holds in equilibrium, after the effects of government tax and spending policies have been endogenized. This has two important implications. First, monetary policy can be shown to not only affect the level of inflation via a target path for the nominal anchor, but also the volatility (and also the level) of inflation via balance sheet operations. Most strikingly, sterilized intervention affects interest rates through its effect on inflation volatility. Second, this provides a theory of currency risk premia and their endogenous determination by fundamentals and monetary policy, including a determination of the conditions under which risk premia are or are not significant.

We identify two factors that affect the effectiveness of sterilized intervention. The first is the prevalence of exogenous fiscal spending shocks, shocks that induce budget balancing exchange rate movements instead of being financed by endogenous tax responses. The second is the central bank's initial balance sheet position - sterilized intervention has the largest effects if the government has only issued small amounts of domestic currency denominated debt. This suggests that the conditions that give rise to this type of imperfect asset substitutability are more likely to be observed in developing countries.

The authors thank Guillermo Calvo, Ken Judd, Carmen Reinhart, Ken Rogoff, Tom Sargent, Martin Schneider and Stephen Turnovsky for helpful comments.

1 INTRODUCTION

This paper analyzes the general equilibrium effects of monetary policy choices on portfolio shares of domestic and foreign currency denominated securities. Concentrating on the small open economy case, it relates the optimal choice of portfolio shares to the domestic-foreign interest rate differential. The first contribution of the paper is to show that there are indeed conditions under which a portfolio balance relationship holds in equilibrium, after the effects of government tax and spending policies have been endogenized. This has two important implications. First, monetary policy can be shown to not only affect the level of inflation via a target path for the nominal anchor, but also the volatility (and also the level) of inflation via balance sheet operations. Most strikingly, sterilized intervention affects interest rates through its effect on inflation volatility. Second, this provides a theory of currency risk premia and their endogenous determination by fundamentals and monetary policy, including a determination of the conditions under which risk premia are or are not significant.

We identify two factors that affect the effectiveness of sterilized intervention. The first is the prevalence of exogenous fiscal spending shocks, shocks that induce budget balancing exchange rate movements instead of being financed by endogenous tax responses. The second is the central bank's initial balance sheet position - sterilized intervention has the largest effects if the government has only issued small amounts of domestic currency denominated debt. This suggests that the conditions that give rise to this type of imperfect asset substitutability are more likely to be observed in developing countries.

The paper is motivated by a curious tension between economic theory and practice on the question of sterilized intervention. Most notably in developing countries, central bankers routinely intervene in foreign exchange markets with offsetting operations in domestic currency debt, with the intention of affecting interest rates and real activity without changing the money supply and therefore inflation. Their thinking might be taken to reflect older, partial equilibrium versions of portfolio balance theory such as Branson and Henderson

(1985). But the economics profession, both theorists and empiricists, has been challenging the validity of such models for some time. We begin by summarizing this critique, and then develop our model.

The standard reference of modern open economy macroeconomics, Obstfeld and Rogoff (1996), dismisses portfolio balance theory as partial equilibrium reasoning because it omits the government budget constraint. This point is made most comprehensively in an important paper by Backus and Kehoe (1989).[1] They show that under complete asset markets, or under incomplete asset markets and a set of spanning conditions, changes in the currency composition of government debt require no offsetting changes in monetary and fiscal policies to both meet the government budget constraint and to leave private budget constraints unaffected. Consequently this 'strong form' of intervention is irrelevant for equilibrium allocations and prices. This result does not depend on Ricardian equivalence, monetary neutrality, or the law of one price, and can be shown using only an arbitrage condition. The authors then go on to argue that weaker forms of government intervention in asset markets generally require offsetting changes in monetary and/or fiscal policies to meet the government budget constraint. Because the impact of such 'weak form' interventions can as easily be attributed to these monetary and/or fiscal changes as to the intervention per se, sterilized intervention cannot be considered a separate, *third* policy instrument.

When the question of the efficacy of sterilized intervention is posed in this most general form, the results of Backus and Kehoe (1989) are very powerful. However, as these authors point out themselves, this leaves open the narrower but practically very important question of precisely how 'weak form' interventions affect the economy. The answer to this question requires taking a stance on the precise form of other government policies. In this context, one important consideration is that fiscal policy is generally not used as a short-term instrument

[1] Other related references include: Sargent and Smith (1988) on the irrelevance of open market operations in foreign currencies; Chamley and Ptolemarchakis (1984), Sargent and Smith (1987), and Wallace (1981) on the irrelevance of domestic open market operations.

to affect asset market equilibria. It is therefore plausible to rule out fiscal behavior that can respond arbitrarily to asset market interventions, and instead to consider only tax and spending rules whose form is independent of such interventions. We can then ask how sterilized intervention affects equilibrium allocations and prices conditional on the precise form of these rules. In other words, we ask whether sterilized intervention is effective as a *second* independent instrument of *monetary* policy. This is in fact a nontrivial exercise, because several papers such as Obstfeld (1982) and Grinols and Turnovsky (1994) have given a negative answer to that question. They show that, once a monetary policy rule such as a money growth rule is specified, sterilized intervention has no further effects on asset market equilibria. In their models domestic and foreign bonds are perfect substitutes in general equilibrium, so that a version of uncovered interest parity holds. In this paper we show that these results depend on the specific form of the fiscal policy rule used by these authors, namely lump-sum redistribution of all government net revenue, and complete absence of exogenous fiscal spending. While this is a convenient and frequently used assumption, it is also very strong, and we contend that in many real world cases it is not very descriptive of actual government behavior. When it is replaced by assuming at least some exogenous fiscal spending, sterilized intervention can become an effective second instrument of monetary policy. Our paper explores the nature of its effects in general equilibrium.

The model assumes stochastic processes for the nominal money supply, velocity, real returns on internationally tradable assets, and government spending. All of these processes generate nominal exchange rate volatility. Domestic currency denominated government debt securities therefore generate a stochastic seigniorage flow, and in partial equilibrium this would give rise to currency risk for private asset holders. But in general equilibrium, fiscal policy, or in other words the use of this seigniorage by the government, is critical. In the case of full lump-sum redistribution to households, we will confirm the well-known result that currency risk is absent in general equilibrium and that uncovered interest parity must hold. But that assumption is not available for exogenous fiscal spending. As long as such spending is not a perfect substitute for private spending, we can then show that in this case domestic currency denominated government bonds are risky even in general equilibrium. They are imperfect substitutes for foreign currency denominated bonds and their portfolio share is determined by a portfolio balance equation.

The focus of this paper on emerging markets is also justified on empirical grounds. As mentioned above, economists have questioned the effectiveness of sterilized intervention not only theoretically but also empirically. Edison (1993) is a good summary of the latter, but her evidence is limited to developed countries. The evidence for emerging markets summarized by Montiel (1993) is thinner but it does suggest some effectiveness of sterilized intervention. A key precondition for this is imperfect substitutability between domestic and foreign currency denominated bonds. An important paper by Bansal and Dahlquist (2000) presents valuable and more recent evidence on this question. These authors find high currency risk premia in emerging markets, and show that country-specific risk factors are much more important than systematic portfolio risk factors in explaining the cross-country variation is risk premia. Our model explores one important country-specific risk factor, fiscal spending volatility. Gavin and Perotti (1997) document that Latin American countries do indeed exhibit much more fiscal volatility than OECD countries.

Our paper is related to a large theoretical literature trying to explain nominal interest rate risk premia. These are generally decomposed into default risk premia and currency risk premia. While there is a well-established and growing literature on interest rate default risk premia[2], currency risk is a less straightforward notion. Engel (1992) and Stulz (1984) show that in flexible price monetary models monetary volatility per se will not give rise to a risk premium. Engel (1999), using the frameworks of Obstfeld and Rogoff (1998, 2000) and Devereux and Engel (1998), shows that sticky prices are required to generate a risk premium. However, the terms that he identifies as being able to generate risk premia are

[2] The early contributions include Eaton and Gersovitz (1981) and Aizenman (1989). More recent contributions include Kehoe and Perri (2001) and Kletzer and Wright (2000).

generally empirically small in industrialized countries. In developing countries this may be different, but the added difficulty there is to distinguish currency risk premia or discounts from sometimes large default risk premia.

Our theory suggests an alternative explanation that depends on the fundamentals of fiscal policy, and that has very different implications for monetary policy, specifically the ability of central bank sterilized intervention to affect interest rates and allocations. In a flexible price setting, it generates a risk discount due to a Jensen's inequality term.[3] We show that this term can in fact be very small if the government has issued a large amount of nominal debt, and if exogenous fiscal spending volatility is small, as may be the case in many industrialized countries. This would be consistent with the empirical evidence mentioned by Engel (1999). But in the opposite scenario this discount can be of the order of several hundred basis points, and it can give the government significant scope for balance sheet operations. The fact that in many developing countries one often observes an overall risk premium is likely to be due to the interaction of discounts of the kind we emphasize with borrowing risk premia and with Peso-problem type premia of the kind emphasized by Obstfeld (1987).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents an illustrative example of the model's results and policy implications using Mexican data. Section 4 concludes. Mathematical details and details of the data used are presented in a number of appendices.

2 The Model

Consider a small open economy composed of a continuum of identical infinitely lived households and a government. Households' consumption c_t is financed from a constant endowment stream y[4] and from the returns on three types of financial assets,

[3] It also incorporates a borrowing risk premium. While this could be used for some interesting policy experiments, in this paper it will only be used to complete the model and to rule out interest rate indeterminacy at extremely high levels of government borrowing.

[4] The endowment stream is not strictly necessary for the theoretical model. But it is critical

domestic currency denominated money M_t with a zero nominal return, domestic currency denominated bonds Q_t with a nominal return $i^q_t dt$, and internationally tradable assets b_t with a real return dr^b_t. The nominal exchange rate E_t floats. Aggregate exchange rate risk cannot be hedged through financial instruments.[5] We will see that this may, but need not, imply that financial markets are incomplete. All goods are tradable and the international price level is normalized to one. Assuming purchasing power parity, domestic goods prices P_t therefore satisfy $P_t = E_t$. Nominal variables are denoted by upper case letters and real variables in terms of tradable goods by lower case letters.

We use a continuous time stochastic monetary portfolio choice model to derive households' optimal consumption and portfolio decisions.[6] In order to determine the equilibrium portfolio share of domestic currency denominated assets in a small open economy, we follow Grinols and Turnovsky (1994) in assuming that these bonds are held exclusively by domestic residents. This is not a restrictive assumption for many emerging markets, where the vast majority of claims by foreigners tends to be denominated in dollars. Figure 1 illustrates this for the case of Mexico, the country that we will use later for a calibration of the model's shock processes.

2.1 Shock Processes

We fix a probability space $(\Omega, \mathcal{F}, P)$. A stochastic process is a measurable function $\Omega \times [0, \infty) : \mapsto \Re$. The value of a process X at time t is the random variable written as X_t. We define a three-dimensional Brownian motion $B_t = [B^M_t \quad B^\alpha_t \quad B^r_t]'$, consisting

for the computed example in Section 3.

5 This requires that the risk of the domestic currency is too idiosyncratic to be internationally diversifiable, or that that market is too small relative to transactions costs. Both are plausible for emerging markets. Of course individual households can hedge domestically if there is heterogeneity among them. But what matters is that households as a whole cannot hedge their aggregate domestic currency exposure vis-a-vis their own government.

6 Useful surveys of the technical aspects of stochastic optimal control are contained in Chow (1979), Fleming and Rishel (1975), Malliaris and Brock (1982), Karatzas and Shreve (1991), and Duffie (1996). The seminal papers using this technique to analyze macroeconomic portfolio selection are Merton (1969, 1971) and Cox, Ingersoll and Ross (1985). Other contributions include Dumas and Uppal (2000), Grinols and Turnovsky (1994), and Stulz (1983, 1984, 1987, 1988).

of shocks B_t^M to the growth rate of the nominal money supply, shocks B_t^α to the growth rate of consumption velocity $\alpha_t = \frac{c_t}{(M_t/E_t)}$, and shocks B_t^r to the real return on tradable assets dr_t^b. We also define a one-dimensional Brownian motion W_t that represents shocks to the growth rate of government spending dg_t. We employ different notation for this shock because, compared to B_t, it assumes a different transmission channel between money and the exchange rate. This difference is critical for our results.

Specifically, all shocks affect the return properties of domestic currency denominated assets through the exchange rate, and this has repercussions for the government budget constraint. The difference between B_t-shocks and W_t-shocks is the nature of the fiscal response. We assume that with respect to B_t-shocks the fiscal policy response is *endogenous*. This means that the government redistributes the net fiscal balance resulting from financial transactions back to households via lump-sum transfers. But W_t-shocks are by assumption *exogenous* shocks to fiscal policy, and here we assume that the exchange rate adjusts to balance the government's budget. This in turn implies that monetary policy is endogenous, specifically that money is allowed to accommodate the exchange rate movements necessitated by fiscal balance.

The nominal money supply follows a geometric Brownian motion with drift process μ_t determined by monetary policy, a constant, exogenous three dimensional diffusion $\sigma_M = [\sigma_M^M \quad \sigma_M^\alpha \quad \sigma_M^r]$ with respect to B_t-shocks, an endogenous diffusion $\sigma_{M,t}^g$ with respect to W_t-shocks:

$$\frac{dM_t}{M_t} = \mu_t dt + \sigma_M dB_t + \sigma_{M,t}^g dW_t \quad . \tag{1}$$

Here we index endogenous drift and diffusion terms by time if they represent possibly time-varying monetary policy choices, or if they are functions of such choices. Finally, we will at a later stage allow for unanticipated, one-off and discontinuous shocks to the stock of money ΔM_t. This will be required to discuss the effects of unsterilized foreign exchange interventions and of domestic open market operations. That discussion will be useful to decompose the effects of sterilized foreign exchange interventions. For the latter, the main topic of this paper, money follows equation (1) without any discontinuities.

The process for velocity is similar, except of course in that velocity does not endogenously respond to fiscal shocks:

$$\frac{d\alpha_t}{\alpha_t} = \nu dt + \sigma_\alpha dB_t \quad . \tag{2}$$

The real return on tradable assets follows a process

$$dr_t^b = rdt + \sigma_r dB_t \quad . \tag{3}$$

It is assumed that the exogenous components of the stochastic processes $d\log(M_t)$, $d\log(\alpha_t)$ and dr_t^b are correlated, with a variance-covariance matrix Σ. Finally, government spending follows a process for which shocks are assumed to be uncorrelated with B_t-shocks:

$$\frac{dg_t}{a_t} = \mathfrak{g}dt + \sigma_g^g dW_t \quad . \tag{4}$$

A critical feature of fiscal spending shocks is that they are assumed to introduce a real risk for holders of government liabilities because of their implications for the government budget. For this risk to matter in general equilibrium, it must be true that government consumption is an imperfect substitute for private consumption. We choose the simplest and most tractable assumption under which this is true, namely government spending does not enter household utility at all.

The tribe $\mathcal{F}_t^{BW}$ includes every event based on the history of the above four Brownian motion processes up to time t. We complete the probability space by assigning probabilities to subsets of events with zero probability. We define $\mathcal{F}_t$ to be the tribe generated by the union of $\mathcal{F}_t^{BW}$ and the null sets. This leads to the standard filtration $\mathcal{F} = \{\mathcal{F}_t : t \geq 0\}$.

The nominal exchange rate process E_t is endogenously determined as a function of the four exogenous stochastic processes. It follows a geometric Brownian motion, with its drift ε_t and diffusions σ_E and $\sigma_{E,t}^g$ to be determined in equilibrium:

$$\frac{dE_t}{E_t} = \varepsilon_t dt + \sigma_E dB_t + \sigma_{E,t}^g dW_t \quad . \tag{5}$$

We assume and later verify that the endogenous drift and diffusion terms are adapted processes satisfying $\int_0^T |\varepsilon_t| dt < \infty$ almost surely for each T, and $\int_0^T (\sigma_E^x)^2 dt < \infty$

$(x = M, \alpha, r)$, $\int_0^T \left(\sigma_{E,t}^g\right)^2 dt < \infty$ almost surely for each T. The corresponding conditions for all exogenous or policy determined drift and diffusion processes hold by assumption - the exogenous terms are constants and policy choices μ_t are assumed to be bounded.

In the ensuing analysis we will use the following shorthand notation for diffusion terms, choosing terms relating to exchange rates and money as the example:

$$(\sigma_E)^2 = \left(\sigma_E^M\right)^2 + (\sigma_E^\alpha)^2 + (\sigma_E^r)^2 \ ,$$

$$\sigma_M \sigma_E = \sigma_M^M \sigma_E^M + \sigma_M^\alpha \sigma_E^\alpha + \sigma_M^r \sigma_E^r \ ,$$

$$(\sigma_M - \sigma_E) = \left[\left(\sigma_M^M - \sigma_E^M\right) \quad (\sigma_M^\alpha - \sigma_E^\alpha) \quad (\sigma_M^r - \sigma_E^r)\right] \ .$$

2.2 Households

The representative household has time-separable logarithmic preferences[7] that depend on his lifetime stochastic path of tradable goods consumption $\{c_t\}_{t=0}^\infty$. It is convenient to model period utility in terms of consumption in excess of the constant endowment stream y. We therefore have

$$E_0 \int_0^\infty e^{-\beta t} \ln(c_t - y) dt \quad , \quad 0 < \beta < 1 \ , \tag{6}$$

where β is the rate of time preference. We denote the real stocks of money and domestic bonds by $m_t = M_t/E_t$ and $q_t = Q_t/E_t$, and total private financial wealth by $a_t = b_t + q_t + m_t$. Portfolio shares of money and domestic bonds will be denoted by $n_t^m = \frac{m_t}{a_t}$ and $n_t^q = \frac{q_t}{a_t}$. Finally, households are subject to a lump-sum tax dT_t levied as a proportion of wealth and following an Itô process with adapted drift process τ_t and diffusion process σ_t^T:

$$\frac{dT_t}{a_t} = \tau_t dt + \sigma_{T,t} dB_t \ . \tag{7}$$

The drift and diffusion terms will be determined in equilibrium from a balanced budget requirement for the government. Note that taxes respond to B_t-shocks but not to W_t-shocks. We assume that $\int_0^T |\tau_t| dt < \infty$ almost surely for each T and $\int_0^T (\sigma_{T,t})^2 dt < \infty$ almost

[7] Logarithmic preferences are commonly assumed in the open economy asset pricing and portfolio choice literature for their analytical tractability, see e.g. Stulz (1984, 1987) and Zapatero (1995).

surely for each T, and will later verify that this is satisfied in equilibrium. The household budget constraint is given by

$$da_t = a_t \left[n_t^m dr_t^m + n_t^q dr_t^q + (1 - n_t^m - n_t^q) dr_t^b\right] + y dt - c_t dt - a_t[\tau_t dt + \sigma_{T,t} dB_t] - \Gamma_t \ , \quad (8)$$

where dr_t^i is the real rate of return on asset i. The final term Γ_t represents a risk-premium on international borrowing, specified as

$$\Gamma_t = \Im \gamma a_t (n_t^m + n_t^q - \phi)^2 \ , \quad (9)$$

where $\Im$ is an index, with $\Im = 1$ if $(n_t^m + n_t^q) > \phi$ and $\Im = 0$ otherwise. Intuitively, as the government expands its debt stock so that $(n_t^m + n_t^q)$ rises, households have to sell part of their internationally tradable assets to the central bank to purchase government liabilities. Beyond some level ϕ of government borrowing, households have to start borrowing from foreigners in order to finance their domestic debt holdings. At that point foreigners demand a risk premium.[8] The assumption of international borrowing costs that depend on the level of foreign indebtedness has become commonplace in the open economy macroeconomics literature. We will follow the convention in that literature, which is based on empirical evidence, to assume a small risk premium γ. While the risk premium channel could be used to generate interesting results of its own for portfolio decisions, in our paper it is mainly adopted for technical reasons. Specifically, it rules out degenerate behavior of the interest rate at extremely high levels of foreign borrowing. At more realistic levels of borrowing we will see that the key driving force of portfolio behavior is endogenous exchange rate volatility. It remains to specify the critical level ϕ. A strict interpretation of the model requires $\phi = 1$, based on the fact that there is no formal limit to sales of internationally tradable assets in the model. A more flexible and realistic assumption realizes that any aggregate net international asset position reflects many offsetting gross positions, meaning gross international borrowing positions will occur at $\phi < 1$. In any event, given a choice of γ near zero the assumption about ϕ does not dominate the final results.

[8] Foreigners therefore do not take into account the government's net asset position when lending to the country's private sector.

Many monetary portfolio choice models introduce money into the utility function separably because this preserves the separability between portfolio and savings decisions found in Merton (1969, 1971). However, as pointed out by Feenstra (1986), without a positive cross partial between money and consumption the existence of money cannot be rationalized through transactions cost savings. We therefore use a cash constraint instead, and show that it is nevertheless possible to obtain very elegant analytical solutions. Specifically, consumers are required to hold real money balances equal to a time-varying multiple α_t of their consumption expenditures:

$$c_t = \alpha_t m_t = \alpha_t n_t^m a_t \quad . \tag{10}$$

The now very common treatment of the cash-in-advance constraint in Lucas (1990) has two aspects, a cash requirement aspect and an in-advance aspect. Our own treatment goes back to the earlier Lucas (1982), which uses only the cash requirement aspect. This is due to the difficulty of implementing the in-advance timing conventions in a continuous-time framework. In the continuous time stochastic finance literature, Bakshi and Chen (1997) have used the same device. Finally, note that equation (10) could also be obtained as an optimality condition by assuming Leontief preferences over consumption and real balances, $\int_0^\infty e^{-\beta t} \ln(\min(c_t, \alpha_t m_t))dt$.

Using Itô's lemma we can derive the real returns in terms of tradable goods on money and domestic bonds (see Appendix I):

$$dr_t^m = \left(-\varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2\right) dt - \sigma_E dB_t - \sigma_{E,t}^g dW_t \quad , \tag{11}$$

$$dr_t^q = \left(i_t^q - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2\right) dt - \sigma_E dB_t - \sigma_{E,t}^g dW_t \quad . \tag{12}$$

Note that the exchange rate affects returns in two ways. First, depreciation $\sigma_E dB_t > 0$ reduces the ex-post real value of nominal domestic assets in terms of tradables. Second, by Jensen's inequality, larger exchange rate volatility $(\sigma_E)^2$ increases their ex-ante real return.

The household's portfolio problem is to maximize present discounted lifetime utility by the appropriate portfolio choice $\{n_t^m, n_t^q\}_{t=0}^{\infty}$:

$$\max_{\{n_t^m, n_t^q\}_{t=0}^{\infty}} \left\{ E_0 \int_0^{\infty} e^{-\beta t} \ln\left(\alpha_t n_t^m a_t - y\right) dt \right\} \qquad s.t.$$

$$\begin{aligned} da_t \;=\; & a_t \left\{ (r - \tau_t - \gamma \Im (n_t^m + n_t^q - \phi)^2) dt \right. \\ & + n_t^m \left[(-\alpha_t - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2) dt - \sigma_E dB_t - \sigma_{E,t}^g dW_t \right] \\ & + n_t^q \left[(i_t^q - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2) dt - \sigma_E dB_t - \sigma_{E,t}^g dW_t \right] \\ & \left. + (1 - n_t^m - n_t^q) \sigma_r dB_t - \sigma_{T,t} dB_t \right\} \; . \end{aligned} \tag{13}$$

We will solve this optimal portfolio problem recursively using a continuous time Bellman equation, as in Merton (1969, 1971). Let $V(a_t, t) = e^{-\beta t} J(a_t, t) \in C^2$ be a solution of the Hamilton-Jacobi-Bellman equation of stochastic optimal control. Let $\dot{J} = \partial J(a_t, t)/\partial t$. Then the Hamilton-Jacobi-Bellman equation is

$$\begin{aligned} \beta J - \dot{J} \;=\; & \sup_{n_{m_t}, n_{q_t}} \left\{ \ln\left(\alpha_t n_t^m a_t - y\right) + J_a a_t [\left(r - \tau_t - \gamma \Im (n_t^m + n_t^q - \phi)^2\right) \right. \\ & + n_t^m \left(-\alpha_t - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2\right) + n_t^q \left(i_t^q - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2\right)] \\ & + \frac{1}{2} J_{aa} a_t^2 \left[\left(n_t^m + n_t^q\right)^2 \left((\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2\right) + \left(1 - n_t^m - n_t^q\right)^2 (\sigma_r)^2 + (\sigma_{T,t})^2 \right. \\ & - 2\left(n_t^m + n_t^q\right) \sigma_E \sigma_r + 2\left(n_t^m + n_t^q\right)^2 \sigma_E \sigma_r \\ & \left. \left. + 2\left(n_t^m + n_t^q\right) \sigma_E \sigma_{T,t} - 2\left(1 - n_t^m - n_t^q\right) \sigma_r \sigma_{T,t} \right] \right\} \end{aligned} \tag{14}$$

with boundary condition

$$\lim_{\tau \to \infty} E_0 \left[e^{-\beta \tau} \left| J(a_\tau, \tau) \right| \right] = 0 \; . \tag{15}$$

Optimality for n_t^q requires

$$n_t^q + n_t^m = \frac{\left(\frac{-J_a}{J_{aa} a_t}\right) \left(i_t^q - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2\right)}{\left((\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2 + (\sigma_r)^2 + 2\sigma_E \sigma_r\right)} \tag{16}$$

$$+\frac{(\sigma_r)^2+\sigma_E\sigma_r+2\gamma\left(\frac{J_a}{J_{aa}a_t}\right)\Im\left(n_t^m+n_t^q-\phi\right)-\sigma_E\sigma_{T,t}-\sigma_r\sigma_{T,t}}{\left((\sigma_E)^2+\left(\sigma_{E,t}^g\right)^2+(\sigma_r)^2+2\sigma_E\sigma_r\right)} \quad ,$$

This expression is not yet in a very informative form, as we first need to solve for equilibrium taxation. On the other hand, the first-order condition for n_t^m is straightforward:

$$\frac{1}{\alpha_t n_t^m a_t - y} = J_a\left(1+\frac{i_t^q}{\alpha_t}\right) \quad . \tag{17}$$

The marginal utility of consumption equals the marginal utility of wealth J_a times the effective price of consumption, where the latter varies with the opportunity cost of holding money balances for transactions purposes. A closed form solution for J, which is required to get an explicit expression, will be derived following the endogenization of taxes in the next section, and following the definition of equilibrium.

2.3 Government

Monetary policy is characterized by two policy variables and by a technical condition on the government budget.

Primary control over the *level of inflation* is achieved through a target path for the nominal anchor consistent with an inflation target. In the present, flexible price model this is a target path $\{\mu_t\}_{t=0}^{\infty}$ for money in equation (1). The diffusion process σ_M in that equation is assumed to be constant and exogenous - the government needs to allow mean zero shocks to the money supply to accommodate exogenous financial market shocks that may or may not be correlated with shocks to velocity or interest rates. On the other hand, the diffusion process $\sigma_{M,t}^g$ is endogenous and determined in equilibrium. Finally, being an Itô process, M_t is continuous, which ensures exchange rate determinacy.

We will show that control of the *volatility of inflation* can be achieved by setting a target path for the stock of nominal government debt $\{Q_t\}_{t=0}^{\infty}$. We will also see that under our assumptions, particularly that of a risk premium at very high levels of government debt issuance, there is a monotonic relationship between Q_t and i_t^q for all $i_t^q > 0$, so that there is an equivalent target path for the nominal interest rate on government debt.$\{i_t^q\}_{t=0}^{\infty}$. As it turns out, this target also has a secondary effect on the level of inflation. To show that the government can indeed control i_t^q independently of μ_t requires that we find a determinate portfolio demand share for domestic currency bonds n_t^q in equation (16), after endogenizing fiscal policy. The first major objective of this paper is to clarify the conditions under which that is possible. If it is possible, it becomes meaningful to consider separately the effects of, first, unsterilized foreign exchange intervention, second, domestic open market operations, and third, sterilized foreign exchange intervention as a combination of the first two. That analysis will contain the main policy relevant conclusions of the paper.

Finally, we need a technical condition on the government budget. Discrete unanticipated policy changes will generally result in discontinuous jumps of the nominal exchange rate on impact[9], denoted $E_0 - E_{0-}$. Here 0_- stands for the instant before the announcement of a new policy at time 0. At such points the government could either spend the associated net seigniorage revenue, or it could fully redistribute it through a one-off transfer of internationally tradable assets. We assume the latter, and this ensures that private financial wealth remains unchanged upon the impact of any new policy. We denote internationally tradable assets held by the government as h_t, and we denote asset transfers to compensate exchange rate jumps by $\Delta h_0 = -\Delta b_0$.[10] Then we can formalize the above as

$$(M_{0-} + Q_{0-}) \left(\frac{1}{E_0} - \frac{1}{E_{0-}} \right) = -\Delta b_0 = \Delta h_0 \quad . \tag{18}$$

Next we consider fiscal policy. The exogenous, spending component of fiscal policy is specified in (4) and the endogenous, lump-sum tax component in (7). We assume that the latter meet three requirements. First, the expected budget balance is always zero. Second, the budgetary effects of shocks to money, velocity and international interest rates, B_t-shocks, are exactly offset by lump-sum taxes. Third, in order for the assumption of exogeneity of spending shocks g_t in (4) to be meaningful, endogenous lump-sum taxes do not adjust to

9 Subsequent discontinuous exchange rate jumps are ruled out by arbitrage.

10 Note that Δh_0 need not equal $h_0 - h_{0-}$, because the policy itself may in addition involve the purchase or sale of foreign exchange reserves against domestic money or bonds at the new exchange rate E_0.

react to these shocks. Instead, the budget balancing role in response to such shocks falls to the exchange rate.

The government's budget constraint is

$$a_t \tau_t dt + a_t \sigma_{T,t} dB_t + h_t dr_t^b = m_t dr_t^m + q_t dr_t^q + dg_t \quad . \tag{19}$$

The assumption of budget balance together with (18) imply that the government's net wealth $W_t = h_t - m_t - q_t$ does not change over time:

$$dW_t = dh_t - dm_t - dq_t = 0 \quad .$$

Therefore we have $dh_t = dm_t + dq_t$. For simplicity we also assume the initial condition $W_{0-} = 0$, or

$$h_{0-} = m_{0-} + q_{0-} \quad , \tag{20}$$

which implies that

$$h_t = m_t + q_t \quad \forall t \quad . \tag{21}$$

This formulation treats government issued and central bank issued domestic currency bonds as perfect substitutes, so that q_t could represent either debt class. Condition (21) therefore states that the consolidated government's net domestic currency denominated liabilities are fully backed by internationally tradable assets.[11] It should also be pointed out that, given perfect international capital mobility, the assumption of instantaneous redistribution in (19) is not restrictive. It is equivalent to redistribution over households' infinite lifetime combined with instantaneous capitalization by households of the expected redistribution stream. Our treatment is analytically more convenient.

To determine the drift τ_t and diffusion $\sigma_{T,t}$ of the tax process dT_t, and the diffusion $\sigma_{E,t}^g$ of the exchange rate process, we equate terms in (19) using (11), (12), (3) and (21), and we use our above three requirements for lump-sum taxes. We obtain the following:

$$\tau_t = n_t^q i_t^q - \left(n_t^m + n_t^q\right) \left(r + \varepsilon_t - (\sigma_E)^2 - \left(\sigma_{E,t}^g\right)^2\right) \quad , \tag{22}$$

[11] Note that q_t could be negative and represent government claims on the private sector. In that case h_t could also be negative.

$$\sigma_{T,t} = -\left(n^m_t + n^q_t\right)\left(\sigma_E + \sigma_r\right) \quad , \tag{23}$$

$$\sigma^g_{E,t} = \frac{\sigma^g_g}{\left(n^m_t + n^q_t\right)} \quad . \tag{24}$$

The first condition ensures that the expected budget balance is always zero. The second condition represents the endogenous response of lump-sum taxes to B_t-shocks. The third condition is critical for our results. It represents the endogenous response of the exchange rate to W_t-shocks. Fiscally induced exchange rate volatility depends on the volatility of the fiscal shocks themselves, but it is decreasing in the amount of domestic currency denominated liabilities the government has succeeded in issuing. This is because the base of the 'inflation tax', the stock of nominal liabilities that can be revalued by nominal exchange rate movements, is larger in that case. As we will see, raising the amount of domestic currency denominated liabilities requires a higher interest rate. In other words, a higher interest rate is (ceteris paribus) associated with lower exchange rate volatility and therefore inflation volatility.

2.4 Equilibrium and Balance of Payments

We define a *government policy* as a list of stochastic processes[12] $\{\mu_t, i^q_t, \tau_t, \sigma_{T,t}\}_{t=0}^{\infty}$ and an initial net compensation Δh_0 such that, given a list of stochastic processes $\{\varepsilon_t, \sigma_E, \sigma^g_{E,t}, n^m_t, n^q_t\}_{t=0}^{\infty}$, initial conditions $b_{0-}, h_{0-}, M_{0-}, Q_{0-}$ and E_{0-}, and an initial exchange rate jump $E_0 - E_{0-}$, the conditions (22), (23), (24) and (18) hold at all times. Then equilibrium is defined as follows:

> *An equilibrium is a set of initial conditions* $b_{0-}, h_{0-}, M_{0-}, Q_{0-}$ and E_{0-}*, exogenous stochastic processes* $\{B_t, W_t\}_{t=0}^{\infty}$*, an allocation consisting of stochastic processes* $\{c_t, b_t, h_t, Q_t, M_t\}_{t=0}^{\infty}$*, a price system consisting of an initial exchange rate jump* $E_0 - E_{0^-}$ *and stochastic processes* $\{\varepsilon_t, \sigma_E, \sigma^g_{E,t}\}_{t=0}^{\infty}$*, and a government policy such that, given the initial conditions, the exogenous stochastic process, the government policy*

[12] In all policy experiments we will assume for simplicity that $\{\mu_t, i^q_t\}_{t=0}^{\infty}$ are deterministic sequences.

and the price system, the allocation solves households' problem of maximizing (6) *subject to* (8) *and (*10*), resulting in optimality conditions (*16) *and (*17*).*

The condition $h_t = m_t + q_t\ \forall t$ ensures that $a_t = b_t + h_t\ \forall t$, i.e. internationally tradable private assets at any point are equal to the economy's net internationally tradable assets. Then the current account can be derived by consolidating households' and the government's budget constraint (8) and (19).

$$da_t = ra_t dt + ydt - c_t dt + a_t\sigma_r dB_t - a_t\sigma_g^g dW_t - a_t\gamma\mathfrak{I}(n_t^m + n_t^q - \phi)^2 \quad . \tag{25}$$

We are now ready to derive a closed form expression for the household value function. The solution proceeds by first substituting (16), (17), (22) and (23), which contain the terms J_a and J_{aa}, back into the Hamilton-Jacobi-Bellman equation (14). That equation is then solved for J by way of a conjecture. Given our specification of the utility function a good conjecture is

$$J(a_t, t) = X\left[\ln(a_t) + \ln(Y(t;x_t))\right] \quad , \tag{26}$$

where X and $Y(t;x_t)$ are to be determined in the process of verifying the conjecture, and where $x_t = \left\{\mu_t, i_t^q, M_t, \alpha_t, r_t^b, g_t\right\}$. Thus, the conjecture allows for $Y(.)$ to be a function of both time and of the economy's exogenous policy and shock processes. To understand this note that the value function represents the present discounted value of expected future consumption streams. First-order condition (17) together with (25) implies that consumption at t is decreasing in i_t^q, increasing in α_t, and proportional to a_t, where the latter in turn is a function of the full set of exogenous policy and shock processes of the economy. The same is true for expected future consumption, given the Brownian nature of shock increments.

In Appendix II we first show that (26) implies

$$X = \beta^{-1} \quad . \tag{27}$$

Therefore, using (22) and (23), the first-order conditions (16) and (17) become

$$n_t^q + n_t^m = \frac{\left(i_t^q - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2 + (\sigma_r)^2 + \sigma_E\sigma_r + 2\gamma\mathfrak{I}\right)}{\left(\left(\sigma_{E,t}^g\right)^2 + 2\gamma\mathfrak{I}\right)} \quad , \tag{28}$$

$$c_t = y + \frac{\beta a_t}{(1 + (i_t^q/\alpha_t))} \quad . \tag{29}$$

Equation (29) is a standard condition in this model class. It states that consumption in excess of the flow endowment is proportional to wealth and, because of the cash constraint, negatively related to nominal interest rates. It is equation (28), the key equation of this paper, that distinguishes this model from other open economy models. What is says is that the portfolio share of domestic currency denominated assets is determinate, even after taxes have been endogenized. In the following discussion we will pass over the risk premium term $2\gamma\mathfrak{I}$, which in our calibration is both very small and which only enters at very high levels of government borrowing. Assume for a moment that the volatility of exogenous fiscal spending is zero ($\sigma_g^g = 0$). Then (28) in conjunction with (24) would imply that

$$i_t^q = r + \varepsilon_t - (\sigma_E)^2 - (\sigma_r)^2 - \sigma_E \sigma_r \quad . \tag{30}$$

Endogenizing taxes in the absence of exogenous fiscal spending therefore results in a version of uncovered interest parity, the only difference being Jensen's inequality terms relating to exchange rate and interest rate volatility. Crucially, the portfolio share $(n_t^m + n_t^q)$ is indeterminate. In such a model the exchange rate drift would be fully determined by money growth, exchange rate volatility would be fully determined by exogenous B_t-shocks, and there would be no scope for monetary policy to choose a second policy instrument such as the interest rate i_t^q (or the stock of domestic bonds Q_t). The currency composition of central bank balance sheets would not matter. Versions of this result have been found in other papers, such as Grinols and Turnovsky (1994) and Kumhof and Van Nieuwerburgh (2003). The reason for this result is that full lump-sum redistribution of the net seigniorage consequences of shocks fully insures agents against exchange rate risk in general equilibrium - the absence of a market for hedging exchange rate risk is immaterial. Private agents may lose from exchange rate movements at the expense of the government, but the economy as a whole does not, and therefore neither do private agents after the government returns to them what it gained at their expense.

With exogenous fiscal spending shocks the situation is entirely different. A spending shock $dW_t > 0$ is a net resource loss to the economy because government spending does not enter private utility. Furthermore, the government passes this loss on to holders of domestic currency denominated assets through exchange rate movements. This risk of nominal exchange rate changes, the source of which is in fact real, is the source of imperfect asset substitutability. Equation (28) shows that the government can induce agents to expand their portfolio share of domestic currency denominated liabilities by offering (ceteris paribus) a higher interest rate. And by equation (24) the higher portfolio share implies that a given fiscal spending volatility translates into a lower inflation volatility because the base of this 'inflation tax' is higher. Because inflation volatility increases the mean return of domestic currency denominated assets by (11) and (12), interest rates have to rise more strongly the more they induce substantial reductions in inflation volatility. The final result is a monotonically increasing relationship between domestic interest rates and the portfolio share of domestic currency denominated assets. The following subsection develops the mathematics behind this argument in more detail.

2.5 Equilibrium Exchange Rate Dynamics

Equilibrium exchange rate dynamics can be determined by noting that consumption has to satisfy two conditions in equilibrium. The first is the cash-in-advance constraint (10), and the second the consumption optimality condition (29). The latter in turn links the evolution of consumption to the evolution of assets, and therefore needs to be analyzed in conjunction with equation (25).

We begin with (10). By Itô's Law, the cash-in-advance constraint can be stochastically differentiated as

$$dc_t = \alpha_t dm_t + m_t d\alpha_t + dm_t d\alpha_t \ . \tag{31}$$

Again by Itô's Law, real money balances evolve as follows:

$$\begin{aligned} dm_t &= m_t \left[\mu_t - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2 - \sigma_M \sigma_E - \sigma_M^g \sigma_{E,t}^g\right] dt \\ &\quad + m_t \left[\sigma_M - \sigma_E\right] dB_t + m_t \left[\sigma_M^g - \sigma_{E,t}^g\right] dW_t \ . \end{aligned} \tag{32}$$

We substitute (32), (2) and (10) into (31) to obtain the following evolution of consumption:

$$\begin{aligned} &\frac{dc_t}{c_t - y} \\ &= \left[\mu_t - \varepsilon_t + \nu + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2 - \sigma_M \sigma_E - \sigma_M^g \sigma_{E,t}^g + \sigma_\alpha \sigma_M - \sigma_\alpha \sigma_E\right] dt \\ &\quad + \left[\sigma_M - \sigma_E + \sigma_\alpha\right] dB_t + \left[\sigma_M^g - \sigma_{E,t}^g\right] dW_t \ . \end{aligned} \tag{33}$$

We rewrite the consumption optimality condition (29) as follows:

$$z_t = (c_t - y)\left(1 + (i_t^q / \alpha_t)\right) - \beta a_t \equiv 0 \ . \tag{34}$$

We differentiate this condition using Itô's Law to obtain

$$\frac{\partial z_t}{\partial c_t} dc_t + \frac{\partial z_t}{\partial \alpha_t} d\alpha_t + \frac{\partial z_t}{\partial a_t} da_t + \frac{1}{2} \frac{\partial^2 z_t}{\partial \alpha_t^2} (d\alpha_t)^2 + \frac{\partial^2 z_t}{\partial c_t \partial \alpha_t} (dc_t d\alpha_t) = 0 \ . \tag{35}$$

For aggregate asset evolution, we substitute (29) into (25) to get

$$da_t = a_t \left[r - \frac{\beta}{(1 + (i_t^q / \alpha_t))}\right] dt + a_t \sigma_r dB_t - a_t \sigma_g^g dW_t - a_t \gamma \Im (n_t^m + n_t^q - \phi)^2 dt \quad ,$$

while velocity evolution is given by (2). Substituting these terms into (35) and simplifying, we obtain the following expression for the evolution of consumption:

$$\begin{aligned} &\frac{dc_t}{c_t - y} \\ &= \left[\left(r - \frac{\beta}{(1 + (i_t^q / \alpha_t))}\right) + \left(\frac{i_t^q}{\alpha_t + i_t^q}\left(\nu - (\sigma_\alpha)^2\right)\right) - \gamma \Im (n_t^m + n_t^q - \phi)^2\right] dt \\ &\quad + \left[\sigma_r + \frac{i_t^q}{\alpha_t + i_t^q} \sigma_\alpha\right] dB_t - \sigma_g^g dW_t + \left[\frac{i_t^q}{\alpha_t + i_t^q} (\nu dt + \sigma_\alpha dB_t)\right] \frac{dc_t}{c_t - y} \end{aligned} \tag{36}$$

To obtain the equilibrium exchange rate dynamics, we substitute (33) into (36) and separately equate the terms multiplying dt, dB_t and dW_t. For simplicity denote the share of domestic currency denominated assets in private portfolios as $n_t = n_t^m + n_t^q$. Then, given

α_t and a_t, the following is the equilibrium system of 10 equations that determines the 10 variables n_t, ε_t, $\sigma_E = [\sigma_E^M \ \sigma_E^\alpha \ \sigma_E^r]$, $\sigma_{E,t}^g$, $\sigma_{M,t}^g$, c_t, E_t, and either M_t or Q_t, given either Q_t or M_t:

$$n_t = \frac{i_t^q - r - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2 + (\sigma_r)^2 + (\sigma_r \sigma_E) + 2\gamma\mathfrak{I}}{\left(\sigma_{E,t}^g\right)^2 + 2\gamma\mathfrak{I}} \quad , \tag{37}$$

$$\begin{aligned} &\mu_t + \nu - \varepsilon_t + (\sigma_E)^2 + \left(\sigma_{E,t}^g\right)^2 - \sigma_M\sigma_E - \sigma_{M,t}^g\sigma_{E,t}^g + \sigma_\alpha\sigma_M - \sigma_\alpha\sigma_E \\ = \quad &\left(r - \frac{\beta}{(1 + (i_t^q/\alpha_t))}\right) + \left(\frac{i_t^q}{\alpha_t + i_t^q}\left(\nu + \sigma_\alpha\sigma_M - \sigma_\alpha\sigma_E\right)\right) - \gamma\mathfrak{I}(n_t - \phi)^2 \quad , \end{aligned} \tag{38}$$

$$\sigma_M + \sigma_\alpha - \sigma_r - \sigma_E = \frac{i_t^q}{\alpha_t + i_t^q}\sigma_\alpha \quad , \tag{39}$$

$$\sigma_{M,t}^g - \sigma_{E,t}^g + \sigma_g^g = 0 \quad , \tag{40}$$

$$\sigma_{E,t}^g = \frac{\sigma_g^g}{n_t} \quad , \tag{41}$$

$$\alpha_t M_t = E_t c_t \quad , \tag{42}$$

$$c_t = y + \frac{\beta a_t}{(1 + (i_t^q/\alpha_t))} \quad , \tag{43}$$

$$n_t = \frac{M_t + Q_t}{E_t} \quad . \tag{44}$$

We assume and later verify, for a specific calibration of the exogenous shock processes, that this system has a unique solution for a bounded setting of the policy variables μ_t and i_t^q, and given constant exogenous drift and diffusion processes of the economy's four shock processes.[13] This means that all endogenous variables including n_t can be written uniquely in terms of the exogenous policy and shock processes. This verifies the regularity conditions

[13] It can be shown that in certain borderline cases the solution may in fact not be unique, specifically when $n_t > 1$ and $\gamma \longrightarrow 0$. We can rule out such cases by choosing a sufficiently positive but still very small γ. But in general, it can also be shown that uniqueness can in such cases be reestablished if the government chooses as its second policy instrument the nominal stock of bonds Q instead of the interest rate on those bonds i^q.

posited earlier for $\varepsilon_t, \sigma_E, \sigma^g_{E,t}, \tau_t$ and $\sigma_{T,t}$. It is also needed to verify that our conjectured value function and resulting policy functions for n^m_t and n^q_t solve the Hamilton-Jacobi-Bellman equation. The steps of that verification are shown in Appendix II.

3 Policy Implications

3.1 An Illustrative Example

To analyze the properties of the model we now turn to data for Mexico, a small open economy with a substantial outstanding stock of domestic currency denominated liabilities. We use quarterly data from the first quarter of 1996 through the second quarter of 2004 (34 observations) to estimate the drifts and variance-covariance matrix of the economy's four shock processes. We then go on to show what these data imply for the behavior of inflation and taxation under different assumptions about government balance sheet operations and therefore interest rates. In doing so we will hold constant the calibrated parameters $\beta = 0.04$, $y = 1$, $\phi = 0.5$ and $\gamma = 0.0001$.[14]

This is a highly stylized model economy, and some of its key features have no easily identifiable counterpart in the data. In addition to estimating the shock processes, we therefore need to make some reasonable assumptions and approximations. The following exercise, while guided by the data, is therefore best seen as illustrative of our theoretical results. Most importantly, consumption in the model is financed from both an endowment income and from income on financial assets, see equation (29). We assume that the fraction x of the latter in overall consumption spending is $x = 0.02$ on average. As we will see, this has reasonable implications in the light of Mexican national accounts and government liabilities data. Given our normalization $y = 1$, this implies a model-consistent relationship between average consumption $\bar{c}$ and average national wealth $\bar{a}$, which is useful because no

[14] The last two assume that a borrowing premium starts to apply when households have invested half their financial wealth in government securities, but that the premium is extremely small.

reliable data exists for the latter. Specifically, $\bar{a} = \bar{c}x/\beta$.[15]

The key shock process in the model is the volatility of fiscal spending, and all spending shocks are assumed to be exogenous, in other words to have no endogenous tax response. This will generally not be true in the data, see Canzoneri, Cumby and Diba (2004) on the difficulties of distinguishing fiscally dominant and monetary dominant policies in the data. We therefore need to make an assumption about the fraction z of observed fiscal spending volatility that is exogenous. We assume $z = 0.0225$ to get a set of benchmark results for Mexico, but we also explore the sensitivity of our results to other assumptions, given that this is a critical parameter for our results.

We now turn to estimating the shock processes. This requires estimating a continuous-time diffusion process from a discrete sample. Aït-Sahalia (2002) shows that this can be quite complex in general settings, but the case of geometric Brownian motion processes is much simpler. As shown in Campbell, Lo and MacKinlay (1997, Ch. 9), such processes can be estimated in a straightforward fashion by maximum likelihood. Note that the four shock processes can be rewritten, using Itô's Law, as:

$$d\log M_t = \left[\mu_t - \frac{1}{2}\left(\sigma_M\right)^2 - \frac{1}{2}\left(\sigma^g_{M,t}\right)^2\right]dt + \sigma_M dB_t + \sigma^g_{M,t}dW_t = \eta_M dt + +\sigma_M dB_t + \sigma^g_{M,t}dW_t \quad , \tag{45}$$

$$d\log \alpha_t = \left[\nu - \frac{1}{2}\left(\sigma_\alpha\right)^2\right]dt + \sigma_\alpha dB_t = \eta_\alpha dt + +\sigma_\alpha dB_t \quad , \tag{46}$$

$$dr^b_t = rdt + \sigma_r dB_t = \eta_r dt + \sigma_r dB_t \quad , \tag{47}$$

$$d\log g_t = \left[\frac{a_t}{g_t} - \frac{1}{2}\left(\frac{a_t}{g_t}\right)^2\left(\sigma^g_g\right)^2\right]dt + \frac{a_t}{g_t}\sigma^g_g dW_t = \eta_g dt + \sigma_g dW_t \quad . \tag{48}$$

15 The effective price of consumption term $(1 + (i^q/\alpha))$ can be neglected because α is very large in the data, i.e. mean velocity is very high.

The model-consistent variance-covariance matrix between these shocks is

$$\tilde{\Sigma} = \begin{array}{cccc} (\sigma_M)^2 + \left(\sigma_{M,t}^g\right)^2 & \sigma_M \sigma_\alpha & \sigma_M \sigma_r & 0 \\ \sigma_M \sigma_\alpha & (\sigma_\alpha)^2 & \sigma_\alpha \sigma_r & 0 \\ \sigma_M \sigma_r & \sigma_\alpha \sigma_r & (\sigma_r)^2 & 0 \\ 0 & 0 & 0 & (\sigma_g)^2 \end{array} , \tag{49}$$

where, from (40) and (41),

$$\left(\sigma_{M,t}^g\right)^2 = \left(\sigma_g^g \frac{1-n_t}{n_t}\right)^2 . \tag{50}$$

We wish to estimate σ_g^g and the 3-by-3 variance-covariance matrix Σ of velocity, interest rate and *exogenous* money shocks, which corresponds to the 3-by-3 submatrix of the first three rows and columns of $\tilde{\Sigma}$ after first extracting *endogenous* money shocks $\left(\sigma_{M,t}^g\right)^2$. We denote the fraction of money shock variance due to fiscal shocks as $f = \left(\sigma_{M,t}^g\right)^2 / \left((\sigma_M)^2 + \left(\sigma_{M,t}^g\right)^2\right)$.

We begin by estimating government spending as a geometric Brownian motion:

$$d \log g_t = \left[\mathfrak{g} - \frac{1}{2} (\sigma_G)^2\right] dt + \sigma_G dW_t = \eta_g dt + \sigma_G dW_t \ . \tag{51}$$

As mentioned above, we assume that a fraction z of the volatility of this process is due to exogenous spending. We therefore have $\widehat{(\sigma_G)^2} = z * \widehat{(\sigma_G^{ML})^2}$. A comparison of (51) and (48) shows that we need the ratio of government spending to financial assets to obtain an estimate $\widehat{\sigma_g^g}$ from $\widehat{\sigma_G}$, and here we make use of the relationship $\bar{a} = \bar{c}x/\beta$ explained above to compute $\widehat{\sigma_g^g} = \frac{\beta}{x}\overline{\left(\frac{g}{c}\right)}\widehat{\sigma_G}$. Finally, to approximate endogenous money volatility $(\sigma_M^g)^2$ in the variance-covariance matrix $\tilde{\Sigma}$, we use $\widehat{\sigma_g^g}$ in (50) and replace n_t by its sample average $\bar{n} = \overline{((\beta(M+Q))/xC)}$, where C is nominal consumption spending. In our sample $\bar{n} = 0.26$, in other words government liabilities represent roughly a quarter of household financial assets. It should be noted that if the real annual financial returns on Mexican government debt are multiplied by four, the resulting figure does indeed correspond to roughly 2% of annual consumption, in line with our earlier assumption. It is in this sense that the series of approximations that we used to get to this point is reasonable.

It is straightforward to estimate the drifts and variance-covariance matrix of M, α and r^b, again by maximum-likelihood. After subtracting our approximation of $(\sigma_M^g)^2$ from the first cell of the variance-covariance matrix, we are left with nine entries can be used to separately identify, including their signs, the nine diffusion processes σ_M, σ_α and σ_r. In doing so we impose identifying restrictions, namely that real interest rates are completely exogenous $\sigma_r^M = \sigma_r^\alpha = 0$, and that the money supply responds instantaneously to velocity shocks but not vice versa $\sigma_\alpha^M = 0$. Finally the estimated diffusions, from (45) and (46), can be used to recover the drift processes μ and ν, while r can be estimated directly.

The results of our estimations and approximations are as follows: $f = 0.5208$, $\mu = 0.0773$, $\nu = -0.0178$, $r = 0.0227$, $\sigma_g^g = 0.0083$, $\sigma_M^M = 0.0220$, $\sigma_M^\alpha = -0.0009$, $\sigma_M^r = 0.0025$, $\sigma_\alpha^\alpha = 0.0294$, $\sigma_\alpha^r = -0.0059$, $\sigma_r^r = -0.0186$.

These results can be used to compute a baseline economy. To do so we also need to fix velocity at its sample average $\bar{\alpha} = 34.4$ and the initial nominal exchange rate at its sample average $\bar{E} = 9.35$. This can then be used to compute the key initial quantities $\bar{a}$, $\bar{M}$ and $\bar{Q}$.

3.2 Monetary Policy

Monetary policy consists of an inflation target and balance sheet operations.[16] It is straightforward that the main tool for achieving an inflation target in the current flexible price model is a target path $\{\mu_t\}_{t=0}^{\infty}$ for money. We will simply hold μ constant at its estimated value for Mexico. The main subject of this paper is balance sheet operations, of which we can distinguish three types. An unsterilized foreign exchange intervention is a sale or purchase of foreign exchange reserves h in exchange for domestic money M. An open market operation is a sale or purchase of domestic currency bonds Q in exchange for domestic money M. Such operations can be used to 'sterilize' the effect of an unsterilized intervention on the money stock. A fully sterilized foreign exchange intervention is therefore a sale or purchase of

[16] For simplicity, and because the relationship $P_t = E_t$ holds, we will conduct our discussion in terms of inflation and the price level instead of exchange rate depreciation and the nominal exchange rate.

domestic currency bonds Q in exchange for foreign currency bonds h, holding M constant.

We analyze the effect of balance sheet operations on equilibrium quantities using the system of equations (37) - (44), holding constant $\alpha_t = \bar{\alpha}$ Because full government compensation for initial exchange rate jumps (18) is assumed, a_t is always constant on impact $a_t = \bar{a}$. The nine panels in Figure 2 show, going from the left to the middle of each panel, the effect of a doubling of the nominal money supply through a foreign exchange purchase. Going from the middle to the right, they show how the economy reacts if this expansion of the money supply is reversed through a domestic open market operation (OMO), specifically a sale of domestic currency bonds against money.

The foreign exchange intervention could be occasioned, for example, by the central bank accommodating, on a one-off basis, a large capital inflow. But the effect would of course be highly inflationary, causing a doubling of the price level. This in turn would negatively affect the real value of domestic bonds and therefore the overall share of domestic assets in agents' portfolios. The government's domestic liabilities however provide a cushion against fiscal shocks, and this operation therefore not only doubles the price level, it also doubles the subsequent volatility of inflation. Because inflation volatility increases the ex-ante return of holders of domestic assets, the equilibrium interest rate can then fall by around 0.5%. The cost to the government of higher inflation volatility turns out to outweigh the interest savings, so that the government has to raise net taxes. Note that consumption is barely affected by the change in interest rates, principally because, for the Mexican data set we used to calibrate the baseline economy, velocity is extremely high. This may not be true for other countries, and a significantly lower interest rate could then clearly stimulate consumption.

The open market operation completely sterilizes the foreign exchange intervention, so that the exchange rate is driven back to its initial value. But importantly, other variables do not return to their baseline values. Most importantly, while real money balances are unchanged after sterilization, the real quantity of outstanding domestic liabilities has increased significantly, thereby lowering inflation volatility, increasing interest rates, and lowering the required tax rate to balance the government budget. Sterilized intervention therefore has significant real effects, and as stated above with a lower velocity those effects would also include lower consumption.

The following Figures 3-5 illustrate the effects of sterilized intervention over a broader range of sterilized interventions. Figure 3 shows the Mexican baseline case. This time we show the domestic asset share n_t in % along the horizontal axis. The first panel shows the relationship between the stock of domestic currency government securities Q_t and n_t. Issuing more nominal debt increases the real debt stock given that full sterilization keeps the nominal money stock constant. As the domestic debt share rises from 10% to 60%, fiscally induced inflation volatility $\sigma^g_{E,t}$ falls from 1% in annual interest rate equivalent terms to near zero, because more debt absorbs fiscal shocks through smaller price level movements. This lowers overall inflation volatility by the same amount, because inflation volatility originating from the three non-fiscal shocks is constant.[17] As investors benefit from inflation volatility, the equilibrium nominal interest rate has to rise. That rise is however very nonlinear, because most of the effects of lower inflation volatility happen at lower levels of debt. From around a 30% debt share onwards, further expansions of the nominal debt stock have quite modest effects on inflation volatility and interest rates. Note that a higher interest rate also has a secondary effect on mean inflation, but this is very minor compared to the effect of the nominal anchor μ_t. Finally, as we saw above, a higher debt stock allows the government to lower the mean tax rate, because the negative effect of higher interest rates on the budget is more than offset by the beneficial effect of less volatile inflation.

Combining the above results, it is also clear that the effect of a given contraction of the money supply depends on the market through which that contraction is implemented. The effects on interest rates are larger if done through the domestic bond rather than the foreign exchange market, because under open market operations the portfolio share n_t expands not just due to a drop in the price level but also due to an expansion of the domestic nominal

[17] It is also significantly smaller at low levels of debt issuance.

bond stock. This requires an even lower interest rate to establish portfolio balance.

Figures 4 and 5 show how these results on the effects of sterilized intervention depend on the volatility of exogenous fiscal shocks, leaving all other parameters of the model unchanged. Figure 4 shows that, when the fiscal shock diffusion σ_g^g is reduced from 0.0083 to 0.002, balance sheet operations over the same range of Q as those reported in Figure 3 have almost no effect on inflation volatility and interest rates, while we see in Figure 5 that raising σ_g^g to 0.024 increases the range over which they are effective, and it increases the size of their effect on interest rates. This may explain why empirical studies of sterilized intervention have found very little evidence for their effect in industrialized countries. In such countries the fiscal situation is generally much more robust, and fiscal dominance is much less of a problem. At the same time, even if there was fiscal dominance, the ability of such countries to issue substantial amounts of domestic currency denominated debt means that the induced inflation volatility would be comparatively low. On the other hand, developing countries routinely engage in sterilized intervention, especially when faced with volatile capital inflows, and they have much more difficulty issuing substantial stocks of domestic currency debt. In such countries sterilized intervention would be a second tool of monetary policy that gives the central bank autonomy to set nominal interest rates independently of the inflation target. The bottom line is that low outstanding stocks of domestic currency debt combined with high fiscal volatility give rise to imperfect asset substitutability between domestic and foreign currency debt, and that this is most likely to be observed in developing countries.

4 CONCLUSION

We have studied a general equilibrium monetary portfolio choice model of a small open economy with floating exchange rates and flexible prices. The model emphasizes the importance of fiscal policies for the number of instruments available to monetary policy, specifically for its ability to affect allocations and prices through balance sheet operations such as sterilized intervention. Conventional results were shown to depend on a particular assumption about fiscal policy, full lump-sum redistribution of stochastic seigniorage income and an absence of exogenous fiscal spending shocks. When this assumption is relaxed, two important results are obtained.

First, government balance sheet operations matters even if they do not affect the money stock. Their primary effect is on the volatility of inflation, because a larger outstanding stock of nominal government debt requires smaller price level movements to balance the budget following an exogenous fiscal spending shock. The volatility of inflation in turn affects the domestic nominal interest rate. Putting this differently, the central bank can set its domestic interest rate and its inflation target independently to affect both the mean and the variance of inflation.

Second, uncovered interest parity fails to hold. Large risk discounts are obtained when a central bank's nominal liabilities are small and the volatility of its fiscal shocks is high, because fiscal shocks induce a high nominal exchange rate volatility that increases investors' ex-ante return. On the other hand, risk premia become possible when a central bank issues very large amounts of nominal liabilities, because of risk premia charged by international lenders. Our paper has not focused on the latter aspect, but it has provided the analytical apparatus for doing so.

A welfare analysis of different policy choices is currently beyond the scope of the paper, but the outlines of the trade-off are clear. Up to some point the government can expand its stock of nominal liabilities and achieve three objectives that, in more detailed models, are all welfare improving. These are a reduction of mean inflation, a reduction of inflation volatility, and a reduction of taxes needed to balance the budget. It can be shown that required taxation begins to increase once households have to start borrowing and paying a risk premium in order to finance further holdings of domestic currency denominated liabilities. The point at which this may occur for a specific country could be earlier than assumed in the present model, but there will nevertheless be a broad range over which a government should be able to increase its issue of nominal liabilities with very positive effects.

Appendix I Returns on Assets

The return on real money balances is derived using Itô's law to differentiate M_t/E_t holding M_t constant:

$$\begin{aligned} m_t dr_t^m &= M_t d\left(\frac{1}{E_t}\right) = \\ & \frac{-M_t}{E_t^2} E_t \left[\varepsilon_t dt + \sigma_E dB_t + \sigma_{E,t}^g dW_t\right] + \frac{1}{2}\frac{2M_t}{E_t^3} E_t^2 \left[(\sigma_E)^2 + (\sigma_{E,t}^g)^2\right] dt \quad , \end{aligned}$$

which yields the return

$$dr_t^m = \left(-\varepsilon_t + (\sigma_E)^2 + (\sigma_{E,t}^g)^2\right) dt - \sigma_E dB_t - \sigma_{E,t}^g dW_t \quad . \tag{A.1}$$

The real return on the domestic bond is given by its nominal interest rate i_t^q, minus the change in the international value of domestic money as in (A.1). We have

$$dr_t^q = \left(i_t^q - \varepsilon_t + (\sigma_E)^2 + (\sigma_{E,t}^g)^2\right) dt - \sigma_E dB_t - \sigma_{E,t}^g dW_t \quad . \tag{A.2}$$

The real return on internationally tradable assets is exogenous and given by (3), which is repeated here for completeness.

$$dr_t^b = r dt + \sigma_r dB_t + \gamma \Im (n_t^m + n_t^q - 1) \quad . \tag{A.3}$$

Appendix II The Value Function

This Appendix verifies the conjectured value function $V(a_t, t) = e^{-\beta t} J(a_t, t) = e^{-\beta t} X[\ln(a_t) + \ln(Y(t; x_t))]$ and derives closed form expressions for X and $Y(t; x_t)$. Substitute the conjecture, the optimality condition (17), and the government policy rules (22), (23) and (24) into the Bellman equation (14). Then cancel terms to get

$$\begin{aligned} & \beta X \ln(a_t) + \beta X \ln(Y(t; x_t)) - X \frac{\dot{Y}(t; x_t)}{Y(t; x_t)} \\ = \; & \ln(a_t) - \ln(X) - \ln(1 + \frac{i_t^q}{\alpha_t}) \\ & + X \left[r - \alpha_t n_t^m - \gamma \Im \left(1 - n_t^m - n_t^q\right)^2\right] \\ & - \frac{X}{2} \left[(\sigma_r)^2 + \left(\sigma_g^g\right)^2\right] \quad , \end{aligned}$$

where $\dot{Y}(t;x_t) = \partial Y(t;x_t)/\partial t$. Equating terms on $\ln(a_t)$ yields

$$X = \beta^{-1} \; . \tag{B.1}$$

This implies the first-order conditions (28) and (29) shown in the paper. We are left with a differential equation in $Y(t;x_t)$ as follows:

$$\begin{aligned} \frac{\dot{Y}(t;x_t)}{Y(t;x_t)} &= \beta\ln(Y(t;x_t)) - \beta\ln(\beta) + \beta\ln(1+(i_t^q/\alpha_t)) - \left(r - \frac{\beta}{1+(i_t^q/\alpha_t)}\right) \\ &\quad + \frac{1}{2}\left((\sigma_r)^2 + \left(\sigma_g^g\right)^2\right) + \gamma\Im\left(n_t^m + n_t^q - \phi\right)^2 \quad . \end{aligned} \tag{B.2}$$

The equilibrium set of equations determining the evolution of the economy are presented in the text as (38) - (41). As argued there, this system has unique bounded solutions for, among others, $(n_t^m + n_t^q)$. This means that all terms on the right-hand side of (B.2) are, or can be expressed uniquely in terms of, exogenous policy or shock variables x_t, as conjectured at the outset. Furthermore,

$$\frac{\partial\dot{Y}(t;x_t)}{Y(t;x_t)}|_{\dot{Y}(t;x_t)=0} = \beta > 0 \; . \tag{B.3}$$

This implies that $Y(t;x_t)$ is saddle path stable for any given x_t, and is therefore uniquely determined for each t and x_t. It is instructive to consider the value of $Y(t;x_t)$ under some simplifying assumptions. Let $\alpha_t = \bar{\alpha}\ \forall t$, let i^q be set such that $r(1+(i^q/\bar{\alpha})) = \beta$ and such that $\Im = 0$. Then we have $(E(da_t))/dt = 0$ and $c_t = ra_t$. We obtain a steady state value $\bar{Y} = r\exp\left(\left(\frac{-1}{\beta}\right)\left(\frac{(\sigma_r)^2}{2} + \frac{\left(\sigma_g^g\right)^2}{2}\right)\right)$. In the absence of the final term, we would have $J(a_t,t;x_t) = \frac{1}{\beta}\ln(c_t)$. However, the actual utility value of a_t is lower because of risk to the return on internationally tradable assets and risk to asset accumulation due to government spending volatility.

Our approach has followed Duffie's (1996, Ch. 9) discussion of optimal portfolio and consumption choice in that we have focused mainly on necessary conditions. This is because the existence of well-behaved solutions in a continuous-time setting is typically hard to prove in general terms. We have adopted the alternative approach of conjecturing a solution and then verifying it, and have found that our conjecture $V(a_t, t)$ does solve the Hamilton-Jacobi-Bellman equation and is therefore a logical candidate for the value function. In the process of doing so we have also solved for the associated feedback controls (n_t^{q*}, n_t^{m*}) and wealth process a_t^*. We now verify that $V(a_t, t)$ and (n_t^{q*}, n_t^{m*}) are indeed optimal.

Note first that our solutions solve the problem

$$\sup_{n_t^m, n_t^q} \{\ln(\alpha_t n_t^m a_t - y) + DJ(a_t, t)\} = 0 \quad , \tag{B.4}$$

where

$$DJ(a_t, t) = J_a(a_t, t) g(a_t, n_t^m, n_t^q) + \frac{1}{2} J_{aa}(a_t, t) \left[h(a_t, n_t^m, n_t^q)\right]^2 - \beta J(a_t, t) + \dot{J}(a_t, t) \quad . \tag{B.5}$$

The functions $g(.,.,.)$ and $h(.,.,.)$ derive from the equilibrium evolution of wealth a_t given the conjectured form of the value function $V(a_t, t)$ and given the associated feedback controls (n_t^{q*}, n_t^{m*}). Let $\sigma_a = [\sigma_r \quad (-\sigma_g^g)]$ and $Z_t = [B_t \ \ W_t]'$. Then we have

$$da_t = g(a_t, n_t^m, n_t^q)dt + h(a_t, n_t^m, n_t^q)dZ_t \quad ,$$

where

$$\begin{aligned} g(a_t, n_t^m, n_t^q) &= r a_t + y - \alpha_t n_t^m a_t \quad , \\ h(a_t, n_t^m, n_t^q) &= a_t \sigma_a \quad , \end{aligned}$$

and where in line with our previous notation $\left[h(a_t, n_t^m, n_t^q)\right]^2 = a_t^2 \left[(\sigma_r)^2 + \left(\sigma_g^g\right)^2\right]$.

Now let (n_t^q, n_t^m) be an arbitrary admissible control for initial wealth a_0 and let a_t be the associated wealth process. By Itô's formula, the stochastic integral for the evolution of the quantity $e^{-\beta t} J(a_t, t)$ can be written as

$$\begin{aligned} e^{-\beta t} J(a_t, t) &= J(a_0, 0) + \int_0^t e^{-\beta s} DJ(a_s, s) ds + \int_0^t e^{-\beta s} \psi_s dB_s \quad , \qquad \text{(B.6)} \\ \text{where} \quad \psi_t &= J_a(a_t, t) h(a_t, n_t^m, n_t^q) \quad . \end{aligned}$$

We proceed to take limits and expectations of this equation. We show first that $E\left(\int_0^t e^{-\beta s} \psi_s dB_s\right) = 0$. To do so we need to demonstrate that $\int_0^t e^{-\beta s} \psi_s dB_s$ is a martingale, which requires that $e^{-\beta s}\psi_s$ satisfies $E\left[\int_0^t \left(e^{-\beta s}\psi_s\right)^2 ds\right] < \infty$, $t > 0$. In our case, we have simply that $\psi_t = \sigma_a/\beta$, where $\beta > 0$. The condition is therefore satisfied, and we have that

$$\begin{aligned} \lim_{t\to\infty} E_0\left\{e^{-\beta t}J(a_t,t)\right\} &= \lim_{t\to\infty} E_0\left\{J(a_0,0) + \int_0^t e^{-\beta s}DJ(a_s,s)ds\right\} \qquad \text{(B.6')} \\ &= J(a_0,0) + \int_0^\infty e^{-\beta s}DJ(a_s,s)ds \ . \end{aligned}$$

The transversality condition (15) can easily be verified because both wealth a_t and the term $Y(t;x_t)$ at most grow at an exponential rate. The left-hand side of (B.6') is therefore zero. Because the chosen control is arbitrary, (B.4) implies that

$$-DJ(a_t,t) \geqq \ln(\alpha_t n_t^m a_t - y) \quad ,$$

and therefore

$$J(a_0,0) \geqq \int_0^\infty e^{-\beta s}\ln(\alpha_s n_s^m a_s - y)ds \quad . \qquad \text{(B.7)}$$

On the other hand, when we do the same calculation for our feedback controls (n_t^{q*}, n_t^{m*}) we arrive at (B.7) but with $\geqq$ replaced by an equality sign:

$$J(a_0,0) = \int_0^\infty e^{-\beta s}\ln(\alpha_s n_s^{m*} a_s^* - y)ds \quad . \qquad \text{(B.8)}$$

We therefore conclude that $J(a_0,0)$ dominates the value obtained from any other admissible control process, and that the controls (n_t^{q*}, n_t^{m*}) are indeed optimal.

Appendix III Data

Quarterly data from the first quarter of 1996 through the second quarter of 2004 are used. We use an interpolated series of annual population figures obtained from International Financial Statistics (IFS) to convert aggregate to per capita series, and we use Peso-dollar exchange rate data obtained from Banco de Mexico to convert nominal to real series. Because the velocity series is a ratio of a stock to a flow, we used as our stock data series the midpoint of each quarter instead of the endpoint. For consistency we did so all series, including base money. The series for nominal base money is from Banco de Mexico, and is converted to per capita terms. The velocity series is the ratio of nominal domestic absorption (from IFS) to the stock of base money, where quarterly absorption is multiplied by four to obtain an annual series. Real interest rates are gross annualized US treasury bill rates divided by gross annualized one-quarter ahead US CPI inflation (from IFS).[18] Government spending is computed as the sum of government consumption and government investment from national accounts data available at Banco de Mexico, and put on a real per capita basis by divided by the exchange rate and population. Finally, we need to compute the sample average portfolio share of Mexican government liabilities. To do so we obtain data for nominal outstanding stocks of base money M and government debt Q from Banco de Mexico. Nearly all Mexican government debt is denominated in local currency.

[18] We have also worked with the log difference of the CPI-deflated S&P500 index. The main results of the paper do not change in that case.

References

Aït-Sahalia, Y. (2002), 'Maximum Likelihood Estimation of Discretely Sampled Diffusions: A Closed-Form Approximation Approach', Econometrica, 70(1), 223-262.

Aizenman, J. (1989), 'Country Risk, Incomplete Information and Taxes on International Borrowing', *Economic Journal*, **99**, 147-161.

Backus, D.K. and Kehoe, P.J. (1989), 'On the Denomination of Government Debt - A Critique of the Portfolio Balance Approach', *Journal of Monetary Economics*, **23**, 359-376.

Bakshi, G.S. and Chen, Z. (1997), 'Equilibrium Valuation of Foreign Exchange Claims', *Journal of Finance*, **52**, 799-826.

Bansal, R. and Dahlquist, M. (2000), 'The Forward Premium Puzzle: Different Tales from Developed and Emerging Economies', *Journal of International Economics*, **51(1)**, 115-144.

Branson, W.H. and Henderson, D.W. (1985), 'The Specification and Influence of Asset Markets', Ch. 15 in: *Handbook of International Economics*, vol. 2.

Cambbell, J.Y., Lo, A.W. and MacKinlay, A.C. (1997), *The Econometrics of Financial Markets*, Princeton University Press, Princeton, New Jersey.

Canzoneri, M.B., Cumby, R.E. and Diba, B.T. (2004),'Is the Price Level Determined by the Needs of Fiscal Solvency?', *American Economic Review* (forthcoming).

Chamley, C. and Ptolemarchakis, H. (1984), 'Assets, General Equilibrium, and the Neutrality of Money', *Review of Economic Studies*, **51**, 129-138.

Chow, G.C. (1979), 'Optimum Control of Stochastic Differential Equation Systems', *Journal of Economic Dynamics and Control*, **1**, 143-175.

Cox, J.C., Ingersoll, J.E. and Ross, S.A. (1985), 'An Intertemporal General Equilibrium Model of Asset Prices', *Econometrica*, **53(2)**, 363-384.

Duffie, D. (1996), *Dynamic Asset Pricing Theory*, 2nd edition, Princeton University Press, Princeton, NJ.

Dumas, B. and Uppal, R. (2000), "Global Diversification, Growth and Welfare with Imperfectly Integrated Markets for Goods", Manuscript, INSEAD and MIT.

Eaton, J. and Gersovitz, M. (1981), "Debt with Potential Repudiation: Theoretical and Empirical Analysis", *Review of Economic Studies*, **48**, 289-309.

Edison, H.J. (1993), 'The Effectiveness of Central Bank Intervention: A Survey of the Literature since 1982', *Special Papers in International Economics* No. 18., Princeton University, Princeton, NJ.

Devereux, M.B. and Engel, C. (1998), 'Fixed versus Floating Exchange Rates: How Price Setting Affects the Optimal Choice of Exchange Rate Regime', *NBER Working Paper* No. 6867.

Engel, C. (1992), 'On the Foreign Exchange Risk Premium in a General Equilibrium Model', *Journal of International Economics*, **32**, 305-319.

Engel, C. (1999), 'On the Foreign Exchange Risk Premium in Sticky-Price General Equilibrium Models', *NBER Working Paper* No. 7067.

Feenstra, R.C. (1986), 'Functional Equivalence between Liquidity Costs and the Utility of Money', *Journal of Monetary Economics*, **17**, 271-291.

Fleming, W.H. and Rishel, R.W. (1975), *Deterministic and Stochastic Optimal Control*, Springer Verlag, New York, NY.

Gavin, M. and Perotti, R. (1997), 'Fiscal Policy in Latin America', (Manuscript, Inter-American Development Bank and Columbia University).

Grinols, E.L. and Turnovsky, S.J. (1994), 'Exchange rate determination and asset prices in a stochastic small open economy', *Journal of International Economics,* **36**, 75-97.

Karatzas, I. and Shreve, S.E. (1991), *Brownian Motion and Stochastic Calculus*, 2nd edition, Springer Verlag, New York, NY.

Kehoe, P.J. and Perri, F. (2001), 'International Business Cycles with Endogenous Incomplete Markets', *Econometrica* (forthcoming).

Kletzer, K.M. and Wright, B.D. (2000), 'Sovereign Debt as Intertemporal Barter', *American Economic Review*, **90(3)**, 621-639.

Kumhof, M. and van Nieuwerburgh, S. (2002), 'A Fiscal Theory of the Currency Risk Premium and of Sterilized Intervention', IMF Working Paper WP/02/29.

Lucas, Robert E. jr. (1982), 'Interest Rates and Currency Prices in a Two-Country World', *Journal of Monetary Economics*, **10**, 335-359.

Lucas, Robert E. jr. (1990), 'Liquidity and Interest Rates', *Journal of Economic Theory*, **50**, 237-264.

Malliaris, A.G. and Brock, W.A. (1982), *Stochastic Methods in Economics and Finance*, 1st edition, Advanced Textbooks in Economics, Volume 17, Elsevier, Amsterdam.

Merton, R.C. (1969), 'Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case', *The Review of Economics and Statistics*, **51**, 239-246.

Merton, R.C. (1971), 'Optimum Consumption and Portfolio Rules in a Continuous-Time Model', *Journal of Economic Theory*, **3**, 373-413.

Montiel, P.J. (1993), 'Capital Mobility in Developing Countries', *The World Bank Working Paper Series* 1103.

Obstfeld, M. (1982), 'The Capitalization of Income Streams and the Effects of Open-Market Policy under Fixed Exchange Rates', *Journal of Monetary Economics*, **9**, 87-98.

Obstfeld, M. (1987), 'Peso Problems, Bubbles, and Risk in the Empirical Assessment of Exchange-Rate Behavior', *NBER Working Paper* No. 2203.

Obstfeld, M. and Rogoff, K. (1996), *Foundations of International Macroeconomics*, 1st edition, MIT Press, Cambridge, MA.

Obstfeld, M. and Rogoff, K. (1998), 'Risk and Exchange Rates', *NBER Working Paper* No. 6694.

Obstfeld, M. and Rogoff, K. (2000), 'New Directions for Stochastic Open Economy Models', *Journal of International Economics*, **50**, 117-153.

Sargent, T.J. and Smith, B.D. (1987), 'The Irrelevance of Government Foreign Exchange Operations', in: Helpman, E., Razin, A. and Sadka, E., eds., *Economic Effects of the Government Budget*, Cambridge, Mass. and London: MIT Press.

Sargent, T.J. and Smith, B.D. (1988), 'Irrelevance of Open Market Operations in Some Economies with Government Currency Being Dominated in Rate of Return', *American Economic Review*, **77(1)**, 78-92.

Stulz, R.M. (1983), 'The Demand for Foreign Bonds', *Journal of International Economics*, **15**, 225-238.

Stulz, R.M. (1984), 'Currency Preferences, Purchasing Power Risks, and the Determination of Exchange Rates in an Optimizing Model', *Journal of Money, Credit and Banking*, **16(3)**, 302-316.

Stulz, R.M. (1987), 'An Equilibrium Model of Exchange Rate Determination and Asset Pricing with Nontraded Goods and Imperfect Information', *Journal of Political Economy*, **95(5)**, 1024-1040.

Stulz, R.M. (1988), 'Capital Mobility and the Current Account', *Journal of International Money and Finance*, **7**, 167-180.

Uribe, M. (2001), 'A Fiscal Theory of Sovereign Risk', (Manuscript, University of Pennsylvania).

Wallace, N. (1981), 'A Modigliani-Miller Theorem for Open Market Operations', *American Economic Review*, **71**, 267-274.

Zapatero, F. (1995), 'Equilibrium Asset Prices and Exchange Rates', *Journal of Economic Dynamics and Control*, **19**, 787-811.

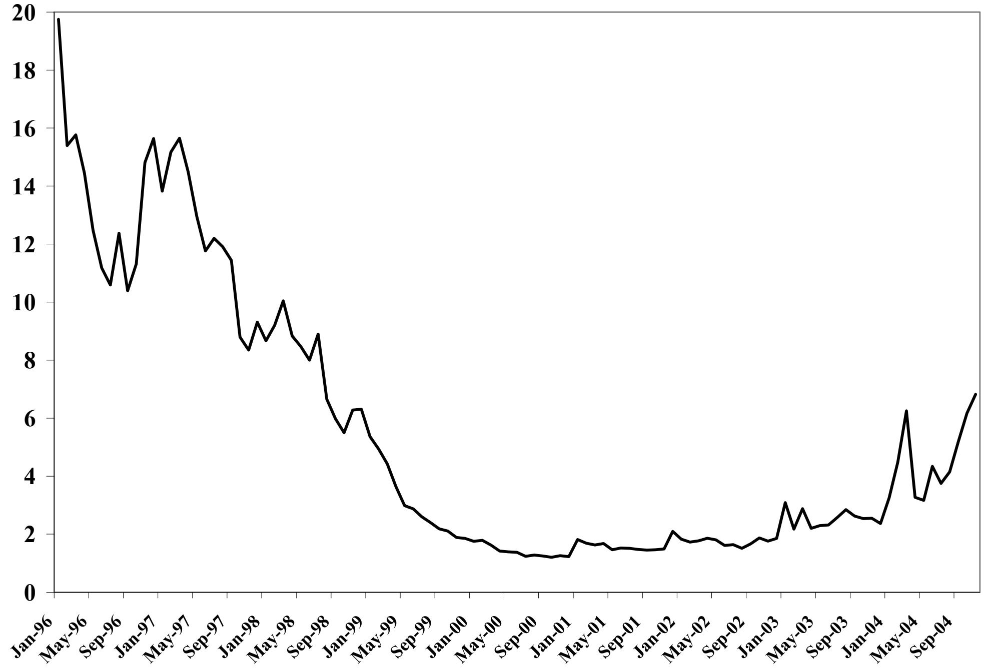

Figure 1: Foreign Holdings of Mexican Government Securities - % of Total (Source: Banco de Mexico)

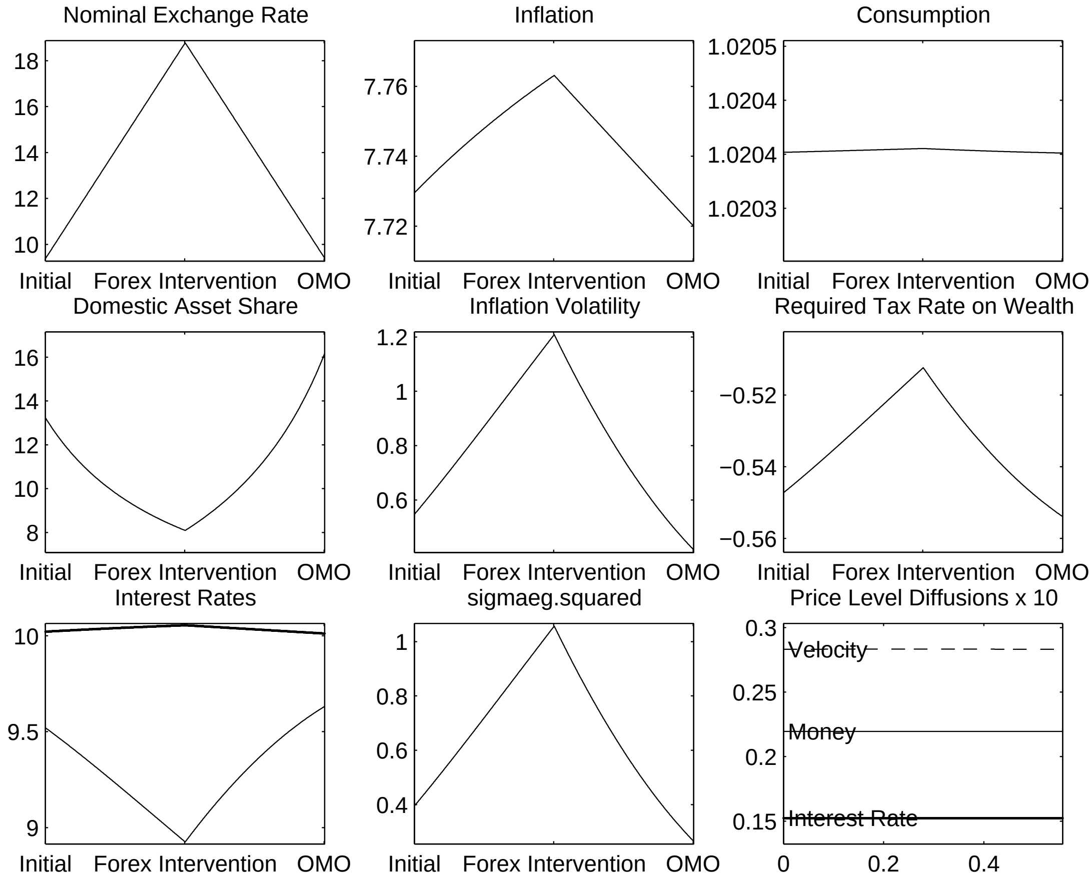

Figure 2: Unsterilized Foreign Exchange Purchase and Open Market Sale

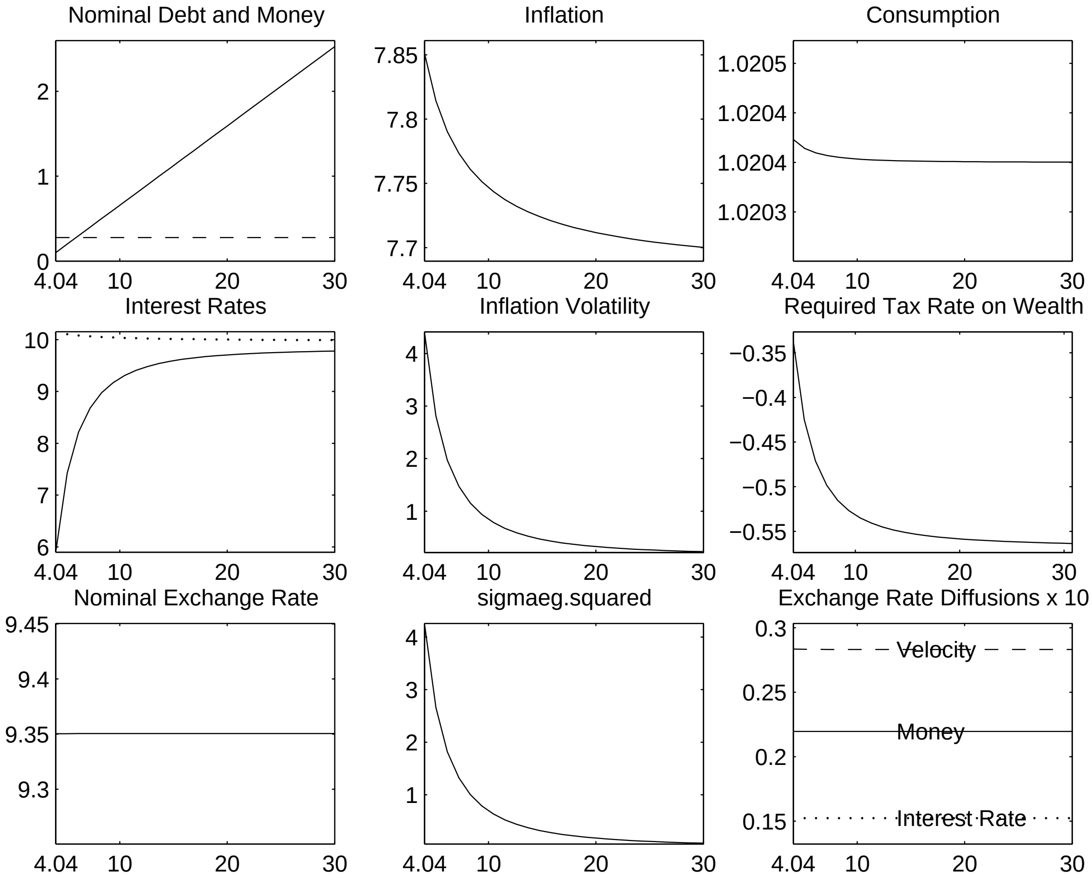

Figure 3: Sterilized Foreign Exchange Intervention - Baseline Case

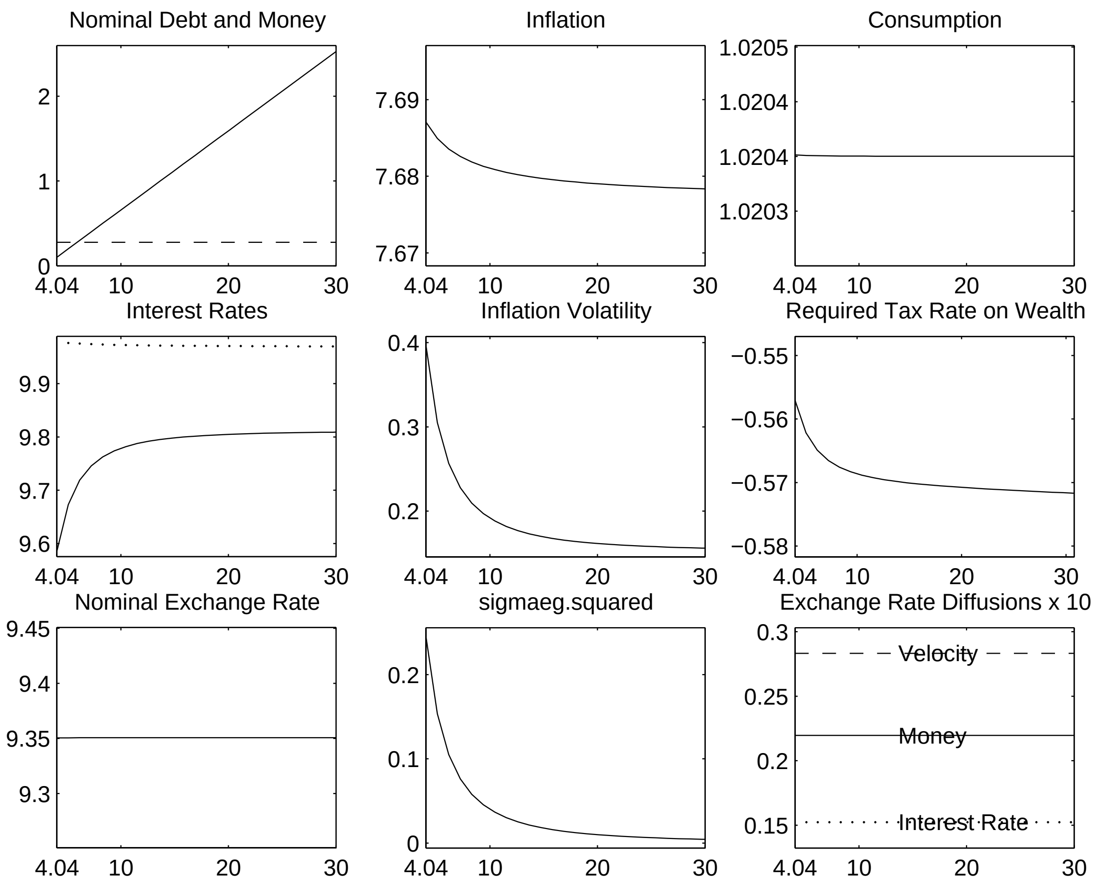

Figure 4: Sterilized Foreign Exchange Intervention - Low Volatility of Fiscal Shocks

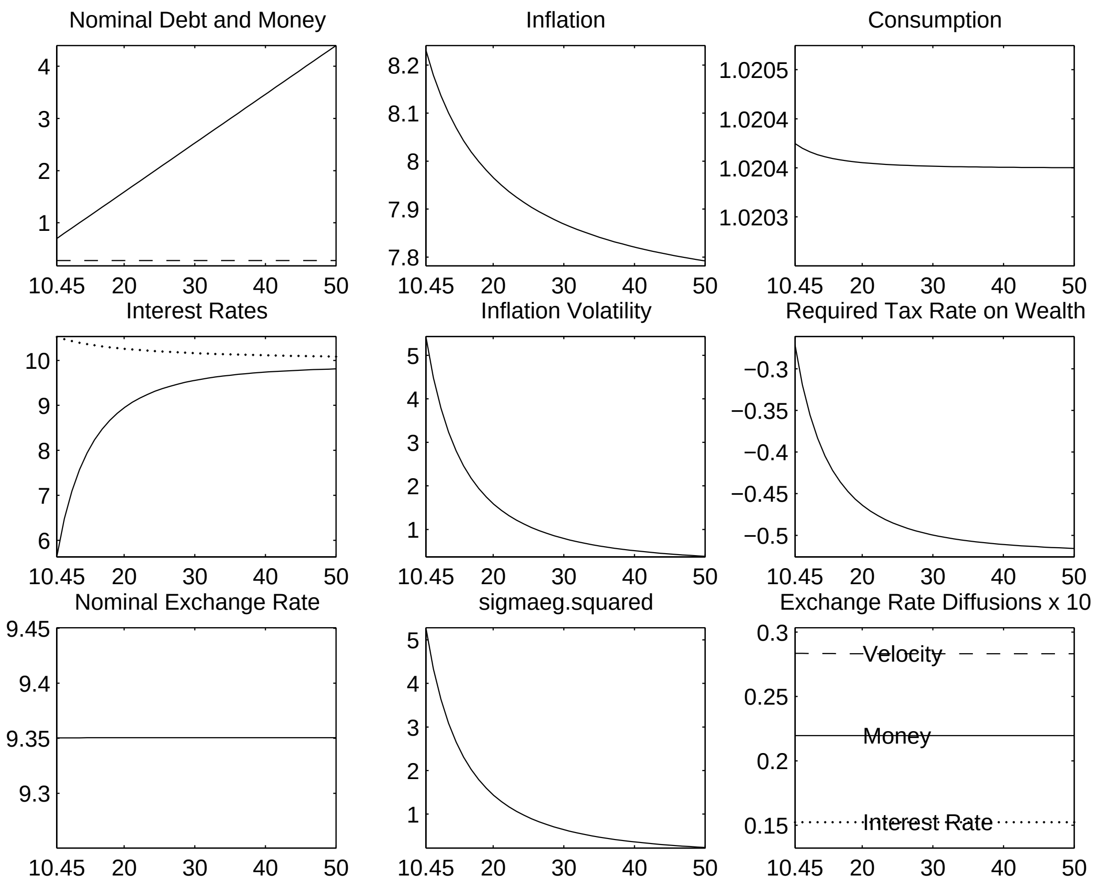

Figure 5: Sterilized Foreign Exchange Intervention - High Volatility of Fiscal Shocks