# Benefits from U.S. Monetary Policy Experimentation in the Days of Samuelson and Solow and Lucas*


Timothy Cogley†
Riccardo Colacito‡
Thomas J. Sargent§


December 9, 2004


**Abstract**

A policy maker knows two models of inflation-unemployment dynamics. One implies an exploitable trade-off, the other does not. The policy maker's prior probability over the two models is part of his state vector. Bayes' law converts the prior into a posterior at each date and gives the policy maker an incentive to experiment. For a model calibrated to U.S. data through the early 1960s, we isolate the component of government policy that is due to experimentation by comparing the outcomes from two Bellman equations, the first of which 'experiments and learns', the second of which 'learns but doesn't experiment'. We interpret the second as an 'anticipated utility' model and study how well its outcomes approximate those from the 'experiment and learn' Bellman equation. The approximation is good.


# 1 Introduction

We quantify the importance of deliberate experimentation when two models that fit the historical data equally well have sharply different operating characteristics that are vital to a policy decision. As our laboratory, we analyze a case in which


*We thank Patrick Bajari and Martin Schneider for useful discussions.


†University of California, Davis. Email: twcogley@ucdavis.edu
‡New York University. Email: rc652@nyu.edu
§New York University and Hoover Institution. Email: ts43@nyu.edu

two competing models of inflation-unemployment dynamics differ with respect to whether they imply an exploitable Phillips curve.

In the late 1960's, a debate raged between advocates of the natural unemployment hypothesis and those who thought that there is an exploitable unemployment-inflation trade-off. To capture this dispute, we imagine that a monetary policy authority has the following two models of inflation-unemployment dynamics:

- Model 1 (Samuelson-Solow):

$$
\begin{aligned}
U_t &= .0023 + .7971 U_{t-1} - .2761 \pi_t + .0054 \eta_{1,t} \\
\pi_t &= v_{t-1} + .0055 \eta_{3t}
\end{aligned}
$$

- Model 2 (Lucas):

$$
\begin{aligned}
U_t &= .0007 + .8468 U_{t-1} - .2489 (\pi_t - v_{t-1}) + .0055 \eta_{2,t} \\
\pi_t &= v_{t-1} + .0055 \eta_{3t}
\end{aligned}
$$

where $U_t$ is the deviation of the unemployment rate from an exogenous measure of a natural rate $U_t^*$,[1] $\pi_t$ the rate of inflation, $v_{t-1}$ is the rate of inflation that at time $t-1$ the monetary authority and private agents had both expected to prevail at time $t$, and, for $i = 1, 2, 3$, $\eta_{it}$ is an i.i.d. Gaussian sequence with mean zero and variance 1. The monetary authority has a Kydland-Prescott (1977) type of loss function $E \sum_{t=0}^{\infty} \beta^t r_t$, where $r_t = -.5(U_t^2 + \lambda v_t^2)$.[2] The monetary authority sets $v_t$ as a function of time $t$ information.[3] The monetary authority knows the parameters of each model for sure and attaches probability $\alpha_0$ to model 1 and probability $1 - \alpha_0$ to model 2.[4]

[1]We use this specification mainly as a device to get good fitting models while keeping the dimension of the state of our model to the minimum required to represent 'natural rate' and 'non-natural rate' theories of unemployment. See appendix D for details.

[2]Alan Blinder (1998) has stressed that this objective function forces a conflict between the policy maker (who prefers an unemployment lower than the natural rate) and the public (which would choose to set unemployment to the natural rate) that is essential to induce the time consistency problem for inflation described by Kydland and Prescott (1977).

[3]Under this timing protocol, there is no time-consistency problem in Kydland and Prescott's model. See Stokey (1989).

[4]We assume that model parameters are known because we want to reduce to a minimum the dimension of the monetary authority's posterior distribution. If we were to treat the parameters as unknown, probability distributions for those parameters would be part of the monetary authority's prior, increasing the dimension of the state beyond what we can manage computationally. See Wieland (2000a,b) and Beck and Wieland (2002) for analysis of the Bellman equation for a decision maker who experiments to learn about parameter values. See El-Gamal and Rangarajan (1993) for an analysis of convergence in a class of models in which agents are learning. Kenneth Kasa (1999) adapts results that earlier researchers had obtained for a monopolist who could learn, but chooses not to learn, his demand curve. Kasa thereby creates a model in which the Fed chooses not to learn objects that could be learned through some different strategy.

Although they fit the U.S. data from 1948:3-1963:I almost equally well, these two models call for very different policies toward inflation under our loss function. Model 1, whose main features many have attributed to Samuelson and Solow (1960), has an exploitable tradeoff between $v_t$ and subsequent levels of unemployment. Having operating characteristics advocated by Lucas (1972, 1973) and Sargent (1973), model 2 has no exploitable Phillips curve: systematic variations in inflation $v_t$ affect inflation but not unemployment. If $\alpha_0 = 0$, then our decision maker should implement the trivial policy $v_t = 0$ for all $t$. However, if $\alpha_0 > 0$, the policy maker is willing to set $v_t \neq 0$ partly in order to exploit a probable inflation-unemployment tradeoff and partly in order to refine $\alpha$. After calibrating the two models to U.S. data before 1963, this paper imputes the same objective to the monetary authority that Kydland and Prescott(1977) used, then solves the Bellman equation

We use the optimal decision rule to study the following questions:

1. Suppose that the Samuelson-Solow model actually governs the data and that before date $T$ the government had assigned probability $\alpha = 1$ to the Samulelson-Solow model and had used the corresponding optimal policy for a long time, so that the economy is in a stochastic steady state. Having been persuaded by an advocate of the natural rate hypothesis, at date $T$ the government suddenly lowers $\alpha$ to a number $\alpha \in (0, 1)$ even though, unbeknownst to the government, the Samuelson-Solow model actually prevails. Under these assumptions, we use our model to quantify the adverse effects on government policy that follow from its attaching some weight to the Lucas model. Lucas's model is subversive in leading to higher unemployment than would have prevailed had it never been invented. We ask how much higher is unemployment, and how long does it take for the government to forget the Lucas model?

2. Suppose that the Lucas model actually governs the data and that before date $T$ the government had assigned probability $1 - \alpha = 1$ to the Lucas model and used the corresponding optimal policy for a long time, so that the economy is in a stochastic steady state at date $T - 1$. At date $T$, having been persuaded by advocates of the Samuelson-Solow model, the government suddenly lowers $1 - \alpha$ to a number in $(0, 1)$ even though, unbeknownst to the government, the Lucas model actually prevails. We use our model to quantify the effects on government policy that follow from its putting some weight on the Samuelson-Solow model. Samuelson and Solow's model is pernicious in leading to higher inflation and no lower inflation than if it had never been thought of. We study how much more inflation is produced by this scenario and how long it takes for the government to retire the Samuelson-Solow model.

3. We want to quantify the role of 'active' as opposed to 'passive' experimentation. We do this by comparing the decision rule and value function for the problem includes $\alpha$ as a state variable and Bayes' law as a transition equation with another Bellman equation that suppresses $\alpha$ as a state variable and ignores Bayes' law as a transition equation. By comparing the associated decision rules, we identify a component of time $t$ decisions that is attributable to intentional experimentation.

## 1.1 Organization

Section 2 formulates Bellman equations, one for a decision maker who consciously experiments, another for an 'anticipated utility' decision maker who does not consciously experiment. These Bellman equations describe alternative states of mind for the policy maker. Section 3 describes alternative ways of modelling how the true data generating model relates to the policy maker's state of mind. Section 4 discusses our numerical approximations to the value functions and decision rules. Section 5 describes quantitative experiments designed to answer the three questions asked above, as well as a variety of statistics on 'waiting times' to learn the truth. Section 6 adds some concluding remarks. Four appendixes contain technical details about how we solved the Bellman equations and calibrated the two models.

# 2 Two Formulations of the Policy Problem Under Model Uncertainty

We map our example into a general setup, then provide Bellman equations for the government under our two alternative assumptions about the government's response to the opportunity to experiment.

## 2.1 The Models

The policy maker has two models

$$s_{t+1} = A_i s_t + B_i v_t + C_i \epsilon_{i,t+1}, \tag{1}$$

$i = 1, 2$, where $s_t$ is a state vector, $v_t$ is a control vector, and $\epsilon_{i,t+1}$ is an i.i.d. Gaussian process with mean zero and contemporaneous covariance matrix $I$. Let $F(\cdot)$ denote the c.d.f. of this normalized multivariate Gaussian distribution. At time $t$, the policy maker has observed a history of outcomes $s^t = s_t, s_{t-1}, \ldots, s_0$ and assigns probability $\alpha_t$ to model 1 and probability $(1 - \alpha_t)$ to model 2. By applying

Bayes' Law, the policy maker updates $\alpha_t$:

$$\alpha_{t+1} = B(\alpha_t, s_{t+1}). \tag{2}$$

In equations (33) and (37) in appendix A, we provide a formula for $B(\alpha_t, s_{t+1})$. The policy maker wants a policy for setting $v_t$ that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t r(s_t, v_t), \quad \beta \in (0,1), \tag{3}$$

where $E_0$ is a mathematical expectation with respect to the distribution over future outcomes induced by the models (1) and the policy maker's opinions about them.

## 2.2 Intentional Experimentation

The policy maker's belief $\alpha_t$ is a component of the time $t$ state vector $(s_t, \alpha_t)$. In choosing $v_t$, it is in the policy maker's interest to recognize the revisions of his beliefs that he foresees will occur through equation (3). Let $V(s_t, \alpha_t)$ be the optimal value in state $(s_t, \alpha_t)$. The Bellman equation is

$$\begin{aligned} V(s_t, \alpha_t) = \max_{v_t} \{ & r(s_t, v_t) \\ & + \beta\alpha_t \int V(A_1 s_t + B_1 v_t + C_1 \epsilon_{1,t+1}, B(\alpha_t, A_1 s_t + B_1 v_t + C_1 \epsilon_{1,t+1})) dF(\epsilon_{1,t+1}) \\ & + \beta(1-\alpha_t) \int V(A_2 s_t + B_2 v_t + C_2 \epsilon_{2,t+1}, B(\alpha_t, A_2 s_t + B_2 v_t + C_2 \epsilon_{2,t+1})) dF(\epsilon_{2,t+1}) \} \end{aligned} \tag{4}$$

The optimal decision rule can be represented recursively as

$$v_t = v(s_t, \alpha_t) \tag{5}$$
$$\alpha_{t+1} = B(s_t, \alpha_t). \tag{6}$$

Repeated substitution of (6) into (5) yields the policy maker's strategy in the form of a sequence of functions

$$v_t = \sigma_t(s^t, \alpha_0), \tag{7}$$

where $s^t = (s_t, s_{t-1}, \ldots, s_0)$. The presence of $B(\alpha_t, A_i s_t + B_i v_t + C_i \epsilon_{t+1})$, $i = 1, 2$, on the right side of (4) imparts a motive to experiment. To choose $v_t$ is to design experiments.

## 2.3 Bellman Equation in Detail

Appendix A derives the function $B(s_t, \alpha_t)$ and thereby obtains a particular version of (4) that we approximate numerically. Let $\Omega_i = C_i C_i'$, $R_t = \frac{\alpha_t}{1-\alpha_t}$, and define

$$\begin{aligned} g(\epsilon_{1,t+1}; s_t, \alpha_t) = {} & \log R_t - \frac{1}{2}\log|\Omega_1| + \frac{1}{2}\log|\Omega_2| - \frac{1}{2}\left(C_1\epsilon_{1,t+1}\right)' \Omega_1^{-1}\left(C_1\epsilon_{1,t+1}\right) \\ & + \frac{1}{2}\left[(A_1 - A_2)s_t + (B_1 - B_2)v_t + C_1\epsilon_{1,t+1}\right]' \\ & \times \Omega_2^{-1}\left[(A_1 - A_2)s_t + (B_1 - B_2)v_t + C_1\epsilon_{1,t+1}\right] \end{aligned} \tag{8}$$

and

$$\begin{aligned} h(\epsilon_{2,t+1}; s_t, \alpha_t) = {} & \log R_t - \frac{1}{2}\log|\Omega_1| + \frac{1}{2}\log|\Omega_2| + \frac{1}{2}\left(C_2\epsilon_{2,t+1}\right)' \Omega_2^{-1}\left(C_2\epsilon_{2,t+1}\right) \\ & - \frac{1}{2}\left[(A_2 - A_1)s_t + (B_2 - B_1)v_t + C_2\epsilon_{2,t+1}\right]' \\ & \times \Omega_1^{-1}\left[(A_2 - A_1)s_t + (B_2 - B_1)v_t + C_2\epsilon_{2,t+1}\right]. \end{aligned} \tag{9}$$

Using (33) in Appendix A, we obtain a law of motion for $\alpha_{t+1}$ under the two models. Then Bellman equation (4) becomes

$$\begin{aligned} V(s_t, \alpha_t) = \max_{v_t} \Bigg\{ & r(s_t, v_t) + \beta\alpha_t \int V\left(A_1 s_t + B_1 v_t + C_1\epsilon_{1,t+1}, \frac{e^{g(\epsilon_{1,t+1})}}{1 + e^{g(\epsilon_{1,t+1})}}\right) dF(\epsilon_{1,t+1}) \\ & + \beta(1-\alpha_t) \int V\left(A_2 s_t + B_2 v_t + C_2\epsilon_{2,t+1}, \frac{e^{h(\epsilon_{2,t+1})}}{1 + e^{h(\epsilon_{2,t+1})}}\right) dF(\epsilon_{2,t+1}) \Bigg\}. \end{aligned} \tag{10}$$

Appendix B describes how we approximate the solution of (10).

## 2.4 Attitudes Toward Experimentation

Although they surely know (4), several prominent macroeconomists have advised against exploiting the opportunity (or succumbing to the temptation) to experiment identified by the right side of Bellman equation (4). Blinder (1998, p. 11) asserts that

> "while there are some fairly sophisticated techniques for dealing with parameter uncertainty in optimal control models with learning, those methods have not attracted the attention of either macroeconomists or policymakers. There is a good reason for this inattention, I think: You don't conduct policy experiments on a real economy solely to sharpen your econometric estimates."

Lucas (1981, p. 288) agrees, remarking that

> "Social experiments on the grand scale may be instructive and admirable, but they are best admired at a distance. The idea, if the marginal social product of economics is positive, must be to gain some confidence that the component parts of the program are in some sense reliable prior to running it at the expense of our neighbors."

These economists argue that conscious experimentation is a bad idea. Perhaps Blinder and Lucas suspect that the decision maker has too few models on the table (e.g., that neither of models in Bellman equation (4) is correct) and that therefore the decision problem is misspecified.

Another reason for not deliberately experimenting is that it is very difficult to approximate the solution of the Bellman equation that corresponds to (4) when there are more dimensions of uncertainty, e.g., unknown coefficients and more models). To sidestep that problem, researchers like Cogley and Sargent (2004) have appealed to Kreps's (1998) 'anticipated utility' model to justify an adaptive approach that we now describe.

A third possible reason for being skeptical about experiments is related to the previous two. We can interpret the fact that Bellman equation (4) is difficult to solve as saying that it is difficult to design optimal experiments. The value function that obeys (4) is maximized over all possible experiments. Suboptimal experiments attain lower values. Many such suboptimal experiments would actually attain lower values than those delivered by the 'don't experiment' rule that solves the alternative Bellman equation (4).

## 2.5 Unintentional Experimentation

Another Bellman equation that appears in the literature on adaptive control lets us quantify how much the policy maker sacrifices by abstaining from the opportunity to experiment. We formulate an optimum problem that ignores the opportunity to experiment by replacing the law of motion (2) for $\alpha_t$ dictated by Bayes' law with the "don't experiment on purpose" specification

$$\alpha_t = \alpha \quad \forall t \geq 0. \tag{11}$$

When he makes a decision at time $t$, the policy maker pretends that he cannot or will not learn about the model from future data. One interpretation of this assumption is that the policy maker believes that nature will draw next period's $s_{t+1}$ from an

$\alpha$-weighted mixture of models 1 and 2. Another interpretation is that the policy maker plans not to revise his views. Under either interpretation, a policy maker with this fixed-$\alpha$ view has a value function $W(s_t;\alpha)$ that solves the Bellman equation

$$
\begin{aligned}
W(s_t;\alpha) = \max_{v_t}\{ & r(s_t, v_t) \\
& + \beta\alpha \int W(A_1 s_t + B_1 v_t + C_1 \epsilon_{t+1};\alpha) dF(\epsilon_{1,t+1}) \\
& + \beta(1-\alpha) \int W(A_2 s_t + B_2 v_t + C_2 \epsilon_{t+1};\alpha) dF(\epsilon_{2,t+1})\}.
\end{aligned} \tag{12}
$$

The decision rule that attains $W(s_t;\alpha)$ is

$$v_t = w(s_t;\alpha). \tag{13}$$

Because this is a feasible policy for the decision maker of subsection 2.2 who is willing to experiment, it follows that

$$V(s_t,\alpha) \geq W(s_t;\alpha) \tag{14}$$

for all values of $s_t, \alpha$. The gap

$$V(s_t,\alpha) - W(s_t;\alpha) \tag{15}$$

measures the value of experimentation and the difference

$$v(s_t,\alpha) - w(s_t;\alpha) \tag{16}$$

measures the component of the time $t$ policy choice that can be attributed purely to the policy maker's motive to experiment.

Appendix C describes our algorithm for solving (12).

#### 2.5.1 Adaptive Interpretation

In the spirit of the adaptive control literature, suppose that the policy maker does indeed revise $\alpha_t$ by applying Bayes' Law even though he uses a policy (13) derived by solving the abstain-from-learning Bellman equation (12). Then his actual decisions can be represented recursively as

$$v_t = w(s_t;\alpha_t) \tag{17}$$

$$\alpha_{t+1} = B(s_t;\alpha_t). \tag{18}$$

These decisions would emerge from a '*don't* experiment but *do* learn' prescription.[5] Equations (17), (18) can be solved by repeated substitution to yield the policy maker's strategy in the form of a sequence of functions

$$v_t = \tilde{\sigma}_t(s^t, \alpha_0). \tag{19}$$

In addition to representing a stylized '*don't* experiment but *do* learn' view, rules like (17)-(18) have been recommended as an alternative or approximation to (5)-(6) to be used in situations in which the curse of dimensionality somehow prevents the policy maker or the analyst from solving Bellman equation (4) or the pertinent counterpart to it. The appeal of this approximation is greatest when the dimension of the prior distribution is large.[6] We have assumed that $A_i, B_i, C_i$ in (1) are known matrices. Had we assumed instead that the policy maker has a nontrivial prior probability distribution over those parameters, those distributions would enter the value function on the left side of (4). The Bellman equation for this value function would be easy to write down but difficult to solve because of the dimension of the state vector.

# 3 The Truth

So far our description has been about the views of the monetary authority that are summarized by equations (1) and $\alpha_0 \in (0, 1)$. We have said everything about what the monetary authority believes and how it chooses $v_t$, but nothing about how the economy actually works. Thus, our description so far is about ideas that are 'just in the head' of the monetary authority.

Under the monetary authority's prior distribution over sequences for unemployment and inflation that is implied by our specification, $\alpha_t$ is a martingale. See section A.1 for a proof. Because $\alpha_t \in [0, 1]$, the martingale convergence theorem implies that $\alpha_t$ converges almost surely under that measure. To say what happens to $\alpha_t$ under the measure that actually generates the economy, we have to say what that true measure is. If we assume that one of our two models, either model 1 (Samuelson and Solow's) or model 2 (Lucas's), or some fixed-$\alpha$ mixture of them, governs the data, then $\alpha_t$ given by (29) converges almost to the true $\alpha$.[7]

[5] It seems fair to say that Blinder (1998, chapter 1) advocates this point of view.

[6] See footnote 4.

[7] Our model has a feature that El-Gamal and Rangarajan (1993) identify as important in promoting convergence, namely, the presence of an exogenous component of randomness that generates 'natural experiments' that can help discriminate between models even if the policy maker decides not to experiment in setting his policy.

Our concern in the next section is to study the *rates* at which $\alpha_t$ converges to the true $\alpha$ under alternative assumptions about which model is the true data generating process and alternative initial conditions for $\alpha, U$. We design alternative scenarios to shed light on the questions stated in section 1 and to determine which of our two models is more difficult to learn about.

# 4 Value Functions and Decision Rules

We have reported calibrated versions of our two models in section 1. For government preference parameters $\beta = .995, \lambda = .1$, figures 1, 2, and 3 display value functions and decision rules associated with our two Bellman equations (4) and (12). As figure 1 and 3, panel a, confirm, $V(U, \alpha) > W(U, \alpha)$ except at the boundaries $\alpha = 1$ and $\alpha = 0$, where $V(U, \alpha) = W(U, \alpha)$. This relationship of the 'experiment and learn' value function $V$ to the 'don't experiment but learn' value function $W$ is as expected: when $\alpha \in (0, 1)$, there is value to intentional experimentation. The policy functions in figure 2 and their difference in 3, panel b, show the different actions called for by the decision rules $v$ and $w$ associated with Bellman equations (4) and (12), respectively.

Overall, the differences between the value functions and the decision rules are both small. Therefore, in this example at least, the type of anticipated utility model used by Cogley and Sargent (2004), which is associated with Bellman equation (12), seems to provide a good approximation to the outcomes from the intentional experimentation model.[8] We study the quality of approximation more fully in the following subsections that analyze the questions posed in section 1.

To bring out their differences, figure 4 shows the decision rules $w(U, \alpha)$ and $v(U, \alpha)$ as functions of $U$ for different values of $\alpha$. As noted, the differences between $v$ and $u$ are always small, but the biggest differences occur for $\alpha$'s away from the boundaries of 0 and 1. The figures reveal that when $\alpha$ is well into the interior of $(0, 1)$, $w$'s call for additional experimentation serves to make it nonlinear and to enhance the countercyclicality of inflation policy. That is, the $v$-policy inflation policy is higher than the $w$-inflation policy when $U$ is high, and lower when $U$ is low. This pattern reveals a kind of 'opportunism': the best time to experiment with Keynesian stimulus is when $U$ is high.[9]

Another interesting feature of figure 4 is that for both the $v$ and $w$ decision rules, policy begins quickly to look more Keynesian even for $\alpha = .2$ (i.e., a small weight

[8] See David Kreps (1998) for a broader defense of this modelling strategy in games and dynamic economic models.

[9] In contrast, Alan Blinder's opportunistic call for more deflation in recessions seems to have been motivated not by an appeal to optimal experimentation but a way for the Fed to find political cover for reducing inflation.

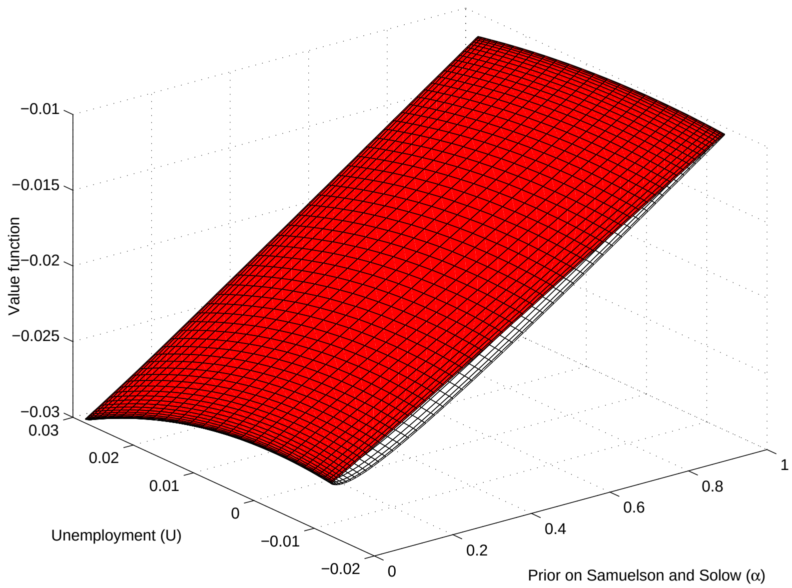


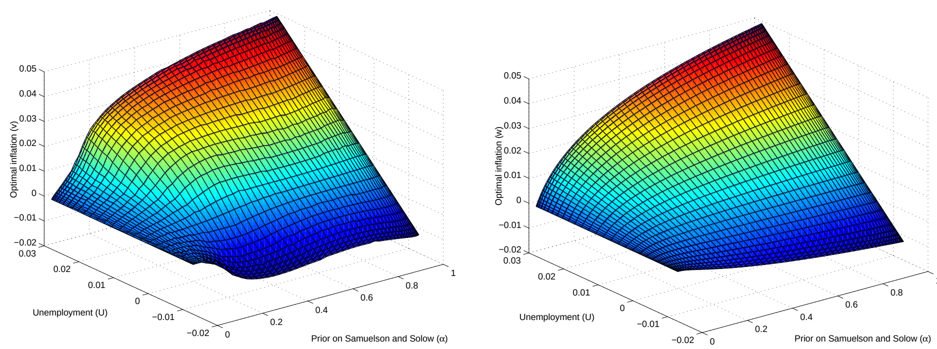

Figure 1: Two value functions: $W(U, \alpha) \leq V(U, \alpha)$

on the Samuelson-Solow model), while it continues to look quite Keynesian when there is a comparable small weight of $1 - \alpha = .2$ on the Lucas model. Thus, a little bit of doubt about the Lucas model makes the policy maker begin to behave like a Keynesian, while a Keynesian has to have bigger doubts about the Samuelson-Solow model to begin behaving as Lucas's model advises.[10] These features of our policy rules will influence outcomes of the experiments that we report in the next section.

# 5 Experiments in Forgetting Pernicious Ideas

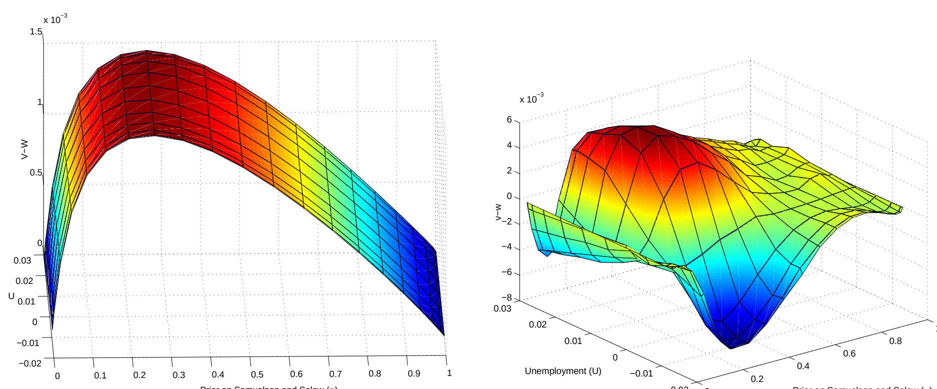
We generate alternative scenarios by specifying an initial condition for $U$, government beliefs $\alpha$, and which of our two models actually generates the data. We use the policy functions in figure 2 to generate histories of outcomes.

[10]This feature of the decision rules conforms to the story about the conquest of American inflation told by Cogley and Sargent (2004).




(a) $v(U, \alpha)$ (b) $w(U, \alpha)$

Figure 2: Policy functions with and without experimentation.




(a) $V(U, \alpha) - W(U, \alpha)$ (b) $v(U, \alpha) - w(v, \alpha)$

Figure 3: Differences in value functions and policies with and without deliberate experimentation.

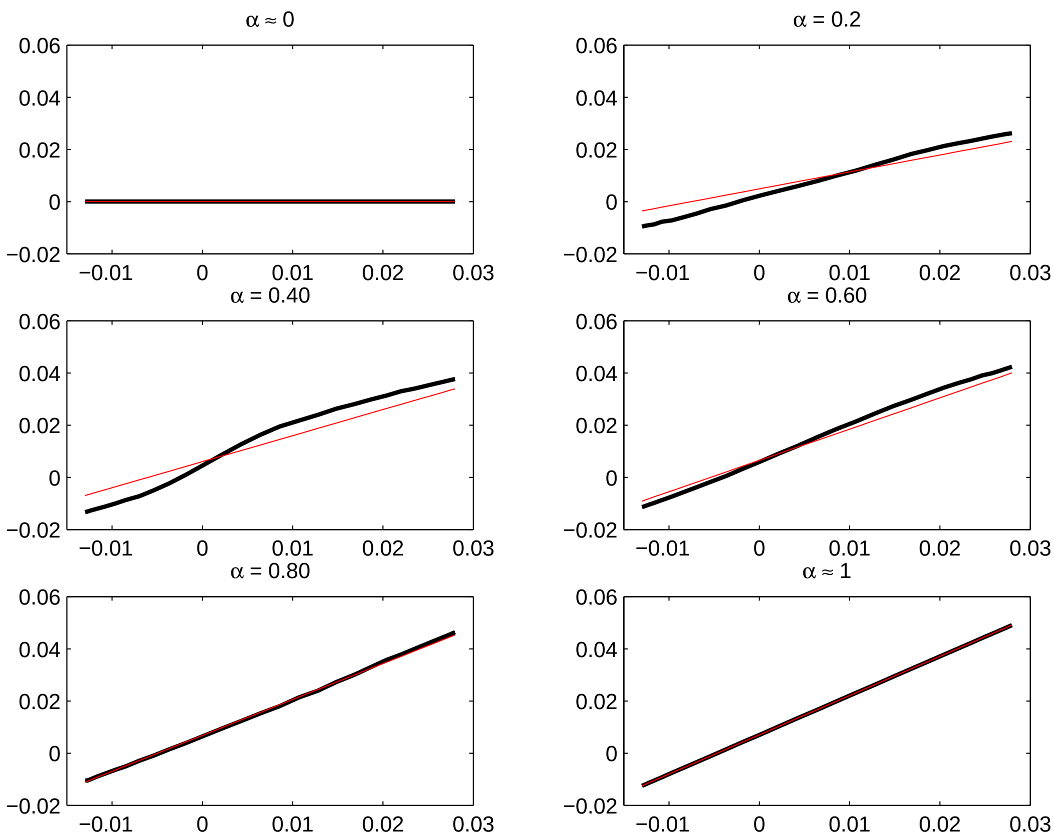



Figure 4: Slices of the optimal decision rules for inflation. The bold line is $v(U, \alpha)$ and the other line is $w(U, \alpha)$.

## 5.1 Misplaced Experimentation When Samuelson And Solow Are Correct

Assume that the data generating process is the Samuelson and Solow model. Figure 5 shows outcomes after the arrival of Lucas and his model prompt the policy maker erroneously to assign some probability to it. For the first 19 periods, the policy maker had $\alpha = 1$ and therefore had optimally exploited the tradeoff between unemployment and inflation given by the Samuelson-Solow model. In period 19, Lucas's model arrives and is assigned a positive probability. Starting from period 19, we model the behavior of three central banks. As a benchmark, the first one (dotted line in the pictures) continues to assign probability one to the Samuelson-Solow model and therefore abstains from experimenting or learning. The second and the third ones attach a prior probability of 75% to his model being true. The second central bank takes into account that this prior will be revised in subsequent periods (black continuous lines), while the third (red lines) does not. The experimenting policy maker keeps inflation high for a while, but the benefit is a sharper decrease in unemployment compared to the anticipated utility central bank. The second bank evidently learns faster than the third.

## 5.2 Misplaced Experimentation When The Lucas Model Is True

Now assume that Lucas's is the true data generating mechanism. Figure 6 shows outcomes when after 19 periods of correct policy under Lucas's model, under the influence of Samuelson and Solow, the monetary policy decision maker assigns a positive probability to their model. As with the previous subsection, we display paths for three types of decision makers. As a benchmark, the first continues to assign probability one to Lucas's model throughout and neither learns nor experiments. The second experiments and learns, while the third learns but does not intentionally experiment. Unemployment behaves in the same way under the three banks' policies because Lucas's model is the true data generating process. However, the experimenting policy maker typically chooses a lower inflation rate than does the non-experimenting bank. Furthermore, the process of forgetting the 'wrong model', as reflected in the convergence of $\alpha_t$ back to 0, appears to be slower than occurred our analysis in the previous subsection where the Samuelson-Solow model prevailed.

The following table presents summary statistics from several related experiments. The variable that we call waiting time in the following table represents the number of time periods that are needed for $\alpha$ to return to within a 0.01 neighborhood of what it should be under the data generating process. For each experiment, we report

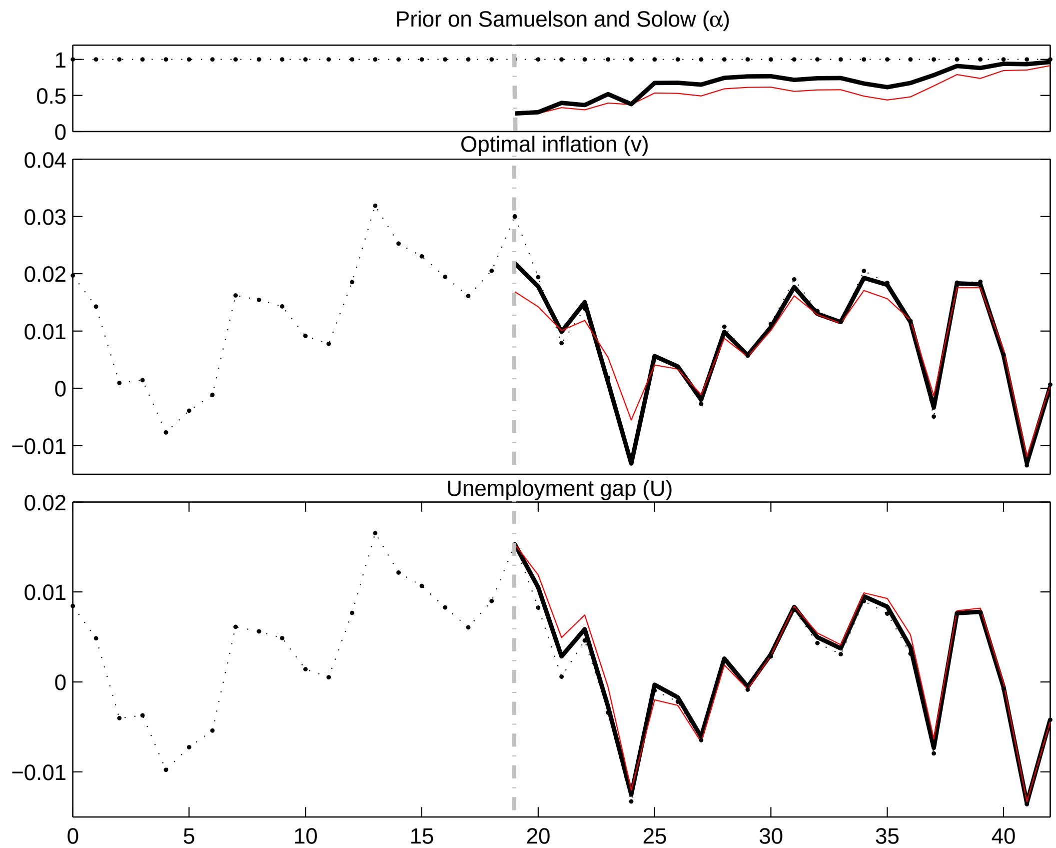



Figure 5: In the three panels: the dotted line represents the behavior of a central bank that attaches probability one to the Samuelson and Solow model, the bold continuous line is the experimenting central bank and the other line is the non-experimenting bank.

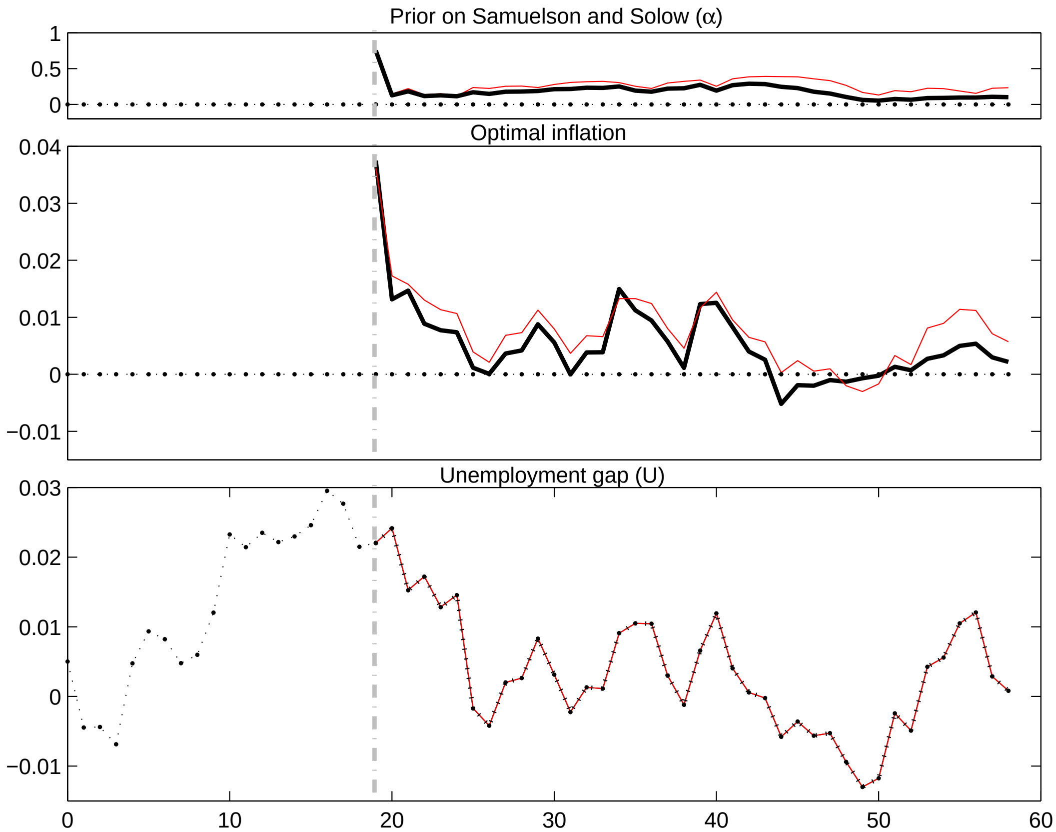



Figure 6: In the three panels: the dotted line represents the behavior of a central bank that attaches probability one to the Samuelson and Solow model, the bold continuous line is the experimenting central bank and the other line is the non-experimenting bank.

the true model, the initial prior, the initial unemployment rate, the median waiting time with and without experimentation and the 10%-90% confidence sets in square brackets. When a '+' appears next to a number it means that the waiting time exceeded the length of the simulated path. A number of things can be learned from this table. If we start with a 50-50 probability on the two models, the Samuelson and Solow model is easier to unveil. Absent unemployment, the experimenting central bank will learn the truth much faster than the anticipated utility bank compared to the cases in which we start from high unemployment. If we start by attaching a probability of almost one to the wrong model, it is easier to learn when Lucas's model is true than when Samuelson and Solow's is true.

| | | | Waiting Time | |
|---|---|---|---|---|
| True Model | $\alpha_0$ | $U_0$ | Experimentation | No Experimentation |
| SS | 0.01 | 0 | 247 [149,486] | 267 [156,500+] |
| SS | 0.01 | 0.025 | 272 [154,498] | 278 [171,500+] |
| Lucas | 0.99 | 0 | 97 [37,242] | 107 [40,244] |
| Lucas | 0.99 | 0.025 | 85 [21,213] | 87 [26,216] |
| SS | 0.5 | 0 | 39 [20,80] | 45 [23,92] |
| SS | 0.5 | 0.025 | 21 [6,54] | 25 [8,69] |
| Lucas | 0.5 | 0 | 87 [32,160+] | 93 [35,160+] |
| Lucas | 0.5 | 0.025 | 65 [14,160+] | 68 [18,160+] |
| SS | 0.28 | 0 | 52 [27,160+] | 65 [35,160+] |
| SS | 0.28 | 0.025 | 35 [15,142] | 46 [19,148] |
| Lucas | 0.28 | 0 | 80 [26,160+] | 90 [28,160+] |
| Lucas | 0.28 | 0.025 | 71 [20,160+] | 73 [22,160+] |

# 6 Concluding Remarks

The value functions and decision rules in figures 1 and 2 reveal that in our example, an anticipated utility model does a good job of approximating outcomes of a Bayesian model in which the monetary policy maker exploits the opportunity to experiment. While the passive learner in the anticipated utility does not design policies in order to experiment, there outcomes of his policies induce enough variation in the data that he is able to discriminate between the two models almost as fast as the Bayesian agent. This outcome is related to features in the environment identified by El-Gamal and Rangarajan (1993), who show how the presence of sufficient 'natural experiments' promotes learning.

Another interesting outcome is captured in the concavity of the decision rules in figure 2. This shape conveys that the decisions of a Samuelson-Solow style Keynesian are more robust to small doubts, i.e., perturbations of $\alpha$ away from 1, than are the decisions of Lucas-style classical economist to small perturbations of $\alpha$ away from 0. That lack of robustness of the classical recommendations to small doubts plays an essential feature in Cogley and Sargent's accounting for U.S. inflation policy during the 1970s.

Our calculations also reveal how long it takes to disabuse a doubtful monetary authority of the wrong model.

To evaluate the lessons of our results, it is important to assess the about reality that we impute to our policy maker to the doubts that one thinks should be in the minds of the policy maker. We have given the policy maker only two models, each of which he knows for sure. Of course, the models have very different operating characteristics. While their differences are important, they are not subtle, so this makes easier the task of generating or waiting for data to discriminate between them. In effect, we have assumed that the monetary authority's doubts are limited to ignorance of the 'correct' value of one hyperparameter, $\alpha$. If in practice one thinks that the monetary authority's doubts are broader and vaguer, we have substantially understated the difficulty of the decision and learning problem that it faces.

# 7 Appendixes

# A Transition Equation For $\alpha_t$

Let $\alpha_{i0} \equiv p(M_i)$ be the prior probability on model $i$, and let $p_i(s_i^t|\theta_i)$ represent its likelihood function. Here we abstract from parameter uncertainty by adopting the shortcut that the parameters $\theta_i$ are known. By Bayes's theorem, the posterior probability on model $i$ is

$$\alpha_{it} \equiv p(M_i|s_i^t, \theta_i) = \frac{p_i(s_i^t|\theta_i)p(M_i)}{\int p_i(s_i^t|\theta_i)p(M_i)dM_i}. \tag{20}$$

The numerator is an unnormalized model weight, which we label $w_{it}$, and the denominator is a normalizing constant that ensures that model probabilities sum to 1. With a finite collection of models, the denominator is just the sum of the unnormalized model weights, $\sum_i w_{it}$.

We start with a simple recursion for the unnormalized weights $w_{it}$. After taking logs and first-differencing, we find

$$\log w_{it} - \log w_{it-1} = \log p_i(s_i^t|\theta_i) - \log p_i(s_i^{t-1}|\theta_i). \tag{21}$$

Note that the prior model weight drops out of the recursion; $\alpha_{i0}$ initializes the sequence but the likelihood is all that matters for updates. Also notice that $\alpha$-updates depend only on the value of the likelihood at the given $\theta_i$. Usually the model probability updates would depend on a marginalized likelihood, but this drops out because we assume that $\theta_i$ is known. We need only to evaluate the likelihood, not marginalize across unknown parameters.

To simplify further, use the prediction error decomposition of the likelihood to write

$$\log p_i(s_i^t|\theta_i) = \sum_{s=1}^t \log p_i(s_{is}|s_i^{s-1}, \theta_i). \tag{22}$$

Subtracting the log-likelihood through $t-1$ from that through $t$, we get

$$\log w_{it} = \log w_{it-1} + \log p_i(s_{it}|s_i^{t-1}, \theta_i). \tag{23}$$

The date $t$ update depends on the value of the conditional log-likelihood. An observation that is likely given the model raises the unnormalized model weight, and a puzzling observation (for that model) lowers it. Notice that $\log w_{it}$ is a martingale if the model residuals are serially uncorrelated.

Now let's specialize to a two-model model. Let $\alpha_t$ be the normalized probability weight for model 1,

$$\alpha_t = \frac{w_{1t}}{w_{1t} + w_{2t}}. \tag{24}$$

The probability weight on model 2 is $1 - \alpha_t$.

The normalizing constant is a nuisance, so we eliminate it by taking the ratio,

$$R_t \equiv \frac{\alpha_t}{1 - \alpha_t} = \frac{w_{1t}}{w_{2t}}. \tag{25}$$

The transition equation for log$R_t$ follows from the transition equations for $\log w_{it}$,

$$\log R_t = \log R_{t-1} + \log \frac{p_1(s_{1t}|s_1^{t-1},\theta_1)}{p_2(s_{2t}|s_2^{t-1},\theta_2)}. \tag{26}$$

Thus, the updating rule for the log odds ratio depends only on the log-likelihood ratio for the two competing models. If we write this in terms of $\alpha_t$, we find

$$\frac{\alpha_t}{1-\alpha_t} = \frac{\alpha_{t-1}}{1-\alpha_{t-1}} \frac{p_1(s_{1t}|s_1^{t-1},\theta_1)}{p_2(s_{2t}|s_2^{t-1},\theta_2)}, \tag{27}$$

or

$$\alpha_t = \frac{\frac{\alpha_{t-1}}{1-\alpha_{t-1}} \frac{p_1(s_{1t}|s_1^{t-1},\theta_1)}{p_2(s_{2t}|s_2^{t-1},\theta_2)}}{1+\frac{\alpha_{t-1}}{1-\alpha_{t-1}} \frac{p_1(s_{1t}|s_1^{t-1},\theta_1)}{p_2(s_{2t}|s_2^{t-1},\theta_2)}}, \tag{28}$$

$$= \frac{\alpha_{t-1}p_1(s_{1t}|s_1^{t-1},\theta_1)}{\alpha_{t-1}p_1(s_{1t}|s_1^{t-1},\theta_1)+(1-\alpha_{t-1})p_2(s_{2t}|s_2^{t-1},\theta_2)}.$$

If the two models involve the same data, we can equate $s_{1t} = s_{2t}$. In that case,

$$\alpha_t = \frac{\alpha_{t-1}p_1(s_t|s^{t-1},\theta_1)}{\alpha_{t-1}p_1(s_t|s^{t-1},\theta_1)+(1-\alpha_{t-1})p_2(s_t|s^{t-1},\theta_2)}. \tag{29}$$

The right side of this equation spells out the function $B(\alpha_{t-1}, s_t)$.

## A.1 Martingale Property Of $\alpha_t$

The updating formula makes $\alpha_t$ a martingale from the point of view of the Bayesian agent (this is an example of Doob's martingale result for Bayesian updating). To see why, take the expectation of $\alpha_t$ with respect to the posterior at $\alpha_{t-1}$,

$$E_{t-1}B(\alpha_{t-1}, s_t) = \int B(\alpha_{t-1}, s_t) f_{t-1}(s_t|s^{t-1}) ds_t. \tag{30}$$

Because model parameters are assumed to be known, there is a single source of uncertainty about next period's $\alpha_t$, viz. what next period's $s_t$ will be. Therefore the expectation is taken with respect to the agent's posterior predictive density for $s_t$, which we denote $f_{t-1}(s_t|s^{t-1})$. This density is a probability weighted average of the predictive densities for the two models,

$$f_{t-1}(s_t|s^{t-1}) = \alpha_{t-1}p_1(s_t|s^{t-1},\theta_1) + (1-\alpha_{t-1})p_2(s_t|s^{t-1},\theta_2). \tag{31}$$

Thus, the conditional expectation for $\alpha_t$ is

$$
\begin{aligned}
E_{t-1}\alpha_t &= \int B(\alpha_{t-1}, s_t) \left[\alpha_{t-1}p_1(s_t|s^{t-1}, \theta_1) + (1-\alpha_{t-1})p_2(s_t|s^{t-1}, \theta_2)\right] ds_t, \\
&= \int \alpha_{t-1}p_1(s_t|s^{t-1}, \theta_1)ds_t, \\
&= \alpha_{t-1} \int p_1(s_t|s^{t-1}, \theta_1)ds_t = \alpha_{t-1}.
\end{aligned} \tag{32}
$$

## A.2 A Different State Space

To get a tractable Bellman equation, it is convenient to rewrite the problem so that the state transition equation is linear. Define:

$$\log R_t = \log \frac{\alpha_t}{1-\alpha_t}$$

then

$$\log R_{t+1} = \log R_t + \log \frac{f_1(s_{t+1}|s_t)}{f_2(s_{t+1}|s_t)}$$

$\alpha_t$ can be obtained back through the following expression

$$\alpha_t = \frac{1}{1 + (\exp \log R_t)^{-1}} \tag{33}$$

In the Bellman equation, we take expectations of functions that involve the log likelihood ratio. These expectations involve the distribution of $\varepsilon_{2,t+1}$ under model 1 and viceversa. We can represent those distributions by exploiting the assumption that $s_t$ is the same across models. This assumption means that the model innovations are related. After subtracting the transition equation for model 2 from that for model 1, we find:

$$C_2\epsilon_{2,t+1} = (A_1 - A_2)s_t + (B_1 - B_2)v_t + C_1\epsilon_{1,t+1} \tag{34}$$

$$C_1\epsilon_{1,t+1} = -(A_1 - A_2)s_t - (B_1 - B_2)v_t + C_2\epsilon_{2,t+1} \tag{35}$$

Define $\Omega_1 = C_1C_1', \Omega_2 = C_2C_2'$. We use (34) and (35) to write the recursion for $\log R_{t+1}$ under models 1 and 2. When model 1 is true, we have

$$
\begin{aligned}
\log R_{t+1} \;=\; & \log R_t - \frac{1}{2}\log|\Omega_1| + \frac{1}{2}\log|\Omega_2| - \frac{1}{2}\left(C_1\epsilon_{1,t+1}\right)' \Omega_1^{-1} \left(C_1\epsilon_{1,t+1}\right) \\
& + \frac{1}{2}\left[(A_1 - A_2)s_t + (B_1 - B_2)v_t + C_1\epsilon_{1,t+1}\right]' \\
& \times \Omega_2^{-1}[(A_1 - A_2)s_t + (B_1 - B_2)v_t + C_1\epsilon_{1,t+1}]
\end{aligned} \tag{36}
$$

When model 2 is true, we have

$$\begin{aligned}\log R_{t+1} &= \log R_t - \frac{1}{2}\log|\Omega_1| + \frac{1}{2}\log|\Omega_2| + \frac{1}{2}\left(C_2\epsilon_{2,t+1}\right)' \Omega_2^{-1}\left(C_2\epsilon_{2,t+1}\right) \\ &\quad -\frac{1}{2}[(A_2-A_1)s_t + (B_2-B_1)v_t + C_2\epsilon_{2,t+1}]' \\ &\quad \times\Omega_1^{-1}[(A_2-A_1)s_t + (B_2-B_1)v_t + C_2\epsilon_{2,t+1}] \end{aligned} \tag{37}$$

It is convenient to use $\alpha_t$ rather than $\log R_t$ as a state variable. So we want to transform (36) and (37) to get laws of motion for $\alpha_t$ under the two models. For the purpose of doing this, define

$$\begin{aligned}g(\epsilon_{1,t+1}; s_t, \alpha_t) &= \log R_t - \frac{1}{2}\log|\Omega_1| + \frac{1}{2}\log|\Omega_2| - \frac{1}{2}\left(C_1\epsilon_{1,t+1}\right)' \Omega_1^{-1}\left(C_1\epsilon_{1,t+1}\right) \\ &\quad +\frac{1}{2}\left[(A_1-A_2)s_t + (B_1-B_2)v_t + C_1\epsilon_{1,t+1}\right]' \\ &\quad \times\Omega_2^{-1}[(A_1-A_2)s_t + (B_1-B_2)v_t + C_1\epsilon_{1,t+1}] \end{aligned} \tag{38}$$

and

$$\begin{aligned}h(\epsilon_{2,t+1}; s_t, \alpha_t) &= \log R_t - \frac{1}{2}\log|\Omega_1| + \frac{1}{2}\log|\Omega_2| + \frac{1}{2}\left(C_2\epsilon_{2,t+1}\right)' \Omega_2^{-1}\left(C_2\epsilon_{2,t+1}\right) \\ &\quad -\frac{1}{2}[(A_2-A_1)s_t + (B_2-B_1)v_t + C_2\epsilon_{2,t+1}]' \\ &\quad \times\Omega_1^{-1}[(A_2-A_1)s_t + (B_2-B_1)v_t + C_2\epsilon_{2,t+1}] \end{aligned} \tag{39}$$

Using (33), we get a law of motion for $\alpha_{t+1}$ under the two models. Then our Bellman equation can be expressed

$$\begin{aligned}V(s_t,\alpha_t) &= \max_{v_t}\Bigg\{ r(s_t,v_t) + \beta\alpha_t \int V\left(A_1 s_t + B_1 v_t + C_1\epsilon_{1,t+1}, \frac{e^{g(\epsilon_{1,t+1})}}{1+e^{g(\epsilon_{1,t+1})}}\right) dF(\epsilon_{1,t+1}) \\ &\quad +\beta(1-\alpha_t)\int V\left(A_2 s_t + B_2 v_t + C_2\epsilon_{2,t+1}, \frac{e^{h(\epsilon_{2,t+1})}}{1+e^{h(\epsilon_{2,t+1})}}\right) dF(\epsilon_{2,t+1})\Bigg\}. \end{aligned} \tag{40}$$

# B Approximating the Bellman Equation

Discretize the support of $\alpha$ and $s$ into $I_\alpha$ and $I_s$ points respectively, to get $I = I_\alpha \cdot I_s$ nodes $(\alpha, s)_i$, $\forall i = 1, ..., I$. In what follows, we will refer to $\alpha_i$ and $s_i$ as the first and the second entry of $(\alpha, s)_i$ respectively. Specify $J$ known linearly independent basis functions $\phi_j\left((\alpha, s)_i\right)$, $j \in \{1, ..., J\}$. In our solution, we employ a third order complete polynomial, implying that $J = 10$. The goal is to find basis coefficients $c_j$, $j = 1, ..., J$ that best approximate the value function

$$V_i = V((\alpha, s)_i) \approx \sum_{j=1}^{J} c_j \phi_j ((\alpha, s)_i) = \sum_{j=1}^{J} c_j \phi_{j,i} \quad (41)$$

$\forall i = 1, ..., I$ or, in the equivalent matrix notation:

$$V \approx \Phi c$$

where $V$ is the $I \times 1$ vector of approximated value functions at each node, $\Phi$ is the $I \times J$ collocation matrix and $c = [c_1, ..., c_J]'$ is the vector of approximation coefficients. We also discretize the support of the two shocks in $K_1$ and $K_2$ points and denote $w_k$ the approximated probability mass associated to each of the resulting $K = K_1 \times K_2$ nodes. Using (41) in the Bellman equation we get for each node $i \in \{1, ..., I\}$:

$$\begin{aligned} V_i = \max_{v_i} \Bigg\{ & r_i(v_i) + \beta\alpha_i \sum_{k=1}^{K} \sum_{j=1}^{J} w_k c_j \phi_j \left( s'_{1,i,k}(v_i), \frac{\exp[g_{k,i}(v_i)]}{1 + \exp[g_{k,i}(v_i)]} \right) \\ & + \beta(1 - \alpha_i) \sum_{k=1}^{K} \sum_{j=1}^{J} w_k c_j \phi_j \left( s'_{2,i,k}(v_i), \frac{\exp[h_{k,i}(v_i)]}{1 + \exp[h_{k,i}(v_i)]} \right) \Bigg\} \end{aligned} \quad (42)$$

where

$$\begin{aligned} r_i(v_i) &= r(s_i, v_i) \\ s'_{1,i,k}(v_i) &= A_1 s_i + B_1 v_i + C_1 \varepsilon_k \\ x'_{2,i,k}(v_i) &= A_2 s_i + B_2 v_i + C_2 \varepsilon_k \end{aligned}$$

and $g_{k,i}(v_i)$ and $h_{k,i}(v_i)$ defined as in (38) and (39) respectively:

$$\begin{aligned} g_{k,i}(v_i) &= g(\varepsilon_k; s_i, \alpha_i, v_i) \\ h_{k,i}(v_i) &= h(\varepsilon_k; s_i, \alpha_i, v_i) \end{aligned}$$

We can now use the following algorithm to solve the Bellman equation recursively:

1. guess an initial vector of basis coefficients $c^1$
2. for each node $(s, \alpha)_i$ compute the right hand side of equation (42) using $c^1$ and call $v(c^1)$ the outcome
3. solve for $c^2 = (\Phi'\Phi)^{-1} \Phi' v(c^1)$
4. replace $c^1$ with $c^2$ and iterate until convergence.

# C The 'Don'T Experiment' Model

This appendix describes how to solve Bellman equation (12) by mapping the problem into what Cogley and Sargent (2004) called a 'Bayesian linear regulator'. Stack the two state space models from (1) as

$$\begin{bmatrix} s_{1,t+1} \\ s_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} v_t + \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \end{bmatrix} \tag{43}$$

or

$$s_{t+1} = As_t + Bv_t + C\epsilon_{t+1} \tag{44}$$

Let $\alpha \in (0, 1)$ be a fixed probability that the decision maker attaches to model 1. Express the time $t$ loss as $r(s_t, v_t) = -.5(s_t' R s_t + v_t' Q v_t)$. The decision maker seeks to maximize

$$L = -.5E \sum_{t=0}^{\infty} \beta^t \Big\{ \alpha s_{1t}' R s_{1t} + (1-\alpha) s_{2t}' R s_{2t} + v_t' Q v_t \Big\} \tag{45}$$

or

$$L = -.5E \sum_{t=0}^{\infty} \beta^t \left\{ s_t' \begin{bmatrix} \alpha R & 0 \\ 0 & (1-\alpha)R \end{bmatrix} s_t + v_t' Q v_t \right\} \tag{46}$$

Cogley and Sargent (2004) note that The problem of choosing a decision rule to maximize (46) with respect to (44) is an optimal linear regulator problem. The optimal decision rule is

$$v_t = -Fs_t = -F_1 s_{1t} - F_2 s_{2t}. \tag{47}$$

# D Description of the Empirical Specification

Here we briefly describe how the two policy models are estimated. Inflation is measured by the log difference of the chain-weighted GDP deflator, and unemployment is the civilian unemployment rate. Both series are seasonally adjusted and are sampled over the period 1948:1 to 1963:1. We stop the estimation there to represent the kind of model uncertainty that Federal Reserve officials would have faced in the years leading up to the Great Inflation.

Both Phillips curve specifications involve the gap between the unemployment rate and a time-varying natural rate of unemployment. In order to keep the size of the state space to a minimum, we approximate the natural rate of unemployment $U_t^*$ by exponentially smoothing the actual unemployment rate $UR_t$,

$$U_t^* = U_{t-1}^* + \mu(UR_t - U_{t-1}^*), \tag{48}$$

with a constant gain parameter $\mu = 0.075$. That makes the unemployment gap a geometrically distributed lag of past changes in unemployment,

$$U_t \equiv UR_t - U_t^* = \frac{(1-\mu)(1-L)}{1-(1-\mu)L} UR_t. \tag{49}$$

This procedure approximates a one-sided high-pass filter that transforms unemployment into the unemployment gap. The decomposition is shown in the following figure.

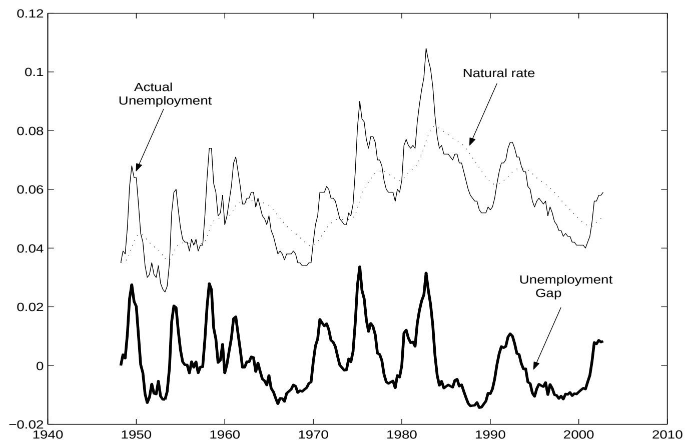



Figure 1: Decomposing Unemployment: The Natural Rate and the Gap

The blue line records actual unemployment, the red line depicts our proxy for the natural rate, and the green line is the unemployment gap, which is the variable that appears in the Phillips curves. This decomposition assigns most of the short-term variation in unemployment to the unemployment gap, and attributes long-term movements in the level to shifts in the natural rate. For the years over which we estimate the models, the natural rate increases only slightly, and most of the variation in $UR_t$ is in the gap measure $U_t$.

For model 1, this is all we need for estimation. We simply project the current unemployment gap onto a constant, current inflation, and one lag of gap, and estimate parameters by OLS. For the period 1948:3-1963:1, the least-squares point estimates and standard errors are as follows.

Table 1: Estimates of Model 1, 1948:3-1963:1

| | Intercept | $U_{t-1}$ | $\pi_t$ |
|---|---|---|---|
| $\hat{\beta}$ | 0.0023 | 0.7971 | -0.2761 |
| $\sigma_{\hat{\beta}}$ | 0.0010 | 0.0699 | 0.1189 |

In model 2, unemployment depends not on inflation but on unexpected inflation, $\pi_t - v_{t-1}$, so to estimate that model we also need a measure of expected inflation $v_{t-1}$. We construct that in the simplest way possible, by projecting current inflation on a constant along with one lag of inflation and unemployment. The fitted value from that regression is our measure of $v_{t-1}$, and the residual is our measure of unexpected inflation, $\pi_t - v_{t-1}$. Then we substitute that variable into the Phillips curve and estimate its parameters by least squares. The estimates and standard errors for model 2 are shown in the next table.

Table 2: Estimates of Model 2, 1948:3-1963:1

| | Intercept | $U_{t-1}$ | $\pi_t - v_{t-1}$ |
|---|---|---|---|
| $\hat{\beta}$ | 0.0007 | 0.8468 | -0.2489 |
| $\sigma_{\hat{\beta}}$ | 0.0008 | 0.0674 | 0.1298 |

We use the point estimates in these tables to calibrate the two policy models. Our central bank takes the point estimates as if they were known with certainty and formulates policy by averaging across the models. Thus, it takes account of model uncertainty, but suppresses parameter uncertainty.

## References


Beck, G. and V. Wieland, 2002, "Learning and Control in a Changing Environment," *Journal of Economic Dynamics and Control* 26, 1359-1377.

Blinder, A.S. 1998, *Central Banking in Theory and Practice*, Cambridge: MIT Press.

El-Gamal, Mahmoud A., and Rangarajan K. Sundaran, 1993, "Bayesian Economists . . . Bayesian Agents: An alternative approach to optimal learning," *Journal of Economic Dynamics and Control*, 17, 355-383.

Kasa, Kenneth, 1999, "Will the Fed Ever Learn?" *Journal of Macroeconomics.* 21(2): 279-92.

Kreps, David, 1998, "Anticipated Utility and Dynamic Choice," 1997 Schwartz Lecture, in *Frontiers of Research in Economic Theory*, Edited by Donald P. Jacobs, Ehud Kalai, and Morton Kamien, Cambridge University Press, Cambridge, England.

Kydland, F.E. and E.C. Prescott, 1977, "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy* 85, 473-91.

Lucas, R.E., Jr., 1972, "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4, 103-124.

Lucas, R.E., Jr., 1972, "Some International Evidence on Output-Inflation Trade-Offs," *American Economic Review* 63, 326-334.

Lucas, R.E., Jr., 1981, "Methods and Problems in Business Cycle Theory," in R.E. Lucas, Jr., (ed.) *Studies in Business-Cycle Theory*, Cambridge, Mass., MIT Press.

Samuelson, P.A., and R.M. Solow, 1960, "Analytical Aspects of Anti-Inflation Policy," *American Economic Review*, 50, 177–184.

Sargent, T.J., 1973, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment," *Brookings Papers on Economic Activity*, 429-472.

Stokey, N.L., 1989, "Reputation and Time Consistency," *American Economic Review Papers and Proceedings* 79, 134-139.

Wieland, V., 2000a, "Monetary Policy, Parameter Uncertainty, and Optimal Learning," *Journal of Monetary Economics* 46, 199-228.

Wieland, V., 2000b, "Learning by Doing and the Value of Optimal Experimentation," *Journal of Economic Dynamics and Control* 24, 501-534.