On-the-Job Search and the Cyclical Dynamics of the Labor Market[*]

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Abstract

We develop a dynamic general equilibrium model where workers can engage in search while on the job. We show that on-the-job search is a key component in explaining labor market dynamics in models of equilibrium unemployment. The model predicts fluctuations of unemployment, vacancies, and labor productivity whose relative magnitudes replicate the data. A standard search and matching model suggests much lower volatilities of these variables. Intuitively, in a boom, rising search activity on the job avoids excessive tightening of the labor market for expanding firms. This keeps wage pressures low, thus further increasing firms' incentives to post new jobs. Labor market tightness as measured by the vacancy-unemployment ratio is as volatile as in the data. The interaction between on-the-job search and job creation also generates a strong internal propagation mechanism.

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	Beveridge curve, business cycle, propagation

1 Introduction

Recent research by Hall (2003) and Shimer (2003) shows that search and matching models along the lines of Mortensen and Pissarides (1994) can explain the cyclical dynamics of the labor market only by assuming implausibly large productivity shocks. In particular, the standard framework underpredicts the volatility of vacancies and unemployment. Both Hall and Shimer explore real wage rigidity as a solution to this shortcoming. With this mechanism, firms' incentives to create new jobs in a boom are kept high since workers do not share the returns through bargaining. Hence, more vacancies are posted, and unemployment falls. This argument rests on the vacancy-unemployment ratio entering wages, reflecting workers' outside options. Thus, when wages are not rigid, but continuously renegotiated, they are excessively volatile.

We argue in this paper that on-the-job search offers an alternative resolution to this puzzle. In a boom, rising search activity by employed workers expands the pool of potential hires for firms, in addition to those searching from unemployment. As a consequence, the bargaining power of incumbent and newly hired workers rises by much less than would be suggested by the standard vacancy-unemployment ratio. Wages are endogenously rigid in the presence of on-the-job search.

To quantitatively assess this argument, we develop a dynamic general equilibrium model with labor market frictions and search by employed and unemployed workers. Search on the job is motivated in a straightforward manner by the presence of two types of jobs, which differ in terms of profitability and thus wages. Workers in low-wage ('bad') jobs search in order to gain employment in high-wage ('good') jobs. Good job vacancies can be matched with employed and unemployed job seekers, whereas firms in the bad job sector only hire unemployed workers. Wages are determined by Nash bargaining for each matched job-worker unit and continuously renegotiated. We calibrate the model to match salient long-run features of job and worker flows.

We find that our model correctly predicts the observed volatility of the vacancy-unemployment ratio. At the same time, the ratio of vacancies to unemployed *and* employed job seekers is substantially less volatile. Employed workers' search activity responds strongly to a positive aggregate shock to take advantage of the increased availability of good employment opportunities. Job-to-job flows increase substantially. But as search on the job rises, and wage increases are muted, the incentive to create vacancies remains large, especially for good jobs. The corresponding fall in unemployment is also large. This is achieved even

though productivity shocks are of plausible magnitude and wages are, a priori, fully flexible. Moreover, on-the-job search yields a powerful internal propagation mechanism in that small aggregate impulses engender large and long-lasting responses of output and employment.

Important for the ability of the model to match the data is the interaction of two features: the endogeneity of on-the-job search and the heterogeneity of jobs. The former amplifies the incentives to create good jobs in a boom, since the likelihood of filling a vacancy remains large, in spite of falling unemployment. The latter, that is, the increasing availability of good vacancies, raises employed workers' search effort. Without either element, the response of job-to-job transitions and the propagation of shocks on output would be much weaker. This complementarity explains the prolonged effect of shocks. Furthermore, not only are more new jobs created, but the job composition shifts towards more productive jobs, which raises aggregate output.

The model's implications are in line with other empirical regularities on worker flows emphasized in the literature. It features a form of vacancy chain, since job-to-job quits induce creation of bad job vacancies (Akerlof, Rose, and Yellen, 1988, and Contini and Revelli, 1997). Thus, hiring into new jobs and replacement hiring are strongly procyclical.[1] Furthermore, as argued by Okun (1973), booms are associated with a larger supply of good jobs. Search on the job facilitates the reallocation of workers from bad to good jobs, and therefore the creation of good jobs in a boom, a point also stressed by Mortensen (1994) and Mortensen and Pissarides (1999). Finally, workers that have been employed for a long time have lower quit rates, since they are more likely to have made the transition to a good job.[2]

To our knowledge, this is the first analysis of on-the-job search in a general equilibrium business cycle model with equilibrium unemployment. The closest precursors are the contributions by Pissarides (1994) and Mortensen (1994). The former studies a deterministic, continuous-time model and qualitatively discusses possible adjustment dynamics. It shares the heterogeneity in job types employed in this paper. The latter conducts a simulation of a stochastic version of the Mortensen and Pissarides (1994) model. Mortensen shows that on-the-job search helps explain the negative correlation between job creation and destruction rates. In both papers, employed search varies through adjustments in the number of searchers, rather than the intensity of search. Finally, the two papers have exogenous interest rates and prices, shutting down important dynamic general equilibrium effects, which affect the dynamics of vacancies and unemployment. Neither of the papers considers these

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[1] See also Albaek and Sorensen (1998) for some direct evidence.
[2] See, for example, Pissarides (1994) for an overview of the relevant empirical findings.

dynamics quantitatively.

The paper proceeds as follows. The next section gives a brief discussion of the relevant evidence on the dynamic behavior of the labor market, in particular the quit rate. Section 3 lays out the model and characterizes the steady state. Section 4 gives the calibration details. The results of the dynamic simulation of the model are presented in section 5, while section 6 contains further discussion and relates the findings to the literature. Section 7 concludes. The log-linearized model and remarks on the solution procedure can be found in the Appendix.

2 Empirical Evidence

This section documents the cyclical behavior of vacancies, unemployment, and labor market tightness for the U.S. labor market and their relation to productivity, output, employment, and real wages. While we use labor market data from 1948 until 2003, some other series cover only a shorter period. In particular, the time series on average hourly earnings which is only available from 1964 on and which we use as our measure of the real wage (deflated by the CPI). All series are available from the website of the U.S. Bureau of Labor Statistics (www.bls.gov), except the series on quits, which has been compiled from the Employment and Earnings publication of the BLS. This series, however, is only available up to 1982, when it was discontinued. Vacancies are constructed from the BLS index of help-wanted advertisements. All variables are quarterly and, where appropriate, detrended using the HP-filter, with the smoothing parameter set to 1600.

The dynamics of vacancies and unemployment follow a familiar pattern. Figure 1 shows vacancies that are highly procyclical whereas unemployment is strongly countercyclical; that is, the two variables exhibit a Beveridge curve with a contemporaneous correlation of −0.95. This pattern implies that a measure of labor market tightness, the vacancy-unemployment ratio, is also highly procyclical. Table 1 presents the standard deviations and cross-correlations of the variables of interest. Real wages are procyclical, the degree of which depends on the time period considered.[3] Particularly the 1970s feature a highly procyclical real wage, while from the 1980s on it appears almost acyclical. In fact, for the full sample, the correlation between output and real wages is 0.57, whereas from 1982 onward it is merely 0.26. For consistency with the theoretical model, we take output per worker as a measure of labor productivity, which has a correlation with output of 0.69.[4]

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[3]These results are not reported, but available from the authors.
[4]Output per hour has a correlation of 0.54 with output.

One of the central variables for the argument considered in this paper is the rate of job-to-job mobility and quits, which we consider to be the outcome of on-the-job search activity. However, there is no direct evidence on the cyclical behavior of on-the-job search that we are aware of, and we have to rely upon somewhat indirect evidence. Two data sets have become available recently, but they only cover relatively short periods of time. The Job Openings and Labour Turnover Survey (JOLTS) by the U.S. Bureau of Labor Statistics was begun in December 2000. This period essentially covers only one mild downturn. Since 1994, the Current Population Survey uses a "dependent interviewing" technique which allows contruction of detailed worker flow series. This series thus comprises the protracted boom of the 1990s as well as the subsequent downturn. This dataset does not allow us, however, to infer unconditional time series properties of the data, but it is at least useful in providing long-run averages.

The longest time series on worker mobility and quits is contained in the BLS labor turnover series for the manufacturing sector from 1926 to 1981, which we use from 1948 on. We follow Blanchard and Diamond (1989) by making two adjustments based more recent numbers. First, quit rates in manufacturing tend to be lower than in the entire economy and therefore need to be adjusted upwards. We use Fallick and Fleischman's (2004) finding based on the CPS data. They find an economy-wide average monthly quit rate of 2.6%. Some caution may be mandated since the data cover only one upswing and one mild downturn. A long-run average which includes a severe contraction might yield somewhat lower rates. Secondly, not all quits are job-to-job flows. Fallick and Fleischman (2004) suggest that job-to-job quits are about half of total quits, while Blanchard and Diamond (1989) postulate 40 percent.

The standard deviation of the adjusted quit series can be found Table 1, based on the sample up to the end of 1981. It is worth noting that the quit rate is eight times as volatile as GDP and about 50 percent more volatile than unemployment.[5] Figure 1 shows that the quit rate appears to comove with the vacancy index, especially between about 1955 and 1975. In fact, the detrended series of vacancies and the quit rate for the whole period have a correlation of 0.94. It appears quite unlikely that the volatilities and short-run relationships between quits, unemployment, and vacancies have significantly changed since 1982. Increasing availability of data from the CPS and JOLTS will allow to be more precise.

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[5] See Petrongolo and Pissarides (2000) for evidence on the relative magnitudes of different quit flows.

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3 A Business Cycle Model with On-the-Job Search

Time is discrete and infinite, and the economy is populated by a representative household, homogeneous workers and heterogeneous firms. The key elements of the model are the heterogeneity of jobs and the endogenously chosen search intensity by employed workers. There are two types of firms, labeled 'good' and 'bad', which differ according to the costs of creating new jobs. In this respect, the model is similar to Pissarides (1994) and Acemoglu (2001).[6] In the presence of labor market frictions, these costs generate rents which give rise to equilibrium wage differentials between job types. The implied differences in the value of employment motivate workers in low-wage jobs to search for employment in high-wage jobs. All workers in low-wage jobs search on the job, but the intensity of their search depends on labor market conditions, in particular, the likelihood of finding a good job and the differentials in the returns to working. Workers direct their search to either good jobs or bad jobs, so employed workers approach only good vacancies, while unemployed workers choose between either job type. Workers in good jobs have no incentive to search as it is costly and does not offer any improvements over their current returns to employment. We first turn to the nature of the product and labor markets, then discuss the optimization problems faced by firms, workers, and the aggregate household.

3.1 Firms and Product Markets

First consider the different job, or firm, types. The cost of creating a job is represented by a flow cost of posting a vacancy, c^g for good firms, and c^b for bad firms, where $c^g > c^b$. Production of a (representative) good firm is given by:

$$y_{gt} = A_t n_t^g, \tag{1}$$

where A_t is aggregate productivity and n_t^g is employment at good firms. Analogously, bad firms produce according to:

$$y_{bt} = A_t n_t^b. \tag{2}$$

Output of good and bad firms is combined in a final goods sector according to a CES aggregator:

$$y_t = (\alpha y_{bt}^\gamma + (1-\alpha)y_{gt}^\gamma)^{1/\gamma} \tag{3}$$

with $1 > \gamma \neq 0$, and the corresponding Cobb-Douglas production function for $\gamma = 0$. The two intermediate goods, y_{gt} and y_{bt}, are sold at competitively determined prices, P_{gt} and

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[6]Krause and Lubik (2004) explore the business cycle properties of a model without on-the-job search that utilizes the two-sector structure in a similar way.

P_{bt}:

$$P_{gt} = (1 - \alpha) \left(\frac{y_{gt}}{y_t} \right)^{-(1-\gamma)}, \qquad (4)$$

$$P_{bt} = \alpha \left(\frac{y_{bt}}{y_t} \right)^{-(1-\gamma)}, \qquad (5)$$

where we have chosen the price of aggregate output as the numeraire. A similar product market structure is used by Acemoglu (2001). It can be interpreted as representing differences across industries or differences across firms within industries.[7]

3.2 The Labor Market

The process of matching workers and firms is subject to frictions, represented by a matching function, which gives the number of per period matches of job searchers and vacancies. Let the matching function be constant returns to scale and homogeneous of degree one.[8] Both high-wage and low-wage firms post vacancies, while employed and unemployed workers search for jobs. All workers in bad jobs search on the job and choose the intensity of search. Unemployed workers either search for good jobs or for bad jobs, depending on the relative returns. The total number of matches between good vacancies and searching workers is given by:

$$m_t^g = m(v_t^g, u_t^g + e_t), \qquad (6)$$

where v_t^g is the measure of good job vacancies, while the measure of unemployed workers looking for good jobs is u_t^g. e_t is the measure of efficiency units of search of employed job seekers, that is, $e_t = s_t n_t^b$ gives the total amount of search activity by the n_t^b workers searching with intensity s_t. All workers in bad jobs engage in search. Correspondingly, the number of matches between bad jobs and unemployed workers is:

$$m_t^b = m(v_t^b, u_t^b). \qquad (7)$$

Note that unemployed workers search in distinct pools for jobs, and have to make an ex-ante decision as to which sector they devote their search effort to. Worker mobility implies that, in equilibrium, the returns to search in either sector have to be equal each period.[9]

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[7]Evidence by Parent (2000), among others, indicates that a large fraction of job-to-job transitions are within industries. This is suggestive of intra-industry differences of jobs motivating worker mobility. Additional evidence comes from Albaek and Sorensen (1998), who find that flows of workers in upturns typically are from small firms to large firms.

[8]This assumption is usually based on empirical findings, such as those by Blanchard and Diamond (1990). Note, however, that these estimates ignore the presence of job-to-job flows. For a thorough discussion of the biases this may lead to, see Petrongolo and Pissarides (2001).

[9]Alternatives to the directed search assumption are discussed below.

The probabilities of finding a match for the participants in the matching market are as follows. Good and bad vacancies are filled with the respective probabilities

$$q_t^g \equiv \frac{m_t^g}{v_t^g} = m\left(1, \frac{1}{\theta_t^g}\right) \quad \text{and} \quad q_t^b \equiv \frac{m_t^b}{v_t^b} = m\left(1, \frac{1}{\theta_t^b}\right),\tag{8}$$

where $\theta_t^g = v_t^g/(u_t^g + e_t)$ and $\theta_t^b = v_t^b/u_t^b$ are measures of labor market tightness in the matching markets for good jobs and bad jobs, respectively. Match probabilities for firms fall *ceteris paribus* with the number of vacancies posted, and rise with the number of job seekers. Unemployed job seekers are assumed to search with fixed search intensity (equal to one). Conseqeuently, the probabilities of finding a good or bad vacancy are given by:

$$p_t^g = \frac{m_t^g}{u_t^g + e_t} = m(\theta_t^g, 1) \quad \text{and} \quad p_t^b = \frac{m_t^b}{u_t^b} = m\left(\theta_t^b, 1\right).\tag{9}$$

Note that employed job seekers and unemployed job seekers cause congestion for each other in the market for good jobs.[10] Employed job seekers choose the intensity of their search, denoted by s_t, taking the aggregate probability of finding a good job as given. Thus, an employed worker's probability of being matched is $s_t p_t^g$.

The evolution of employment in good and bad jobs is described by the equations:

$$\begin{aligned} n_{t+1}^g &= (1-\rho)[n_t^g + m_t^g],\tag{10} \\ n_{t+1}^b &= (1-\rho)[n_t^b + m_t^b - s_t p_t^g n_t^b], \end{aligned}$$

where ρ is the probability of matches breaking up, which is exogenous and identical for both types of jobs. It comprises both job destruction events and separations of workers for reasons other than quits to another employer. The last term in the second equation can also be expressed as $s_t p_t^g n_t^b = \frac{e_t}{u_t + e_t} m_t^g$, that is, as the fraction of new good matches with employed searchers.

3.3   Wages and Search Intensity

A worker and a firm split the joint surplus that their match generates. The size of each party's share is determined by the Nash bargaining solution, depending on their relative bargaining powers. Wages are determined by taking the search intensity of workers as given, while search intensity itself is chosen by workers taking as given the current wage.[11]

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[10] This observation is consistent with empirical evidence, for example Burgess (1995), but also the discussion in Petrongolo and Pissarides (2001). In Pissarides' (1994) model with on-the-job search, workers cannot direct their search and are randomly matched across good and bad vacancies.

[11] Note that since contracts are renegotiated at each point in time, firms cannot reduce quits by promising higher wages. We discuss this point further below.

We begin by specifying the asset values for workers and firms. The asset value of a good firm with a job filled with a worker is given by the Bellman equation:

$$J_t^g = P_{gt}A_t - w_t^g + E_t\beta_t\left[(1-\rho)J_{t+1}^g + \rho V_{t+1}^g\right]. \tag{11}$$

w_t^g is the wage paid to the worker, E_t the expectation operator conditional on the information set at time t, and β_t the discount factor, to be determined further below. Jobs survive into the next period with probability $(1-\rho)$, and become vacant otherwise. The value V_t^g of a vacancy for good jobs is:

$$V_t^g = -c^g + E_t\beta_t\left[(1-\rho)q_t^g J_{t+1}^g + (1-(1-\rho)q_t^g)V_{t+1}^g\right]. \tag{12}$$

With probability $(1-\rho)q_t^g$ the vacancy is filled and survives the separation shock. The corresponding equations for bad jobs are:

$$
\begin{aligned}
J_t^b &= P_{bt}A_t - w_t^b \\
&\quad + E_t\beta_t\left[(1-\rho)(1-s_tp_t^g)J_{t+1}^b + (\rho + (1-\rho)s_tp_t^g)V_{t+1}^b\right],
\end{aligned}
\tag{13}
$$

and:

$$V_t^b = -c^b + E_t\beta_t\left[(1-\rho)q_t^b J_{t+1}^b + (1-(1-\rho)q_t^b)V_{t+1}^b\right]. \tag{14}$$

Note that $s_tp_t^g$ reduces the likelihood of a bad job remaining matched in the next period.

Free entry implies that the values of good and bad vacancies are driven to zero at any point in time, such that $V_t^g = V_t^b = 0$. With these conditions, solving the asset equations for vacancies yields two job creation conditions:

$$
\begin{aligned}
\frac{c^g}{q_t^g} &= (1-\rho)E_t\beta_t J_{t+1}^g = (1-\rho)E_t\beta_t\left[P_{gt+1}A_{t+1} - w_{t+1}^g + \frac{c^g}{q_{t+1}^g}\right], \tag{15} \\
\frac{c^b}{q_t^b} &= (1-\rho)E_t\beta_t J_{t+1}^b = (1-\rho)E_t\beta_t\left[P_{bt+1}A_{t+1} - w_{t+1}^b + (1-s_{t+1}p_{t+1}^g)\frac{c^b}{q_{t+1}^b}\right].
\end{aligned}
$$

The equations relate the expected cost of a posted vacancy to the expected benefit of a filled job. If, for example, the left-hand side of either equation were smaller than the right hand side, entry is profitable, so that the number of vacancies posted increases. This leads to a fall in the probability of finding a worker q_t, which depends on labor market tightness, until no ex-ante profits from posting vacancies remain.

For workers, the asset values of employment in good jobs and employment in bad jobs are:

$$W_t^g = w_t^g + E_t\beta_t\left[(1-\rho)W_{t+1}^g + \rho U_{t+1}\right], \tag{16}$$

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$$W_t^b = w_t^b - k(s_t) + E_t\beta_t \left[(1-\rho)(1-s_tp_t^g)W_{t+1}^b + (1-\rho)s_tp_t^gW_{t+1}^g + \rho U_{t+1}\right].$$

$k(s_t)$ denotes the strictly convex cost of search in terms of intensity s_t, with $k(0) = 0$, $k' > 0$, and $k'' > 0$. The higher the search intensity, the more likely the worker is matched with a good job. Convexity of the effort function guarantees uniqueness of the optimal search effort. For $s_t = 0$, the worker either stays on the job or returns to unemployment after an exogenous separation. Note that the worker enjoys the value of a good job only if that job survives into the next period, with probability $1 - \rho$.

The Nash bargaining solution divides the surplus of a match between the two parties at each point in time. Depending on the type of job, the surplus for workers is $W_t^i - U_t$ and for firms J_t^i, $i = g, b$.[12] Denoting the total surplus of a match by $S_t^i = J_t^i + W_t^i - U_t$, the wage has to be such that workers obtain a share $W_t^i - U_t = \eta S_t^i$, with the bargaining weight $0 < \eta < 1$. Firms receive the remainder $J_t^i = (1-\eta)S_t^i$.

The assumption that unemployed workers direct their search activity between the two types of jobs yields a restriction on the costs of job creation and the match probabilities. To see this, note that search for good jobs has the value:

$$U_t^g = z + E_t\beta_t[p_t^g(1-\rho)W_{t+1}^g + (1 - p_t^g(1-\rho))U_{t+1}^g], \tag{17}$$

while for bad jobs:

$$U_t^b = z + E_t\beta_t[p_t^b(1-\rho)W_{t+1}^b + (1 - p_t^b(1-\rho))U_{t+1}^b]. \tag{18}$$

In equilibrium, arbitrage by workers between sectors implies that $U_t^g = U_t^b = U_t$, for all t. Setting the two equations equal, and using the bargaining equations results in $p_t^g(1-\rho)E_t\beta_tJ_{t+1}^g = p_t^b(1-\rho)E_t\beta_tJ_{t+1}^b$. Inserting the job creation condition gives:

$$p_t^g\frac{c^g}{q_t^g} = p_t^b\frac{c^b}{q_t^b} \qquad \Longleftrightarrow \qquad \theta_t^g c^g = \theta_t^b c^b \tag{19}$$

from the definitions of the match probabilities. The measures of labor market tightness for both types of jobs are exactly proportional to the relative costs of job creation.

To derive expressions for wages, consider first bargaining in good jobs, and write $(1-\eta)(W_t^g - U_t) = \eta J_t^g$. Insert the respective equations, and solve for the wage, then collect terms for the surplus of workers at time $t + 1$, and use the bargaining equation to replace these terms with the expressions for J_{t+1}^g or J_{t+1}^b. Finally, use the job creation conditions and simplify. This yields the wage equation for good jobs:

$$w_t^g = \eta P_{gt}A_t + (1-\eta)z + \eta p_t^g\frac{c^g}{q_t^g}. \tag{20}$$

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[12]Recall that $V_t^i = 0$, from the free entry condition.

The wage is a function of the flow return to production and the outside option of the worker. Intuitively, the last term reflects the worker's labor market prospects, should negotiations break down. The worker obtains a fraction $(1-\eta)$ of his share $\eta/(1-\eta)$ of the value c^g/q_t^g of an alternative good job. Such a job is found with probability p_t^g. The last term can also be written as $\eta p_t^g c^g/q_t^g = \eta \theta_t^g c^g$, with $\theta_t^g = v_t^g/(u_t^g + e_t)$. The outside option of the worker not only depends on the number of unemployed workers searching for the same type of job, but also on the number of employed job seekers. Note at this point that there is no recall. That is, wages in previous jobs are not part of the outside options of a worker.

The wage equation for bad jobs is found in an analogous manner, using $(1-\eta)(W_t^b - U_t) = \eta J_t^b$. It contains additional terms that reflect the presence of on-the-job search:

$$w_t^b = \eta P_{bt} A_t + (1-\eta)(z + k(s_t)) + \eta \left((1 - s_t)p_t^g \frac{c^g}{q_t^g} \right), \tag{21}$$

making use of equation (19).[13] On the one hand, searching workers incur the search cost $k(s_t)$, which reduces their surplus from the match, relative to the situation without search. This increases the wage that firms need to pay the worker. On the other hand, the increased likelihood of leaving the firm requires workers to accept a lower wage as a compensating differential for firms. Another way to look at this is to realize that search on the job is undertaken only if it raises total match value, as search gives workers the option to earn more in the future, while the firm faces the additional risk of separation. Through bargaining, firms obtain part of that option value.

Search intensity is chosen by the worker taking the wage as given, on the ground that firms cannot directly observe the search effort of workers. However, firms anticipate the optimal choice that workers will make in equilibrium. The optimal search intensity is found by maximizing the asset value of employment in a bad job with respect to individual search intensity, s_t. That is:

$$W_t^b = \max_{s_t} \left(w_t^b - k(s_t) + E_t\beta_t \left[(1-\rho)(1 - s_t p_t^g)W_{t+1}^b + (1-\rho)s_t p_t^g W_{t+1}^g + \rho U_{t+1} \right] \right). \tag{22}$$

Maximization yields:

$$k'(s_t) = \frac{\eta}{1-\eta} p_t^g \left(\frac{c^g}{q_t^g} - \frac{c^b}{q_t^b} \right). \tag{23}$$

Search intensity rises with the probability of finding a good job, with the value of good jobs, and falls with the value of bad jobs. If $c^g/q_t^g \le c^b/q_t^b$ no search would take place on

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[13]The last term is a simplification of $\eta \left(p_t^b c^b/q_t^b - s_t p_t^g c^g/q_t^g \right)$, showing that the wage in bad jobs depends in a familiar way on the prospects of finding a similar job again, minus the expected value of finding a good job.

bad jobs. The factor $\eta/(1-\eta)$ reflects the fact that workers only obtain a share of the total value of a job.

The reader might expect to see a role of the wage for reducing the likelihood of workers quitting. There is no such effect because of the timing structure of the model and the nature of Nash bargaining. Wages are continuously renegotiated so that currently paid wages have no implications for wages paid next period, which will be newly negotiated. But next period's payments are what motitvates worker search this period. If firms could commit to wages for more than a period, then adjusting today's wage would have an effect on search intensity and thus quitting. However, we do observe in the model a negative correlation between wages and the likelihood of quitting. This arises from the incentive to search on low wage jobs, not because firms raise wages to maximize profits by reducing turnover.[14]

3.4   Closing the Model

Households choose consumption to maximize lifetime utility. Each household is endowed with a unit of labor which is supplied inelastically to the labor market. The optimization problem of a representative household is:

$$\max_{\{C_t\}_{t=0}^{\infty}} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\tau} - 1}{1 - \tau} + (1 - \chi_t)z - \chi_t h \right], \tag{24}$$

subject to

$$c_t = y_t^l + \pi_t, \tag{25}$$

where c_t is consumption, y_t^l is income earned from providing labor services to firms in good and bad sectors, and π_t is residual profits from the firms. Labor is supplied inelastically, with a disutility of h suffered if the agents works ($\chi_t = 1$) and a value of leisure or household production z, enjoyed if unemployed. We normalize $h = 0$ since this choice does not affect the search and matching process. $0 < \beta < 1$ is the household's discount factor, and $\tau > 0$ is the inverse of the intertemporal elasticity of substitution.

We assume perfect risk-sharing among the households and a complete asset market.[15] This implies that firms use the household's intertemporal rate of substitution to evaluate their profit streams. Using the household's first order condition for consumption, the utility-

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[14]This logic applies however to models of wage posting, such as Burdett and Mortensen (1998).

[15]To avoid complications from heterogeneity in workers' asset positions, we follow Merz (1995) and Andolfatto (1996) in assuming a large number of members of families which perfectly insure each other against fluctuations in income. This is a commonly used device in business cycle models with (possibly) heterogeneous agents.

based and time-varying discount factor used by firms and workers is given by:

$$\beta_t = \beta \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}. \tag{26}$$

In the absence of capital or, for instance, government borrowing, households cannot engage in consumption smoothing in the usual manner. However, the intertemporal trade-off inherent in the creation of vacancies allows substitution of consumption over time through increases in employment. We will discuss this notion in more detail below.

In equilibrium, the income that acrues to the household is:

$$y_t^l + \pi_t = y_t - c^g v_t^g - c^b v_t^b, \tag{27}$$

where the resources lost by posting vacancies are subtracted. Since sectoral production technologies as well as the aggregator function are constant returns to scale, residual profits are identicaly equal to zero. Income therefore consists of wage payments to employed workers. The equations describing the model economy are collected in the Appendix.

4   Calibration and Model Solution

We now proceed by computing the non-stochastic steady state around which the equation system is then linearized. The resulting linear rational expectations model is solved by the method described in Sims (2002). We assign numerical values to the structural parameters in order to conduct a quantitative analysis. Since pertinent information may not be available for some parameters, we compute these indirectly from the steady-state values of quantifiable endogenous variables. The calibration is somewhat more difficult than in models without on the job search, as aggregate statistics can not easily be matched with corresponding model statistics. In what follows we describe our benchmark parameterization, which we then modify in subsequent sections. The calibration is summarized in Table 2.

We start with the separation rate and set $\rho = 0.1$. This value covers both exogenous job destruction as well as quits into unemployment or movements out of the labor force. The unemployment rate is set to 12%, i.e., $u = 0.12$. The corresponding mass of workers participating in the production process is given by $n = 1 - u$. The unemployment rate is chosen higher than that commonly observed in the data to take into account workers that are only loosely attached to the labor force, such as discouraged workers or workers engaging in home production. Once the opportunity arises, these (potential) workers participate in

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the matching market.[16]

We calibrate the steady-state job-to-job transition rate as 0.06 which corresponds in our modeling framework to the variable ep_g/n, the number of workers in bad jobs who move on to good jobs relative to total employment. For the matching function itself, we choose a Cobb-Douglas functional form that is identical in both sectors with elasticity parameter $\mu = 0.4$, so that $m_g = M_g v_g^{1-\mu} (u_g + e)^\mu$ and $m_b = M_b v_g^{1-\mu} u_b^\mu$.[17] The level parameters M_g, M_b are chosen to imply an average firm matching probability of 0.7, which is a commonly used value in the literature. This leads to $M_g = 0.6$ and $M_b = 0.6$. The implied steady state sectoral matching rates, that is, the probability that a firm in the good or bad sector finds an employee, are, respectively, 0.77 and 0.63.

Existence of a high wage ('good') sector rests on the assumption that $c_g > c_b$. We assume that job creation costs for good firms are three times as large as for bad firms and set $c_b = 0.2$. The aggregator function of sectoral production into economy-wide output is of the CES-type. For simplicity, we choose $\gamma = 0$. Furthermore, impose that prices are about equal, and that wages are higher in the good job sector than in the bad job sector. This implies a weight of $\alpha = 0.4$ on production from bad jobs. It can be interpreted as a productivity differential.

The costs of searching on the job are assumed to be strictly increasing and convex in the search intensity. We use $k(s) = \kappa s^\sigma$, where $\kappa > 0$, $\sigma > 1$. In our benchmark calibration we choose $\sigma = 1.1$. However, this is one of our main parameters of interest and we will present and discuss the implications of variations in the search elasticity. A value close to one appears most plausible for reasons discussed below. The scale parameter κ is not chosen independently, but is computed implicitly to be consistent with the calibrated steady state. We find $\kappa = 0.04$.[18]

The parameters describing the household are standard. We choose a coefficient of relative risk aversion $\tau = 1$, and a discount factor $\beta = 0.98$. The worker's share in the surplus of the match is $\eta = 0.5$. There is no independent information available on the utility value of household production z. Reverse calibration of the unemployment rate, however, implies that $z = 0.39$.

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[16] This argument follows Blanchard and Diamond (1990).

[17] It needs to be pointed out that empirical estimates of this elasticity parameter are biased if there is on-the-job search (see Petrongolo and Pissarides, 2001, for the estimation). We are aware of no empirical study of the matching function that takes on-the-job search into account.

[18] Incidentally, this also implies a steady state value for the search intensity of employed job seekers of $s = 0.26$. Mortensen (1994) reports that their search intensity is 1/5 of that of unemployed seekers (which we assume to be one).

Finally, we need to calibrate the shock process. The (logarithm of the) aggregate productivity shock is assumed to follow an AR(1) process with coefficient $\rho_A = 0.90$. As is common in the literature we choose an innovation variance such that the baseline model's predictions match the standard deviation of U.S. GDP, which is 1.62%. While this is not a robust procedure, it is not essential for our approach since we do not evaluate the model along this dimension. What matters are the relative volatilities of the variables of interest. Consequently, the standard deviation of technology is set to $\sigma_\varepsilon = 0.0049$.

5   Model Analysis

This section reports the main findings of our benchmark model. To recapitulate, we have developed a two-sector labor market model of search and matching, where workers can engage in on-the-job search. First, we discuss the implications of on-the-job search for the model's steady state. Secondly, we report impulse responses to technology shocks and discuss their robustness. Finally, summary statistics from the data are compared with the corresponding statistics from the simulated model.

5.1   Steady State Implications

In the non-stochastic steady state equilibrium, about 30% of jobs are bad, and search intensity is about one third. In other words, 10% of the labor force are effectively searching on the job. There is a relatively low number of unemployed workers looking for good jobs (1.3%), while the remainder of the unemployed (10.7%) search for bad jobs. Note that this is an endogenous response of the unemployed to the competition for good jobs that face with employed seekers. The measure of vacancies is expressed in relation to the labor force and is 7.5 percent for good jobs, and 15.6 percent for bad jobs. Remember that the labor force is normalized to one. The resulting probabilities to be matched within the following quarter is for firms 0.75 (good vacancies) and 0.57 (bad vacancies), while for workers, the probabilities to be matched with a good job are 0.43 and for a bad job 0.67. In line with intuition, the *queue* of workers looking for good employment is longer, in the sense that match probabilities are lower. The flow of new good matches per period is 0.057 and for new bad matches is 0.092. The larger amount of bad matches reflects the fact that the workers flowing from bad to good jobs are being replaced at the industry level.[19] Finally, note that wages for good jobs are slightly higher than for bad jobs, the difference is about 4

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[19] The flows in the bad job sector can be interpreted as either reflecting replacement hiring at the firm level, or as job destruction in some firms, while others expand, holding total industry employment steady.

percent. The wage differerence is not essential though for on-the-job search. What matters is the difference between the asset values of employment in the two sectors.

5.2 Impulse Response Analysis

The importance of on-the-job search for the dynamics of the economy becomes strikingly apparent in the impulse responses. For illustration, consider a positive, one percent shock to productivity. First of all, observe that this leads to an increase of employment in both sectors, but a relatively stronger increase in employment in good jobs. The reason is that relative to aggregate productivity, the cost differential for creating either job type has now become lower. This stimulates good job creation. Accordingly, output by good jobs rises somewhat more.

Next consider worker flows. Search intensity, and with it the effective amount of on-the-job searchers, e, shoots up, leading to a strong increase in subsequent job-to-job transitions.[20] Search activity rises mainly due to the increased availability of good employment opportunities, but also due to a rising wage differential. The increased availability of searching workers further stimulates the opening of good vacancies. At the same time, rising quits also stimulate posting of bad vacancies, to find replacements. Unemployed workers react to the competition with employed workers by directing their search to bad jobs. Thus, in a boom, the fraction of unemployed workers finding employment in good jobs falls, and of those finding bad jobs rises. This effect would be absent without search on the job. Overall, unemployment falls substantially.

Interestingly, wages rise only by half as much as productivity, a direct result of the rising competition between employed and unemployed job seekers. Wages in bad jobs rise but by less than in good jobs. Remember that both good and bad wages depend on a job's output, the outside benefit of workers, which is constant, and the expected value of good jobs, due to unemployed workers' directed search. The wage in bad jobs rises by less, however, because of the higher search intensity. While search has a positive impact on the present value of the match for workers, it reduces the value of the match to firms. But there is a net rise in the joint match surplus, of which firms obtain a slice.

On impact, output rises as much as productivity. However, after that, output does not move in line with the process for productivity. It continues to rise, until, after about 5

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[20]Suppose the shock were permanent. Then the initial increase in on-the-job search is above the higher level that obtains in the new long-run steady state. We thus observe an "overshooting" behavior as noted by Pissarides (1994). Of course, with an transitory aggregate shock, the economy converges back to the original steady state.

quarters, it begins to fall. Thus changes in productivity have persistent effects, indicating that search on the job adds substantial propagation to the model. Similarly, employment has a hump-shaped response. It is important to realize that this is not caused by the job heterogeneity in the model. Simulations of the model without employed search (not reported here) show that the impulse responses of that model are very similar to those of a standard one-sector model, such as those by Andolfatto (1996) or Merz (1995).

It is important to recognize the role of the aggregate household's discount factor, β_t, which firms use to discount future profits. Time variation in the discount factor smoothes consumption over time. This is achieved through its effect on vacancy creation. In a boom, high labor productivity induces expansion of employment, and thus the posting of vacancies. Since consumers have to give up consumption for firms' investment in employment, the interest rate rises, mitigating some of the effect. Still, vacancies rise substantially in response to the shock. In a partial equilibrium model with a constant interest rate, the effect would be even larger. In future periods, the interest rate falls, allowing higher consumption, and depletion of the employment stock back to the steady-state.

5.3   Simulation Results

We now turn to a discussion of the business cycle statistics computed from our benchmark model. Table 3 shows sample moments for the labor market variables of interest. We first evaluate the success of our benchmark specification in matching the standard deviations in the data conditional on aggregate technology shocks. Since we calibrate the variance of technology shocks to match the volatility of U.S. GDP we only evaluate the model's predictions based on relative volatilities. We find that, in general, the variables in the model are only slightly less volatile than in the data, in particular, vacancies, unemployment, and labor market tightness.

The data yield strong predictions with respect to contemporaneous correlations. First and foremost is the Beveridge-curve, the negative correlation of unemployment and vacancies over the business cycle. In U.S data this correlation is -0.95. Our benchmark calibration comes extremely close in matching this stylized fact.[21] We are also able to replicate the negative comovement of unemployment with all other aggregate variables of interest. For instance, the unemployment rate is highly negatively, though not perfectly, correlated with the job-to-job transition rate. When an adverse technology shock raises unemployment, search intensity falls due to declining probability of finding jobs. Workers

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[21]For their model, Mortensen and Pissarides (1994) report a correlation of only -0.26. See also the interesting discussion in Shimer (2003).

are therefore less likely to engage in on-the-job search so that relatively fewer workers in bad jobs move on to better ones. Interestingly, our two measures of labor market tightness are perfectly correlated on account of the strong comovement of search intensity with GDP. However, the inclusive measure is substantially less volatile.

The volatility of the quit rate comes very close to what is observed in the data, a result of the highly responsive search intensity. The supply of additional searchers holds the ratio of vacancies to unemployment plus employed search relatively stable. At the same time, it keeps the incentives high for firms to post vacancies. We also see the very high procyclicality of job-to-job quits in terms of the correlation with output. A noteworthy exception is the high correlation of wages and on the job search in the model, in contrast to the data.

Note at this point that the match between wages in model and data cannot be perfect for two reasons. One is empirical: wages are measured for heterogeneous workers, and thus combine aggregate and composition effects. The second reason is theoretical. Wages are best understood as a dividend payment on an asset, employment. The Nash bargaining approach yields a certain value of this asset, and a corresponding payment stream. However, many payment streams are consistent with that asset value. What matters for the economics of the model is how the asset values and bargaining positions of agents change in response to shocks, irrespective of the currently paid wages.

6 Discussion

6.1 The Role of Search Intensity and Job Heterogeneity

Why does the cyclicality of job-to-job quits change the behavior of the economy so substantially? This is best understood as the result of an interaction between rising search effort and the heterogeneity of posted vacancies. On the one hand, rising search effort raises good firms' incentives to post vacancies. Without employed searchers, the creation of good jobs is constrained by the fall in the number of unemployed searchers and the strong rise in wages. On the other hand, the increasing availability of good jobs further encourages on-the-job search. Thus a small rise in productivity leads to large changes in the incentives to search and posting vacancies, which explains that unemployment falls substantially even though competition with employed job seekers rises. Only slowly do these incentives fall back to their steady state levels.

The role of search intensity can be further illustrated by varying the elasticity of search effort. The results are depicted in Figure 3. As σ approaches one from above, the quit rate and labor market tightness become exceedingly volatile. Since the responsiveness of

search costs to changing search effort declines, the volatility of job-to-job quits rises. Even though the standard and our modified measures of labor market tightness, $\theta = v/u$ and $\theta^* = v/(u+e)$, are almost perfectly correlated, their volatility is strikingly different. While the former is very responsive to changes in σ, the latter is not at all. The reason is that as unemployment falls, employed search rises, keeping the incentives for vacancy creation high after a favorable aggregate shock. The theoretical counterpart in our model, $v^g/(u^g + e)$, behaves similarly. Since this measure of labor market tightness affects wages, they are much less volatile than in the case without on-the-job search. In this sense, on-the-job search endogenously generates wage rigidity.

Even though the choice of the search cost elasticity of $\sigma = 1.1$ is favorable to our results, it appears also most plausible. Merz (1995) chooses a value of one for unemployed searchers.[22] After paying a fixed cost of beginning to search, the marginal cost of search not likely to rise substantially: sending out one more application cannot add much cost. For large numbers, there is obviously a time constraint. Thus average search cost may be declining at first before rising at very high intensity. Another argument for a low σ is that on-the-job search activity in the model represents both the intensive and the extensive margin. Thus, also the number of searchers may change substantially as aggregate conditions change.[23]

As is evident from the impulse responses the presence of time-varying on-the-job search activity leads to persistent movements of output after shocks to technology. We investigate this issue further by analyzing modifications to our benchmark specification. First, we shut down on-the-job search over the business cycle. That is, we impose $s_t = s$, $\forall t$. While there is still employed search in the steady state – and optimally chosen according to Eq. (23) –, workers are not allowed to adapt their search intensity to changing business cycle conditions. Secondly, we remove the possibility of on-the-job search entirely, thereby only preserving the two-sector, good job/bad job structure.[24]

Figure 4 depicts impulse responses of output to a 1% productivity shock. The effect of on-the-job search on magnification and persistence is strikingly evident. The output response for the model without employed search essentially reflects the underlying productivity process.[25] It is this inability of the search and matching model that has been widely

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[22]While this leads to a unique equilibrium of the planner solution she presents, the choice of individual workers is not determined.

[23]Pissarides (1994) models search at the extensive margin by having workers with different levels of specific human capital choose to search depending on their outside options.

[24]The resulting specification can be thought of as a dynamic general equilibrium version of Acemoglu (2001). Krause and Lubik (2004) discuss its business cycle implications in more detail.

[25]Incidentally, the behavior of this model specification is virtually identical to a standard, one-sector search

discussed in the literature (see, for instance, Den Haan et. al., 2000). In contrast, on-the-job search provides strong amplification as well as persistence effects on output as adjustment to the steady state is much slower. With constant search effort the peak response is reached two periods after impact, and after three periods with time-varying search effort. Amplification occurs as workers in bad jobs move onto good jobs so that bad vacancies are posted. This attracts searchers from the unemployed even in the case of constant search intensity. When workers in the bad sector can optimally choose their search effort, this transmission mechanism is amplified further, as discussed above.

The endogenous persistence due to on-the-job search is therefore helpful in explaining the autocorrelation patterns in U.S. data. Figure 5 depicts the autocorrelation functions of U.S. GDP growth rates over the period 1948:1-2002:4 and for the three model specifications discussed above. The lack of propagation in the model without on-the-job search is well documented by a flat autocorrelation function around zero. The benchmark model, on the other hand, captures U.S. output dynamics remarkably well, even slightly overpredicting the first-order autocorrelation. In contrast, the search and matching model of Den Haan, Ramey, Watson (2000) yields magnification and more realistic autocorrelations of output by endogenizing the job destruction rate and including capital accumulation. Here, we obtain very similar results with a fixed job destruction rate but employed search.[26]

6.2 Relation to Previous Work

The literature that confronted the Mortensen and Pissarides (1994) model with the data typically focused on the performance of the model along the dimension it was designed to explain, namely the behavior of job creation and destruction. A well-known example is Cole and Rogerson (1999), who find that the model performs well if the steady-state unemployment rate is high. The argument is that the relevant pool of searchers in the labor market is high, based on the findings of Blanchard and Diamond (1990). Den Haan, Ramey, and Watson (2000) achieve plausible job flows by modeling endogenous job destruction along with capital. As mentioned, Hall (2003) and Shimer (2003) are the first to consider the ability of the search and matching framework to quantitatively match the cyclical behavior of unemployment and vacancies. It appears that in all papers, the performance of the model is enhanced by an assumption that reduces the cyclicality of hiring costs or wages. This can either be a large unemployment pool, or, in the case of Hall and Shimer, wage rigidity.

and matching framework. Results for the latter are not reported in order to conserve space.

[26]Note that Den Haan et al. focus on the behavior of job creation and destruction, but they do not report results concerning vacancies and unemployment.

In our model, it is on-the-job search motivated by job heterogeneity which endogenously reduces the volatilities of both wages and hiring costs.

Comparing our model with other approaches in the literature, consider first Pissarides' (1994) search model with on-the-job search. Our model shares with it the existence of two job types. In his deterministic model, jobs also differ in terms of creation costs and productivity, but technology is linear, and prices and the interest rate are constant. Workers differ in their job-specific skills, and in response to permanent changes in productivity, more workers choose to search on the job, at constant search intensity. In contrast to Pissarides, we find that employed job search does not reduce the volatility of unemployment, but increases it. One reason is that employed and unemployed workers can direct their search effort to the sectors where the prospect of finding a match are highest, rather than being randomly matched. This makes the reallocation of labor more efficient and leads to an amplification of shocks.

Pissarides (2000) has a different structure. Jobs differ by idiosyncratic productivity levels, drawn from a continuous distribution. With workers' choice to search or not, this implies two thresholds in terms of productivity. Below one threshold workers have an incentive to search for better employment, participating in the common matching market. New matches start at the highest possible productivity. Below the second threshold, which is lower than the first, the joint value of the match with the firm is below the parties' outside option, leading to job destruction. Since all jobs are created at the highest possible productivity level, all vacancies are the same for employed and unemployed workers. Thus quits are not replaced by firms and always lead to job destruction.

Mortensen (1994) simulates a stochastic version of the Mortensen-Pissarides model, with the addition of on-the-job search, modeled at the extensive margin. The presence of employed search helps in explaining the negative correlation between job creation and destruction, which we discussed above. The model also features a procyclical quit rate, with workers being randomly matched to the most productive jobs. Both Pissarides (1994, 2000) and Mortensen do not explore the link between vacancies, unemployment and job-to-job flows or the effects on wage setting.

On-the-job search is a central element of theories of the wage distribution based on job competition and wage posting, such as Mortensen (2003) or Cahuc, Postel-Vinay, Robin (2003). These models derive an endogenous steady-state wage distribution from the competition of firms for workers. Paying a high wage reduces the likelihood of workers quitting which offsets the wage cost. Firms that pay high wages and low wages co-exist, which gives

unemployed workers incentives to search more intensively. The equilibrium is more efficient than the one with no wage dispersion. However, wages are posted and are kept constant throughout employment. This is a key difference to the approach used here. We are not aware, though, of attempts to model the out-of-steady state dynamics of such frameworks.

Another approach to motivate search on the job is to introduce varying workers preferences or heterogeneous match quality. Employed workers then either search because of a deterioration of their satisfaction with their job (or an improvement in outside options), or they search because the quality of their match with the firm turns out to be unsatisfactory. The latter may be the result of learning about the job, and mostly applies to workers at the beginning of their life cycle.

6.3 Further Robustness Discussion and Potential Extensions

By modeling search intensity as the main margin of adjustment of employed job search, we leave the extensive margin out of the picture. It is clear that empirically the number of searchers varies over the business cycle, with searchers being inactive until prospects brighten. This mirrors the behavior of discouraged workers and those out of the labor force, which may begin search in good times. However, the two job types in the present model are sufficiently different so that all workers in bad jobs search. Changes in the number of searchers come about exclusively through changes in employment in bad jobs.

In other models employed search is mainly varied at the extensive margin and a lump sum is paid for searching.[27] Proceeding along those lines would require specifying a distribution of idiosyncratic match productivities and result in an optimal threshold below which workers earn a return to work so low that incuring the search cost is justified. It would be possible to generate similar results as with our model, albeit at the cost of increasing complexity. Particularly tracking the distribution of idiosyncratic productivities should prove cumbersome if they are persistent. Furthermore, it would naturally require consideration of an endogenous job destruction threshold for productivities below which jobs are destroyed. An advantage would be that it may help to explain the joint dynamics of firm-initiated separations (layoffs due to job destruction) and worker-initiated separations (quits into unemployment or to another job). It is well-known that the sum of the two is relatively stable over the cycle.[28] For clarity, we chose to exclude this possibility. However, all the important

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[27] Examples are Pissarides (1994) and Mortensen (1994). In steady-state labor models, Cahuc, Postel-Vinay, Robin (1999) make similar assumptions.

[28] The high procylicality of job-to-job quits is the a logical consequence of the countercyclical job destruction and quits for other reasons. See Davis, Haltiwanger, and Schuh (1996), Akerlof et al. (1988).

effects are captured by variation in search intensity.

We abstain from any experiments involving exogenous job destruction shocks. These may be interpreted as reallocation shocks, which stimulate job reallocation in the absence of aggregate disturbances. The reason is that these are intrinsically endogenous choices by firms, a response to changed economic conditions. However, experimenting with such shocks is instructive. Shimer (2003) shows that a negative job destruction shock increases vacancies, but at the same time unemployment as well. The larger pool of searchers makes finding worker for firms cheaper. We identify the same problem in a GE model with endogenous job destruction where the positive correlation between unemployment and vacancies dominates the negative correlation that increased job creation should induce. In fact, even job creation and destruction are positively correlated.[29] A deeper reason is that the cost of adjusting employment at the firing margin is cyclically less sensitive for firms with homogeneous labor than that of using the hiring margin. From this perspective, the job heterogeneity in the present model can explain why net employment expansions may not be accomodated by merely firing less workers in bad jobs.

The CES aggregator leaves open whether worker mobility is between or within industries. The literature seems to suggest the latter. Using Danish data, Albaek and Sorensen (1998) find that job to job transitions are frequent between different firm-size classes, but less frequent between industries. They find that a substantial amount of worker reallocation of between manufacturing establishments is from small to large firms. In light of the well-known firm size-wage effect this appears plausible.

While the model has no capital, it nevertheless features a form of investment, namely in vacancies. Consumption is in fact output minus the costs of vacancy posting. This investment needs to be made in order to increase the stock of workers. In that sense, labor and capital are perfect complements. For the household, it is a means to smooth consumption over time. Inclusion of capital would introduce an additional smoothing element. It is possible that an interaction between capital and on-the-job search would further strengthen the propagation of shocks in the model, since capital would allow carrying the effects of productivity into the future. The incentive for employed search would then also be maintained for a longer time, with the effects described earlier.

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[29]See Krause and Lubik (2003). Also Cooley and Quadrini (1999) and Cole and Rogerson (1999) discuss this issue. The former resolve it by assuming a lower elasticity of the matching function with respect the unemployment. The latter suggest increasing the relevant pool of searchers to include those not in the labor force. This amounts to twice the unemployment pool. We make a similar assumption here.

7 Conclusion

We have presented a model of labor market dynamics in which search on the job plays a crucial role. The main conclusion is that it is possible to explain the joint dynamics of vacancies, unemployment, and productivity without resorting to any imperfection other than search and matching frictions. In particular, we do not require wages to be rigid in order to bring the model closer to the data. Instead, increased search effort by employed workers serves to hold their outside options tame. This endogenously delivers wage rigidity, and thus maintains strong incentives for firms to post vacancies in a boom.

However, the findings are not meant to deny an potentially important role for (real) wage rigidity. Hall (2003) and Shimer (2003) suggest this as a solution to the empirical difficulties they identified with Mortensen-Pissarides model. Also in our model, wage rigidity would further amplify the cyclical response of vacancies, unemployment and job-to-job flows. Hall (2004) has made an interesting advance modeling wage setting based on social norms, which allows wages even for new hires to be rigid. In previous work, we applied this idea in a monetary business cycle model with search frictions.[30]

The model delivers a rich description of the labor market over the business cycle. Booms are times which allow employed workers to upgrade into better jobs, while opening jobs for unemployed workers, albeit of lower quality. The reallocation of labor to more productive units is facilitated by direct job-to-job transitions, rather than requiring movements of workers through the unemployment pool.[31] One fundamental reason for worker mobility is the heterogeneity of jobs which gives rise to differences in the returns to workers. The creation of good jobs is amplified by the rising intensity of search by employed workers.

Even though the model with on-the-job search explains important dimensions of the data surprisingly well, other aspects of reality may be worthwhile incorporating. Introduction of worker heterogeneity would allow to track which types of workers are hired into which types of jobs. In recessions, skilled worker may be parked in bad jobs only to transit to better jobs when conditions improve. Less skilled workers might find employment in good sectors, but only as long as favorable conditions prevail and face higher separation risk.

The propagation that the model implies may have important implications for business

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[30] Krause and Lubik (2003).

[31] A different interpretation of the demand structure also comes to mind. The good job-bad job distinction might better be reflecting old and new jobs in a vintage model. In that case, search on the job could accelerate the creation of new vintages at the technological frontier. It would also induce destruction of less productive units, with different implications for the efficiency of creative destruction. See Caballero and Hammour (1995).

cycle analysis. In the response to a positive productivity shock, output peaked after a number of quarters, not in the first period, as the process for productivity suggests. Higher labor productivity induces employed workers to search for better jobs. This feeds back into the incentives for firms to continue posting vacancies for a protracted period. Only slowly does this effect appear to fade. Interestingly, we obtain a magnifaction of shocks that is about as large as in Den Haan, Ramey, and Watson (2000), even though we do not include capital or a variable destruction rate. We intend to explore the propagation properties of on-the-job search in future work.

Appendix

A Equation System

1. Job creation conditions:

$$\frac{c^g}{q_t^g} = (1-\rho)E_t\beta\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}\left[P_{g,t+1}A_{t+1} - w_{t+1}^g + \frac{c^g}{q_{t+1}^g}\right],$$

$$\frac{c^b}{q_t^b} = (1-\rho)E_t\beta\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}\left[P_{b,t+1}A_{t+1} - w_{t+1}^b + (1-s_{t+1}p_{t+1}^g)\frac{c^b}{q_{t+1}^b}\right].$$

2. Wages determination:

$$w_t^g = \eta P_{g,t}A_t + (1-\eta)z + \eta p_t^g\frac{c^g}{q_t^g},$$

$$w_t^b = \eta P_{b,t}A_t + (1-\eta)(z + \kappa s_t^\sigma) + \eta(1-s_t)p_t^g\frac{c^g}{q_t^g}.$$

3. Optimal search intensity:

$$\kappa\sigma s_t^{\sigma-1} = \frac{\eta}{1-\eta}p_t^g\left(\frac{c^g}{q_t^g} - \frac{c^b}{q_t^b}\right).$$

4. Evolution of employment:

$$n_{t+1}^g = (1-\rho)\left(n_t^g + m_t^g\right),$$

$$n_{t+1}^b = (1-\rho)\left(n_t^b + m_t^b - s_t p_t^g n_t^b\right).$$

5. Unemployment:

$$u_t = u_t^g + u_t^b = 1 - n_t^g - n_t^b.$$

6. Employed searchers:

$$e_t = s_t n_t^b.$$

7. Matching functions:

$$m_t^g = m(v_t^g, u_t^g + e_t) = M_g(v_t^g)^{1-\mu}(u_t^g + e_t)^\mu,$$

$$m_t^b = m(v_t^b, u_t^b) = M_b(v_t^b)^{1-\mu}(u_t^b)^\mu.$$

8. Firm and worker match probabilities:

$$q_t^g \equiv m_t^g/v_t^g, \quad q_t^b \equiv m_t^b/v_t^b,$$

$$p_t^g = m_t^g/(u_t^g + e_t), \quad p_t^b = m_t^b/u_t^b.$$

9. Arbitrage condition ($U_t^g = U_t^b = U_t$):

$$p_t^g \frac{c^g}{q_t^g} = p_t^b \frac{c^b}{q_t^b}.$$

10. Sectoral and aggregate output:

$$y_{g,t} = A_t n_t^g, \quad y_{b,t} = A_t n_t^b,$$

$$y_t = (\alpha y_{b,t}^\gamma + (1-\alpha) y_{g,t}^\gamma)^{1/\gamma}.$$

11. Prices

$$
\begin{aligned}
P_{g,t} &= (1-\alpha) \left(\frac{y_{g,t}}{y_t} \right)^{-(1-\gamma)}, \\
P_{b,t} &= \alpha \left(\frac{y_{b,t}}{y_t} \right)^{-(1-\gamma)}.
\end{aligned}
$$

12. Aggregate consumption:

$$c_t = y_t - c_g v_{g,t} - c_b v_{b,t}.$$

B   Linearized System

1. Good jobs creation, with $X^g = (1 - \beta(1-\rho))/(P_g A - w^g)$

$$
\begin{aligned}
&\tau \widehat{c}_t - X^g P_g A \widehat{P}_{gt} + X^g w^g \widehat{w}_t^g + \beta(1-\rho) \widehat{q}_t^g \\
= \ &\widehat{q}_{t-1}^g + \tau \widehat{c}_{t-1} + X^g P_g A \rho_A \widehat{A}_{t-1} \\
&+ \tau \eta_t^c - X^g P_g A \eta_t^{P_g} + X^g w^g \eta_t^{w^g} + \beta(1-\rho) \eta_t^{q^g}
\end{aligned}
$$

2. Bad jobs creation, with $X^b = (1 - (1 - sp^g)\beta(1-\rho))/(P_b A - w^b)$

$$
\begin{aligned}
&\tau \widehat{c}_t - X^b P_b A \widehat{P}_{bt} + X^b w^b \widehat{w}_t^b + \beta(1-\rho)sp^g \widehat{s}_t \\
&+ \beta(1-\rho)sp^g \widehat{p}_t^g + \beta(1-\rho)(1-sp^g) \widehat{q}_t^b \\
= \ &\widehat{q}_{t-1}^b + \tau \widehat{c}_{t-1} + X^b P_b A \rho_A \widehat{A}_{t-1} + X^b P_b A \rho_b \widehat{A}_{bt-1} \\
&+ \tau \eta_t^c - X^b P_b A \eta_t^{P_b} + X^b w^b \eta_t^{w^b} + \beta(1-\rho)sp^g \eta_t^s \\
&+ \beta(1-\rho)sp^g \eta_t^{p^g} + \beta(1-\rho)(1-sp^g) \eta_t^{q^b}
\end{aligned}
$$

3. Wages good jobs

$$w^g \widehat{w}_t^g - \eta p^g \frac{c^g}{q^g} \widehat{p}_t^g + \eta p^g \frac{c^g}{q^g} \widehat{q}_t^g - \eta P_g A \widehat{A}_t - \eta P_g A \widehat{P}_{gt} = 0$$

4. Wages bad jobs

$$w^b \widehat{w}_t^b - \eta(1-s)p^g \frac{c^g}{q^g}\widehat{p}_t^g + \eta(1-s)p^g \frac{c^g}{q^g}\widehat{q}_t^g - \left[(1-\eta)ks^\sigma\sigma - s\eta p^g \frac{c^g}{q^g}\right]\widehat{s}_t$$

$$-\eta P_b A \widehat{A}_t = 0$$

5. Optimal search

$$(\sigma - 1)\widehat{s}_t - \widehat{p}_t^g + \frac{c^g/q_t^g}{c^g/q_t^g - c^b/q_t^b}\widehat{q}_t^g - \frac{c^b/q_t^b}{c^g/q_t^g - c^b/q_t^b}\widehat{q}_t^b = 0$$

6. Employment good jobs

$$\widehat{n}_t^g = (1-\rho)\widehat{n}_{t-1}^g + \rho\widehat{m}_{t-1}^g$$

7. Employment bad jobs

$$\widehat{n}_t^b = (1-\rho)(1-sp^g)\widehat{n}_{t-1}^b + (1-\rho)\frac{m^b}{n^b}\widehat{m}_{t-1}^b - (1-\rho)sp^g\widehat{s}_{t-1} - (1-\rho)sp^g\widehat{p}_{t-1}^g$$

8. Unemployment and employment

$$u^b\widehat{u}_t^b + u^g\widehat{u}_t^g + n^b\widehat{n}_t^b + n^g\widehat{n}_t^g = 0$$

9. Effective search

$$\widehat{e}_t - \widehat{s}_t - \widehat{n}_t^b = 0$$

10. Good job match probability (matching function)

$$\widehat{q}_t^g - \mu\widehat{u}_t^g - \mu\widehat{e}_t + \mu\widehat{v}_t^g = 0$$

11. Bad job match probability (matching furnction)

$$\widehat{q}_t^b - \mu\widehat{u}_t^b + \mu\widehat{v}_t^b = 0$$

12. Output good jobs

$$\widehat{y}_{gt} - \widehat{n}_t^g - \widehat{A}_t = 0$$

13. Output bad jobs

$$\widehat{y}_{bt} - \widehat{n}_t^b - \widehat{A}_t = 0$$

14. Aggregate output

$$\widehat{y}_t - \frac{\alpha y_b^\gamma}{\alpha y_b^\gamma + (1-\alpha)y_g^\gamma}\widehat{y}_{bt} - \frac{(1-\alpha)y_g^\gamma}{\alpha y_b^\gamma + (1-\alpha)y_g^\gamma}\widehat{y}_{gt} = 0$$

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15. Price good output

$$\widehat{P}_{gt} - (1 - \gamma)\widehat{y}_t + (1 - \gamma)\widehat{y}_{gt} = 0$$

16. Price bad output

$$\widehat{P}_{bt} - (1 - \gamma)\widehat{y}_t + (1 - \gamma)\widehat{y}_{bt} = 0$$

17. Directed search condition

$$\widehat{p}_t^g + \widehat{q}_t^b - \widehat{p}_t^b - \widehat{q}_t^g = 0$$

18. Aggregate Income (= consumption)

$$c\widehat{c}_t - y\widehat{y}_t + c^g v^g \widehat{v}_t^g + c^b v^b \widehat{v}_t^b = 0$$

19. Bad matches defined

$$\widehat{q}_t^b - \widehat{m}_t^b + \widehat{v}_t^b = 0$$

20. Good matches defined

$$\widehat{q}_t^g - \widehat{m}_t^g + \widehat{v}_t^g = 0$$

21. Unemployment in bad jobs

$$\widehat{p}_t^b - \widehat{m}_t^b + \widehat{u}_t^b = 0$$

22. Unemployment in good jobs and effective search

$$\widehat{p}_t^g - \widehat{m}_t^g + \frac{u^g}{u^g + e}\widehat{u}_t^g + \frac{e}{u^g + e}\widehat{e}_t = 0$$

23. Aggregate technology

$$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \varepsilon_{At}$$

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Table 1: U.S. Business Cycle Statistics

	Y	W	N	$\frac{Y}{N}$	U	V	θ	QR
				Standard Deviation				
	1.62	0.69	0.81	0.83	6.90	8.27	14.96	9.81
				Cross-Correlations				
	Y	W	N	$\frac{Y}{N}$	U	V	θ	QR
Y	1	0.57	0.82	0.69	-0.87	0.92	0.91	0.91
W	–	1	0.27	0.66	-0.42	0.51	0.47	0.05
N	–	–	1	0.16	-0.93	0.88	0.91	0.91
$\frac{Y}{N}$	–	–	–	1	-0.34	0.49	0.43	0.44
U	–	–	–	–	1	-0.95	-0.98	-0.93
V	–	–	–	–	–	1	0.99	0.94
θ	–	–	–	–	–	–	1	0.95
QR								1

Table 2: Model Parameters and Calibration

Parameter	Value	Description
μ	0.4	Match Elasticity
M_g	0.6	Level Parameter
M_b	0.6	Level Parameter
c_g	0.16	Good Job Creation Cost
c_b	0.04	Bad Job Creation Cost
ρ	0.1	Separation Rate
σ	1.1	Search Elasticity
η	0.5	Nash Bargaining Share
α	0.4	CES-Weight
γ	0	Substitution Elasticity
β	0.98	Discount Factor
τ	1	Intertemporal Substitution Elasticity
u	0.12	Unemployment Rate
ζ	0.06	Quit Rate
z	0.39	Value of Home Production
κ	0.04	Search Cost Function Parameter

Table 3: Benchmark Simulation

Standard Deviations								
Y	W	N	$\frac{Y}{N}$	U	V	θ	θ_n	QR
				(rel. to Y)				
1.62	0.19	0.56	0.27	6.09	5.43	11.17	2.57	10.05

Cross-Correlations								
	Y	W	N	$\frac{Y}{N}$	U	V	θ	QR
Y	1	0.83	0.99	0.97	-0.99	0.93	0.99	0.96
W	-	1	0.81	0.86	-0.75	0.97	0.88	0.94
N	-	-	1	0.54	-1.0	0.87	0.97	0.92
Y/N	-	-	-	1	-0.54	0.84	0.87	0.96
U	-	-	-	-	1	-0.87	-0.97	-0.92
V	-	-	-	-	-	1	0.96	0.99
θ	-	-	-	-	-	-	1	0.98
QR	-	-	-	-	-	-	-	1
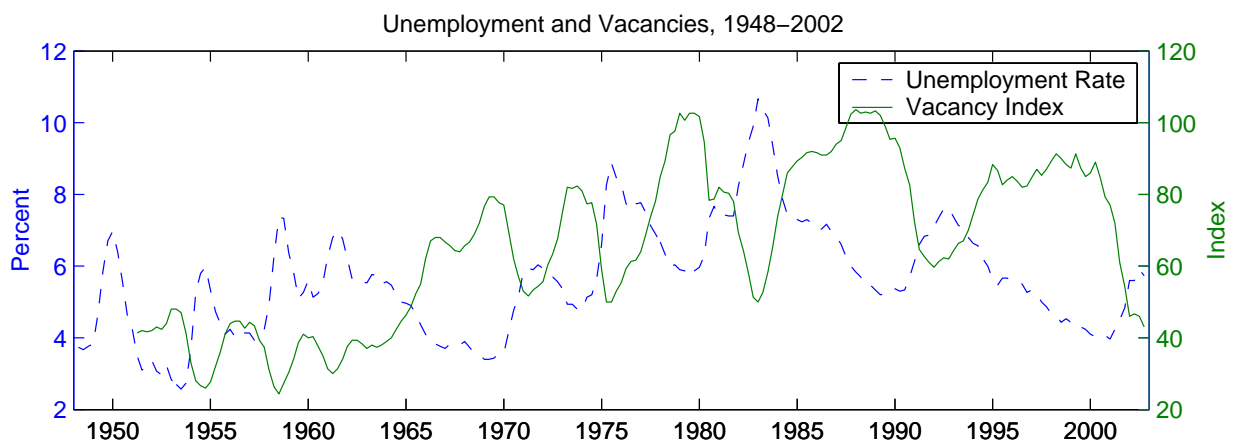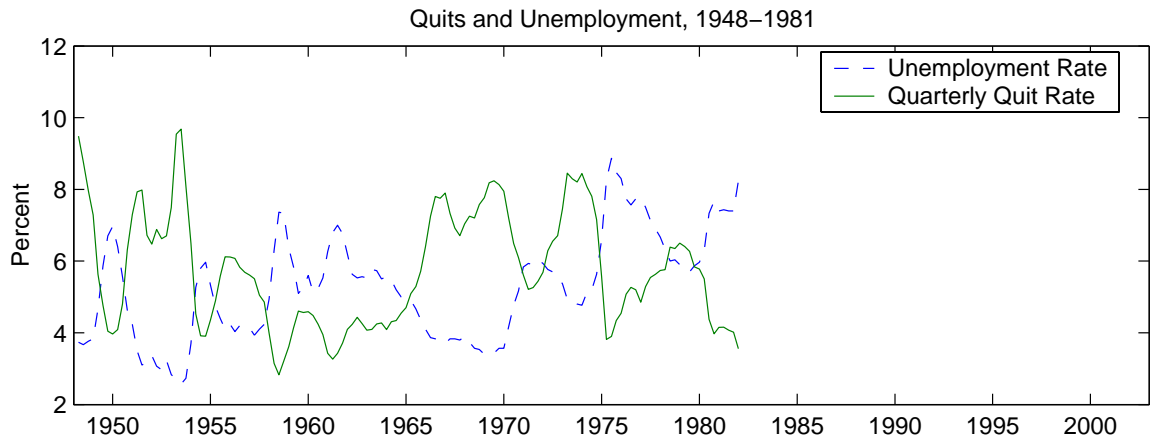
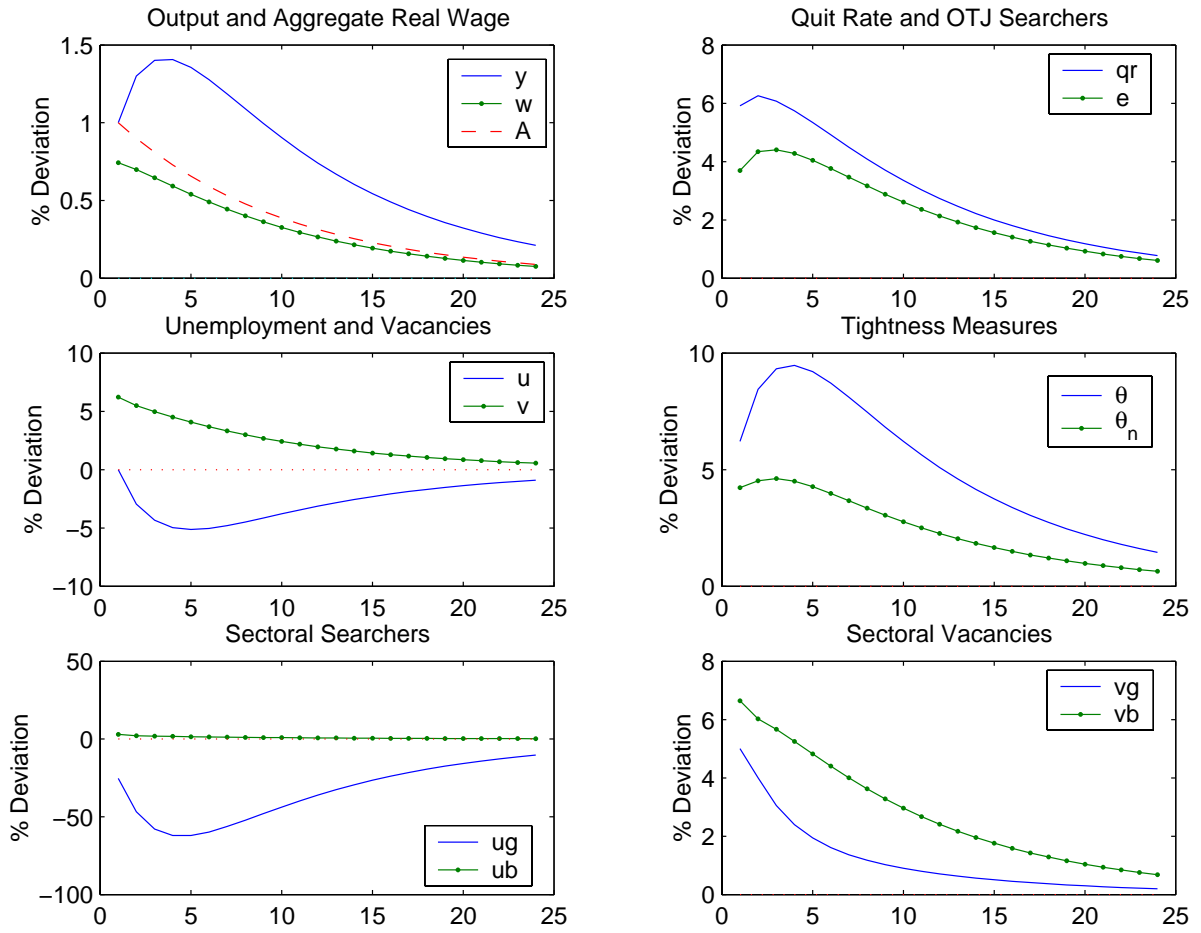Figure 1: Vacancies, Unemployment and Quits

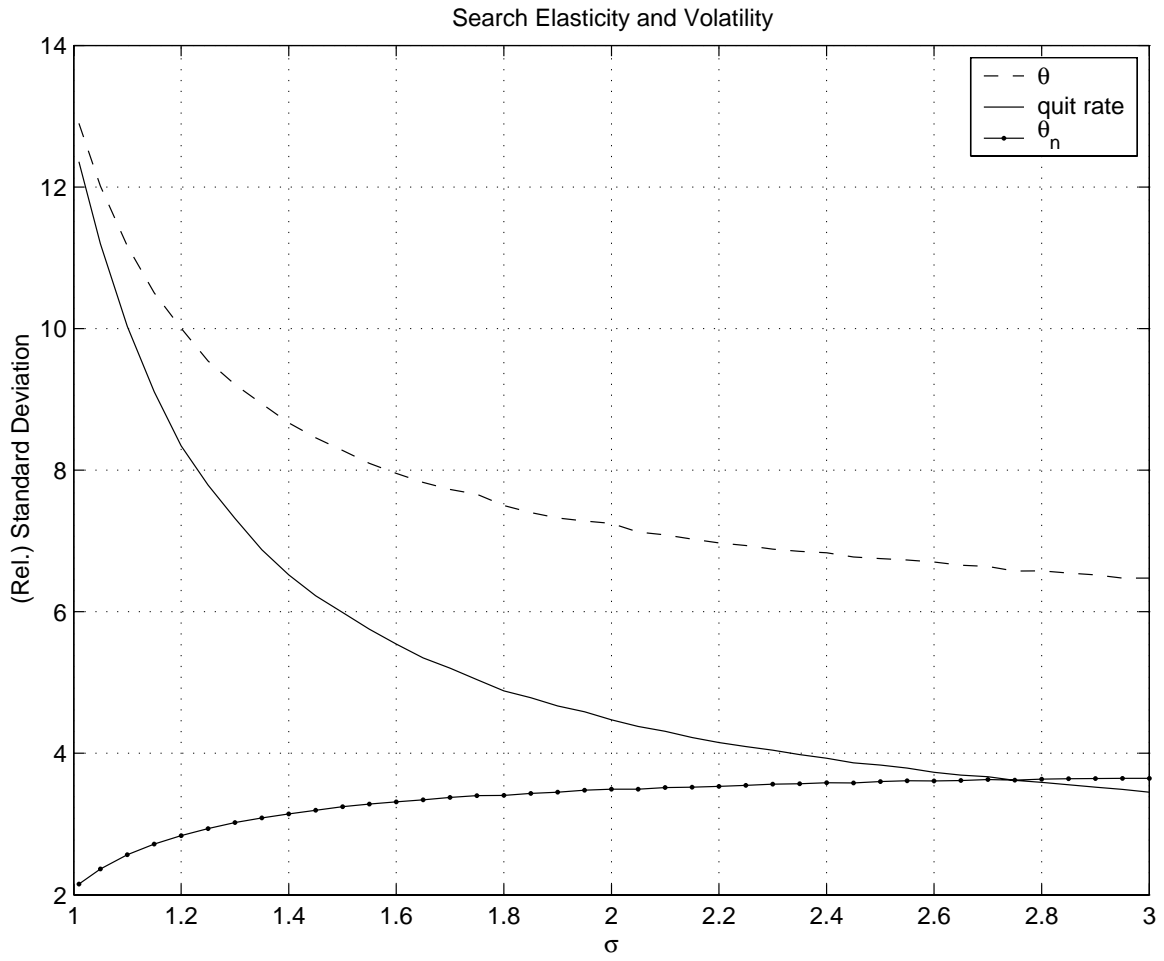Figure 2: Impulse Response Functions to a 1% Productivity Shock

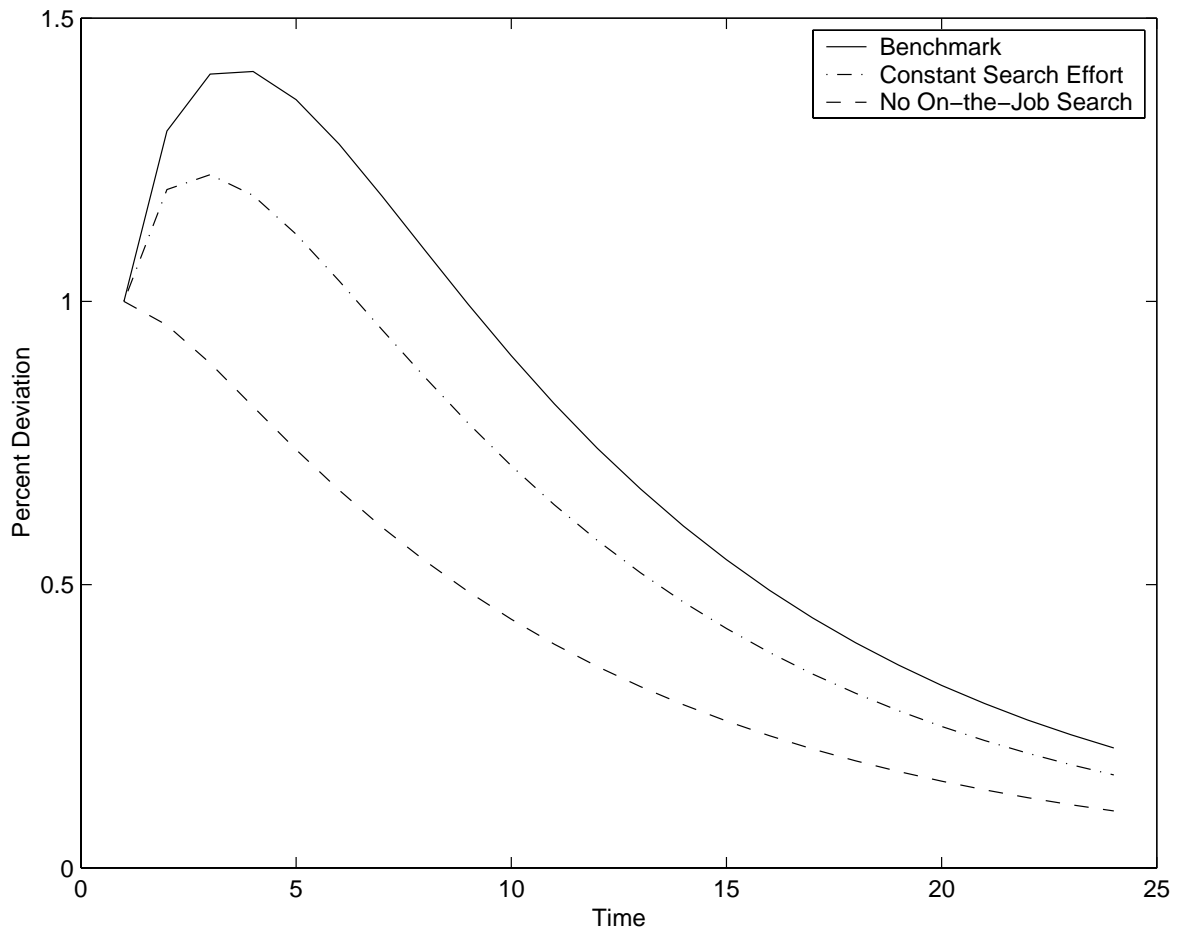Figure 3: Search Elasticity and Aggregate Volatilities
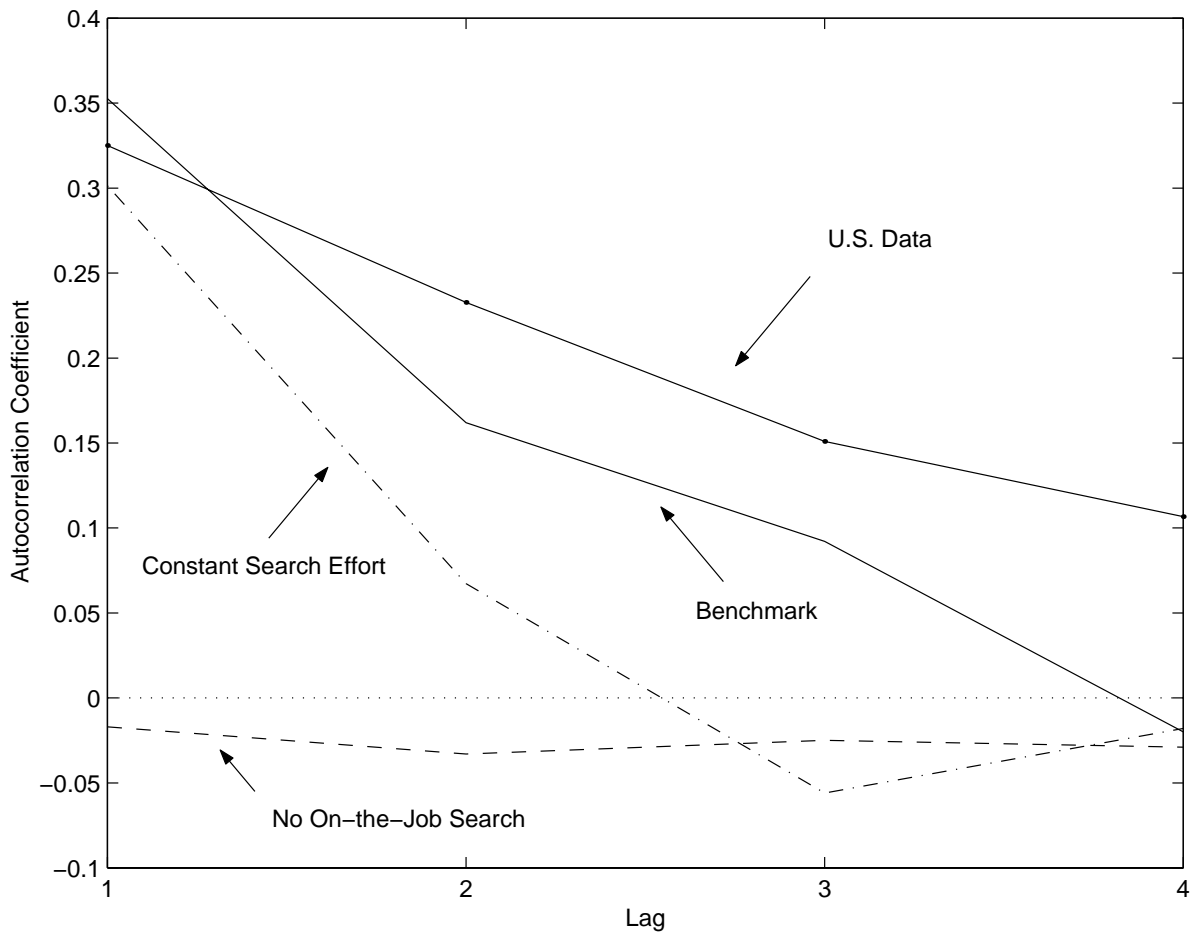
Figure 4: Impulse Responses of Output to a 1% Productivity Shock

Figure 5: Autocorrelations of Output Growth Rates