# Minimum Wage and Compliance in a Model of Search on-the-Job* (Preliminary Draft)

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**Abstract**

We estimate a job search model with four main ingredients: (i) search on-the-job; (ii) wage growth on-the-job; (iii) minimum wages, with potentially imperfect compliance; and, (iv) exogenous wage growth on-the-job. We use data drawn from the NLSY79 to estimate the parameters of our job search model and, in particular, the extent of compliance to the minimum wage. The model is solved numerically and we use simulated moments to estimate the parameters. The estimated parameters are consistent with the model and they provide a good fit for the observed level and trend of main labor market moments. Furthermore, the arrival rate of job offers below the minimum wage is 40% lower than the arrival rate of job offers above the minimum wage.

Keywords: minimum wages, compliance, job search, wage growth
JEL: J42, J63, J64

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# 1 Introduction

Wage data from National Longitudinal Survey of the Youth of 1979 (NLSY79) indicate that during the first six months after graduation from high school about 25 percent of white males earn an hourly wage below the US federal legal minimum wage. This percentage is reduced to 15 percent after a year following graduation, about 9 percent after three years and from the tenth year about three percent of the high school graduates earn an hourly wage below the minimum wage (see Figure 14). Figures reported by U.S. Bureau of Census (1997, p. 433) show that as many as 40% of workers who qualify are paid less than the minimum wage.

The evidence on compliance of workers and firms with the minimum wage law goes back to Ashenfelter and Smith (1979). They compute compliance as the fraction of workers earning less than the minimum wage before the enactment of the law, and who earned exactly the minimum wage after the enactment of the law. They conclude that "for the country as a whole the point estimate of the compliance rate is 69%, although a conventional confidence interval would include the range in the 63-75%". More recent work by Cortes (2004) studies whether immigrants are more likely to be paid less than the minimum wage than natives and, overall, she finds no systematic pattern of noncompliance between immigrants and natives. Finally, Weil et. al. (2004) use data on apparel contractors in the Los Angeles area, and find that 54% of employers in 2000 did not comply with minimum wage laws, and that 27% of employees were paid below the minimum wage. This evidence suggest that a large proportion of wages reported in the NLSY79 to below the minimum wage are due to noncompliance rather than just measurement error in wages.

This paper uses the standard Burdett (1978) and Lucas and Prescott (1974) search on-the-job models to estimate the extent of compliance to the minimum wage in the US. Observed wages below the minimum wage can result from both non-compliance and/or measurement error (Ashenfelter and Smith, 1979). But while measurement error should apply throughout the wage distribution, non-compliance, by definition, applies to actual accepted wages below the minimum wage. This distinction will be the basis of our identification strategy of the extent of compliance with the US federal minimum wage.

We construct a continuous time search model in a stationary labor market environment with the following ingredients: (i) search on the job; (ii) minimum wages with imperfect compliance by firms; (iii) endogenous search effort; (iv) exogenous wage growth on-the-job.[1] The model is solved numerically and we use simulated moments to estimate the parameters. The estimated parameters are consistent with the model and they provide a good fit for the level, the trend and the fluctuations of several moments that are used for estimation. We find that the arrival rate of job offers below the minimum wage is 40% lower than the arrival rate of job offers above the minimum wage. There are good reasons to believe that this is an over-estimate of the true non-compliance of firms with the minium wage federal law.

The literature on the minimum wage policy is large and we certainly do not attempt to cover it here. However, it is well recognized that the analysis of this policy requires an

[1] Bowlus and Neuman (2004) use a search equilibrium model to empirically analyze the wage growth using NLSY data.

equilibrium model where firms and workers respond to the change in policy. This basic claim provides the reason that minimum wage policy was analyzed by Eckstein and Wolpin (1990), van den Berg and Ridder (1998) and Flinn (2003, 2004), among others, in estimable search equilibrium models. Here we use a simple search model where the equilibrium is interpreted as in Lucas and Prescott (1974). The main reason for our choice of the model is that by using data such as the NLSY79 it is not clear that one can empirically distinguish between the different search models (see. e.g., Eckstein and van den Berg, 2004). The simple search model is a benchmark specification where the observed wage dispersion and labor market mobility of workers are due to productivity difference across firms, worker heterogeneity and worker search decisions. In this simple model wage dispersion and worker mobility are not a result of the response of firms and the endogenous wage formation within an equilibrium labor market with frictions (e.g., Burdett and Mortensen, 1998). Here, the wage dispersion is due to the assumed productivity differences between potentially homogenous workers, but where aditional disperssion is due to the worker dynamic search decision rules.

The paper is organized as follows. Section 2 presents our job search model. Section 3 describes our data set. Section 4 gives the estimation method, and Section 5 presents preliminary results.

# 2 The Model

We construct a continuous time search model in a stationary labor market environment with the following ingredients: (i) search on the job; (ii) minimum wages with imperfect compliance by firms; (iii) endogenous search effort; (iv) exogenous wage growth on-the-job.

Agents are infinitely lived, and at each moment in time they can be either non-employed (a state denoted by $n$) or employed (a state denoted by $e$). When they are unemployed, they enjoy some real return $b$ (typically including the value of leisure and unemployment insurance benefits), and receive job offers at a Poisson rate $\lambda_n$. Generating job offers at rate $\lambda_n$ requires some search effort, with related search costs $c_n(\lambda_n)$, with $c_n'(\lambda_n) > 0$ and $c_n''(\lambda_n) > 0$. When employed, they enjoy a real wage $w$, which is growing at an exogenous rate $g$, receive job offers at a Poisson rate $\lambda_e$, and bear search costs $c_e(\lambda_e)$, with $c_e'(\lambda_e) > 0$ and $c_n''(\lambda_e) > 0$. Existing jobs are hit by idiosyncratic shocks, which occur at a Poisson rate $\delta$. The instantaneous discount rate is $r$. New wage offers for employed and unemployed are randomly drawn from some known, fixed distribution $F(w)$. As said above, once the individual accept a wage $w$, his wage on the same jobs grow with tenure, $\tau$, on the job such that $w_\tau = we^{g\tau}$.

Our modelling of the wage offer distribution closely resembles Lucas and Prescott (1974) islands' model as it is presented in Mortensen (1986). In the Lucas and Prescott formulation, the distribution of wage offers represents productivity differentials across different islands. As productivity in each island is subject to idiosyncratic shocks, workers need to spend some effort in order to locate better matching opportunities and eventually relocate across islands in pursuit of wage gains. In our model, the wage offer distribution represents productivity differentials across firms. Each firm productivity is given, but better matching opportunities arise to workers through search on-the-job.

There is an exogenously set minimum wage in the economy, denoted by $w_M$. However, there is no full compliance of firms with the minimum wage. We assume that firms with lower than the minimum wage productivity continue to offer jobs for wages below the minimum wage. Hence, workers still face some positive probability to receive an offer which pays below the minimum wage. We denote by $\lambda_n$ and $\lambda_e$ the arrival rates of job offers above the minimum wage for the unemployed and the employed, respectively, and by $\alpha\lambda_n$ and $\alpha\lambda_e$ the corresponding arrival rates of job offers below the minimum wage, with $0 \leq \alpha \leq 1$. When $\alpha = 0$ there is full compliance of firms and workers with the economy minimum wage. When $\alpha = 1$ there is no effective minimum wage regulation in the economy. Hence, in this model the minimum wage policy has two parameters, the level on the minimum wage, $w_M$, and the level of compliance, $\alpha$.

In this paper we focus on high school graduate males that do not attend college and we follow them since they leave school.[2] Below we compute lifetime utilities for the employed and the non-employed. The value of employment $\tau$ periods on the same job is denoted by $V_e(w_\tau)$ and the value of non-employment clearly does not depend on specific job attributes and is denoted by $V_n$.

A worker who is currently non-employed enjoys a net flow of income $b - c_n(\lambda_n)$, receives job offers above or below the minimum wage at rates $\lambda_n$ and $\alpha\lambda_n$, respectively, which are accepted if the value attached to them exceeds the value of non-employment:

$$\begin{aligned} rV_n &= b - c_n(\lambda_n) + \lambda_n\{E_{w\geq w_M} \max[0, V_e(w) - V_n]\} \\ &\quad + \alpha\lambda_n \{E_{w<w_M} \max[V_e(w) - V_n, 0]\}. \end{aligned} \tag{1}$$

A worker currently employed in a job with starting wage $w$ and tenure $\tau$ receives net income $w_\tau - c_e(\lambda_e)$, enjoys wage growth at rate $g$, is forcibly separated from her employer at rate $\delta$, and receives job offers above or below the minimum wage at rates $\lambda_e$ and $\alpha\lambda_e$, respectively, which are accepted if the value attached to them exceeds the lifetime utility in the current job:

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta[V_n - V_e(w_\tau)] \\ &\quad + \lambda_e E_{w\geq w_M} \max[0, V_e(w) - V_e(w_\tau)] + \alpha\lambda_e E_{w<w_M} \max[0, V_e(w) - V_e(w_\tau)] \\ &\quad + g w_\tau V_e'(w_\tau) \text{ for } w_\tau < w_M \end{aligned}$$

and

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta[V_n - V_e(w_\tau)] \\ &\quad + \lambda_e E_w \max[0, V_e(w) - V_e(w_\tau)] + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M, \end{aligned}$$

[2] We suppose for now that each individual starts search for a job at the month he leaves school, and he is assumed to be unemployed. Later we will relax this assumption for the empirical application.

where the last term in each case represents the change in value on the job, that is, $\frac{\partial V_e(w_\tau)}{\partial \tau}$. In either labor market state, agents set an acceptance rule for job offers and the optimal level of search effort. As job switching involves no cost, the optimal acceptance rule for the employed consists in accepting any job that pays more than their current wage $w$. For the non-employed, the optimal acceptance rule consists in accepting all job offers which pay at least some reservation wage $w^*$, such that $V_n = V_e(w^*)$. Note that such reservation wage exists and is unique because, while the value of search is constant, the value of employment is monotonically increasing in $w$. If $w_M \leq w^*$, then minimum wages have no impact on agents' decisions or equilibrium outcomes. Therefore, we assume that the minimum wage is binding , i.e. $w_M > w^*$, such that the value functions in this model can be rewritten as:

$$rV_n = b - c_n(\lambda_n) + \lambda_n \int_{w_M} \left[V_e\left(w\right) - V_n\right] dF(w) + \alpha\lambda_n \int_{w^*}^{w_M} \left[V_e\left(w\right) - V_n\right] dF(w) \tag{2}$$

and

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\ &\quad + \lambda_e \int_{w_\tau} \left[V_e\left(w\right) - V_e\left(w_\tau\right)\right] dF(w) + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M & (3) \\ rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] + \lambda_e \int_{w_M} \left[V_e\left(w\right) - V_e\left(w_\tau\right)\right] dF(w) \\ &\quad + \alpha\lambda_e \int_{w_\tau}^{w_M} \left[V_e\left(w\right) - V_e\left(w_\tau\right)\right] dF(w) + g w_\tau V_e'(w_\tau) \text{ for } w_\tau < w_M, & (4) \end{aligned}$$

Note that in (3) the probability of getting an offer below the minimum wage ($\alpha\lambda_e$) does not affect the value of employment, as any such offer would be rejected by someone employed at $w \geq w_M$.

A non-employed worker will choose $\lambda_n$ in order to maximize (2). The first-order condition for this optimization problem is given by

$$c_n'(\lambda_n) = \int_{w_M} \left[V_e\left(w\right) - V_n\right] dF(w) + \alpha \int_{w^*}^{w_M} \left[V_e\left(w\right) - V_n\right] dF(w), \tag{5}$$

thus equating the marginal cost of an extra job offer to its marginal benefit. Similarly, the first order condition for the choice of search intensity for the employed is given by

$$\begin{aligned} c_e'(\lambda_e) &= \int_{w} \left[V_e\left(w'\right) - V_e\left(w\right)\right] dF(w'), \text{ if } w \geq w_M & (6) \\ c_e'(\lambda_e) &= \int_{w_M} \left[V_e\left(w'\right) - V_e\left(w\right)\right] dF(w') + \alpha \int_{w^*}^{w_M} \left[V_e\left(w\right) - V_n\right] dF(w), \text{ if } w < w_M. & (7) \end{aligned}$$

By convexity of the search cost function, the unemployed will have a higher incentive to search for jobs, and, keeping everything else equal, raise their arrival rate of job offers above the one of the employed. Among the employed, search effort decreases with the current wage: in particular, those employed below the minimum wage will search more intensively than those employed above.

Given the acceptance rule $rV_n = rV_e(w^*)$, we can solve for the value of the reservation wage by setting equation (2) equal to equation (4) evaluated at $w = w^*$ and $\tau = 0$ (exploiting the continuity of $V_e(w_\tau)$ at $w_M$, a property that only holds if the wage offer distribution is the same above or below the minimum wage):

$$\begin{aligned} w^* &= b - c_n(\lambda_n) + c_e(\lambda_e) + (\lambda_n - \lambda_e)\int_{w^*} [V_e(w) - V_n]\, dF(w) \\ &\quad - (1-\alpha)(\lambda_n - \lambda_e)\int_{w^*}^{w_M} [V_e(w) - V_n]\, dF(w). - gw^* V_e'(w^*) \qquad (8) \\ &= b - c_n(\lambda_n) + c_e(\lambda_e) + (\lambda_n - \lambda_e)\int_{w^*} [1 - F(w)] V_e'(w)\, dw \\ &\quad - (1-\alpha)(\lambda_n - \lambda_e)\int_{w^*}^{w_M} [1 - F(w)] V_e'(w)\, dw - gw^* V_e'(w^*) \qquad (9) \end{aligned}$$

using integration by parts.

To solve equation (9) one needs to know the function $V_e'()$. We prove in the appendix that

$$V_e'(w_\tau) = e^{R(w_0,\tau)}\left[V_e'(w_0) - \int_0^\tau e^{-R(w_0,t)} dt\right] \qquad (10)$$

and

$$V_e'(w_0) = \int_0^\infty e^{-R(w_0,\tau)} d\tau, \qquad (11)$$

where

$$R(w_0,\tau) = (r + \delta - g)\tau + \lambda_e \int_0^\tau [1 - F(w_\tau)]\, d\tau, \text{ for } w_\tau \geq w_M \qquad (12)$$

$$R(w_0,\tau) = (r + \delta - g)\tau + \lambda_e \int_0^\tau [1 - \alpha F(w_\tau) - (1-\alpha) F(w_M)]\, d\tau, \text{ for } w_\tau < w_M. \qquad (13)$$

The reservation wage can be numerically calculated substituting (10)-(13) into (9). If the job offer arrival rates are set exogenously, then the model is fully solved by calculating the reservation wage $w^*$ using the solution to (9). Otherwise, the joint solution of $w^*, \lambda_n$ and $\lambda_e$ is found by solving jointly (9), (5), (6) and (7), and this solution enable us to simulate the dynamic decision sequence of the worker. This solution provides a joint dynamic distribution of labor market mobility from non-employment to work, from job-to-job and back to non-employment. Furthermore, this solution enables us the calculate by simulations the probability of all labor market states conditional on observed wages and, in particular, on whether the observed wage is below or above the minimum wage. Finally, the model provides a characterization of the probability to observe worker that are employed at a wage below the minimum wage and above the minimum wage. When $\alpha = 0$ the probability to observe a wage below the minimum wage is zero, hence, the only way to justify this observation if there is full compliance with the law, is to assume that wages are reported with error. This is the main alternative hypothesis to the claim that there is no full compliance ($0 < \alpha \leq 1$) with the minimum wage law.

# 3 Data

We use data drawn from the National Longitudinal Survey of Youths, which contains information on a sample of 12686 respondents who were between 14 and 21 years of age in January 1979 (NLSY79). Our sample is restricted to white males who are high school gradutes, and never returned to school. Specifically, we select non-black, non-hispanic, males who have completed at most 12 years of schooling and declare to hold high school degree. We exclude from our sample those who (i) ever went to the army; (ii) ever declared to be in college; (iii) ever declared to have a college or professional degree. We further restrict our sample to those who completed high school between age 17 and 19. These restrictions leave us with a sample of 577 individuals.

Information on selected respondents is available from calendar time January 1978. We construct individual monthly work histories using answers to retrospective questions. We assume that market entry coincides with calendar month an individual completed high school. Individuals in our sample completed high school between 1974 and 1984. More than 95% of them graduated in either May or June. We follow the individuals for 18 years after high school graduation and the data is organized to be consistent with the model's definitions and assumptions.

**Labor Market States** From the NLSY79 work history file, we obtain the monthly employment and non-employment (labor force status) from the first month of 1978 to the last month of 1998. We define an individual as employed in a month if he works at least 10 hours per week and at least three weeks per month, or during the last two weeks in the month. Otherwise, the individual is non-employed, and we do not further distinguish between unemployed and out of the labor force. Figure 1 shows the monthly proportion of employed and non-employed by the years since high school graduation. The data shows clear pattern of seasonality.[3]

Among those who were found in employment upon finishing high school, some started working before graduation. As Figure 2 shows, on average 55% of individuals in our sample worked during the year preceding graduation. This may happen because job search starts while in school or, more likely, because high school students may take up temporary and part time jobs while in school. The latter explanation seems also supported by the clear

[3]We focus here on the growth of employment on the extensive margine. It should be noted that average hours per-employed worker also shows a positive trend during the first eight years (Table 1). This intensive margine is not part of this paper but could be added to the search framework discussed here.

seasonal pattern of employment rates during the last year before graduation. We assume that individuals employed before graduation enter the "official" labor market upon graduation, but we will treat the proportion of individuals employed at labor market entry as an initial condition in our simulations.

The employment history information is employer-based. All references to a "job" should be understood as references to an employer. Multiple jobs held contemporaneously are treated as new jobs altogether: the associated wage is the average of the two hourly wages and the associated hours are the sum of the hours worked on the different jobs. Duration of a given job is considered as completed when a new job is recorded or the work is terminated and the individual is back to non-employment.

Table 1 gives employment statistics by years of labor market experience after graduation. Both the average number of months worked and average annual hours increase with experience. As expected, the average yearly cumulative number of jobs decreases with experience from 1.52 to 1.21 jobs, while to cumulative jobs per worker reach about 6.5 after 18 years.[4]

Table 2 reports the duration of non-employment spells leading to the first 10 jobs in individual careers, which seems to fall roughly monotonically with the job rank. 103 observations had more than 10 jobs, 12 of whom had more than 20 jobs. The maximum number of jobs held is 27. As we have several censored spells in our sample, the sample mean duration is downward biased. In column 5 we therefore also present the Kaplan-Meier nonparametric durations estimates.[5] If the observation with the largest associated duration is censored, the Kaplan-Meier survivor function does not go to zero as duration goes to infinity. Consequently, the area under the curve still underestimates the mean duration. We thus extrapolate the survivor function using an exponential density function to compute the area under the entire curve, and from this one we obtain the Kaplan-Meier extended mean duration. This is reported in column 5 of table 2, and the mean duration between for each job is estimated to be about six to one month longer using the extended (column 5) rather than the restricted (column 4) Kaplan-Meier estimates.[6]

Figure 3 plots the job separation hazard by job tenure. We do not distinguish among job ranks, due to the insufficient number of observations for each rank. Duration is truncated at 10 years (and consequently 83 out of 2539 job spells are dropped). The monthly job hazard rate decreases significantly with duration, consistently with the above search model with

[4]In the Bureau of Labor Statistics report on "Number of jobs held, labor market activity, and earnings growth among younger baby boomers: results from more than two decades of a longitudinal survey" (BLS 2002, Table 1), the average number of jobs held by white high school graduates is 9.2, which is higher than our figure. Such discrepancy stems from the different definitions of jobs. BLS define a job as an uninterrupted period of work with a particular employer, excluding recalls from temporary layoffs. In our definition we do not exclude recalls.

[5]Let $n_t$ be the population alive at time $t$ and $d_t$ the number of failures. The nonparametric maximum likelihood estimate of the survivor function is: $\widehat{S}(t) = \Pi_{j|t_j \leqslant t}(\frac{n_j - d_j}{n_j})$. The Kaplan-Meier restricted mean duration is computed as the area under the Kaplan-Meier survivor function. And the associated standard error is given by the Greenwood formula: $\widehat{Var}\{\widehat{S}(t)\} = \widehat{S}^2(t) \sum_{j|t_j \leqslant t} \frac{d_j}{n_j(n_j - d_j)}$.

[6]Using job identifiers, individuals recalled by old employers after a nonemployment spell are considered as staying in the same job.

Table 1: Employment statistics by labor market experience

| Year after graduation | Ave. months worked* | Ave. annual hours** | Ave. cumulative no. of jobs | Cumulative jobs |
|---|---|---|---|---|
| 1 | 8.74 (3.96) | 2031 (458) | 1.52 (0.74) | 1.52 (0.74) |
| 2 | 9.54 (3.42) | 2082 (440) | 1.40 (0.64) | 1.99 (1.14) |
| 3 | 9.79 (3.51) | 2130 (489) | 1.36 (0.68) | 2.41 (1.52) |
| 4 | 10.10 (3.23) | 2153 (443) | 1.33 (0.65) | 2.88 (1.84) |
| 5 | 10.32 (3.07) | 2200 (506) | 1.31 (0.64) | 3.28 (2.17) |
| 6 | 10.48 (2.86) | 2212 (469) | 1.33 (0.65) | 3.72 (2.53) |
| 7 | 10.55 (2.90) | 2195 (490) | 1.29 (0.62) | 4.07 (2.79) |
| 8 | 10.86 (2.58) | 2206 (492) | 1.24 (0.55) | 4.39 (3.03) |
| 9 | 10.96 (2.49) | 2226 (510) | 1.29 (0.68) | 4.72 (3.34) |
| 10 | 10.86 (2.66) | 2256 (569) | 1.29 (0.64) | 5.06 (3.61) |
| 11 | 11.06 (2.34) | 2292 (555) | 1.21 (0.50) | 5.29 (3.81) |
| 12 | 11.16 (2.27) | 2309 (548) | 1.23 (0.56) | 5.52 (4.00) |
| 13 | 10.98 (2.44) | 2337 (552) | 1.21 (0.49) | 5.74 (4.17) |
| 14 | 10.93 (2.70) | 2288 (549) | 1.21 (0.56) | 5.95 (4.32) |
| 15 | 11.15 (2.32) | 2352 (677) | 1.21 (0.48) | 6.15 (4.53) |
| 16 | 11.10 (2.48) | 2342 (616) | 1.19 (0.47) | 6.31 (4.68) |
| 17 | 10.96 (2.84) | 2360 (621) | 1.19 (0.49) | 6.45 (4.77) |
| 18 | 11.00(2.79) | 2356 (615) | 1.14 (0.44) | 6.55 (4.87) |

Standard errors are in paratheses. * The value is conditional on observations where all states are available in all months.** Average hours are conditional on working in all months.

Table 2: Duration of non-employment spells and job spells ( 1 to 10 jobs) since high school gradaution

| Job No. | No. of obs. | Sample Mean duration (s.d.) | Kaplan-Meier restricted Mean duration (s.d.) | Kaplan-Meier extended Mean duration |
|---|---|---|---|---|
| NE* | 148 | 8.86 (23.90) | 9.93 (2.37) | 10.74 |
| 1 | 574 | 32.74 (53.94) | 38.69 (2.76) | 44.85 |
| 2 | 508 | 33.77 (49.05) | 41.03 (2.85) | 47.66 |
| 3 | 457 | 27.82 (38.35) | 39.37 (3.10) | 46.98 |
| 4 | 387 | 24.71 (33.28) | 34.06 (2.77) | 37.73 |
| 5 | 337 | 22.22 (30.91) | 31.23 (2.73) | 35.05 |
| 6 | 276 | 20.38 (28.84) | 32.33 (3.83) | 37.95 |
| 7 | 236 | 23.23 (31.30) | 34.59 (3.40) | 40.31 |
| 8 | 182 | 17.92 (25.78) | 24.82 (3.07) | 29.51 |
| 9 | 150 | 17.58 (19.22) | 21.51 (2.12) | 22.12 |
| 10 | 131 | 16.96 (22.31) | 24.19 (3.40) | 28.10 |

* Mean duration of non-employment conditional on non-employment in $\tau = 1$.

wage growth on-the-job and/or endogenous efforts (see Mortensen, 1986).

Figures 4–6 show labor market transition rates by experience. The probability of staying on the same job increases slightly, that of staying non-employed decreases, and that leaving non-employment decreases. Both the employment exit and job-to-job flows have relatively large fluctuations with relatively low negative trend at the early years. All transitions, in Figures 4-6, seem to reach a constant rate after ten years. Furthermore, there are large monthly fluctuations with some evidence for seasonality. The search model should be able to fit the trends and levels of these transition rates, but is not likely to fit the monthly fluctuations.

**Wages and Employment Cycles** We next define employment cycles, in order to set the data in a way that is consistent with the search model (see Wolpin, 1992). Each cycle starts with non-employment and terminates with the last job before a subsequent non-employment spell. Since 55% of individuals in the sample started working before graduation, their first cycle started with their first job instead of non-employment. For an individual $i$, the sequence of cycles is denoted by

$$\{c_i^1(ne_i^1, J1_i^1, J2_i^1, \cdots), c_i^2(ne_i^2, J1_i^2, J2_i^2, \cdots), \cdots\}.$$

For each cycle $j$ and individual $i$, we record the non-employment duration, the wage in the first job, the first job duration, the wage in the second job, the second job duration, and so on.

The NLSY collects data on respondents' usual earnings (inclusive of tips, overtime, and bonuses, before deductions) during every survey year for each employer for whom the respondent worked since the last interview date. The amount of earnings, reported in dollars and cents, is coupled with information on the applicable unit of time, e.g., per day, per hour, per week, per year, etc. Combining earnings and time unit data, the variable "hourly rate of pay job #1-5" in the work history file provides the hourly wage rate for each job. We use coded real hourly wage in 2000 dollars. Nominal wage data are deflated by monthly CPI from BLS CPI-U. We top code the hourly wage at 150$ until 1990, and at 200$ afterwards. Note that, given the way in which the NLSY constructs wage information, we do not exactly have monthly wages. In particular, an individual's hourly wage is constant within a year unless he moves job. Clearly, when we convert nominal wage in real terms, real wages may decrease with inflation, but this is may or may not be the correct actual pay for each month.

We focus on the data of accepted real hourly wage. Table 3 reports the mean accepted wage on the first five jobs in the first three cycles. As expected, mean wages increase with job moves within cycles. When the second cycle starts, the mean accepted wage on the first job is lower than the accepted real wage on the third to fifth jobs in the first cycle. When new cycles start, the mean accepted wage on the first job is lower than the accepted real wage on late jobs of previous cycles. Furthermore, the real accepted wage increases by jobs during the second cycle but this doesn't hold clearly during the cycle. These patterns are potentially consistent with the prediction of our search model, but the empirical analysis requires more formal analysis.

Table 3: Mean wages in the first three cycles of labor market careers

| | Mean wage | Mean wage above $w_M$ | Mean wage below $w_M$ |
|---|---|---|---|
| First Cycle | | | |
| Job 1 | 8.22 (306) | 9.16 (239) | 4.89 (67) |
| Job 2 | 10.37 (192) | 10.75 (179) | 5.17 (13) |
| Job 3 | 11.85 (132) | 11.96 (130) | 4.99 (2) |
| Job 4 | 12.43 (77) | 13.24 (70) | 4.26 (7) |
| Job 5 | 12.64 (42) | 12.83 (41) | 4.87 (1) |
| Second Cycle | | | |
| Job 1 | 10.22 (311) | 11.15 (264) | 4.99 (47) |
| Job 2 | 11.02 (178) | 11.50 (165) | 4.96 (13) |
| Job 3 | 11.49 (84) | 12.21 (77) | 3.47 (7) |
| Job 4 | 13.13 (56) | 13.27 (55) | 5.46 (1) |
| Job 5 | 15.27 (29) | 15.27 (29) | - |
| Third Cycle | | | |
| Job 1 | 9.95 (242) | 10.61 (217) | 4.29 (25) |
| Job 2 | 11.29 (125) | 11.86 (115) | 4.72 (10) |
| Job 3 | 11.81 (71) | 12.22 (67) | 5.02 (4) |
| Job 4 | 10.98 (39) | 11.16 (38) | 4.38 (1) |
| Job 5 | 14.20 (24) | 14.20 (24) | - |

Table 4: Number of months working below the minimum wage

| No. of months | No. of obs. | % |
|---|---|---|
| 0 | 309 | 53.55 |
| 1-6 | 125 | 21.67 |
| 7-12 | 62 | 10.74 |
| 13-24 | 43 | 7.45 |
| 25-36 | 21 | 3.64 |
| 36+ | 17 | 2.95 |
| Total | 577 | 100.00 |

The federal minimum wage for covered nonexempt employees is currently at $5.15 an hour.[7] From Figure 7 we see that between 1978 and 2002, the nominal federal minimum wage increased from $2.65 to $5.15. However, the real minimum wage, deflated by monthly CPI-U and expressed in 2000 dollars, has been decreasing during this sample period. Several states also have minimum wage laws. Where an employee is subject to both the state and federal minimum wage laws, he or she is entitled to the higher of the two. Seven states have no minimum wage law, namely Alabama, Arizona, Florida, Louisiana, Mississippi, South Carolina and Tennessee. Four states have minimum wage rates lower than the Federal level, namely Kansas, New Mexico, Ohio and Virgin Islands. All other states have minimum wage rates that are equal or higher than the Federal level. For the moment, we only take into account the time path of the federal minimum wage in our estimates.

Various minimum wage exceptions apply under specific circumstances to workers with disabilities, full-time students, youths under 20 in their first 90 consecutive calendar days of employment, tipped employees and student learners. A minimum wage of $4.25 per hour applies to young workers under the age of 20 during their first 90 consecutive calendar days of employment with an employer. After 90 days or when the employee reaches age 20, he or she must receive a minimum wage of $5.15. Full-time students can be paid not less than 85% of the minimum wage before they graduate or leave school for good. Student learners aged 16 or more can be paid not less than 75% of the minium wage for as long as they are enrolled in the vocational education program. Again in our baseline analysis, we do not take exemptions into consideration.

Tables 4 and 5 show statistics on pay below the minimum wage. 47 percent of the individuals are observed to work for a wage below the minimum wage for at least one month. For these workers that had a job below minimum wage, the average number of months worked below the minimum wage is 13.5, and the average number of jobs held below the minimum wage is 1.5. The mean job duration below the minimum wage is 8.9 months. These facts indicate that if wages are reported without an error the violation of the minimum wage law is substantial. Mainly among young high school graduate workers.

[7] The federal minimum wage provisions are contained in the Fair Labor Standards Act (FLSA).

Table 5: Number of jobs paying below the minimum wage

| No. of jobs | No. of obs. | % |
|---|---|---|
| 0 | 309 | 53.55 |
| 1 | 172 | 29.81 |
| 2 | 66 | 11.44 |
| 3 | 22 | 3.81 |
| 4 | 6 | 1.04 |
| 6 | 2 | 0.35 |
| Total | 577 | 100.00 |

Table 6 presents the mean durations of non-employment and the first five jobs in the first three cycles, conditional on wages above or below the minimum wage. The mean duration from non-employment to the first job is lower for jobs paying at least the minimum wage. Also, mean duration on jobs paying at least the minimum wage is always longer than mean duration on jobs paying less than the minimum wage.

Table 7 gives the number of individuals making transitions from non-employment to jobs, and between jobs, again conditional on wages above or below the minimum wage. Most transitions to jobs paying less than the minimum wage originate in non-employment, and most workers earn wages above the minimum wage once they switch job. Very few workers move from a job paying more than the minimum wage to one paying less than the minimum wage, and in the model we assume that this could be only a result of measurement error in wages.

The data show evidence of wage growth on-the-job, and we assume a constant wage growth rate $g$ in all jobs, which can be interpreted as the return to both general and job-specific experience. In order to estimate $g$, we consider wage observations in a given job:

$$\ln w_{i\tau} = \ln w_{i0} + \tau \ln(1+g)$$

where $w_{i0}$ is the first wage observation for individual $i$ and $\tau$ is job tenure. The OLS estimate of $g$ is 0.2%. The corresponding annual growth rate is $(1+g)^{12} - 1 = 2.43\%$. (see Figure 8a). However, if we only use the first and last wage observations on each job to compute the growth rate and take the average across jobs, the resulting annual growth rate is -0.75%. (see Figure 8b). The two estimates are very different, as jobs with low or negative wage growth tend to last relatively shorter, and are thus assigned a lower weight when using the first method. In particular, jobs with tenure shorter than two years have negative annual growth on average (-1.62% for shorter tenures than one year, -0.19% for tenures between one and two years). Jobs with tenure longer than two years have an average growth rate 1.74% annual.[8]

[8] It should be noted that the focus of this paper is not on wage growth, which requires a separate analysis.

Table 6: Mean duration of nonemployment and jobs in months

| | First Cycle | Second Cycle | Third Cycle |
|---|---|---|---|
| NE | 7.3 (116) | 5.8 (311) | 5.8 (242) |
| To job 1 above $w_M$ | 7.2 (87) | 5.3 (264) | 5.0 (217) |
| To job 1 below $w_M$ | 7.7 (29) | 8.7 (47) | 12.6 (25) |
| Job 1 | 29.6 (306) | 27.5 (311) | 23.9 (242) |
| Above $w_M$ | 31.0 (239) | 29.4 (264) | 25.0 (217) |
| Below $w_M$ | 24.5 (67) | 17.3 (47) | 14.4 (25) |
| Job 2 | 40.3 (192) | 27.5 (178) | 27.8 (125) |
| Above $w_M$ | 42.3 (179) | 27 (165) | 29.8 (115) |
| Below $w_M$ | 12.9 (13) | 34.4 (13) | 5.2 (10) |
| Job 3 | 39.4 (132) | 25.9 (84) | 21.1 (71) |
| Above $w_M$ | 39.8 (130) | 26.9 (77) | 22.1 (67) |
| Below $w_M$ | 13.5 (2) | 14.9 (7) | 3.5 (4) |
| Job 4 | 31.2 (77) | 21.5 (56) | 19.6 (39) |
| Above $w_M$ | 31.0 (70) | 21.7 (55) | 18.9 (38) |
| Below $w_M$ | 32.7 (7) | 12 (1) | 45 (1) |
| Job 5 | 25.2 (42) | 18.5 (29) | 20.6 (24) |
| Above $w_M$ | 25.7 (41) | 18.5 (29) | 20.6 (24) |
| Below $w_M$ | 5 (1) | - | - |

# 4 Estimation

*Specification.* We estimate the model using simulated moments. We first restrict the model to have exogenous arrival rates without search efforts and no wage growth on the job, so $g = 0$. We allow for unobserved heterogeneity in arrival rates and the value of unemployment such that $\lambda_{n1}, \lambda_{e1}, \lambda_{n2}, \lambda_{e2}$ are the non-employment and employment arrival rates of the two types, $b_1$ and $b_2$ are the value of unemployment of the two types (later we will make it also a function of unobserved and observed variables) and $\pi$ is the proportion of type one. The wage density function is assumed to be log normal, $\ln w_i \sim N(\mu_i, \sigma_w^2)$, $\mu_i = \beta' x_i$, for the application now we assume that $\mu_i = \mu$ . The time preference parameter $r$ is known to be 4% annually, which is 0.3% monthly. There is a measurement error to the observed wages, such that the true wage is $w$ and the observed wage is, $w^o = w + u$, where $\ln u \sim N(0, \sigma_u^2)$. Let $\delta$ be the rate of job destruction and $\alpha$ be the level of compliance with the minimum wage. 55% of our sample worked before graduation. We assume that a separate labor market exists during pre-graduation periods, and we characterize it by the initial reservation wage $w_0^*$[9]. The parameters of the model to be estimated are in the vector

[9] We make this assumption to match the initial wage. In the model, we assume 45% of the individuals non-employed and 55% of them employed at $\tau = 0$. To simulate their labor market status and wage in $\tau = 1$, we first draw one wage from the log normal distribution for each individual. If he is non-employed, we compare the wage to his reservation wage, which is determined by the underlying parameters of the model,

Table 7: Transitions to employment and from job-to-job (no. of obs.)

| | First Cycle | Second Cycle | Third Cycle |
|---|---|---|---|
| Unemployed | (116) | (311) | (242) |
| UE to J1 above $w_M$ | 87 | 264 | 217 |
| UE to J1 below $w_M$ | 29 | 47 | 25 |
| First Job above $w_M$ | (239) | (264) | (217) |
| Move to J2 above $w_M$ | 93 | 123 | 88 |
| Move to J2 below $w_M$ | 4 | 6 | 6 |
| First Job below $w_M$ | (67) | (47) | (25) |
| Move to J2 above $w_M$ | 22 | 14 | 11 |
| Move to J2 below $w_M$ | 4 | 2 | 2 |
| Second Job above $w_M$ | (179) | (165) | (115) |
| Move to J3 above $w_M$ | 86 | 67 | 50 |
| Move to J3 below $w_M$ | 2 | 5 | 2 |
| Second Job below $w_M$ | (13) | (13) | (10) |
| Move to J3 above $w_M$ | 4 | 5 | 5 |
| Move to J3 below $w_M$ | 0 | 2 | 1 |
| Third Job above $w_M$ | (130) | (77) | (67) |
| Move to J4 above $w_M$ | 59 | 48 | 33 |
| Move to J4 below $w_M$ | 6 | 0 | 1 |
| Third Job below $w_M$ | (2) | (7) | (4) |
| Move to J4 above $w_M$ | 2 | 2 | 1 |
| Move to J4 below $w_M$ | 0 | 0 | 0 |
| Fourth Job above $w_M$ | (70) | (55) | (38) |
| Move to J5 above $w_M$ | 31 | 28 | 18 |
| Move to J5 below $w_M$ | 1 | 0 | 0 |
| Fourth Job below $w_M$ | (7) | (1) | (1) |
| Move to J5 above $w_M$ | 6 | 0 | 1 |
| Move to J5 below $w_M$ | 0 | 0 | 0 |

$\theta = [\lambda_{n1}, \lambda_{e1}, \lambda_{n2}, \lambda_{e2}, \pi, b_1, b_2, \mu, \sigma_w, \sigma_u^2, \delta, \alpha, w_0^*]'$.

*Data:* We have a sample of white male high school graduates index by $i = 1, ..., 577$. We observe their employment status and wage if employed every month after high school graduation. The data do not differentiate between unemployment and out of labor force thus employment and non-employment are the only labor market states. Let $d_{i\tau_i} = 1$ if the individual is working and $d_{i\tau_i} = 0$ if the individual is not employed, where $\tau_i$ is the month after graduation or the month in the labor market. We observe the following data: $[d^D_{i\tau_i}, w^{Do}_{i\tau_i}]$ for $i = 1, \cdots, 577$ and $\tau_i = 1, \cdots, T_i$, where $d^D_{i\tau_i}, w^{Do}_{i\tau_i}$ are the observed labor market states and observed wages. It should be noted that we may miss some observations on states as well as observations on wages for many periods even if employment is observed.

*Simulations:* We simulate employment status and wages from the model in a consistent way as in the data. We simulate both a conditional monthly predicted values that depend on the observed (data) values in the relevant state (previous month) and an unconditional monthly predicted values that depend *only* on the simulated values in the relevant state. Consider first all the individuals that have observations from $\tau_i = 1, ...T_i$. These are observations without left censuring. In a conditional simulation $s$, the model predicts $d^s_{i\tau_i}$ and $w^s_{i\tau_i}$ conditional on $d^D_{i\tau_i-1}$ and $w^D_{i\tau_i-1}$ in the data. Specifically, at the starting period, $\tau_i = 1$, the individual is either employed or non-employed, with 55% employed at graduation as we observe in the data. Consider individual $i$ in this sub-sample. Given the value of parameters, we solve for the reservation wage and simulate the values of $[d^s_{i\tau_i=1}, w^s_{i\tau_i=1}]$ for $N^S = 25$ simulations. The simulations should also consider the unobserved uncertainty of types and the measurement error. At $\tau_i = 2$, if the individual is working and the wage is observed in the first period, simulate the measurement error to get the "true" wage from the data following $w^D_{i\tau_i} = w^{Do}_{i\tau_i} - u$. Conditional on that "true" wage in $\tau_i = 1$, simulate the outcome for $\tau_i = 2$, i.e. $[d^s_{i\tau_i=2}, w^s_{i\tau_i=2}]$. Follow that for $N^S = 25$ simulations. Now at $\tau_i = 3$, we repeat the same as in the previous case. Now we generate a sequence of simulated ($N^S = 25$) observations $[d^s_{i\tau_i}, w^s_{i\tau_i}]$ for $\tau_i = 1, ..., T_i$, that follow the true sequence $[d^D_{i\tau_i-1}, w^D_{i\tau_i-1}]$ for $\tau_i = 1, ..., T_i$ such that in each case that the wage is not observed, the simulated wage is dropped from the simulated sample for all simulations. In the unconditional simulation, the prediction of $[d^s_{i\tau_i}, w^s_{i\tau_i}]$ does not conditional on data in the last period $[d^D_{i\tau_i-1}, w^D_{i\tau_i-1}]$ but on the last period simulations $[d^s_{i\tau_i-1}, w^s_{i\tau_i-1}]$.

For the left censored observations, suppose that the first observation is at $\tau_i = 2$. Now the simulation for that period is based on the sequence of two simulations. First, simulate period $\tau_i = 1$, and conditional on the outcome in the first period, simulate $\tau_i = 2$. The outcome is the $[d^s_{i\tau_i=2}, w^s_{i\tau_i=2}]$. The period to the future are the same as above for the observations that are not left censored. For the case that the first observation is at $\tau_i = 3$, the first simulation is for 3 periods, etc. etc. Now we have ($N^S = 25$) simulations based on the vector of parameters $\theta$, based on the independent uncertainty of the model that is described above, that follows the sample exactly for each individual at each period.

*Moments:* We use moments from both the conditional and the unconditional simulations.

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and he accepts the offer as long as the wage draw exceeds the reservation wage. If he was employed, we draw his wage such that it is at least greater than $w_0^*$.

The conditional moments include the non-employment rate, $mne$; the proportion of individuals that move from non-employment to employment, $mtr_1$; the proportion of individuals that move from old job to new job, $mtr_2$; mean wage, $mw_1$; and the variance of wage, $mw_2$. All the moments are by month in the labor market.[10] We first compute all these moment from the data, namely, $mne^D, mtr_1^D, mtr_2^D, mw_1^D, mw_2^D$. For each simulation, the same moments are computed. Then the simulated moments are the averages over all simulations, which we denote by $mne^S(\theta)$, $mtr_1^S(\theta)$, $mtr_2^S(\theta)$, $mw_1^S(\theta)$, $mw_2^S(\theta)$. The unconditional moment we used is the proportion of individuals that work below the minimum wage. $mp^D$ and $mp^S$ denote the moment in the data and in the simulation respectively.[11]

*Implementation:* We actually implement the SGMM by using a two-step iterative estimation procedure. In the first step, we fix $\theta_2 = [\alpha, w_0^*]$, and use the one-period-ahead conditional moments to estimate $\theta_1 = [\lambda_{n1}, \lambda_{n2}, \lambda_{e1}, \lambda_{e2}, b_1, b_2, \pi, \mu, \sigma_w, \sigma_u^2, \delta]$. The conditional moments used are the non-employment rate, the proportion moving from non-employment to employment, the proportion moving from job to job, the mean wage and the wage variance in each period. We put all these moments in a vector,

$$\begin{aligned} g_1(\theta_1)' &= [mne^{D\prime} - mne^S(\theta_1;\theta_2)', mtr_1^{D\prime} - mtr_1^S(\theta_1;\theta_2)', mtr_2^{D\prime} - mtr_2^S(\theta_1;\theta_2)', \\ & \quad mw_1^{D\prime} - mw_1^S(\theta_1;\theta_2)', mw_2^{D\prime} - mw_2^S(\theta_1;\theta_2)']. \end{aligned}$$

Now the objective function to be minimized with respect to $\theta_1$ is,

$$J_1(\theta_1) = g_1(\theta_1)' W g_1(\theta_1),$$

where the weighting matrix $W$ is set to be diagonal. The weight on each moment is set to be one over its sample mean.

In the second step $\theta_1$ is fixed at its estimated values from the first step and we use the unconditional moments. We first estimate $w_0^*$ by matching the mean wage and proportion of individuals working below the minimum wage at $\tau_i = 1$. Then we estimate $\alpha$ using the moments of proportion of individuals working below the minimum wage in each period. That is, we define

$$g_2(\alpha)' = [mp^{D\prime} - mp^S(\alpha;\theta_1, w_0^*)'],$$

and we minimize the objective function

$$J_2(\alpha) = g_2(\alpha)' g_2(\alpha),$$

with respect to $\alpha$.

This procedure is replicated until convergence between the estimates for $\theta_2$ obtained in the second step and $\theta_1$ obtained in the first step. We chose to identify $\lambda_{n1}, \lambda_{n2}, \lambda_{e1}, \lambda_{e2}, b_1, b_2, \pi, \mu, \sigma_w^2, \sigma_u^2$ and $\delta$ using conditional moments, since unconditional moments would not do as good a job in matching wage profiles, as wages would be rising fast upon labor market entry and then stay basically unchanged.

[10] We have 18 years' data. So each moment is a vector of 216 elements.

[11] See Appendix B for the exact defenitions of the moments.

*The Asymptotic Theory of the SGMM :* Let $y_{i\tau} = [d^D_{i\tau_i}, w^{Do}_{i\tau_i}]$ be a vector of observed data for individual $i$ at some point $\tau$ over labor market experience. Given the observed state vector at $\tau$ for individual $i$, $z_{i\tau}$, the simulated values of the random events at simulation $s$, $\varepsilon_s(z_{i\tau})$, and the value of the parameters $\theta^*$ the model implies that,

$$y^s_{i\tau} = G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta^*).$$

The function $G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta^*)$ is given by the solution to the model. We assume that the data $\{y_{i\tau}, z_{i\tau}\}^I_{i=1}$ for all $\tau$ are *i.i.d.* By the independence of the simulated random variables we have the orthogonality condition that $E[G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta^*) - y_{i\tau} \mid z_{i\tau}] = 0$. Now for $N^S$ simulations of $\varepsilon_s(z_{i\eta})$, we define $h(y_{i\tau})$ as the contribution of individual $i$ for the vector of data moments at time $\tau$, and $h(y^s_{i\tau})$ as the contribution of simulation $s$ of individual $i$ for the vector of simulated moments at time $\tau$.

$$g_{\tau I}(\theta) = [\frac{1}{I_\tau}\sum_{i=1}^{I_\tau} h(y_{i\tau}) - \frac{1}{N^S}\sum_{s=1}^{N^S}(\frac{1}{I_\tau}\sum_{i=1}^{I_\tau} h(y^s_{i\tau})]$$

and we have the result that $g_{\tau I}(\theta) \longrightarrow 0$ as $I \longrightarrow \infty$. And under the standard regularity conditions $\theta \longrightarrow \theta^*$. Note that for any function of $z_{i\tau}$ that multiply $y_{i\tau} - \frac{1}{N^S}\sum_{s=1}^{N^S} G(z_{i\tau}, \varepsilon_s(z_{i\tau}); \theta)$ and the average of this product converges to zero as $I$ converges to infinity.[12]

# 5 Results

*Parameters:* The estimates of the parameters are presented in Table 8.[13]

The parameter estimates have plausible magnitudes and are in line with previous estimates of the parameters of a search model with search on-the-job. That is, the arrival rates of job offers is higher for non-employed than employed individuals and these rates are different by the two types of individuals, such that the mean hazard rate is decreasing. Type 1 individuals, who are about 76 percent of workers, have lower non-employment utility and lower arrival rates of offers both as non-employed and employed. Furthermore, type 1 workers have a lower value for non-employment and a reservation wage for non-employment of about \$4.49. Type 2 workers have a higher value for non-employment and a reservation wage of \$5.61.[14] Hence, type 2 may possibly work below the minimum wage only in the first six years following the graduation of high school (see Figure 7). The estimated reservation wage for workers before high school graduation is \$4.71, which is just above the reservation wage of type 1 and much below the mean reservation wage for high school graduates.

[12]For a recent survey of the asymptotic distribution of the estimated parameters, tests and references see Carrasco and Florens (2002).

[13]It turned out at the measurement error varinace, $\sigma_u$, could not be well identified and it seemed to be equal to zero at the point estimates we report here. This result requires some additional analysis. The stadard errors are not presented as these are preliminary results.

[14]It should be noted that the reservation wage depends on the level of the minmum wage, but using the data it has a negligible impact on the actual value of the reservation wage.

Table 8: Estimates of the parameters of a search model with $g = 0$.

| Parameters | Estimates |
|---|---|
| $\lambda_{n1}$ | 0.457 |
| $\lambda_{n2}$ | 0.847 |
| $\lambda_{e1}$ | 0.137 |
| $\lambda_{e2}$ | 0.188 |
| $\pi$ | 0.764 |
| $b_1$ | 4.487 |
| $b_2$ | 5.612 |
| $\mu$ | 1.170 |
| $\sigma_w$ | 0.953 |
| $\delta$ | 0.032 |
| $w_0^*$ | 4.741 |
| $\alpha$ | 0.604 |

Notes. The sample includes male high-school graduates from the NLSY. Number of observations: 577. Estimation methods: Simulated GMM.

The novelty of our results consists in providing an estimate for the extent of compliance of firms' job offers to the minimum wage regulations. We find that the arrival rate of job offers below the minimum wage is 40% lower than the job offers above the minimum wage. This looks very close to the results in Ashenfelter and Smith (1979) and to more recent figures reported by the U.S. Bureau of Census (1997, p. 433).

However, there are good reasons to believe that our estimate for $\alpha$ is an over-estimate of the true non-compliance of firms. First, observed wages below the minimum wage may stem from measurement error, such that the true wage is at or above the minimum wage, but due to measurement error the observed wage is not. As we implicitly assume that wages are observed without error, any wage observation below the minimum wage is attributed to non-compliance. Second, there are (limited) categories of workers who are exempt from minimum wage regulations. And, finally, we use as minimum wage the current \$5.15 federal value, and assume that it is applied equally in all states. However, there are states that do not have minimum wage regulations, or that adopt a minimum wage below the federal level. On the other hand, there are also states that adopt a minimum wage above the federal level, and this should deliver an overestimate of $\alpha$ in our empirical model. All these possibilities will be considered as extensions to the current bare-bones empirical model.

*Fit:* To assess the goodness of fit of the estimated model we show the model predictions with data moments used to estimate the parameters. Figures 9–13 report the one-period-ahead model predictions and data moments that we used to estimate all the parameters but $\alpha$. Both the model simulated moments and the corresponding data moments is computed for each month following high-school graduation. The model one-period-ahead simulated predictions of the non-employment rate (Figure 9), the mean wage (Figure 10) and the wage variance (Figure 11) fit very well the level, the trend and the fluctuations in the data. The fit to the fluctuations is a result that we look at the cross-section conditional means where the change in actual choice in the previous period affects dramatically the predicted response in the current period. When we compare unconditional means of these moments the model does not fit the large fluctuations but is able to fit the levels and less accurately to fit the trend. The predicted transitions from non-employment to work (Figure 12) and job-to-job fit well the levels and trends but not the very large fluctuations in the actual transitions moments.[15]

*Counterfactuals:* We perform two counterfactual policy simulations. The first is an increase of 20% in the minimum wage over the sample period and, the second is an increase in compliance from 0.604 to 0.302. We use the parameter estimates reported above and we look at the impact on the predicted moments of the model as a result of this policy. Figure 15 shows that as a result of the first policy, the proportion of workers that are paid below the minimum wage increases dramatically. The non-employment rate increases, but by a negligible rate and the mean accepted wage increases by 10 to 20 cents per hour. Since the lower arrival rates of jobs below the minimum wage covers a larger range of wages, the probability of getting a job falls. however, it falls only very slightly from 2.0 to 1.9% per month on average. The effect of the second policy simulations is that the proportion of workers that are paid below the minimum wage decreases and the mean accepted wage increases slightly (see Figure 16).

[15] We will aslo check the fite to unconditional moments and to moments related to the employment cycle data.

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# Appendix A: Computation of $V_e'()$

Using integration by parts, (3) and (4) can be rewritten as:

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] + \\ \lambda_e \int_{w_\tau} [1-F(w)]V_e'(w)\,dw + g w_\tau V_e'(w_\tau) \text{ for } w_\tau &\geq w_M \end{aligned} \tag{14}$$

and

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\ &\quad +\lambda_e \int_{w_M} [1-F(w)]V_e'(w)\,dw \\ &\quad +\lambda_e \int_{w_\tau}^{w_M} V_e'(w)\left[1-\alpha F(w) - (1-\alpha)F(w_M)\right]dw \\ +g w_\tau V_e'(w_\tau) \text{ for } w_\tau &< w_M \end{aligned} \tag{15}$$

By differentiating (14) and (15):

$$g w_\tau V''(w_\tau) = \left\{r + \delta + \lambda_e\left[1 - F(w_\tau)\right] - g\right\} V'(w_\tau) - 1, \text{ for } w_\tau \geq w_M \tag{16}$$

$$g w_\tau V''(w_\tau) = \left\{r + \delta + \lambda_e\left[1 - \alpha F(w_\tau) - (1-\alpha)F(w_M)\right] - g\right\} V'(w_\tau) - 1, \text{ for } w_\tau < w_M \tag{17}$$

which implies:

$$V_e'(w_\tau) = e^{R(w_0,\tau)}\left[A - \int_0^\tau e^{-R(w_0,t)}dt\right] \tag{18}$$

where $A$ is an arbitrary constant and

$$\begin{aligned} R(w_0,\tau) &= \int_{w_0}^{w_\tau} \frac{r+\delta+\lambda_e[1-F(w)]-g}{gw}dw \\ &= \int_0^\tau \left[r+\delta+\lambda_e[1-F(w_\tau)]-g\right]d\tau \\ &= (r+\delta-g)\,\tau + \lambda_e \int_0^\tau \left[1-F(w_\tau)\right]d\tau, \text{ for } w_\tau \geq w_M \end{aligned} \tag{19}$$

$$R(w_0,\tau) = (r+\delta-g)\,\tau + \lambda_e \int_0^\tau \left[1-\alpha F(w_\tau) - (1-\alpha)F(w_M)\right]d\tau, \text{ for } w_\tau < w_M \tag{20}$$

Having set $\tau = 0$ in (18), one obtains $V_e'(w_0) = A$ and thus

$$V_e'(w_\tau) = e^{R(w_0,\tau)}\left[V_e'(w_0) - \int_0^\tau e^{-R(w_0,t)}dt\right]. \tag{21}$$

A solution to the forward differential equations (16) and (17) therefore exists for all $w_0$ if $r+\delta>g$ and then

$$V_e'(w_0)=\int_0^{\infty}e^{-R(w_0,\tau)}d\tau. \tag{22}$$

To check that (22) is correct, imagine that, in general, the starting time of the process is $s$, and compute the corresponding solution:

$$V_e'(w_s)=\int_s^{\infty}e^{-R(w_s,\tau)}d\tau. \tag{23}$$

If (22) is correct, differentiation of (23) with respect to $s$ should give the initial differential equations (16) and (17):

$$\begin{aligned}
V_e''(w_s)gw_s &= -\int_s^{\infty}\frac{dR(w_s,\tau)}{ds}e^{-R(w_s,\tau)}d\tau-e^{-R(w_s,s)} && (24)\\
&= \int_s^{\infty}\left[r+\delta+\lambda_e[1-F(w_\tau)]-g\right]e^{-R(w_s,\tau)}d\tau-1\\
&= \left[r+\delta+\lambda_e[1-F(w_\tau)]-g\right]V_e'(w_s)-1,\text{ for } w_\tau\geq w_M && (25)\\
V_e''(w_s)gw_s &= \left[r+\delta+\lambda_e\left[1-\alpha F(w_\tau)-(1-\alpha)F(w_M)\right]-g\right]V_e'(w_s)-1,\text{ for } w_\tau<w_M && (26)
\end{aligned}$$

As (25) and (26) are equivalent to (16) and (17), the hypothesized solution (22) is correct.

Finally, using (22):

$$V_e(w_0)=\int_{w^*}^{w_0}V_e'(w)dw=\int_0^{\infty}e^{-R(w,t)}d\tau dw. \tag{27}$$

# 6 Appendix B: Moments

To compute the moments in the data, we use following formulas. Note that all moments are calculated by each month in the labor market $\tau=1,2,\cdots,216$. For example, $mne^D$ is a column vector of 216 dimensions and each element $mne^D(\tau)$ is determined by

$$mne^D(\tau)=\frac{\sum_i I(d_{i\tau}^D=0)}{\sum_i I(d_{i\tau}^D=0)+\sum_i I(d_{i\tau}^D=1)};$$

$I(\cdot)$ is an indicator function, which equals one if the condition is satisfied and equals zero otherwise. Similarly

$$mtr_1^D(\tau)=\frac{\sum_i I(d_{i\tau-1}^D=0,d_{i\tau}^D=1)}{\sum_i I(d_{i\tau-1}^D=0)+\sum_i I(d_{i\tau-1}^D=1)};$$

$$mtr_2^D(\tau)=\frac{\sum_i I(d_{i\tau-1}^D=1,d_{i\tau}^D=1)}{\sum_i I(d_{i\tau-1}^D=0)+\sum_i I(d_{i\tau-1}^D=1)},$$

where $d^D_{i\tau-1} = 1$ and $d^D_{i\tau} = 1$ refer to two different jobs;

$$mw^D_1(\tau) = \frac{\sum_i (w^D_{i\tau} | w^D_{i\tau} > 0)}{\sum_i I(d^D_{i\tau} = 1 | w^D_{i\tau} > 0)};$$

$$mw^D_2(\tau) = \frac{\sum_i ((w^D_{i\tau} - mw^D_1(\tau))^2 | w^D_{i\tau} > 0)}{\sum_i I(d^D_{i\tau} = 1 | w^D_{i\tau} > 0)};$$

$$mp^D(\tau) = \frac{\sum_i I(w^D_{i\tau} < MW(\tau) | w^D_{i\tau} > 0)}{\sum_i I(d^D_{i\tau} = 1 | w^D_{i\tau} > 0)},$$

where $MW(\tau)$ is the minimum wage.

Similarly, the formulas for the simulated moments are

$$mne^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d^s_{i\tau} = 0)}{\sum_i I(d^s_{i\tau} = 0) + \sum_i I(d^s_{i\tau} = 1)};$$

$$mtr^S_1(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d^s_{i\tau-1} = 0, d^s_{i\tau} = 1)}{\sum_i I(d^s_{i\tau-1} = 0) + \sum_i I(d^s_{i\tau-1} = 1)};$$

$$mtr^S_2(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d^s_{i\tau-1} = 1, d^s_{i\tau} = 1)}{\sum_i I(d^s_{i\tau-1} = 0) + \sum_i I(d^s_{i\tau-1} = 1)},$$

where $d^s_{i\tau-1} = 1$ and $d^s_{i\tau} = 1$ refer to two different jobs;

$$mw^S_1(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i (w^s_{i\tau} | w^s_{i\tau} > 0)}{\sum_i I(d^s_{i\tau} = 1 | w^s_{i\tau} > 0)};$$

$$mw^S_2(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i ((w^s_{i\tau} - mw^s_1(\tau))^2 | w^s_{i\tau} > 0)}{\sum_i I(d^s_{i\tau} = 1 | w^s_{i\tau} > 0)};$$

$$mp^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(w^s_{i\tau} < MW(\tau) | w^s_{i\tau} > 0)}{\sum_i I(d^s_{i\tau} = 1 | w^s_{i\tau} > 0)},$$

where $MW(\tau)$ is the minimum wage and $N^S = 25$ is the total number of simulations.

**Figure 1: Monthly employment and nonemployment rates**

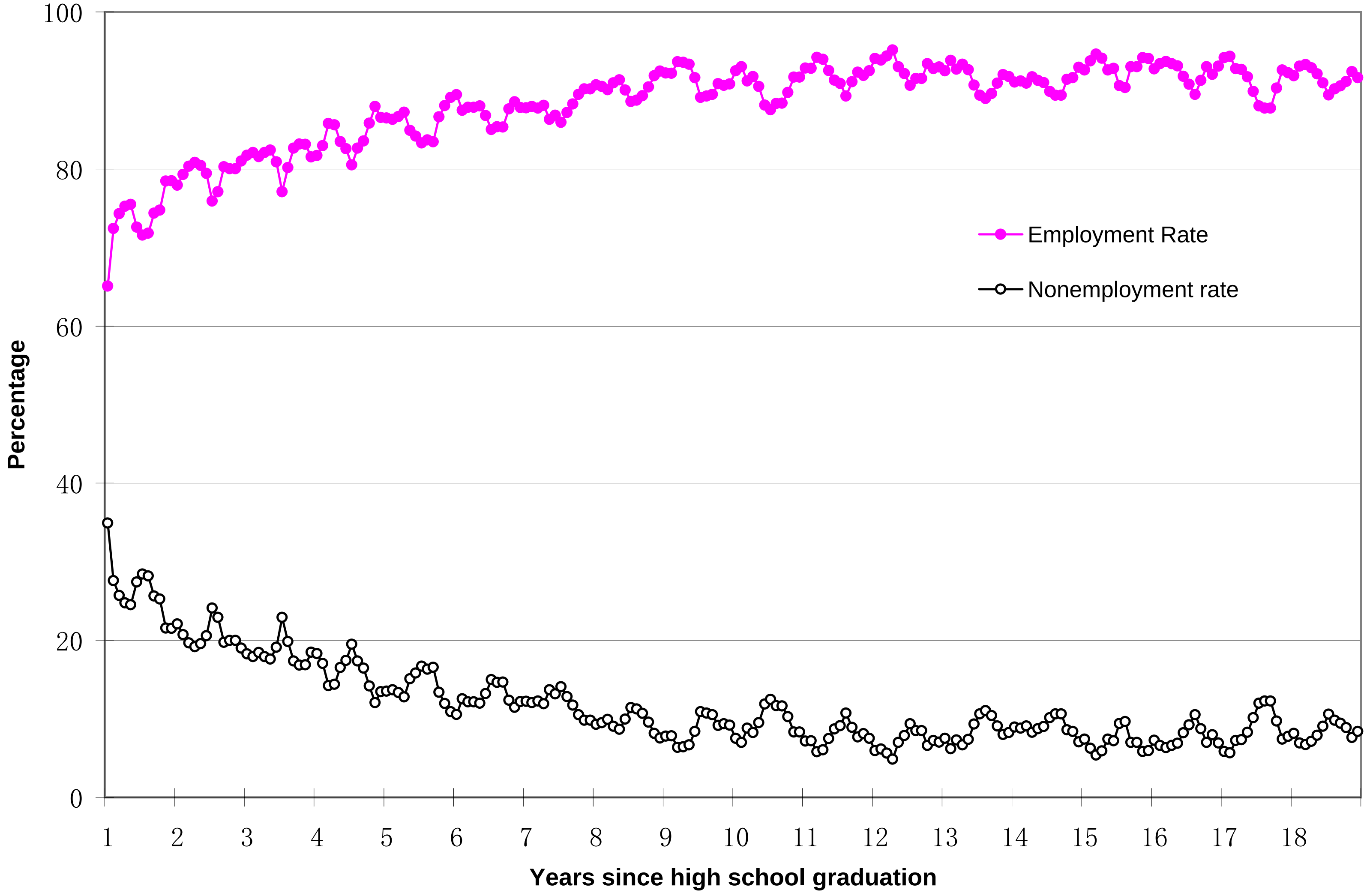


**Figure 2: Proportion of employed by months preceding high school graduation**

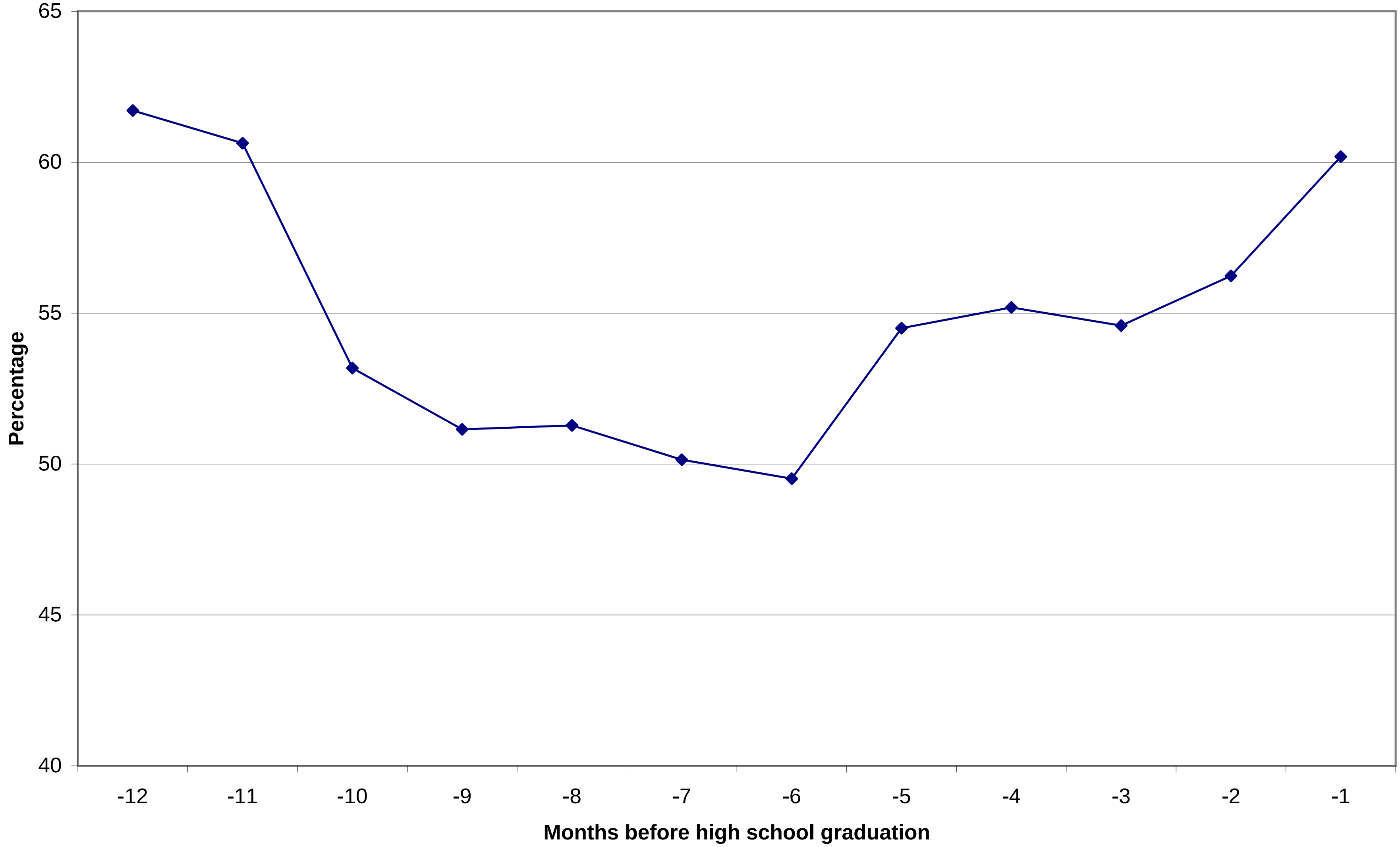


Figure 3: The job separation hazard, by job tenure (in months)

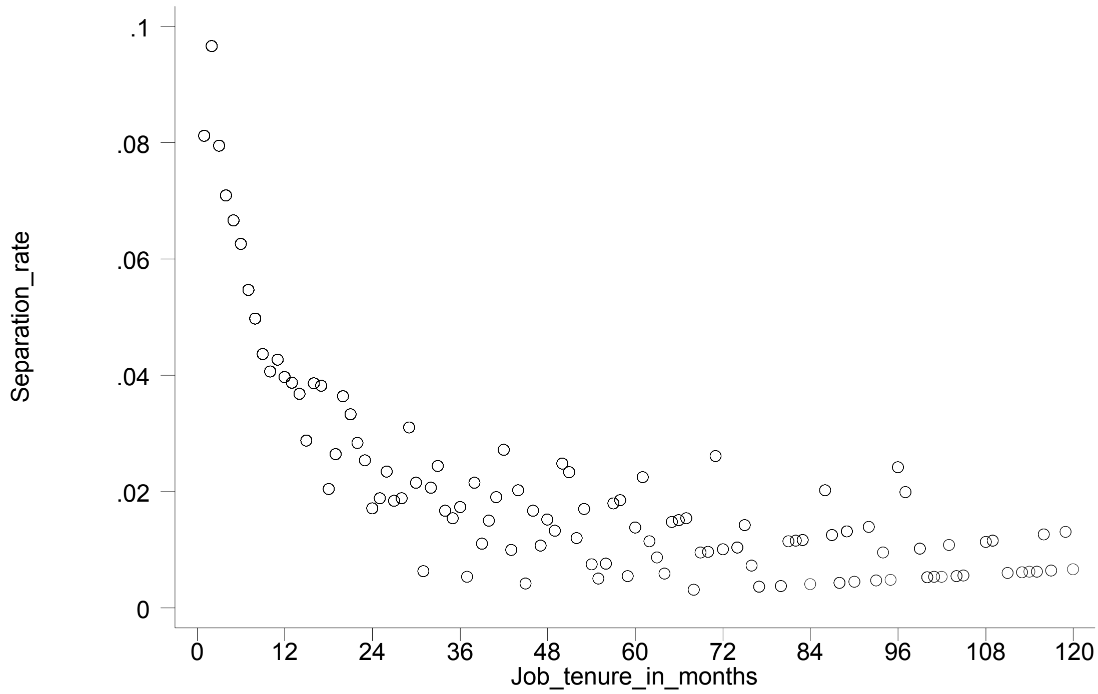


**Figure 4: The proportion of workers who stay on the same job, stay nonemployed, and move from employment to nonemployment per month**

**Figure 5: The proportion of workers who move from nonemployment to employment per month**

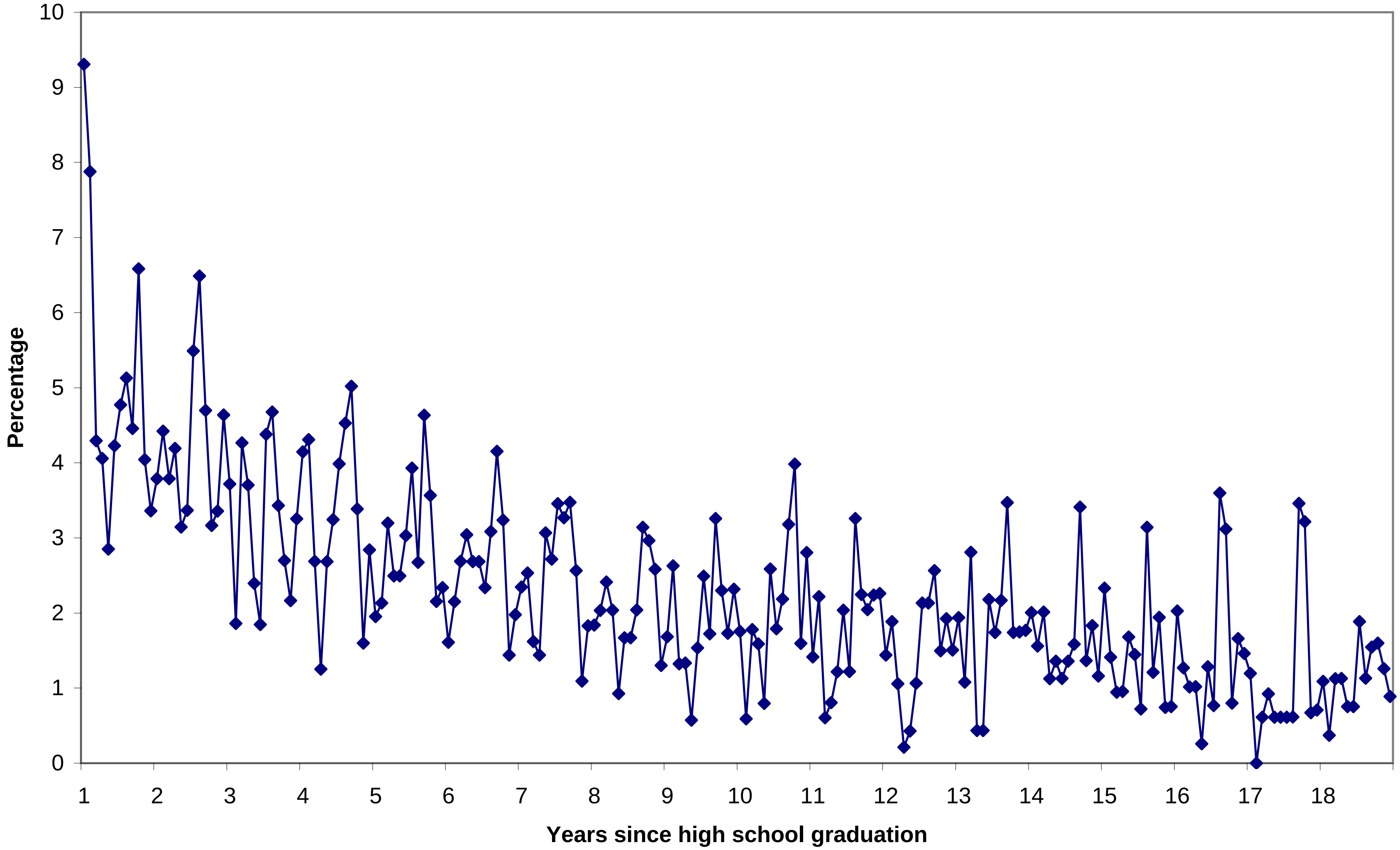


**Figure 6: The proportion of workers moving from job to job per month**

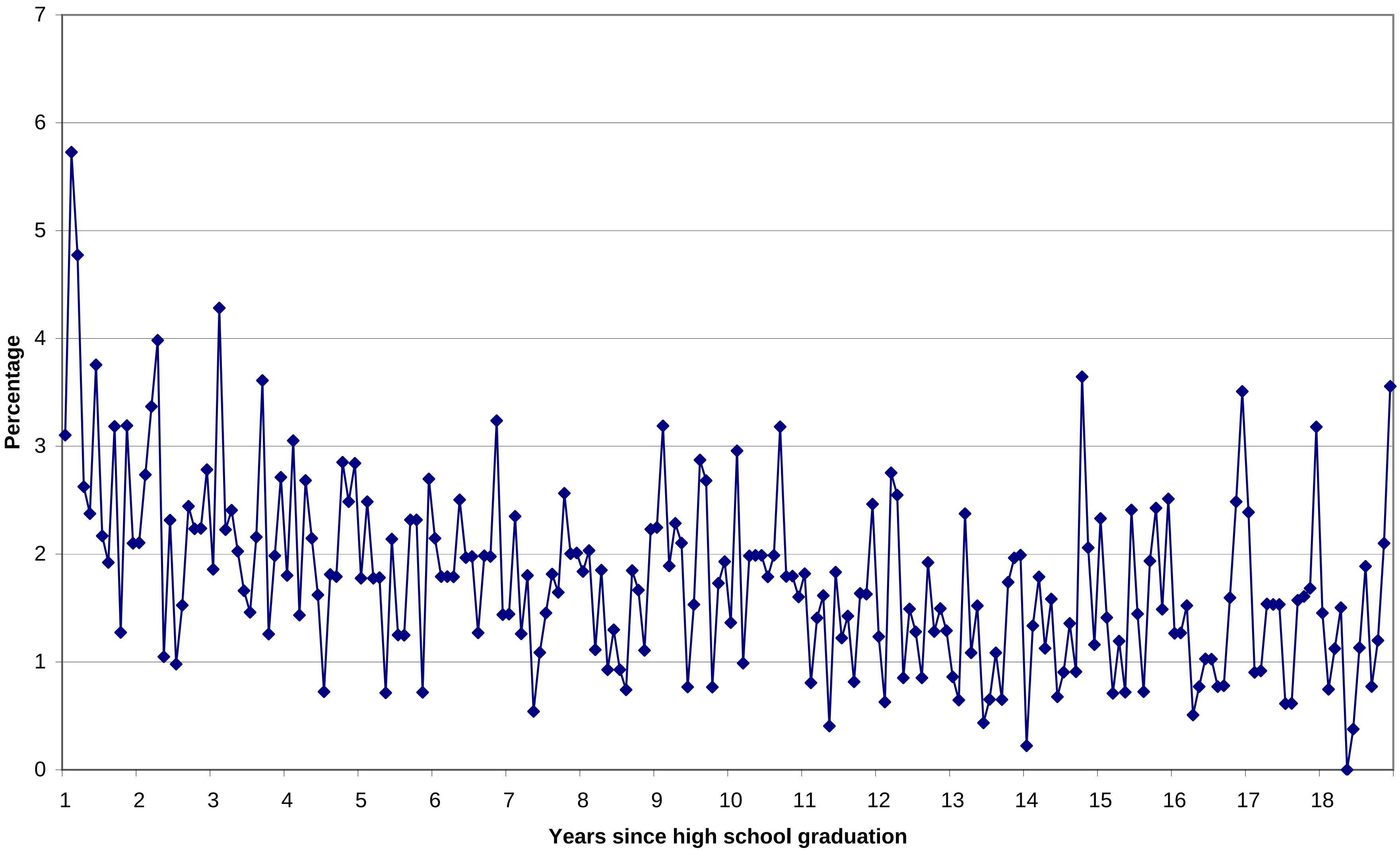
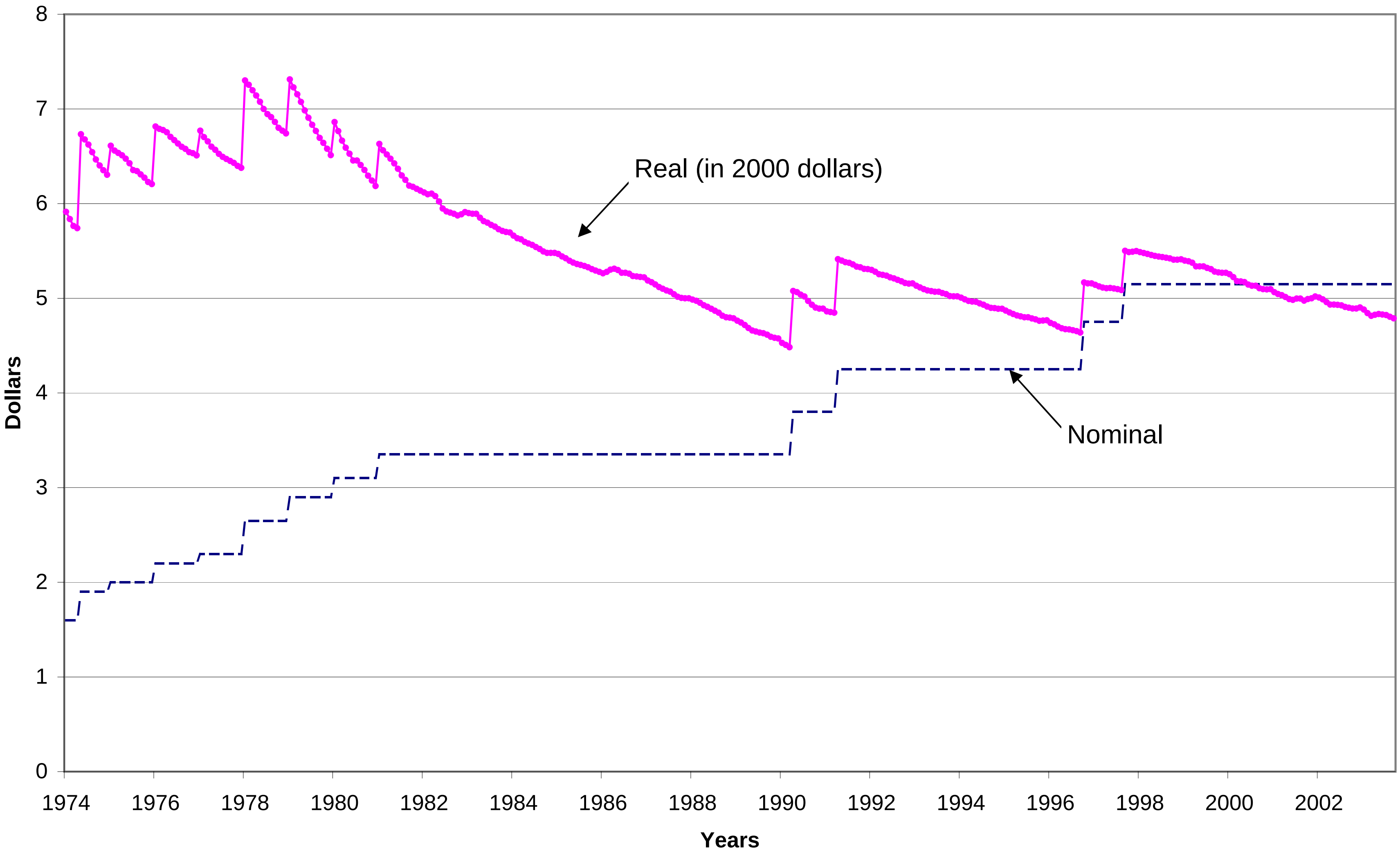


**Figure 7: Federal minimum wage under the fair labor standards act**



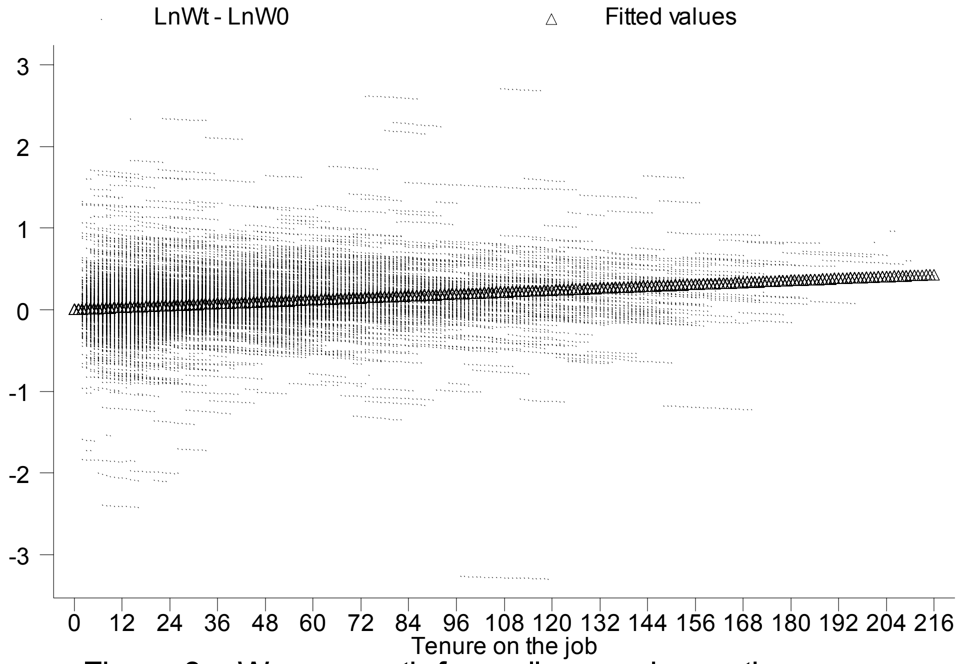



Figure 8a: Wage growth from all wage observations

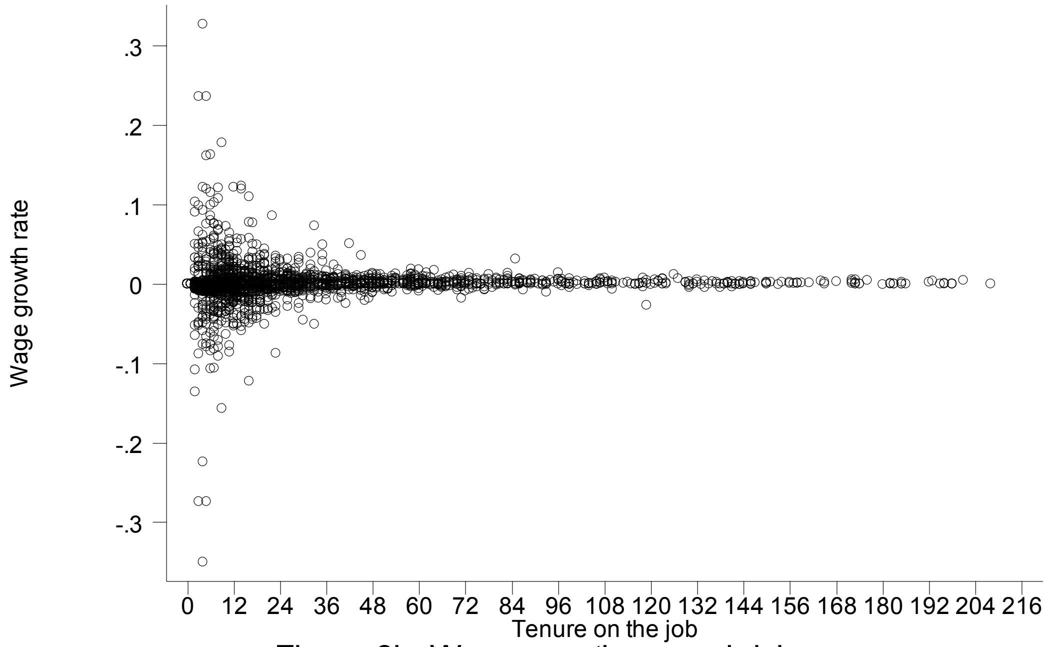



Figure 8b: Wage growth on each job

Figure 9: Actual and predicted monthly nonemployment rate

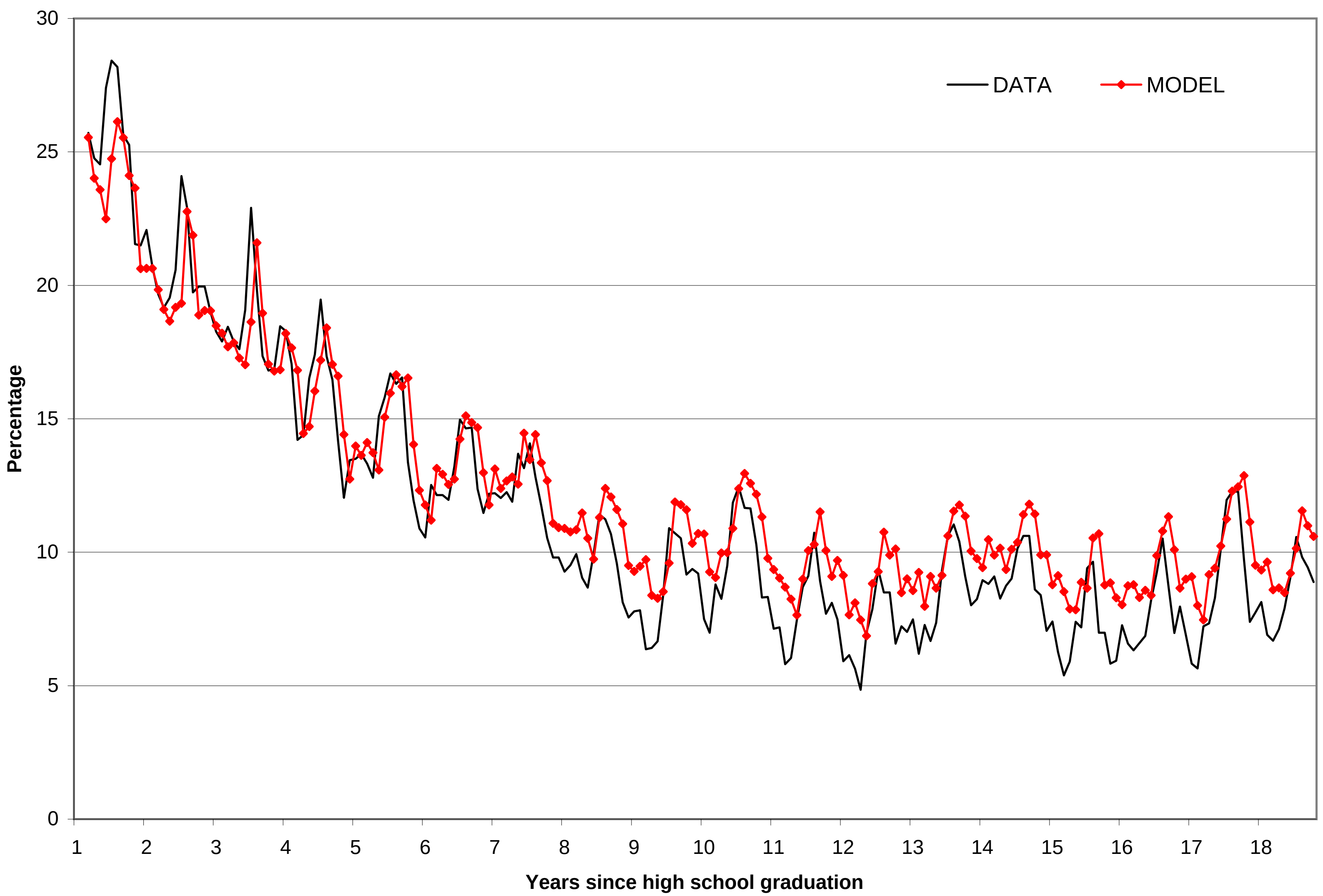


**Figure 10: Actual and predicted monthly mean wage**

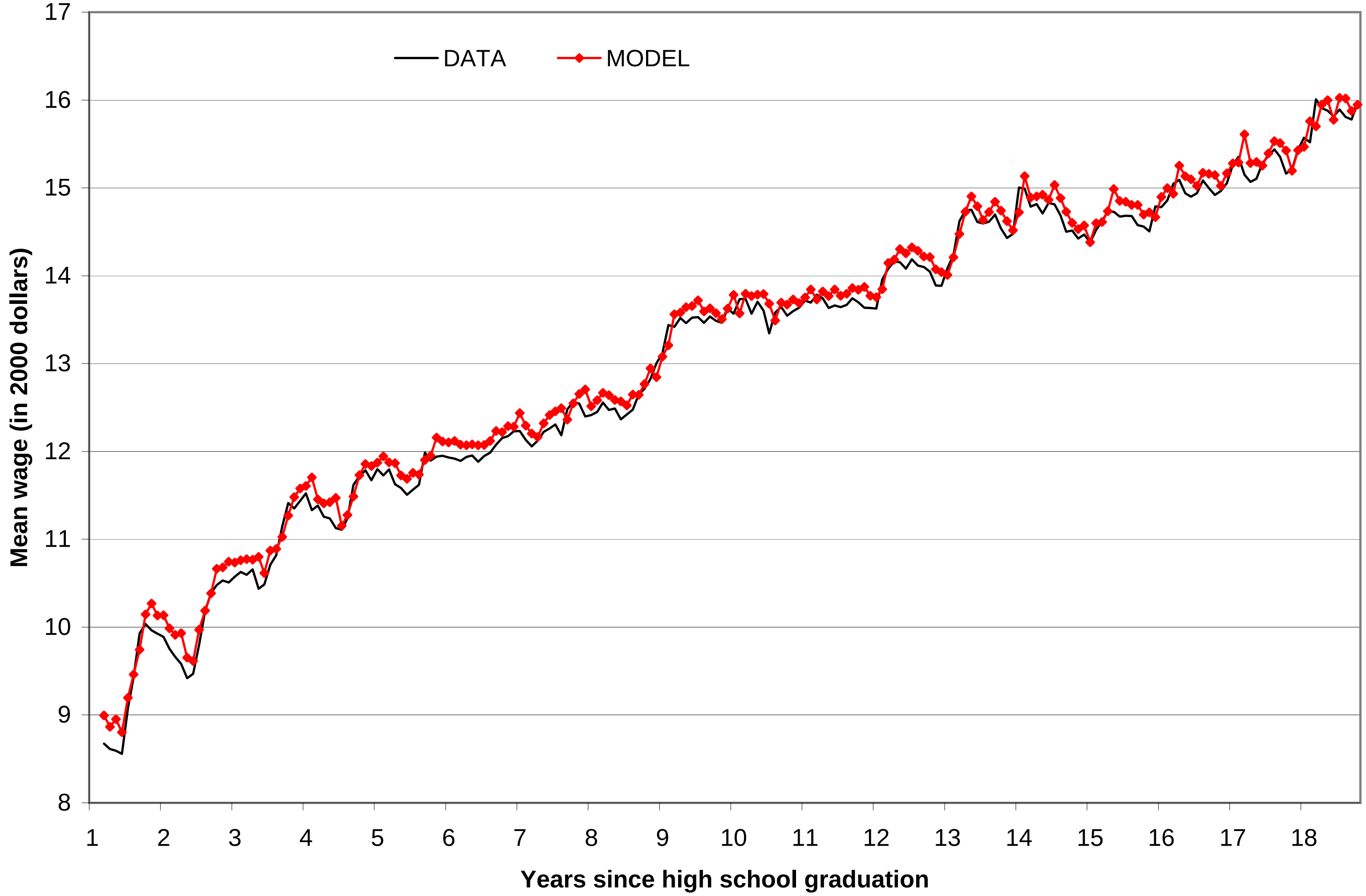


**Figure 11: Actual and predicted monthly wage variance**

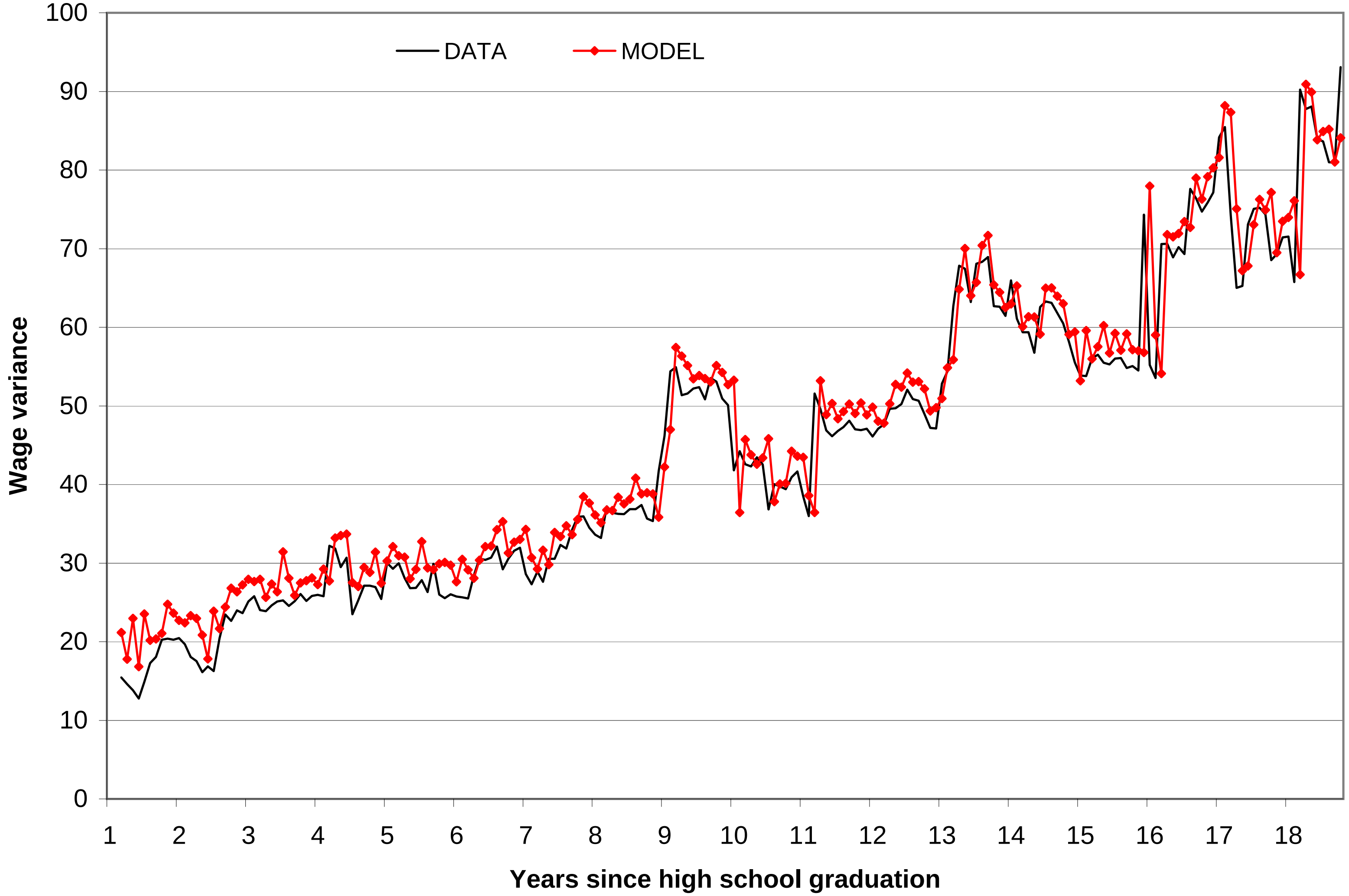


**Figure 12: Actual and predicted monthly transition rate from nonemployment to work**

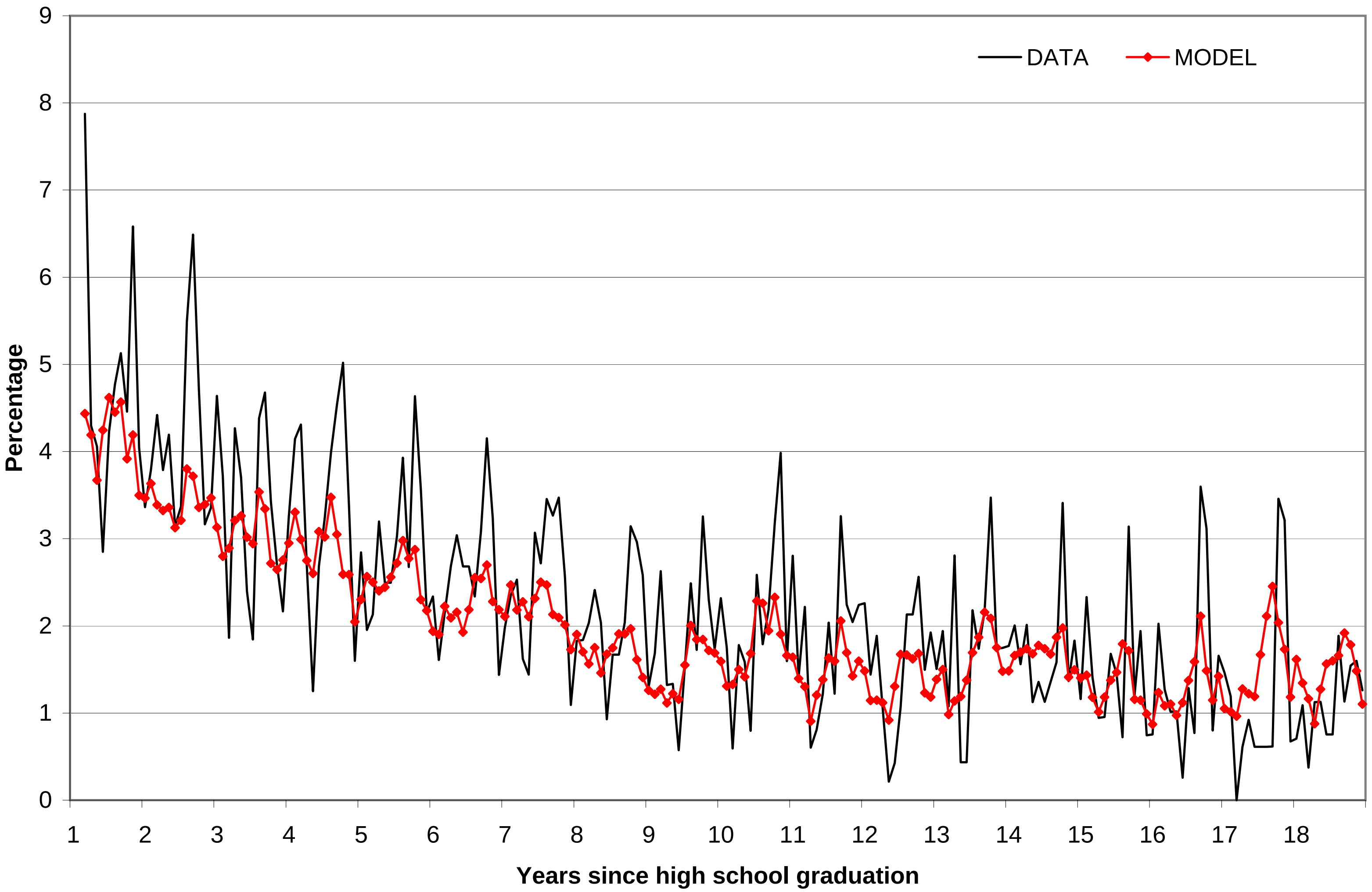


**Figure 13: Actual and predicted monthly transition rate from job to job**

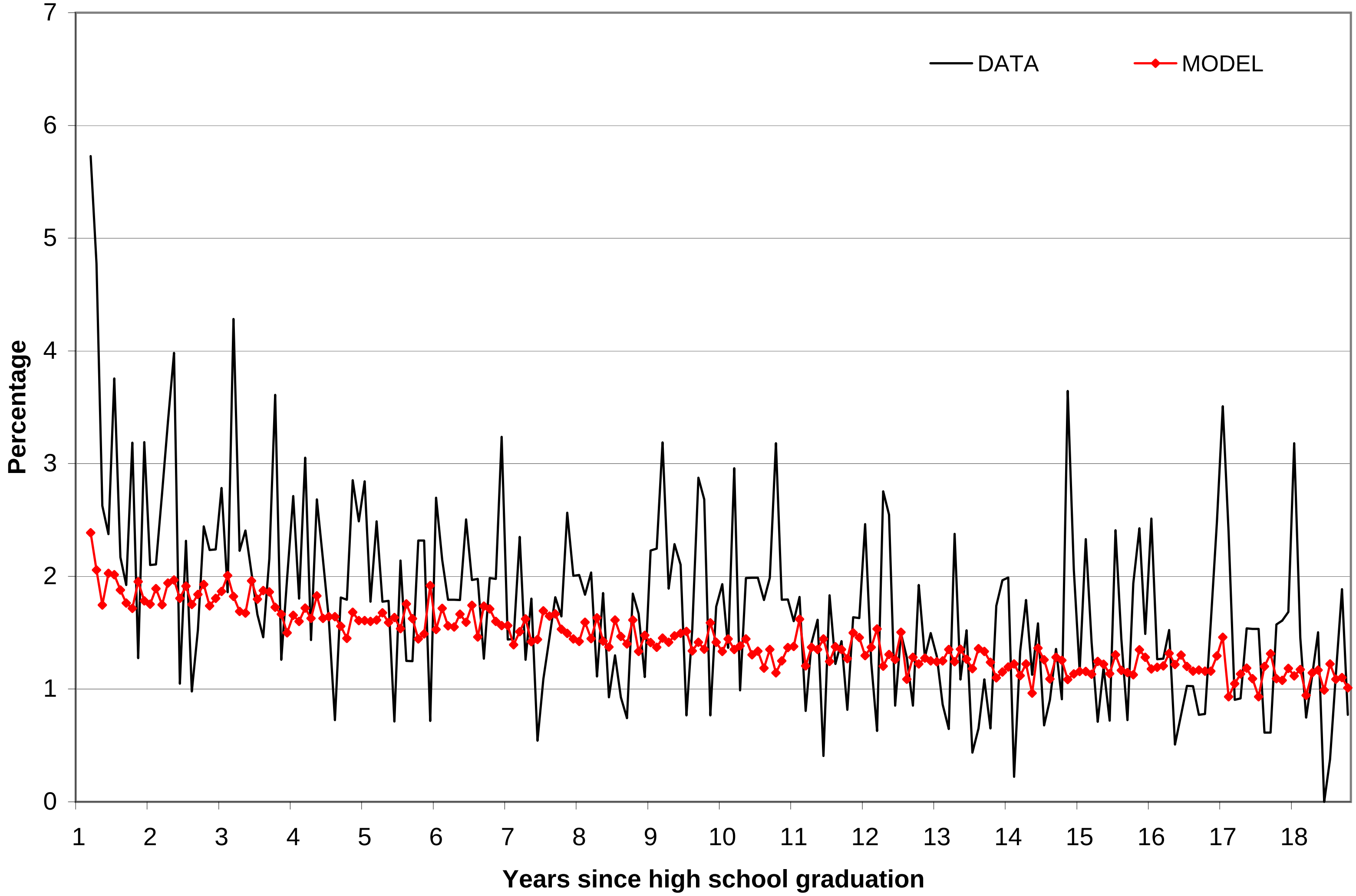


**Figure 14: Actual and predicted percentage of workers paid below the minimum wage**

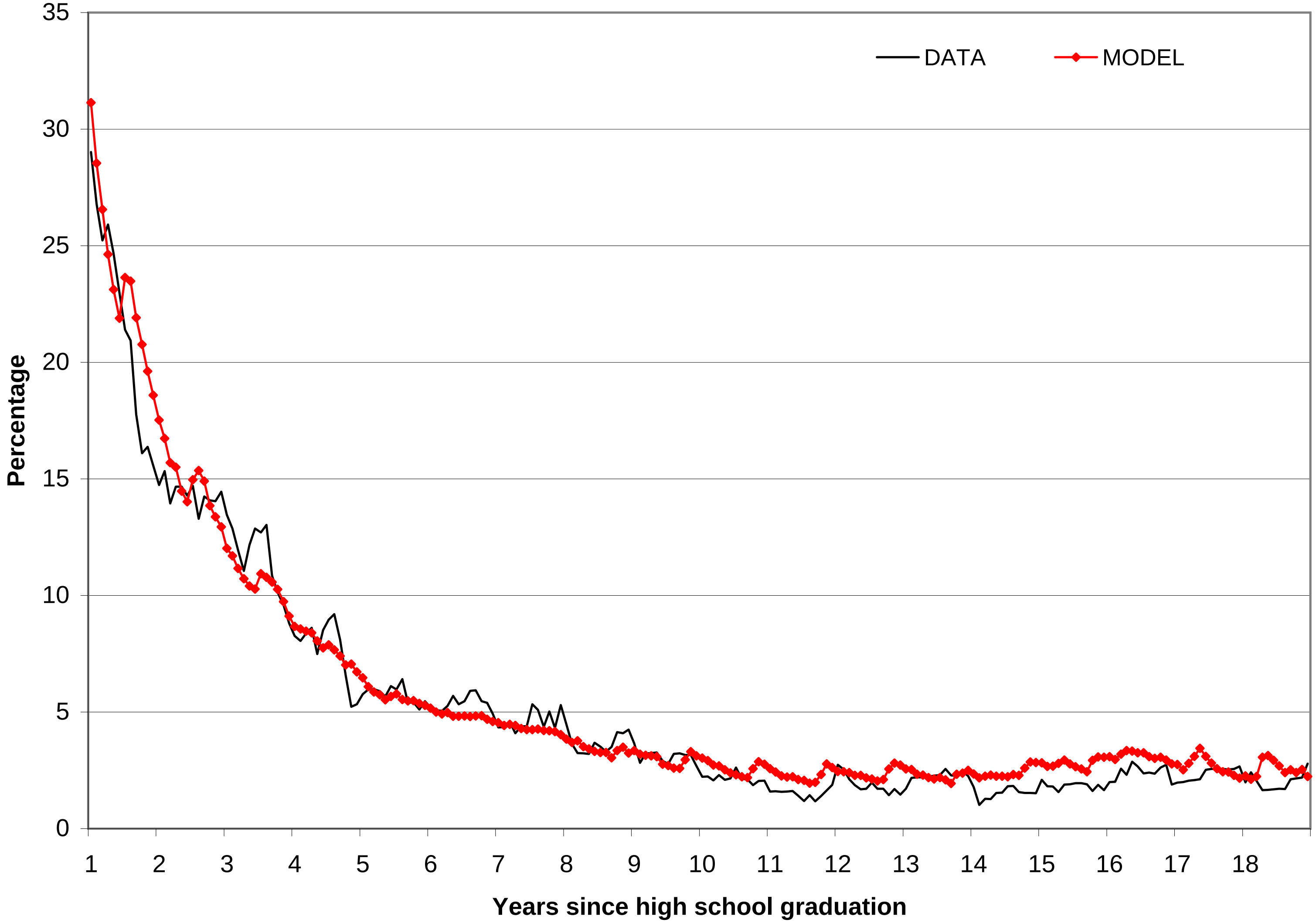


**Figure 15: Predicted percentage of workers paid below the minimum wage before and after 20% increase in the minimum wage**

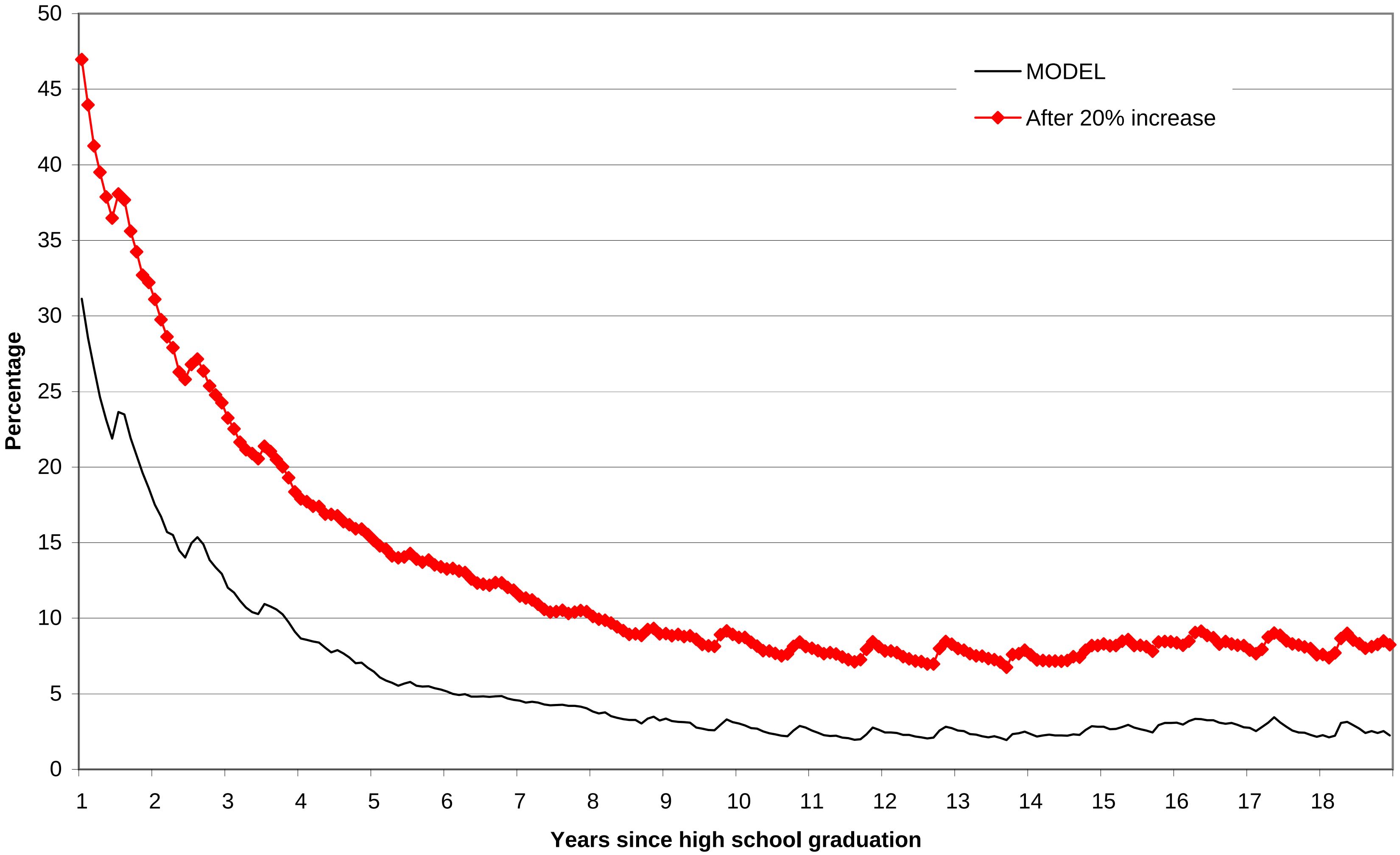


**Figure 16: Predicted monthly mean wage (unconditional) with increasing compliance from 0.604 to 0.302**