Lots of Heterogeneity in a Matching Model

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Abstract

This paper develops a dynamic general equilibrium matching model with many types of workers, many types of jobs and many possible aggregate productivity states. Wages are determined by labor auctions and workers can continue to search on-the-job for better employment opportunities. The model is solved using two systems of linear equations - the first gives a solution for wages given a vacancy creation rule and the second uses the solution of the first to solve for equilibrium vacancies. Consequently, the numerical calculation of the decentralized

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equilibrium is both accurate and fast. The model is evaluated quantitatively using micro data on wages and worker productivity and macro data on vacancies and unemployment.

1 Introduction

It is becoming increasingly apparent that the empirical study of the labor market requires a dynamic general equilibrium model capable of explaining a multitude of facts (ref: Browning, Hansen and Heckman 1999, Sargent and Ljungqvist 2004). However, Walrasian models cannot explain unemployment because of the assumption of market clearing. And, matching models, which do explain unemployment, give only limited insights into the questions of who works with who, and who gets paid what, because the answers to these questions are largely imposed by the exogenous matching technology and sharing rule assumed. The acknowledgement of such problems has led to research in recent years to develop alternative theoretical frameworks.

One promising theoretical framework is the model of wage posting advanced by Burdett, Shi and Wright (2000). This matching model endogenizes both the matching technology and wage formation. However, efforts to derive more general wage posting models suitable to the study of a complex matching environment has proven to be difficult and presently there does not exist a wage posting model that combines heterogenous jobs and workers, on-the-job search and aggregate fluctuations. The basic problem is that a posted wage is a strategic variable that influences both the arrival rate of workers and the incentives of employed workers to conduct on-the-job search. The equilibrium equations of such complicated models are non-linear and implicit. Therefore, it is not at all obvious to see how one can compute the solution of wage posting games - steady state or otherwise - for all but the simplest examples.

An alternative to wage posting is the theory of competing labor auctions (ref: Julien Kennes and King 2000) [1] The competing auction model shares a very similar structure to wage posting models, but instead of a posted wage by firms, workers choose a reserve wage. Julien, Kennes and King (2004a) show that complex matching models based on competing labor auctions are easy to solve, because the equilibrium reserve wage is equal to the worker's outside option. Therefore, considerable progress can be made towards the development of analytical solutions for equilibrium wage dispersion and on-the-job search. Moreover, just like price posting models, the equilibrium wages and matching technology in the competing labor auction model are the outcome of a non-cooperative game.

The present paper has two main goals. The first goal is to show that there exists reliable numerical methods to calculate the equilibrium solution of competing auction models in complex stochastic environments. Thus we will consider an economy with many types of workers and firms, on-the-job search and aggregate shocks to productivity. The second goal is to evaluate the performance of the competing auction model against empirical evidence. Here, we use recent advances in empirical methods to do statistical inference. The two goals are complementary: recent develops in microeconometrics require the efficient and reliable numerical solution of economic models, and more complex theoretical models are needed to make a serious attempt at explaining microeconometric data.

This paper makes considerable progress towards developing a general

[1] The seminal research on competing auctions is by McAfee (1993). See also Shimer (1999).

method of efficiently and accurately computing the numerical solution of competing auction models in complex stochastic economic environments. In particular, we show that these models can be solved as systems of linear equations for which exact solutions are available using standard numerical methods.

In the model, workers rank jobs on the basis of two attributes - labor productivity and the possibility for career advancement. One aspect of this problem is that workers might rank a very low productivity job (say unemployment) higher than a middle productivity job if the former job gives a higher transition probabilility to even higher productivity jobs. Consequently, if opportunities for on-the-job search are limited, there is generally an optimal stopping rule in which workers reject low quality jobs.

The paper is organized as follows. In the next section we present the model and show how the solution can be solved using linear equations. The following section develops the alogirithm by which the model is evaluated numerically. The last section provides some concluding remarks.

2 The Model

There is a large number of identical risk neutral workers facing an infinite horizon, perfect capital markets, and a common discount factor β. In each time period, each worker has one indivisible unit of labor to sell. Since we focus on recursive equilibria, we drop the time subscript and, whenever needed, we use an apostrophe to refer to period $t+1$. In each time period, a worker produces

$$y = F^h(\theta, \lambda) \tag{1}$$

units of output, where h is the worker's type, θ is the job type employing the worker and λ is an aggregate shock. There are N worker types, $1, ..., N$, of fixed quantities $n_i \in \{n_1, ..., n_N\}$; M job types, $1, ..., M$ of endogenous quantities (defined later); and S aggregate shocks, $1, ..., S$. Job types are ordered by productivity with

$$F^h(i, \lambda) \geq F^h(j, \lambda), \tag{2}$$

for all $i > j$. Furthermore, the least productive job type - a type 1 job - denotes home production, i.e. unemployment. The labor productivity of each worker also depends on the aggregate technology parameter, λ. These aggregate technology shocks follows a first order Markov process that evolves according to the following transition function,

$$X(\lambda_j|\lambda_i) = \Pr(\lambda' = \lambda_j|\lambda = \lambda_i). \tag{3}$$

In general, we assume that a higher realization of the aggregate shock raises worker productivity.

The supply of new job types is determined by free entry. If a new type k job vacancy is created by a firm, the firm must pay a recruiting cost, $C^h(k|\theta, \lambda)$, in order to direct it to a type h worker who is currently employed in a type θ job in aggregate state λ.

At the end of each period, an employed worker faces a chance of becoming unemployed. This exogenous displacement shock occurs with probability, δ^h.

2.1 Job ranking

A job is ranked above other jobs if, in equilibrium, the worker chooses it from the available set of alternatives. In this model, jobs are ranked according to a combination of two characteristics; job type, which is determined by labor productivity, and the possibility of career advacement, which is determined by recruting costs. The rank, ξ, of a type θ job is described by a single valued ranking function, $\xi(\theta)$, where $\xi(\theta) \in \{1, ..., M\}$. The rank of a type 1 job - unemployment - is denoted by

$$\xi(1) = \xi^*.$$

Of course, it is possible that $\xi^* > 1$. For example, a worker might choose unemployment over a type 2 job if unemployment permits job search for the highest type jobs while employment in a type 2 job does not. Likewise, the ranking of middle level job types will depend on the relatively efficiency of job search in different employment states.

2.2 Local markets

The allocation of new job vacancies to workers is determined by a matching game in which workers auction their labor services- The solution of this game requires a description of the 'local market' valuations of each worker's services by firms (ref: Julien, Kennes and King 2004b). This section introduces some of the important features of these local markets and how such features change over time.

Let a and b denote the *recruitment characteristics* of a worker at the start of the period just prior to the assignment of new jobs, where a is the job rank of the worker's current employer and b is the job rank of the second best job offer made to this worker during the worker's present job tenure. After the assignment of new jobs, the relevant *wage negotiation characteristics* are given by A and B, where A and B correspond to updated versions of a and b taking into account new job opportunities created by the assignment of new job vacancies to workers.

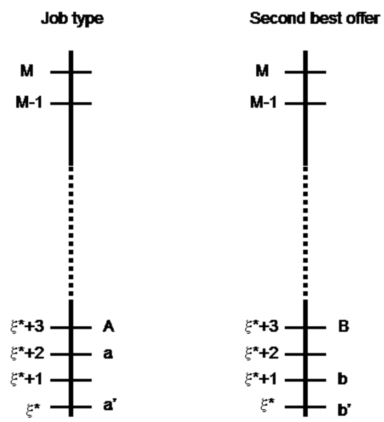

Figure 1: The Career ladder

Figure 1 gives an example of the evolution of the local market characteristics of a type h worker. This figure depicts two job ladders that track the rank of the worker's job and second best offer. The ladder on the left hand side gives the rank of the worker's job and the ladder on the right hand side gives the rank of the worker's second best offer. In this example, the worker starts in a rank $\xi^* + 2$ job and has a rank $\xi^* + 1$ second best offer. The worker is then recruited by better jobs and, in this example, advances to a rank $\xi^* + 3$ job and a rank $\xi^* + 3$ second best offer. At the end of the period, the worker faces an exogenous probability of displacement. In this example, the worker is displaced and enters the next period in a recruitment state involving a rank ξ^* job and a rank ξ^* second best offer. However, if the displacement did not occur, the worker would have moved to the next period in a recruitment state equal to this period's wage negotiation state.

The supply of job vacancies determines the relationship between the recruitment and wage negotiation characteristics. At the start of the period there are $n^h(a, b)$ type h workers in recruitment state, a, b. New job vacancies are then created and the ratio of the number of rank z job vacancies directed at type h workers in state, a, b, to the number of such workers is given by a job creation rule

$$\phi_z^h(a, b, \lambda) = v_z^h(a, b, \lambda)/n^h(a, b) \tag{4}$$

where the supply of rank z job vacancies, $v_z^h(a, b, \lambda)$, depends on the workers' recruitment characteristics and the realization of the aggregate shock. Note that the cost a type z job vacancy depends on the job vacancy type and the job type of the workers current empoyer. Therefore, we rewrite the cost of the rank z job vacancy by $C^h(\theta(z)|\theta(a), \lambda)$ where $\theta(z)$ is the job type of a z rank job and $\theta(a)$ is the job type of the worker's current employer.

We also assume that the worker's current second best offer, b never influences the job creation rule, because in the event of a better offer, the worker's current best offer becomes the worker's second best offer and the worker's old second best offer is irrelevant (to the labor auction). Therefore,

$$\phi_z^h(a, b, \lambda) = \phi_z^h(a, \lambda) \;\; \forall \; z. \tag{5}$$

Finally, we assume that workers never accept jobs ranked below the rank assigned to unemployment. Therefore, the rank of the worker's job and second best offer at the start of the period always satisfies

$$a, b \geq \xi^* \tag{6}$$

The new jobs created by firm are simultaneously assigned to each relevant group of similar workers. Therefore, as in other directed search models, the mixed strategy equilibrium of this assigment game is a random assignment of job vacancies over these workers (ref: Montgomery 1991) The probability that a worker gets m new offers from jobs of rank z is given by

$$\omega_z^m(a,\lambda;h) = \frac{\left(\phi_z^h(a,\lambda)\right)^m}{m!} e^{-\phi_z^h(a,\lambda)} \tag{7}$$

Using 7, we can characterize the probability function, $Z^h(A,B|a,b,\lambda)$, that a type h worker in recruitment state, a,b moves to wage negotiation state, A,B. For integer values $i,j>0$, $a+i+j\leq M$, we have

$$Z^h(A,B|a,b,\lambda)$$

$$= \begin{cases} \prod_{k=a+1}^{M} \omega_k^0(a,\lambda;h) \text{ if } A=a, B=b; \\ \left(\prod_{k=a+1}^{M} \omega_k^0(a,\lambda;h)\right)\left(\frac{\omega_{a+i}^1(a,\lambda;h)}{\omega_{a+i}^0(a,\lambda;h)}\right) \text{ if } A=a+i, B=a; \\ \left(\prod_{k=a+i+1}^{M} \omega_k^0(a,\lambda;h)\right)\left(1-\omega_{a+i}^0(a,\lambda;h)-\omega_{a+i}^1(a,\lambda;h)\right) \text{ if } A=B=a+i; \\ \left(\prod_{k=a+i+1}^{M} \omega_k^0(a,\lambda;h)\right)\frac{\omega_{a+i+j}^1(a,\lambda;h)}{\omega_{a+i+j}^0(a,\lambda;h)}\left(1-\omega_{a+i}^0(a,\lambda;h)\right) \text{ if } A=a+i+j, B=a+i; \\ \text{and } 0, \text{otherwise} \end{cases} \tag{8}$$

The functional form of this distribution is defined over five cases: the first case is the event that the worker receives no new job offers; the second case is the event that the worker receives only one new job offer; the third case is the event that the worker receives a new best offer and a new second best offers, both of which are identical; the fourth case is the event that worker receives a new best offer and a new second best offer, one of which is greater than the other; and the final case indicates that al other realizations of A and B are not possible - thus the wage negotiation state is never lower than the recruitment state either in terms of job rank or quality of second best offer.

At the end of the period, the worker loses their job with probability, δ^h. Therefore, there exists a chance that the wage negotiation characteristics this period, A, B, will be greater than their recruitment characteristics next period, a', b'. Let $\zeta^h(a', b'|A, B)$ denote the probability function over these 'end of the period' events. This function is given by

$$\zeta^h(a', b'|A, B) = \begin{cases} 1 - \delta^h & \text{if } a' = A, b' = B, \\ \delta^h & \text{if } a' = \xi^*, b' = \xi^*, \text{ and} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The transition rule, $P^h(A', B'|A, B, \lambda')$, giving the probability a worker moves from any particular wage negotiation state this period, A, B, to any particular wage negotiation state, A', B', next period is simply

$$P(A', B'|A, B, \lambda') = Z^h(A', B'|a', b', \lambda')\zeta^h(a', b'|A, B) \tag{10}$$

where $Z^h(A, B|a', b', \lambda')$ and $\zeta^h(a', b'|A, B)$ are given by equations (9) and (8). Moreover, the probability that a worker in state A, B will be in state B' next period is the summation of all possible realizations of A' given state B'. Thus

$$P(B'|A, B, \lambda') = \sum_{A'=0}^{M} P(A', B'|A, B, \lambda') \tag{11}$$

The task at hand is to show how wages are determined in the context of a competing labor auction given the expected movements of workers accross wage negotiatons states. Here we will establish which ranking function is valid. Once wages are determined, we can then turn to the equilibrium supply of jobs and characterize the equilibrium movement of workers across different wage negotiation states.

2.3 Wage determination

In a recursive equilibrium, the expected present value of a worker at the time of wage negotiations is dependent on the worker's wage negotiation state and the probability that the worker will be in a particular wage negotiation state next period. The expected present value, $W_h(A, B, \lambda)$, for a type h worker with wage negotiation characteristics, A, B, λ, is given by the discrete Bellman equation

$$W^h(A, B, \lambda) =$$
$$w^h(A, B, \lambda) + \beta \sum_{A'=\xi^*,}^{M} \sum_{B'=\xi^*,}^{M} \sum_{\lambda'=0}^{S} X(\lambda'|\lambda) P(A', B'|A, B, \lambda') W^h(A', B', \lambda') \quad (12)$$

where $w^h(A, B, \lambda)$ is the equilibrium wage of the worker in this state. In the set of discrete Bellman equations of the worker, both $w^h(A, B, \lambda)$ and $W^h(A, B, \lambda)$ are unknown. Therefore, even if we take $P(A', B'|A, B, \lambda')$ as given, this is a linear system of $S(M + 1)(M + 2)$ equations and $S(M + 1)(M + 2)/2$ unknowns, because (i) A and B can each take one of $M + 1$ values and λ can take on one of S values and (ii) $A \geq B$.

Wages are determined by auction. This mechanism imposes two important properties on the possible outcomes for wages and the expected value functions of workers. The first important property of the labor auction is that the value of the worker's wage contract is entirely dependent on the worker's second best available job at the time of wage contract negotiations. Thus

$$W^h(i, B, \lambda) = W^h(j, B, \lambda) \quad (13)$$

for all $i >, j$. A second important property of the labor auction is that the worker earns a wage equal to the entire output of the job if the worker has two identical best offers. Thus .

$$w^h(A, A, \lambda) = F^h(A, \lambda) \tag{14}$$

for all $A \in \{1, ..., M\}$. These two properties can be substituted into the Bellman equations of the workers giving us a linear system of $SM(M+1)$ equations and $SM(M+1)$ unknowns.

Equations 12, (13) and (14) are sufficient to get a solution for wages and value functions. However, a third property must be satisfied if the solution for $w^h(A, B, \lambda)$ and $W^h(A, B, \lambda)$ is to be an equilibrium outcome of the worker's auction It must be the case that the worker accepts a new job offer only if it pays a higher expected return than the maximum possible return of continuing in their old job. If not, the old job will bid appropriately on the worker's services and the worker will not accept the new offer. This means that each new job offer that is accepted must represent a higher outside option for the worker. Therefore, we have the following *job acceptance constraint*:

$$W^h(i, i, \lambda) \geq W^h(j, j, \lambda) \tag{15}$$

for all $i > j$. The satisfaction of the job acceptence constraint for an ordering of jobs on the basis of productive requires that being employed in a more productive job does not excessively reduce the chances for advancement through continued search. For example, a worker might choose to remain unemployed rather than accept a low quality job if the latter employment prospect does not permit on-the-job search (see example 1). In this case, only rank 2 jobs would be offered in equilibrum because type 0 jobs would be prefered to type 1 jobs and equation 8 implies that the probability of transition of workers into a type 1 jobs would now be zero, since its new ranking is below type 0. The method to handle any violation in the job acceptance constraint is to appropriately re-rank the jobs, substitute them using their new order into equation 8 and then resolve for wages and value functions using equations 12, (13) and (14).

One implication of the job acceptance constraint, 15, is that the maximum wage of a worker is equal to their productivity, because 5 implies the workers in a rank A job with $B < A$ enjoy the same transitions to better jobs as workers in the wage negotiation state $A, B = A$. Moreover, these workers have a lower expected earning as given by 15. Therefore, we have shown that

$$w^h(A, B, \lambda) \leq F^h(A, \lambda) \tag{16}$$

must be satisfied for all $B \leq A$. This result will be used in the next section on equilibrium job entry.

2.4 Equilibrium job entry

Consider a firm that directs a job vacancy of rank z at a worker in recruiting state a, b when the aggregate state is λ. The probability, $Q^h(z, B|a, b, \lambda)$, that this firm will find the worker in wage negotiation state $A = z, B \leq z$ is given by

$$Q^h(z, B|a, b, \lambda) = \begin{array}{l} (1 - \omega_B^0(a, \lambda; h)) \prod\limits_{k=B+1}^{M} \omega_k^0(a, \lambda; h) \text{ if } z > \xi^* \\ 0 \text{ otherwise} \end{array} \tag{17}$$

where $\omega_B^0(a, \lambda; h)$ is given by 7. Once recruited the wage negotiation state of an employer-employee match never changes and seperation occur only if the worker quits to a better job or the match is destroyed by the exogenous displacement shock. Therefore, the probability, $d^h(z, \lambda')$, that employer-employee match continues in the next period is given by

$$d^h(z, \lambda') = (1 - \delta) \prod_{k=z+1}^{M} \omega_k^0(z, \lambda'; h) \tag{18}$$

The value of creating a rank z job vacancy and directing it at a type h worker in recruitment state a, b, λ is given by

$$C^h(\theta(z)|\theta(a), \lambda) = \sum_{B=a}^{M} Q_z^h(B|a, b, \lambda) J^h(z, B, \lambda) \tag{19}$$

where $C_z^h(a, \lambda)$ is the cost of the vacancy and the remaining terms are the expected flow of income from this vacancy in a free entry equilibrium. The value of an existing job is given by the return each period,

$$J^h(z, B, \lambda) = F^h(z, \lambda) - w^h(z, B, \lambda) + \beta \sum X(\lambda'|\lambda) d^h(z, \lambda') J^h(z, B, \lambda')\}, \tag{20}$$

which depends on the probability of a continued match and changes in the aggregate state. Note the expected value of an ongoing job is positive, $J_z^h(B, \lambda') \geq 0$, because we have shown previously that $F^h(z, \lambda) - w^h(z, B, \lambda) \geq 0$. This condition also implies that vacancy costs must also be positive, $C_z^h(a, \lambda) \geq 0$,which is, of course, what is assumed.

2.5 Labor force dynamics

The wage distribution for each type of worker in each period is given by the equilibrium set of values, $\{w(A, B, \lambda), n(A, B)\}$. The number of workers in each wage negotiation state evolves according to the following transition equation,

$$n^h(A', B') = \sum_{A=\xi^*}^{M} \sum_{B=\xi^*}^{M} P^h(A', B'|A, B, \lambda') n^h(A, B) \tag{21}$$

The number of workers in each rank of job need not be stationary, because $P^h(A', B'|A, B, \lambda')$ will generally fluctuate with λ'. The quantity of workers in each recruiting state is given by

$$n^h(a', b') = \begin{cases} n^h(A, B)(1 - \delta^h) \text{ if } A, B \geq 1 \\ n^h(0, 0) + (n_h - n^h(0, 0))\delta^h \text{ otherwise} \end{cases} \tag{22}$$

Also, the values of market tightness, $\phi_z^h(a, b, \lambda)$, over all possible job vacancies are uniquely determined by λ However, the supply of job vacancies can fluctuate over time as

$$v_z^h(a, b, \lambda) = \phi_z^h(a, b, \lambda)n^h(a, b) \tag{23}$$

depends on the fluctuating value of $n^h(a, b, \lambda)$.

3 Numerical Analysis

The properties of the equilibrium can be illustrated by numerical analysis. This section briefly introduces the step for numerical analysis, however this scetion is highly preliminary. The various steps in the numerical analysis for a simple example are as follows. Step (1) *Parameterization:* Let $N = 1, M = 3, S = 1$. The productivity of each job type, the discount rate and the job destruction rate are given by

$$\begin{aligned} F(1) &= 0 \\ F(2) &= 1000 \\ F(3) &= 2000 \\ \beta &= .99 \\ \delta &= .04 \end{aligned}$$

Step (2) *Job ranking conjecture:* Conjecture that $\xi(1) = 1, \xi(2) = 2, \xi(3) = 3$; Step (3) *Job creation rule.* Let

$$\begin{aligned} \phi_{\xi(2)}(\xi(1)) &= 1 \\ \phi_{\xi(3)}(\xi(1)) &= 2 \\ \phi_{\xi(3)}(\xi(2)) &= 0 \end{aligned}$$

where the supply of jobs for home production is indeterminant reflecting the fact that such jobs are always available. The assumption that $\phi_{\xi(3)}(\xi(2)) = 0$ means that we rule out on-the-job search.; Step (4) *Calculation of the transition probabilities between wage negotiation states.* Here we can use equation (9) and (8). to calculate the probability each period, $P(A', B'|A, B)$ that a worker moves between any two wage negotiation states. Note that equation (8). implies that wage bargaining states with jobs ranked below $R(0)$ are zero probability.events. Therefore, if the bad jobs were ranked below home producton, neither the worker's employer nor the worker's second best offer will ever be a bad job. Step (5) *Worker wage and value function calculation.* Here, the wage and value function of the worker in different wage negotiation states is determined. The appropriate equation with two job types ranked above home producton is given by a solution to the system of linear equations of the form $Ax = b$ where

$$A = \begin{bmatrix} -\beta P(1|1,1)+1 & -\beta P(2|1,1) & -\beta P(2|1,1) & 0 & 0 & 0 \\ -\beta P(1|2,2) & -\beta P(2|2,2)+1 & -\beta P(3|2,2) & 0 & 0 & 0 \\ -\beta P(1|3,3) & -\beta P(2|3,3) & -\beta P(3|3,2)+1 & 0 & 0 & 0 \\ -\beta P(1|2,1) & -\beta P(2|2,1) & -\beta P(3|1,1) & -1 & 0 & 0 \\ -\beta P(1|3,1) & -\beta P(2|3,1) & -\beta P(3|1,1) & 0 & -1 & 0 \\ -\beta P(1|3,2) & -\beta P(2|3,2) & -\beta P(3|1,1) & 0 & 0 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} W(1,1) \\ W(2,2) \\ W(3,3) \\ w(2,1) \\ w(3,1) \\ w(3,2) \end{bmatrix}$$

$$b = \begin{bmatrix} F(1) \\ F(2) \\ F(3) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where, the other values of wages and value function in the different wage negotiation states are given by

$$\begin{aligned} w(1,1) &= F(1) \\ w(2,2) &= F(2) \\ w(3,3) &= F(3) \\ W(2,1) &= W(1,1) \\ W(3,1) &= W(1,1) \\ W(3,2) &= W(2,2) \end{aligned}$$

(Step 6) *Check Consistency of the Ranking:* The ranking of job is consistent if $W(\xi(\theta), \xi(\theta))$ satisfies

$$W(3,3) \geq W(2,2) \geq W(1,1)$$

If not then, we have to choose a new ranking of jobs and go back to step 1. Otherwise we continue to step (7); (step 7) *Vacancy costs:* The wages given in step 3 are substituted into the following expression and we get a solution for vacancy costs

$$\begin{bmatrix} 1 & 0 & -Q_2(1|1) & 0 & 0 \\ 0 & 1 & 0 & -Q_3(1|1) & -Q_3(2|1) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2(1) \\ C_3(1) \\ J_2(1) \\ J_3(1) \\ J_3(2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{F(2)-w(2,1)}{1-\beta d(2)} \\ \frac{F(3)-w(3,1)}{1-\beta d(2)} \\ \frac{F(3)-w(3,2)}{1-\beta d(2)} \end{bmatrix}$$

(Step 8) *Repeat:* Consider other possible values for labor market tightness at step 2 stage. If all possible values are exhausted, go to step 1 and try a different parameterization.

4 Empirical Analysis

This section is under development.

5 Conclusions

This paper takes some first steps towards deriving a matching model with lots of heterogeneity and showing how such a model can be solved numerically. The next steps for this paper are to consider a broad set of simulations and to confront the model with empirical evidence.

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