# Redistribution and Disability 

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## 1 Introduction

- Design of welfare programs
- Several types of insurance
- Insurance against permanent low ability shocks redistribution.
- Disability insurance
- Incentives
- Analyze interaction in optimal design in simple model
- Evaluate consequences of lack of coordination (multiple agencies)


## 2 The model

- Two periods
- Two types of agents with productivities $\left\{x_{l}, x_{h}\right\}$, shares $(1-\pi), \pi$
- Agent's type private (Mirrlees.)
- No disutility of work first period.
- Second period independent shock to disutility of work $e^{\sim} F(e)$.
- Utilitarian Principal.


### 2.1 Design problem and incentives

- Contracts specify $\left\{c_{1 h}, c_{2 h}, c_{d h}\right\},\left\{c_{1 l}, c_{2 l}, c_{d l}\right\}$.
- Employment decision in second period:

$$
\begin{aligned}
u\left(c_{2 h}\right)-e_{h} & =u\left(c_{d h}\right) \\
u\left(c_{2 l}\right)-e_{l} & =u\left(c_{d l}\right)
\end{aligned}
$$

- Simplified notation for second period utility:

$$
\begin{aligned}
U_{2}\left(c_{2}, c_{d}\right)= & \max _{e}\left(1-F\left(e_{j}\right)\right) u\left(c_{d}\right) \\
& +\int_{0}^{e}(u(c)-a) F(d a)
\end{aligned}
$$

- Self selection constraint:

$$
u\left(c_{1 h}\right)+U_{2}\left(c_{2 h}, c_{d h}\right) \geq u\left(c_{1 l}\right)+U_{2}\left(c_{2 l}, c_{d l}\right)
$$

### 2.2 The optimal contract

- For convenience take $\pi=\frac{1}{2}$
$\max u\left(c_{1 h}\right)+U_{2}\left(c_{2 h}, c_{d h}\right)+u\left(c_{1 l}\right)+U\left(c_{2 l}, c_{d l}\right)$
subject to:
$u\left(c_{1 h}\right)+U_{2}\left(c_{2 h}, c_{d h}\right) \geq u\left(c_{1 l}\right)+U_{2}\left(c_{2 l}, c_{d l}\right)$
$0 \leq x_{h}-c_{1 h}+\left(x_{h}-c_{2 h}\right) F\left(e_{h}\right)-\left(1-F\left(e_{h}\right)\right) c_{d h}$
$+x_{l}-c_{1 l}+\left(x_{l}-c_{2 l}\right) F\left(e_{l}\right)-\left(1-F\left(e_{l}\right)\right) c_{d l}$


### 2.3 Some results

- First order condition for first period consumption:

$$
\begin{aligned}
u^{\prime}\left(c_{1 h}\right) & =\lambda-\mu \\
u^{\prime}\left(c_{1 l}\right) & =\lambda+\mu
\end{aligned}
$$

- $c_{1 l}<c_{1 h}$ if and only if self-selection binds $(\mu>0)$.
- $e_{h}>e_{l}$
- If $\mu>0$, then:

1. $c_{2 h}>c_{2 l}>c_{d l}>c_{d h}$
2. $U_{2 h}<U_{2 l}$

- Remark: with no second period incentive constraint $\rightarrow$ full insurance $\rightarrow$ all consumptions identical.
- Incentives for disability limit redistribution.


## 3 Numerical results

- Calibration $\left(\pi, x_{l}, x_{h}, F, u\right)$

1. $\pi=0.25$
2. $x_{h}=3 x_{l}$
3. $u(c)=\ln c$
4. $F$ exponential hazard rate $\lambda \in\{0.5,1,2\}$

## Median Disutility of Effort

(equivalent \% loss in wages)
Hazard disutility
$\lambda=0.5 \quad 75 \%$
$\lambda=1 \quad 50 \%$
$\lambda=2 \quad 29 \%$

Consumption (Constrained/Optimal)

$$
\lambda=0.5 \quad \lambda=1 \quad \lambda=2
$$

| C_11 | 85 | 89 | 96 |
| :--- | ---: | ---: | ---: |
| C_2l | 127 | 118 | 110 |
| C_dl | 70 | 65 | 58 |
| replacement | $55 \%$ | $55 \%$ | $53 \%$ |
| C_1h | 106 | 123 | 110 |
| C_2h | 169 | 135 | 110 |
| C_dh | 40 | 13 | 0 |
| replacement | $24 \%$ | $9 \%$ | $0 \%$ |
| Avg. Replacement | $45 \%$ | $50 \%$ | $53 \%$ |

- Limited replacement ratios
- Very low for $H$ types.
- Less redistribution first period.
- Replacement rates decreasing with $\lambda$.


## Employment and Disability

|  | $\lambda=0.5$ |  | $\lambda=1$ | $\square$ | $\lambda=2$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Optimal | Constrained | Optimal | Constrained | Optimal | Constrained |
| F(e_l) | 35.1\% | 25.8\% | 53.9\% | 45.1\% | 75.9\% | 72.3\% |
| F(e_h) | 72.7\% | 51.2\% | 90.2\% | 90.6\% | 98.6\% | 100.0\% |
| $\begin{aligned} & \text { \% } \\ & \text { disabled } \end{aligned}$ | 55.5\% | 67.8\% | 37.0\% | 43.5\% | 18.4\% | 20.7\% |
| autharky |  | 53\% |  | 22\% |  | 0\% |

- Lower employment of low types.
- Increase in \% disabled.
- Much more than under autharky.


## Welfare

$$
\lambda=0.5 \quad \lambda=1 \quad \lambda=2
$$

$\begin{array}{llll}\text { First Best } & 100.0 & 100.0 & 100.0\end{array}$

- Big gains relative to autharky.
- Considerable difference to first best for low $\lambda$.


## 4 Uncoordinated decisions

- Advantages of coordinated redistribution and disability policies.
- Two principals.
- First principal:

1. Decides on wage taxes
2. Budget for disability insurance office.

- Second principal - disability insurance office:
- Decides $c_{d h}$ and $c_{d l}$.


# 4.1 Coordination problem 

- Free riding on self-selection.
- Does not internalize changes in tax revenue.
- Dynamic game.


### 4.2 Disability insurance office

- Takes as given $c_{2 h}, c_{2 l}$ (follows from taxes)
- Can discriminate between $h, l$.
- Offers $c_{d h}, c_{d l}$ to solve:

$$
\begin{aligned}
& \max _{c_{d h}, c_{d l}} F\left(e_{h}\right) u\left(c_{2 h}\right)-\int_{0}^{e_{h}} a f(a) d a+\left(1-F\left(e_{h}\right.\right. \\
& +F\left(e_{l}\right) u\left(c_{2 l}\right)-\int_{0}^{e_{l}} a f(a) d a+\left(1-F\left(e_{l}\right)\right) u(c \\
& \text { subject to } \\
e_{j}= & u\left(c_{2 j}\right)-u\left(c_{2 j}\right), j=h, l \\
B= & F\left(e_{h}\right) c_{d h}+F\left(e_{l}\right) c_{d l}
\end{aligned}
$$

- Marginal cost of increasing $c_{d j}$ :

$$
\begin{aligned}
& 1-F\left(e_{j}\right)-f\left(e_{j}\right) c_{d j} \frac{\partial e_{j}}{\partial c_{d j}} \\
= & F\left(e_{j}\right)+f\left(e_{j}\right) c_{d j} u^{\prime}\left(c_{d j}\right)
\end{aligned}
$$

- Marginal benefit: $F\left(e_{j}\right) u^{\prime}\left(c_{d j}\right)$
- Optimal rule equate Mg benefit $/ \mathrm{Mg}$ cost on both types.

$$
\frac{\left(1-F\left(e_{j}\right)\right) u^{\prime}\left(c_{d j}\right)}{\left(1-F\left(e_{j}\right)\right)+f\left(e_{j}\right) u^{\prime}\left(c_{d j}\right) c_{d j}}=\lambda
$$

where $\lambda$ satisfies budget constraint.

### 4.3 First Principal's problem

- Same as before with the additional constraint:

$$
=\begin{aligned}
& \frac{\left(1-F\left(e_{h}\right)\right) u^{\prime}\left(c_{d h}\right)}{\left(1-F\left(e_{h}\right)\right)+f\left(e_{h}\right) u^{\prime}\left(c_{d h}\right) c_{d h}} \\
= & \frac{\left(1-F\left(e_{l}\right)\right) u^{\prime}\left(c_{d l}\right)}{\left(1-F\left(e_{l}\right)\right)+f\left(e_{l}\right) u^{\prime}\left(c_{d l}\right) c_{d l}}
\end{aligned}
$$

- Rewriting:

$$
\frac{1}{\frac{1}{u^{\prime}\left(c_{d j}\right)}+\frac{f\left(e_{j}\right)}{1-F\left(e_{j}\right)} c_{d j}}=\lambda
$$

- Decreasing in $c_{d j}$ and increasing (decreasing) in $e_{j}$ if and only if hazard rate is decreasing (increasing).
- If $F$ is exponential, then $c_{d h}=c_{d l}$ is only additional constraint. If hazard rate is increasing $c_{d h}<c_{d l}$ and $e_{h}>e_{l}$. If hazard rate is decreasing, opposite!


# 4.4 Two principals - numerical results 

- Same case as before.
- $F$ is exponential, so only add constraint $c_{d h}=c_{d l}$.

Consumption (two planners/one planner)

$$
\lambda=0.5 \quad \lambda=1 \quad \lambda=2
$$

| C_1I | 104 | 104 | 104 |
| :--- | ---: | ---: | ---: |
| C_2l | 93 | 96 | 94 |
| C_dl | 81 | 89 | 70 |
| replacement | 47.6 | 51.2 | 39.5 |
| one planner | $55 \%$ | $55 \%$ | $53 \%$ |
| C_1h |  |  |  |
| C_2h | 75 | 83 | 91 |
| C_dh | 117 | 109 | 116 |
| replacement | 414 | 155 | - |
| one planner | $33 \%$ | $34 \%$ | $32 \%$ |
| Avg. Replacement | $24 \%$ | $9 \%$ | $0 \%$ |
| one planner | $42 \%$ | $44 \%$ | $37 \%$ |

- More redistribution first period (same consump-
tion!)
- Replacement increases for $h$ and decreases for $l$.


## Employment and Disability

|  | $\lambda=0.5$ |  | $\lambda=1 \quad \square$ |  | $\lambda=2$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Constrained | 2 Principals | Constrained | 2 Principals | Constrained | 2 Principals |
| F(e_l) | 25.8\% | 28.5\% | 45.1\% | 52.4\% | 72.3\% | 84.4\% |
| F(e_h) | 51.2\% | 41.8\% | 90.6\% | 66.5\% | 100.0\% | 89.9\% |
| \% <br> disabled | 67.8\% | 68.2\% | 43.5\% | 44.0\% | 20.7\% | 14.2\% |
| autharky |  | 53\% |  | 22\% |  | 0\% |

- $e_{l}$ goes up and $e_{h}$ goes down.


## Welfare

$$
\lambda=0.5 \quad \lambda=1 \quad \lambda=2
$$

First Best
Constrained
Two Principals
Autharky
$\begin{array}{lll}100.0 & 100.0 & 100.0\end{array}$
$\begin{array}{lll}92.7 & 95.4 & 98.3\end{array}$
$91.7 \quad 93.1 \quad 96.5$
$\begin{array}{lll}75.9 & 77.5 & 83.6\end{array}$

## - Effects not negligible but small.

# 4.5 Redistribution and incentives 

| Redistribution (avg. Taxes on H) |  |  |  |
| :--- | ---: | ---: | ---: |
| $\qquad$ $\lambda=0.5$ $\lambda=1$ | $\lambda=2$ |  |  |
| Optimal | $55 \%$ | $55 \%$ | $53 \%$ |
| One planner | $46 \%$ | $44 \%$ | $48 \%$ |
| two planners | $45 \%$ | $45 \%$ | $46 \%$ |

- Interaction with disability insurance incentives leads to lower income redistribution.
- Less so in later period.
- Disability much lower for high wage workers.
- Lack of coordination can lead to more equal disability.

