The Complementarity and Self-Productivity of Investments in Human Capital in a Stochastic Dynamic General Equilibrium Economy with Altruism and Lifetime Liquidity Constraints*

Flavio Cunha†

January 26th, 2005

Abstract

1 Introduction

This paper presents formal models of child development that capture the essence of recent findings from the empirical literature on child development. The goal is to provide a theoretical framework for interpreting the evidence from a vast empirical literature, for guiding the next generation of empirical studies and for formulating policy.

Recent empirical research in a variety of fields has substantially improved our understanding of how skills and abilities are formed over the life cycle. The early human capital literature (Becker, (1964)) showed how the complementarity between ability and human capital investments could be explored in order to explain many features of earnings distributions and earnings dynamics that models of innate cognitive ability could not. This point of view underlies many recent economic models of family influence. (*e.g.* Becker and Tomes, 1979, 1986; Aiyagari, Greenwood, Seshadri, 2002; Laitner, 1992, 1997). Later work (Ben-Porath, 1967 and Griliches, 1977) emphasized that innate ability was an input into the production of human capital, although it was ambiguous about its effect on human capital accumulation. More innate ability could lead to less schooling if all schooling did was teach one what an able person could learn without formal schooling. On the other hand, more innate ability might make learning easier and promote schooling. The signalling literature made the latter interpretation in developing models of schooling that emphasized that higher levels of schooling signalled higher innate ability. In one extreme form, this literature suggested that there was no learning content in schooling.

The literature in economics focuses on liquidity constraints and heritability as the principal sources of parental influence on child development. Becker and Tomes (1979, 1986) initiated a large literature that emphasizes the importance of credit constraints and family income on the schooling and earnings of children. Important developments of this work by Laitner (1992, 1997), Benabou (2000, 2002), Aiyagari, Greenwood, Seshadri (2002) and Seshadri and Yuki (2003), emphasize the role of credit constraints and altruism in forming the skills of children. In this literature, the lifecycle of the child at home is collapsed into a single period so that there is no distinction between early and late investments in children. Becker and Tomes (1986) suggest that there is no trade-off between equity and efficiency in government transfer policy because the return to human capital investment is high due to the presence of credit constraints.

*I thank Fernando Alvarez, Gary Becker, Francisco Buera, Pedro Carneiro, Lars Hansen, James Heckman, John Laitner, Lance Lochner, Salvador Navarro, Robert Pollak, Alejandro Rodriguez, Paul Schrimpf, Raghu Suryanarayanan, and Robert Townsend for comments. I thank James Heckman for encouragement and guidance. Research support by CAPES grant 1430/99-8 is gratefully aknowledged. All remaining errors are mine.

†Graduate student at the Department of Economics of The University of Chicago.

Recent research, summarized in Carneiro and Heckman (2003), presents a much richer picture of schooling, life cycle skill formation and earnings determination. It recognizes the importance of both cognitive and noncognitive abilities in explaining schooling and socioeconomic success. These abilities are themselves produced by family and personal actions. Both genes and environments produce these abilities and environments affect genetic transmission mechanisms (See Turkheimer et al., 2003). This interaction has important theoretical and empirical implications developed in this paper.

Complementarity of investments and self productivity, two distinct ideas often folded into one, are essential features of the skill and ability formation process that was neglected in the early human capital literature.[1] Skill begets skill; ability begets ability. Strong complementarity leads to a trade-off between efficiency and equity in considering investments made at an early age. Diminishing returns would argue in favor of equalization of investment across persons. Complementarity and self productivity are forces toward specialization of investments made after the early years to certain groups. Disadvantaged young adults with low levels of cognitive and noncognitive skills have lower rates of return to schooling and job training than more advantaged young adults. Due to complementarity, remediation for neglected investment is costly, and may be prohibitively so for the most disadvantaged.

One contribution of this analysis is to place the child development process in a multiperiod context, disaggregating the one period of family influence assumed in a variety of current models into multiple periods. Complementarity and self-productivity of human capital imply an equity-efficiency trade-off for late child investments but no equity-efficiency trade-off for early investments. This has important consequences for the design and evaluation of public policies toward families. In particular, the returns to late childhood investment and remediation for persons from disadvantaged backgrounds is low.

A second contribution of the analysis is to emphasize the secondary importance of credit constraints in the college going years, as traditionally conceived in applied economics in explaining child schooling attainment. Permanent income plays an important role, not income in adolescent years. As shown by Carneiro and Heckman (2003), the important market failure is the inability of children to buy their parents and not the inability of families to secure loans for a child's education. This has major implications for the way we design family policy.

Controlling for cognitive ability, in American society with current meritocratic policies in place, family income plays only a minor role in determining college enrollment decisions although much public policy is predicated on the opposite point of view. Yet ability itself seems to be determined by early family environments. Permanent income matters in determining schooling and ability, but "cash in advance" credit constraints facing parents in the child's teenage years do not. Ability has both environmental and genetic components, and environments affect the expression of the genes. Evidence from interventions on disadvantaged populations demonstrate that interventions can raise measured ability but their major impact is on noncognitive abilities. These features are missing from the current literature in economics on child development and the aim is to redress these gaps. They are also ignored in current empirical studies of family and genetic influence. Measured ability is determined in part by environmental factors.

Any finished model that is faithful to the evidence summarized in this paper will stress that (a) parental endowments are key constraints governing family influence in American society; (b) early child investments must be distinguished from late child investments and that an equity-efficiency trade-off exists for late investments but not for early investments. These insights change the way we interpret the evidence and design public policy. Point (a) is emphasized in many papers. Point (b) is ignored by models that consider only one period of childhood investment.

The paper is organized in the following way: section 2 describes the evidence on the importance of complementarity and self-productivity of investments in human capital as summarized by Carneiro and Heckman (2004), and presents the technology of skill formation introduced by Carneiro, Cunha and Heckman (2004) that captures the empirical findings from the literature on child development. Section 3 reviews and provides new evidence that supports that while liquidity constraints at college-going years are secondary in explaining college attendance, it

[1] Heckman, Lochner and Taber (1998) develop a model in which ability determines schooling and both ability and schooling determine post school investment. While Ben-Porath (1967) emphasized the self-productivity of human capital, he assumed human capital was homogeneous and did not develop models of heterogeneous skills and abilities. We contrast our model with his in Section **??**.

also shows that lifetime resources matter. In fact, the evidence suggests a model such as Laitner (1992). In section 4, the technology of skill formation of Carneiro, Cunha and Heckman (2004) is embedded in a extended Laitner (1992) economy. Section 5 develops one example of the model introduced in section 4 and discusses possible applications of the model. Section 6 contains a discussion of future research.

2 The Technology of Skill Formation

This section emphasizes key features of the human capital accumulation technology. Some of them have not yet been fully incorporated in economic models. I also report some examples that illustrate the empirical importance of these features. A more complete review of this evidence is provided by Carneiro and Heckman (2003).

Human capital accumulation and skill formations are dynamic processes. The skills acquired in one stage of the life cycle affect both the initial conditions and the technology of learning at the next stage. Human capital is produced over the life cycle by families, schools, and firms, although most discussions of skill formation focus on schools as the major producer of abilities and skills, despite a substantial body of evidence that families and firms are also major producers of abilities and skills. Skill formation starts in the womb and takes place throughout the whole life of the individual.

A major determinant of successful schools is successful families. Schools work with what parents bring them. They operate more effectively if parents reinforce them by encouraging and motivating children. Job training programs, whether public or private, work with what families and schools supply them and cannot remedy twenty years of neglect. Children from disadvantaged families may suffer from a lack of resources invested in them, or they may have parents that lack the information necessary to make adequate investments in their children, even if resources are made available (for example, through state programs), because of poor education or the like. It is easier to substitute against low current funds (if parents borrow against future consumption to finance current investments in their children) than against low parental human capital.

Abilities are both inherited and created. As summarized in Shonkoff and Phillips (2000), the "long standing debate about the importance of nature versus nurture, considered as independent influences, is overly simplistic and scientifically obsolete". They write: "Scientists have shifted their focus to take account of the fact that genetic and environmental influences work together in dynamic ways over the course of development. At any time, both are sources of human potential and growth as well as risk and dysfunction. Both genetically determined characteristics and those that are highly affected by experience are open to intervention. The most important questions now concern how environments influence the expression of genes and how genetic make-up, combined with children's previous experiences, affects their ongoing interactions with their environments during the early years and beyond."

A study of human capital policy grounded in economic and scientific fundamentals improves on a purely empirical approach to policy evaluation that relies on evaluations of the programs and policies in place or previously experienced. Although economic policy analysis should be grounded in data, it is important to recognize that the policies that can be evaluated empirically are only a small subset of the policies that might be tried. If we base speculation about economic policies on economic fundamentals, rather than solely on estimated "treatment effects" that are only weakly related to economic fundamentals, we are in a better position to think beyond what has been tried to propose more innovative solutions to human capital problems. Carneiro and Heckman (2003) investigate the study of human capital policy by placing it in the context of economic models of life cycle learning and skill accumulation rather than focusing exclusively on which policies have "worked" in the past. This paper extends their analysis by presenting formal models of the investment process.

Figure 1 (source: Carneiro and Heckman (2004)) summarizes the major finding of Carneiro and Heckman (2004) and the motivation for this paper. It plots the rate of return to human capital at different stages of the life cycle for a person of given abilities. The horizontal axis represents age, which is a surrogate for the agent's position in the life cycle. The vertical axis represents the rate of return to investment assuming the same amount of investment is made at each age. Ceteris paribus the rate of return to a dollar of investment made while a person is young is higher than the rate of return to the same dollar made at a later age. Early investments are harvested over a longer horizon than those made later in the life cycle. In addition, because early investments raise the productivity (lower the costs) of later investments, human capital is synergistic. Learning begets learning; skills (both cognitive and noncognitive) acquired early on facilitate later learning. Early deficits make later remediation difficult. Finally, young children's cognition and behavior are more easily malleable than cognition and behavior in adults: even in the absence of dynamic complementarity, early investments are more productive than late investments. For an externally specified opportunity cost of funds r (represented by the horizontal line with intercept r in figure 1), an optimal investment strategy is to invest less in the old and more in the young. At any age, investment is more profitable for persons with higher innate ability. Figure 2 (source: Carneiro and Heckman (2004)) presents the optimal investment quantity counterpart of figure 1.

Carneiro and Heckman (2003) develop an alternative interpretation of figure 1 as an empirical description of the economic returns to investment at current levels of spending in the American economy. The return to investment in the young is high; the return to investments in the old and less able is quite low. A socially optimal investment strategy would equate returns across all investment levels. A central empirical conclusion of their analysis is that at current investment levels, efficiency in public spending would be enhanced if human capital investment were directed more toward the young and away from older, less-skilled, and illiterate persons for whom human capital is a poor investment. The idea of self-productivity of human capital investments is rather old in economics and is present in the work of Ben-Porath (1967) who specifies a production function where the stock human capital increases the productivity of additional investments in human capital: human capital is a crucial input in the production of more human capital. The stock of human capital and investments in human capital are complementary inputs in the production of skill. This complementarity also means that the costs of remediating neglect of early investments in human capital can be very high, if remediation investments have no solid (human capital) base to build on. It also means that if early investments are not followed up by later investments then their effect on the amount of skill accumulated by early adulthood may be small.

Carneiro and Heckman (2003) summarize a body of evidence that suggests that this complementarity is empirically important. Two conclusions are drawn from their paper. First, individuals with higher levels of cognitive ability have higher returns to college than individuals with lower levels of cognitive ability. Further, individuals with higher ability and education are more likely to participate in company training than those with lower ability and education levels. Second, Currie and Thomas (1995) study the Head-Start program and conclude that the overall effects of the program on test scores are lower for black than for white children. In fact, they show that the effect of the program on test scores at the age the program ends is about the same for blacks and whites, but the fade out effects after exit from Head Start are much larger for black children. In a companion paper, Currie and Thomas (1999) suggest that this may be due to the fact that black Head Start children go on to attend much lower quality schools than white Head Start children. Head Start investments are followed up by very poor schooling for black children and therefore it is not surprising that the final effects of Head Start on test scores of blacks is small.[2]

The central theme of this paper is that skills generate more skills. Investments made in the past raise the productivity (or reduce the costs) of current investments. With this in mind, let x denote investment in human capital that occurs during the childhood years, and z denote late investment in human capital that occurs at adolescent years (such as a college education). Heckman (2003) proposes the following nested CES production function of the adult human capital h:

$$h = \left\{ \gamma_0 x^{\phi_1} + (1-\gamma_0) \left[\gamma_1 x^{\phi_2} + (1-\gamma_1) z^{\phi_2} \right]^{\frac{\phi_1}{\phi_2}} \right\}^{\frac{\rho}{\phi_1}} \tag{1}$$

$$\text{with} \quad 0 < \gamma_0, \gamma_1, \rho < 1, \quad \phi_1, \phi_2 \leq 1,$$

In the next section, I add parental human capital, public investment in the early childhood and the kids ability to the production function.

This production function generalizes the Ben Porath model by allowing for qualitatively different investments to be made at different stages of the life cycle while retaining the key notion of the self productivity of human

[2]In other analysis of the Head Start data, Currie, Garces and Thomas (2003) show that Head Start has important effects on high school graduation, wages and criminal behavior of adults.

capital investment. Previous investments make future investments more productive and less costly. The elasticity of substitution across investment stages may change over time. It could be the case that investments become more complementary (or substitutable) later in life.

If there is symmetry of all inputs at all stages so $\gamma_0 = \gamma_1 = \frac{1}{2}$,and $\phi_1 = \phi_2 = \phi$, the technology becomes:

$$h = \left[\frac{3}{4}x^{\phi} + \frac{1}{4}z^{\phi}\right]^{\frac{\theta}{\phi}} \tag{2}$$

Under these assumptions, early investment is especially important for the formation of human capital, and this fact is reflected by assigning x a higher weight in the production function. Technology (2) is a stand-in for the more general recursive technology and the higher weight on x captures the crucial notion that early investment is more productive than later investment through self productivity. Recall that the Hicks-Allen elasticity of substitution is $\mathcal{E} = \frac{1}{1-\phi}$. As $\phi \longrightarrow 1$, we obtain perfect substitutes. As $\phi \longrightarrow -\infty$, we obtain perfect complements. $\phi = 0$ is the Cobb-Douglas case.

3 The Importance of Parental Resources

In this section I review the empirical findings on the role of family income in promoting schooling achievement. I also present new evidence that challenges the view that parents face a complete set of Arrow-Debreu securities. Indeed, I show that an economy such as Laitner (1992) seems to be the one that is consistent with the evidence.

There is a strong relationship between family income and college attendance. Figure 3, reproduced here from Carneiro and Heckman (2004), displays aggregate time series college participation rates for eighteen to twenty-four year old American males classified by their parental income, as measured in the child's late adolescent years. There are substantial differences in college participation rates across family income classes in each year. This pattern is found in many other countries (see the essays in Blossfeld and Shavit 1993). In the late 1970s or early 1980s, college participation rates start to increase in response to increasing returns to schooling, but only for youth from the top family income groups. This differential educational response by income class promises to perpetuate or widen income inequality across generations and among racial and ethnic groups.

There are two not necessarily mutually exclusive interpretations of this evidence. The common interpretation of the evidence, and the one that guides current policy, is the obvious one. Credit constraints facing families in a child's adolescent years affect the resources required to finance a college education. A second interpretation emphasizes more long-run factors associated with higher family income. It notes that family income is strongly correlated over the child's life cycle. Families with high income in a child's adolescent years are more likely to have high income throughout the child's life at home. Better family resources in a child's formative years are associated with higher quality of education and better environments that foster cognitive and noncognitive skills.

Carneiro and Heckman (2003) argue on quantitative grounds that the second interpretation of figure 3 is by far the more important one. Controlling for ability formed by the early teenage years, parental income plays only a minor role. The evidence from the U.S. presented in their research suggests that at most 8 percent of American youth are subject to short-term liquidity constraints that affect their postsecondary schooling. Most of the family income gap in enrollment is due to long-term factors that produce the abilities needed to benefit from participation in college.

In this paper, I follow the path of Carneiro and Heckman (2003) and build on their analysis by providing new evidence that long-run family resources are indeed important. In order to carry out this task, I start by merging two data sets, the NSLY/1979 and the Children of the NLSY/1979. From the former, I use the information on education, AFQT, earnings and demographics for females. From the latter I get information on children's education, ability (PIAT math test scores), and number of members in the household. Note that mother's income is reported yearly in the NLSY. I convert this information in terms of child's age and per capita terms. For example, consider a child that is born in 1979 from a mother with income of U$10,000.00 in 1979 and U$20,000.00 in 1980 in a household with 3 other people (so there are 5 people total). I report that the mother's income is U$2,000.00 at age 0 of the child, and U$4,000.00 at age 1 of the child. We refer the reader to appendix B for the details on the construction of the per capita permanent income variables used below.

Using the merged data set I replicate an exercise described in Carneiro and Heckman (2003). Table 3.1 shows the results of probits whose dependent variable is college attendance. In column 1, we show the results for the probit when we consider the child's ability, household per capita permanent income between ages 0 and 18, mother's education, mother's AFQT and demographic controls (not reported). All of these variables, but mother's AFQT, help predict schooling attainment of the child. We remark that even after controlling for permanent income, mother's schooling is an important determinant of schooling achievement of the offspring.

The additional probits also control for the timing of the income. For example, column 2 presents all the controls from column 1 plus the household per capita permanent income between ages 0 and 5. In all cases, with the exception of probit 6, the timing of income does not help predict schooling attainment. Moreover, in all cases, mother's education is an important variable to determine schooling attainment. The same is true for household per capita permanent income between ages 0 and 18, with the exception of probit 6. In that case, after we control for permanent income between ages 0-5, then permanent income between ages 0-18 is no longer significant, but neither is permanent income between ages 0-5. One possible explanation for the reason this odd results arises is because both variables are highly colinear, as indicated in table 3.4. When we run an OLS regression of household per capita permanent income between ages 0 and 18 against a constant and household per capita permanent income between ages 0 and 5, we get an adjusted R^2 of almost 1. As it is widely known, the coefficient estimators of two colinear variables have large standard deviations, and thus very small $t-$statistics, making it impossible to test whether the timing of the income matters or not.

This exercise is an evidence in favor of the view that, given the current set of tuition policies, credit constraints in college going years do not seem to play an important role in determining college attendance. The result of table 3.1 is consistent with three economies. First, if schooling is a normal consumption good, it should be the case that parents with higher permanent income are going to buy more of that good. One consequence of schooling as a consumption good is that, unless its income elasticity is very small, the returns to schooling for high-ability children of rich parents should be very small. This implication, however, is at odds with the evidence of Carneiro and Heckman (2004), who show that these are precisely the agents that have the highest returns.

Second, the results from table 3.1 are also consistent with parents facing a complete set of Arrow-Debreu securities. Even if markets are complete, the positive coefficient on lifetime resources may be capturing some unobserved trait that is correlated with parental resources, thus a classical problem of omitted variables. Appendix B shows the argument analytically.

Third, the result from table 3.1 is also consistent with an economy in which parents can transfer resources within their lifetime, but cannot borrow against the income of their offspring in order to finance investments in human capital of the child. This is an economy with markets arrangement such as Laitner (1992), which I describe next.

4 Altruism, Lifetime Liquidity Constraints and Human Capital Investments.

Laitner (1992) analyses an economy in which altruistic households are subject to lifetime liquidity constraints. More specifically, markets are incomplete: agents cannot buy insurance claims against exogenous (ability or productivity) shocks. The credit market works in the following way: while parents can borrow against their income when old, they cannot borrow against the income of their offspring. Laitner (1992) uses this model to understand the pattern of intergenerational transfers. However, he does not consider investments in human capital. In this section, I show how to extend Laitner (1992) to allow for investment in human capital.

4.1 A Description of the Model.

4.1.1 Generational Structure, Ability and the Human Capital Production Function.

The environment is an overlapping generations economy with an infinite number of periods, each one denoted $t \in \{0, 1, 2, ...\}$. Each generation consists of a continuum of agents. Each agent lives for four periods[3]. At age 1,

[3]In what follows, I use period to describe the evolution of time of the economy, and age to describe that of the agent. Obviously, an agent lives for 4 ages, or 4 periods.

the agent is a child, at age 2 the agent is an adolescent, at age 3 the agent becomes a young adult (and has a child of his own), at age 4 the agent becomes an old adult. At the end of age 4, the agent dies and is replaced by the generation of his grandchild. Note that at every period there are agents of every possible age. Life goes on in the future in similar fashion. Table 4.1 describes the demographics of the economy.

Next, I describe the stochastic process of ability. To make matters simple, I assume that child's ability a' follows a first-order Markov process of the form:

$$\log a' = \mu \log a + \nu$$

where a is the parent's ability, $\mu \in (0,1)$ and ν is distributed according to F_ν. I denote by F_a the invariant distribution of ability and by $\mathcal{A}$ its support. The distribution function F_a is a primitive of the model. The invariance assumption generates the implication that the cross-sectional distribution of ability will be given by $F_{a,t} = F_a$ for all t. The ability of a child is perfectly observed once the child is born.

Adults differ in terms of their human capital h. The parent can influence the productivity of his offspring by investing in the education of the kid during childhood and adolescent periods. Let h denote the human capital of the parent. Assume that the parent invests resources $x, z \geq 0$ during the childhood and adolescent years of his offspring. In the application below, I interpret $z > 0$ (positive investment in human capital at adolescent years) as "college". The public provision of education is given by a uniform per-child supply g of education goods that is financed with an income tax. I use τ to denote the income tax rate. Any investment $x, z \geq 0$ made by the parent is in addition to the public amount. I assume that there is no public provision of college, so the only way the agent can attend college is by parent expenditures through $z > 0$. Therefore, the kid with ability a', parental human capital h, public supply of education g and private expenditures x, z will grow up and become an adult person with human capital h' as described by

$$h' = \left\{ \gamma_a \left(a'\right)^{\phi_1} + \gamma_h h^{\phi_1} + \gamma_x (x+g)^{\phi_1} + (1-\gamma_x-\gamma_a-\gamma_h) \left[\gamma_z (x+g)^{\phi_2} + (1-\gamma_z)(z+g)^{\phi_2} \right]^{\frac{\phi_1}{\phi_2}} \right\}^{\frac{\rho}{\phi_1}}$$

for $\phi_1, \phi_2 \leq 1, 0 < \rho < 1, 0 < \gamma_h < 1, \gamma_a > 0$.

4.1.2 The Problem of the Agent

The Budget Constraint Next, I describe the way agents go through life. At age 1 and 2, agents are children and do not work. They only go to school. The amount of schooling the child gets is as described above. When the agents reach age 3, they become young adults and also a parent (that is, each agent has one child). The important point here is that child and adolescent individuals have no volition and make no economic decisions.

The young parent starts age 3 with a stock of human capital (or efficiency units) h, inheritance in form of physical assets b, and gives birth to a child of ability a'. I assume that labor supply is perfectly inelastic, so that labor income of the parents is given by wh, where w is the wage rate of one efficiency unit. Young agents are subject to productivity innovations ε when young. It is assumed that shocks ε are independently and identically distributed across individuals and over time. The distribution of the productivity innovation ε has support $[\varepsilon_{\min}, \varepsilon_{\max}]$ with $\varepsilon_{\min} > 0$ and $\varepsilon_{\max} < \infty$. We denote by F_ε the distribution of ε, and note that $E(\varepsilon) = 1$ and $Var(\varepsilon) = \sigma_\varepsilon^2 < \infty$. To finance the public supply of education, the government charges an income tax τ. Therefore, the after-tax disposable income of the young parent is $(1-\tau)(wh\varepsilon)$. The individual state variables for the young parent are given by the vector $\xi^y = (\varepsilon, a', b, h)$. Given vector ξ^y, the young parent chooses consumption when young c^y, savings s (given interest rate r), and early investment x (whose relative price is p) in the human capital of his child. Because the parent cannot leave negative bequests to his child, the parent cannot borrow more than his earnings when old (the natural borrowing limit). Let $V(\varepsilon, a', b, h)$ denote the value function of the young parent. The problem of the period-t young parent is:

$$V\left(\varepsilon, a', b, h\right) = \max \left\{ u\left(c^y\right) + \beta \int W\left(\eta, a', s, h, x\right) dF_\eta(\eta) \right\}$$

$$c^y + px + \frac{s}{1+r} = (1-\tau)(wh\varepsilon) + b \tag{3}$$

$$s \geq -(1-\tau)(wh\eta_{\min}), x \geq 0 \tag{4}$$

where $W(\eta, a, s, h, x)$ is the value function of the old parent (see discussion below). Equation (4) represents the natural borrowing limit of Aiyagari(1994). The first-order conditions for savings s, and early investment x are:

$$\frac{\partial u(c^y)}{\partial c^y} \frac{1}{(1+r)} = \beta \int \frac{\partial W(\eta, a, s, h, x)}{\partial s} dF_\eta(\eta) \tag{5}$$

$$\frac{\partial u(c^y)}{\partial c^y} p = \beta \int \frac{\partial W(\eta, a, s, h, x)}{\partial x} dF_\eta(\eta) \tag{6}$$

Upon reaching age 4 the agents become old adults (or old parents). The parents start age 4 with the same stock of human capital h (so that before tax labor income is still wh), and the child's ability is a'. The parents have savings given by s, and a total x of early investments. Old agents are subject to productivity innovations η when old, too. It is assumed that shocks η are independently and identically distributed across individuals and over time. The distribution of the productivity innovation η has support $[\eta_{\min}, \eta_{\max}]$ with $\eta_{\min} > 0$ and $\eta_{\max} < \infty$. We denote by F_η the distribution of η, and note that $E(\eta) = 1$ and $Var(\eta) = \sigma_\eta^2 < \infty$. Therefore, the after-tax disposable income of the young parent is $(1-\tau)(wh\eta)$. Note that the old parent's after-tax disposable earnings are always greater than or equal to $(1-\tau)(wh\eta_{\min})$. It is this value that determines the natural borrowing limit (4). The individual state variables for the young parent are given by the vector $\xi^o = (\eta, a', s, h, x)$. Given state vector ξ^o the old parent makes decisions regarding consumption c^o, the late investment z on his child's human capital and the bequests b' his child inherits once the old parent dies. The parent may decide to leave positive bequests to his child because of parental altruism. We use θ to capture the degree of altruism of the parent toward the child. Note that $\theta = 0$ means parents are not altruistic at all. When deciding on z the parents are also deciding on the human capital stock of the child h', since everything else is already determined. If the parent decides to invest $z > 0$ it has to pays p dollars per unit of z plus a fixed cost φ (tuition). At the end of age 4, the parent dies, the child becomes a young adult and gives birth to the generation of the grandchild. Let a'' denote the ability of the grandchild. Formally, the problem of the period-$t+1$ old parent is

$$W(\eta, a', s, h, x) = \max\left\{u(c) + \beta\theta \int\int V(\varepsilon', a'', b', h')\, dF_\varepsilon(\varepsilon')\, dF_a(a'')\right\}$$

$$c^o + (pz + \varphi) + \frac{b'}{1+r} = (1-\tau)(wh\eta) + s,\ if\ z > 0 \tag{7}$$

$$c^o + \frac{b'}{1+r} = (1-\tau)(wh\eta) + s,\ if\ z = 0 \tag{8}$$

$$b' \geq 0 \tag{9}$$

$$h' = \left\{\gamma_a (a')^\phi + \gamma_h h^\phi + \gamma_x (x+g)^\phi + (1-\gamma_x-\gamma_a-\gamma_h)(z+g)^\phi\right\}^{\frac{\rho}{\phi}}$$

The first-order conditions for bequests b' and late investments z are:

$$\frac{\partial u(c^o)}{\partial c^o} \frac{1}{(1+r)} = \beta\theta \int\int \frac{\partial V(\varepsilon', a'', b', h')}{\partial b'} dF_\varepsilon(\varepsilon')\, dF_a(a''),\ if\ b' > 0 \tag{10}$$

$$\frac{\partial u(c^o)}{\partial c^o} p = \beta\theta \int\int \frac{\partial V(\varepsilon', a'', b', h')}{\partial h'} \frac{\partial h'}{\partial z} dF_\varepsilon(\varepsilon')\, dF_a(a''),\ if\ z > 0 \tag{11}$$

To finish the characterization of the household problem, I show the envelope conditions:

$$\frac{\partial W(\eta, a, s, h, x)}{\partial s} = \frac{\partial u(c^o)}{\partial c^o} \tag{12}$$

$$\frac{\partial W(\eta, a, s, h, x)}{\partial x} = \beta\theta \int\int \frac{\partial V(\varepsilon', a'', b', h')}{\partial h'} \frac{\partial h'}{\partial x} dF_{\varepsilon}(\varepsilon')\, dF_a(a'') \tag{13}$$

$$\frac{\partial V(\varepsilon', a'', b', h')}{\partial b'} = \frac{\partial u(c^y)}{\partial c^y} \tag{14}$$

$$\frac{\partial V(\varepsilon', a'', b', h')}{\partial h} = \frac{\partial u(c^y)}{\partial c^y}(1-\tau) w + \beta \int \frac{W(\eta, a, s, h, x)}{\partial h} dF_{\eta}(\eta) \tag{15}$$

$$\frac{W(\eta, a, s, h, x)}{\partial h} = \frac{\partial u(c^o)}{\partial c^o}(1-\tau) w + \beta\theta \int\int \frac{\partial V(\varepsilon', a'', b', h')}{\partial h'} \frac{\partial h'}{\partial h} dF_{\varepsilon}(\varepsilon')\, dF_a(a'') \tag{16}$$

4.1.3 Consumption Good Production

I use capital letters to denote aggregate levels. Because I focus on stationary equilibrium, I do not use time subscripts. There are two inputs in the production function of goods: physical capital and labor, which is measured in efficiency units. Let $\mathbf{K}, \mathbf{L}$ denote the aggregate quantities of physical capital and labor, respectively. Let $\mathbf{Y}$ denote the aggregate output. The production technology is represented by the production function F:

$$\mathbf{Y} = F(\mathbf{K}, \mathbf{L})$$

Assumption A1 The production function of aggregate output has the following properties: F presents constant returns to scale, is strictly increasing in all its arguments, satisfies the Inada Conditions, and is twice-continuously differentiable.

The problem of the firm in the good production sector is:

$$\pi_Y = \max\{F(\mathbf{K}, \mathbf{L}) - w\mathbf{L} - (r+\delta)\mathbf{K}\}$$

The first-order conditions are:

$$w = \frac{\partial F(\mathbf{K}, \mathbf{L})}{\partial \mathbf{L}}$$

$$(r+\delta) = \frac{\partial F(\mathbf{K}, \mathbf{L})}{\partial \mathbf{K}}$$

4.1.4 Education Good Sector

There is an educational sector that produces goods for investment in human capital. Let $\mathbf{E}$ denote the total supply of educational goods. This sector does not use physical capital as input, only labor $\mathbf{U}$.

Assumption A2 This sector does not use physical capital as input, only labor $\mathbf{U}$. The production technology is:

$$\mathbf{E} = \mathbf{U}$$

The problem of the firm is:

$$\pi_E = \max\{p\mathbf{E} - w\mathbf{U}\}$$

As is commonly known, this problem has a solution with limited, positive production if, and only if:

$$p = w$$

4.1.5 Feasibility

Let $\mathbf{C}^y, \mathbf{C}^o$ denote the aggregate consumption of young and old adults, respectively. Let $\mathbf{I}$ denote the aggregate investment in physical capital. Feasibility in the goods production sector implies:

$$\mathbf{C}^y + \mathbf{C}^o + \mathbf{I} = \mathbf{Y}$$

At every period there is a cohort of children and a cohort of adolescents (see table 4.1). Let $\mathbf{X}, \mathbf{Z}$ denote the aggregate levels of early and late investment, respectively. At this point, I remind the reader that there is no population growth and the size of each generation is normalized to one. Each cohort of children and adolescent is entitled to a public supply of g units of education goods. Therefore, the total supply of education goods at every period is $2g$. Consequently, the market clearing condition for this sector is:

$$\mathbf{X} + \mathbf{Z} + 2g = \mathbf{E}$$

Next, let $\mathbf{B}, \mathbf{S}$ denote the aggregate stock of bequests and savings held by young and old parents, respectively. I use $\mathbf{K}$ to denote the aggregate stock of physical capital. Feasibility of physical capital allocation implies:

$$\frac{\mathbf{B} + \mathbf{S}}{1 + r} = \mathbf{K}$$

Let $\mathbf{H}, \mathbf{L}, \mathbf{U}$ denote the aggregate amount of efficiency units[4] (or human capital), the aggregate efficiency units allocation to the goods sector, and the aggregate supply of efficiency units in the educational sector. Feasibility of the human capital (or efficiency units) allocation implies:

$$\mathbf{L} + \mathbf{U} = \mathbf{H}$$

The government budget is balanced[5]:

$$pg = (1 - \tau) w \int h dF_h (h)$$

4.2 Definition of Stationary General Equilibrium with Uninsurable Income Shocks

In what follows, we denote by $\zeta^y = (\varepsilon, a, b, h, F_h, F_b, F_s)$ and $\zeta^o = (\eta, a, s, h, x, F_h, F_b, F_s)$ the set of state variables for the young and old parents, respectively. Next, I define the equilibrium.

[4]Let $\mathbf{H}^y, \mathbf{H}^o$ denote the period-t aggregate stock of efficiency units of old and young parents, respectively. The aggregate stock of human capital is given byThree remarks should be made at this point: a) there is no population growth, b) there is no investment in human capital once the agent becomes an adult, c) there is no depreciation of human capital. These three points imply that in a stationary equilibrium, the steady-state distribution of human capital of the old and young adults are the same. Thus:

$$\mathbf{H}^y = \int_{\mathcal{H}} h dF_h (h)$$

$$\mathbf{H}^o = \int_{\mathcal{H}} h dF_h (h)$$

Consequently:

$$\mathbf{H}^y = \mathbf{H}^o$$

Normalizing the size of each cohort to 1 and assuming there is no population growth, it follows that the period-t aggregate stock of human capital is:

$$\mathbf{H} = 2\mathbf{H}^y$$

[5]Note that since the shocks are idiosyncratic, $\int \varepsilon dF_\varepsilon (\varepsilon) = \int \eta dF_\eta (\eta) = 1$.

5 One Illustration of the Theory

5.1 Computation of the Steady State

I start by describing the algorithm to calculate the steady state.

Step 1: Guess equilibrium interest rate and wages r^j, w^j.

Step 2: Guess equilibrium expenditures in education: $g^{j,i}$. The goal is that in equilibrium $g = 2\%$ of the GDP (see discussion below).

Step 3: Guess value functions $V^{j,i,t}$, and $W^{j,i,t}$ and calculate $V^{j,i,t+1}$, and $W^{j,i,t+1}$ by discretizing the state-space and perform Bellman Iteration with Policy Function Iteration.

Step 4: Check if $max\left\{\left|V^{j,i,t}-V^{j,i,t+1}\right|,\left|W^{j,i,t}-W^{j,i,t+1}\right|\right\}<\epsilon_{VW}$ convergence of value functions has been achieved. If not, go back to Step 3.

Step 5: Draw a, ε, η from stationary distributions 210,000 times. Discard the first 200,000 to allow for series to converge to stationary distributions of b, s, h. Given the simulated series, compute expenditures $g^{i,j+1}$. Check if $\left|g^{j,i}-g^{j,i+1}\right|<\epsilon_g$ convergence of the public expenditure has been achieved. If not, go back to Step 2.

Step 6: Using the simulated series generated in Step 5, compute r^{j+1}, w^{j+1}. Check if $max\left\{\left|r^j-r^{j+1}\right|,\left|w^j-w^{j+1}\right|\right\}<\epsilon_{r,w}$ convergence of prices achieved. If not, go back to Step 1.

5.2 Description of the Data and Making up Numbers

To contrast the model with the data, I use a sample of white males from the NLSY data. Following the preceding theoretical analysis, I consider only two schooling choices: high school and college graduation. I take each period in the model to represent ten years. Thus agents are assumed to live for 40 years. I follow Carneiro, Heckman and Masterov (2004) and model the ability process by:

$$\log a' = 0.4 \log a + \nu$$

$$\nu_t \sim N(0,1)$$

These two conditions imply that the invariant distribution of ability F_a is $N\left(0, \frac{1}{1-(0.4)^2}\right)$. Figure 4 plots the stationary density of ability. I approximate the Markov process for log ability according to the procedure developed in Tauchen(1986), using a 15-point grid to approximate the process. Thus, for this example $\mathcal{A} = \{a_1, ..., a_{15}\}$ and $a_1 < a < a_{15}$ for $\forall a \in \mathcal{A}$.

The production function for human capital is:

$$h' = A_H\left\{\gamma_a\left(a'\right)^\phi+\gamma_h h^\phi+\gamma_x(x+g)^\phi+\left(1-\gamma_x-\gamma_a-\gamma_h\right)(z+g)^\phi\right\}^{\frac{\rho}{\phi}}$$

and we use $\gamma_a = 0.2$, $\gamma_h = 0.2, \rho = 0.7, \gamma_x = 0.35, \gamma_z = 0.25, \phi = -1.5$.

The distributions of productivity shocks are both lognormal. That is, $\log\varepsilon \sim N\left(0,\sigma_\varepsilon^2\right)$ and $\log\eta \sim N\left(0,\sigma_\eta^2\right)$. In this example, $\sigma_\varepsilon^2 = \sigma_\eta^2 = 0.1$. I approximate the distribution of the log of productivity shocks using the procedure developed in Tauchen(1986), with a 15-point grid.

The value of g is set such that in equilibrium it represents 2% of the GDP. The GDP of this economy is given by $GDP = Y + wE$. Tuition φ is taken to be the average local tuition for four-year college from the NLSY/1979 data, which corresponds to around two-thousand one-hundred dollars in (2000 dollars). This corresponds to the amount of public expenditures by the federal government in public primary and elementary education in the US, according to the Statistical Abstract of the US (1999). I take $X = \{0, 0.5\}$ and $Z = \{0, 2.8\}$. Again, the child is labeled college graduate if $z = 2.8$, and high-school graduate otherwise. The value $z = 2.8$ is meant to capture opportunity costs of going to college. In this example, I assume that every adolescent that does not go to college can get a job that pays earnings $w.\underline{\mathrm{h}}$, where $\underline{\mathrm{h}}$ is the solution of:

$$\underline{\mathrm{h}} = A_H\left\{\gamma_a\left(a'\right)^\phi+\gamma_h\underline{\mathrm{h}}^\phi+\gamma_x g^\phi+\left(1-\gamma_x-\gamma_a-\gamma_h\right)g^\phi\right\}^{\frac{\rho}{\phi}}$$

that is, $\underline{h}$ is the lowest amount of human capital attainable in this economy (and I remind the reader that $a_1 < a$ for $\forall a \in \mathcal{A}$). For the particular example I show here, it turns out that $\underline{h} \simeq 7$. Because each period of the model represents 10 years, and going to college only takes 4 years, it follows that $z = \frac{4*7}{10} = 2.8$. In terms of dollars, this implies that, in equilibrium, the present value of going to college is around fifty-six thousand dollars. Figure 5 shows the stationary density of human capital. As one can see, it is a bimodal distribution.

The utility function is:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

with $\sigma = 2$.

The aggregate production function is:

$$F(\mathbf{K}, \mathbf{L}) = A_Y \mathbf{K}^{\alpha} \mathbf{L}^{1-\alpha}$$

with $\alpha = 0.36$.

5.3 Results

5.3.1 How the Model Fits the Data

I start by performing a variety of checks of fit of predictions against the data. First, I compare the proportions of people who choose each schooling level. In the NLSY data, 54.5% choose high school and 45.5% choose college. From table 4 one can see that the baseline economy predicts 56.8% and 43.2%, respectively. The example baseline economy replicates the observed proportions well.

In figure 6 I show the densities of the predicted and actual present value of earnings for the overall sample. Figures 7 and 8, show the same densities restricted to the sample of those who choose high school (7) and college (8). The fit is reasonable, although it does not pass a goodness of fit test.

5.3.2 Selection on Ability and Credit Constraints

The empirical analysis of Carneiro, Hansen and Heckman (2003), and Cunha, Heckman and Navarro (2004) show that there is selection on ability: more able agents are more likely to graduate from college. From figure 9 one can easily see that the example also generates the same pattern here.

According to Carneiro and Heckman (2003), the evidence from the U.S. presented in their research suggests that at most 8 percent of American youth are subject to short-term liquidity constraints that affect their postsecondary schooling. As I show below, in a deterministic economy in which parent could borrow against their offspring earnings, all agents which are above the median ability group (ability group 8) would become college graduates, while all agents below the median ability group would become high-school graduates. In this economy, in which agents are subject to lifetime liquidity constraints and earnings shocks, about 6.5% of the agents above the median ability group do not go on to college (table 5). This number is thus remarkably consistent with the empirical findings of Carneiro and Heckman (2003).

5.3.3 Counterfactuals and Returns

In estimating the distribution of earnings in counterfactual schooling states within a policy regime (e.g., the distributions of college earnings for people who actually choose to be high school graduates) Carneiro, Hansen and Heckman (2003), and Cunha, Heckman and Navarro (2004) find that college graduates tend to have higher earnings than high school graduates in both possible schooling states. Figures 10 and 11 present the marginal distributions of fitted and counterfactual distributions of earnings for high school (10) and college (11). When we compare the densities of present value of earnings for persons who choose college, both in the college sector (factual) and high school sector (counterfactual), the density of the present value of earnings for college graduates is to the right of the counterfactual density of the present value of earnings of high school graduates if they were college graduates. Figure reveals the same pattern for actual high school graduates. This is yet another empirical finding that the baseline economy replicates.

Figure 12 plots the density of returns to college for agents who are high school graduates (the solid curve), and the density of returns to education for agents who are college graduates (the dashed curve). College graduates have returns distributed "to the right" of high school graduates. An economic interpretation of figure 12 is that agents who choose a college education are the ones who tend to gain more from it. Again, this pattern is consistent with the findings of Carneiro, Heckman and Hansen (2003), and Cunha, Heckman and Navarro (2004).

Figure 13 plots observed (ex-post) internal rate of return to college. The same pattern of selection repeats itself here. Table 7 shows that while college graduates have an internal rate of return to college of around 9% per year, the high school graduates are only 1.11% per year. Even though the model replicates the pattern of selection on returns, it underpredicts the mean returns conditioned on college and high school as summarized by Carneiro and Heckman (2004). These returns could be made greater if I also included earnings from ages 41 through 65 (the retirement age).

It should be noted that agents choose in terms of expected (or ex-ante), and not observed (ex-post), returns. Figure 14 shows that even in terms of ex-ante returns there is selection.

5.3.4 Inequality

How well does the model predict the inequality in lifetime earnings? Lillard (1977) reports Ginni coefficients for lifetime earnings around 19%. The model underpredicts Lillard's estimation. According to the baseline economy, the Ginni coefficient is only 11.14%. Figure 17 plots the Lorenz curve generated by the model. One possible explanation to the discrepancy between the baseline economy and Lillard (1977) findings is that here we are only fitting the data from ages 21 to 40. In principle some of the understatement of inequality generated by the model can be recovered by allowing agents to "live longer".

5.4 Counterfactual Economy 1: Neither Productivity Shocks, Nor Lifetime Liquidity Constraints.

Next, I show how the model can be used to generate counterfactual environments. I start by computing the stationary equilibrium of an economy with deterministic earnings and no lifetime liquidity constraints. Formally, the problem of the agent becomes:

$$V\left(a',b,h\right)=\max\left\{u\left(c^{y}\right)+\beta u\left(c^{o}\right)+\beta\theta\int V\left(a'',b',h'\right)dF_{a}\left(a''\right)\right\}$$

$$c^{y}+px+\frac{c^{o}}{1+r}+\frac{(pz+\varphi)}{1+r}+\frac{b'}{(1+r)^{2}}=(1-\tau)(wh)+b, if\ z>0$$

$$c^{y}+px+\frac{c^{o}}{1+r}+\frac{b'}{(1+r)^{2}}=(1-\tau)(wh)+b, if\ z=0$$

$$h'=\left\{\gamma_{a}\left(a'\right)^{\phi}+\gamma_{h}h^{\phi}+\gamma_{x}(x+g)^{\phi}+(1-\gamma_{x}-\gamma_{a}-\gamma_{h})(z+g)^{\phi}\right\}^{\frac{\rho}{\phi}}$$

It is easy to show that in this economy the steady-state interest rate is $r=\frac{1}{\beta}$. The fact that the production function presents constant returns to scale allows us to recover the steady-state wage rate. Given prices, we solve the problem of the agent and obtain the decision rules on savings, early investment in human capital, bequests and late investments in human capital. We draw a series of random shocks in ability from its stationary distribution and generate the simulated series by applying the decision rules. I then check to see if the aggregates produced by the simulated series are consistent with stationary prices. They are. This simulation produces the following results that I next describe.

First, when I eliminate the productivity shocks that generate earnings uncertainty, and allow parents to borrow against the earnings of the kid, there is not much increase in college attendance. Table 4 compares the figures.

While the baseline economy indicates that around 43.2% of the agents choose college, the economy without earnings uncertainty and no liquidity constraints shows that only 45.55% of the agents would choose college. This is roughly an increase of two percentage points in college attendance as a result of the elimination of all uncertainty in earnings and the introduction of the possibility of leaving large debts to the kids.

To understand how these results arise I start by showing the profile of selection implied by the model. Table 8 shows the distribution of ability by education group. As one can easily see from table 8, the stationary equilibrium is almost a separating one. All the agents below the median ability group (ability group 8) are high-school graduates. All the agents above the median ability group are college graduates. The median ability group has agents in both groups, though. Agents that have high parental human capital become college graduates. Those who do not have high parental human capital go on to become high-school graduates. This can also be seen from table 9 that plots the equilibrium internal rate of returns for college. High-returns agents (those with high ability) become college graduates. Low-return agents remain high-school graduates. This constrasts with the selection profile generated by the baseline economy, in which there was pooling of ability in equilibrium.

Two reasons provide the intuition for the "mixing" equilibrium generated by the baseline economy which is actually found empirically. First, in an economy a la Laitner (1992) the steady state equilibrium interest rate is below that of an economy without liquidity constraints and earnings uncertainty. This fact is established by Laitner (1992), and Aiyagari (1994). The combination of earnings productivity shocks and incomplete markets generates precautionary savings, which causes the equilibrium interest rate to plunge. This effect makes it profitable for lower ability agents to invest in human capital.

The second reason is a classical result of portfolio allocation. In the baseline economy, agents are subject to productivity shocks. These shocks make human capital a risky asset. Because we are analyzing steady states, the interest rate is constant. Thus, physical capital is a risk-free investment. When an asset is risky, agents will invest part of their portfolio on it even if expected returns are below that of a risk-free asset. This effect acts on the same direction of the Laitner (1992) interest-rate effect and makes investment in human capital attractive for lower ability agents.

All in all, eliminating earnings uncertainty and allowing parents to borrow against the income of their children does not have strong effects on the number of agents who become college graduates. As in Carneiro, Hansen and Heckman (2003), the effect is small. However, it does affect strongly the decisions of who goes to college.

5.5 Counterfactual Economy 2: Introducing a Full Tuition Subsidy

As shown before, about 6.5% of the agents with ability above the mean (and who would become college graduates) do not go to college in the baseline economy, because the parents receive a negative income shock. The interpretation of this evidence is the one that guides current policy: families are constrained in college going years. The solution proposed by policy makers is to introduce tuition subsidies financed by increasing tax rates to ammeliorate the problem. The hope is that with the tuition subsidy parents will be able to send the able kids to college. To analyze this question, I change the model in the following way. First, all parents that send the kids to college do not have to pay tuition. Second, in order to finance the tuition subsidy, the government increases tax to keep the budget balanced. Thus, the model becomes:

$$V\left(\varepsilon, a', b, h\right) = \max \left\{ u\left(c^{y}\right) + \beta \int W\left(\eta, a', s, h, x\right) dF_{\eta}\left(\eta\right) \right\}$$

$$c^{y} + px + \frac{s}{1+r} = \left(1 - \tilde{\tau}\right)\left(wh\varepsilon\right) + b$$

$$s \geq -\left(1 - \tilde{\tau}\right)\left(wh\eta_{\min}\right), x \geq 0$$

where

$$W\left(\eta, a', s, h, x\right) = \max \left\{ u\left(c\right) + \beta\theta \int \int V\left(\varepsilon', a'', b', h'\right) dF_{\varepsilon}\left(\varepsilon'\right) dF_{a}\left(a''\right) \right\}$$

$$c^o + pz + \frac{b'}{1+r} = (1-\tilde{\tau})(wh\eta) + s, \text{ if } z > 0$$

$$c^o + \frac{b'}{1+r} = (1-\tilde{\tau})(wh\eta) + s, \text{ if } z = 0$$

$$b' \geq 0$$

$$h' = \left\{\gamma_a (a')^\phi + \gamma_h h^\phi + \gamma_x (x+g)^\phi + (1-\gamma_x-\gamma_a-\gamma_h)(z+g)^\phi\right\}^{\frac{\rho}{\phi}}$$

where $\tilde{\tau}$ is the tax rate so that in equilibrium the government budget is balanced at every period:

$$\tilde{\tau} = \frac{2wg + \Pr(z>0)\varphi}{w\int hdF_h(h)}$$

and $\Pr(z>0)$ denotes the proportion of agents that attend college under the tuition policy. The algorithm to compute the steady-state of the economy with full-tuition subsidy is similar to that of the baseline economy, so we skip the discussion here. Next, I show the predictions of the model of the full-tuition subsidy and compare it to what is found in the empirical literature.

First, when a tuition subsidy is implemented, there is a substantial increase in college attendance, as one can see from table 4. While in the baseline economy about 43% of the agents become college graduates, in the economy with a tuition subsidy that figure is almost 56%. This result is different from what Keane and Wolpin (2002) find in their model. The difference may be due to the fact that in the Keane and Wolpin (2002) economy agents are allowed to work and study. The possibility of working while in college makes credit constraints less severe. As a result, Keane and Wolpin (2002) find that when tuition subsidies are implemented, there is little change in college and a large increase in leisure.

Figure 18 shows the densities of ability for high-school graduates in the baseline economy and the economy with full-tuition subsidy. Note that the density of the economy with full-tuition subsidy is to the "left" of that implied by the baseline economy. This means that the agents that become college graduates as a response to the tuition subsidy are prone to have above-averege ability in the high school distribution of ability. Figure 22 shows that the same pattern is true for the density of present value of lifetime earnings: most of the agents that move to college are above average earners in the high school distribution of present value of lifetime earners.

But how does the ability of movers compare to the ability of those who are college graduates in the baseline economy? Figure 19 plots the densities of ability for college graduates as implied by the baseline and full-tuition economies. Note that again the density generated by the full-tuition economy is to the "left" of the one predicted by the baseline economy. This fact implies that the agents that move to college as a response to the tuition subsidy are likely to have below average ability in the college distribution of ability. This fact is consistent with the evidence summarized in Carneiro and Heckman (2004). Note that the same result holds when one compares the densities of the present value of lifetime earnings for college graduates (figure 23): the tuition subsidy attracts to college agents that tend to be below-average earners.

Another prediction of the model that is confirmed by the empirical evidence of Carneiro and Heckman (2004) is the pattern of returns to college as a response to the tuition subsidy. Figures 24 - 27 compares the densities of the ex-post internal rate of returns to college in the baseline and full-tuition subsidy economies for the high-school and college sample. Note that while the agents that leave the high-school sector are likely to have above average returns in the high school distribution, they tend to have below average returns in the college distribution. This fact is consistent with the returns to college estimated in the literature using the marginal treatment effects (see Carneiro and Heckman (2003)).

Another reason policymakers may defend a tuition subsidy is because of its redistributive benefits. Figure 28 compares the Lorenz curves generated by the baseline and full-tuition subsidy economies: there is basically no difference. While the Ginni coefficient in the baseline economy is 0.1114, the economy with a full-tuition subsidy is 11.07, thus a marginal reduction in inequality.

How does the agents fare the tuition-subsidy versus the baseline economy? Let (c_b^y, c_b^o) denote the consumption profile of a given agent in the steady-state equilibrium of the baseline economy. Let (c_t^y, c_t^o) denote the consumption profile of that same given agent in the steady-state equilibrium of the economy with full tuition subsidy. The equivalent variation is the constant amount of dollars λ such that:

$$u\left(c_b^y + \lambda\right) + \beta u\left(c_b^o + \lambda\right) + \beta^2 \theta E\left(V_b\right) = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left(V_t\right)$$

The present value of the equivalent variation is defined as:

$$\Lambda = \lambda + \frac{\lambda}{(1+r)}$$

Agents that prefer the baseline economy to the economy with full-tuition subsidy present $\lambda < 0$, since they are willing to forego consumption in the baseline economy not to move to an economy with tuition subsidy. Figure 29 plots the density of the present value of the equivalent variation. Note that around 82% of the agents actually have $\lambda > 0$, so they prefer the full-tuition subsidy economy to the baseline economy. The average of the present value of the equivalent variation Λ is around U$17,000.00. When we consider Λ relative to the present value of lifetime earnings, this means agents should be given, on average, 3.10% of their lifetime earnings in the baseline steady-state economy in order to be indifferent to the steady-state of the full-tuition subsidy economy.

I go beyond and investigate who tends to gain the most with such a policy. Do the high ability agents benefit the most? Table 10 shows the average present value of the equivalent variation by ability group. Note that the agents in the median ability group are the ones that tend to gain the most from the tuition subsidy. Figure 30 shows the graph associated with table 10. In terms of the distribution of lifetime earnings, it is not the parents in the bottom of the distribution that benefit the most. Table 11 and figure 31 show the average present value of the equivalent variation by deciles of the present value of lifetime earnings. It is the agents on and above the fifth-decile of the distribution that tend to gain the most from the policy. These results show that is the middle class, middle ability agents - and not the bright kids from poor parents - that benefit from the tuition policy.

Why does the tuition policy fails to attain its goals? The tuition policy reduces the price of the late investment. However, parents that suffer a bad income shock when the kid is still a child will most certainly underinvest in the child. This low early investment affects the rates of returns to the late investment in a way that the tuition subsidy alone is not enough to make it attractive to invest. On the other hand, consider the parents of children with mean ability. The mean ability children are the ones close to the indifference between high-school and college careers. The tuition subsidy decreases the price of the late investment. Because the price of the late investment is lower, and late and early investment are complements, the parents of mean ability children that can afford it increase their early and late investments to take advantage of the now higher returns to investment in human capital. That is, parents not only increase their late investment because of the tuition subsidy, but since they know they will have the late investment partially subsidized, they consider it now profitable to invest more early.

6 Final Considerations and Future Research

7 Appendix A - Description of the Data

We use white males from NLSY79. In the original sample there are 2439 individuals. We consider the information on these individuals from age 19 to age 41. We discard 663 individuals because they have observations missing for at least one of the covariate variables we use in the analysis. Tables 2a-b contain a description of the number of missing observations per variable. For example, we discard 50 individuals because we do not observe whether they were living in the South when they were 14 years old or not. Then we discard another 6 for not having information on whether they lived in urban area at age 14, other 5 for not reporting the number of siblings, 221 for not indicating parental education. We then restrict the NLSY sample to white males with a high school or college degree. We define high school graduates as individuals having a high school degree or having completed 12 grades and never reporting college attendance. We define participation in college as having a college degree or having completed more than 16 years in school. We exclude the oversample of poor whites. Experience is Mincer experience (age-12 if high-school graduate, age-16 for college graduate). The variables that we include in the outcome and choice equations are number of siblings, parental years of schooling, AFQT, year of birth dummies, average tuition of the colleges in the county the individual lives in at 17 (which we use to simulate the policy), distance to the nearest college at 17, average local blue collar wage in state of residence at 17 (or in 1979, for individuals entering the sample at ages older than 17) and local unemployment rate in county of residence in 1979. Tuition at age 17 is average tuition in colleges in the county of residence at 17. If there is no college in the county then average tuition in the state is taken instead. For details on the construction of this variable see Cameron and Heckman (2001). State average blue collar wages are constructed using data from the BLS. For a description of the NLSY sample see BLS (2001).

In 1980, NLSY respondents were administered a battery of ten achievement tests referred to as the Armed Forces Vocational Aptitude Battery (*ASVAB*) (See Cawley, Conneely, Heckman and Vytlacil (1997) for a complete description). The math and verbal components of the *ASVAB* can be aggregated into the Armed Forces Qualification Test (AFQT) scores.[6] Many studies have used the overall AFQT score as a control variable, arguing that this is a measure of scholastic ability. We argue that AFQT is an imperfect proxy for scholastic ability and use the factor structure to capture this. We also avoid a potential aggregation bias by using each of the components of the *ASVAB* as a separate measure.

For our analysis, we use the random sample of the NLSY and restrict the sample to 1062 white males for whom we have information on schooling, several parental background variables, test scores and behavior. Distance to nearest college at each date is constructed in the following way: Take the county of residence of each individual and all other counties within the same state. The distance between two counties is defined as the distance between the center of each county. If there exists a college (2 year or 4 year) in the county of residence where a person lives then the distance to the nearest college (2 year or 4 year) variable takes the value of zero. Otherwise we compute distance (in miles) to the nearest county with a college. Then we construct distance to nearest college at 17 by using the county of residence at 17. However for people who were older than 17 in 1979 we use the county of residence in 1979 for the construction of this variable.

Local labor market variables for the county of residence are computed using information in the 5% sample of the 1980 Census. For each county group in the census we compute the local unemployment rate and average wage for high school dropouts, high school graduates, individuals with some college and four year college graduates. We do not have this variable for years other than 1980 so, for each county, we assume that it is a good proxy for local labor market conditions in all the other years where NLSY respondents are assumed to be making the schooling decisions we consider in this paper.

We also use the variable annual labor earnings. We extract this variable from the NLSY79 reported annual earnings from wages and salary. Earnings (in thousands of dollars) are discounted to 2000 using the Consumer Price Index reported by the Bureau of Labor Statistics. Missing values for this variable may occur here for two reasons: First, because respondents do not report earnings for wages/salary, and second, because the NLSY becomes biannual after 1994 and this prevents us from observing respondents when they reach certain ages. For example, because the NLSY79 was not conducted in 1995, we do not observe individuals born in 1964 when they are 31 year-old. In this case we input missing values.

To predict missing log earnings between ages 21 and 40 we pool NLSY and PSID data. From the latter, we use the sample of white males that are household heads and that are either high-school or college graduates according to the definition given above. This produces a sample of 3,043 individuals from PSID. To get annual earnings, we multiply the reported CPI-adjusted (2000 =100) hourly wage rate by the annual hours worked and divide the outcome by 1000. Then we have an NLSY-comparable variable. Similarly to NLSY, we generate the mincerian experience according to the rule given above. We also generate dummy variables for cohorts. The first (omitted) cohort consists of individuals born between 1896 and 1905, the second consists of individuals born between 1906 and 1915, and so on up to the last cohort which is made up of PSID respondents born between 1976 and 1985. We pool NLSY and PSID by merging the NLSY respondents in the PSID cohort born between 1956 and 1965.

Let Y_{ia} denote log earnings of agent i at age a. For each schooling choice s, we model the earnings-experience profile as

$$Y_{ia}(s) = \alpha + \beta_0 X_{ia} + \beta_1 X_{ia}^2 + D\gamma + \varepsilon_{ia} \tag{17}$$

[6] Implemented in 1950, the AFQT score is used by the army to screen draftees.

$$\varepsilon_{ia} = \eta_i + v_{ia} \tag{18}$$

$$v_{ia} = \rho v_{ia-1} + \kappa_{ia} \tag{19}$$

where X is Mincer Experience, D is a set of dummy variables that indicate cohort, η_i is the individual effect, and κ_{ia} is white noise.

Now, let $\hat{\varepsilon}_{ia}$ be the estimated residual of the earnings-experience profile. An estimator of the individual effect η_i is

$$\begin{aligned} \hat{\eta}_i &= \frac{1}{\sum\limits_{a=21}^{40} \phi_{ia}} \sum_{a=21}^{40} \phi_{ia} \hat{\varepsilon}_{ia}, \\ \text{where } \phi_{ia} &= \mathbf{1}(\text{if individual } i \text{ is observed at age } a) \end{aligned}$$

Then, we can obtain an estimator of v_{ia} by computing

$$\hat{v}_{ia} = \hat{\varepsilon}_{ia} - \hat{\eta}_i$$

Now, given $\widehat{v}_{ia}$ we can run equation (19) and then compute ρ. From this we obtain an estimator of κ_{ia} according to

$$\hat{\kappa}_{ia} = \hat{v}_{ia} - \hat{\rho}\hat{v}_{ia-1}$$

we can then predict earnings for missing observations for ages 21 to 40 by computing for each individual

$$\begin{aligned} \hat{Y}_{ia}(s) &= \hat{\alpha} + \hat{\beta}_0 X_{ia} + \hat{\beta}_1 X_{ia}^2 + D\hat{\gamma} + \hat{\varepsilon}_{ia} \\ &= \hat{\alpha} + \hat{\beta}_0 X_{ia} + \hat{\beta}_1 X_{ia}^2 + D\hat{\gamma} + \hat{\eta}_i + \hat{\rho}\hat{v}_{ia-1} + \hat{\kappa}_{ia} \end{aligned}$$

Note that to get $\hat{\varepsilon}_{ia}$ we do not set $\hat{\kappa}_{ia}$ equal to zero. Instead, we sample ten draws from its distribution and average them for each individual, for each time period.

The next step is to get the present value of earnings at age 21 for each agent. In order to do it we discount earnings at each period using a discount rate of 3%. To make the data consistent with the theoretical model, we break each individual's working-life in two periods. The first one goes from age 21 to age 30. The second period goes from age 31 through 40. This produces a panel in which the first observation for each agent is the present value of earnings from age 21 to 30 and the second is the present value of log earnings from 31 to 40.

In order to compute the permanent income of the household between ages t_0, and t_1, we proceed in two stages. The first stage is to compute the mean earnings between these two periods $\mu_y(t_0, t_1)$. If earnings are missing for any years between these two periods, because of parental unemployment, or non-interview, we replace the missing earnings with $\mu_y(t_0, t_1)$. The second stage is to compute the present value of earnings between ages t_0, and t_1, if the agent had constant earnings $\mu_y(t_0, t_1)$ for every year between ages t_0, and t_1 of the child. This last variable is taken to be the permanent income of the household between ages t_0, and t_1, $y^p_{t_0,t_1}$. Thus:

$$y^p_{t_0,t_1} = \sum_{s=t_0}^{t_1} \left(\frac{1}{1+r}\right)^{t-t_0} \mu_y(t_0, t_1)$$

8 Appendix B - The Correlation between Permanent Income and Investments in Human Capital in a Complete Markets Economy

Consider the problem of a parent that has to decide how much to invest in the human capital of the child. Suppose that the dynasty only exists for two periods. After the child dies, the dynasty disappears. Assume that all kids have the same amount of cognitive ability and that all parents have the same amount of human capital (both normalized to one). The argument can be extended easily by allowing heterogeneity in ability and parental human capital at the cost of more notation. The only uncertainty arises from the child's productivity ζ_k that is unobserved by the parent when the child is still young. Once the child becomes an adult, his productivity is publicly observed. Further, assume ζ_k is distributed according to F. Let $q\left(\zeta_k|\zeta_p\right)$ denote the price of an Arrow-Debreu security that pays 1 unit of consumption good if the child's earnings productivity is ζ_k and nothing otherwise, conditional on the parent's productivity being ζ_p. Let $b\left(\zeta_k|\zeta_p\right)$ denote the quantity of such claims that the parent purchases. Let u denote the Bernoulli utility function. Let c_p, c_k the consumption of the parent and child, respectively. Let x_k denote the amount of parental investment in the human capital of the child. Let y_p, y_k denote the parent's and the child's income, respectively. The problem of the parent is:

$$\max u\left(c_p\right)+\beta E\left[u\left(c_k\right)|\zeta_p\right]$$

$$\text{subject to : } c_p+\int q\left(\zeta_k|\zeta_p\right)b\left(\zeta_k|\zeta_p\right)dF\left(\zeta_k|\zeta_p\right)+x_k=y_p$$

$$c_k=y_k+b\left(\zeta_k|\zeta_p\right)$$

$$y_k=\zeta_k x^{\alpha}$$

$$\log\zeta_k=\rho\log\zeta_p+\eta$$

$$\eta\sim F_{\eta},\ E\left(\eta\right)=0,\ E\left(\eta^2\right)=\sigma_{\eta}^2$$

The first-order conditions are:

$$\frac{\partial u}{\partial c_p}=\alpha\beta x^{\alpha-1}E\left(\zeta_k\frac{\partial u}{\partial c_k}|\zeta_p\right) \tag{20}$$

$$\frac{\partial u}{\partial c_p}q\left(\zeta_k|\zeta_p\right)=\beta\frac{\partial u}{\partial c_k}dF\left(\zeta_k|\zeta_p\right) \tag{21}$$

With idiosyncratic shocks, there is perfect insurance. Assuming that the price of an Arrow-Debreu security sells at its actuarially fair price:

$$q\left(\zeta_k|\zeta_p\right)=\frac{dF\left(\zeta_k|\zeta_p\right)}{1+r}$$

If $\beta\left(1+r\right)=1$

$$x^{1-\alpha}=\alpha\zeta$$

where $\zeta=E\left(\zeta_k|\zeta_p\right)$. Therefore, once we control for the conditional expectation of the productivity of the child, the parental investment in the human capital of the child is constant. In other words, if we run

$$\log x=\beta_0+\beta_1\log\zeta+\beta_2\log y_p+\varepsilon$$

a test of whether $\beta_2=0$ is indeed a test of existence of complete markets in this particular economy. The key point here is to control for ζ. Next, I show how the permanent income of the household may be capturing the effects of ζ in the investment in human capital when we do not control for ζ. To see this, let ζ_p, x_p denote the parent ability and investment in human capital performed on the parent by the grandparent, so that the earnings of the parent is given by:

$$y_p=\zeta_p x_p^{\alpha}$$

If we assume that:

$$\log\zeta_k=\rho\log\zeta_p+\eta$$

and run the regression:

$$\log x=\beta_0+\beta_2\log y_p+\varepsilon$$

then, it is easy to show that:

$$\text{plim }\hat{\beta}_2=\rho$$

In this sense, the statistically significant coefficient on permanent income from ages 0 to 18 of the child may be capturing some uncontrolled ability whose expected value affects the amount of ability a child gets. This is the classical problem of omitted variables.

9 Appendix C - The Effect of Permanent Income in Investments in Human Capital in an Economy with Incomplete Markets

Aiyagari/Bewley/Laitner/Shechtmann Economy

Assume now that markets are incomplete and parents may not leave debts for their children. Parents can transfer resources to their children through investment in human capital or physical assets s. However, they cannot buy claims contingent on the ability of their children. This situation may arise because the ability is private information to the children. The parental problem becomes:

$$Max\ u(c_p) + \pi_1 u(c_{k,1}) + \pi_2 u(c_{k,2})$$

$$\begin{aligned} subject\ to: c_p + s + x &= y_p \\ c_{k,1} &= y_{k,1} + (1+r)\, s \\ c_{k,2} &= y_{k,2} + (1+r)\, s \\ y_{k,1} &= \zeta_1 x^\alpha \\ y_{k,2} &= \zeta_2 x^\alpha \\ s &\geq 0 \end{aligned}$$

The first-order conditions become:

$$\frac{\partial u}{\partial c_p} = \alpha\beta x^{\alpha-1}\left(\pi_1\zeta_1\frac{\partial u}{\partial c_{k,1}} + \pi_2\zeta_2\frac{\partial u}{\partial c_{k,2}}\right)$$

$$\frac{\partial u}{\partial c_p} \geq \beta(1+r)\left(\pi_1\frac{\partial u}{\partial c_{k,1}} + \pi_2\frac{\partial u}{\partial c_{k,2}}\right)$$

To characterize the relationship between parental income y_p and investment x we break the analysis in two cases. First, assume that the optimal solution implies $s = 0$. Then, from the budget constraint $c_p = y_p - x$. This implies that the optimal investment x is given by:

$$G(y_p, x) = \frac{\partial u(y_p - x)}{\partial c_p} - \alpha\beta x^{\alpha-1}\left(\pi_1\zeta_1\frac{\partial u(\zeta_1 x^\alpha)}{\partial c_{k,1}} + \pi_2\zeta_2\frac{\partial u(\zeta_2 x^\alpha)}{\partial c_{k,2}}\right) \equiv 0$$

Note that:

$$\frac{\partial G}{\partial y_p} = \frac{\partial^2 u(y_p - x)}{\partial c_p^2} < 0$$

$$\frac{\partial G}{\partial x} = -\frac{\partial^2 u(y_p - x)}{\partial c_p^2} - \alpha(\alpha-1)\beta x^{\alpha-2} E\left(\frac{\partial u}{\partial c_k}\right) - \alpha\beta\left(x^{\alpha-1}\right)^2 E\left(\zeta^2\frac{\partial^2 u}{\partial c_k^2}\right) > 0$$

Therefore, we can apply the implicit function theorem to conclude that

$$\frac{dx}{dy_p} = -\frac{\frac{\partial G}{\partial y_p}}{\frac{\partial G}{\partial x}} > 0$$

that is, if $s = 0$ a small increase in the parental income y_p generates an increase in the parental investment in human capital even after we control for ability ζ.

Now, we focus on the case when $s > 0$. Then, the solution is characterized by:

$$G(y_p, x, s) = \begin{bmatrix} \frac{\partial u}{\partial c_p} - \alpha\beta x^{\alpha-1}\left(\pi_1\zeta_1\frac{\partial u}{\partial c_{k,1}} + \pi_2\zeta_2\frac{\partial u}{\partial c_{k,2}}\right) \\ \frac{\partial u}{\partial c_p} - \beta(1+r)\left(\pi_1\frac{\partial u}{\partial c_{k,1}} + \pi_2\frac{\partial u}{\partial c_{k,2}}\right) \end{bmatrix}$$

Note that:

$$D_{s,x}G\left(y_p,x,s\right)=\left[\begin{array}{cc}-\frac{\partial^2 u}{\partial c_p^2}-\alpha\left(\alpha-1\right)\beta x^{\alpha-2}E\left(\zeta\frac{\partial u}{\partial c_k}\right)-\alpha\beta\left(x^{\alpha-1}\right)^2E\left(\zeta^2\frac{\partial^2 u}{\partial c_k^2}\right) & -\frac{\partial^2 u}{\partial c_p^2}-\alpha\beta\left(1+r\right)x^{\alpha-1}E\left(\zeta\frac{\partial^2 u}{\partial c_k^2}\right)\\ -\frac{\partial^2 u}{\partial c_p^2}-\alpha\beta\left(1+r\right)x^{\alpha-1}E\left(\zeta\frac{\partial^2 u}{\partial c_k^2}\right) & -\frac{\partial^2 u}{\partial c_p^2}-\beta\left(1+r\right)^2E\left(\frac{\partial u}{\partial c_k}\right)\end{array}\right]$$

Note that $-\frac{\partial^2 u}{\partial c_p^2}-\alpha\left(\alpha-1\right)\beta x^{\alpha-2}E\left(\zeta\frac{\partial u}{\partial c_k}\right)-\alpha\beta\left(x^{\alpha-1}\right)^2E\left(\zeta^2\frac{\partial^2 u}{\partial c_k^2}\right)>0$, $-\frac{\partial^2 u}{\partial c_p^2}-\beta\left(1+r\right)^2E\left(\frac{\partial u}{\partial c_k}\right)>0$. Optimality implies $\left|D_{s,x}G\left(y_p,x,s\right)\right|<0$. Therefore, we can again apply the implicit function theorem. Then:

$$\left[\begin{array}{c}\frac{dx}{dy_p}\\ \frac{ds}{dy_p}\end{array}\right]=-\frac{1}{\left|D_{s,x}G\left(y_p,x,s\right)\right|}D_{s,x}G\left(y_p,x,s\right)^{-1}D_yG\left(y_p,x,s\right)$$

where $D_yG\left(y_p,x,s\right)=\left[\begin{array}{c}\frac{\partial^2 u}{\partial c_p^2}\\ \frac{\partial^2 u}{\partial c_p^2}\end{array}\right]$

Note that .

References

[1] Aiyagari, S.R, J.Greenwood and A.Seshadri, "Efficient Investment in Children," *Journal of Economic Theory*, 2002, Vol.102, pp.290-321.

[2] Altonji, Joseph, and Thomas Dunn. 1996. "The Effects of Family Characteristics on the Return to Education," *Review of Economics and Statistics* 78, no.4: 692-704.

[3] Akabayashi, Hideo. 1996. "On the Role of Incentives in the Formation of Human Capital in the Family," Ph.D. Dissertation, University of Chicago.

[4] Barnett, W. Steven. 2002. Personal Communication with author, February 26.

[5] Becker, Gary. 1964. *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education.* New York: Columbia University Press.

[6] Becker, G. and N.Tomes, 1979. "An Equilibrium Theory of The Distribution of Income and Intergenerational Mobility," *Journal of political Economy*, Vol 87, pp.1153-1189.

[7] —— 1986. "Human Capital and The Rise and Fall of Families," *Journal of Labor Economics* Vol.4(3): S1-S39.

[8] Benabou, Roland. 2000. "Unequal Societies: Income Distribution and the Social Contract," *American Economic Review* 90, March 2000, 96-129.

[9] Benabou, Roland. 2002. "Tax and Education Policy in a Heterogeneous Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica* 70(2), March 2002, 481-517.

[10] Ben-Porath, Yoram. 1967. "The Production of Human Capital and the Life Cycle Earnings," *Journal of Political Economy*, 75, no.4 (part 1): 352-365.

[11] Blossfeld, Hans-Peter, and Yossi Shavit. 1993. *Persistent Inequality: Changing Educational Attainment in Thirteen Countries*. Boulder, Colo.: Westview.

[12] Bowles, Samuel and Howard Gintis. 1976. *Schooling in Capitalist America*. New York: Basic.

[13] Bowles, Samuel, Howard Gintis, and Melissa Osborne. 2001. "The Determinants of Earnings: A Behavioral Approach" *Journal of Economic Literature* 39, no. 4: 1137-1176.

[14] Bureau of Labor Statistics (BLS) 2001. *NLS Handbook 2001*. Washington, D.C.: U.S. Department of Labor.

[15] Cameron, S., and J. Heckman. 1998. "Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts of American Males" *Journal of Political Economy*, 106, no.2: 262-333.

[16] Cameron, S., and J. Heckman. 1999. "Can Tuition Policy Combat Rising Wage Inequality?" In *Financing College Tuition: Government Policies and Educational Priorities*, Martin Kosters, ed. Washington, D.C.: American Enterprise Institute Press.

[17] Cameron, S., and J. Heckman. 2001. "The Dynamics of Educational Attainment for Black, Hispanic, and White Males" *Journal of Political Economy* 109, no.3: 455-499.

[18] Cameron, Stephen and Christopher Taber. 2000 "Borrowing Constraints and the Returns to Schooling." NBER working paper no. 7761.

[19] Card, David. 1999. "The Causal Effect of Education on Earnings." In *Handbook of Labor Economics*, vol. 3A, Orley Ashenfelter, and David Card, eds. Amsterdam: Elsevier Science/North-Holland.

[20] Card, D.2001. "Estimating the Return to Schooling: Progress on Some Persistent Econometric Problems." *Econometrica* 69, no.5: 1127-1160.

[21] Card, D. and A. Krueger. 1992. "Does School Quality Matter? Returns to Education and the Characteristics of Public Schools in the United States," *Journal of Political Economy* 100(1): 1-40.

[22] Carneiro, Pedro, Karsten Hansen, and James J. Heckman. 2001. "Removing the Veil of Ignorance in Assessing the Distributional Impacts of Social Policies." *Swedish Economic Policy Review* 8: 273-301.

[23] Carneiro, P., K. Hansen, and J. Heckman. 2003. "Estimating Distributions of Treatment Effects with an Application to the Returns to Schooling." *International Economic Review*, 44(2): 361-422.

[24] Carneiro, Pedro, and James J. Heckman. 2002. "The Evidence on Credit Constraints in Post-secondary Schooling." *Economic Journal* 112, no.482: 705-734.

[25] Carneiro, Pedro, James J. Heckman and Dimitriy Masterov (2003) "Racial Gaps in Pre-Market Factors", NBER working paper.

[26] Carneiro, Pedro, James J. Heckman, and Edward Vytlacil. 2003. "Understanding what Instrumental Variables Estimate: Estimating Marginal and Average Returns to Education." University of Chicago working paper.

[27] Cawley, John, James J. Heckman, Lance Lochner, and Edward Vytlacil. 2000. "Understanding the Role of Cognitive Ability in Accounting for the Recent Rise in the Return to Education." In *Meritocracy and Economic Inequality*, Kenneth Arrow, Samuel Bowles, and Steven Durlauf, eds. Princeton: Princeton University Press.

[28] Currie, Janet, and Duncan Thomas. 1995. "Does Head Start Make a Difference?" *American Economic Review* 85, no.3: 341-364.

[29] Donohue, John, and Peter Siegelman. 1998. "Allocating Resources among Prisons and Social Programs in the Battle against Crime." *Journal of Legal Studies* 27, no.1: 1-43.

[30] Duncan, Greg, and Jeanne Brooks-Gunn. 1997. "Income Effects Across the Life Span: Integration and Interpretation." In *Consequences of Growing Up Poor*, Greg Duncan and Jeanne Brooks-Gunn, eds. New York: Russell Sage.

[31] Dynarski, Susan. 2000. "Hope for Whom? Financial Aid for the Middle Class and Its Impact on College Attendance," *National Tax Journal* 53, no. 3 (part 2): 629-662.

[32] Dynarski, S. 2001. "Does Aid Matter? Measuring the Effects of Student Aid on College Attendance and Completion." Harvard University working paper.

[33] Edwards, Richard. 1976. "Individual Traits and Organizational Incentives: What Makes a Good Worker?" *Journal of Human Resources* 11, no.1: 51-68.

[34] Ellwood, David, and Thomas Kane. 2000. "Who is Getting a College Education?: Family Background and the Growing Gaps in Enrollment." In *Securing the Future: Investing in Children from Birth to College*, Sheldon Danziger and Jane Waldfogel, eds. New York: Russell Sage.

[35] Garces, Eliana, Duncan Thomas and Janet Currie. 2002. "Longer-Term Effects of Head Start." *American Economic Review* 92, no.4: 999-1012.

[36] Griliches, Zvi. 1977. "Estimating the Returns to Schooling: Some Econometric Problems," *Econometrica* 45, no.1: 1-22.

[37] Hansen, Karsten, James J. Heckman, and Kathleen Mullen. 2002. "Educational Attainment, Ability and Test Scores," University of Chicago working paper.

[38] Heckman, James J. 1995. "A Life-Cycle Model of Earnings, Learning and Consumption." In *Labor Economics*, vol. 2, Orley Ashenfelter and Kevin Hallock, eds. Brookfield, Vt.: Elgar/Ashgate.

[39] Heckman, James J.1999. "Education and Job Training: Doing It Right" *Public Interest*, No.135: 86-107.

[40] Heckman, James J. 2000. "Policies to Foster Human Capital." *Research in Economics* 54, no.1: 3-56.

[41] Heckman, James J., Jingjing Hsee, and Yona Rubinstein. 2002. "The GED is a 'Mixed Signal': The Effect of Cognitive and Noncognitive Skills on Human Capital and Labor Market Outcomes." University of Chicago working paper.

[42] Heckman, James J., Robert LaLonde, and Jeffrey Smith. 1999. "The Economics and Econometrics of Active Labor Market Programs." In *Handbook of Labor Economics*, vol. 3A, Orley Ashenfelter and David Card, eds. Amsterdam: Elsevier.

[43] Heckman, James J., and Lance Lochner. 2000. "Rethinking Myths about Education and Training: Understanding the Sources of Skill Formation in a Modern Economy." In *Securing the Future: Investing in Children from Birth to College*, Sidney Danziger and Jane Waldfogel, eds. New York: Russell Sage.

[44] Heckman, James J., Lance Lochner and Christopher Taber. 1998a. "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Earnings with Heterogeneous Agents." *Review of Economic Dynamics* 1, no. 1: 1-58.

[45] Heckman, J., Lance Lochner, and Christopher Taber. 1998b. "General Equilibrium Treatment Effects: A Study of Tuition Policy" *American Economic Review*, 88, no.2: 381-386.

[46] Heckman, J., Lance Lochner, and Christopher Taber. 1998c. "Tax Policy and Human Capital Formation," *American Economic Review*, 88, no.2: 293-297.

[47] Heckman, James J., Lance Lochner, and Petra Todd. 2003. "Fifty Years of Mincer Regressions." University of Chicago working paper.

[48] Heckman, James J., and Yona Rubinstein. 2001. "The Importance of Noncognitive Skills: Lessons from the GED Testing Program." *American Economic Review* 91, no.2: 145-149.

[49] Heckman James J., and Jeffrey Smith. 1998. "Evaluating the Welfare State." In Econometrics and Economic Theory in the 20th Century: The Ragnar Frisch Centennial Symposium, S. Strøm, ed. Cambridge: Cambridge University Press.

[50] Herrera, Carla, Cynthia Sipe, Wendy McClanahan, Amy Arbreton, and Sarah Pepper. 2000. *Mentoring School-Age Children: Relationship Development in Community-Based and School-Based Programs.* Philadelphia: Public/Private Ventures.

[51] Johnson, Amy. 1996. *An Evaluation of the Long-Term Impacts of the Sponsor-a-Scholar Program on Student Performance.* Princeton: Mathematica Policy Research.

[52] Judd, Kenneth. 2000. "Is Education as Good as Gold? A Portfolio Analysis of Human Capital Investment," Stanford University working paper.

[53] Kane, Thomas. 1994. "College Entry by Blacks since 1970: The Role of College Costs, Family Background, and the Returns to Education." *Journal of Political Economy* 102, no.5: 878-911.

[54] Kane, Thomas. 1996. "College Costs, Borrowing Constraints and the Timing of College Entry, " *Eastern Economic Journal* 22, no.2: 181-194.

[55] Kane, Thomas. 1999. *The Price of Admission: Rethinking How Americans Pay for College.* Washington, D.C.: Brookings Institution.

[56] Karoly, Lynn. 2001. "Investing in the Future: Reducing Poverty through Human Capital Investments." In *Understanding Poverty*, S. Danzinger and R. Haveman, eds. New York: Russell Sage.

[57] Keane, Michael, and Kenneth Wolpin. 2001. "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment." *International Economic Review* 42, no.4: 1051-1103.

[58] Klein, Roger, Richard Spady, and Andrew Weiss. 1991. "Factors Affecting the Output and Quit Propensities of Production Workers." *Review of Economic Studies* 58, no.2: 929-954.

[59] Laitner, J. 1992, "Random Earnings Differences, Lifetime Liquidity Constraints and Altruistic Intergenerational Transfers," *Journal of Economic Theory*, Vol.58, pp.135-170.

[60] Laitner, J. 1997, "Intergenerational and Interhousehold Economic Links," in M.Rosenzweig and Oded Stark, eds, *Handbook of Population and Family Economics*, Amsterdam: North Holland.

[61] Lally, J. Ronald, Peter Mangione, and Alice Honig. (1988). "The Syracuse University Family Development Research Program: Long-Range Impact on an Early Intervention with Low-Income Children and Their Families," In Parent Education as Early Childhood Intervention, Douglas Powell, ed. Norwood, N.J.: Ablex.

[62] Meghir, Costas, and Marten Palme. 1999. "Assessing the Effect of Schooling on Earnings Using a Social Experiment." IFS working paper no. W99/10.

[63] National Center for Education Statistics (NCES) (1993). *The 1993 Digest of Education Statistics*. Washington, D.C.: NCES.

[64] Orfield, Gary. 1992. "Money, Equity, and College Access." *Harvard Educational Review* 2, no.3: 337-372.

[65] Ramey, Craig, Donna Bryant, Frances Campbell, Joseph Sparling, and Barbara Wasik. 1988. "Early Intervention for High-Risk Children: The Carolina Early Intervention Program." In *14 Ounces of Prevention: A Casebook for Practitioners*, Richard Price, Emory Cowen, Raymond Lorion, and Julia. Ramos-McKay, eds. Washington, D.C.: American Psychological Association.

[66] Schweinhart, Lawrence, Helen Barnes, and David Weikart. 1993. *Significant Benefits: The High-Scope Perry Pre-school Study through Age 27.* Ypsilanti, Mich.: High Scope Press.

[67] Seitz, Victoria. 1990. "Intervention Programs for Impoverished Children: A Comparison of Educational and Family Support Models." In *Annals of Child Development: A Research Annual*, vol. 7, Ross Vasta, ed. London: Kingsley.

[68] Seshadri, A. and K.Yuki, "Equity and Efficiency Effects of Redistributive Policies," University of Wisconsin, Madison, June, 2003.

[69] Shonkoff, Jack, and Deborah Phillips, eds. 2000. *From Neurons to Neighborhoods: The Science of Early Childhood Development.* Washington, D.C.: National Academy Press.

[70] Stanley, Marcus. 1999. "Education, Opportunity, and the Mid-Century G.I. Bills." Harvard University working paper.

[71] Sternberg, Robert. 1985. *Beyond IQ: A Triarchic Theory of Human Intelligence*. Cambridge: Cambridge University Press.

[72] Taber, Christopher. 2001. "The Rising College Premium in the Eighties: Return to College or Return to Unobserved Ability?" *Review of Economic Studies* 68, no.3: 665-691.

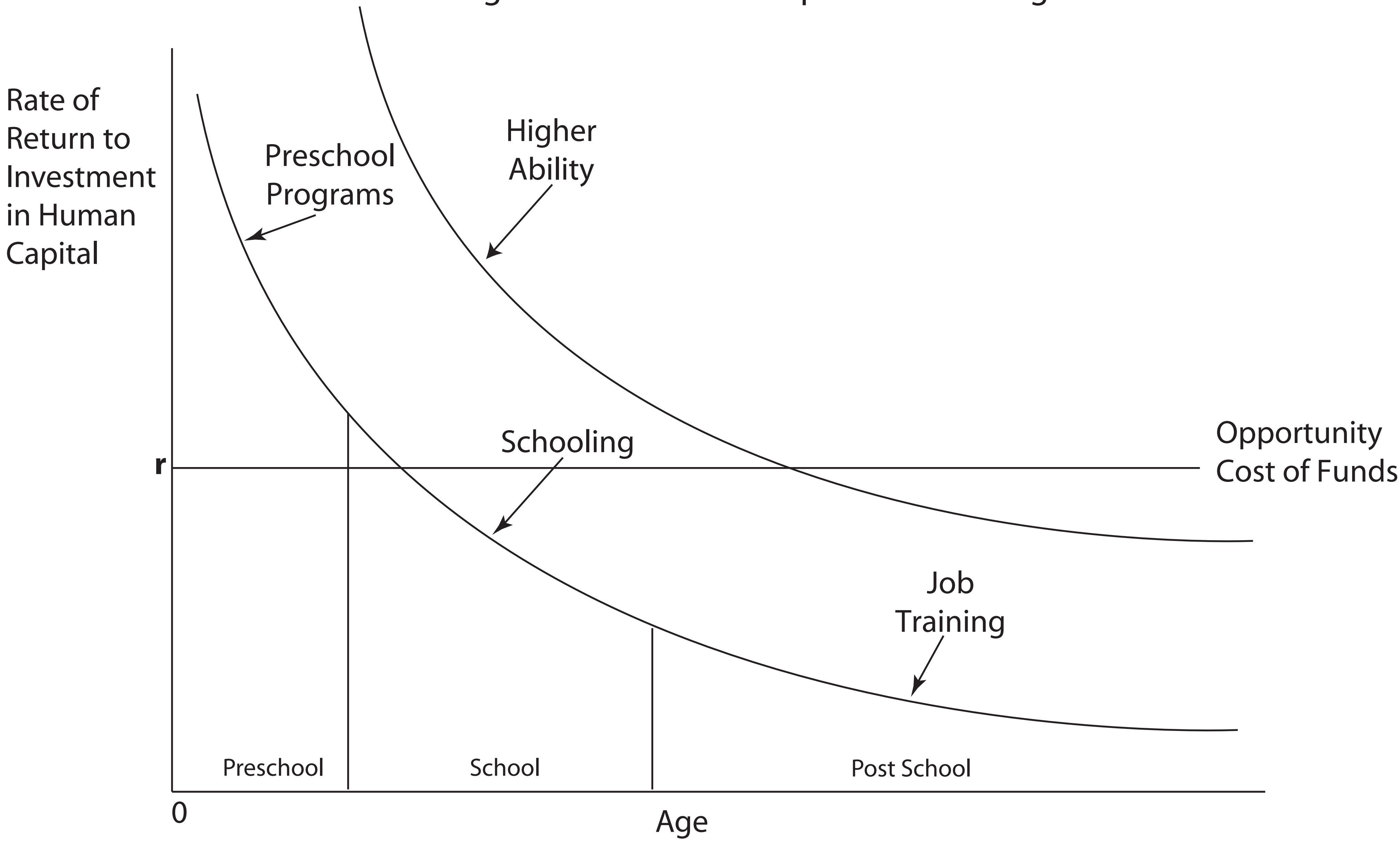
[73] Taggart, Robert. 1995. *Quantum Opportunity Program*. Philadelphia: Industrialization Center of America.

[74] Tierney, Joseph, and Jean Grossman. 1995. *Making a Difference: An Impact Study of Big Brothers/Big Sisters.* Philadelphia: Public/Private Ventures.

[75] Turkheimer, E., A.Haley, M.Waldron, B.D'Onofrio and I. Gottesman 2003, "Socioeconomic Status Modifies heritability of IQ in Young Children," *Psychological Science* Vol.14, #6 pp.623-628.

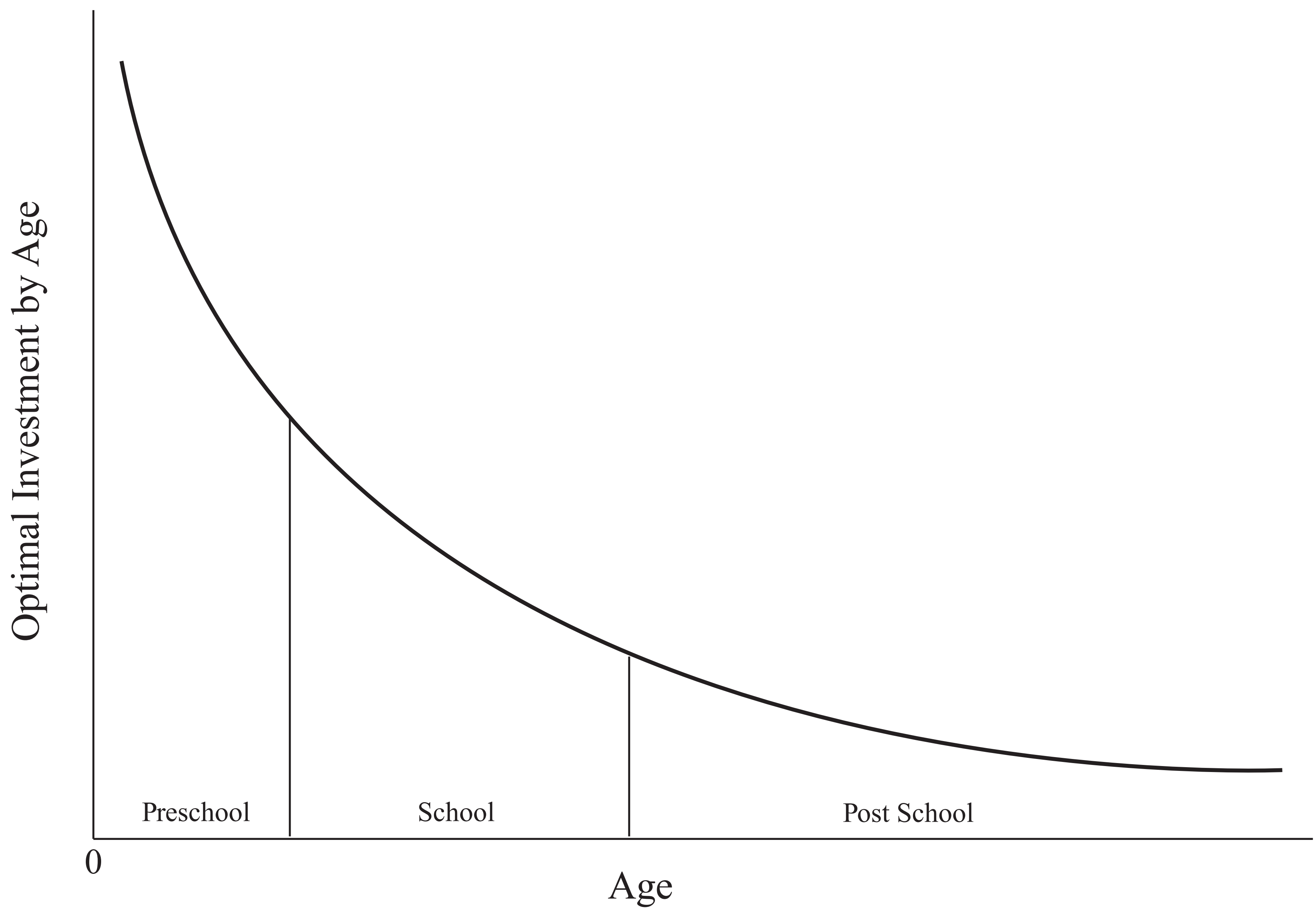
[76] Willis, Robert. 1986. "Wage Determinants: A Survey and Reinterpretation of Human Capital Earnings Functions." In *Handbook of Labor Economics*, vol. 1, Orley Ashenfelter and David Card, eds. New York: North-Holland.

Figure 1
Rates of Return to Human Capital Investment Initially
Setting Investment to be Equal Across all Ages



Rates of Return to Human Capital Investment Initially Setting Investment to be Equal Across all Ages

Figure 2
Optimal Investment Levels



College Participation, 18 to 24 Yrs, HS Grads and GED Holders

White Males

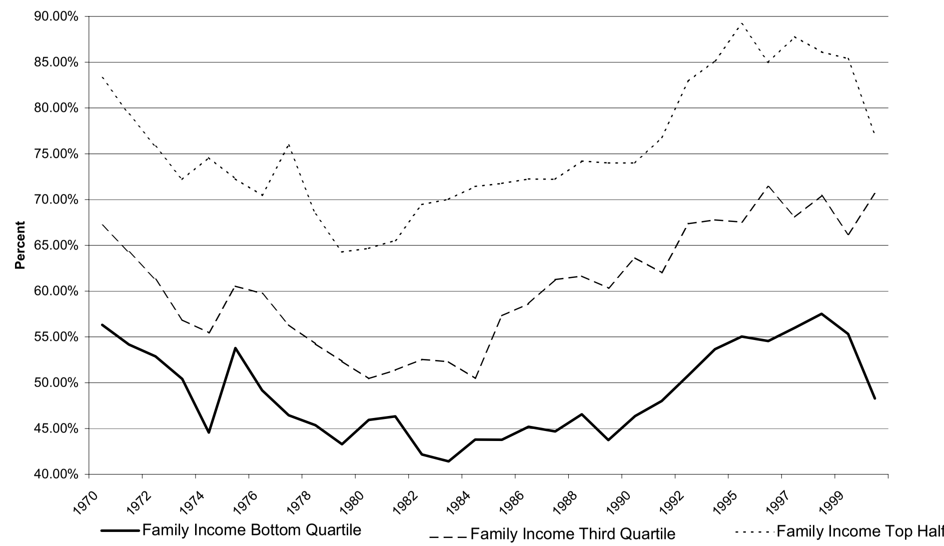


Table 3.3

Timing of Per Capita Permanent Income Controlling for Mother's Earnings Ability Factor

Dependent Variable: College = 1, High School = 0

Variable Name	Probit 1	Probit 2	Probit 3	Probit 4	Probit 5	Probit 6	Probit 7
Per Capita Permanent Income 0-18	**1.2982**	**1.0654**	**1.5845**	**1.3837**	**1.4571**	1.1285	**1.4974**
t-statistic	5.1400	2.9100	3.3000	2.9400	4.5300	1.7000	3.5400
Per Capita Permanent Income 0-5		0.5801					
t-statistic		0.8700					
Per Capita Permanent Income 6-10			-0.6072				
t-statistic			-0.4500				
Per Capita Permanent Income 11-15				-0.4089			
t-statistic				-0.2200			
Per Capita Permanent Income 16-18					-1.5816		
t-statistic					-0.7100		
Per Capita Permanent Income 0 - 11						0.2228	
t-statistic						0.2800	
Per Capita Permanent Income 12 - 18							-0.5609
t-statistic							-0.5800
Mother's Schooling	**0.1455**	**0.1473**	**0.1438**	**0.1458**	**0.1483**	**0.1464**	**0.1477**
t-statistic	3.2200	3.2500	3.1800	3.2200	3.2600	3.2300	3.2500
Child Ability (Math at age 12)	**0.0288**	**0.0290**	**0.0287**	**0.0288**	**0.0286**	**0.0289**	**0.0289**
t-statistic	3.9000	3.9300	3.8800	3.9000	3.8600	3.9100	3.9100
Mother's AFQT	-0.0278	-0.0307	-0.0265	-0.0286	-0.0281	-0.0289	-0.0296
t-statistic	-0.4700	-0.5200	-0.4500	-0.4800	-0.4700	-0.4900	-0.5000
Mother's Earnings Ability Factor	-0.0483	-0.0502	-0.0517	-0.0482	-0.0326	-0.0479	-0.0468
t-statistic	-0.8700	-0.9100	-0.9400	-0.8700	-0.5900	-0.8700	-0.8500
Constant	-0.4852	-0.5104	-0.4811	-0.4870	-0.4972	-0.4896	-0.4895
t-statistic	-1.5800	-1.6500	-1.5700	-1.5900	-1.6200	-1.6000	-1.6000
Observations	668	668	668	668	668	668	668

Per capita permanent income between child's age s and t is constructed in the following way. For each age x of the child, I get household earnings and divide by the number of household members at age x, to compute the per capita household earnings at age x. Then, I calculate the mean earnings between ages s and t by simple average. Next, I discount per capital mean household eaernings at age x to dollars at age 0. I add the resulting values to compute the present value. This variable is the permanent income between ages s and t.

Dummies for year of birth, sex, race, urban and south birth are included as additional controls, but not reported here.

The sample consists of children of NLSY/1979 born between 1975 and 1981, for whom we observe all of the variables. In order to obtain household earnings, and parental background variables, I use the respondents of the NLSY/1979 sample.

Table 3.4

Regression of Per Capita Permanent Income 0-18 on Per Capita Permanent Income 0-11

	Coefficient	Standard Error	t-statistic	p-value	95% Confidence Interval	
Per Capita Permanent Income 0 - 11	1.3555	0.0011	1263.9300	0.0000	1.3534	1.3576
Constant	-0.0062	0.0011	-5.4800	0.0000	-0.0084	-0.0040
R Square			**0.9943**			

Per capita permanent income between child's age s and t is constructed in the following way. For each age x of the child, I get household earnings, and divide by the number of household members at age x, to compute the per capita household earnings at age x. Then, I calculate the mean earnings between ages s and t by simple average. Next, I discount per capita mean household earnings at age x to dollars at age 0. I add the resulting values to compute the present value. This variable is the permanent income between ages s and t.

The sample consists of children of NLSY born between 1975 and 1981, for whom we observe all of the variables. Household earnings and parental background are from NLSY/1979

Table 4

Proportions

Schooling Choice	Data*	Baseline Economy	No Productivity Shock, No Lifetime Liquidity Constraint	Full-Tuition Economy	Partial Tuition and Partial Early Investment Subsidy Economy
High School	54.55	56.76	54.46	44.10	47.38
College	45.45	43.24	45.54	55.90	52.62

Table presents the proportions of agents that choose high school and college according to the data and the model. Source: NLSY - 1979, White Males that are high school or college graduates. We discard agents that make other schooling decisions from the sample.

Table 4.1

Average Returns to College (Percentage Returns)

Parameter	Baseline Economy	No Productivy Shock, No Lifetime Liquidity Constraint	Full-Tuition Subsidy	Partial Tuition, Partial Early Investment Subsidy
Average Treatment Effect (ATE)	21.3684	21.3168	21.3763	21.5671
Treatment on the Untreated (TU)	15.3980	14.3642	13.2908	14.3159
Treatment on the Treated (TT)	29.2075	29.6296	27.7536	28.0947

Table presents the returns to college as implied by the model with idiosyncratic innovations in productivity and lifetime liquidity constraints (baseline economy), and the economy with full-tuition subsidy financed by an increase in the income tax-rate (so the government budget is balanced at every period in the steady state). The first row presents the average returns for the population, the second row presents the implied returns for only those who do not go to college and the last row shows the returns for those who graduate from college. Formally, let y_0, y_1 denote the present value of high school and college earnings, respectively. Let C denote total costs of college (i.e., tuition and opportunity costs). Then, in the first row we show

$$E\,(\ returns\ ) = E\left[\frac{y_1 - y_0 - C}{y_0 + C}\right]$$

The second row shows:

$$E\,(returns \mid \text{high school}) = E\left[\frac{y_1 - y_0 - C}{y_0 + C} \mid \text{high school}\right]$$

The third row we show:

$$E\,(returns \mid \text{college}) = E\left[\frac{y_1 - y_0 - C}{y_0 + C} \mid \text{college}\right]$$

Table 5

Density of Ability by Skill Group

Ability Group	Conditioned on not Choosing College	Conditioned on Choosing College
1	0.0271	0.0000
2	0.0389	0.0000
3	0.0617	0.0000
4	0.0974	0.0005
5	0.1344	0.0044
6	0.1802	0.0099
7	0.1963	0.0190
8	0.1987	0.0377
9	0.0197	0.2642
10	0.0178	0.2272
11	0.0130	0.1719
12	0.0062	0.1147
13	0.0037	0.0733
14	0.0025	0.0421
15	0.0023	0.0352

Table shows the pattern of selection on ability. Note that low ability agents (groups 1, 2 and 3) do not go to college. On the other hand, there are agents with high ability (groups 9 - 15) that are not going to college because of liquidity constraints. These agents represent 6% of the total number of high school graduates. This figure is consistent with the empirical findings of Carneiro and Heckman (2003).

Table 6

The Returns to College by Ability Group

Ability Group	Conditioned on Not Choosing College	Conditioned on Choosing College
1	0.0247	-
2	0.0336	-
3	0.0456	-
4	0.0660	0.0851
5	0.1143	0.1150
6	0.1495	0.1498
7	0.1869	0.1875
8	0.2246	0.2250
9	0.2590	0.2593
10	0.2862	0.2885
11	0.3115	0.3120
12	0.3294	0.3297
13	0.3193	0.3427
14	0.3508	0.3518
15	0.3386	0.3580

Table presents the returns to college by ability group as implied by the model with idiosyncratic innovations in productivity and lifetime liquidity constraints.The second column presents the implied returns for only those who do not go to college and the third column shows the returns for those who graduate from college. Formally, let y_0, y_1 denote the present value of high school and college earnings, respectively. Let C denote total costs of college (i.e., tuition and opportunity costs). Let a denote ability. Then, in the first row we show

$$E\left(\; returns \mid a\right) = E\left[\frac{y_1 - y_0 - C}{y_0 + C} \mid a\right]$$

The second row shows:

$$E\left(returns \mid \text{high school}, \, a\right) = E\left[\frac{y_1 - y_0 - C}{y_0 + C} \mid \text{high school}, \, a\right]$$

The third row we show:

$$E\left(returns \mid \text{college}, \, a\right) = E\left[\frac{y_1 - y_0 - C}{y_0 + C} \mid \text{college}, \; a\right]$$

Table 7

The Internal Rate of Return

	High School Graduates	College Graduates	Overall Sample
Mean	0.0111	0.0895	0.0278
Std Error	0.0600	0.0278	0.1910
Max	0.1790	0.1910	0.1910
Min	-0.1614	-0.0429	-0.1614

Let $y_{s,t+1}, y_{s,t+2}$ denote the earnings at periods t+1, and t+2 at schooling level s, $s \in \{0,1\}$ for college and high school, respectively. Let C_t denote the total costs (i.e., tuition and opportunity costs) of going to college. The internal rate of return is the constant value i that solves:

$$\frac{y_{1,t+1} - y_{0,t+1}}{(1+i)} + \frac{y_{1,t+2} - y_{0,t+2}}{(1+i)^2} - C_t = 0$$

Here we report the anualized internal rate of return. That is, since each period in the model corresponds to a 10-year period in the data, here we report the number $(1+i)^{\frac{1}{10}} - 1$. Therefore, the internal rate of return to college is about 9% per year for a person that chooses college, and only 1.11% a year for a person that chooses high school in the stationary steady state.

Table 8

Density of Ability by Schooling Choice: The Economy with Negative Bequests and No Productivity Shocks

Ability Group	High School	College
Group 1	0.0283	0.0000
Group 2	0.0406	0.0000
Group 3	0.0643	0.0000
Group 4	0.1020	0.0000
Group 5	0.1437	0.0000
Group 6	0.1958	0.0000
Group 7	0.2197	0.0000
Group 8	0.2056	0.0378
Group 9	0.0000	0.2752
Group 10	0.0000	0.2377
Group 11	0.0000	0.1793
Group 12	0.0000	0.1165
Group 13	0.0000	0.0742
Group 14	0.0000	0.0430
Group 15	0.0000	0.0362

Table shows the pattern of selection on ability in the economy with no liquidity constraints, and no idiosyncratic shocks in productivity. Note that there is a separation: Low ability agents become high-school graduates, while high-ability agents become college graduates.

Table 9

The Internal Rate of Return: The Economy with Negative Bequests and No Productivity Shocks

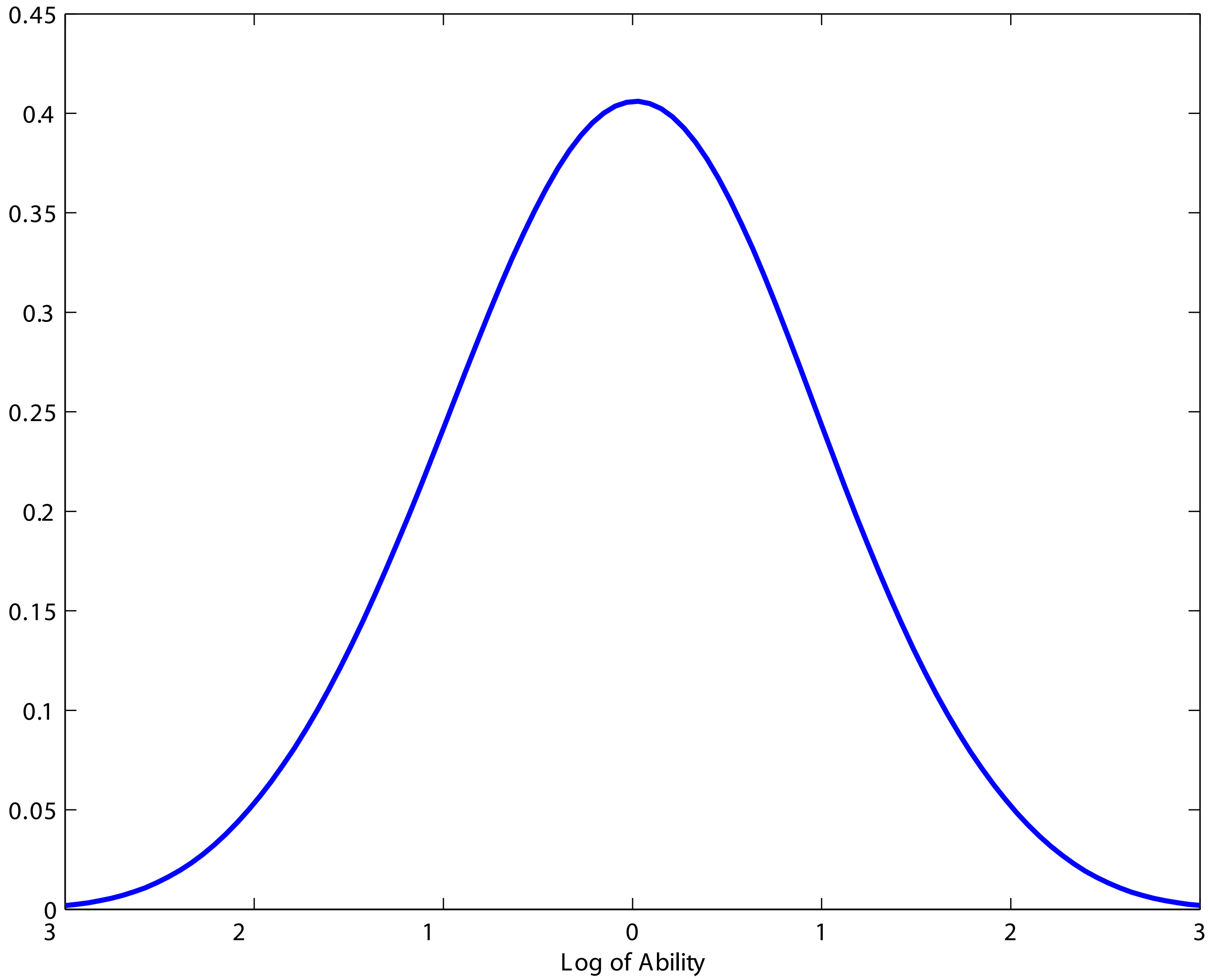
	High School Graduates	College Graduates	Overall Sample
Mean	0.0066	0.0924	0.0457
Std Error	0.0560	0.0130	0.0601
Max	0.0446	0.1149	0.1149
Min	-0.1266	0.0617	-0.1266

Let $y_{s,t+1}, y_{s,t+2}$ denote the earnings at periods t+1, and t+2 at schooling level s, $s \in \{0,1\}$ for college and high school, respectively. Let C_t denote the total costs (i.e., tuition and opportunity costs) of going to college. The internal rate of return is the constant value i that solves:

$$\frac{y_{1,t+1} - y_{0,t+1}}{(1+i)} + \frac{y_{1,t+2} - y_{0,t+2}}{(1+i)^2} - C_t = 0$$

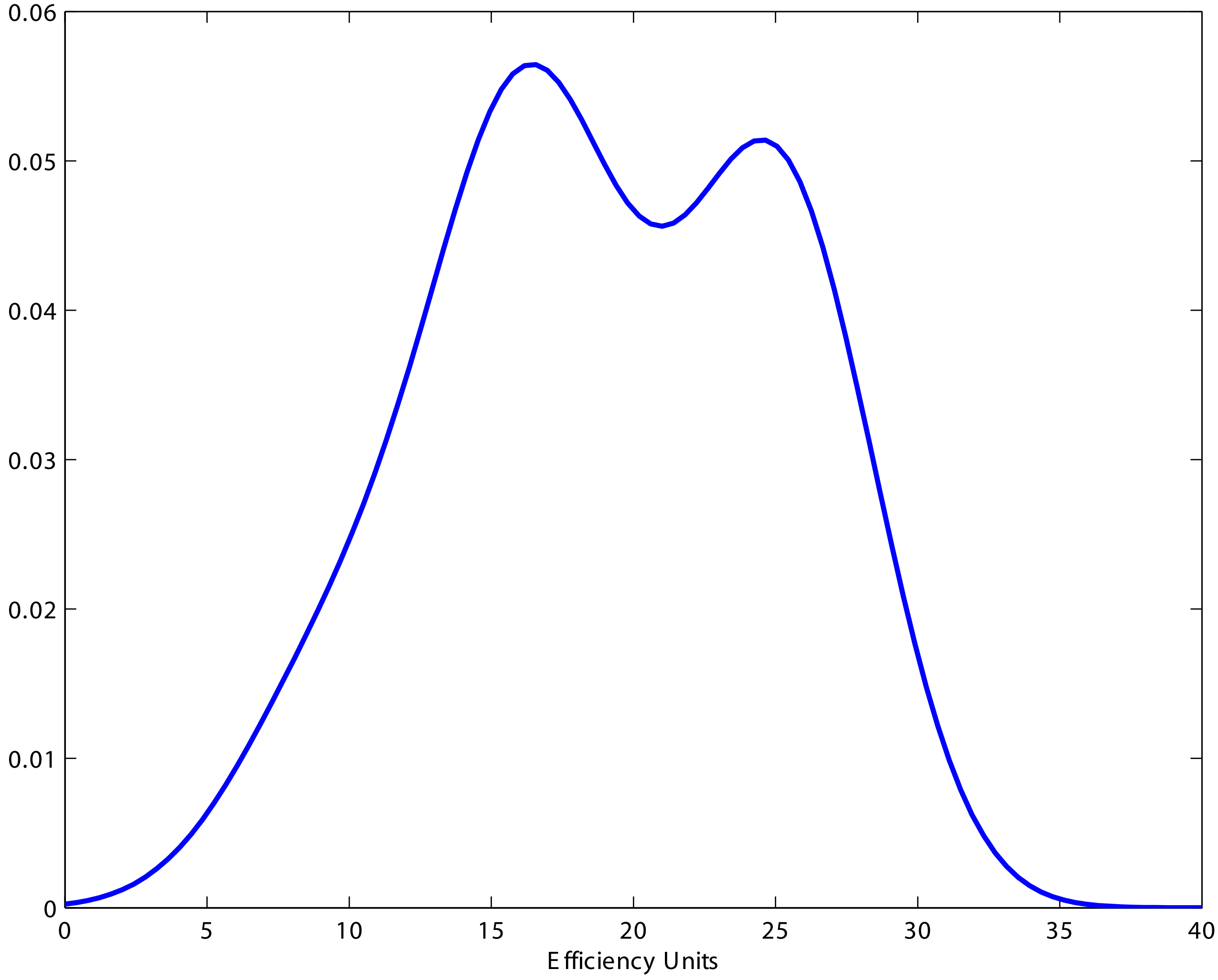
Here we report the anualized internal rate of return. That is, since each period in the model corresponds to a 10-year period in the data, here we report the number $(1+i)^{\frac{1}{10}} - 1$. Therefore, the internal rate of return to college is about 9% per year for a person that chooses college, and less than 1% a year for a person that chooses high school in the stationary steady state.

Figure 4
Density of Log Ability for Overall Sample



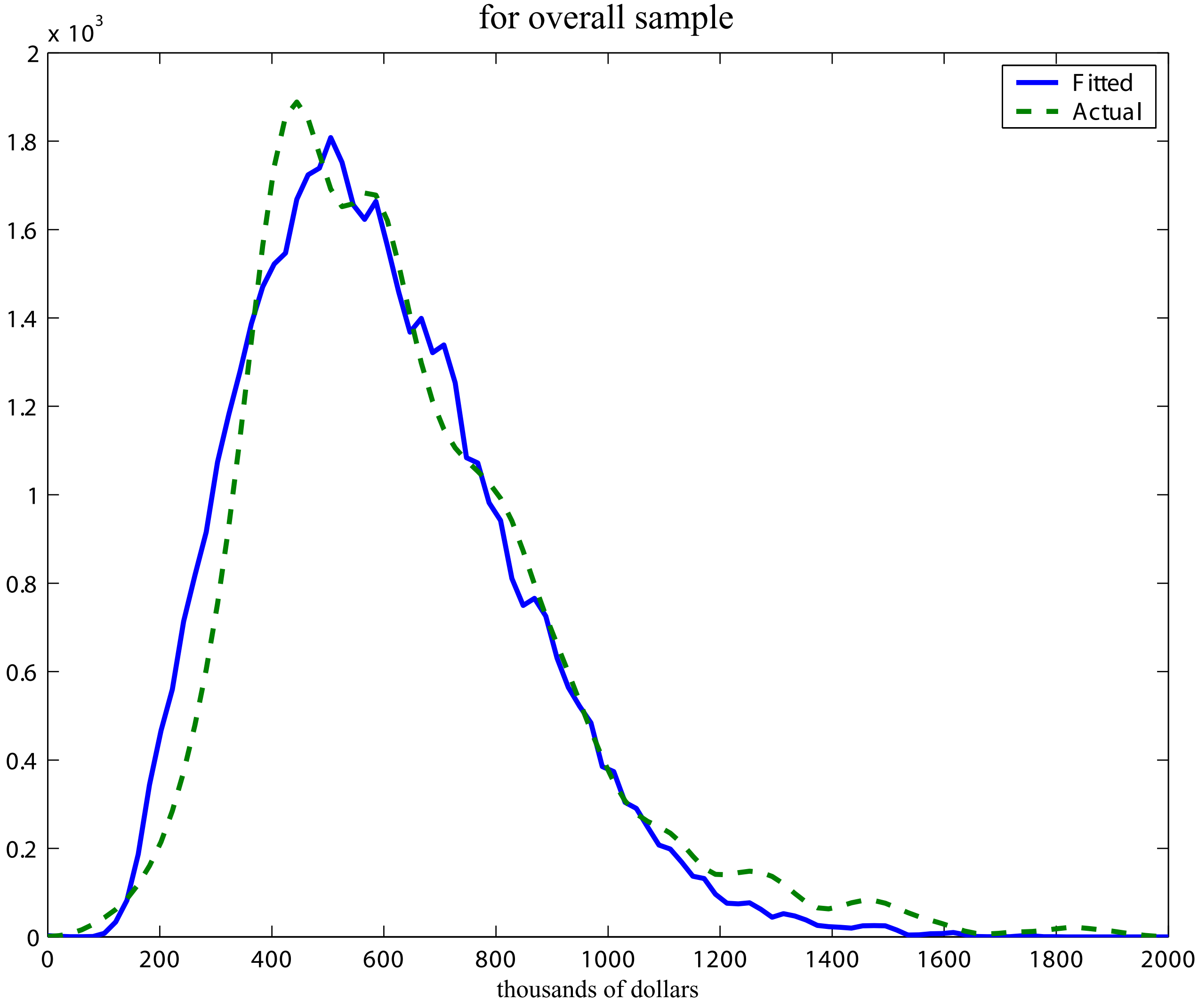
This figure displays the density associated with the invariant distribution of log ability, F(log a). To calculate the steady state of the baseline economy I approximate F using the procedure described in Tauchen (1986) with a 15-point discrete Markov process. Let a_k The log ability is assumed to follow the process: $\log a_k = 0.4 \log a_p + \nu_k$ with ν_k distributed as N(0,1).

Figure 5
Stationary Density of Human Capital



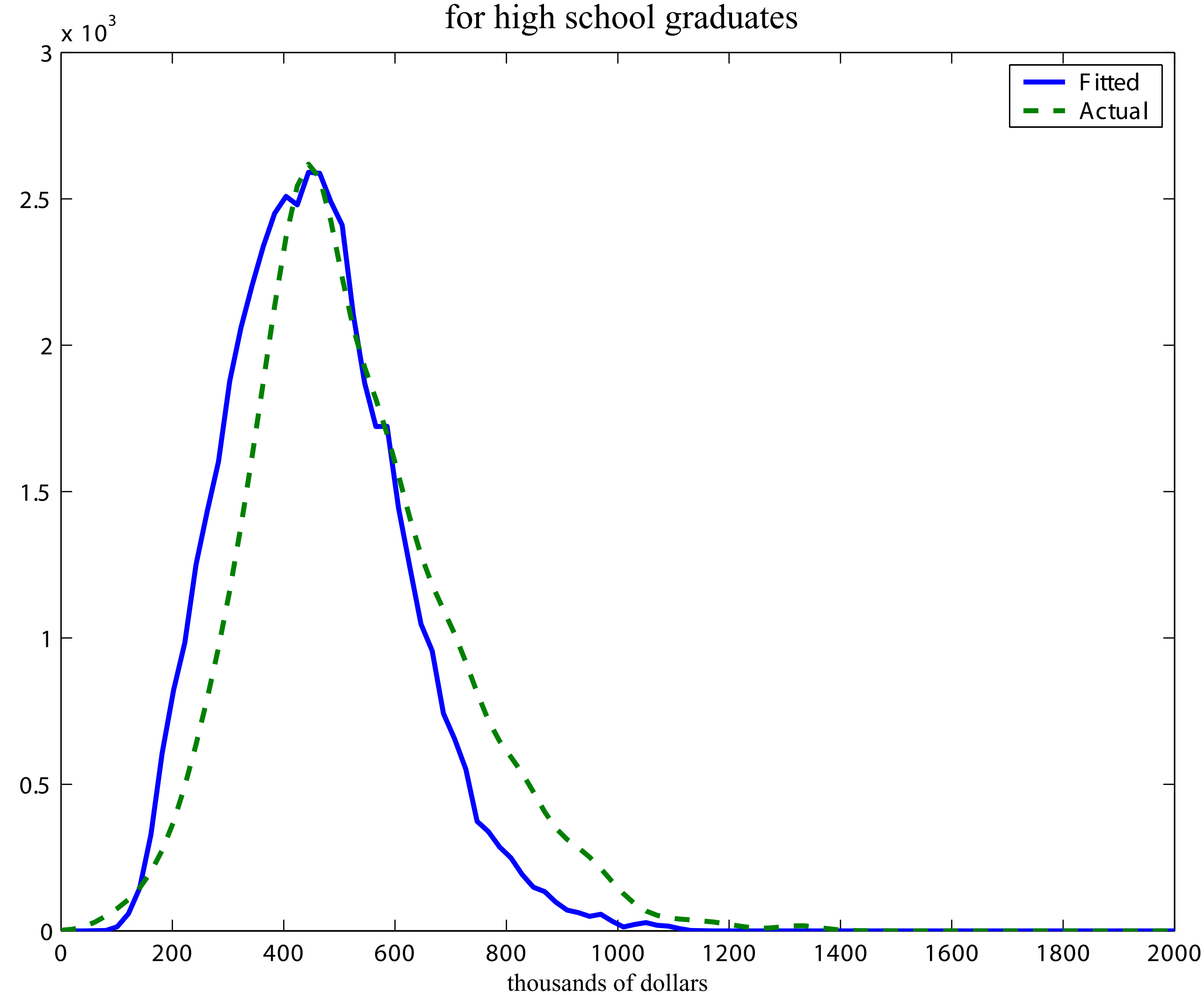
This figure shows the stationary density of efficiency units of the baseline economy. Tho produce this figure, one proceeds in 5 steps. First, one solves the problem of the agent in the extended Laitner (1992) economy to obtain the decision rules for savings, bequest, early and late investments. Second, one draws shocks in ability and productivity and simulates the economy using the decision rules. The third step is to check whether the simulated series is an actual steady state of the economy. If not, one has to return to step 1 until convergence to stationary state is achieved. This figure plots the density simulated series of human capital in steady state.

Figure 6
Density of predicted and actual present value of earnings
for overall sample



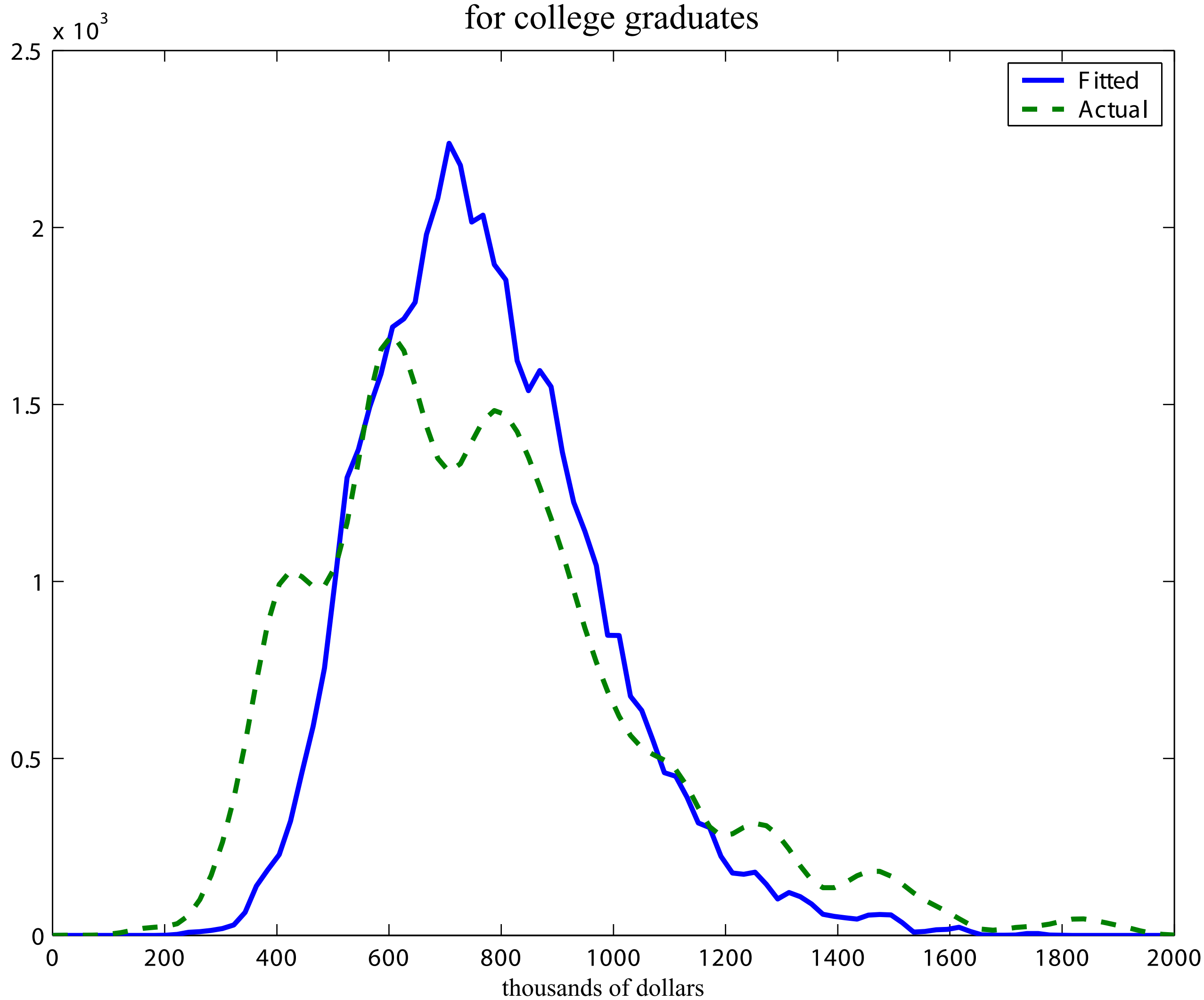
Let y, f(y) denote the present value of earnings from ages 21 to 40 and its steady-state density, respectively. The dashed curve is the density of actual present value of earnings for white males from the NLSY/1979. The solid curve is the steady-state distribution of earnings as implied by the model.

Figure 7
Density of predicted and actual present value of earnings
for high school graduates



Let y, f(y) denote the present value of earnings from ages 21 to 40 and its steady-state density, respectively. Let s = 0 denote agents that are high school graduates. Thus, f(y | s = 0) is the density of present value of earnings conditioned on choosing high school. In the figure above are plotted the density of actual present value of earnings (dashed curve) for white males from the NLSY/1979 and the steady-state distribution of earnings (solid curve) as implied by the model.

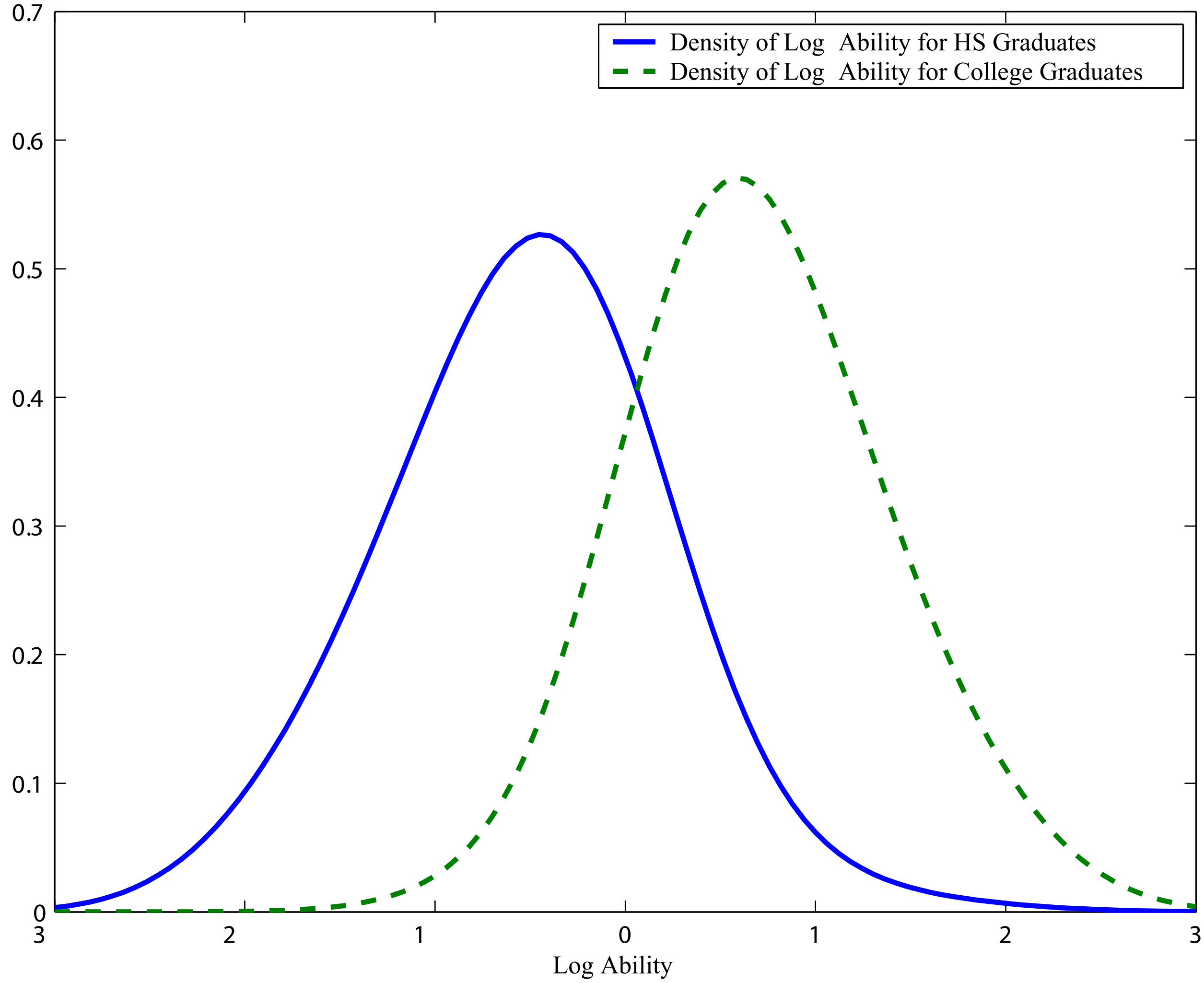
Figure 8
Density of predicted and actual log of present value of earnings
for college graduates



Let y, f(y) denote the present value of earnings from ages 21 to 40 and its steady-state density, respectively. Let s = 1denote agents that are college graduates. Thus, f(y | s = 1) is the density of present value of earnings conditioned on choosing college. In the figure above are plotted the density of actual present value of earnings (dashed curve) for white males from the NLSY/1979 and the steady-state distribution of earnings (solid curve) as implied by the model.

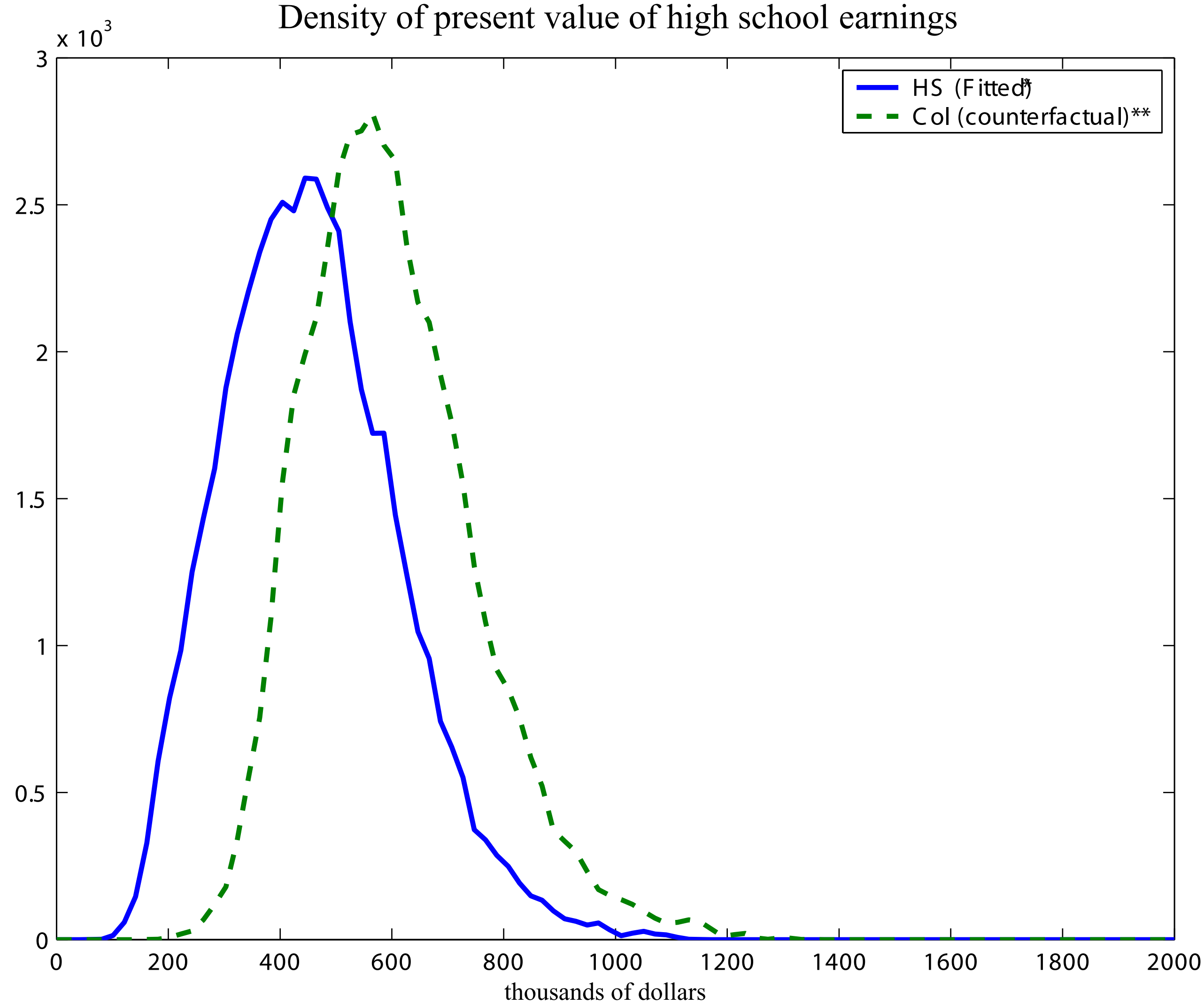
Figure 9

Density of Log Ability for HS and College Graduates



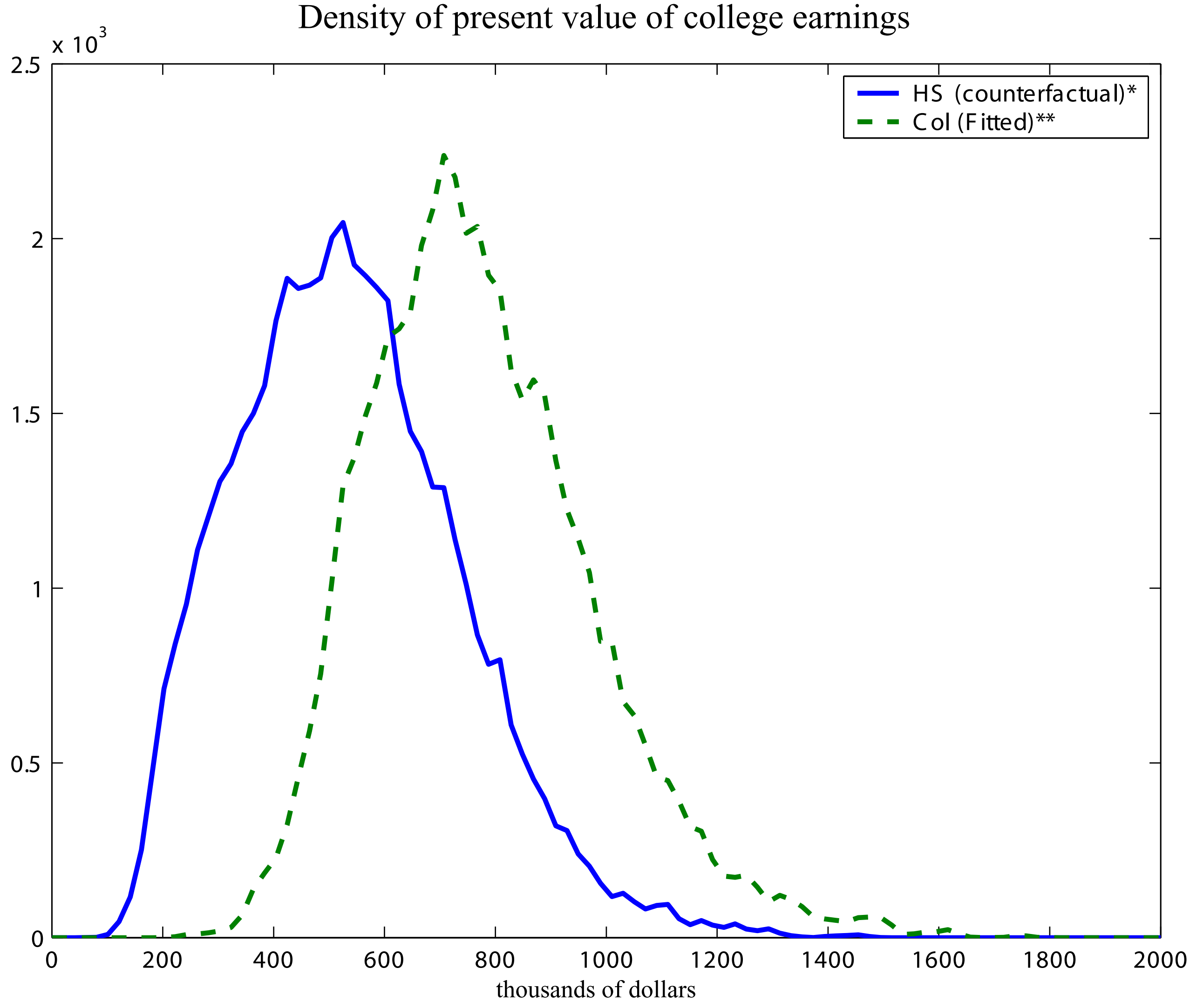
Let f(log a) denote the density of log ability. Let s = 0 denote agents whose parents do NOT send to college, while s = 1 denotes agents that are send to college. Thus, f(log a | s = 0) and f(log a | s = 1) are the densities of log ability conditioned on being a high-school and college graduate, respectively. These densities are plotted in the figure above. Note that the model is able to reproduce the empirical finding of selection in ability in schooling choices.

Figure 10
Density of present value of high school earnings



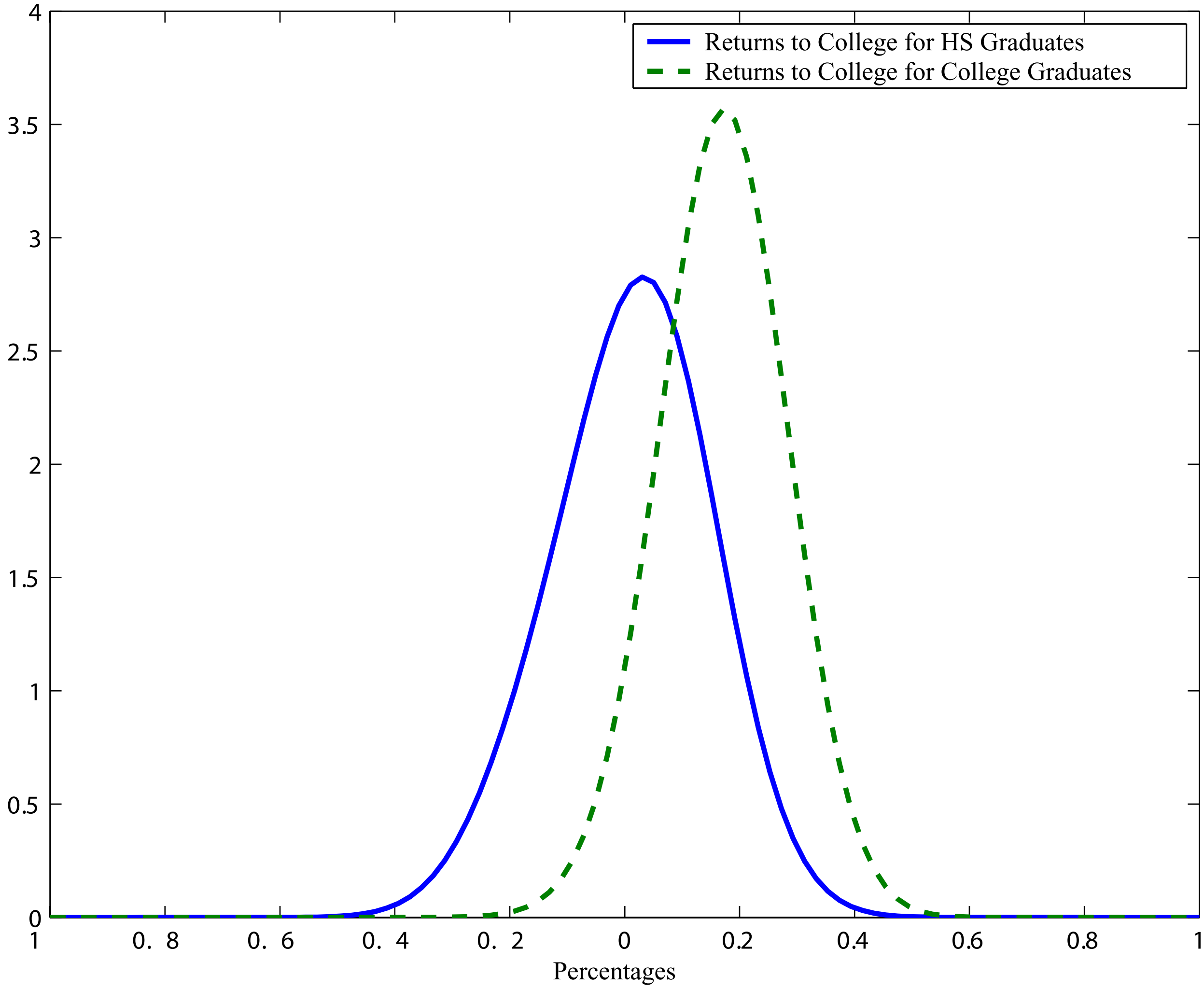
Let y_0 denote the present value of high school earnings. Let $f(y_0)$ denote its density function. Let $s = 0$ denote those agents that are high-school graduates and $s = 1$ those that are college graduates. This figure plots the densities of present value of high school earnings for high school graduates $f(y_0 \mid s = 0)$, (fitted, solid curve) and college graduates $f(y_0 \mid s = 1)$, (counterfactual, dashed curve), as predicted by the model. This pattern is consistent with the empirical evidence from Carneiro, Hansen and Heckman (2003) and Cunha, Heckman and Navarro (2004).

Figure 11
Density of present value of college earnings



Let y_1 denote the present value of college earnings. Let $f(y_1)$ denote its density function. Let $s = 0$ denote those agents that are high-school graduates and $s = 1$ those that are college graduates. This figure plots the densities of present value of college earnings for high school graduates $f(y_1 \mid s = 0)$, (counterfactual, solid curve) and college graduates $f(y_1 \mid s = 1)$, (fitted, dashed curve), as predicted by the model. This pattern is consistent with the empirical evidence from Carneiro, Hansen and Heckman (2003) and Cunha, Heckman and Navarro (2004).

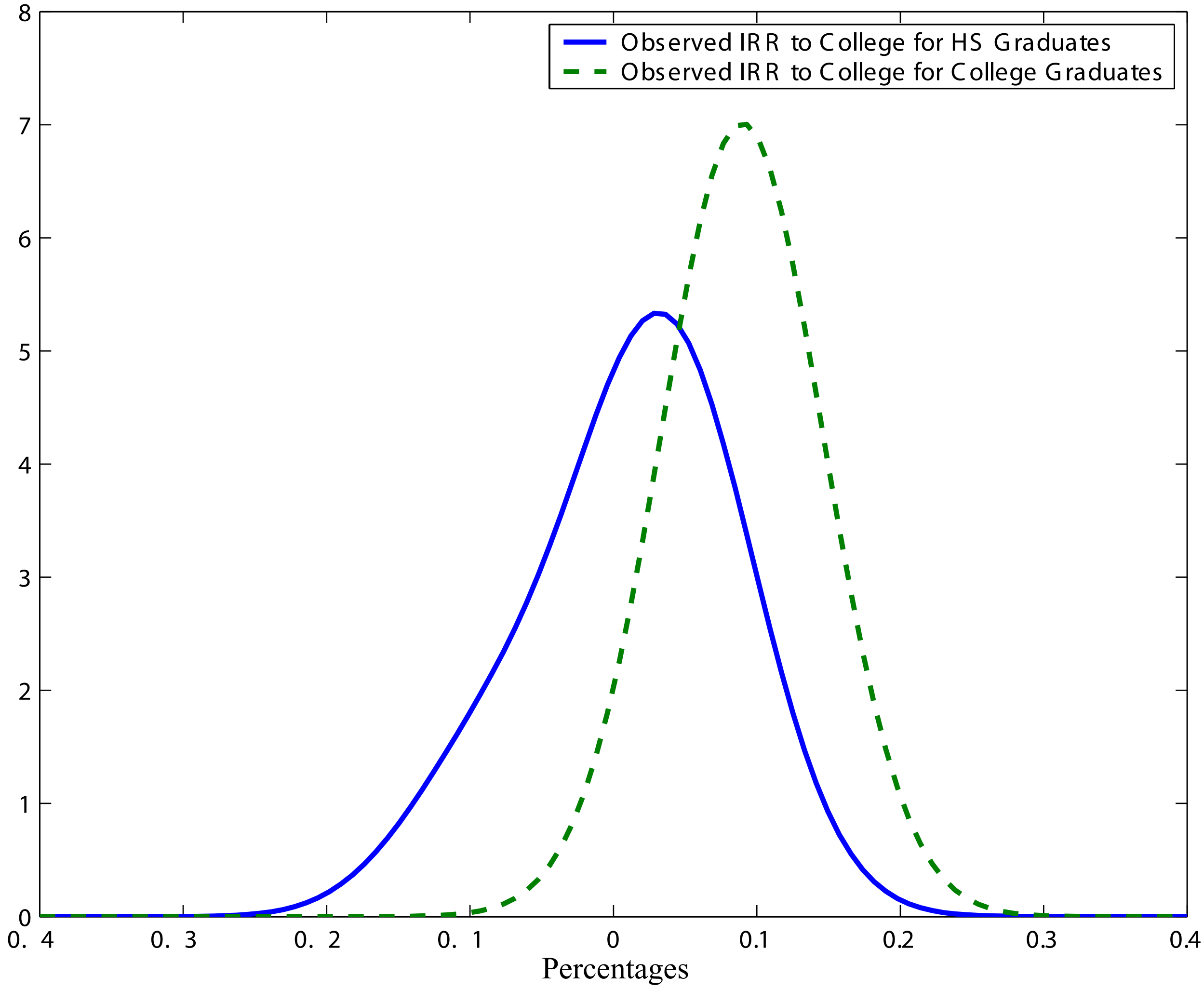
Figure 11
Returns to College



Let y_0, y_1 denote the present value of high school and college earnings, respectively. Let C denote the total costs (opportunity and tuition costs) of going to college. The returns to college R is defined as:
$R = (y_1 - y_0 - C)/(y_0 + C)$.
Let f(r) denote the density of the returns to college. Let s = 0, s = 1 denote high-school and college graduates, respectively. This picture plots the density of returns to college for high-school graduates, $f(r \mid s = 0)$, (solid curve), and the density of returns to college for college graduates, $f(r \mid s = 1)$, (dashed curve). Note that the model generates selection on returns: agents that can benefit more from college are more likely to enroll in college. Again, this pattern is consistent with the findings of Carneiro, Hansen and Heckman (2003), and Cunha, Navarro, and Heckman (2004).

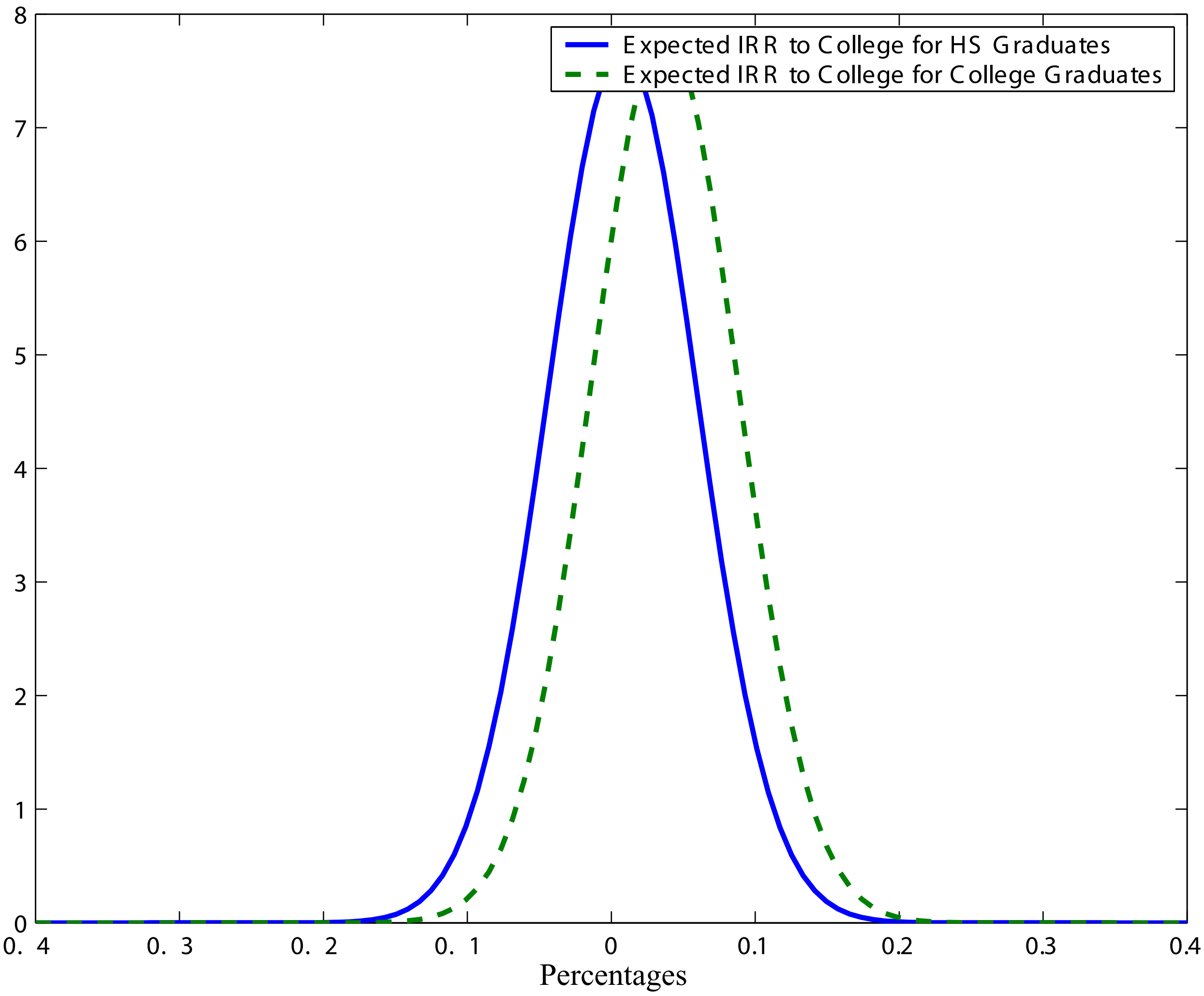
Figure 13

The Density of the Observed Internal Rate of Return to College



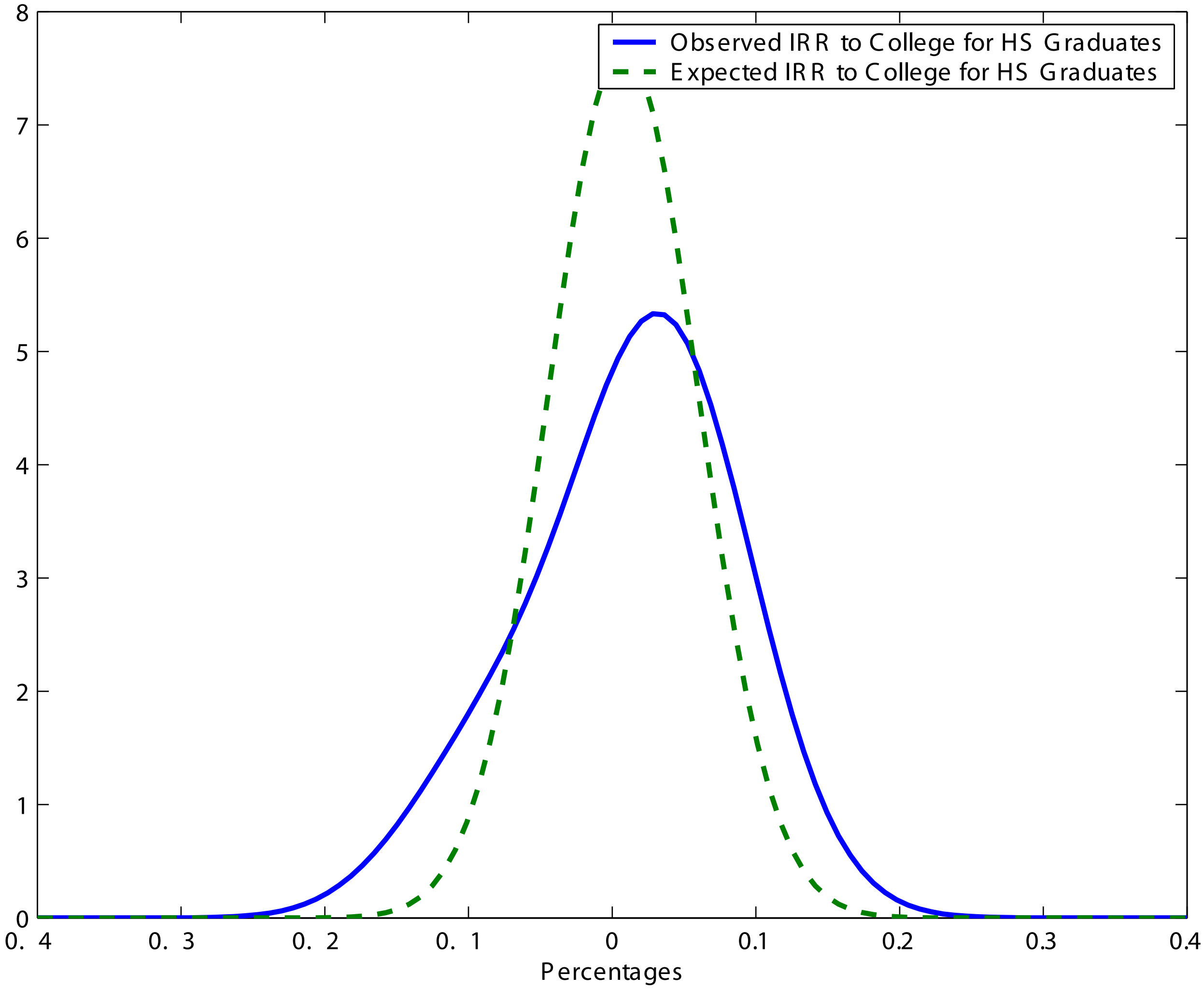
Let $y_{1,t+1}$, $y_{1,t+2}$ denote the college earnings in periods t+1, and t+2, respectively. Let $y_{0,t+1}$, $y_{0,t+2}$ denote the high-school earnings in periods t+1, and t+2, respectively. Let C_t denote the total costs of going to college (tuition and opportunity costs). The observed (or ex-post) internal rate of return is the constant value i that solves: $-C_t + (y_{1,t+1} - y_{0,t+1})/(1 + i) + (y_{1,t+2} - y_{0,t+2})/(1 + i)^2 = 0$. Let f(i) denote the density of the observed internal rate of return i. Again, let s = 0, s = 1 denote high-school and college graduate agents. This figure plots the density of the internal rate of return i for high-school agents, $f(i \mid s = 0)$, (the solid curve), and the density of the internal rate of return i for college graduates, $f(i \mid s = 1)$, (the dashed curve). Again, there is selection on returns: agents that can benefit more from college are more likely to graduate from college. This is consistent with the evidence presented by Carneiro, Hansen and Heckman (2003), and Cunha, Heckman and Navarro (2004).

Figure 14
The Density of the Expected Internal Rate of Return to College



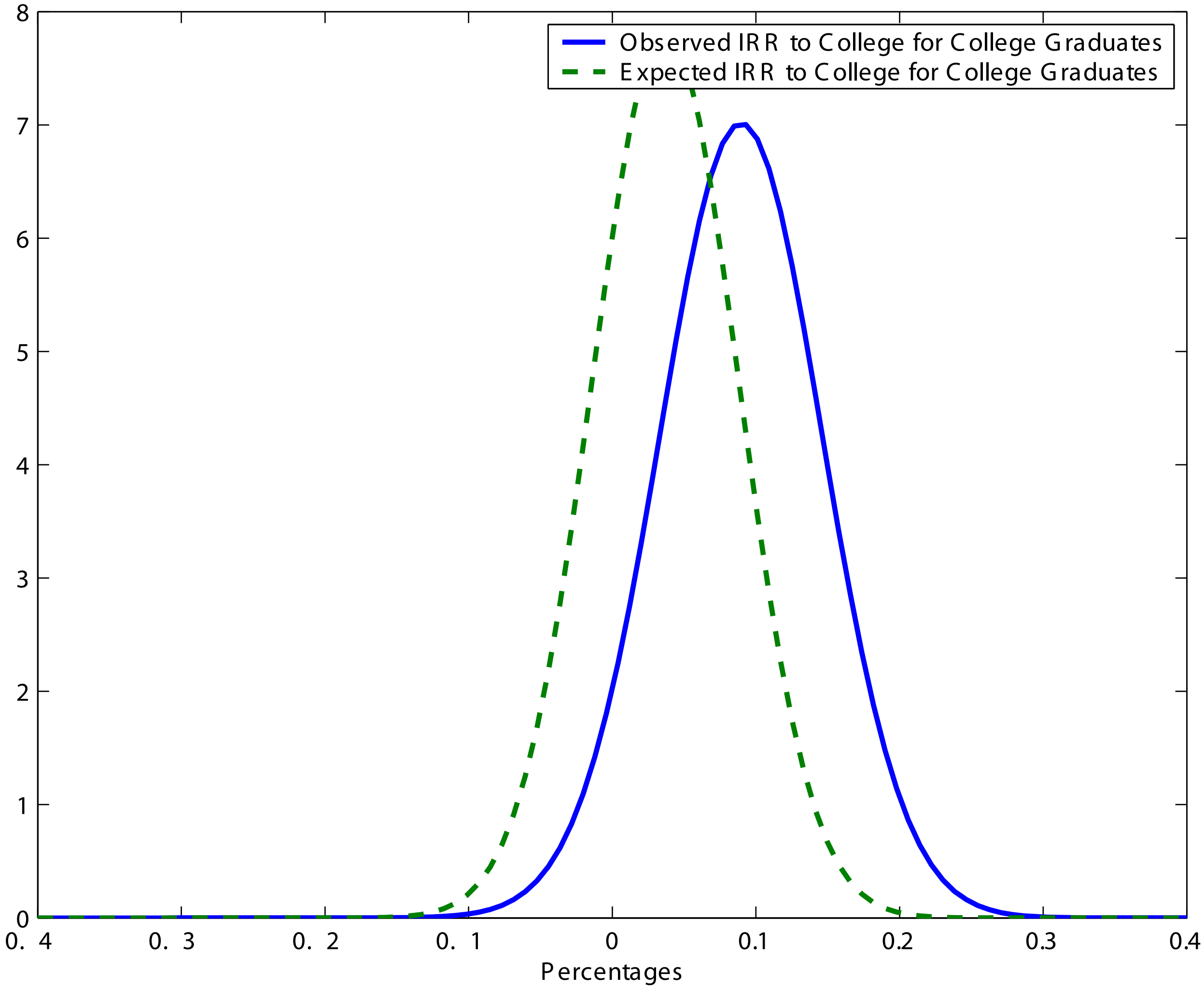
Let $y_{1,t+1}$, $y_{1,t+2}$ denote the college earnings in periods t+1, and t+2, respectively. Let $y_{0,t+1}$, $y_{0,t+2}$ denote the high-school earnings in periods t+1, and t+2, respectively. Let C_t denote the total costs of going to college (tuition and opportunity costs). The expected (or ex-ante) internal rate of return is the constant value i that solves: $-C_t + E_t[(y_{1,t+1} - y_{0,t+1})/(1 + i) + (y_{1,t+2} - y_{0,t+2})/(1 + i)^2]= 0$. Let f(i) denote the density of the expected internal rate of return i. Again, let s = 0, s = 1 denote high-school and college graduate agents. This figure plots the density of the internal rate of return i for high-school agents, f(i | s = 0), (the solid curve), and the density of the internal rate of return i for college graduates, f(i | s = 1), (the dashed curve). Again, there is selection on EXPECTED returns: agents that can benefit more from college are more likely to graduate from college. This is consistent with the evidence presented by Carneiro, Hansen and Heckman (2003), and Cunha, Heckman and Navarro (2004).

Figure 15
The Densities of the Observed and Expected
Internal Rate of Return to College for HS Graduates



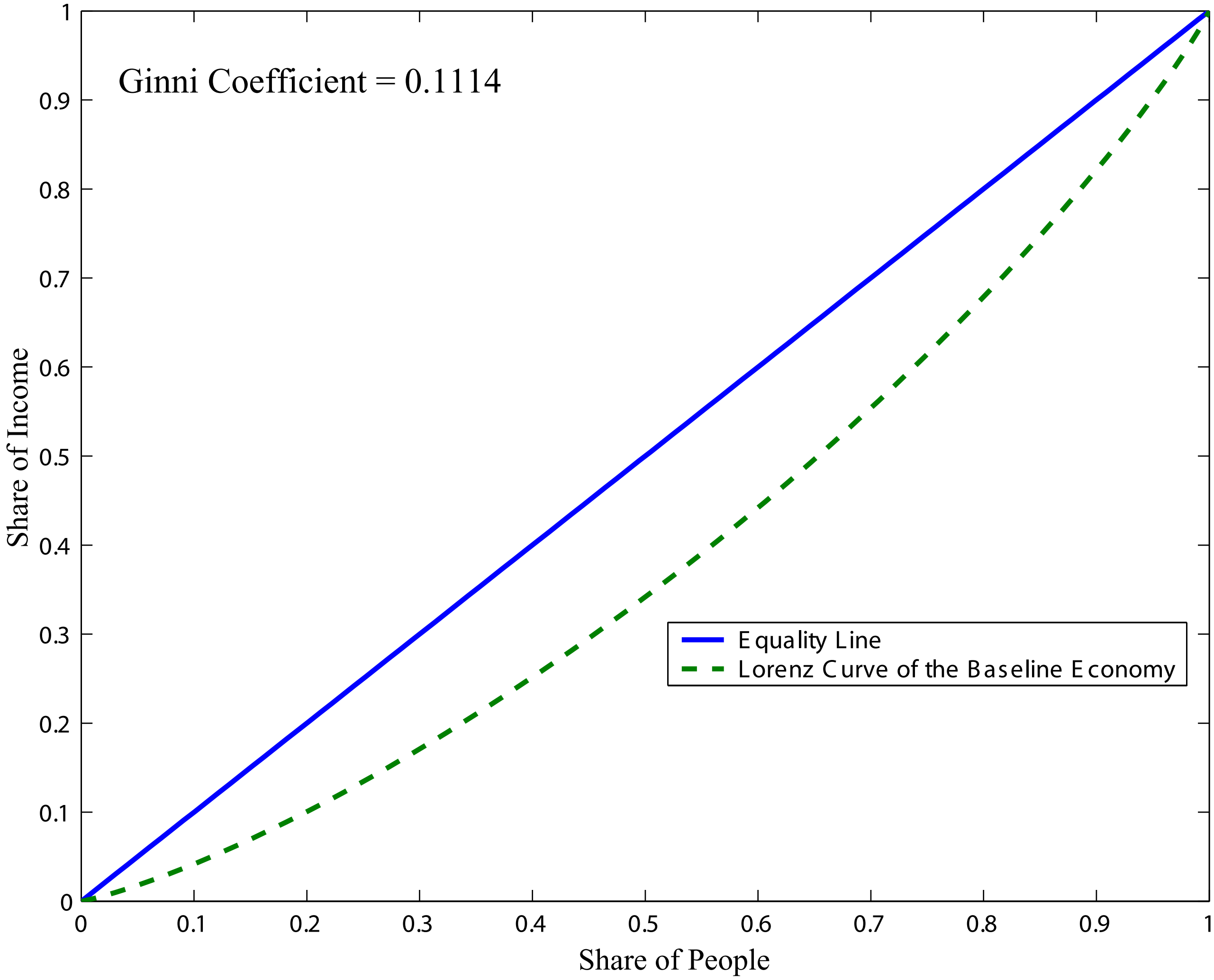
Let $y_{1,t+1}$, $y_{1,t+2}$ denote the college earnings in periods t+1, and t+2, respectively. Let $y_{0,t+1}$, $y_{0,t+2}$ denote the high-school earnings in periods t+1, and t+2, respectively. Let C_t denote the total costs of going to college (tuition and opportunity costs). The observed (or ex-post) internal rate of return is the constant value i_p that solves: $-C_t + (y_{1,t+1} - y_{0,t+1})/(1 + i_p) + (y_{1,t+2} - y_{0,t+2})/(1 + i_p)^2 = 0$. The expected (or ex-ante) internal rate of return is the constant value i_e that solves: $-C_t + E_t[(y_{1,t+1} - y_{0,t+1})/(1 + i_e) + (y_{1,t+2} - y_{0,t+2})/(1 + i_e)^2] = 0$. Let s = 0 denote high-school graduates. Let $f(i_p)$, and $g(i_e)$ denote the density functions of i_p, and i_e, respectively. This figure plots the density of the observed internal rate of returns to college for high-school graduates, $f(i_p \mid s = 0)$, (the solid curve) and the density of the expected internal rate of returns to college for high-school graduates, $g(i_e \mid s = 0)$, (the dashed curve).

Figure 16
The Densities of the Observed and Expected
Internal Rate of Return to College for College Graduates



Let $y_{1,t+1}$, $y_{1,t+2}$ denote the college earnings in periods t+1, and t+2, respectively. Let $y_{0,t+1}$, $y_{0,t+2}$ denote the high-school earnings in periods t+1, and t+2, respectively. Let C_t denote the period t total costs of going to college (tuition and opportunity costs). The observed (or ex-post) internal rate of return is the constant value i_p that solves: $-C_t + (y_{1,t+1} - y_{0,t+1})/(1 + i_p) + (y_{1,t+2} - y_{0,t+2})/(1 + i_p)^2 = 0$. The expected (or ex-ante) internal rate of return is the constant value i_e that solves: $-C_t + E_t[(y_{1,t+1} - y_{0,t+1})/(1 + i_e) + (y_{1,t+2} - y_{0,t+2})/(1 + i_e)^2] = 0$. Let $s = 1$ denote college graduates. Let $f(i_p)$, and $g(i_e)$ denote the density functions of i_p, and i_e, respectively. This figure plots the density of the observed internal rate of returns to college for college graduates, $f(i_p \mid s = 1)$, (the solid curve) and the density of the expected internal rate of returns to college for college graduates, $g(i_e \mid s = 1)$, (the dashed curve).

Figure 17
Lorenz Curve of Present Value of Earnings

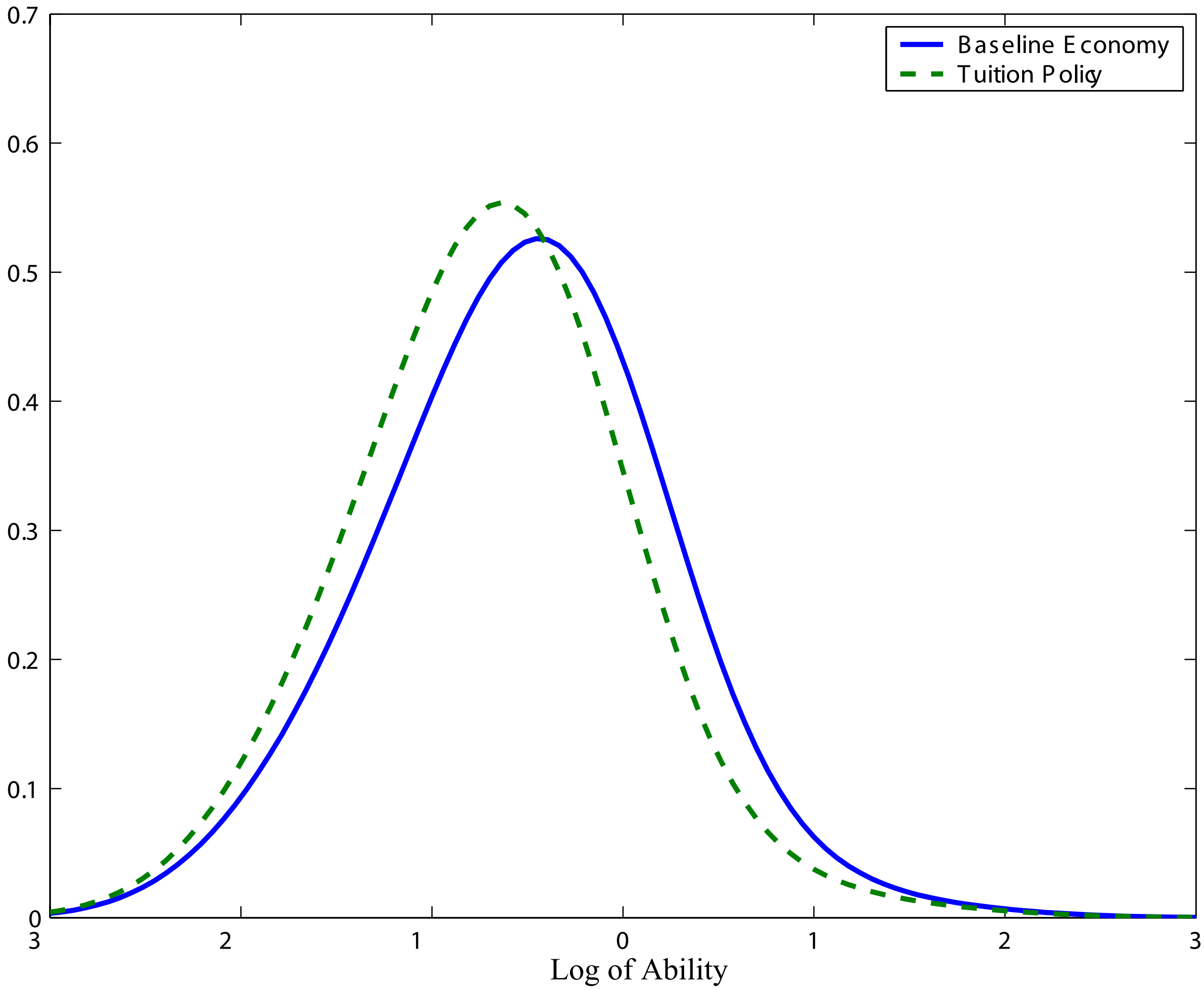


This figure plots the Lorenz curve of the present value of earnings for the overall sample in the extended Laitner economy. Note that the Ginni coefficient for lifetime earnings is 0.11, below of the range reported by Lillard(1977). One possible explanation of the underprediction of lifetime inequality of the model is because the data on earnings is only from ages 21 to 40.

Figure 18

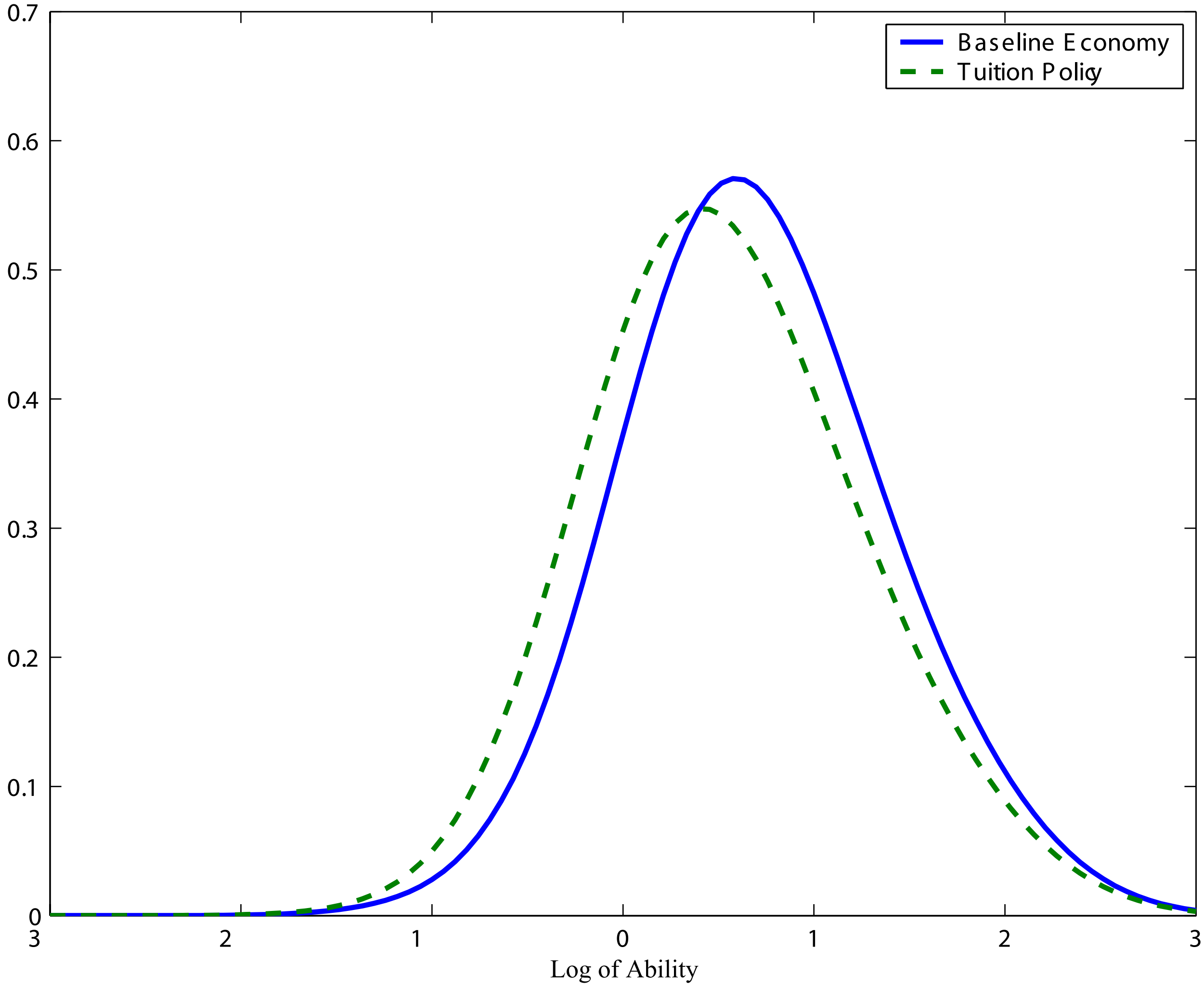
Density of Log Ability for HS Graduates

Baseline vs. under Tuition Policy



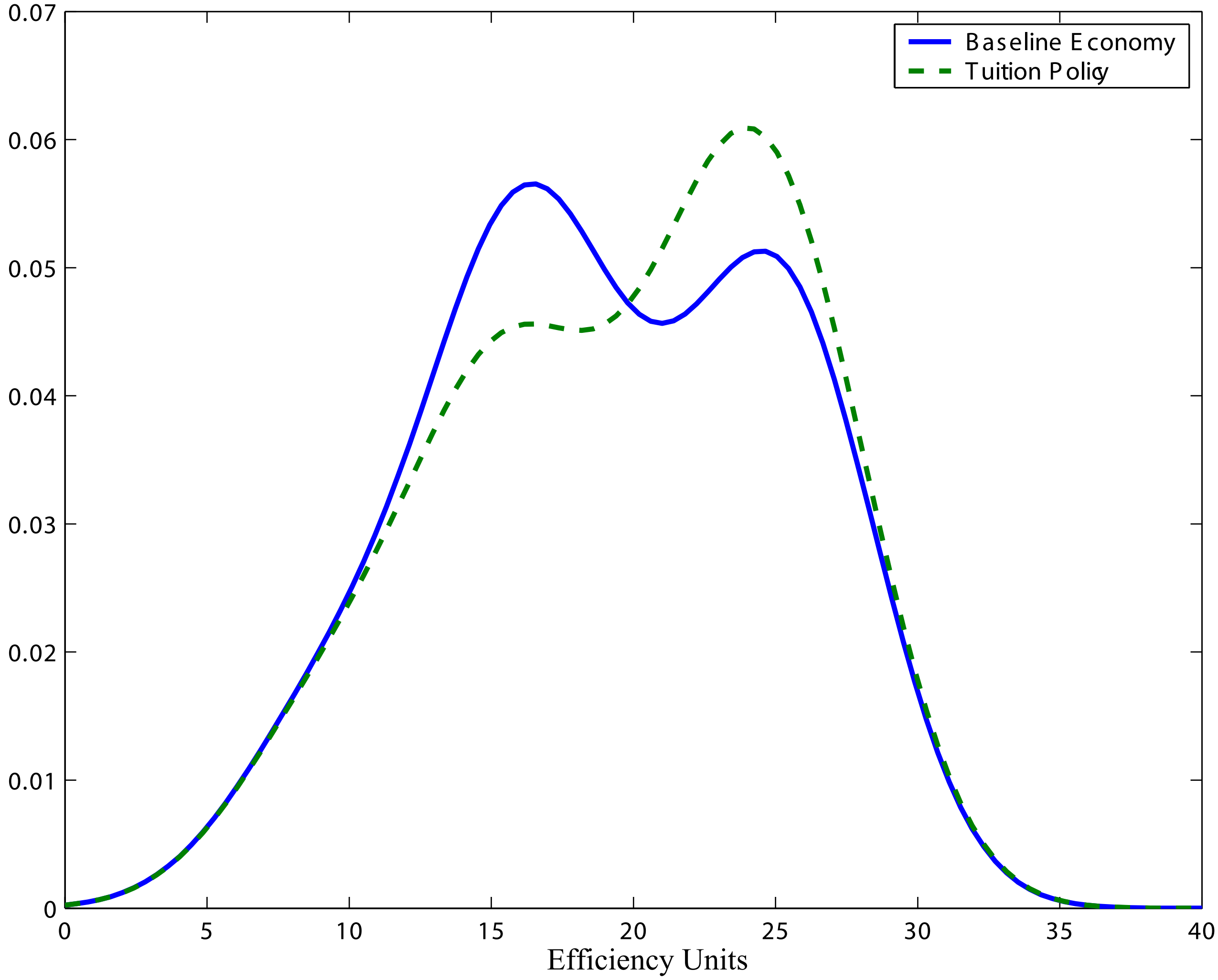
Let log a and f(log a) denote log ability and its stationary distribution. Let s = 0 denote high-school graduate agents. This figure plots the stationary denstity of log ability for high-school graduates, f(log a | s = 0) in the steady state of the baseline economy (solid curve) against that of the economy with a full tuition subsidy financed through an increase in the income tax (dashed curve). Note that the density of log ability in the economy with tuition policy is to the "left" of the density associated with the baseline economy. The tuition policy affects people that are above the average of the stationary density of ability for high school graduates.

Figure 19
Density of Log Ability for College Graduates
Baseline vs. under Tuition Policy



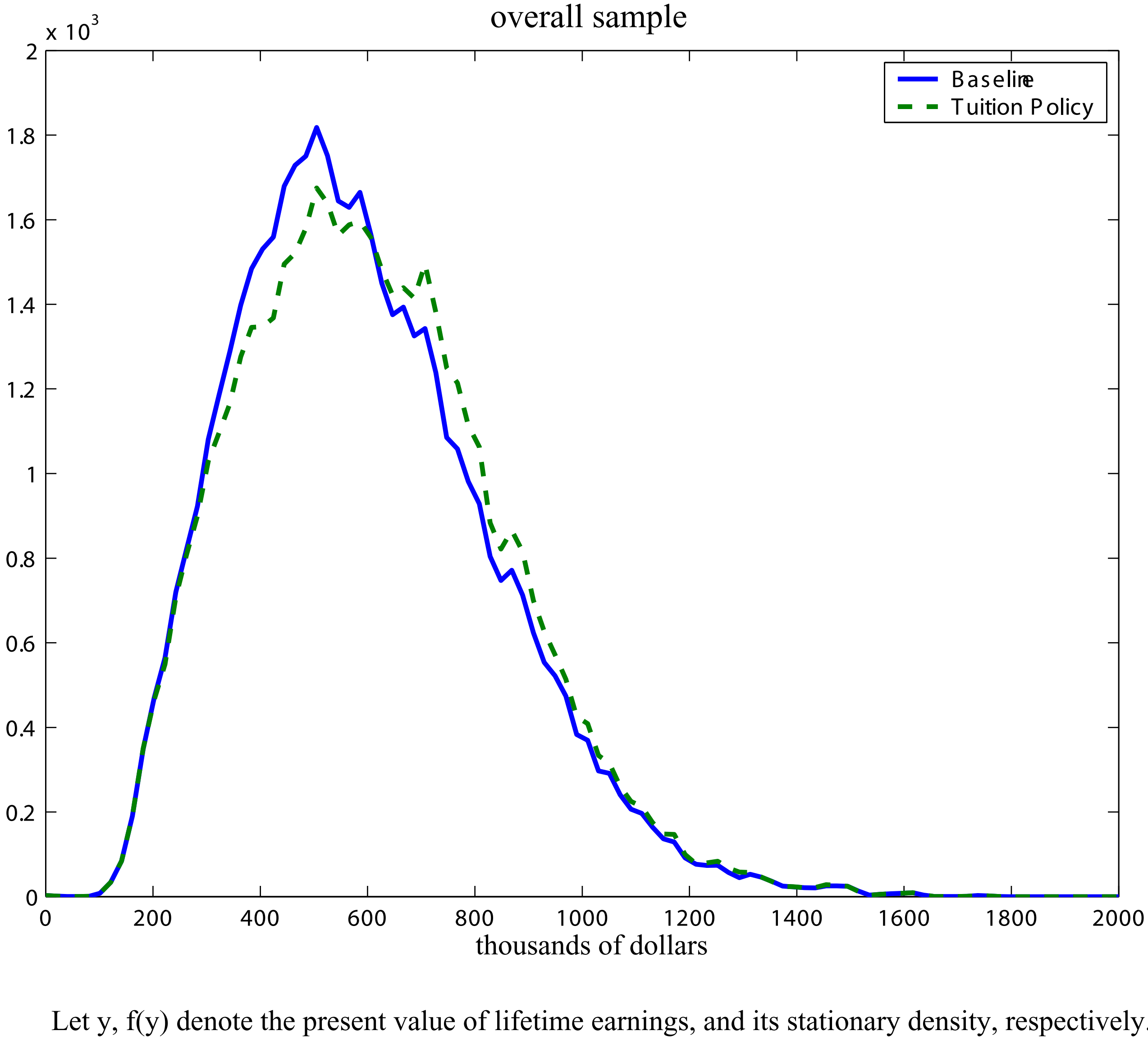
Let log a and f(log a) denote log ability and its stationary distribution. Let s = 1 denote college graduate agents. This figure plots the stationary denstity of log ability for college graduates, f(log a | s = 1) in the steady state of the baseline economy (solid curve) against that of the economy with a full tuition subsidy financed through an increase in the income tax (dashed curve). Note that the density of log ability in the economy with tuition policy is to the "right" of the density associated with the baseline economy. The tuition policy affects people that are below the average of the stationary density of ability for college graduates.

Figure 20
Stationary Density of Human Capital
Baseline vs Tuition Policy



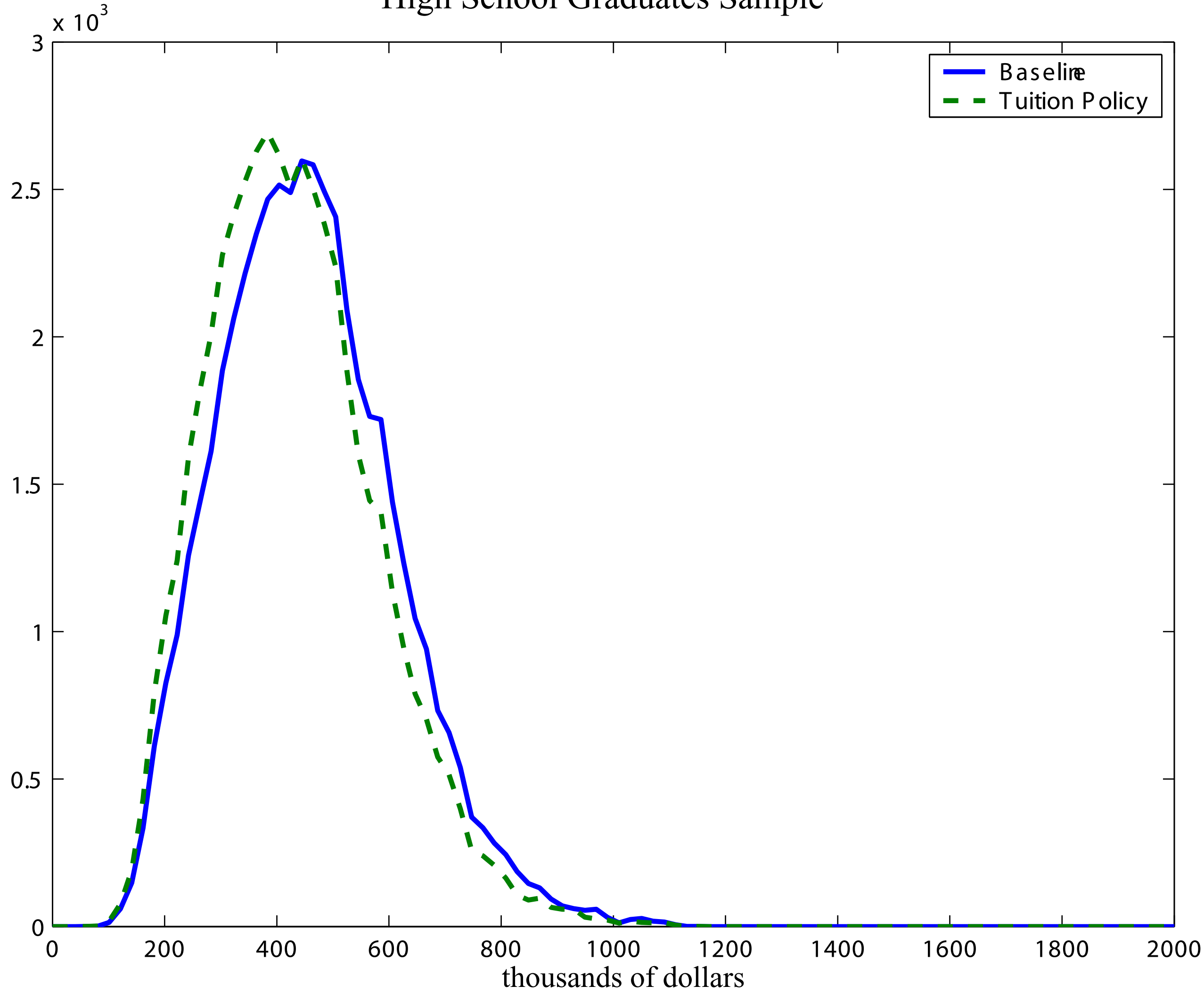
Let h, f(h) denote human capital and its stationary density. This figure plots the stationary density implied by the baseline economy (solid curve) versus the one implied by the economy with full tuition subsidy. Note that the tuition subsidy affects people that are in the middle of the baseline stationary density of human capital, and not so much agents that are in the tails of the density.

Figure 21
Density of present value of earnings
Baseline vs Tuition Policy
overall sample



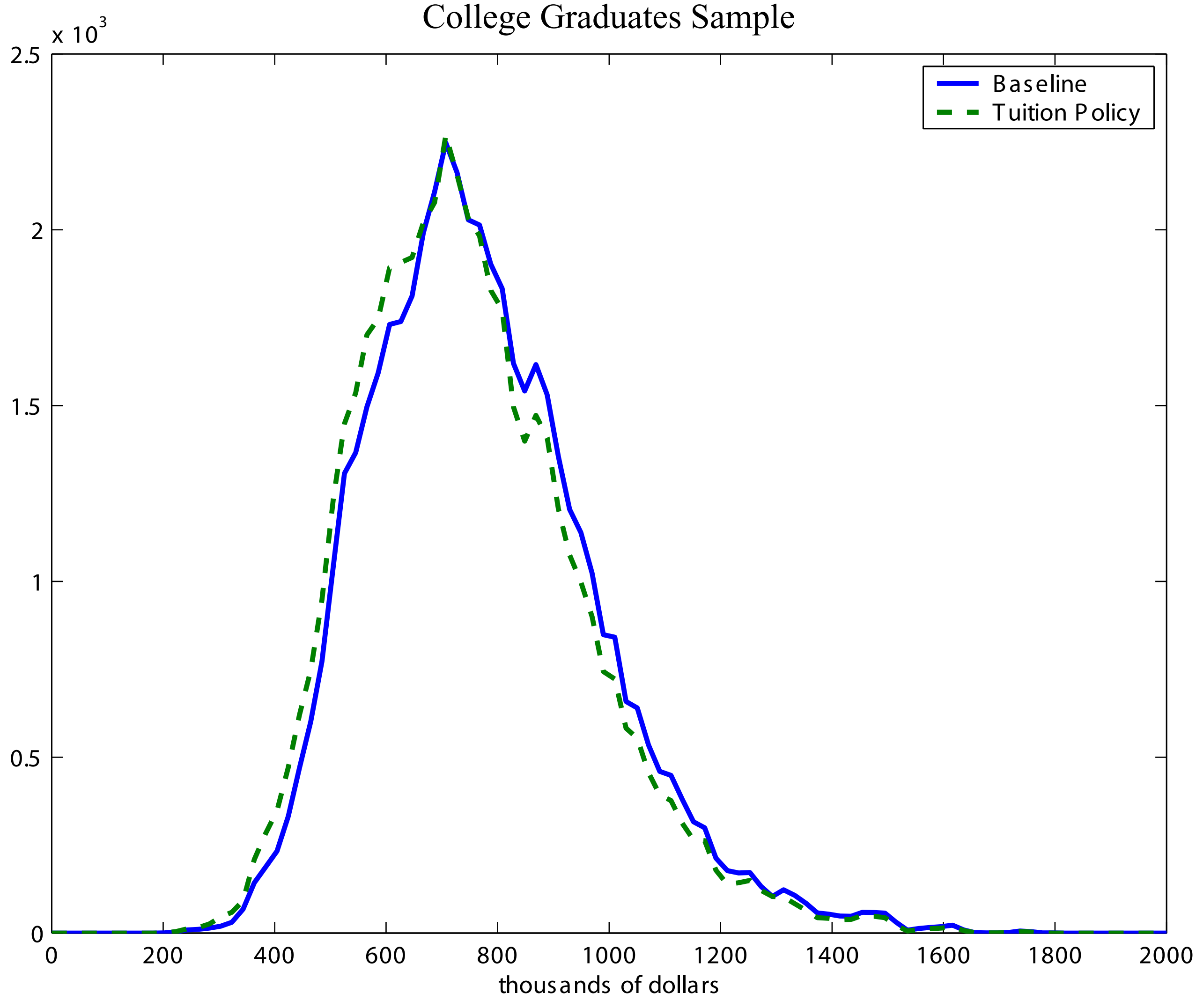
Let y, f(y) denote the present value of lifetime earnings, and its stationary density, respectively. This figure plots the stationary density implied by the baseline economy (the solid curve) against that implied by the economy with full tuition subsidy. Again, the tuition subsidy affects primarily those agents that are in the middle of the present value of earnings distributions.

Figure 22
Density of present value of earnings
Baseline vs Tuition Policy
High School Graduates Sample



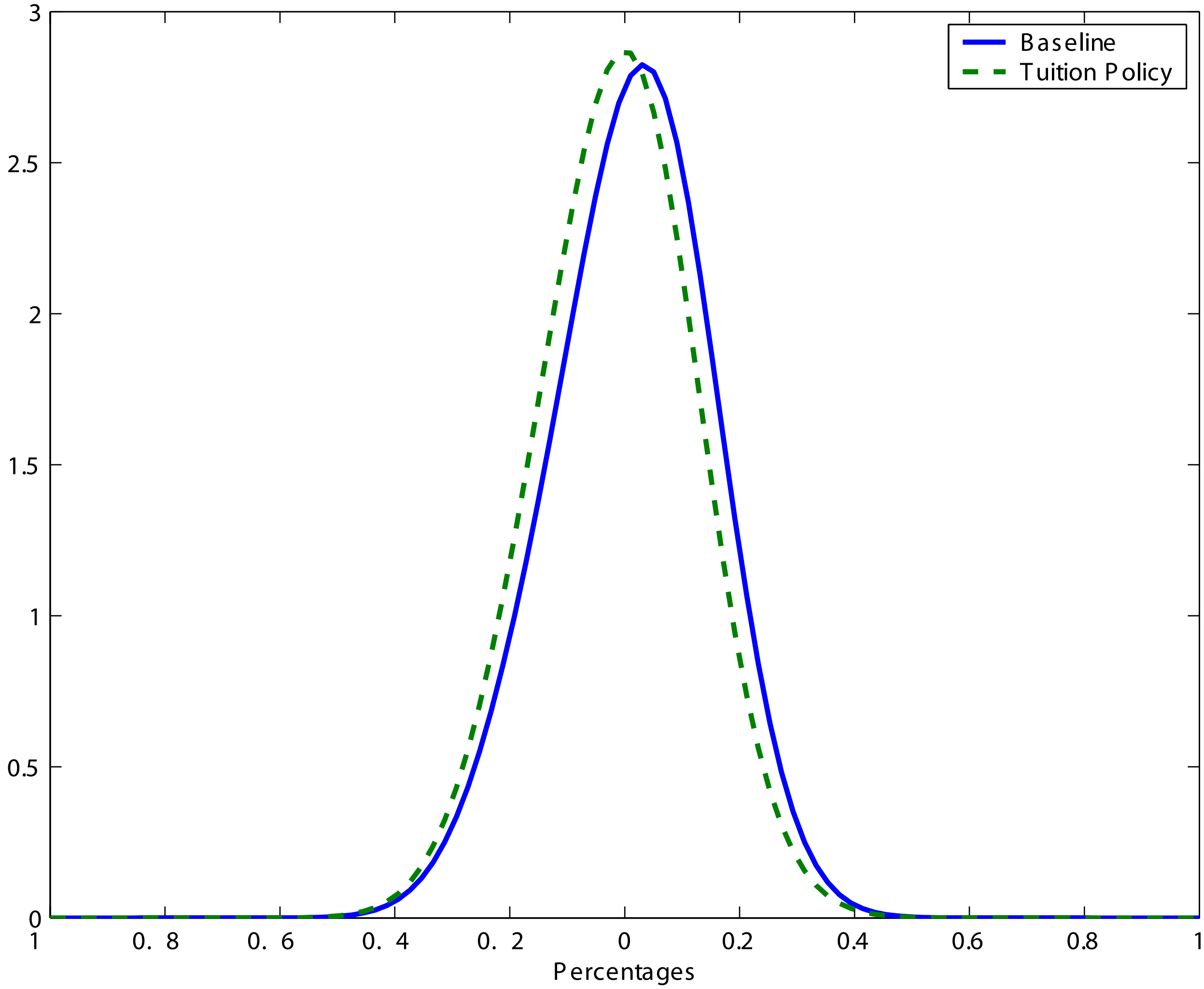
Let y, f(y) denote the present value of lifetime earnings, and its stationary density, respectively. Let s = 0 denote high school graduate agents. This figure plots the stationary density of y for the high school sample, f(y | s = 0), implied by the baseline economy (the solid curve) against that implied by the economy with full tuition subsidy. The tuition subsidy moves agents that are above-average earners in the high school distribution of present value of earnings.

Figure 23
Density of present value of earnings
Baseline vs Tuition Policy
College Graduates Sample



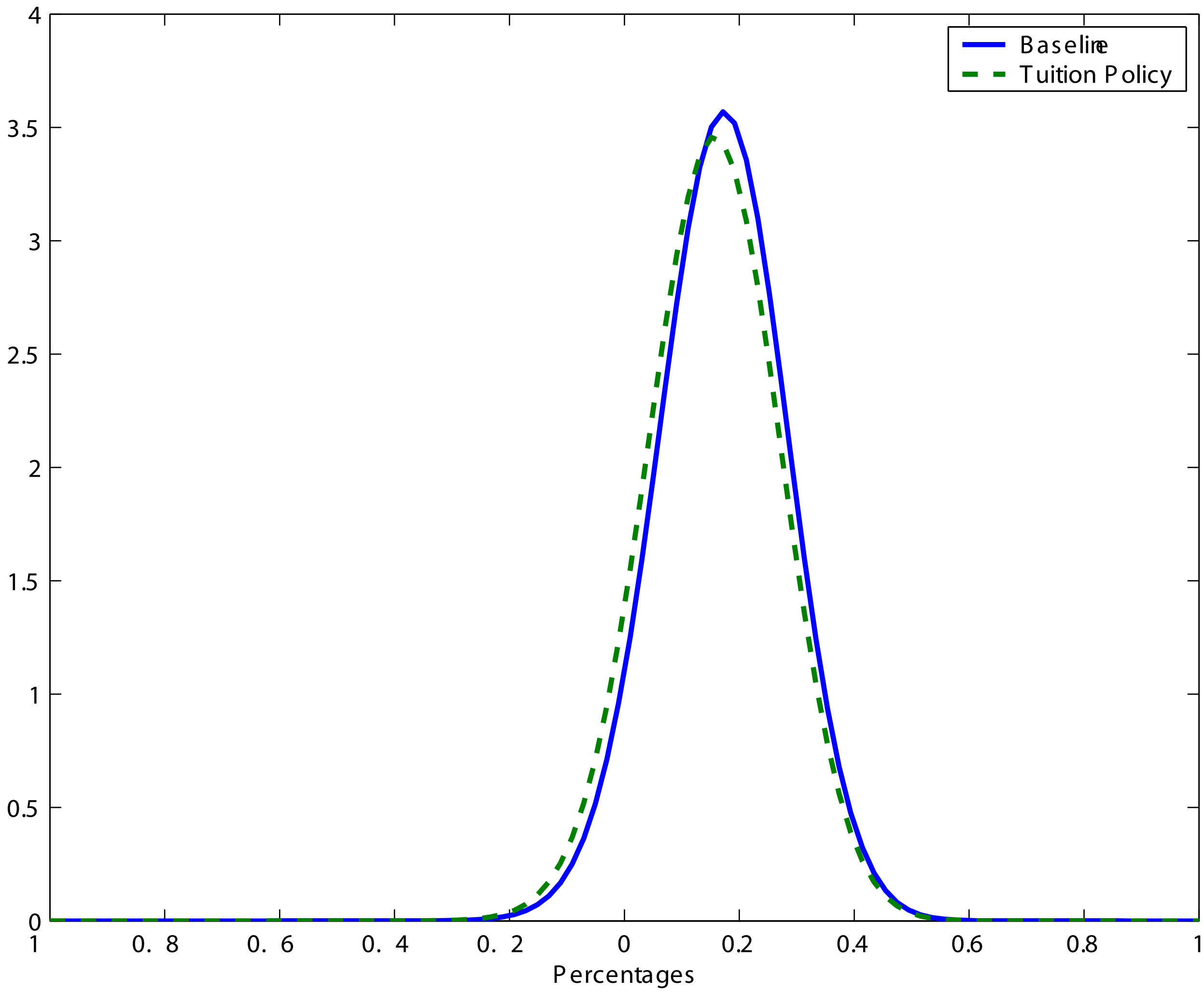
Let y, f(y) denote the present value of lifetime earnings, and its stationary density, respectively. Let s = 1 denote college graduate agents. This figure plots the stationary density of y for the high school sample, f(y | s = 1), implied by the baseline economy (the solid curve) against that implied by the economy with full tuition subsidy. The tuition subsidy moves agents that are below-average earners in the college distribution of present value of earnings.

Figure 24
Gross Returns to College for HS Graduates
Baseline vs. Tuition Policy



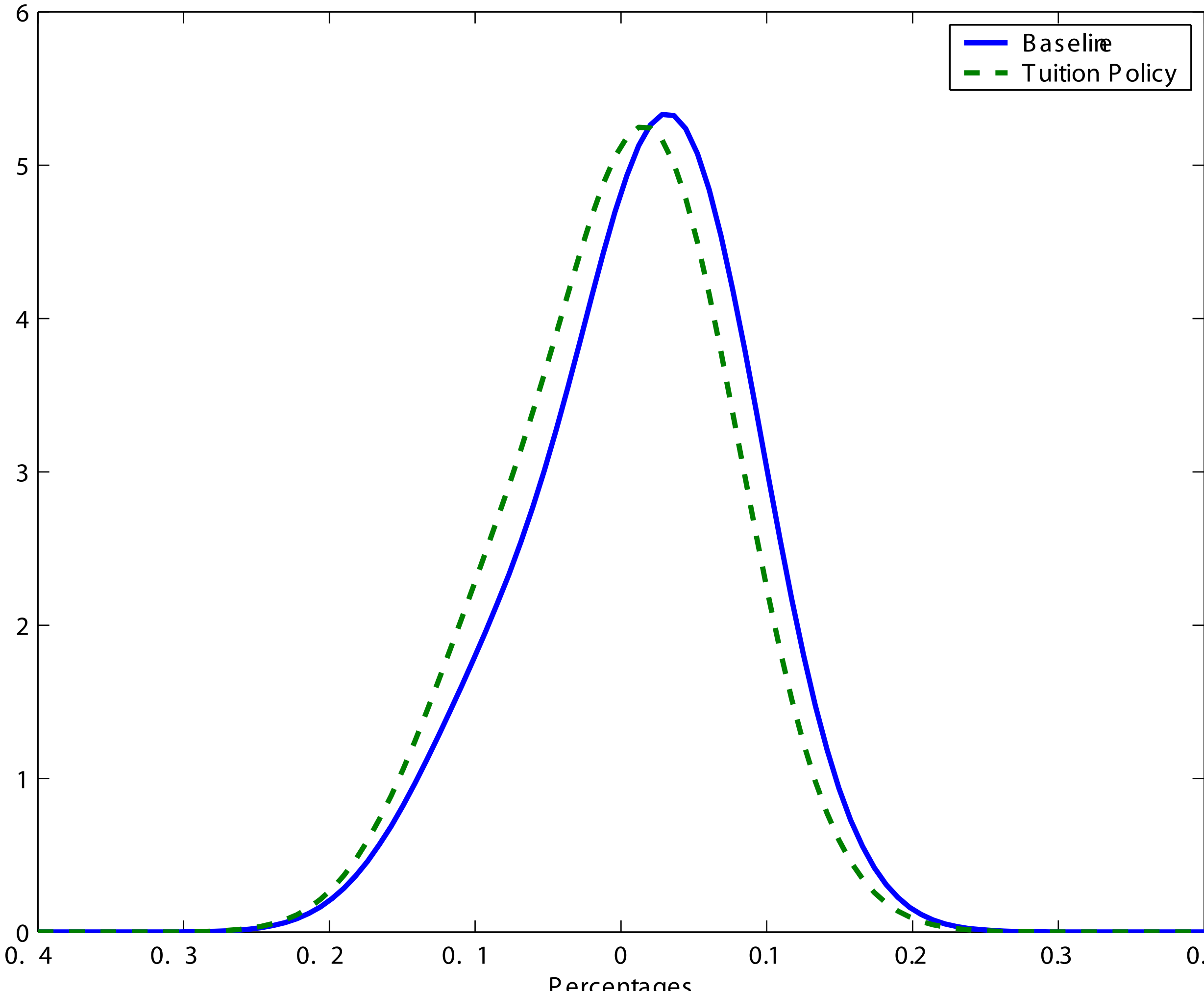
Let y_0, y_1 denote the present value of high school and college earnings, respectively. Let C denote the total costs (opportunity and tuition costs) of going to college. The returns to college R is defined as:
$R = (y_1 - y_0 - C)/(y_0 + C)$. Let $f(r \mid s)$ denote the density of returns conditioned on schooling choice s.
This picture plots the density of returns to college for high-school graduates in the baseline economy (solid curve), against that implied by the economy with full tuition subsidy (dashed curve). Note that the model predicts that the tuition policy affects agents that have returns above average in the high school distribution of returns.
This pattern is consistent with the findings of Carneiro, Heckman and Vytlacil (2003), Carneiro, Hansen and Heckman (2003), and Cunha, Navarro, and Heckman (2004).

Figure 25
Gross Returns to College for HS Graduates
Baseline vs. Tuition Policy



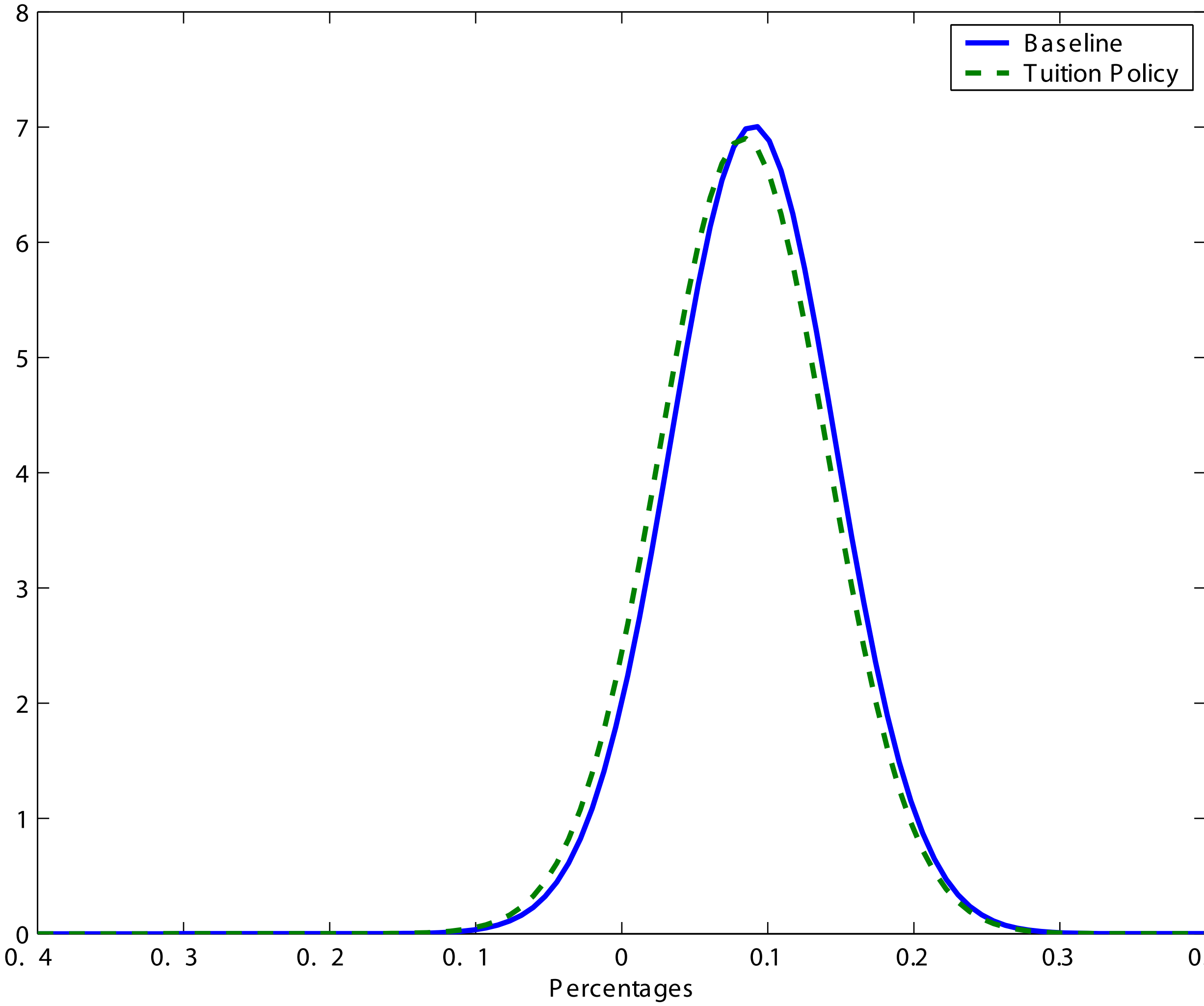
Let y_0, y_1 denote the present value of high school and college earnings, respectively. Let C denote the total costs (opportunity and tuition costs) of going to college. The returns to college R is defined as:
$R = (y_1 - y_0 - C)/(y_0 + C)$. Let $f(r \mid s)$ denote the density of returns conditioned on schooling choice s.
This picture plots the density of returns to college for college graduates in the baseline economy (solid curve), against that implied by the economy with full tuition subsidy (dashed curve). Note that the model predicts that the tuition policy affects agents that have returns below average in the college distribution of returns.
This pattern is consistent with the findings of Carneiro, Heckman and Vytlacil (2003), Carneiro, Hansen and Heckman (2003), and Cunha, Navarro, and Heckman (2004).

Figure 26
Ex post Internal Rate of Return to College for HS Graduates
Baseline vs Tuition Policy



Let $y_{1,t+1}$, $y_{1,t+2}$ denote the college earnings in periods t+1, and t+2, respectively. Let $y_{0,t+1}$, $y_{0,t+2}$ denote the high-school earnings in periods t+1, and t+2, respectively. Let C_t denote the total costs of going to college (tuition and opportunity costs). The observed (or ex-post) internal rate of return is the constant value i that solves: $-C_t + (y_{1,t+1} - y_{0,t+1})/(1 + i) + (y_{1,t+2} - y_{0,t+2})/(1 + i)^2 = 0$. Let f(i) denote the density of the observed internal rate of return i. Again, let s = 0, s = 1 denote high-school and college graduate agents. This figure plots the density of the internal rate of return i for high-school agents, f(i | s = 0), (the solid curve), in the baseline economy against that implied by the economy with full tuition subsidy (the dashed curve). Again, the tuition subsidy affects agents that are above-average in the high school distribution of returns. This is consistent with the evidence presented Carneiro and Heckman (2003), by Carneiro, Hansen and Heckman (2003), and Cunha, Heckman and Navarro (2004).

Figure 27
Ex post Internal Rate of Return to College for College Graduates
Baseline vs Tuition Policy



Let $y_{1,t+1}$, $y_{1,t+2}$ denote the college earnings in periods t+1, and t+2, respectively. Let $y_{0,t+1}$, $y_{0,t+2}$ denote the high-school earnings in periods t+1, and t+2, respectively. Let C_t denote the total costs of going to college (tuition and opportunity costs). The observed (or ex-post) internal rate of return is the constant value i that solves: $-C_t + (y_{1,t+1} - y_{0,t+1})/(1 + i) + (y_{1,t+2} - y_{0,t+2})/(1 + i)^2 = 0$. Let f(i) denote the density of the observed internal rate of return i. Again, let s = 0, s = 1 denote high-school and college graduate agents. This figure plots the density of the internal rate of return i for high-school agents, $f(i \mid s = 0)$, (the solid curve), in the baseline economy against that implied by the economy with full tuition subsidy (the dashed curve). Again, the tuition subsidy affects agents that are below average in the college distribution of returns. This is consistent with the evidence presented Carneiro and Heckman (2003), by Carneiro, Hansen and Heckman (2003), and Cunha, Heckman and Navarro (2004).

Figure 28
Lorenz Curve
Baseline vs Tuition Policy

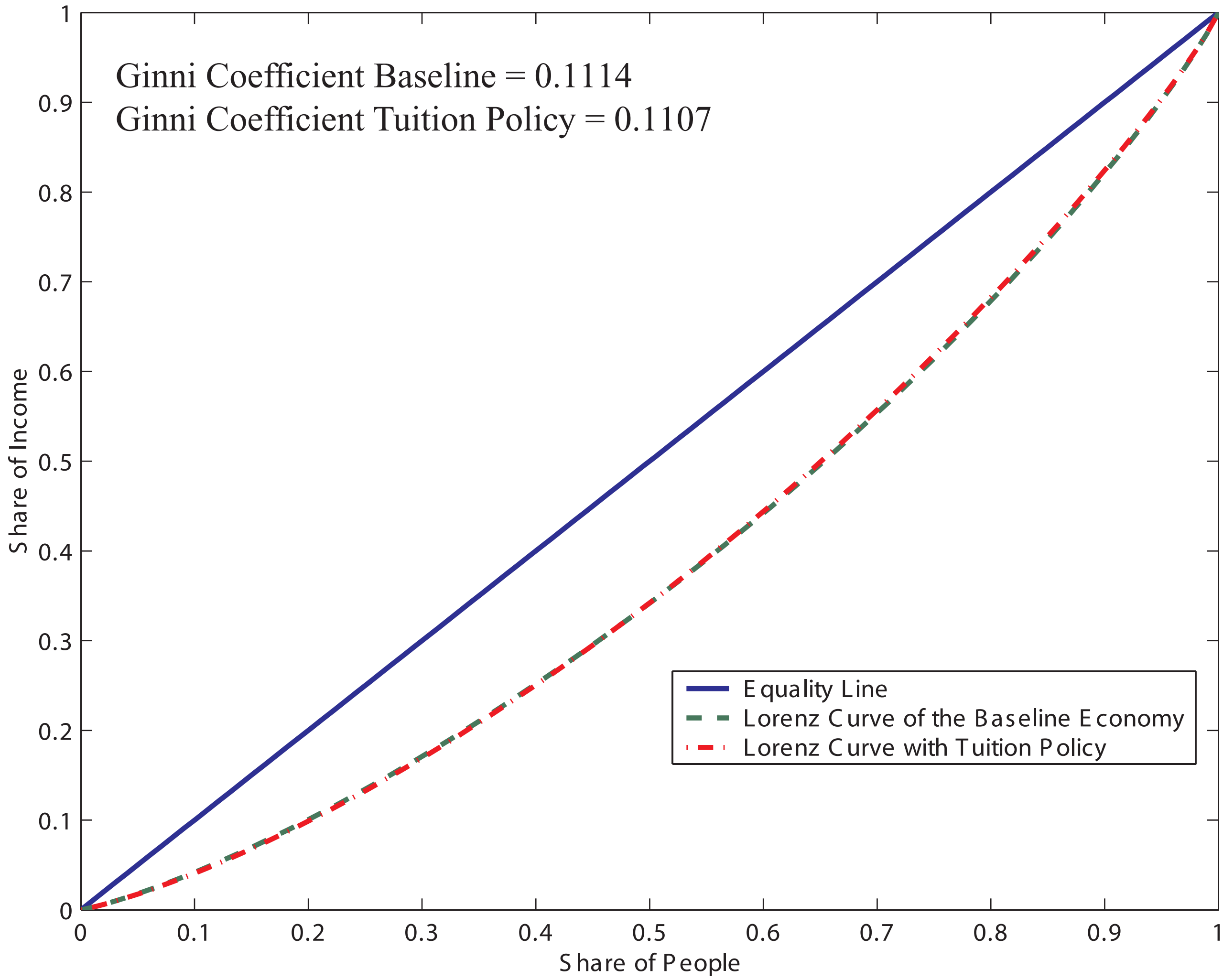
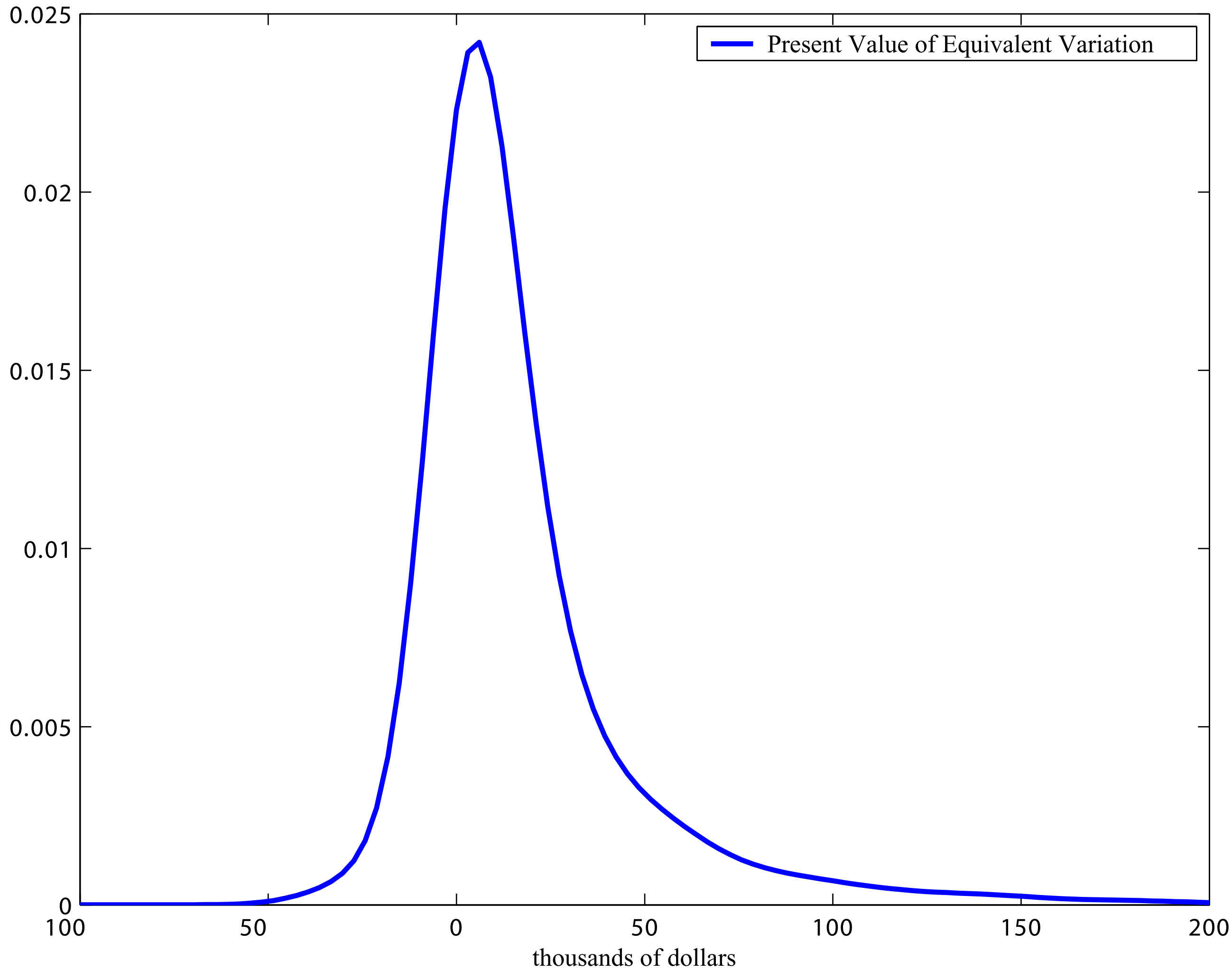


Figure 29
Density of the Present Value of the Equivalent Variation for a Tuition Policy
Proportion of agents with Positive Present Value of Equivalent Variation = 82,26%
Average Present Value of Equivalent Variation = U$ 16,992.60
Average Present Value of Equivalent Variation as Fraction of Lifetime Earnings = 3.10%



Let c_y, and c_o denote the steady state consumption when young and old of a given agent in the baseline economy. Let Let d_y, and d_o denote the steady state consumption of the very same given agent in the economy with full tuition subsidy. The equivalent variation are constant dollars λ that solve: $u(c_y + \lambda) + \beta u(c_o + \lambda) + \beta E(V) = u(d_y) + \beta u(d_o) + \beta E(V)$. 60% of the agents have $\lambda > 0$. This means that they must be given consumption in the baseline economy to be indifferent to the economy with a full-tuition subsidy. The average present value of the equivalent variation, $\lambda + \lambda/(1 + r)$, is close to U$ 12,000.00. Let y denote the present value of lifetime earnings. Then, the average present value of the equivalent variation as a fraction of lifetime earnings, $[\lambda + \lambda/(1 + r)]/y$, is 2.10%.

Table 10

The Average Present Value of Equivalent Variation by Deciles of the Present Value of Lifetime Earnings Distribution

Ability Group	Average Present Value of Equivalent Variation
1	10549.19
2	10830.92
3	13453.89
4	14199.94
5	19391.02
6	19972.97
7	23422.12
8	19039.78
9	20384.81
10	18970.84

Let c_b^y, c_b^o denote the baseline-economy consumption of a given agent when young and old, respectively. Let c_t^y, c_t^o denote the full–tuition-subsidy-economy consumption of the very same given agent when young and old, respectively. The ex-post lifetime utility of an agent in the baseline economy is:

$$V_b = u\left(c_b^y\right) + \beta u\left(c_b^o\right) + \beta^2 \theta E\left[V_b\right]$$

while in the economy with the full tuition subsidy is:

$$V_t = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The equivalent variation is the constant dollar amount λ that makes the agent indifferent between the baseline-economy and the economy with full-tuition subsidy:

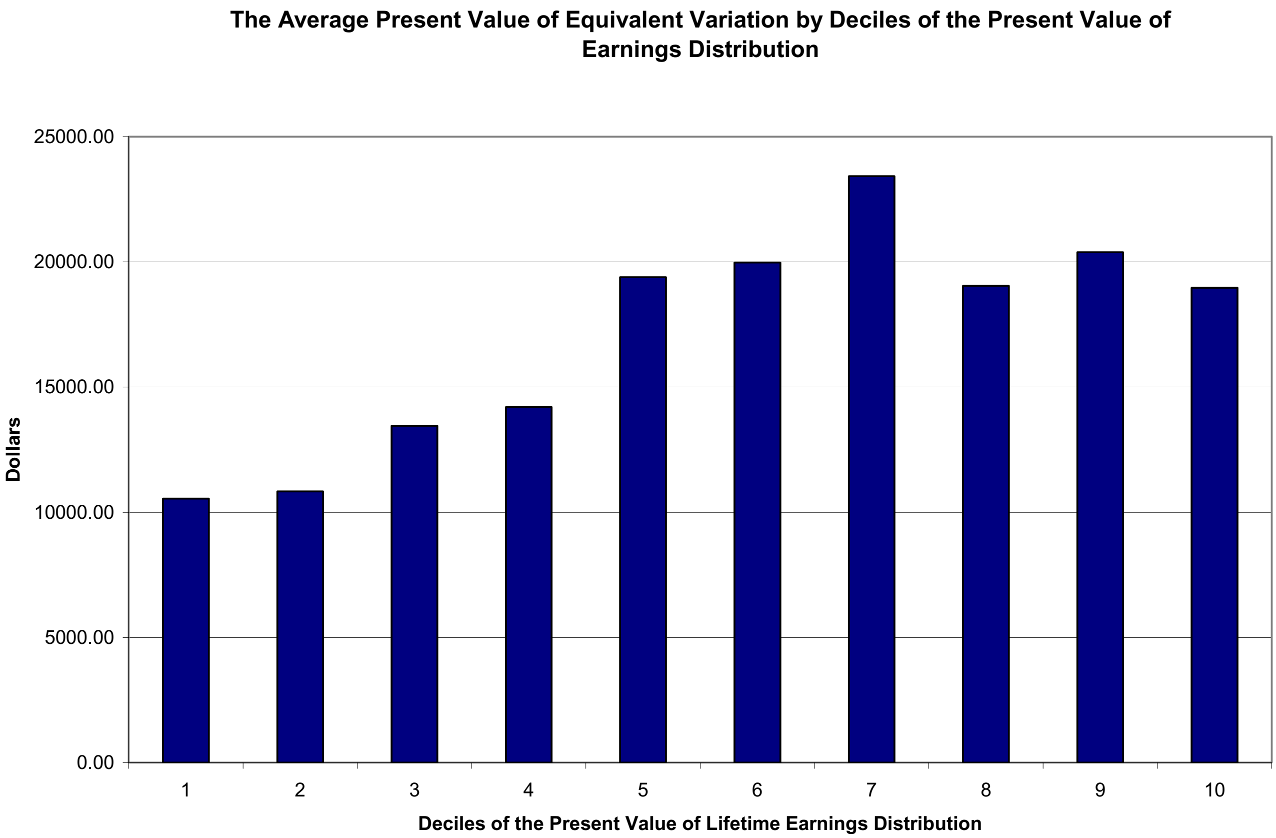
$$u\left(c_b^y + \lambda\right) + \beta u\left(c_b^o + \lambda\right) + \beta^2 \theta E\left[V_b\right] = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The present value of the equivalent variation Λ is defined as:

$$\Lambda = \lambda + \frac{\lambda}{1+r}$$

The table above shows the average present value of the equivalent variation by deciles of the present value of lifetime earnings. For example, agents in the top decile of present value of lifetime earnings must get roughly U$ 19,000.00 in the baseline economy to be indifferent to an economy with full-tuition subsidy.

Figure 30



Let c_b^y, c_b^o denote the baseline-economy consumption of a given agent when young and old, respectively. Let c_t^y, c_t^o denote the full–tuition-subsidy-economy consumption of the very same given agent when young and old, respectively. The ex-post lifetime utility of an agent in the baseline economy is:

$$V_b = u\left(c_b^y\right) + \beta u\left(c_b^o\right) + \beta^2 \theta E\left[V_b\right]$$

while in the economy with the full tuition subsidy is:

$$V_t = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The equivalent variation is the constant dollar amount λ that makes the agent indifferent between the baseline-economy and the economy with full-tuition subsidy:

$$u\left(c_b^y + \lambda\right) + \beta u\left(c_b^o + \lambda\right) + \beta^2 \theta E\left[V_b\right] = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The present value of the equivalent variation Λ is defined as:

$$\Lambda = \lambda + \frac{\lambda}{1+r}$$

The figure above shows the average present value of the equivalent variation by deciles of the present value of lifetime earnings. For example, agents in the top decile of present value of lifetime earnings must get roughly U\$ 19,000.00 in the baseline economy to be indifferent to an economy with full-tuition subsidy.

Table 11

The Average Present Value of Equivalent Variation by Ability Group

Ability Group	Average Present Value of Equivalent Variation
1	9646.01
2	7776.95
3	8476.42
4	13286.29
5	11640.72
6	13007.55
7	17545.01
8	23637.45
9	19283.89
10	18617.71
11	19213.51
12	19619.89
13	16849.19
14	17123.33
15	16223.17

Let c_b^y, c_b^o denote the baseline-economy consumption of a given agent when young and old, respectively. Let c_t^y, c_t^o denote the full–tuition-subsidy-economy consumption of the very same given agent when young and old, respectively. The ex-post lifetime utility of an agent in the baseline economy is:

$$V_b = u\left(c_b^y\right) + \beta u\left(c_b^o\right) + \beta^2 \theta E\left[V_b\right]$$

while in the economy with the full tuition subsidy is:

$$V_t = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The equivalent variation is the constant dollar amount λ that makes the agent indifferent between the baseline-economy and the economy with full-tuition subsidy:

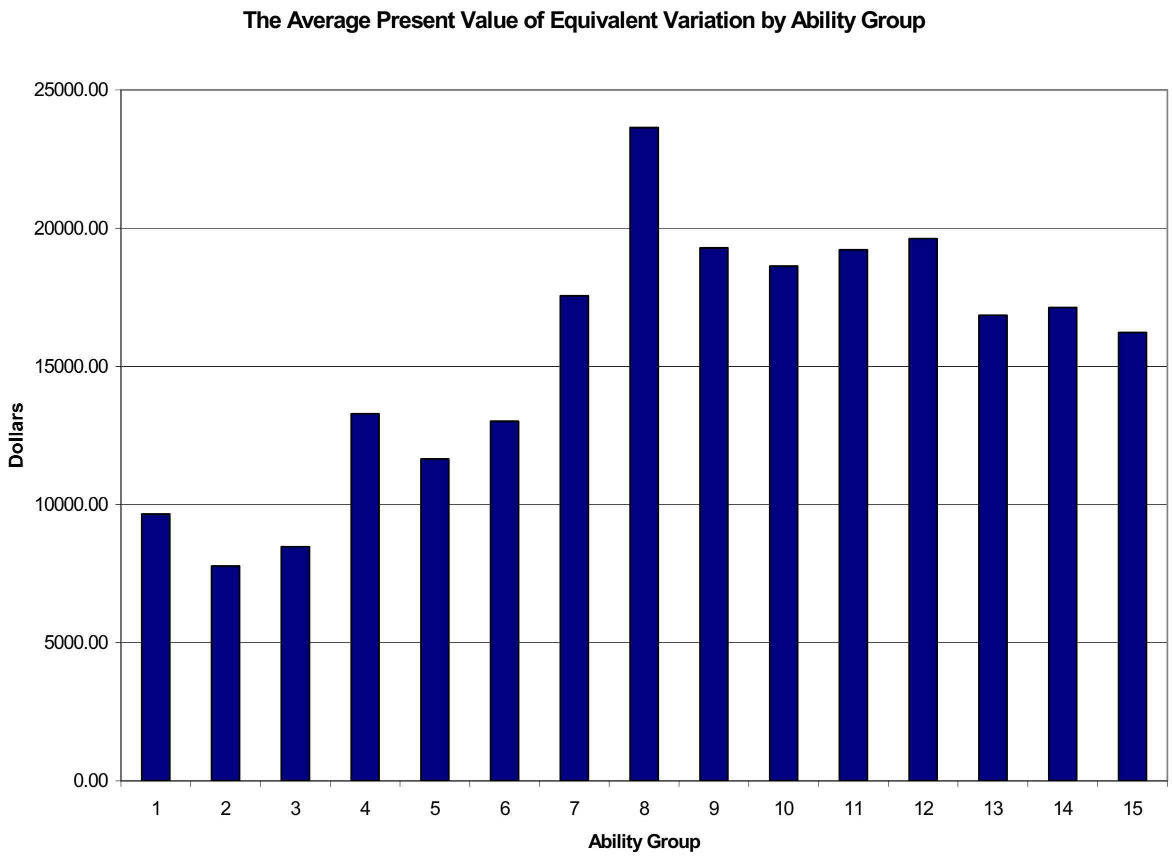
$$u\left(c_b^y + \lambda\right) + \beta u\left(c_b^o + \lambda\right) + \beta^2 \theta E\left[V_b\right] = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The present value of the equivalent variation Λ is defined as:

$$\Lambda = \lambda + \frac{\lambda}{1+r}$$

The table above shows the average present value of the equivalent variation by ability group. For example, agents in the top ability group must get roughly U\$ 16,000.00 in the baseline economy to be indifferent to an economy with full-tuition subsidy.

Figure 31



Let c_b^y, c_b^o denote the baseline-economy consumption of a given agent when young and old, respectively. Let c_t^y, c_t^o denote the full–tuition-subsidy-economy consumption of the very same given agent when young and old, respectively. The ex-post lifetime utility of an agent in the baseline economy is:

$$V_b = u\left(c_b^y\right) + \beta u\left(c_b^o\right) + \beta^2 \theta E\left[V_b\right]$$

while in the economy with the full tuition subsidy is:

$$V_t = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The equivalent variation is the constant dollar amount λ that makes the agent indifferent between the baseline-economy and the economy with full-tuition subsidy:

$$u\left(c_b^y + \lambda\right) + \beta u\left(c_b^o + \lambda\right) + \beta^2 \theta E\left[V_b\right] = u\left(c_t^y\right) + \beta u\left(c_t^o\right) + \beta^2 \theta E\left[V_t\right]$$

The present value of the equivalent variation Λ is defined as:

$$\Lambda = \lambda + \frac{\lambda}{1+r}$$

The table above shows the average present value of the equivalent variation by ability group. For example, agents in the top ability group must get roughly U$ 16,000.00 in the baseline economy to be indifferent to an economy with full-tuition subsidy.