Bargaining in Monetary Economies.*

Guillaume Rocheteau† Christopher Waller‡

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Abstract

Search models of monetary exchange have typically relied on Nash (1950) bargaining, or strategic games that yield an equivalent outcome, to determine the terms of trade. By considering alternative axiomatic bargaining solutions in a simple search model with divisible money, we show how this choice matters for important results such as the ability of the optimal monetary policy to generate an efficient allocation. We show that the quantities traded in bilateral matches are always inefficiently low under the Nash (1950) and Kalai-Smorodinsky (1975) solutions whereas under strongly monotonic solutions, such as the egalitarian solution (Luce and Raiffa, 1957; Kalai, 1977), the Friedman rule achieves the first-best allocation. We evaluate quantitatively the welfare cost of inflation under the different bargaining solutions, and we extend the model to allow for endogenous market composition.

Keywords : Money, Bargaining, Search, Inflation.

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†Federal Reserve Bank of Cleveland. E-mail: Guillaume.Rocheteau@clev.frb.org.

‡University of Notre Dame. E-Mail: cwaller@nd.edu.

1 Introduction

Bargaining theory is a cornerstone of the search-theoretic paradigm of decentralized markets (e.g., Osborne and Rubinstein, 1990). The usual approach to determine terms of trade in bilateral matches is to impose the generalized Nash solution, or to use a strategic bargaining game that yields a similar outcome. This paradigm has been applied to monetary economies by Shi (1995) and Trejos and Wright (1995).[1] By investigating alternative axiomatic bargaining solutions in a search model of money, we will show that the choice of the Nash solution is far from innocuous in models of monetary economies and that important results, such as the (in)efficiency of monetary equilibrium, hinge on this choice.

For example, a key result in Lagos and Wright (2004) – hereafter denoted LW – is that the Friedman rule cannot replicate the first-best allocation and the quantity of goods traded is inefficiently low unless buyers have all the bargaining power. This too-little-trade inefficiency has been attributed to a holdup problem in the bargaining, which suggests that it is a robust feature of models with bargaining. However, at the Friedman rule there is no sunk cost of acquiring money – agents can sell a unit of money in the next period for more goods than they gave up to acquire it and this exactly compensates them for discounting. Since this too-little-trade inefficiency is not generated by a holdup problem, it seems important to check whether it has something to do with the particular bargaining solution used in LW.

Given the concerns on how the bargaining solution may affect the efficiency of the intensive margin (the quantity traded in a match), one may also wonder how the bargaining solution affects the efficiency of the extensive margin (the number of matches). For example, some models have shown that deviations from the Friedman rule could be optimal when the composition of trades is endogenous (Shi, 1997). The basic idea is that inflation affects not only agents' choice of real balances but also their participation decisions. In some cases, the positive effects of inflation on the extensive margin outweigh the negative effects on the intensive margin. Although intuitive, this result is difficult to establish analytically, and it seems to be present only for a small set of parameter values (Rauch, 2000). Also, the trade-off between intensive and extensive margins

[1] Shi (1995) and Trejos and Wright (1995) use alternating-offer bargaining games and show the equivalence with the Nash solution. Coles and Wright (1998) have shown that this equivalence breaks down when one looks at dynamic equilibria. While many papers in the literature use the dictatorial solution where buyers have all the bargaining power, Rupert *et al.* (2001) have investigated the generalized Nash solution. One exception is Diamond (1984), who uses the egalitarian solution. Also, other papers have relied on alternative pricing mechanisms such as price posting, e.g.,Curtis and Wright (2004) and Rocheteau and Wright (2003).

depends on whether the intensive margin is efficient at the Friedman rule. Again, exploring how alternative bargaining solutions affect the extensive margin appears to be needed.

The objective of this paper is precisely to examine how alternative axiomatic bargaining solutions affect the intensive and extensive margins in a monetary search model. We demonstrate that the source of the too-little trade inefficiency in LW is the lack of monotonicity of the Nash solution, i.e., the fact that the Nash solution does not require agents' payoffs to be monotonic as the bargaining set expands. Since the monotonicity of the buyer's surplus matters for his choice of real balances, we carry out our analysis by comparing the Nash solution to two other standard axiomatic bargaining solutions based on notions of monotonicity — the Kalai-Smorodinsky (1975) solution based on *individual monotonicity* and the egalitarian solution (Kalai, 1977) based on *strong monotonicity*.[2]

We show that, at the Friedman rule, the quantity traded is always inefficiently low under the Kalai-Smorodinsky solution, whereas under the egalitarian solution the Friedman rule achieves the first best. We generalize these results by considering asymmetric bargaining solutions and show more generally that the monetary equilibrium is efficient at the Friedman rule under any strongly monotonic bargaining solution. With regards to the extensive margin, we are able to establish a simple condition under which a deviation from the Friedman rule is optimal when terms of trade are determined according to a strongly monotonic bargaining solution. Finally, we also investigate some quantitative implications of alternative symmetric bargaining solutions. We show that the bargaining solutions matter greatly for the welfare costs of small inflation (say 2%); but, for substantial inflation rates (10%), the welfare costs of inflation are of similar magnitude across bargaining solutions.

Before proceeding, we want to justify the use of axiomatic bargaining solutions over non-cooperative bargaining games in models of monetary exchange. The axiomatic approach of bargaining games consists of imposing properties that one thinks are desirable for a solution to a bargaining problem (e.g., Pareto optimality, individual rationality, monotonicity...) and then investigating the solution, or the class of solutions, that satisfy these properties. For our purpose, the axiomatic approach has the advantage of focusing directly on the properties of the solutions that matter for the (in)efficiency of monetary equilibrium. While one may want to have the explicit protocol of

[2]For surveys on axiomatic bargaining solutions, see Roth (1979), Peters (1992) and Thomson (1994). The different notions of monotonicity (strong, weak and individual) for bargaining solutions and their importance for economic applications are discussed in Chun and Thomson (1988).

the game that generates a particular solution in order to guarantee that this protocol does not conflict with the frictions that make monetary exchange essential, such as the lack of commitment, it is in general difficult to dismiss a trading mechanism that satisfies agents' individual rationality constraints.[3]

The paper is organized as follows. Section 2 describes the environment. In Section 3 we define the bargaining problem in a match and the axiomatic bargaining solutions we consider. Section 4 contains the characterization of steady-state monetary equilibria and results regarding efficiency. Section 5 generalizes the results by considering a larger class of bargaining solutions. Section 6 investigates how the bargaining solution matters for the effects of inflation on the extensive margin. Finally, in Section 7 we explore some quantitative implications of alternative bargaining solutions for the welfare cost of inflation. All proofs are in the Appendix.

2 The model

The basic environment is similar to the one in LW. Time is discrete and continues forever. Each period is divided into two subperiods, called *day* and *night*, in which different activities take place. There is a continuum of agents with measure one who are specialized in terms of the goods they produce and consume during the day, but all agents produce and consume the same good at night. During the day, trading is decentralized, and agents are matched bilaterally. Each agent meets someone who produces a good he wishes to consume with probability $\sigma \leq 1/2$ and meets someone who likes the good he produces with the same probability σ. For simplicity, we rule out double-coincidence-of-wants meetings. In Section 6, we will assume that agents choose to be buyers or sellers in the decentralized market in order to endogenize market composition (See Rocheteau and Wright, 2003, 2004). At night there is a centralized Walrasian market, where agents can trade goods and money. All goods are nonstorable and perishable.

Agents' preferences are represented by the following utility function

$$\mathcal{U}(q^b, q^s, x, y) = u(q^b) - c(q^s) + U(x) - y, \tag{1}$$

where q^b and q^s are the quantities consumed and produced during the day, and x and y are the quantities produced and consumed at night. We assume $U'(x) > 0$, $U''(x) < 0$, $u'(q) > 0$, $u''(q) < 0$, $u(0) = c(0) = c'(0) = 0$, $c'(q) > 0$, $c''(q) > 0$, and $c(\bar{q}) = u(\bar{q})$ for some $\bar{q} > 0$. Let q^* denote the

[3] The mechanism-design approach of monetary exchange considers as admissible all trading mechanisms that satisfy agents' individual rationality constraints (Kocherlakota, 1998; Wallace, 2001).

solution to $u'(q^*) = c'(q^*)$ and x^* the solution to $U'(x^*) = 1$; $q^* \in (0, \bar{q})$ exists by the previous assumptions, and we assume such an $x^* > 0$ also exists. All agents have the same discount factor $\beta \equiv (1+r)^{-1} \in (0,1)$.

Agents trade *anonymously*; hence, they cannot get credit in the decentralized market because they could default without fear of punishment. Let the quantity of fiat money per capita at the beginning of period t be $M_t > 0$ and assume $M_{t+1} = \gamma M_t$, where $\gamma \equiv 1 + \pi$ is constant and new money is injected by lump-sum transfers. The price of goods in terms of money in the centralized market is p_t. In the following, we will omit time indices and will replace $t+1$ by $+1$, $t+2$ by $+2$ and so on.

We restrict our attention to steady-state equilibria where the real value of aggregate money balances M/p is constant. This implies $p_{+1} = \gamma p$. Bellman's equation for an agent in the decentralized market holding $z = m/p$ units of real balances is

$$V(z) = \sigma \int \{u[q(z,\tilde{z})] + W[z - d(z,\tilde{z})]\} \, dF(\tilde{z})$$

$$+\sigma \int \{-c[q(\tilde{z},z)] + W[z + d(\tilde{z},z)]\} \, dF(\tilde{z}) + (1-2\sigma) W(z), \tag{2}$$

where $F(\tilde{z})$ is the distribution of real balances across agents, and $W(z)$ is the value function of the agent in the centralized market. Equation (2) has the following interpretation. An agent meets someone who produces a good he likes with probability σ. He consumes q units of goods and delivers d units of real balances to his trading partner, where q and d depend on his real balances z and the real balances $\tilde{z}$ of his partner in the match. With probability σ, the agent meets someone who likes his good. He is then the seller in the match. With probability $1-2\sigma$, no trade takes place. In the centralized market the problem of the agent is

$$W(z) = \max_{\hat{z},x,y} \{U(x) - y + \beta V(\hat{z})\} \tag{3}$$

$$\text{s.t.} \quad x + \gamma \hat{z} = y + z + T, \tag{4}$$

where T is the lump-sum transfer (expressed in general goods), and $\hat{z}$ is the real balances taken into the next day.[4] In the budget identity (4), we have used the fact that the relative price of real balances in the next period in terms of the general good is $p_{+1}/p = \gamma$. Substituting y from (4) into

[4]Note that $\hat{m}/p = (p_{+1}/p)(\hat{m}/p_{+1}) = \gamma \hat{z}$.

(3) we obtain[5]

$$W(z)=\max_{\hat{z},x}\left\{U(x)-x-(\gamma\hat{z}-T-z)+\beta V(\hat{z})\right\}. \tag{5}$$

From (5), $x=x^*$, the maximizing choice of $\hat{z}$ is independent of z, and W is linear in z with $W_z=1$. Substituting $V(\hat{z})$ by its expression given by (2), we can reformulate the buyer's problem as

$$\max_{\hat{z}}\left\{-i\hat{z}+\sigma\int\left\{u\left[q(\hat{z},\tilde{z})\right]-d\left(\hat{z},\tilde{z}\right)\right\}dF(\tilde{z})+\sigma\int\left\{d(\tilde{z},\hat{z})-c[q(\tilde{z},\hat{z})]\right\}dF(\tilde{z})\right\}, \tag{6}$$

where we have used the fact that $1+i=(1+\pi)(1+r)$, where i is the nominal interest rate.[6] According to (6), agents choose their real balances in order to maximize their expected surplus in the search market minus the opportunity cost of carrying real balances.

3 Bargaining

In this section, we describe the bargaining problem in a match between a buyer holding z units of real balances and a seller holding $\tilde{z}$ units of real balances, and we apply three standard bargaining solutions – the Nash, Kalai-Smorodinsky and egalitarian – to this problem.[7]

3.1 The bargaining problem

An agreement is a pair (q,d) where q is the amount of goods produced by the seller and d is the amount of real money transferred by the buyer to the seller. The monetary transfer is constrained by the real balances of the buyer and the seller, i.e., $-\tilde{z}\leq d\leq z$. The utility of the buyer if an agreement is reached is $u^b=u(q)+W(z-d)$, whereas the utility of the seller is $u^s=-c(q)+W(\tilde{z}+d)$. If no agreement is reached, the utility of the buyer is $u_0^b=W(z)$ and the utility of the seller is $u_0^s=W(\tilde{z})$. While u_0^b and u_0^s are taken as given within the bargaining problem, they are endogenously determined in the equilibrium of the economy.

From the linearity of $W(z)$, $u^b=u_0^b+u(q)-d$, and $u^s=u_0^s+d-c(q)$. Assume the buyer commits to spend no more than $\tau\leq z$ real balances. The set $\mathcal{S}(\tau)$ of feasible utility levels associated with this problem is

$$\mathcal{S}(\tau)=\left\{\left(u(q)-d+u_0^b,d-c(q)+u_0^s\right):\ d\in[-\tilde{z},\tau]\ \text{and}\ q\geq 0\right\}.$$

[5] We do not impose nonnegativity on y, but it is easy to choose fundamentals in order to guarantee that $y\geq 0$ in equilibrium. Also, for most of the analysis, one can simply assume $U(x)=x$.

[6] One could introduce bonds in the model and let agents trade these bonds in the centralized market but not take them into the decentralized market. The nominal interest rate would then be given by the Fisher equation in the text.

[7] These axiomatic bargaining solutions are reviewed in detail in Roth (1979) and Thomson (1994).

In Figure 1, we represent the bargaining game $\mathcal{S}(\tau)$ for three values of τ, i.e., $\tau_3 > \tau_2 > \tau_1$. Note that $\mathcal{S}(\tau) \subset \mathcal{S}(\tau')$ for all $\tau < \tau'$. (See the Appendix for details.) In the following, we will assume that money holdings are common knowledge in a match, and agents cannot commit to spend only a fraction of the money they bring into a match, i.e., $\tau = z$.

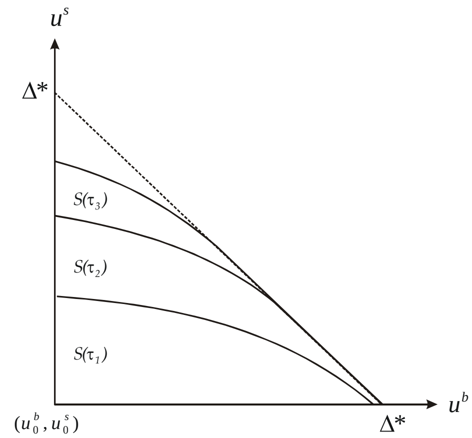

Figure 1: The bargaining set

Formally, a bargaining game is a pair $(\mathcal{S}, u_0)$ where $\mathcal{S}$ is the set of feasible utility levels and $u_0 = (u_0^b, u_0^s)$ is the disagreement outcome. A solution to the bargaining problem is a function μ that assigns a pair of utility levels to every bargaining game.

3.2 The Nash solution

The Nash (1950) solution, μ^N, is the unique solution that satisfies the axioms of Pareto optimality, scale invariance, symmetry and independence of irrelevant alternatives. It is given by

$$\mu^N(\mathcal{S}, u_0) = \arg \max_{(u^b, u^s) \in \mathcal{S}} (u^b - u_0^b)(u^s - u_0^s). \tag{7}$$

Since $u^b - u_0^b = u(q) - d$ and $u^s - u_0^s = -c(q) + d$, (q, d) satisfies

$$(q, d) = \arg \max_{q,d} [u(q) - d][-c(q) + d]$$

subject to $d \leq z$. The solution is $q = q^*$ and $d = [u(q^*) + c(q^*)]/2$ if $z \geq z^* \equiv [u(q^*) + c(q^*)]/2$, and $d = z$ and

$$z = z(q) \equiv \frac{u'(q)c(q) + c'(q)u(q)}{u'(q) + c'(q)}, \tag{8}$$

otherwise. Note that $u(q) - z(q) = \Theta(q)[u(q) - c(q)]$, where $\Theta(q) = u'(q)/[u'(q) + c'(q)]$. It is easy to show that $u(q) - z(q)$ is nonmonotonic in q and negatively sloped in the vicinity of $q = q^*$. This is illustrated in Figure 2, where the buyer's utility falls as the bargaining set expands.

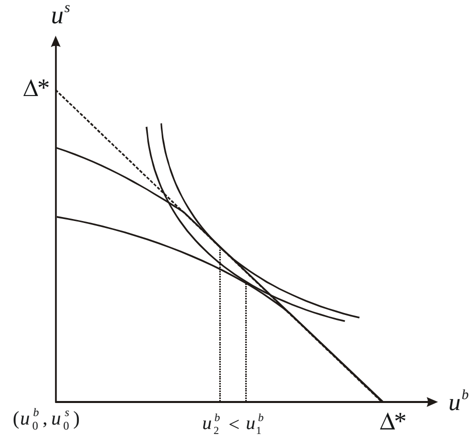

Figure 2: Nash solution.

3.3 The Kalai-Smorodinsky solution

The Kalai-Smorodinsky solution preserves all the axioms of the Nash solution except the independence of irrelevant alternatives, which is replaced by the axiom of *individual monotonicity*.[8] Consider two bargaining problems $(\mathcal{S}_1, u_0)$ and $(\mathcal{S}_2, u_0)$ such that $\mathcal{S}_1 \subset \mathcal{S}_2$, where the range of utility levels attainable by j is the same in $\mathcal{S}_1$ and $\mathcal{S}_2$. Individual monotonicity implies that the utility of player $i \neq j$ is higher in the second bargaining problem. In words, an expansion of the bargaining set $\mathcal{S}$ in a direction favorable to agent i always benefits i. The Kalai-Smorodinsky solution, μ^K, satisfies

$$\mu^K(\mathcal{S}, u_0) = u_0 + \lambda^K \left(\hat{u} - u_0\right), \tag{9}$$

where λ^K is the maximum value of λ such that $u_0 + \lambda\left(\hat{u} - u_0\right) \in \mathcal{S}$, and where $\hat{u} = \left(\hat{u}^b, \hat{u}^s\right) \geq u_0$ specifies the best alternative in $\mathcal{S}$ for each player. From (9),

$$\frac{u^s - u_0^s}{u^b - u_0^b} = \frac{\hat{u}^s - u_0^s}{\hat{u}^b - u_0^b}. \tag{10}$$

The best alternative of the buyer in $\mathcal{S}$ is $\hat{u}^b$ such that $\hat{u}^b = u_0^b + \max_{q,d}[u(q) - d]$, subject to $-c(q) + d = 0$ and $d \leq z$. Therefore,

$$\hat{u}^b = \begin{cases} u_0^b + u(q^*) - c(q^*) & \text{if} \quad z \geq c(q^*), \\ u_0^b + u\left[c^{-1}(z)\right] - z & \text{otherwise.} \end{cases}$$

[8] Like the Nash solution, the Kalai-Smorodinsky solution has strategic foundations. See Moulin (1984) and Peters (1992, ch 9).

Similarly, $\hat{u}^s$ satisfies

$$\hat{u}^s = \begin{cases} u_0^s + u(q^*) - c(q^*) & \text{if} \quad z \geq u(q^*), \\ u_0^s + z - c\left[u^{-1}(z)\right] & \text{otherwise.} \end{cases}$$

From (10), (q,d) satisfies $d = z$, and

$$\frac{u(q) - z}{u\left[c^{-1}(z)\right] - z} = \frac{-c(q) + z}{z - c\left[u^{-1}(z)\right]}, \quad \text{if} \;\; z \leq c(q^*), \tag{11}$$

$$\frac{u(q) - z}{u(q^*) - c(q^*)} = \frac{-c(q) + z}{z - c\left[u^{-1}(z)\right]}, \quad \text{if} \;\; z \in \left[c(q^*), z^*\right], \tag{12}$$

where $z^* \in [c(q^*), u(q^*)]$ is the value of z that satisfies (12) when $q = q^*$. If $z \in [z^*, u(q^*)]$, (q,d) satisfies $q = q^*$ and

$$\frac{u(q^*) - d}{u(q^*) - c(q^*)} = \frac{-c(q^*) + d}{z - c\left[u^{-1}(z)\right]}. \tag{13}$$

For all $z \in [0, z^*]$, (11)-(12) define an implicit relationship between q and z, i.e., $z = z(q)$. Despite the axiom of individual monotonicity, it can be checked that for our bargaining problem the buyer's surplus falls as q approaches q^* (see Figure 3).

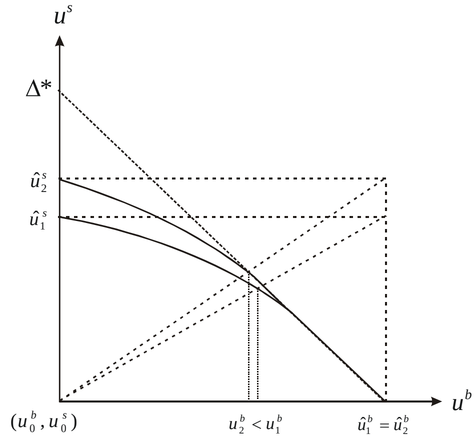

Figure 3: Kalai-Smorodinsky solution.

3.4 The egalitarian solution

The egalitarian solution (Luce and Raiffa, 1957; Kalai, 1977) imposes a stronger notion of monotonicity, the *strong monotonicity*, according to which no players are made worse-off if additional alternatives are made available to the players. Consider two bargaining problems $(\mathcal{S}_1, u_0)$ and $(\mathcal{S}_2, u_0)$ such that $\mathcal{S}_1 \subseteq \mathcal{S}_2$. Then, a solution μ is strongly monotonic iff $\mu(\mathcal{S}_1) \leq \mu(\mathcal{S}_2)$. Kalai (1977) showed that the unique solution that satisfies Pareto optimality, symmetry and strong monotonicity is the egalitarian solution.[9] In our context, the egalitarian solution implies

$$u^b - u^b_0 = u^s - u^s_0. \tag{14}$$

From (14), (q, d) satisfies $-c(q) + d = u(q) - d$, and $d = z$ if $q < q^*$. Therefore, $q = q^*$ and $d = [u(q^*) + c(q^*)]/2$ if $z \geq z^* \equiv [u(q^*) + c(q^*)]/2$, and $d = z$ with

$$z = z(q) \equiv \frac{c(q) + u(q)}{2}, \tag{15}$$

otherwise.

Interestingly, this bargaining solution is invariant under decomposition of the bargaining process into stages (Kalai, 1977). As an illustration, suppose the buyer has z units of money. Agents could first bargain over the quantity to produce in exchange for τ_1 units of money, and in a second step they would bargain over the quantity to produce in exchange for the $z - \tau_1$ units that are left. The status quo point in the second step would be the utility that players would reach if they would agree in the first step. Under the egalitarian solution, this procedure by steps is equivalent to a one-time bargaining (see Figure 4).

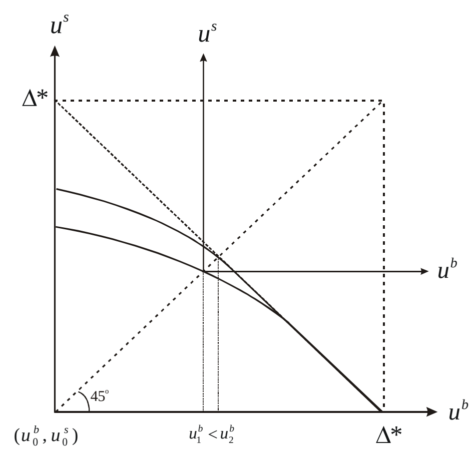

Figure 4: Proportional solution.

[9]In contrast to the Nash and Kalai-Smorodinsky solutions, the egalitarian solution is not scale invariant – it is invariant only under simultaneous rescaling of the utility functions of the two players with the same rescaling factor. Therefore, it allows for interpersonal comparisons of utility. See the discussions in Kalai (1977) and Kalai and Samet (1985). Also, if the set of feasible utilities $\mathcal{S}$ is not strongly comprehensive, the axiom of Pareto optimality has to be weakened or, alternatively, one can use the lexicographic egalitarian solution to maintain Pareto optimality. See Peters (1992) and Thomson (1994).

4 Equilibrium and efficiency

In the following, we derive results regarding the efficiency of monetary equilibrium for a class of bargaining solutions that encompasses the three solutions described in the previous section.

Assumption 1 *The bargaining solution has the following properties for the bargaining problem described in Section 3: (i) Terms of trade* (q, d) *are independent of the seller's real balances; (ii) There exists a threshold* z^* *for the buyer's real balances such that* $q = q^*$ *for all* $z \geq z^*$*; (iii)* d *is a nondecreasing function of* z*; (iv) For all* $z < z^*$*,* $z = z(q)$ where $z(.)$ is a continuous function.

Conditions (i)-(iv) are fairly unrestrictive and, in particular, are satisfied for the three bargaining solutions presented in Section 3 as well as the other bargaining solutions in Section 5. Condition (i) holds for all bargaining solutions since the set of incentive-feasible surpluses for the buyer and the seller is independent of the seller's real balances in the match. Obviously, the fact that terms of trade are independent of the seller's real balances hinges on the linearity of agents' payoffs.[10] Condition (ii) will be satisfied for any solution that is Pareto-efficient. Indeed, if $z > u(q^*)$ any Pareto-efficient outcome is such that $q = q^*$. Regarding condition (iii), Pareto efficiency requires that $d = z$ whenever $q < q^*$ so that d is indeed increasing in z. We will use condition (iv) to show the existence of monetary equilibrium.

Lemma 1 *Under Assumption 1, the agent's problem (6) can be reformulated as*

$$\max_{q \in [0, q^*]} \{-iz(q) + \sigma [u(q) - z(q)]\} \tag{16}$$

The maximization problem in (16) has a simple interpretation. The agent chooses the quantity q to trade in the decentralized market in order to maximize his expected surplus as a buyer minus the cost of holding real balances. If (16) has more than one solution, we restrict our attention to symmetric equilibria where all agents choose the same real balances.[11]

From (16), one can make the following key observation. At $i = 0$ there is no holdup problem because there is no sunk cost associated with acquiring money. So any inefficiencies at $i = 0$ are

[10] This condition would not hold if the trading mechanism is such that the bargaining solution in a match is contingent on the match type. For instance, one could use the generalized Nash solution in all matches but assume that the buyer's bargaining power depends on the seller's real balances. Also, this condition would not hold in a model with nondegenerate distribution of money balances since the seller's marginal value of money would depend on his real balances.

[11] There would be no conceptual difficulty in considering asymmetric equilibria. More generally, an equilibrium is a list (q_i), one for each agent, where each q_i is a solution to (16).

not the result of a holdup problem. However, for $i > 0$ there is a sunk cost – even though an agent can dispose of his money in the following centralized market, the value of money does not increase at the time rate of discount. Thus, the holdup problem is not eliminated by any of our bargaining solutions – rather it is the Friedman rule that eliminates it.

Definition 1 *A steady-state monetary equilibrium is a $q > 0$ solution to (16).*

Proposition 1 *If $u(q^*) - z(q^*) > 0$, then there exists an $\bar{\imath} > 0$ such that an equilibrium exists for all $i < \bar{\imath}$.*

We now investigate some implications of the choice of the bargaining solution for the efficiency of monetary equilibrium. The following proposition is a direct consequence of Lemma 1.

Proposition 2 *(i) Equilibrium at $i = 0$ is efficient iff q^* is the unique maximizer of $u(q) - z(q)$. (ii) If $u(q) - z(q)$ is (strictly) increasing, then $q = q^*$ is an (the) equilibrium at $i = 0$. (iii) Assuming $z(q)$ is differentiable, if $u'(q^*) < z'(q^*)$, then $q = q^*$ is not an equilibrium for any $i \geq 0$.*

If a bargaining solution is such that the buyer's payoff is monotonic in q, then the Friedman rule achieves the first-best allocation. Any strongly monotonic bargaining solution satisfies this requirement. In contrast, if the solution is nonmonotonic, and if the buyer's surplus decreases when q gets close to q^*, then the Friedman rule fails to achieve the efficient allocation.

Corollary 1 *For all $i \geq 0$, equilibria under the Nash and Kalai-Smorodinsky solutions are inefficient, $q < q^*$. Equilibria under the egalitarian solution are efficient iff $i = 0$.*

As noticed in LW, the quantity traded under Nash bargaining is inefficiently low even at the Friedman rule ($i = 0$). The reason for the inefficiently low q comes from the fact that the Nash solution is non-monotonic and the buyer's surplus $u(q) - z(q)$ reaches a maximum at $q < q^*$. A similar inefficiency occurs under Kalai-Smorodinsky bargaining. The reason is as follows. Whenever $z > c(q^*)$, an increase in the buyer's real balances leaves the buyer's maximum surplus unaffected, $\hat{u}^b - u_0^b = u(q^*) - c(q^*)$, whereas the maximum surplus of the seller increases (assuming $z < u(q^*)$). As a consequence, the seller's share of the match surplus increases. When q is close to q^*, this implies that the buyer's surplus falls.

Under the egalitarian solution, the Friedman rule achieves the first-best allocation. This is a consequence of the strong monotonicity axiom of the egalitarian solution. Since the buyer's surplus increases with his money holdings, and strictly increases if $z < z^*$, the buyer will invest up to z^* when $i = 0$.

Aside from efficiency arguments, monotonicity is important for another reason – it eliminates incentives for the buyer to hide his cash balances when meeting a seller. If the bargaining solution yields a nonmonotonic payoff for the buyer, then he has an incentive to hide some of his cash balances whenever it would cause his surplus to decline.

5 More on bargaining solutions

In this section, we give additional examples of bargaining solutions that are strongly monotonic and we show how the results of Section 4 can be generalized to asymmetric bargaining solutions.

5.1 Monotonic solutions

Thomson and Myerson (1980) have characterized the class of bargaining solutions, named the *monotone path solutions*, that satisfy Pareto optimality and strong monotonicity. Consider a normalized bargaining game where $u_0 = (0, 0)$. Given a strictly monotone path $\mathcal{P}$ in $\mathbb{R}^2_+$, i.e., a strictly increasing function from $\mathbb{R}^+$ into $\mathbb{R}_+$, the monotone path solution relative to $\mathcal{P}$ chooses the maximal point of $\mathcal{S}$ along $\mathcal{P}$. The egalitarian solution is the monotone path solution relative to the 45^o-line. From Proposition 2, the equilibrium at the Friedman rule is efficient under monotone path solutions.

One method to choose a monotone path in the context of a gradual bargaining problem, i.e., a family of bargaining problems that varies continuously with one parameter, has been proposed by Wiener and Winter (1998) and O'Neill et al. (2004). This is the so-called *gradual Nash* (or *ordinal*) *solution*. The idea, related to the negotiation by steps of Kalai (1977), is to envision the bargaining as a sequence of small steps, where in each step agents would use the Nash solution and would take as the status quo point the outcome of the previous step. (Despite this description, remember that this solution is axiomatic and not based on a strategic game.) In our context, one can think of the bargaining as a sequence of negotiations, one for each unit of money that the buyer holds. The gradual Nash solution corresponds then to a path of agreements that satisfies the following differential equation

$$\frac{du^s}{du^b} = \frac{\partial H(u^b, u^s, \tau)/\partial u^b}{\partial H(u^b, u^s, \tau)/\partial u^s}, \tag{17}$$

where $H(u^b, u^s, \tau) = 0$ is the equation of the Pareto frontier of $\mathcal{S}(\tau)$. According to (17), at each point of the agreement path, the ratio of the buyer's and the seller's marginal utility gains is the rate of substitution of their utility on the current efficient frontier.[12] The determination of the bargaining path is illustrated in Figure 5.

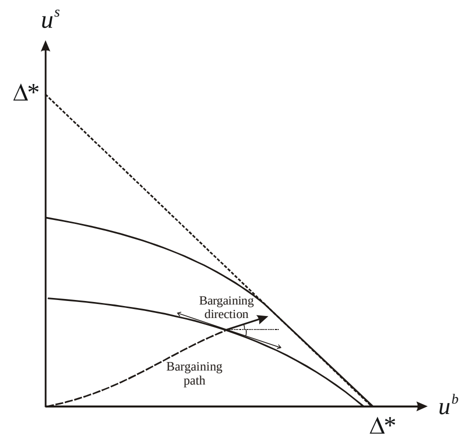

Figure 5: Gradual Nash solution

The gradual Nash solution applied to our problem implies that the change in the buyer's surplus along the bargaining path is $d(u^b - u^b_0)/dq = \Theta(q)\,[u'(q) - c'(q)]$, where $\Theta(q) = \frac{u'(q)}{u'(q)+c'(q)}$ (see the Appendix). So gradual Nash is essentially applying the Nash solution at the margin. The expression for real balances as a function of q is given by

$$z = z(q) = \int_0^q \frac{2c'(x)u'(x)}{u'(x) + c'(x)} dx. \tag{18}$$

Since $u'(q) - z'(q) > 0$, Proposition 2 implies that $q = q^*$ at $i = 0$.

5.2 Asymmetric solutions

The class of solutions that satisfy Pareto efficiency, scale invariance and independence of irrelevant alternatives are the so-called generalized Nash solutions. In our context, the generalized Nash solution satisfies

$$(q, d) = \arg\max\,[u(q) - d]^{\theta}\,[-c(q) + d]^{1-\theta}\,,$$

[12] This solution has several interesting properties. First, if one interprets the bargaining as a bargaining in different stages, the solution used in each stage that would be consistent with gradual Nash could be the Nash solution, the Kalai-Smorodinsky solution or any solution that is scale invariant. Second, the solution is ordinal in the sense of being covariant with monotonic transformations of each player's utility.

where $\theta \in [0, 1]$ is the buyer's bargaining weight. Then,

$$z(q) = \frac{\theta u'(q)c(q) + (1-\theta)c'(q)u(q)}{\theta u'(q) + (1-\theta)c'(q)}. \tag{19}$$

Unless $\theta = 1$, the quantity traded is always too low at $i = 0$. If $\theta = 1$, the buyer can extract the entire surplus of the match and the Friedman rule is optimal. The same logic applies to the asymmetric Kalai-Smorodinsky solution.

The generalization of the egalitarian solution is the proportional solution studied by Kalai (1977). In our context, proportional solutions satisfy $(1-\theta)[u(q) - d] = \theta[-c(q) + d]$. If $\theta = 1/2$, the proportional solution is the egalitarian one. Then,

$$z(q) = (1-\theta)u(q) + \theta c(q). \tag{20}$$

Thus, the buyer's surplus is $u(q) - z(q) = \theta[u(q) - c(q)]$ and, from Proposition 2, $q = q^*$ iff $i = 0$ for any $\theta > 0$.

Finally, the asymmetric gradual Nash solution (see Wiener and Winter, 1998) obeys

$$u'(q) - z'(q) = \Theta(q)[u'(q) - c'(q)],$$

where $\Theta(q) = \theta u'(q) / [\theta u'(q) + (1-\theta)c'(q)]$. This gives

$$z(q) = \int_0^q \frac{u'(x)c'(x)}{\theta u'(x) + (1-\theta)c'(x)} dx. \tag{21}$$

Since $u'(q) - z'(q) > 0$, $q = q^*$ at the Friedman rule for any $\theta > 0$.

6 Extensive margin effects

As shown by Shi (1997), search models of money can be extended to endogenize the frequency of trades. We examine the effect of inflation on the extensive margin under strongly monotonic bargaining solutions. As discussed before, the strong monotonicity property is appealing and it avoids problems related to agents' incentives to hide money. Also, unlike the Nash or Kalai-Smorodinsky solutions, strongly monotonic solutions yield simple analytical conditions as to when the composition of buyers and sellers, and therefore the number of trades, is efficient.

Following Rocheteau and Wright (2004), we extend the model of Section 2 to let each agent choose to be either a buyer or a seller in the decentralized market. Let n be the fraction of sellers.[13]

[13] Instead of assuming that agents can choose their types, one can follow Rocheteau and Wright (2004) and assume that agents need to specialize in a production technology. Either they produce an intermediate good traded in the decentralized market (in which case they are sellers) or they engage in home production that requires them to use the intermediate good traded in the decentralized market (in which case they are buyers).

Following Kiyotaki and Wright (1993), the matching probabilities of buyers and sellers are $\sigma^b = n$ and $\sigma^s = 1-n$, respectively. Substituting $V(z)$ by its expression given by (2) into (5), and using the fact that buyers do not produce in the decentralized market, the value of a buyer in the centralized market with z units of real balances satisfies

$$W^b(z) = U(x^*) - x^* + z + \max_{q\in[0,q^*]} \{-(\gamma-\beta)z(q) + \beta n[u(q) - z(q)]\} + \beta W^b(0). \tag{22}$$

Similarly, the value of being a seller with z units of real balances is given by

$$W^s(z) = U(x^*) - x^* + z + \beta \{(1-n)[z(q) - c(q)] + W^s(0)\}. \tag{23}$$

Since both $W^b(z)$ and $W^s(z)$ are linear in z, the choice of being a buyer or a seller is independent of z. In any active equilibrium, agents must be indifferent between being a seller or a buyer. Consequently, $W^b(z) = W^s(z)$ and, from (22) and (23), n satisfies

$$(1-n)[z(q) - c(q)] = n[u(q) - z(q)] - iz(q). \tag{24}$$

The left-hand side of (24) is the seller's expected surplus in the decentralized market, whereas the right hand-side is the buyer's expected surplus minus the cost of holding real balances. Solving for n we get

$$n = \frac{(1+i)z(q) - c(q)}{u(q) - c(q)}. \tag{25}$$

From (22), q solves

$$\max_{q\in[0,q^*]} \{-iz(q) + n[u(q) - z(q)]\}. \tag{26}$$

Definition 2 *A steady-state monetary equilibrium is a pair (q, n) such that q is solution to (26) and n satisfies (25).*

Assumption 2 *$u(q) - z(q)$ is maximum at $q = q^*$ and $u''(q^*) - z''(q^*) < 0$.*

Strongly monotonic bargaining solutions, such as the egalitarian or the gradual Nash solutions, satisfy the requirements in Assumption 2. Since $q = q^*$ at $i = 0$, the effects of a change in i in the neighborhood of $i = 0$ are given by

$$\begin{aligned} \frac{dq}{di} &= \frac{u'(q^*)}{n^*[u''(q^*) - z''(q^*)]} < 0, \\ \frac{dn}{di} &= \frac{z(q^*)}{u(q^*) - c(q^*)} > 0. \end{aligned}$$

Inflation has a direct effect by raising the cost of holding real balances. Therefore, buyers reduce their real balances and q falls. Also, n tends to increase.

Welfare is measured by the sum of the instantaneous utilities of buyers and sellers, i.e., $\mathcal{W} = n(1-n)[u(q) - c(q)]$. It is maximized for $q = q^*$ and $n = 0.5$. In the following proposition, we give a necessary and sufficient condition for a deviation from the Friedman rule to raise $\mathcal{W}$.

Proposition 3 *Under assumption 2, a deviation from the Friedman Rule is optimal iff* $\frac{u(q^*)-z(q^*)}{u(q^*)-c(q^*)} > 1/2$.

Proposition 3 provides an intuitive condition under which the Friedman rule is suboptimal. If the buyer's share in the total surplus of a match at $q = q^*$ is more than a half, then there are too many buyers in the market. In this case, a social planner would be willing to trade off efficiency on the intensive margin to improve the extensive margin by raising the number of sellers. Note that $\frac{u(q^*)-z(q^*)}{u(q^*)-c(q^*)} = 1/2$ is the Hosios (1990) condition for efficiency in search models.[14]

The following corollary reformulates Proposition 3 in the case where terms of trade in bilateral matches are determined according to an egalitarian bargaining solution that assigns a fraction $\theta \in (0,1)$ of the surplus to buyers – see (20) – or according to the gradual Nash solution where θ is the buyer's bargaining power – see (21).

Corollary 2 *(i) Under egalitarian bargaining, a deviation from the Friedman rule is optimal whenever* $\theta > 0.5$*. (ii) Under gradual Nash bargaining, a deviation from the Friedman rule is optimal whenever* $\theta > \bar{\theta}$*, where* $\bar{\theta} < 0.5$*.*

If $\theta > 0.5$, then inflation is welfare improving under both egalitarian and gradual Nash solutions. How does this compare to the Nash solution? To make the comparison, we need to specify functional forms and parameterize them. If we let $u(q) = q^{0.7}/0.7$ and $c(q) = q$, we find that the threshold $\bar{\theta}$ for the gradual Nash solution above which inflation is welfare improving is approximately 1/3, while under the Nash solution, a deviation is optimal for $\theta > 2/3$. Therefore, the Friedman rule is more likely to be optimal under the Nash solution than under the gradual Nash solution. This is so because under Nash bargaining the Friedman rule fails to achieve the efficient q, which makes a deviation from the Friedman rule more costly.

[14] The argument according to which a deviation from the Friedman rule could be optimal when the Hosios (1990) condition is violated has been spelled out by Berentsen, Rocheteau and Shi (2001) for a particular bargaining protocol. Also, despite some similarities, our results are in sharp contrast with those of Shi (1997), where an increase in the money growth rate raises the number of buyers. Therefore, a deviation from the Friedman rule in Shi's model is welfare improving when the number of buyers is too low.

7 Quantitative implications

The result derived in Corollary 1, namely, under monotonic bargaining solutions the Friedman rule generates the efficient q, has important implications for the welfare effects of inflation. In this section we quantify the welfare costs of inflation under the bargaining solutions presented in Section 3 as well as the gradual Nash solution.

Our quantitative experiments follow the methodology in LW, which is based on Lucas (2000). We calibrate the model in order to match money demand in the data. We set a period to a year and $\beta^{-1} = 1.03$. The functional forms are $U(x) = A \ln x$ so that $x^* = A$, $u(q) = q^{1-a}/(1-a)$ and $c(q) = q$. Finally, we set $\sigma = 0.5$. We choose (a, A) to match the money demand data as defined by $L = M/PY = L(i)$, where P is the nominal price level and Y real output. We measure i by the short-term commercial paper rate, Y by GDP, P by the GDP deflator, and M by $M1$, as in Lucas. We consider the period 1900-2000.

In the model, L is constructed as follows. Nominal output in the decentralized market is $0.5M$. Nominal output in the centralized market is px^*. Hence, $PY = 0.5M + px^*$. Using the fact that $x^* = A$, we have

$$L = \frac{M/p}{A + 0.5M/p}. \tag{27}$$

We measure the welfare cost of a π percent inflation by asking how much buyers' total consumption should be reduced to in order to have the same welfare at $i = (1+\pi)/\beta - 1$ and $i = 0$. Expected utility for an agent given i is measured by $\mathcal{W}_i$. Suppose we reduce i to 0 but also reduce the buyer's consumption of all goods by a factor Δ. Expected utility becomes

$$\mathcal{W}_0(\Delta) = \sigma[u(q_0\Delta) - c(q_0)] + U(x^*\Delta) - x^*,$$

where q_i is the equilibrium values for q given i. The welfare cost of inflation is the value of Δ that solves $\mathcal{W}_0(\Delta) = \mathcal{W}_i(1)$. In the following, we let $\bar{\Delta} = 100(1-\Delta)$; i.e. $\bar{\Delta}$ is the percentage they would give up to have the Friedman Rule instead of i.

We first calibrate the model assuming Nash bargaining. We find $(a, A) = (0.297, 1.91)$. Keeping (a, A) unchanged, we vary the bargaining solutions in order to evaluate the welfare cost associated with different values for i. The labels N, K, E and G indicate the Nash, Kalai-Smorodinsky, egalitarian and gradual Nash solutions, respectively.

	i	0	0.03	0.05	0.08	0.13
q	N	0.78	0.50	0.39	0.27	0.16
	K	0.78	0.50	0.39	0.27	0.16
	E	1	0.67	0.51	0.34	0.17
	G	1	0.68	0.54	0.39	0.25
$\bar{\Delta}$	N	0	0.85	1.52	2.49	3.83
	K	0	0.86	1.53	2.50	3.83
	E	0	0.4	0.96	2	3.79
	G	0	0.36	0.82	1.60	2.84

Table 1. Welfare cost of inflation under alternative bargaining solutions.

The key insight of this quantitative exercise is that the welfare costs of small deviations from the Friedman rule are much bigger for the nonmonotonic bargaining solutions than the monotonic ones. For instance, for $i = 0.03$, the welfare costs under Nash and Kalai-Smorodinsky are more than double those for the egalitarian and gradual Nash solutions. Interestingly, for large deviations, the welfare costs are very similar. For $i = 0.13$, the first three solutions are nearly identical, while the gradual Nash is about one percentage point lower. Nevertheless, the magnitudes are very large compared to the estimates of Lucas (2000). This reflects the fact that a holdup problem exists for all symmetric bargaining solutions, and the inefficiency associated with it is exacerbated as inflation increases.

In the following table, we recalibrate (a, A) for each bargaining solution and we compute the welfare cost associated with $i = 0.13$.

	a	A	$\Delta_{0.03}$	$\Delta_{0.05}$	$\Delta_{0.08}$	$\Delta_{0.13}$
N	0.297	1.91	0.85	1.52	2.49	3.83
K	0.298	1.90	0.86	1.54	2.51	3.85
E	0.292	2.41	0.34	0.81	1.67	3.14
G	0.271	2.21	0.34	0.77	1.48	2.56

Table 2. Welfare cost of inflation under alternative bargaining solutions.

The welfare cost of inflation is almost identical under Nash and Kalai-Smorodinsky bargaining solutions. The smallest welfare cost of inflation is obtained under gradual Nash bargaining. Nevertheless, the estimates are not that much different than in Table 1.

The lesson we take away from this quantitative exercise is the following. For small deviations from the Friedman rule, the bargaining solution that is used matters for measuring the welfare cost of inflation. This is due to the monotonicity properties of the bargaining solution near the

Friedman rule. However, for large deviations, all of our symmetric bargaining solutions give similar numbers, and these numbers show that the welfare costs of inflation are quite large.

8 Conclusion

Bargaining has become an integral part of monetary search models. Yet very little work has been done to understand how various bargaining solutions affect the qualitative and quantitative predictions of the models. In this paper we examined a series of bargaining solutions to do just that. Our qualitative analysis provides insight as to how nonmonotonic payoffs in some bargaining solutions affect the equilibrium of the model in important ways. By studying bargaining solutions other than the Nash solution, we were able to separate effects due to holdup problems from those occurring because of the nonmonotonicity of payoffs. This had not been done before and, as a result, it was not well understood why the Friedman rule could not replicate the first best allocation.

On the quantitative side, we showed that monotonic bargaining solutions are associated with lower welfare costs of inflation near the Friedman rule than nonmonotonic bargaining solutions. However, the costs are very similar for inflation rates sufficiently far away from it. We also showed how alternative bargaining solutions affect the extensive margin of trading and that deviations from the Friedman rule may be optimal over a wide range of parameter values.

Appendix

A1. Pareto frontier of $\mathcal{S}$.

The equation for $\bar{\mathcal{S}}$ is derived from the program $u^b = \max_{q,d}\left[u(q) - d\right] + u_0^b$ s.t. $-c(q) + d \geq u^s - u_0^s$ and $d \leq z$ for some u^s. Similarly

$$u^s - u_0^s = \begin{cases} u(q^*) - c(q^*) - (u^b - u_0^b) & \text{if } u^s - u_0^s \leq z - c(q^*) \\ z - c\left[u^{-1}(u^b - u_0^b + z)\right] & \text{otherwise} \end{cases} \tag{28}$$

Therefore, $d^2u^s/(du^b)^2 = 0$ if $u^s \leq z - c(q^*)$ and $d^2u^s/(du^b)^2 < 0$ otherwise.

A2. Proof of Lemma 1

Since (q, d) only depends on the real balances of the buyer in the match, we can omit the dependence on sellers' real balances and (6) yields

$$\max_{\hat{z}}\left\{-i\hat{z} + \sigma\left\{u\left[q(\hat{z})\right] - d\left(\hat{z}\right)\right\}\right\}.$$

Furthermore, $q(z) = q^*$ and $d(z) \geq d(z^*)$ for all $z \geq z^*$. Therefore, $z \leq z^*$ for all $i > 0$. Finally, since there is a one-to-one relationship between q and z, it is equivalent to express the agent's problem as a choice of q.

A3. Proof of Proposition 1

The function $-iz(q)+\sigma\left[u(q) - z(q)\right]$ is continuous and maximized over the compact set $[0, q^*]$, so a solution exists. At $i = 0$, the solution to (16) is strictly positive since $\max_{q\in[0,q^*]}\left\{\sigma\left[u(q) - z(q)\right]\right\}$ $\geq u(q^*) - z(q^*) > 0$. From the Theorem of the Maximum, $\max_{q\in[0,q^*]}\left\{-iz(q) + \sigma\left[u(q) - z(q)\right]\right\}$ varies continuously with i. Denote $\bar{\imath} = \sup_i \max_{q\in[0,q^*]}\left\{-iz(q) + \sigma\left[u(q) - z(q)\right]\right\} > 0$. Then $\bar{\imath} > 0$, and for all $i < \bar{\imath}$ there exists a $q > 0$ solution to (16).

A4. Proof of Proposition 2

Direct from (16).

A5. Proof of Corollary 1

From Proposition 2, it is sufficient to show that $u'(q^*) < z'(q^*)$ for the Nash and Kalai-Smorodinsky solutions. For the Nash solution, $z'(q^*) = \Theta'(q^*)[u(q^*) - c(q^*)] + u'(q^*) < u'(q^*)$. For the Kalai-Smorodinsky solution,

$$\frac{dq}{dz} = \frac{\hat{u}^s - u^b\left(1 - c'[u^{-1}(z)]/u'[u^{-1}(z)]\right) + \hat{u}^b}{u'(q)\hat{u}^s + c'(q)\hat{u}^b}, \qquad \text{if } z \in [c(q^*), z^*].$$

Therefore, $z'(q^*) = u'(q^*)\left(\hat{u}^s + \hat{u}^b\right) / \left[\hat{u}^s - u^b\left(1 - c'[u^{-1}(z)]/u'[u^{-1}(z)]\right) + \hat{u}^b\right] < u'(q^*)$. Consider next the egalitarian solution. The first-order condition for q gives

$$\frac{i}{\sigma} = \frac{u'(q) - c'(q)}{c'(q) + u'(q)}. \tag{29}$$

From (29), $q = q^*$ iff $i = 0$.

A6. Gradual Nash bargaining solution.

To apply the gradual Nash solution to our problem, from (28), the equation for the Pareto frontier is given by

$$H(u^b, u^s, \tau) = \begin{cases} (u^s - u_0^s) + (u^b - u_0^b) - [u(q^*) - c(q^*)] & \text{if } \tau \geq u^s - u_0^s + c(q^*) \\ u^s - u_0^s + c\left[u^{-1}(u^b - u_0^b + \tau)\right] - \tau & \text{otherwise.} \end{cases} \tag{30}$$

If $\tau \geq u^s - u_0^s + c(q^*)$ then $q = q^*$ so that an increase in τ does not allow for mutual gains. Therefore, $du^s/d\tau = du^b/d\tau = 0$. If $\tau < u^s - u_0^s + c(q^*)$ then $q < q^*$ and $d = \tau$ and (17) and (30) imply

$$\frac{du^s}{du^b} = \frac{c'(q)}{u'(q)}. \tag{31}$$

Differentiating $u^b = u(q) - \tau + u_0^b$ and $u^s = \tau - c(q) + u_0^s$, one can rewrite (31) as

$$\frac{d\tau}{dq} = \frac{2c'(q)u'(q)}{u'(q) + c'(q)} \quad \text{for all } q < q^*. \tag{32}$$

Integrating (32), and using the fact that $q = 0$ at $z = 0$, one obtains (18).

A7. Proof of Proposition 3

Since $q = q^*$ at $i = 0$, the welfare effect of a deviation from the Friedman rule is given by

$$\begin{aligned} \frac{d\mathcal{W}}{di} &= \frac{dn}{di}\left[u(q^*) - c(q^*)\right](1 - 2n^*) \\ &= z(q^*)(1 - 2n^*). \end{aligned}$$

From (25), $n^* = \frac{z(q^*) - c(q)}{u(q) - c(q)}$. Therefore, $\frac{d\mathcal{W}}{di} > 0$ iff $\frac{z(q^*) - c(q^*)}{u(q^*) - c(q^*)} < 1/2$ or, equivalently, $\frac{u(q^*) - z(q^*)}{u(q^*) - c(q^*)} > 1/2$.

A8. Proof of Corollary 2

Under egalitarian bargaining, $\frac{u(q^*)-z(q^*)}{u(q)-c(q^*)} = \theta$. Under gradual Nash bargaining,

$$\frac{u(q^*) - z(q^*)}{u(q^*) - c(q^*)} = \frac{\int_0^{q^*} \Theta(q)[u'(q) - c'(q)]dq}{u(q^*) - c(q^*)},$$

where $\Theta(q) = \theta u'(q)/[\theta u'(q) + (1-\theta)c'(q)] > \theta$ for all $q < q^*$. Therefore, $\frac{u(q^*)-z(q^*)}{u(q^*)-c(q^*)} > \theta$. Let $\bar{\theta}$ be the value of θ such that $\frac{u(q^*)-z(q^*)}{u(q^*)-c(q^*)} = 1/2$. Then, $\bar{\theta} < 0.5$.

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