Commitment Problems in the Political Economy of States and Mafias*

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PRELIMINARY

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Abstract

1 Introduction

The enforcement of contractual arrangements is generally regarded as one of the essential public goods provided by governments. However, in many societies, governmental weakness and corruption create a situation in which extra-governmental organizations, such as Mafias, compete with governments to provide contract enforcement and revenue protection

*We have benefited from comments from Scott Ashworth and Dimitri Landa.

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(Gamebetta 1994, Vareses 2000). The emergence of a strong Mafia on which the smooth functioning of the economy depends can introduce important forms of inefficiency and corruption into society. Understanding why such Mafias emerge, the extent to which voters and governments can prevent the development of a corrupt economy, and how the possibility of extortion affects economic performance is essential to the analysis of the political economy of transitions (Hay and Shleifer 1998).

We present a model of the relationship between the three key types of players in transition economies: governments, mafias, and two economically productive agents (Firms). The mafia is understood as an alternative provider of contract enforcement and revenue protection, rather than simply as an extortionist or thief. The government is modeled as having certain law enforcement responsibilities that it cannot fully avoid. However, the government can act corruptly by misappropriating tax revenues intended for law enforcement. The Firms endogenously choose the level of taxation, whether or not to hire the Mafia, and whether to reelect the government.

The Firms face a commitment problem. In each period, they must each choose whether to depend on the government for contract enforcement or whether to hire the Mafia for a fee. The commitment problem arises because, if only one Firm hires the Mafia, that Firm may be able to use the Mafia to extort money from the other Firm. If both Firms hire the Mafia, however, this cannot happen. The Firms prefer for neither to hire the Mafia than for both to hire the Mafia, in order to avoid paying fees, but they do not trust one another. It is here that the government comes in.

The Firms, acting as voters, endogenously choose how well to fund the government through taxation. They are willing to pay taxes because law enforcement can solve the commitment problem. Government law enforcement can potentially make illegal activity so costly that it drives the Mafia out of business. Even when the Mafia is entirely eliminated, law enforcement can reduce the fess the Mafia is able to demand. Of course, the weakening of the Mafia comes at a price to the Firms—taxation. Moreover, the Firms also have to worry about government corruption—the government may expropriate resources rather than spend

them on law enforcement. The Firms use electoral incentives to try to solve this moral hazard problem.

These trade-offs drive several key results of our model. First, both the extent of government corruption and the probability of the government's success in open confrontation with the mafia are non-monotonic in tax revenue. For low revenue levels, the electoral incentives are sufficient to insure that the government invests all of the revenue into law enforcement activities. Hence, for low levels of taxation, increasing taxation results in higher levels of law enforcement, a higher probability of government success against the mafia, and a smaller proportion of the firms' after-tax profits going to the mafia. However, as government revenue grows, the temptation to be corrupt also grows, and (if the benefits of office remain the same) electoral incentives are no longer adequate to prevent government corruption. The government skims a larger and larger proportion of revenues as taxation grows. For intermediate levels of taxation, the level of investment in law enforcement and the probability of government success against the mafia plateau, but for very high rates of taxation, government corruption becomes so extensive that investment into law enforcement, in absolute terms, decreases, and hence the government's ability to defeat the mafia actually decreases as well.

Ironically, the proximate cause of the decrease (in absolute terms) in law enforcement is the reduced presence of the mafia. While the amount that the mafia is able to demand from the firms is decreasing the level of law enforcement, it remains ubiquitous at low and intermediate levels of taxation. But for sufficiently high levels of taxation and sufficiently punitive measures by the government, the mafia becomes more rare, choosing on some occasions to exit the market rather than to risk a confrontation with the government. Because the mafia is less prevalent, the government has less occasion to use its law enforcement apparatus, and hence reduces its investment into it.

The model also provides insights into the role pro-active law enforcement efforts, in which the government attempts to monitor mafia activities and intervene independently, rather than simply respond to complaints. We show that the latter, purely passive, approach to

law enforcement results in the government being displaced entirely by the mafia. In an extension of the model, we consider the possibility of collusion between the mafia and the government.

2 The Extant Literature

To be added.

3 The Setup

There are four players: two Firms, the Mafia, and a Government official.[1] The sequence of play is as follows. At the beginning of the game, the firms have a contractual relationship, the total value of which is normalized to 1. Nature chooses the division of the benefits of the contract, and determines which Firm (called "Firm 1" or F1) gets proportion $\alpha \in \left(0, \frac{1}{2}\right)$ of those benefits, and which Firm ("Firm 2" or F2) gets $(1 - \alpha)$. Both the Firms and the Mafia observe Nature's selections, but the government does not. Next, the Firms choose the tax rate $\tau \in [0, 1]$. The tax rate is determined through a weighted average of the two firms preferred tax rates, with Firm 1 receiving weight q and Firm 2 the complementary weight. After the Firms choose the tax rate, the Government official chooses a proportion of the tax revenue collected to commit to law enforcement, $\lambda \in [0, 1]$. Neither the Firms nor the Mafia observe λ. Next, the Mafia sets the fees ($\boldsymbol{\phi} = (\phi_1, \phi_2)$) that each Firm must pay for its services and makes a take-it-or-leave-it offer to each Firm of the corresponding fee. Notice that we allow the Mafia to price discriminate between the two Firms. Following these offers, Firms simultaneously choose whether to hire the Mafia. When a Firm hires the Mafia, it pays the fee regardless of outcome.

[1] As will be shown below, because Firms' equilibrium choices are identical, we can think of the present model as being one with any number of randomly matched Firms with the equilibrium we solve for interpreted as the symmetric equilibrium of the corresponding game.

Let μ_{Fi} be the probability that $Fi \in \{F1, F2\}$ hires the Mafia. If one Firm does not hire the Mafia while the other does, the former Firm appeals to the Government, which then challenges the Mafia. If both Firms accept the Mafia's offers, the Government official must *choose* whether to challenge the Mafia or not. Let γ be probability that the Government challenges the Mafia, given that both Firms hire the Mafia. The idea underlying these assumptions is that, if just one Firm hires the Mafia, the Government is obliged to attempt to provide that Firm with protection from extortion. However, if both Firms hire the Mafia, then the Government has the option of whether or not to engage in a fight against the Mafia's protection racket.

The probability that the Government defeats the Mafia when they are in conflict is given by a function of the Government's level of investment in law enforcement ($\lambda\tau$), $f : [0, 1] \to [0, 1]$. We assume that $f(\cdot)$ is increasing, concave, and satisfies $\lim_{x\to 0} f'(x) = \infty$.

If there is a conflict between the Mafia and Government, and the Mafia loses, the Mafia bears a cost k, which we interpret as the punishment imposed by the government.[2] The winner of this conflict determines the division of post-tax benefits $(1-\tau)$ between the Firms. If the Government wins, it imposes the outcome that corresponds to the actual realization of the contract, i.e., $(\alpha(1-\tau), (1-\alpha)(1-\tau))$. If the Mafia wins, it also imposes the actual realization if it was hired by both Firms. However, if only one Firm hired the Mafia, and the Mafia wins, it gives the entire surplus $(1-\tau)$ to the Firm that hired it. The idea, here, is that Government enforcement is essentially fair, reflecting the agreed upon contract. A Firm hires the Mafia to try to extort more than its rightful share from the other Firm. If both Firms hire the Mafia, this has an offsetting effect—neither Firm can extort from the other.[3] If only one Firm hires the Mafia, and this Mafia is able to prevail over Government

[2]We assume that the Mafia's activities, including fighting the government, are financed by the fees paid by the Firms, and that the Mafia's claims of its ability corresponding to the informational assumptions of the model are "credible" - guaranteed by some background repeated interaction that generates the Mafia's reputation.

[3]We justify this assumption in more detail later in the analysis.

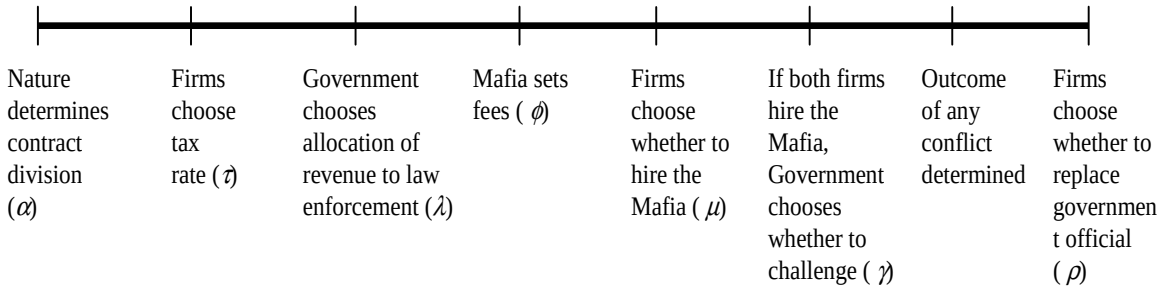

Figure 1: Timeline of the game

law enforcement, then extortion occurs.

After the outcome of any conflict is determined and benefits are divided, Firms choose whether or not to reelect the government, which is understood to compete against an otherwise identical challenger. The probability of reelection $\boldsymbol{\rho}$ can be represented by a finite-dimensional vector specifying the probability of reelection in each observationally distinct (for the Firms) situation. The government receives a payoff, R, if it is reelected and 0 if it is not reelected. The timeline of the game is summarized in Figure 1.

3.1 Payoffs

The Mafia's utility is its net revenue (fees collected minus costs imposed in defeat). The Government's utility is any tax revenue not spent ($(1-\lambda^t)\tau$) plus any electoral payoff. Each Firm's utility is the benefit it realizes from the contract (as enforced) net of taxes, minus the fee paid.

4 Equilibrium

Our solution concept is Perfect Bayesian Equilibrium. Additionally, we require that, when a player is capable of credibly committing to any of a set of behavioral strategies, she play a behavioral strategy that maximizes her *ex ante* expected utility at the earliest point in the history of the game at which such a credible commitment is possible. This requirement

restricts our attention to those equilibria in which the Firms induce, via their choice of a reelection rule ($\boldsymbol{\rho}$), that behavior on the part of the Mafia and Government that the Firms prefer. Because Firms are indifferent between reelecting the Government official or not, such a choice of $\boldsymbol{\rho}$ is credible.

4.1 Reelection

Firms' reelection decisions do not affect their future utilities, and so they are indifferent between reelecting the Government and not doing so. Although the Firms' reelection rule does not affect future payoffs, it does impact first-period payoffs by altering the first-period incentives of the Government and Mafia. Since, at the time of the reelection decision the Firms are indifferent, any reelection rule is credible. We focus, as discussed above, on reelection rules that maximize the Firms' expected utilities. In particular, we assume that Firms behave in a manner that induces "good" behavior on the part of Government. There are four observably distinct situations under which the Firms must decide whether or not to reelect:

1. There is conflict between the Mafia and Government and the Mafia wins.
2. There is conflict between the Mafia and Government and the Government wins.
3. Both Firms hire the Mafia and there is no conflict.
4. Neither Firm hires the Mafia and there is no conflict.

Let M be the event that the Mafia wins and G be the event that the Government, given that there is conflict between the Mafia and Government. Let NG be the event that no conflict between the Mafia and Government occurs after neither Firm has hired the Mafia. Let NM be the event that no conflict between the Mafia and Government occurs after the Firms have hired the Mafia. The set $\{M, G, NM, NG\}$ can be thought of as the range of the outcome function, whose arguments include $\boldsymbol{\mu}, \gamma, \lambda$, and τ; to simplify notation, we suppress the functional representation below.

Let ρ_M, ρ_G, ρ_{NM}, and ρ_{NG} be the probabilities of reelection that correspond to each of the four outcomes enumerated above, respectively, where $\boldsymbol{\rho} = (\rho_M, \rho_G, \rho_{NM}, \rho_{NG}) \in [0,1]^4$. Given that the Firms are indifferent over $\boldsymbol{\rho}$ at the time of the election, their optimal choice is induced by their preferences over the effects of that choice on other players' behavior. Accordingly, we return to the determination of the reelection rule after deriving the Mafia's and the Government's best responses to it.

4.2 Will the Government Challenge?

As noted earlier, the Government makes two choices: the level of investment into law enforcement (λ) and the probability of challenging the Mafia when both Firms hire the Mafia (γ). Somewhat surprisingly, the Firms' induced preferences over γ at the time of the Government's action are such that, once both Firms have paid the Mafia, they are expectationally indifferent between the Government challenging with certainty ($\gamma = 1$) and the Government not challenging ($\gamma = 0$). This does not, of course, mean that Firms are indifferent over the Government's strategy. Rather, Firms care only to the extent that Government behavior impacts the fees the Mafia charges. This intuition is formalized in the following lemma which is instrumental in solving for equilibrium behavior.

Lemma 1 *The Firms' preferences over Government action γ are completely induced by the effects of that action on the Mafia's choices.*

We can, thus, think of the Government as choosing the probability of challenging the Mafia (γ) to maximize the probability of reelection, given the level of investment in law enforcement ($\lambda\tau$) and the reelection rule ($\boldsymbol{\rho}$). With this in mind, we can characterize the Government's best-response correspondence with respect to its choice of γ. The Government's expected utility from challenging with certainty is:

$$E[u_G(\gamma = 1, \boldsymbol{\rho}^*, \lambda | \boldsymbol{\mu} = (1,1))] = f(\lambda\tau)\rho_G R + (1 - f(\lambda\tau))\rho_M R.$$

The Government's expected utility from never challenging is:

$$E[u_G(\gamma = 0, \boldsymbol{\rho}^*, \lambda | \boldsymbol{\mu} = (1,1))] = \rho_{NM} R.$$

Comparing these, we find that the Government's best response correspondence is

$$\gamma^*(\lambda, \boldsymbol{\rho}; \cdot) = \begin{cases} 1 & \text{if } \rho_M(1 - f(\lambda\tau)) + \rho_G f(\lambda\tau) > \rho_{NM} \\ \gamma' \in [0,1] & \text{if } \rho_M(1 - f(\lambda\tau)) + \rho_G f(\lambda\tau) = \rho_{NM} \\ 0 & \text{if } \rho_M(1 - f(\lambda\tau) + \rho_G f(\lambda\tau) < \rho_{NM}. \end{cases} \quad (1)$$

From this condition it is clear that the Firms can always choose a ρ_{NM} such that the government will challenge.

A Benchmark

An instructive question, within this context, is whether it is important that the Government have the ability to challenge the Mafia even when neither Firm solicits the Government's help. What, for instance, would happen if the Government could only engage in conflict with the Mafia if appealed to by one of the Firms? This would be equivalent to restricting γ to be equal to 0. It turns out that if the Government does not have the authority to challenge the Mafia on its own, then regardless of tax policy or investment in law enforcement, the Mafia dominates the economy. The combination of the Firms' ability to appeal to the Government for protection against the Mafia and the arbitrarily stiff penalty that the Government can impose on the Mafia are never sufficient to induce the Firms to finance the Government if the Government cannot also be expected to challenge the Mafia when the Firms do not appeal to the Government directly.

Proposition 1 *If the government cannot challenge the Mafia unless appealed to by one of the Firms (that is, γ is restricted to be 0), the game has a unique equilibrium in which the Mafia completely replaces the Government as the provider of enforcement services and the Firms choose not to fund the government at all ($\tau = 0$).*

Proof. See appendix. ■

As we will see in the remainder of the paper, allowing the Government to challenge the Mafia, unsolicited, radically alters players' equilibrium play.

4.3 Firms' Choice of Whether to Hire the Mafia

Consider now the Firms' (possibly mixed) choice of whether to hire the Mafia ($\boldsymbol{\mu}$). We use two facts in deriving the Firms expected utilities over their choice of whether to hire the Mafia. First, as demonstrated in Lemma 1, if both Firms hire the Mafia, their expected utility from the contract is the same whether or not the Government challenges the Mafia. Second, if both Firms hire the Mafia they get the same contract division that the Government would enforce. This second fact may seem like a strong symmetry assumption. To see why it is a reasonable reduced form, suppose that the Mafia favored one Firm or the other in its enforcement of the contract when hired by both Firms. In this case, the disadvantaged Firm would have an incentive to appeal to the Government rather than hire the Mafia. To counter-act this incentive, the Mafia would have to lower the fee it charged the disadvantaged Firm or lose that Firm as a customer and be forced into conflict with the Government. The Mafia's fee maximizing strategy, then, is to treat the two Firms equally if hired by both.

We can write F1's expected utility from hiring the Mafia with certainty as:

$$E[u_1(\mu_1 = 1, \mu_2)] = \mu_2\alpha(1-\tau) + (1-\mu_2)[(1-f(\lambda\tau))(1-\tau) + f(\lambda\tau)\alpha(1-\tau)] - \phi_1.$$

F1's expected utility from appealing to the Government with certainty is:

$$E[u_1(\mu_1 = 0, \mu_2)] = \mu_2[(1-f(\lambda\tau)) \times 0 + f(\lambda\tau)\alpha(1-\tau)] + (1-\mu_2)\alpha(1-\tau).$$

Similarly, we can write for F2:

$$E[u_2(\mu_1, \mu_2 = 1)] = \mu_1(1-\alpha)(1-\tau) + (1-\mu_1)[(1-f(\lambda\tau))(1-\tau) + f(\lambda\tau)(1-\alpha)(1-\tau)] - \phi_2,$$

and

$$E[u_2(\mu_1, \mu_2 = 0)] = \mu_1[(1-f(\lambda\tau)) \times 0 + f(\lambda\tau)(1-\alpha)(1-\tau)] + (1-\mu_1)(1-\alpha)(1-\tau).$$

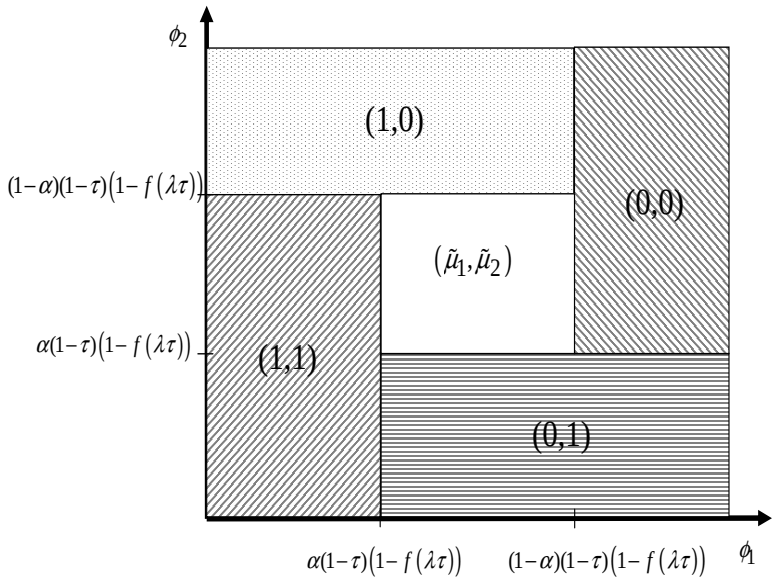

Figure 2: When the Firms Hire the Mafia as a Function of the Mafia's Fees.

Comparing expected utilities, we obtain that $\mu_2 = 1$ is the best response for F2 if

$$(1-\tau)(1-f(\lambda\tau))[\mu_1(1-2\alpha)+\alpha) > \phi_2 \tag{2}$$

and $\mu_1 = 1$ is the best response for F1 if

$$(1-\tau)(1-f(\lambda\tau))[\mu_2(2\alpha-1)+(1-\alpha)] > \phi_1. \tag{3}$$

Rearranging terms shows that F2 is indifferent if

$$\mu_1 = \frac{\phi_2}{(1-2\alpha)(1-\tau)(1-f(\lambda\tau))} - \frac{\alpha}{(1-2\alpha)} \equiv \tilde{\mu}_1,$$

and that F1 is indifferent if

$$\mu_2 = -\frac{\phi_1}{(1-2\alpha)(1-\tau)(1-f(\lambda\tau))} + \frac{(1-\alpha)}{(1-2\alpha)} \equiv \tilde{\mu}_2.$$

Figure 2 illustrates the actions taken by the two Firms in equilibrium for all possible fees charged by the Mafia in the first period ($\boldsymbol{\phi}$). If the fees charged to each Firm are sufficiently high, then neither Firm hires the Mafia (region $(0,0)$). Similarly, if the fees are sufficiently low, both Firms hire the Mafia (region $(1,1)$). The reason these two regions are

not symmetric is that Firm 2 is the Firm disadvantaged in the contract. As a result, Firm 2 is more inclined to hire the Mafia and will do so for higher fees. Of course, since the Mafia can price discriminate, it can also set the fees such that only one Firm hires it or it can choose moderate fees for both Firms that induce a mixed strategy response.

It is instructive to notice that the two Firms have different motivations underlying their behavior. Recall that Firm 1 is financially disadvantaged relative to Firm 2 ($\alpha < \frac{1}{2}$). As such, Firm 1 is tempted to hire the Mafia in order to extort Firm 2, since Firm 2 controls the bulk of economic resources. Firm 2, on the other hand, benefits greatly from economic transactions with Firm 1 and is, consequently less inclined toward extortion. Firm 2, then, is tempted to hire the Mafia not in order to extort Firm 1's resources but, rather, to provide itself with protection from extortion by Firm 1. This logic gives rise to the following proposition.

Proposition 2 *If ϕ_i is in the interval $(\alpha(1-\tau)(1-f(\lambda\tau)), 1-\alpha(1-\tau)(1-f(\lambda\tau))$, the financially disadvantaged Firm (F1) is predatory, hiring the Mafia if the other does not hire the Mafia (i.e., for the purpose of extorting the other Firm). In contrast, the financially advantaged Firm (F2) is defensive, hiring the Mafia if the other Firm also hires the Mafia (i.e., for the purpose of protection from extortion).*

4.3.1 The Mafia's Choice of Fees

Consider next the Mafia's choice of what fee to charge each Firm ($\boldsymbol{\phi}$). If the Mafia chooses $\boldsymbol{\phi}$ such that $\boldsymbol{\mu} = (\tilde{\mu}_1, \tilde{\mu}_2)$, then its expected utility is

$$E[u_M(\boldsymbol{\phi}, \boldsymbol{\mu} = (\tilde{\mu}_1, \tilde{\mu}_2);, \lambda^*, \gamma^*, \tau)] = \tilde{\mu}_1\phi_1 + \tilde{\mu}_2\phi_2 - kf(\lambda\tau)\left[\tilde{\mu}_1(1-\tilde{\mu}_2) + \tilde{\mu}_2(1-\tilde{\mu}_1) + \gamma\tilde{\mu}_1\tilde{\mu}_2\right],$$

which is linear in ϕ_1 and in ϕ_2. This linearity implies that if the Mafia chooses fees that will induce the mixed strategy it will only consider fees that induce one of the four corners of the mixed strategy region in Figure 2. Since these four corners correspond to the pure strategy equilibria, we can restrict attention to the optimal fee choices that induce each of the four

pure strategy combinations. Moreover, all fee choices that induce only one Firm to hire the Mafia are dominated by the optimal fee choice that induces both Firms to hire the Mafia. The reason for this is two-fold. First, the Mafia extracts higher total fees when it is hired by both Firms. Second, the Government and Mafia are certain to be in conflict if only one Firm hires the Mafia, whereas if both Firms hire the Mafia, then the Government and Mafia are in conflict only if the Government choose to challenge, which occurs with probability γ. Consequently, we can restrict attention to the optimal fee that induces both Firms to hire the Mafia and fees that induce neither Firm to hire the Mafia.

If the Mafia chooses $\boldsymbol{\phi}$ such that both Firms pay, i.e., $\boldsymbol{\mu} = (1, 1)$, then it must prefer the highest possible fees such that they do. Hence, from (2) and (3), we get $\phi_1 = \alpha(1-\tau)(1 - f(\lambda^*\tau))$ and $\phi_2 = (1-\alpha)(1-\tau)(1-f(\lambda^*\tau))$, and so:

$$E[u_M(\boldsymbol{\phi}, \boldsymbol{\mu} = (1,1);, \lambda^*, \gamma^*, \tau)] = (1-\tau)(1-f(\lambda^*\tau)) - k\gamma^* f(\lambda^*\tau) \tag{4}$$

If the Mafia chooses $\boldsymbol{\phi}$ such that neither Firm hires it ($\boldsymbol{\mu} = (0, 0)$), then

$$E[u_M(\boldsymbol{\phi}, \boldsymbol{\mu} = (0,0); \lambda^*, \gamma^*, \tau)] = 0. \tag{5}$$

From these expected utilities we determine the optimal choice of fees by the Mafia. The Mafia will choose fees that induce the Firms not to hire it if the threat of Government challenge and censure is sufficiently large to more than offsets the benefit associated with collecting fees. Otherwise, the Mafia will choose fees that induce both Firms to hire it. Formally,

$$\boldsymbol{\phi} = (\alpha(1-\tau)(1-f(\lambda^*\tau)), (1-\alpha)(1-\tau)(1-f(\lambda^*\tau))) \tag{6}$$

if $E[u_M(\boldsymbol{\phi}, \boldsymbol{\mu} = (1,1); \cdot)] \geq E[u_M(\boldsymbol{\phi}, \boldsymbol{\mu} = (0,0); \cdot)]$, which, from equations (4) and (5), is true only if

$$\gamma^* k < (1-\tau)\frac{(1-f(\lambda^*\tau))}{f(\lambda^*\tau))}. \tag{7}$$

Otherwise, the Mafia is indifferent over any pair (ϕ_1, ϕ_2) such that neither Firm hires the Mafia.

4.4 The Government's Resource Allocation Decision

The Government chooses the amount of tax revenues to allocate to law enforcement (λ) such that

$$\lambda \in \arg\max E[u_G(\lambda, \tau, \boldsymbol{\rho}^*(\tau, \cdot), \boldsymbol{\mu}^*(\boldsymbol{\phi}^*(\cdot), \cdot), \gamma^*(\boldsymbol{\rho}^*(\cdot), \lambda, \cdot)], \tag{8}$$

where

$$\begin{aligned} E[u_G(\lambda, \cdot)] = (1-\lambda)\tau + [&(1-\mu_1^*(\cdot))(1-\mu_2^*(\cdot))\rho_{NG}^*(\cdot) \\ &+ (\mu_1^*(\cdot)(1-\mu_2^*(\cdot)) + \mu_2^*(\cdot)(1-\mu_1^*(\cdot)))((1-f(\lambda^*\tau))\rho_M^*(\cdot) + f(\lambda^*\tau)\rho_G^*(\cdot)) \\ &+ \mu_1^*(\cdot)\mu_2^*(\cdot)((\gamma^*(\cdot)(1-f(\lambda^*\tau))\rho_M^*(\cdot) + f(\lambda^*\tau)\rho_G^*(\cdot)) + (1-\gamma^*(\cdot))\rho_{NM}^*(\cdot))]R. \end{aligned} \tag{9}$$

In order to determine the possible equilibrium paths of play in all subgames beginning with the Government's choice of how much to invest in law enforcement (λ), recall that, in equilibrium, either both Firms hire the Mafia ($\boldsymbol{\mu} = (1,1)$) or neither does ($\boldsymbol{\mu} = (0,0)$). Thus, we can restrict attention to the equilibrium behavioral strategy profiles in which $\boldsymbol{\mu} = (0,0)$ and $\boldsymbol{\mu} = (1,1)$.

Notice, from equation (1) that the Government's choice of whether or not to challenge the Mafia (γ) is a function of the Government's resource allocation decision (λ). It turns out that the Government always chooses a resource allocation such that it will challenge the Mafia with probability 0 or 1. This is summarized in the following Lemma.

Lemma 2 *The Government will never choose λ such that $\gamma \in (0,1)$*

Proof. See appendix. ■

This result implies that we can restrict attention to cases where, if both Firms hire the Mafia, the government never challenges ($\gamma = 0$) or challenges with certainty ($\gamma = 1$). Moreover, if the government never challenges in equilibrium, then it has no incentive to invest in law enforcement since it is never be called upon to fight. Thus, when considering the possibility that investment in law enforcement is positive ($\lambda > 0$), we can restrict attention to cases where $\gamma = 1$, which, from equation (1) implies that $\rho_{NM} < (1-f(\lambda\tau))\rho_M + f(\lambda^*\tau)\rho_G$.

Recall from equation (7) that the Mafia charges fees that lead both Firms to hire the Mafia only if $k < \frac{(1-\tau)}{\gamma}\frac{(1-f(\lambda^*\tau))}{f(\lambda^*\tau)}$. Clearly, if the Mafia chooses fees such that neither Firm hires it, then the Government will again choose not to invest in law enforcement. Hence, in considering the possibility of positive λ we can further restrict attention to cases where equation (7) is satisfied.

If the Government chooses a positive level of investment in law enforcement, it will make this choice to maximize its expected utility. Using the fact that $\gamma = 1$, this expected utility is given by:

$$E[u_G|\lambda, \boldsymbol{\mu} = (1,1), \cdot] = (1-\lambda)\tau + [((1-f(\lambda^*\tau))\rho_M + f(\lambda^*\tau)\rho_G)]\, R.$$

At an interior solution, the optimal level of investment in law enforcement, labeled λ', satisfies the following first-order condition:

$$f'(\lambda'\tau) = \frac{1}{(\rho_G - \rho_M)R}, \tag{10}$$

which implies that, if it is interior, $\lambda' = \frac{(f')^{-1}(\frac{1}{(\rho_G-\rho_M)R})}{\tau}$. Notice, further, that if λ' is interior, then $\lambda'\tau = (f')^{-1}\left(\frac{1}{(\rho_G-\rho_M)R}\right) \equiv \overline{\lambda'\tau}$ is invariant to the tax rate.

Remark 1 *If λ' is interior, then $\lambda'\tau$ is constant with respect to the tax rate. Label this fixed value of $\lambda'\tau$ as $\overline{\lambda'\tau}$.*

We must also consider corner solutions. The assumption that $\lim_{x\to 0} f'(x) = \infty$ rules out $\lambda' = 0$. However, if $f'(\lambda\tau) > \frac{1}{(\rho_G-\rho_M)R}$, for all $\lambda \leq 1$, then there is a corner solution at $\lambda' = 1$.

λ' is the optimal choice of investment in law enforcement, given that the government challenges. However, no investment in law enforcement ($\lambda = 0$) could be optimal if the government chooses not to challenge. In order to determine when λ' is preferred to $\lambda = 0$, we need to consider two cases.

Case 1: $\rho_{NM} < (1 - f(0))\rho_M + f(0)\rho_G$.

In this case, if the Government chooses to deviate from $\lambda = \lambda'$ to $\lambda = 0$, γ nonetheless

remains equal to 1. From the concavity of $f(\cdot)$ and the definition of an optimum, it follows that $\lambda = 0$ cannot be optimal in this case unless λ' is itself equal to 0, which is never true.

Case 2: $\rho_{NM} \in ((1 - f(0))\rho_M + f(0)\rho_G, (1 - f(\lambda'\tau))\rho_M + f(\lambda'\tau)\rho_G)$.

In this case, if the Government chooses to deviate from $\lambda = \lambda'$ to $\lambda = 0$, this will also lead it to switch from $\gamma = 1$ to $\gamma = 0$. Thus, we must compare:

$$E[u_G(\lambda', \gamma = 1)] = (1 - \lambda')\tau + ((1 - f(\lambda'\tau))\rho_M + f(\lambda'\tau)\rho_G)\, R$$

to

$$E[u_G(0, \gamma = 0)] = \tau + \rho_{NM} R.$$

Comparing these two conditions, we find that the Government will choose $\lambda = \lambda'$ in this case only if

$$(((1 - f(\lambda'\tau))\rho_M + f(\lambda'\tau)\rho_G) - \rho_{NM})\, R > \lambda'\tau, \tag{11}$$

otherwise it will choose $\lambda = 0$.

We have established the conditions under which λ' is preferred to no investment in law enforcement, conditioned on the Firms hiring the Mafia. Notice that, given Remark 1, the condition in equation (11) is purely a function of parameters, so that the case where λ' is optimal and the case where $\lambda = 0$ is optimal are mutually exclusive.

Now it remains to consider the consistency of these conditions with the conditions under which the Firms hire the Mafia. There are three possibilities. From equation (7), if $k > (1 - \tau)\frac{1-f(0)}{f(0)}$, then the Firms never hire the Mafia. If so, the Government never challenges, and so $\lambda = 0$. If $k < (1 - \tau)\frac{1-f(\lambda\tau)}{f(\lambda\tau)}$, then the Firms always hire the Mafia; the Government challenges and chooses $\lambda = \lambda'$ if equation (11) is satisfied and the Government does not challenge and does not invest in law enforcement if it is not satisfied. Finally, we need to consider when $k \in \left((1 - \tau)\frac{1-f(\lambda\tau)}{f(\lambda\tau)}, (1 - \tau)\frac{1-f(0)}{f(0)}\right)$.

In this case, there is no pure strategy equilibrium. If the government chooses $\lambda = \lambda'$, the Mafia will charge a fee that leads neither Firm to hire it. But then the Government's choice

of λ was not optimal, it should deviate to no investment in law enforcement. But, when it does so, the Mafia now wants to charge fees that lead both Firms to hire the Mafia, which again makes the Government's resource allocation decision sub-optimal. Thus, we look for mixed strategy equilibria.

Define $\hat{\lambda}$ as the choice of λ such that

$$k = (1-\tau)\frac{1-f(\hat{\lambda}\tau)}{f(\hat{\lambda}\tau)}. \tag{12}$$

At this choice of investment in law enforcement, the Mafia is exactly indifferent between charging a fee that induces both Firms both to hire it and charging a fee that induces neither Firm to hire it. Let π be the probability that the Mafia choose $\boldsymbol{\phi}$ such that neither Firm hires it and $1-\pi$ be the probability that the Mafia chooses fees such that both Firms hire it. Then, in equilibrium the Mafia must choose this probability such that $\hat{\lambda}$ is optimal for the Government. The Government's expected utility is:

$$E[u_G(\lambda|\tau,\pi)] = (1-\lambda)\tau + (\pi\rho_{NG} + (1-\pi)((1-f(\lambda\tau))\rho_M + f(\lambda\tau)\rho_G))\,R.$$

The Mafia chooses π such that the following holds:

$$(1-\pi)f'(\hat{\lambda}\tau) = \frac{1}{R(\rho_G-\rho_M)} \iff \pi = 1 - \frac{1}{Rf'(\hat{\lambda}\tau)(\rho_G-\rho_M)} \tag{13}$$

Combining all these cases, we can characterize investment in law enforcement.
If $(((1-f(\lambda'\tau))\rho_M + f(\lambda'\tau)\rho_G) - \rho_{NM})\,R > \overline{\lambda'\tau}$, then when have the following:

$$\lambda^* = \begin{cases} 1 & \text{if } \tau < (f')^{-1}(\frac{1}{(\rho_G-\rho_M)R}) \\ \frac{(f')^{-1}(\frac{1}{(\rho_G-\rho_M)R})}{\tau} & \text{if } \tau \in \left[(f')^{-1}(\frac{1}{(\rho_G-\rho_M)R}), 1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}\right] \\ \hat{\lambda} & \text{if } \tau \in \left(1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}, 1-k\frac{f(0)}{1-f(0)}\right) \\ 0 & \text{if } \tau > 1-k\frac{f(0)}{1-f(0)} \end{cases} \tag{14}$$

where $\hat{\lambda}$ is implicitly defined by equation (12).

If $(((1-f(\lambda'\tau))\rho_M + f(\lambda'\tau)\rho_G) - \rho_{NM})\,R < \overline{\lambda'\tau}$, then when have that $\lambda^* = 0$ for all tax rates.

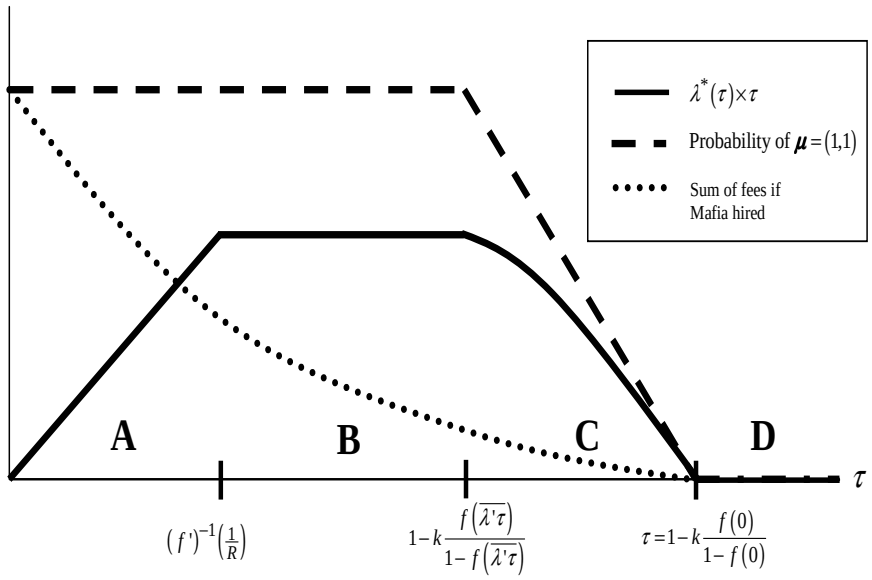

Figure 3: The Effect of Increasing Taxes on Law Enforcement Expenditures, the Frequency with Which the Mafia is Hired, and Fees Paid to the Mafia.

The level of Government spending on law enforcement, the fees charged by the Mafia, and the likelihood of the Firms hiring the Mafia are represented as a function of the tax rate in Figure 3.

4.4.1 Taxation, Government Corruption, and Law Enforcement

The Firms fund the Government through taxation in order to increase law enforcement and, thereby, weaken the Mafia. The question arises, then, whether increasing government funding will actually lead to an increase in law enforcement, given the moral hazard problem that the Firms face vis-a-vis the government.

The government, in choosing how much to invest in law enforcement, balances two types of incentives. On the one hand, it is tempted to expropriate tax revenues. On the other hand, it has electoral incentives to invest in law enforcement. These electoral incentives come from the Firms' threat not to reelect the Government should it fail to challenge and defeat the Mafia. The Government, then, will act in an increasingly uncorrupt manner as the the electoral threat associated with losing to the Mafia increases relative to the appeal

of expropriating tax revenues.

As is clear from the discussion above, and from Figure 3, the level of Government corruption (understood as the percentage of revenues not spent on law enforcement) is not monotonic in the level of funding. For low levels of taxation (region A), the Government is completely non-corrupt, spending all of its revenues on law enforcement. This is because, when the budget is small, reelection incentives loom large relative to the fairly modest opportunities for expropriation. At a somewhat higher level of taxation, the incentives for corruption become sufficiently strong that the Government begins to expropriate tax resources. Within this range (region B), corruption is increasing in taxation—the Government keeps total law-enforcement spending constant, expropriating the surplus. Once taxes become high enough (region C), the level of corruption increases even faster as tax revenue increases. This is because, as taxes increase in this range the Firms hire the Mafia less frequently. This weakens electoral incentives because conflict between the Government and Mafia (which is the source of the electoral incentives) becomes less frequent. In this range, not only is corruption increasing with taxation, but total expenditures on law enforcement are decreasing—the more resources the Government has, the fewer resources it spends on law enforcement. Finally, when taxes are high enough (region D), the Mafia can not extract fees that make it worth begin in business, so the Government is never called on to challenge the Mafia, and therefore it expropriates all tax revenues.

Proposition 3 *Government corruption (λ) is not monotonic in the level of taxation. In region A the government spends all tax revenues on law enforcement, in region B the government expropriates all taxes beyond $(f')^{-1}\left(\frac{1}{R}\right)$. In region C government corruption is increasing in the tax rate, and in region D the government expropriates all tax revenues.*

Proof. Proof is in the appendix. ■

When Government corruption increases or decreases, it is not just the percentage of tax revenues that changes, but the absolute magnitude of resources invested in law enforcement. Thus, we have the following result.

Corollary 1 *The strength of Government law enforcement, measured as the probability that the Government defeats the Mafia when they are in conflict, is not monotonic in the level of taxation. It is increasing in region A, flat in region B, decreasing in region C, and flat again in region D.*

Proof. The probability of government victory is $f(\lambda\tau)$. Since $f(\cdot)$ is monotonic, the result follows from Proposition 3. ■

4.4.2 Taxation and Mafia Viability

The dashed line in Figure 3 shows that the frequency with which the Firms hire the Mafia is weakly decreasing in the tax rate. When taxes are low (in regions A and B), the Mafia dominates the economy in the sense that neither Firm relies on the Government for protection. However, Government policy in these regions does have an effect on the Mafia. In particular, Government investment in law enforcement and the threat of punishment decrease the fees that the Mafia charges the Firms. Thus, in these regions the Firms have used tax and electoral policy to successfully limit the strength, if not the ubiquity, of the Mafia. As taxes increase even further, into region C, both the Mafia and Government are active in enforcing contracts. Finally, if taxes become high enough (region D), the Mafia is entirely eradicated. In order to achieve this outcome, the Firms must turn over enough money to the state in the form of taxes that the amount that the Mafia is able to charge in fees is no sufficient to overcome the risk of punishment that the Mafia faces when it provides revenue protection services.

The level of taxation that drives the Mafia out of business is $\bar{\tau} = 1 - k\frac{f(0)}{1-f(0)}$. Two comparative statics are evident. First, $\bar{\tau}$ is decreasing in the Government's natural advantage relative to the Mafia ($f(0)$). That is, in societies where extant Government institutions make the Government strong relative to Mafias, it is relatively inexpensive to drive the Mafia out of business. Second, $\bar{\tau}$ is decreasing in k. The larger the penalty the Government is able to impose on the Mafia, the easier it is to eradicate the Mafia. In the conclusion, we speculate

briefly about possible extensions to the model that would allow us to endogenize and better interpret these parameters.

Proposition 4 *The frequency with which the Firms hire the Mafia is weakly decreasing in the tax rate. Moreover, if taxes are high enough, $\bar{\tau}$, the Mafia is entirely eradicated. The level of taxation necessary to eradicate the Mafia is decreasing in the Government's natural advantage relative to the Mafia ($f(0)$) and in the penalty the Government is able to impose on the Mafia (k).*

Proof. The proof is in the appendix. ■

4.5 The Firms' Reelection Rule

In light of the best responses, what reelection rule will the Firms adopt? Because any reelection rule is credible in equilibrium, we look for the reelection rule that maximizes the Firms' expected utilities given the paths of play described above. First, notice that ρ_{NM} affects the Government's choice of whether to challenge, but does not directly enter into the Government's choice of investment in law enforcement. In particular, the Government will challenge the Mafia only if $\rho_{NM} < \rho_M(1 - f(\lambda^*\tau)) + \rho_G f(\lambda^*\tau)$. Since the Firms want the Government to challenge, they will always choose a ρ_{NM} that satisfies this constraint, which implies that we can disregard the case, above, where the Government chooses an investment level of $\lambda^* = 0$ because $(((1 - f(\lambda'\tau))\rho_M + f(\lambda'\tau)\rho_G) - \rho_{NM})\, R < \overline{\lambda'\tau}$. Further note that the Firms choice of ρ_{NG} has no effect on any decisions and so any ρ_{NG} is optimal.

Finally, consider ρ_M and ρ_G. λ^* is itself weakly increasing in ρ_G and weakly decreasing in ρ_M. Since the Firms want λ^* to be as large as possible, increasing $\rho_G - \rho_M$ makes the Firms better off. Thus, the Firms want to choose $\rho_G - \rho_M$ as large as possible, which implies $\rho_G = 1$ and $\rho_M = 0$.

4.6 The Optimal Tax Rate

The only remaining action to be determined is the Firms' choice of the tax rate (τ). Given the best response correspondences $\boldsymbol{\rho}^*, \lambda^*, \boldsymbol{\phi}^*, \boldsymbol{\mu}^*$, and γ^*, we have that a firm who will receive a share x of the contract has an expected utility given by:

$$E[u_F(\tau,\cdot)] = \begin{cases} x(1-\tau)f(\tau) & \text{if } \tau < (f')^{-1}\left(\frac{1}{R}\right) \\ x(1-\tau)f(\overline{\lambda'\tau}) & \text{if } \tau \in \left[(f')^{-1}(\frac{1}{R}), 1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}\right] \\ x(1-\tau)\left(\pi+(1-\pi)f(\hat{\lambda}\tau)\right) & \text{if } \tau \in \left(1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}, 1-k\frac{f(0)}{1-f(0)}\right) \\ x(1-\tau) & \text{if } \tau > 1-k\frac{f(0)}{1-f(0)}. \end{cases} \tag{15}$$

Notice, that, regardless of the realization of α, the firms are unanimous in their preferences over tax rate. However, the expected utility changes in each of the regions from Figure 3 because the Government's allocation of the tax resources is different in each region. In order to determine the optimal tax rate, the Firms compare the locally optimal tax rate in each region and choose the one that maximizes their utility. Label the locally optimal tax rate in each region (including the boundaries) τ_j^*, $j \in \{A, B, C, D\}$. The following result will be useful in finding the optimal tax rate.

Lemma 3 *The optimal tax rate is either τ_A^* or τ_C^*.*

Proof. The proof is in the appendix. ■

The intuition behind this lemmas is that, because Government investment in law enforcement is flat in regions B and D, the Firms' expected utility is decreasing in the tax rate in these regions. Thus, the optimal tax rate can never be in the interior of B or D. It is feasible, however, for the optimal tax rate to be in the interiors of A or C or on either of their upper boundaries. In order to determine which it is, we must find the local optima and compare them. This intuition is illustrated in Figure 4.

In region A, if the local optimum is interior, it is given by the following first-order condition:

$$(1-\tau_A^*)\frac{f'}{f}(\tau_A^*) = 1.$$

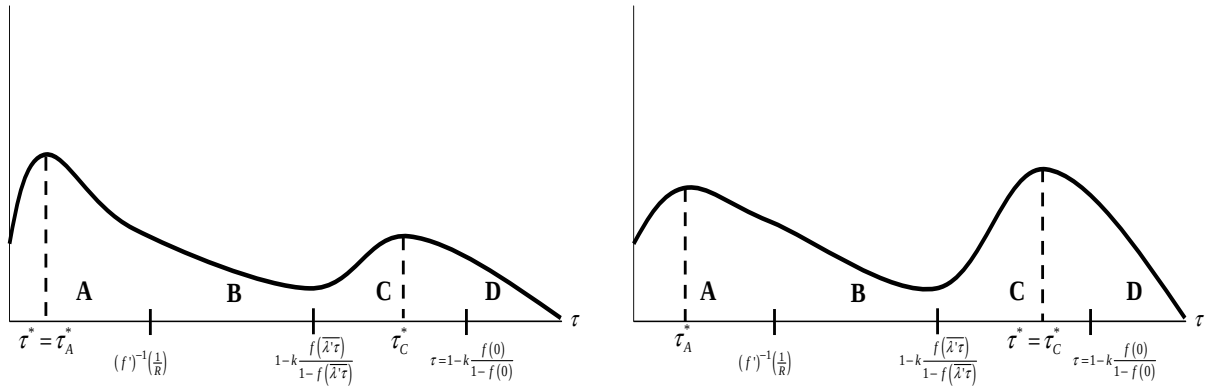

Figure 4: The Firms' expected utility as a function of the tax rate. The optimal tax rate can be in region A (left-hand figure) or region C (right-hand figure), but never regions B or D.

If $(1-\tau)\frac{f'}{f}(\tau) > 1$, for all $\tau < (f')^{-1}\left(\frac{1}{R}\right)$, then there is a corner solution, denoted

$$\bar{\tau}_A^* = (f')^{-1}\left(\frac{1}{R}\right).$$

At the interior optimum, the locally optimal tax rate in region A balances the marginal benefit of increased law enforcement that comes with increased government funding against the marginal cost of increased taxation.

In order to find the local optimum in region C, we will use the following results.

Lemma 4 *π is increasing in τ.*

Lemma 5 *$\hat{\lambda}\tau$ is decreasing in τ.*

The proofs are in the appendix. Lemma 4 points out that, in region C, the higher the tax rate, the greater the probability that the Mafia charges fees such that neither firm hires it. This is because, as the tax rate increases, the Mafia is able to extract relatively less from the firms, making being hired relatively less attractive. Lemma 5 shows that, in region C, as the tax rate increases total spending on law enforcement decreases. This is because the government only realizes electoral benefits from law enforcement spending when the firms hire the Mafia. Since the frequency with which the firms hire the Mafia decreases as taxes increase, spending on law enforcement also decreases.

We can now find the local optimum in region C. If the local optimum is interior, it is given by the following first-order condition:

$$(1-\tau)\left(\frac{\partial\pi}{\partial\tau}\left(1-f(\hat{\lambda}\tau_C^*)\right)+(1-\pi)\frac{\partial f(\hat{\lambda}\tau_C^*)}{\partial\lambda}\right)-\left(\pi\left(1-f(\hat{\lambda}\tau_C^*)\right)+f(\hat{\lambda}\tau_C^*)\right)=0.$$

Increasing the tax rate in region C has three effects on the Firms' expected utility. First, it diminishes the revenues associated with economic activity, which is a cost from the Firms' perspective ($-\left(\pi\left(1-f(\hat{\lambda}\tau_C^*)\right)+f(\hat{\lambda}\tau_C^*)\right)<0$). Second, it changes the probability that Mafia charges fees that lead both firms to hire it ($((1-\tau)\frac{\partial\pi}{\partial\tau}\left(1-f(\hat{\lambda}\tau_C^*)\right)$). Lemma 4 implies that this effect is positive—increasing taxes decreases the probability that the firms hire the Mafia, which makes the Firms better off. Finally, as show in Lemma 5, increasing taxes decreases total spending on law enforcement ($((1-\tau)(1-\pi)\frac{\partial f(\hat{\lambda}\tau_C^*)}{\partial\lambda}<0$), which makes the Firms worse off. The optimal tax rate balances these marginal benefits and marginal costs.

If the first-order condition does not hold with equality for any tax rate in region C, then the locally optimal tax rate is either the lower corner or upper corner, respectively denoted

$$\underline{\tau}_C^*=1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}$$

and

$$\overline{\tau}_C^*=1-k\frac{f(0)}{1-f(0)}.$$

The globally optimal tax rate is found by comparing the expected utilities at these local optima (see Figure 4). This gives rise to the following result.

Proposition 5 *The optimal tax rate is characterized by*

$$\tau^*=\begin{cases}\tau_A^* & \text{if } E[u_F(\tau_A^*)]\geq\max\{E[u_F(\tau_C^*)],E[u_F(\underline{\tau}_C^*)],E[u_F(\overline{\tau}_C^*)]\} \text{ and } \tau_A^*\leq(f')^{-1}\left(\frac{1}{R}\right)\\ \overline{\tau}_A^* & \text{if } E[u_F(\overline{\tau}_A^*)]\geq\max\{E[u_F(\tau_C^*)],E[u_F(\underline{\tau}_C^*)],E[u_F(\overline{\tau}_C^*)]\} \text{ and } \tau_A^*>(f')^{-1}\left(\frac{1}{R}\right)\\ \tau_C^* & \text{if } E[u_F(\tau_C^*)]>\max\{E[u_F(\tau_A^*)],E[u_F(\overline{\tau}_A^*)]\} \text{ and } \tau_C^*\leq 1-k\frac{f(0)}{1-f(0)}\\ \overline{\tau}_C^* & \text{if } E[u_F(\tau_C^*)]>\max\{E[u_F(\tau_A^*)],E[u_F(\overline{\tau}_A^*)]\} \text{ and } \tau_C^*>1-k\frac{f(0)}{1-f(0)}\end{cases}$$

Proof. Lemma 3 implies that the optimum must be in region A or C. The same Lemma implies that the expected utility is decreasing from the local optimum in region A until the boundary between region B and C. Further, if $\underline{\tau}_C^*$ is the local optimum in region C, then the expected utility is decreasing for all tax rates greater than the local optimum in region A. Thus $\underline{\tau}_C^*$ can never be the global optimum. The rest of the proposition follows from the argument in the text. ■

According to Proposition 5, the optimal tax rate can be in either region A or region C. What determines whether the Firms prefer the lower tax rate or the higher tax rate?

In region A, the firms pay relatively low taxes, all of which are directed, by the Government, toward law enforcement. However, because taxes are low, the Mafia can extract fairly high fees from the firms, with relatively little threat of successful law enforcement by the modestly funded Government. Thus, if the Firms choose the lower tax rate, they reap the benefits of relatively low taxation and a non-corrupt government, but they bear the costs of a thriving Mafia.

In region C, the Firms pay higher taxes, only some of which are directed by the Government toward law enforcement. Because taxes are high, the Mafia cannot extract as high fees from the Firms. Moreover, because the Mafia cannot extract high fees, the threat of law enforcement makes it relatively less attractive to the Mafia to be in business in the first place. As a result, the Mafia sometimes chooses to price itself out of the market.

Thus, moving from region A to region C has a variety of effects on the Firms' welfare. On the one hand, it increases the taxes they pay and increases government corruption, making them worse off. On the other hand, it decreases the fees they are charged when hiring the Mafia and decreases the frequency with which they hire the Mafia. Whether lower taxes (region A) or higher taxes (region C) are optimal depends on the relative magnitude of these tradeoffs.

4.6.1 Taxation and Commitment

Economic agents in this model face a commitment problem. For any given level of taxation, it is Pareto inefficient for both Firms to hire the Mafia. This is because the enforced contract is the same whether they both hire the Mafia or both rely on the Government, but when they both hire the Mafia they also pay fees. The problem, of course, is that they do not trust each other not to individually hire the Mafia in an attempt to extort the entire value of the contract.

The Firms fund the government in order to solve this commitment problem. The threat of Government challenge, should both Firms hire the Mafia, makes it relatively less attractive to the Mafia to be hired. Consequently, government law enforcement and taxation can sometimes diminish the fees the Mafia can charge, thereby mitigating the commitment problem. Of course, weakening the Mafia comes at a price to the Firms—taxation. The Firms face a trade-off. They would like to create a situation where neither hires the Mafia. However, funding the government sufficiently to achieve this goal is costly. Consequently, even though appealing to the Mafia is *ex post* inefficient, the Firms will allow the Mafia to persist. That is, the Firms could drive the Mafia out of business by funding the government sufficiently, but they choose not to do so because the increased tax burden would be more costly than the inefficiency of hiring the Mafia. This intuition is summarized in the following proposition.

Proposition 6 *For any tax rate the Firms prefer jointly not to hire the Mafia. Although the firms can always choose a tax rate,* $\tau = 1 - k\frac{f(0)}{1-f(0)}$*, that would lead them not to hire the Mafia, there are conditions under which they choose a lower tax rate that leads both of them to hire the Mafia.*

5 An Extension: The Possibility of Collusion

A common theme in the literature on state/mafia relations that we have not yet touched on involves collusion between the government and the mafia. In this section, we consider a

simple extension to our model that allows us to explore the implications of introducing the possibility of such collusion.

Consider an extension in which, at the time when the Government chooses whether or not to challenge the Mafia, the Mafia and the Government also have the choice to enter into a credible agreement whereby the Government does not challenge the Mafia in exchange for a payment from the Mafia (β). Thus, the Mafia and the Government can collude to prevent the Government from breaking up the Mafia's protection racket. How does this possibility effect equilibrium play?

Without repeating the full analysis, the intuitions can be seen by repeating the backward induction. At the point where this decision is made, the government's expected utility from not colluding with the Mafia is precisely as before:

$$E[u_G(\gamma = 1)] = (1 - \lambda^*)\tau + f(\lambda^*\tau)R.$$

If the Government does collude, it is sure not to be reelected, so its expected utility is:

$$E[u_G(\gamma = 0)] = (1 - \lambda^*)\tau + \beta.$$

The Government, then, will collude if

$$\beta \geq f(\lambda^*\tau)R \equiv \beta^*.$$

Clearly, if the Mafia chooses to collude with the Government, it will pay the lowest possible bribe, β^*. The Mafia's payoff from colluding with the Government is:

$$E[u_M(\beta^*)] = \phi_1 + \phi_2 - \beta^*$$

The Mafia's expected utility from not colluding with the Government is as before:

$$E[u_M(\beta = 0)] = \phi_1 + \phi_2 - f(\lambda^*\tau)k.$$

The Mafia, then, is willing to collude with the Government as long as

$$\beta^* < f(\lambda^*\tau)k \Rightarrow R < k.$$

This gives rise to our first result.

Proposition 7 *If $R > k$, then equilibrium play in the extended model with the possibility of collusion is identical to equilibrium play in the model without the possibility of collusion.*

If electoral benefits are large relative to the punishment the government is capable of meeting out, then collusion is not in the interest of the Mafia and so, will not happen. However, the question remains as to what happens if $R < k$.

As is clear from the earlier analysis, the Firms willingness to hire the Mafia is not a function of the Government's decision over whether or not to challenge the Mafia, thus Firm hiring strategies are the same as in the game without collusion. However, the point at which the Mafia is willing to charge fees that induce the Firms to hire it does change.

The Mafia's utility from being hired by the Firms, given that there will be collusion, is

$$E[u_M(\boldsymbol{\mu} = (1,1))] = \phi_1^* + \phi_2^* - \beta^* = (1-\tau)(1 - f(\lambda^*\tau)) - f(\lambda^*\tau)R.$$

The expected utility from pricing itself out of the market is 0. Thus, the Mafia will charge fees that induce the Firms to hire it if

$$R < (1-\tau)\frac{1 - f(\lambda^*\tau)}{f(\lambda^*\tau)}. \tag{16}$$

Notice that, since $R < k$, for a fixed $\lambda^*\tau$, this condition is easier to fulfill than the condition in equation (7).

The final question that must be answered is how much will be spent on law enforcement when collusion is possible. Even though the Government will never challenge the Mafia, it may have an incentive to invest money in law enforcement to increase the bribe it can extract from the Mafia. Moreover, the bribe the Government can extract is given by $\beta^* = f(\lambda^*\tau)R$, precisely the expected payoff the Government associated with the possibility of reelection in the earlier model. Thus, with the exception of the change in when the Mafia is hired expressed in equation (16), the Government's maximization problem is identical to before. Thus, without redoing the analysis, it is clear that in the extended model with collusion, if $R < k$, then the optimal investment decision is given by

$$\lambda^C = \begin{cases} 1 & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\ \frac{(f')^{-1}(\frac{1}{R})}{\tau} & \text{if } \tau \in \left[(f')^{-1}(\frac{1}{R}), 1 - R\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}\right] \\ \hat{\lambda} & \text{if } \tau \in \left(1 - R\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}, 1 - R\frac{f(0)}{1-f(0)}\right) \\ 0 & \text{if } \tau > 1 - R\frac{f(0)}{1-f(0)} \end{cases} \tag{17}$$

The fact that the Government's optimal investment is the same (but for the cut-points) as it was in the previous model, implies that the Firm's choice of optimal tax rate will also be similar. PROPOSITIONS TO BE ADDED

6 Conclusion

To be added.

7 Appendix

7.1 Proof of Lemma 1

We compare the Firms' expected utilities associated with $\gamma = 1$ and $\gamma = 0$, respectively.

$$E[u_{1,2}(\gamma = 1, \boldsymbol{\mu} = (1,1), \cdot)] = (1 - f(\lambda\tau))\frac{1-\tau}{2} + f(\lambda\tau))\frac{1-\tau}{2} = \frac{1-t}{2} = E[u_{1,2}(\gamma = 0, \boldsymbol{\mu} = (1,1), \cdot)]$$

■

7.2 Proof of Proposition 1

Expected utilities, in the modified game, are the same as in the unmodified game, so the Firms strategies with respect to hiring the Mafia are the same and so is the Mafia's expected utility, evaluated at $\gamma = 0$. It is clear that equation (4) evaluated at $\gamma = 0$ is always greater than equation (5), so equation 6 describes the optimal choice of $\boldsymbol{\phi}$. On the equilibrium path,

$\phi_1 > 0, \phi_2 > 0$, and both Firms hire the Mafia. Because the Government never fights the Mafia in equilibrium, $\lambda^* = 0$, for all $\boldsymbol{\rho}$, which implies that $\tau^* = 0$. ■

7.3 Proof of Lemma 2

There are two cases to consider:

1. $\rho_{NM} < Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G$
2. $\rho_{NM} \geq Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G$.

Since $Pr(M|\lambda,\tau)$ is decreasing in λ, we know from equation (1) that in case 1, $\gamma = 1$, regardless of λ^1.

Now consider case 2. From equation (1) we know that if $\lambda^1 = 0$, then $\gamma = 0$. This yields the following expected utility for the Government:

$$E[u_G(\lambda^1 = 0, \gamma = 0)] = \tau + \rho_{NM}\delta\tau$$

If, however, $\lambda^1 > 0$ and $\gamma \in (0,1)$, then the Government's expected utility is:

$$E[u_G(\lambda^1 > 0, \gamma \in (0,1))] = (1 - \lambda^1)\tau + \gamma\left(Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\rho_G\right)\delta\tau.$$

Note from equation (1) that if $\gamma \in (0,1)$, then $\rho_{NM} = Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))$. Consider, then, the deviation from $(\lambda^1 > 0, \gamma \in (0,1))$ to $(\lambda^1 = 0, \gamma = 0)$. We have that:

$$E[u_G(\lambda^1 = 0, \gamma = 0)] = \tau + \rho_{NM}\delta\tau = \tau + \left(Pr(M|0,\tau)\rho_M + (1 - Pr(M|0,\tau))\right)\delta\tau,$$

which is clearly larger than $E[u_G(\lambda^1 > 0, \gamma \in (0,1))]$. Hence, if $\lambda^1 > 0$, then $\gamma \notin (0,1)$. Moreover, if $\gamma = 0$, then λ^1 must be 0. Thus , if $\lambda > 0$, then $\gamma = 1$. ■

7.4 Proof of Lemma 3

If $\tau \in \left[(f')^{-1}(\frac{1}{R}), 1 - k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}\right]$, the expected utility is $x(1-\tau)f(\overline{\lambda'\tau})$. Lemma 1 shows that $\overline{\lambda'\tau}$ is constant in τ. Thus, it is clear that the expected utility is decreasing in τ, so $E[u_F(\tau_B^*)] \leq E[u_F(\tau_A^*)]$.

If $\tau > 1-k\frac{f(0)}{1-f(0)}$, the expected utility is $x(1-\tau)$, which is decreasing in τ, so $E[u_F(\tau_D^*)] \leq E[u_F(\tau_C^*)]$. ■

7.5 Proof of Lemma 4

From equation (13), $\frac{\partial \pi}{\partial \tau} = \frac{f''(\hat{\lambda}\tau)\frac{\partial \hat{\lambda}\tau}{\partial \tau}}{R(f'(\hat{\lambda}\tau))^2} > 0$, where the inequality follows from $f'' < 0$ and $\frac{\partial \hat{\lambda}\tau}{\partial \tau} < 0$. ■

7.6 Proof of Lemma 5

From equation (12)

$$\frac{f}{1-f}\left(\hat{\lambda}\tau\right) = \frac{1-\tau}{k}.$$

Since the right hand side is obviously decreasing in τ, the left-hand side must be as well. Define $g(\cdot) = \frac{f}{1-f}(\cdot)$. Then $g' = \frac{f'}{1-f} + \frac{ff'}{(1-f)^2} > 0$. Thus, in order for the left-hand side to be decreasing in τ, $\hat{\lambda}\tau$ must be decreasing in τ. ■

7.7 Proof of Proposition 3

The optimal Government resource investment is given by equation (14):

$$\lambda^* = \begin{cases} 1 & \text{if } \tau < (f')^{-1}(\frac{1}{R}) \\ \frac{(f')^{-1}(\frac{1}{R})}{\tau} & \text{if } \tau \in \left[(f')^{-1}(\frac{1}{R}), 1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}\right] \\ \hat{\lambda} & \text{if } \tau \in \left(1-k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}, 1-k\frac{f(0)}{1-f(0)}\right) \\ 0 & \text{if } \tau > 1-k\frac{f(0)}{1-f(0)}. \end{cases}$$

In the first region, $\lambda^*\tau = \tau$, which is increasing in τ. In the second region, $\lambda^*\tau = \overline{\lambda'\tau}$, which is constant in τ, by remark 1. In the third region, $\lambda^*\tau = \hat{\lambda}\tau$ which, by Lemma 5, is decreasing in τ. In the fourth region, $\lambda^*\tau = 0$, which is constant in τ. ■

7.8 Proof of Proposition 4

From equation 7, if $\tau < 1 - k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}$, the Mafia charges a fee such that both firms hire it. If $\tau \in \left(1 - k\frac{f(\overline{\lambda'\tau})}{1-f(\overline{\lambda'\tau})}, 1 - k\frac{f(0)}{1-f(0)}\right)$, the Mafia charges a fee that induces the Firms to hire the Mafia with probability $1 - \pi$ which, by Lemma 4 is decreasing in τ. By equation (7) if $\tau > 1 - k\frac{f(0)}{1-f(0)}$, the Mafia charges a fee such that neither firm hires it. ■

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