***TBA* manuscript No.**
(will be inserted by the editor)

Liquidity Shocks and Optimal Monetary and Exchange Rate Policies in a Small Open Economy*

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May 19, 2005(Very Preliminary Draft)

Abstract This paper studies the potential for liquidity crises and their impact on the course of monetary and exchange rate policies in a microfounded general equilibrium dynamic model in the tradition of Diamond and Dybvig (1983) and Chang and Velasco (2000). We produce a small open economy pure exchange overlapping generations model with random relocation along the lines of Smith (2002). The combination of random relocation and the assumed role of currency in interlocation trade creates random location - and country - specific liquidity needs. Banks naturally arise to provide much-desired insurance against these liquidity shocks. In this setting, if withdrawal demand for the bank's deposits is high enough, the bank will exhaust all its cash reserves and a liquidity shortage will occur. We provide a complete characterization of optimal interest rate policies in this setting. In a deterministic set up, we find that nominal interest rates that are desirable from a welfare perspective may also lower the probability of a liquidity crisis. We go on to study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes by introducing time-varying random endowments into the above structure. This makes the banks' portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks' portfolio choice is deterministic and constant over time and this is supported by the injection/removal of nominal balances by the central bank. Under a flexible exchange rate regime, the money supply stays constant but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. Our results show that a flexible exchange rate regime is superior in an ex-ante welfare sense relative to the fixed exchange rate regime; however, the ordering is reversed when it comes to the likelihood of liquidity crisis.

* Very preliminary draft prepared for submission to conferences. The latest version can be downloaded from http://www.econ.iastate.edu/faculty/singh/WorkingPapers/OMPLP.pdf

1 Introduction

A bank's liabilities are typically short-term deposits, while most of its assets are usually held in long-term, less liquid investments. As has been recognized at least since Diamond and Dybvig (1983), while the maturity transformation activity of banks produces major social benefits, it also exposes the banking system to the possibility of liquidity shocks. Such shocks have long been understood to be an integral component of banking crises; indeed as early as Noyes (1907), the defining characteristics of a banking panic have been known to "include the suspension of cash payments by banks to depositors, the depletion of cash reserves at banks, the emergence of a currency premium,..." (Champ, Smith, and Williamson, 1996). More recently, the importance of liquidity shocks in the creation and propagation of financial crises has been underscored by Chang and Velasco (1998) who identify "international illiquidity" as a "necessary and sufficient condition for financial crashes" especially in the context of the 1997 Asian crisis.[1]

This paper studies the potential for liquidity crises and their impact on the course of monetary and exchange rate policies. The principal focus, as in Chang and Velasco (2000), is on the liquidity-provision and maturity transformation functions of banks. In our setting, banks face uncertain liquidity demand for a foreign currency from their depositors. [2]When such demand is "sufficiently high", banks run out of cash reserves in the sense of Noyes (1907), and a liquidity crisis occurs. In an small open economy setting with free financial flows, policy-induced changes in domestic as well as world interest rates determine the opportunity cost of liquidity, thereby influencing the possibility of such crises. Policy makers weigh the social benefits of liquidity provision and maturity transformation by banks against the potential costs of exposing the economy to a crushing liquidity crisis. The kinds of monetary and exchange rate policies that a benevolent policy maker ought to follow in the wake of this tension constitutes the principal subject matter of our paper.

We produce a single good, pure exchange overlapping generations, small open economy model, that is a marriage of Smith (2002) with Betts and Smith (1997). In our set up, there are two spatially separated locations in the domestic economy. Agents are initially assigned to a location within the country. We start by studying a setting in which they receive a fixed non-stochastic endowment only when young and care only about their consumption when old. Near the end of a period, a fixed fraction of these agents must move to the other location within the same country

[1] Banking crises, or more generally, substantial banking sector woes, are by no means rare. Lindgren et al. (1996) have reported that over the 1980–96 period at least two-thirds of IMF member countries experienced significant banking sector problems.

[2] Chang and Velasco (1998) define a country's financial system to be "internationally illiquid" if its "potential short term obligations in foreign currency exceed the amount of foreign currency it can have access to on short notice" [pg. 2]

(henceforth “home movers”); additionally a random fraction of agents will have to relocate to the rest of the world (henceforth “foreign movers”). The sole real asset is a storage technology which, unlike in Smith (2002), cannot be prematurely liquidated. The only asset that individuals may transport across domestic locations is the domestic currency (henceforth "pesos"); when relocating to the rest of the world, individuals must carry the foreign currency (henceforth "dollars") with them. The domestic central bank issues its own fiat currency pesos whereas dollars can be obtained from the rest of the world. Each currency is dominated in its return by storage. The combination of random relocation and the assumed function of currency in interlocation exchange creates random location – and country – specific liquidity needs. Banks naturally arise to provide much-desired insurance against the possibility of relocation. At the start of any period, they take in deposits and divide their portfolio between pesos, dollars, and storage. Once the “relocation shock” is realized, they provide payments to depositors that are contingent on their relocation status.

As originally studied by Champ, Smith, and Williamson (1996) and more recently by Smith (2002), such a setting is conducive to the occurrence of liquidity crises. Indeed, if demand for dollars turns out to be too high, the bank will exhaust all its reserves of the dollars, and a liquidity crisis will appear. Of course the banks’ portfolio depends crucially on the returns to the domestic and foreign currencies, which in turn, are determined by the money growth rates in the domestic country and the rest of the world. It is here that the international dimension added to Smith (2002) starts to show some action.

To fix ideas, let us fix our attention on the case that we study extensively below, one where a fixed known fraction of agents in the economy move to other within-country locations but a *random* fraction of agents in the home country move to the rest of the world (for simplicity of exposition, assume no one from the rest of the world moves to the home country). In this setting, when the stationary nominal interest in the domestic economy (I^h) is higher than a cutoff I^p i.e., when the peso has a sufficiently low return relative to storage, we can show that banks use up all peso reserves to pay the home movers. Clearly, non-movers get a higher return than home movers. When the realized fraction of foreign movers is below an endogenous threshold, it is not efficient to exhaust all dollar reserves on the few foreign movers. Instead, it makes sense for the bank to equalize the ex-post returns to foreign movers and non-movers. This they can achieve by selling some of its dollar reserves and paying the non-movers in goods in the following period. Above the aforementioned threshold, all the dollar reserves of the banks are used up to pay the foreign movers. The latter get a lower return than the non-movers. As discussed in Smith (2002) and Champ, Smith, and Williamson (1996), this situation can be labelled a “banking crisis” or a liquidity crisis. As long as the ex-post rate of return is equalized between foreign movers and non-movers, both obtain a higher rate of return relative to home movers. However, as the fraction of foreign movers increases, the return obtained by foreign movers is lowered while that of non-movers is raised. If too many foreign movers are realized, then the return obtained by foreign movers may even fall below that received by home movers.

For a uniform distribution for the foreign relocation probability, we can analytically characterize the portfolio weights for dollar and peso reserves. In particular, we can show that the higher is the foreign nominal interest rate (I^f), the lower is the portfolio weight on dollars, and lower is the threshold foreign relocation probability at which banks exhaust all their foreign cash reserves. In short, the higher the opportunity cost of foreign funds, the higher is the probability of a liquidity crunch. This result is in line with the empirical evidence on banking crises as presented in Eichengreen and Rose (1998) where they find that a one percent increase in the "Northern" interest rates "is associated with an increase in the probability of Southern banking crises of around three percent."

When $I^h < I^p$ obtains, matters are substantially different; indeed, there are *two* endogenous thresholds for the foreign relocation shocks that we have to contend with. Below the first threshold, the bank is able to provide complete insurance; non-movers, foreign and domestic movers all receive the same return ex-post. This requires banks to not use up all their home currency reserves to pay the domestic movers. Instead, for such realizations of relocation shocks, some pesos are held back to buy goods for the non-movers. Similarly, not all the dollar reserves of the home banks are used up to pay the foreign movers. Once the first threshold is crossed, the banks can no longer protect the foreign movers even as they end up exhausting all their dollar reserves. At this point, the non-movers and the domestic movers continue to earn the same return, a return that is higher than what the banks can offer the foreign movers. After the second threshold has been reached, the return to the foreign movers continues to decline; the return to non-movers keeps increasing as the same amount of storage is now being divided among lesser and lesser number of non-movers. In turn now, it may not be necessary to transfer pesos to non-movers in the same amount as before. Banks exhaust all pesos to pay the domestic movers whose return is now lower than what the non-movers get.

We find that the portfolio weight attached to pesos (dollars) falls (rises) with the domestic nominal interest rate. Interestingly, the portfolio weight attached to pesos is higher than the known fraction of home movers implying that banks optimally hold *more* of the pesos than could be justified by the purpose of paying the domestic movers. We can also show that the probability of a dollar liquidity crisis is decreasing in the domestic nominal interest rate.

If the rest of the world follows a zero dollar inflation policy, ex ante welfare, in general equilibrium, is maximized when the domestic nominal interest rate equals the rate of storage, i.e., the net peso inflation rate is zero. This result underscores a tension between welfare and crisis probabilities first recognized in Smith (2002): while monetary policy can reduce the crisis probability by reducing the nominal interest rate (and approach the Friedman rule), such policies may hurt aggregate welfare.

We go on to study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes. To that end, we introduce time-varying random endowments into the above structure. This makes the banks' portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks' portfolio choice is deterministic and constant over time and is supported by the central bank injecting or withdrawing nominal balances. This in turn generates a novel inter-generational income redistribution. Under a flexible exchange rate regime, the money supply stays constant (hence no income redistribution) but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. Our results show that a flexible exchange rate regime is welfare superior relative to fixed exchange rate. However, the ordering is reversed when it comes to dollar liquidity crisis.

To get an intuitive albeit informal sense of this, we start by noting that under the flexible rate regime, the exchange rate adjustments and consequent interest rate changes induce banks to tilt their portfolios towards the highest paying assets. Suppose for instance that markets anticipate a low devaluation in any period. Banks will then tilt their portfolio towards pesos, which in turn will induce a current revaluation. As a result, the expected devaluation will now be higher. Thus, in equilibrium, exchange rate adjustments will be muted due to changes in portfolio demand. Thus, even though the flexible exchange rate regime suffers from the rate-of-return uncertainly, the price mechanism ensures that the range of fluctuations is diminished. Fixed exchange rate regime, on the other hand, eliminates rate-of-return uncertainty, but the inefficiency that stems from income redistribution makes it worse. Essentially, the exchange-rate adjustment mechanism along with portfolio choice implies that banks will be allocating their assets in the most efficient way. However, note that, despite these adjustments the domestic assets sometimes may become too attractive, and banks will put less weight on dollar reserves under such cases. As a result, the probability of crisis under flexible exchange rates is higher than under fixed exchange rates.

Our work is one in a line of papers that follow the spirit of the original modeling insights of Diamond and Dybvig (1983) and create microfounded general equilibrium structures in which banks arise "naturally" and are susceptible to crises. The paper that is closest in spirit to our work is Chang and Velasco (2000). There the authors study a small open Diamond-Dybvig economy with three periods where a subset of agents (the "patient") derive direct utility from holding the domestic currency. We make progress by "generalizing [their] results to truly dynamic settings and to more satisfactory specifications of money demand." (Chang and Velasco, 2000).

Additionally, a major advantage of our analysis is that we can quantify the exact probability of a banking crisis; Chang and Velasco (2000) are only able to show conditions under which a banking crisis will occur. A point of contrast with their results is the following. Chang and Velasco find that “a flexible rate system implements the social optimum and eliminates runs” while we find that the probability of crisis under flexible exchange rates is not zero and indeed is higher than under fixed exchange rates.

The rest of the paper is organized as follows. In Section 2, we extend Smith (2002) to an small open economy environment and then characterize banks’ equilibrium portfolio allocations. Section 3 turns to studying optimal monetary policy in a deterministic environment. Section 4 introduces endowment uncertainty into our basic framework and then goes on to comparing fixed and flexible exchange rate regimes. Some conclusions are offered in Section 5. Proofs of all major results are in the appendices.

2 The Model

2.1 The environment

Consider a small open economy with a single tradable good whose price is determined in the world markets in terms of the world currency. The economy has two symmetric spatially separated locations. Each location is the home of an infinite sequence of two-period lived overlapping generations. At each date $t = 1, 2, 3, ..$ a continuum of ex ante identical young agents of unit mass is born. A young agent receives an endowment of w; old agents receive no endowment. For the analysis in this section we assume that w is fixed over time. Agents care only about their second period consumption (c) ordered by $\ln(c)$. In what follows, variables superscripted with $f\,(h)$ represent the rest of the world (domestic) variables.

At the start of each date, newly born agents are assigned to one of two locations in the home country. Towards the end of the period, after asset markets shut, some young agents will have to move across locations domestically and some will have to travel to the rest of the world. The rest stay put (the “non-movers”) in the location they were born in. Specifically, let π be the fraction of agents who get relocated within the country (henceforth “home movers”) and λ denote the fraction of agents who get relocated to the rest of the world (henceforth “foreign movers”). We assume that any agent’s realization of the shock is public information. To keep analytics simple, we posit that π is known and constant. However, λ is assumed to be a *random* variable with a known distribution $f(\lambda)$.

Following Smith (2002), the only real asset is a (commonly available in the world) storage technology: $\mathcal{F}(\kappa) = \rho\kappa$, i.e., 1 unit invested in this technology yields a sure gross real return of

ρ the following period. Relocated agents cannot transport goods to their new homes. Within the country, agents can carry the domestic currency (hereafter "pesos") to the other location and subsequently trade their currency against consumption goods. Relocations abroad require agents to carry with them the world currency (hereafter "dollars"). Unlike Smith (2002), we disallow investments in storage to be liquidated (scrapped) even at a cost. As will become clear below, this assumption along with the limited communication assumption implies that towards the end of a period, any "excess" currency may be converted into goods to give non-movers in the same location, but any "excess" stored goods cannot be given to movers since they are at a different location by then. This asymmetry will have important implications below.

The role of banks is motivated as follows. At the start of a period, young agents at any location receive their endowment; at that point asset markets open. Agents potentially make decisions on how much currency and how much storage to hold. After this is done, asset markets close. The relocation shock is subsequently revealed. At this point, agents who are relocated within and across nations will need to acquire the appropriate currency. Only the currency of their destination is valuable to them; all their holdings of other assets, for lack of an open market for trade, is worthless to them. In short, agents in this economy face considerable private risks associated with relocation. We assume competitive banks arise to supply insurance to these risk averse agents. It will be evident that all young agents will choose to deposit their entire endowment with a bank, i.e., all saving activity will be intermediated. As Nash competitors on both the deposit and the asset sides, banks will make portfolio choices about storage and the two currencies so that these choices maximize ex-ante expected utility of a representative depositor subject to balance sheet constraints described below.

Before proceeding further, some definitions and notation are in order:

$$m_t^h \equiv \frac{M_t^h}{p_t^h},\ m_t^f = \frac{M_t^f}{p_t^f},\ I_t^h \equiv \frac{p_{t+1}^h}{p_t^h}\rho,\ I_t^f \equiv \frac{p_{t+1}^f}{p_t^f}\rho, \tag{1}$$

where M^h denotes the supply of pesos, M^f denotes the amount of dollars held the domestic economy; and where p^h and p^f denote the peso and dollar price of consumption good, respectively.

It is assumed that banks acquire dollars in each period by selling goods, deposited by households, to the rest of the world at a price p^f. Assuming that there are no other transactions costs, it is then implied that $p^h = ep^f$, where e denotes the nominal exchange rate denominated in units of domestic currency per unit of world currency. Using (1) with purchasing power parity condition yields

$$I_t^h = I_t^f \frac{p_{t+1}^h}{p_{t+1}^f}\frac{p_t^f}{p_t^h} = I_t^f \frac{e_{t+1}}{e_t} = I_t^f \left(1 + \varepsilon_{t+1}\right) \tag{2}$$

where ε_{t+1} denotes the (gross) rate of exchange rate devaluation between t and $t+1$. Henceforth, we will focus on situations where $I_t^h, I_t^f \geq 1$ holds, implying that storage dominates the real return to holding either of the currencies.

2.2 The bank's problem

The problem of portfolio selection faced by banks is as follows. As all agents are ex-ante identical, they deposit their endowment at banks who in turn make portfolio allocations between pesos, dollars, and storage as given by the per depositor resource constraint

$$w + \tau_t \geq m_t^h + m_t^f + s_t; \tag{3}$$

here $\tau_t = \frac{M_t^h - M_{t-1}^h}{p_t^h}$ denotes the net government transfer (seigniorage) to the young domestic agents, m and m^f denote the goods value of peso and dollar reserves respectively, and s denotes storage. No transfers are made to the old. For future reference, define

$$\gamma_t^h \equiv \frac{m_t^h}{w + \tau_t}, \; \gamma_t^f \equiv \frac{m_t^f}{w + \tau_t}. \tag{4}$$

Then γ^h (γ^f) represents the peso (dollar) reserve to deposit ratio of the home banks. Notice that $1 - \gamma^h - \gamma^f = \frac{s}{w+\tau}$ must hold (this is the portion invested in real storage).

Banks, at the start of a period, and upon receiving deposits decide what their γ^h and γ^f should be. At this stage, banks know π, but λ has not yet been realized. After asset markets close until the next period, the foreign relocation shock λ is revealed. At that point, the banks will have to pay domestic (foreign) movers in pesos (dollars). Much will depend on the exact realization λ. In this environment, unlike in Betts and Smith (1997), banks announce a schedule of returns for the three types of agents that is contingent on λ. Let $r^h(\lambda)$ and $r^f(\lambda)$ denote the gross real returns promised to agents relocated domestically and abroad, respectively; let $r^n(\lambda)$ denote the gross real return offered to the non-movers. Finally let $\alpha(\lambda)$ denote the λ-contingent fraction of dollar reserves (per depositor) that is used to pay foreign movers; similarly, let $\beta(\lambda)$ denote the λ-contingent fraction of peso reserves (per depositor) that is used to pay domestic movers. These returns must satisfy (see Appendix 6.1)

$$\lambda r_t^f(\lambda) \leq \alpha_t(\lambda)\gamma_t^f \left(\frac{p_t^f}{p_{t+1}^f}\right), \tag{5a}$$

$$\pi r_t^h(\lambda) \leq \beta_t(\lambda)\gamma_t^h \left(\frac{p_t}{p_{t+1}}\right), \text{ and} \tag{5b}$$

$$(1-\pi-\lambda)\, r_t^n(\lambda) \leq (1-\alpha_t(\lambda))\,\gamma_t^f \left(\frac{p_t^f}{p_{t+1}^f}\right) + (1-\beta_t(\lambda))\,\gamma_t^h \left(\frac{p_t}{p_{t+1}}\right) + \left(1-\gamma_t^h-\gamma_t^f\right)\rho. \tag{5c}$$

These constraints have standard interpretations. Consider as an example, constraint (5a). Once λ is realized, the banks becomes aware that it has to pay $r^f(\lambda)$ to each of these foreign movers. It uses up a fraction α of its per depositor dollar reserves (γ^f) for this purpose. Foreign movers in total receive \$ $\alpha(\lambda)\gamma^f$ which when brought to the rest of the world yields $\left(\frac{p_t^f}{p_{t+1}^f}\right)$ per unit in the form of goods next period. The constraint (5b) is simpler. Banks know π upfront and so they know in advance that a fraction π of their clientele will move domestically and will have to be given $r^h(\lambda)$. They can finance this payout by (partially) using their reserves of pesos, $\beta(\lambda)\gamma^h$. In the new location, this cash is worth $\beta(\lambda)\gamma^h\frac{p_t}{p_{t+1}}$ in goods. Finally, note that γ^h, γ^f, $\alpha(\lambda)$ and $\beta(\lambda)$ are chosen at the beginning of period, and can potentially be time-dependent; hence, we represent them with time subscripts.[3] On the other hand, λ is realized after all choices have been made. Hence, in order to economize on notation, we suppress time subscripts from λ.

As noted earlier, banks are Nash competitors in the deposit market, which implies that they maximize the expected utility of a young agent. Once the deposits are made, banks' choice problem each period is identical, and we can drop time subscripts. The banks' problem can be stated as

$$\max_{\gamma_h,\gamma^f,\alpha,\beta}\int_0^1\left[\pi\ln r^h(\lambda)+\lambda\ln r^f(\lambda)+(1-\pi-\lambda)\ln r^n(\lambda)\right]f(\lambda)\,d\lambda \tag{6}$$

subject to (3) - (5c) and non-negativity of $\gamma_h, \gamma^f, \alpha$,and β. Moreover, none of these choices can exceed unity.

For future reference, note that in steady states, for stationary government policies, we can use (1) and (4) to rewrite constraints (5a) - (5c) more compactly as

$$r^f(\lambda)=\frac{\alpha(\lambda)}{\lambda}\gamma^f\frac{\rho}{I^f}, \tag{7a}$$

$$r^h(\lambda)=\frac{\beta(\lambda)}{\pi}\gamma^h\frac{\rho}{I^h}, \tag{7b}$$

$$r^n(\lambda)=\frac{\rho}{1-\pi-\lambda}\left[(1-\alpha(\lambda))\frac{\gamma^f}{I^f}+(1-\beta(\lambda))\frac{\gamma^h}{I^h}+\left(1-\gamma^h-\gamma^f\right)\right], \tag{7c}$$

where, after invoking optimality, it is assumed that (5a)-(5c) will hold with equality. To reiterate, banks maximize (6) subject to (3) and (7a) - (7c).

It is convenient to conceptualize the bank's problem as a two stage problem and work backwards: 1) in the second stage, given γ^h and γ^f, the banks choose $\alpha(\lambda)$ and $\beta(\lambda)$ for each λ, and 2) given the schedules $\alpha(\lambda)$ and $\beta(\lambda)$, they choose γ^h and γ^f so as to maximize (6). Herein lies the seed of much of what is to come. Banks choose γ^h and γ^f by maximizing ex-ante expected utility. Since agents are risk-averse, the bank chooses γ^h and γ^f to try and provide insurance

[3] As will be clear below, these rules are time-invariant in the deterministic case with stationary government policies. However, when endowment uncertainty is introduced in 4, these rules will depend on the endowment realization and hence will vary over time.

against the upcoming relocation shock. It faces a tension. On the one hand, since the return on cash is lower than that on storage, the bank will want to economize on cash holdings. However, since storage cannot be liquidated, and insurance provision requires cash, the bank does not want too illiquid a portfolio. Since asset markets close right after these choices have been made, the bank is "stuck" with its choices as it awaits the realization of λ.

Ex post, the bank may or may not succeed in providing complete insurance. Too high a withdrawal demand will compromise this function and precipitate a liquidity crisis. In particular, the bank may end up holding too many dollars or pesos which cannot be converted into goods until the following period (when asset markets will open again). Limited communication implies that any excess currency (converted next period into goods) can only be used to pay the non movers who are the only depositors still present in the same location; the home and foreign movers have left and cannot be reached any more. At the same time, if there are excess stored goods, the bank cannot use them to pay any of the relocated agents for the same reason.

We start by focusing on the second stage problem. The stock of peso and dollar reserves is predetermined at this point. It is easy to verify that the first-order-condition for the choice of $\alpha(\lambda)$ is:

$$r^n(\lambda) \geq r^f(\lambda), \text{ “} = \text{” if } \alpha(\lambda) < 1 \tag{8}$$

Equation (8) states that agents relocated abroad will get the same return as those staying put (the bank provides complete insurance), *only* if there is sufficient dollar reserves in stock for the emigrants. Of course this does not mean that in the event the (ex-post) realized λ is low, the bank would still distribute its entire dollar reserves equally among the foreign movers. In that case, as we will see below, it may choose to convert some of its dollar reserves into goods and give to the non movers. If the ex-post λ is sufficiently high, each foreign mover will get a low return and all dollar reserves will be exhausted. This is because the number of non-movers is small and so their per capita return (from storage) will be high relative to the foreign movers. However, it bears emphasis that since storage cannot be liquidated, banks cannot transfer stored goods to agents who have already moved out. For future reference note that using (1), (2), (7a), and (7c), equation (8) can be rewritten as

$$\frac{\rho}{1-\pi-\lambda}\left[\frac{(1-\alpha(\lambda))\gamma^f}{I^f}+\frac{(1-\beta(\lambda))\gamma^h}{I^h}+\left(1-\gamma^h-\gamma^f\right)\right] \geq \frac{\alpha(\lambda)}{\lambda}\frac{\rho\gamma^f}{I^f}, \quad \text{“} = \text{” if } \alpha(\lambda) < 1 \tag{9}$$

Similarly, the optimal choice of $\beta(\lambda)$, i.e., how much of peso reserves to pay out, yields:

$$r^n(\lambda) \geq r^h(\lambda), \text{ “} = \text{” if } \beta(\lambda) < 1 \tag{10}$$

Equation (10) has an interpretation similar to that of (8): the bank provides complete insurance against the risk of domestic relocation, *only* if there is sufficient peso reserves in stock for the home movers. Again, using (1), (2), (7b), (7c), equation (10) can be rewritten as

$$\frac{\rho}{1-\pi-\lambda}\left[\frac{(1-\alpha(\lambda))\gamma^f}{I^f}+\frac{(1-\beta(\lambda))\gamma^h}{I^h}+\left(1-\gamma^h-\gamma^f\right)\right] \geq \frac{\beta(\lambda)}{\pi}\frac{\rho\gamma^h}{I^h}, \quad \text{“ = ” if } \beta(\lambda)<1 \tag{11}$$

Obviously, the decision to exhaust reserves of any currency (as evident from (9) or (11)) partly depends on how “good” one currency is relative to the other. To foreshadow, we find that there is an (endogenously determined) nominal interest rate, I^p, that serves as an important threshold rate. If the domestic interest rate, I^h, is greater than this I^p (i.e., the return to pesos is “low enough”) the following are true and consistent in an equilibrium: a) foreign movers and non-movers receive a return equal to ρ/I^p if the realized λ is below a threshold, b) banks hold a peso-reserve to deposit ratio $\left(\gamma^h\right)$ equal to the fraction of home movers (π), and subsequently exhaust their peso reserves $(\beta(\lambda)=1, \forall\lambda)$ for any realization of λ, c) the home movers get a return equal to $\frac{\rho}{I^h}$ [see (7b)] which is independent of I^f and any realization of λ.

As I^h approaches I^p and falls below it, the equilibrium changes; now the home movers join the other two types and get a common return (pooled return from pesos, dollars, and storage) if the realized λ is below a threshold. In this case, however, the bank's choice of γ^h and $\beta(\lambda)$ will also depend on I^f.

2.3 The case of $I^h > I^p$

Suppose $I^h > I^f$. Then pesos are the worst assets. There would be no gain in involving peso holdings into insuring movers or non-movers under varios λ scenarios. Given log preferences, then banks would like to simply allocate a share π of current deposits to pesos. How about when $I^h < I^f$? It turns out that even when I^h is smaller than I^f, but greater than the endogenously determined common return on dollar holdings and storage, defined as I^p, banks still separate home-movers portfolio allocation problem from the rest. In this case $\gamma^h=\pi$, and $\beta(\lambda)=1, \forall\lambda$. These conjectures are verified in equilibrium below.

Proposition 1 *The following constitute an equilibrium. Define*

$$I^p \equiv \frac{1}{1-\frac{\gamma^f}{1-\pi}\left(1-\frac{1}{I^f}\right)},$$

and

$$\hat{\lambda} \equiv \frac{(1-\pi)\gamma^f}{\gamma^f+\left(1-\gamma^h-\gamma^f\right)I^f}. \tag{12}$$

Then if $I^h \geq I^p$,

$$\alpha(\lambda)=\begin{cases}\frac{\lambda}{1-\pi}\left[1+\frac{1-\gamma^h-\gamma^f}{\gamma^f}I^f\right] \equiv \frac{\lambda}{\hat{\lambda}}, & \text{if } \lambda \leq \hat{\lambda}, \\ 1, & \text{if } \lambda > \hat{\lambda}\end{cases}, \tag{13}$$

$$\beta(\lambda) = 1, \quad \forall \lambda,$$

and the state contingent returns to the different types are given by

$$r^f(\lambda) = \begin{cases} \frac{\rho}{I^p}, & \text{if } \lambda \leq \hat{\lambda}, \\ \frac{\gamma^f}{\lambda} \frac{\rho}{I^f}, & \text{if } \lambda > \hat{\lambda}, \end{cases} \tag{14}$$

$$r^n(\lambda) = \begin{cases} \frac{\rho}{I^p}, & \text{if } \lambda \leq \hat{\lambda}, \\ \frac{1-\gamma^h-\gamma^f}{1-\pi-\lambda}\rho, & \text{if } \lambda > \hat{\lambda}. \end{cases} \tag{15}$$

and

$$r^h(\lambda) = \frac{\gamma^h}{\pi} \frac{\rho}{I^h} < r^n(\lambda) = r^f(\lambda) \qquad \text{if } \lambda \leq \hat{\lambda} \tag{16}$$

Additionally,

$$\gamma^h = \pi,$$

and

$$\hat{\lambda} < \bar{\lambda} \text{ and } \gamma^f > \hat{\lambda}$$

obtains.

The state-contingent returns offered by the banks are easily computed using (13) in (7a) and (7c). Using our conjecture that $\beta = 1$, it is clear from (7b) that $r^h(\lambda) = \frac{\gamma^h}{\pi}\frac{\rho}{I^h}$, which does not depend on I^f or on the realization of λ.

Then, using (14) - (16) in (6), along with some rearrangement yields

$$\begin{aligned} W = & \int_0^{\hat{\lambda}_a} (1-\pi) \ln\left(\frac{\gamma^f}{\hat{\lambda}} \frac{\rho}{I^f}\right) f(\lambda)\, d\lambda + \int_{\hat{\lambda}_a}^{\bar{\lambda}} \lambda \ln\left(\frac{\gamma^f}{\lambda}\frac{\rho}{I^f}\right) f(\lambda)\, d\lambda \\ & + \int_{\hat{\lambda}_a}^{\bar{\lambda}} (1-\pi-\lambda) \ln\left(\frac{1-\gamma^h-\gamma^f}{1-\pi-\lambda}\rho\right) f(\lambda)\, d\lambda + \int_0^{\bar{\lambda}} \pi \ln\left(\frac{\gamma\rho}{I^h \pi}\right) f(\lambda)\, d\lambda \end{aligned} \tag{17}$$

The ex-ante choice of γ^h and γ^f is derived by maximizing the expression for stationary welfare in (17).

How is I^p computed? Note that as long as $\lambda < \hat{\lambda}$, the rate of return obtained by foreign movers and non-movers are equal. Using (9) and (13) and $\gamma^h = \pi$, we get

$$\frac{\gamma^f}{\hat{\lambda}} \frac{\rho}{I^f} = \frac{1-\pi-\gamma^f}{1-\pi}\rho + \frac{\gamma^f}{1-\pi}\frac{\rho}{I^f} \equiv \frac{\rho}{I^p}$$

where $\frac{\rho}{I^p}$ is the common rate of return to foreign movers and non-movers for all $\lambda \leq \hat{\lambda}$. This implies that $I^p \equiv \left[1 - \frac{\gamma^f}{1-\pi}\left(1 - \frac{1}{I^f}\right)\right]^{-1}$.

We are now in a position to verify our conjecture, that in equilibrium, the bank will exhaust its peso reserves. Indeed $\beta(\lambda) = 1$ for all λ is indeed true for all $I^h > I^f$. To see this, note from (9) and (11) that $\beta(\lambda) = 1$ for all λ if and only if

$$\frac{\alpha(\lambda)}{\lambda}\frac{\gamma^f}{I^f} \geq \frac{1}{\pi}\frac{\gamma^h}{I^h} \text{ for all } \alpha(\lambda) \leq 1 \tag{18}$$

Further, using (13) and the last part of Proposition 1, eq. (18) can be rewritten as

$$\frac{\gamma^f}{\hat{\lambda}}\frac{\rho}{I^f} \geq \frac{\rho}{I^h} \tag{19}$$

That is, $\beta(\lambda) = 1$ for all λ if and only if (19) holds. Using the last part of Proposition 1, it follows that (19) trivially holds for all $I^h \geq I^f$.

In sum, the qualitative features of the equilibrium described in Proposition 1 above, are as follows. When the domestic nominal interest rate is greater than I^p (i.e., the return to pesos is "low enough"), we find that a) foreign movers and non-movers receive a return equal to ρ/I^p if the realized λ is below a threshold; otherwise the foreign movers receive much less, b) banks hold just enough pesos to pay the home movers and subsequently exhaust their peso reserves ($\beta(\lambda) = 1, \forall\lambda$) for any realization of λ, c) the home movers get a return equal to $\frac{\rho}{I^h}$ [see (7b)] which is insulated from I^f and realizations of λ.

Intuitively, the following issues are at the heart of the matter. Banks would like to provide insurance to the three types, home movers. foreign movers, and non movers, but only upto what is allowable by the efficiency conditions. Holding too much cash balances hurts storage and holding too little hurts the relocated agents. The question central to all this is: when the return to pesos is low enough, why do banks hold just the right amount of pesos and no more? in other words, why are home movers kept insulated from the insurance problem that the bank is solving for the other two types?

In our setting, banks know the exact number of home movers, but they do not know how many non movers and foreign movers will be realized. Consumption efficiency requires them to provide insurance to the foreign movers. The bank is aware that there is always a chance that there may be too many foreign movers. Since the foreign movers are risk averse, their utility falls more from a marginal loss in consumption than it rises from a marginal gain. As such, the bank reserves more dollars than what the expected population share of foreign movers would justify. This comes at a cost; reserving "extra" dollars impedes production efficiency since it is associated with too low investment in storage. Furthermore, given the aforediscussed asymmetric nature of the payouts, any excess dollars remaining (if ex post there are too few foreign movers) can only be used to pay the non movers but not the home movers. In this sense, the solution to the bank's insurance problem involves insulating the home movers from the relocation risk of foreign movers.

Overall, the bank's optimal scheme gives the same return (ρ/I^p) to foreign movers and non movers and a lower return to home movers (ρ/I^h) if few foreign movers are realized. If too many foreign movers are realized, the bank gives relatively more to non movers; the foreign movers suffer and may even get a return lower than the home movers. As discussed above, if too high a foreign relocation probability is realized, the bank cannot convert pesos to dollars (since the asset market does not reopen until the following period by which time, the foreign movers cannot be contacted any more). If a low enough foreign relocation probability is realized, given the low return on the pesos, the foreign movers get a higher return anyway. As such, under no circumstances can the bank use its peso reserves to offer additional consumption protection to the foreign movers. Consequently, it simply holds just enough pesos to pay the home movers and no more.

2.4 The case of $I^h < I^p$

When the domestic currency yields a relatively superior return, i.e., $I^h < I^p$, banks may not always want to exhaust their reserves of domestic currency. Indeed, there are *two* endogenous thresholds for the foreign relocation probability that become relevant. Below the first threshold, the bank is able to provide complete insurance; non-movers, foreign and domestic movers all receive the same return ex-post. This requires banks to not use up all their peso reserves to pay the domestic movers. Instead, some pesos are converted into goods the following period to pay the non-movers. Similarly, not all the dollar reserves of the banks are used up to pay the foreign movers. Once the first threshold is crossed, the banks can no longer protect the foreign movers even as they end up exhausting all their dollar (the weaker currency) reserves. At this point, the non-movers and the home movers continue to earn the same return, a return that is higher than what the banks can offer the foreign movers. After the second threshold has been reached, the return to the foreign movers gets lower and lower; the return to non-movers will keep increasing as the same amount of storage is now being divided among lesser and lesser number of non-movers. In turn now, it may not be necessary to transfer peso reserves to non-movers in the same amount as before. Banks exhaust all peso reserves to pay the home movers whose return is now lower than what the non-movers get.

Formally, for all $\lambda \in [0, \hat{\lambda}]$, (9) and (11) will hold with equality. Manipulating them, one can rewrite

$$\lambda \left[\gamma^f + (1-\beta(\lambda))\gamma^h \frac{I^f}{I^h} + \left(1-\gamma^h-\gamma^f\right) I^f\right] \geq (1-\pi)\,\alpha(\lambda)\,\gamma^f, \quad \text{“}=\text{”} \text{ if } \alpha(\lambda) < 1 \tag{20a}$$

$$\pi \left[(1-\alpha(\lambda))\gamma^f \frac{I^h}{I^f} + \gamma^h + \left(1-\gamma^h-\gamma^f\right) I^h\right] \geq (1-\lambda)\,\beta(\lambda)\,\gamma^h, \quad \text{“}=\text{”} \text{ if } \beta(\lambda) < 1 \tag{20b}$$

The optimal rules for use of currency reserves are collected in the next proposition

Proposition 2 *When* $I^h < I^p \equiv \frac{1-\pi}{(1-\pi-\gamma^f)+\frac{\gamma^f}{I^f}}$, γ^f *is as obtained in Proposition 1*

$$\alpha = \begin{cases} \frac{\lambda}{\hat{\lambda}} \text{ if } \lambda \leq \hat{\lambda}, \\ 1 \text{ if } \lambda > \hat{\lambda}, \end{cases} \tag{21}$$

$$\beta = \begin{cases} \bar{\beta} & \text{if } \lambda \leq \hat{\lambda}_b \\ \frac{1-\tilde{\lambda}}{1-\lambda}, & \text{if } \lambda \in \left[\hat{\lambda}_b, \tilde{\lambda}\right] \\ 1 & \text{if } \lambda \geq \tilde{\lambda}, \end{cases} \tag{22}$$

where

$$\tilde{\lambda} \equiv (1-\pi) - \frac{\pi}{\gamma^h}\left(1-\gamma^h-\gamma^f\right) I^h \tag{23}$$

and

$$\hat{\lambda} \equiv \frac{\gamma^f \frac{I^h}{I^f}}{\gamma^f \frac{I^h}{I^f} + \gamma^h + (1-\gamma^h-\gamma^f) I^h} \tag{24}$$

$$\bar{\beta} \equiv \pi \frac{\gamma^f \left(\frac{I^h}{I^f}\right) + \gamma^h + \left(1-\gamma^h-\gamma^f\right) I^h}{\gamma^h} \tag{25}$$

Note that the value of $\hat{\lambda}$ under $I^h < I^p$ is different from that in the case when $I^h > I^p$. But we continue to use the same notation; this is due to the fact that any λ below $\hat{\lambda}$ gets a constant return that equals that of non-movers. This holds under both cases.

Figure 1 exhibits equations (21) and (22) graphically.

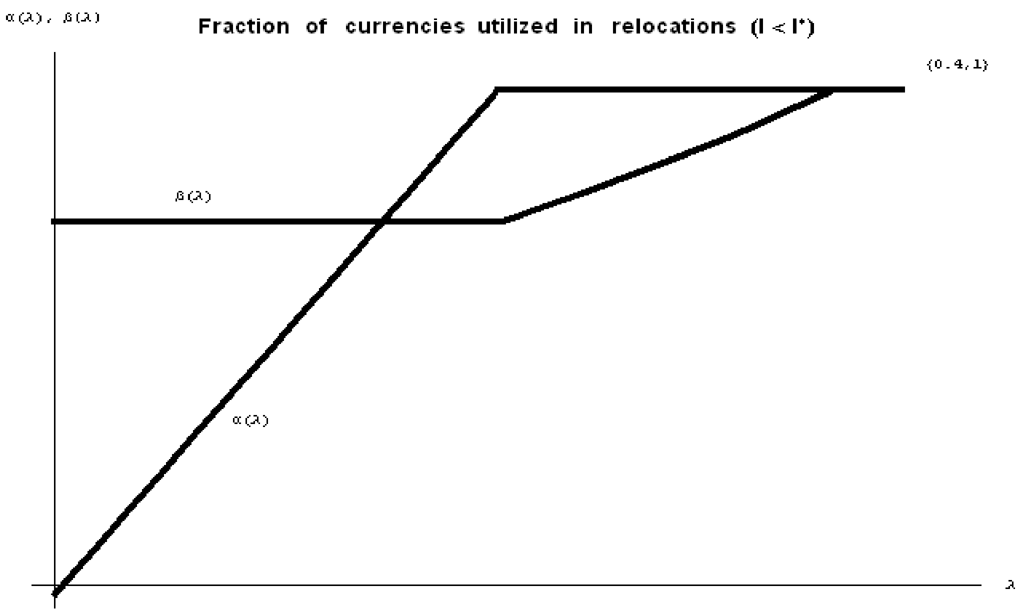

Figure 1

For future reference note,

$$\begin{aligned}
r^f &= r^h = r^n = \frac{\gamma^f}{\hat{\lambda}} \frac{\rho}{I^f} = \frac{\bar{\beta}}{\pi} \frac{\rho\gamma}{I^h}, \text{ for } \lambda \leq \hat{\lambda}_b \\
r^f &= \frac{\gamma^f}{\lambda} \frac{\rho}{I^f} < \ r^h = r^n = \frac{1-\tilde{\lambda}}{1-\lambda} \frac{\rho\gamma}{I^h\pi}, \quad \text{for } \lambda \in \left[\hat{\lambda}_b, \tilde{\lambda}\right] \\
r^f &= \frac{\gamma^f}{\lambda} \frac{\rho}{I^f} < \ r^h = \frac{\rho\gamma^h}{I^h\pi} < r^n = \frac{1-\gamma^f}{1-\pi-\lambda}\rho, \text{ for } \lambda \in \left[\tilde{\lambda}, \bar{\lambda}\right]
\end{aligned} \tag{26}$$

We are now in a position to start investigating the optimal reserve holdings of the two currencies. Using (26), (6) can be rewritten as

$$\begin{aligned}
W = & \int_0^{\hat{\lambda}_b} \ln\left(\frac{1}{\hat{\lambda}} \frac{\rho\gamma^f}{I^f}\right) f(\lambda)\, d\lambda + \int_{\hat{\lambda}_b}^{\tilde{\lambda}} (1-\lambda) \ln\left(\frac{1-\tilde{\lambda}}{1-\lambda} \frac{\rho\gamma}{I^h\pi}\right) f(\lambda)\, d\lambda \\
& + \int_{\tilde{\lambda}}^{\bar{\lambda}} (1-\pi-\lambda) \ln\left(\frac{1-\gamma^h-\gamma^f}{1-\pi-\lambda}\rho\right) f(\lambda)\, d\lambda + \int_{\tilde{\lambda}}^{\bar{\lambda}} \pi \ln\left(\frac{\rho\gamma}{I^h\pi}\right) f(\lambda)\, d\lambda \\
& + \int_{\hat{\lambda}_b}^{\bar{\lambda}} \lambda \ln\left(\frac{1}{\lambda} \frac{\rho\gamma^f}{I^f}\right) f(\lambda)\, d\lambda
\end{aligned}$$

We proceed to study how the optimal reserve holdings respond to the domestic nominal interest rate, holding the foreign interest rate fixed. Suppose we choose the following parametric specification: $\pi = 0.3, \bar{\lambda} = 0.6 < 1-\pi$, and $I^f = 1.3$, $f(\lambda)$ is uniform with support $[0, 0.4]$and let I^h vary between 1.01 and 1.1. Figure 2 plots γ^h and γ^f as I^h varies. Several items deserve mention. First, not surprisingly, γ^h and γ^f respond in exactly opposite ways; while γ^h falls with I^h, γ^f increases with it. As I^h increases with I^f held fixed, and I^h stays below I^f, banks decrease (increase) their domestic (foreign) currency reserves. As I^h increases, banks want to economize on currency holdings; since the domestic opportunity cost is relatively increasing with I^h, the bank reduces its peso holdings.

Second, $\gamma^h > \pi$ for this range of I^h and I^f implying that banks optimally hold more pesos than they need solely for the purposes of paying the home movers.

Finally, as a comparative static exercise, if I^f is raised to 1.35, the γ^h $\left(\gamma^f\right)$ locus shifts up (down).

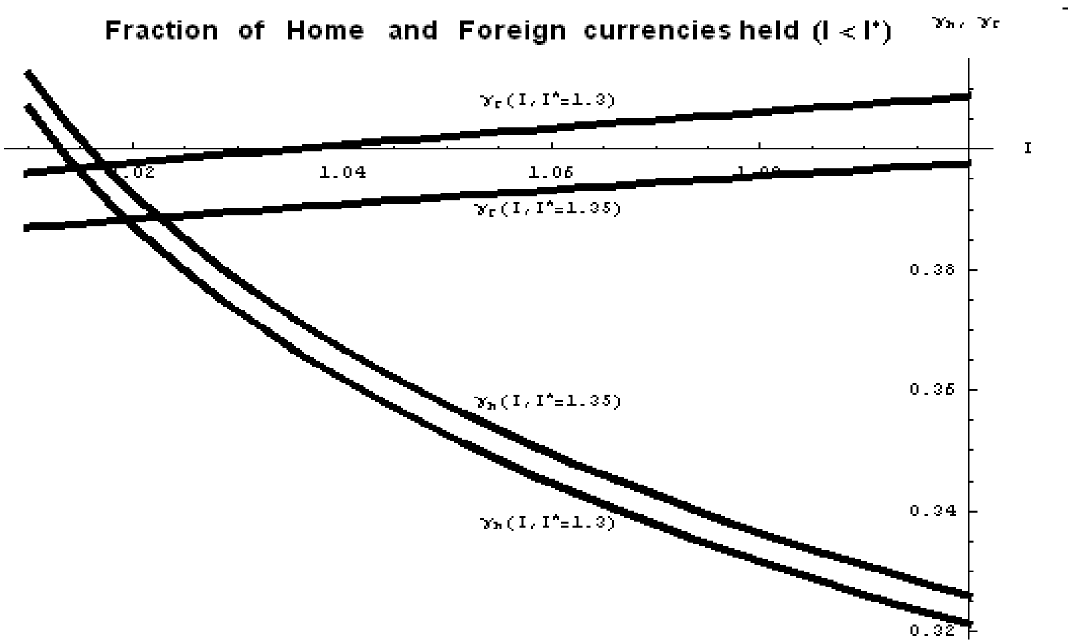

Figure 2

Discussion Contrast the two cases $I^h > I^p$ against $I^h < I^p$. In general, banks would like to equate MRT with MRS between the three groups in expected terms. While they know the number of home movers, they do not know how many non-moverss and foreign movers are going to be out of the remaining $1 - \pi$. They would like to insure foreign movers in case too many of them realize. This requires reserving more dollars than their expected population share. Why? So that if too many come, they don't end up with too low a consumption. But, then, reserving "extra" comes at the cost of sacrificing storage returns. Furthermore, given the asymmetric nature of payouts, any dollars left when too few foreign movers arrive can only be handed out to non-movers; but not to home movers. In this sense, the problem is jointly of foreign movers and non-movers. The scheme gives same return to everyone if low λ is realized, gives more to non-movers when λ is large. As long as the common return (for small λ) is higher than what pesos get, there is nothing that home movers can do: even when too many foreign movers arrive and their return falls below home movers, the latter are unable to help them. And of course, with few foreign movers, home movers have a lower return than the rest anyway. Then, the question is what share to reserve as pesos.? Here, one can argue that, given the log utility, it is best to give π to π number of home movers.

However, if the home movers' return is high enough (i.e., I^h is low enough) and λ is small, home movers can surely help by sharing some pesos with non-movers (and the latter can let foreign movers have more dollars in turn). But what is the gain for the home movers in this scheme? Suppose peso share is still π, then home movers will be net losers. So, in order to get them involved, which ex-post makes sense, they must be compensated by a peso share higher than π. Thus, when when λ is high, non-movers get high returns, home movers no longer need to pay (reinsure) them, and they can also enjoy a higher return (higher than peso returns ($\gamma^h \frac{\rho}{\pi I^h} > \frac{\rho}{I^h}$). Thus, a peso share higher than π compensates homemovers under states of nature with low λ.

3 General equilibrium, welfare, and policy

We allow the government to conduct monetary policy by changing the nominal stock of fiat currency at a fixed non-stochastic gross rate $\mu > 0$ per period, so that $M_t = \mu M_{t-1}$ for all t. If the net money growth rate is positive then the government uses the additional currency it issues to purchase goods, which it gives to current young agents (at the start of a period) in the form of lump-sum transfers. If the net money growth rate is negative, then the government collects lump-sum taxes from the current young agents, which it uses to retire some of the currency. The tax (+) or transfer (−) is denoted τ_t. The budget constraint of the government is

$$\tau_t = \frac{M_t^h - M_{t-1}^h}{p_t^h} = m_t^h - m_{t-1}^h \left(\frac{p_t^h}{p_{t+1}^h} \right) \tag{27}$$

for all $t \geq 1$.

In a stationary equilibrium,

$$\left(\frac{p_t^h}{p_{t+1}^h} \right) = \frac{1}{\mu} \text{ and } I^h = \mu\rho;$$

then $\tau = \left(1 - \frac{1}{\mu}\right) m$ holds. For future reference, note that τ maybe thought of as the amount of seigniorage. We assume that seigniorage is paid as lump sum transfers to only young domestic agents. Since $\gamma^h (w + \tau) = m$, we have

$$\tau = \left(1 - \frac{1}{\mu}\right) \left[\frac{\gamma^h w}{1 - \left(1 - \frac{1}{\mu}\right) \gamma^h} \right],$$

and

$$w + \tau = \frac{w}{1 - \left(1 - \frac{1}{\mu}\right) \gamma^h}.$$

Recall, however, that banks take τ as given and therefore their equilibrium allocations $\left\{\gamma^h, \gamma^f\right\}$ are independent of the fact that equilibrium τ is a function of γ^h.

Fix $I^f = 1.04$, $\rho = 1.04$, $\pi = 0.2, \bar{\lambda} = 0.6$. The following Figure 3 shows how the welfare of the economy varies with the money growth rate μ.

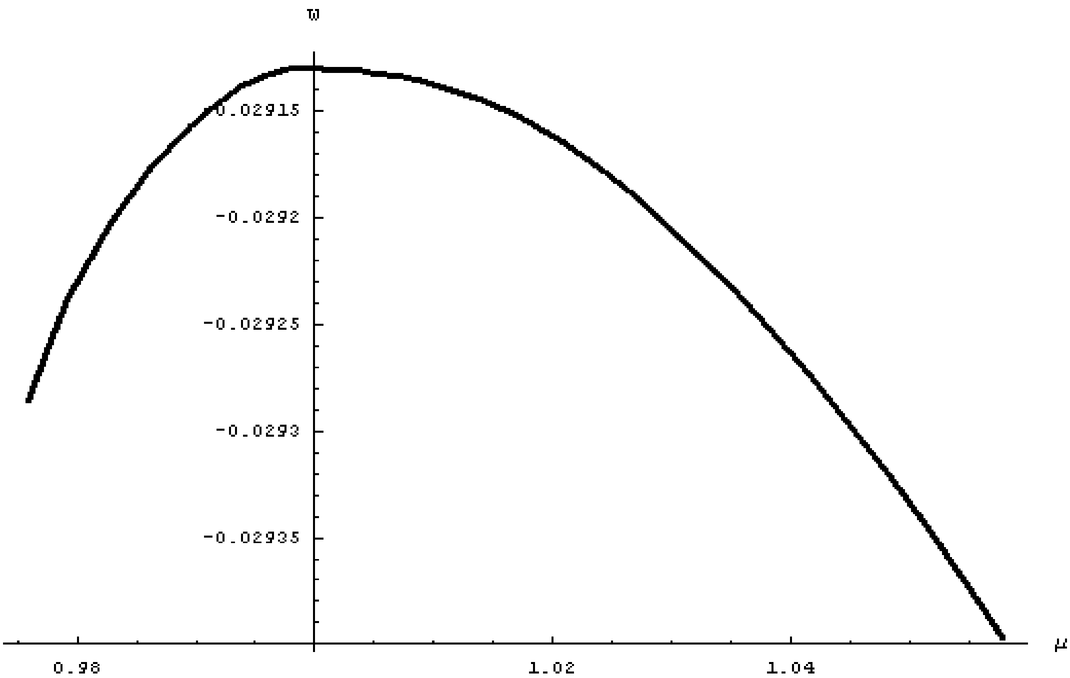

Figure 3

Proposition 3 *Let the world rate of inflation be zero. Then the optimal monetary policy is a fixed money supply, i.e.,* $\tilde{\mu} = 1$.

The intuition behind this result is simple. Given the fact that reserving domestic or foreign cash for movers is dominated by storage in the rate of return, the optimal allocation simply attempts to equate the marginal rate of substitution between consumption of different types of agents with their relative marginal rates of transformation. Any monetary policy other than a fixed money supply introduces further distortions through transfers from home-movers to the young of the next generation. Irrespective of whether these transfers are positive or negative, resulting allocations put a further wedge in the planner's allocations and, therefore, are inefficient.

How does $\hat{\lambda}$ behave as a function of $\mu\left(I^h\right)$, i.e., how does the probability of a banking crisis change with the domestic monetary policy or, in turn, nominal interest rate? For the same parameter specifications as in Figure 3, the following Figure 4 presents $\hat{\lambda}(\mu)$. Recall the higher the value of $\hat{\lambda}$, the lower the probability of a crisis. Notice that domestic interest rate can affect $\hat{\lambda}$ only when $I^h < I^p$. Notice that I^p lies between 1.05 and 1.075.

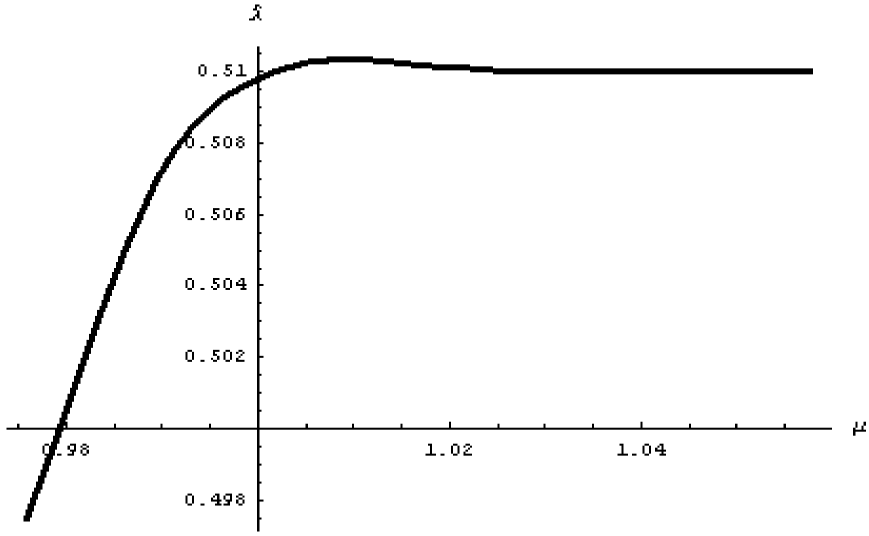

Figure 4

Discussion It is clear that the welfare is maximized unambiguously by keeping the money supply or the price level constant, i.e., $\mu = 1$. However, the probability of a dollar liquidity crisis may or may not be minimized at $\mu = 1$. It has been shown that as I^h increases, the crisis probability decreases. For simplicity, let's assume that the monetary policy in the rest of the world is a constant money supply rule (it can be easily justified that such a rule is optimal for the rest of the world too). Then, $I^f = \rho$. Then we have the following Lemma.

Lemma 1 *When $\mu^f = 1$, $\mu = 1$ minimizes crisis probability in addition to maximizing domestic welfare.*

4 Uncertainty and alternative exchange rate regimes

As observed above, with the rest of the world following an optimal monetary policy, the domestic optimal policy calls for a fixed money supply. In our determinstic set up, this also implies that the price level or the nominal exchange rate is also fixed. Note, however, that in an uncertain world where agents face exogenous uncertainty, either in terms of their endowment or preferences, the exchange rates will fluctuate if a fixed money supply policy is adopted. Then, the question arises: which exchange rate regime, fixed or flexible, is welfare superior? Does the regime that obtains a higher welfare also provides a better insurance against a dollar liquidity crisis? In order to meaningfully analyze these issues, in what follows, we introduce intrinsic uncertainty in the model.

In particular, we propose that young agents in each period arrive with with uncertain endowment w. Specifically w is i.i.d with a known distribution over support $[\underline{w}, \bar{w}]$. Now there are two sources of uncertainty. While w_t is realized before banks make portfolio allocations, λ is realized afterwards. Hence, banks' intial portfolio allocation rules γ^f and γ^h as well as second stage reserve distribution rules α and β will be contingent on w. In addition, α and β will also be λ-contingent. As a result, uncertain endowments will induce a stochastic distribution of both peso and dollar real balances. Below, we solve for stationary stochastic allocations as a fixed point of a rational expectations equilibrium.

As before we continue to use λ without time subscript since it is realized only after all λ-contingent choices are made. However, all the rules will now depend on the current realization of w_t. In order to denote their w_t dependence, we use time subscripts for all choices made at the beginning of t.

After w is realized, the banks' problem can be stated as

$$\max_{\left\{\gamma_{tt}^f, \gamma_t^h, \alpha_t(\lambda), \beta_t(\lambda)\right\}} \int_{w_{t+1}} \left\{ \int_{\lambda} \left[\pi \ln r_t^h(\lambda) + \lambda \ln r_t^f(\lambda) + (1-\pi-\lambda) \ln r_t^n(\lambda) \right] f(\lambda)\, d\lambda \right\} f(w_{t+1})\, dw_{t+1} \tag{28}$$

subject to (7a) - (7c) which are repeated below for convenience:

$$\begin{aligned}
r_t^f(\lambda) &= \frac{\alpha_t(\lambda)}{\lambda} \gamma_t^f \frac{\rho}{I_t^f}, \\
r_t^h(\lambda) &= \frac{\beta_t(\lambda)}{\pi} \gamma^h \frac{\rho}{I_t^h}, \\
r_t^n(\lambda) &= \frac{\rho}{1-\pi-\lambda} \left[(1-\alpha_t(\lambda)) \frac{\gamma_t^f}{I_t^f} + (1-\beta_t(\lambda)) \gamma^h \frac{1}{I_t^h} + \left(1 - \gamma_t^h - \gamma_t^f\right) \right],
\end{aligned}$$

Note that w_{t+1} denotes next period's endowment and $f(w_{t+1})$ is its density function. In addition, $\gamma_t^f, \gamma_t^h \geq 0$, $\gamma_t^h + \gamma_t^f \leq 1$, and $\alpha_t, \beta_t \in [0,1]$. Notice that portfolio allocations and reserve distribution rules carry a subscript t which denotes that they are w-contingent. As in the deterministic case, the problem is solved in two stages. In the second stage, after λ is realized, $\alpha_t(\lambda)$ and $\beta_t(\lambda)$ are optimally chosen given γ_t^f and γ_t^h. The first stage takes these optimal ex-post state-contingent rules into account and determines γ_t^h and γ_t^f.

Without loss of generality, we assume that dollar inflation rate is zero; then $I^f = \rho$. Since γ^h and γ^f are now w-contingent, a constant money supply rule will imply that the nominal exchange rate will be stochastic. On the other hand, if the goverment sets an exchange rate peg, the money supply will be stochastic. Potentially, the equilibrium allocations and welfare under the two scenarios will be different, which is the subject of study in the next section.

A word on notation will be in order. Note that all ex-ante $\left\{\gamma^h, \gamma^f\right\}$ and $\{\alpha, \beta\}$ rules will now be w-contingent. These rules will be chosen after forming expectations of next period's real balances that in equilibrium, that itself will depend upon the aggregate rules $\{\gamma^h, \gamma^f\}$ and $\{\alpha, \beta\}$. In turn, w-contingent rules will induce these aggregate rules. In a stationary rational expectations equilibrium agents except these rules to be followed next period and form their expectations of the aggregate variables on this basis. It is in this sense that allocation rules and induced distributions constitute a rational expectations equilibrium. In equilibrium, current allocation rules coincide with future rules. Hence, given a realized value of w, each period's problem is identical. Essentially, the solution entails finding a fixed point of ex-ante portfolio functions $\gamma^h(w)$, $\gamma^f(w)$, $\alpha(w;.)$ and $\beta(w;.)$.

4.1 Flexible exchange rate regime

Under a flexible exchange rate system, the money supply is kept constant, i.e., $M_t^h = \bar{M}^h$ for all t. Then, $\tau_t = \frac{M_t^h - M_{t-1}^h}{p_t} = 0$. Hence, $m_t^h = \gamma_t^h w_t$. Then $I_t^h = \rho\frac{e_{t+1}}{e_t} = \rho\frac{m_t^h}{m_{t+1}^h} = \rho\frac{\gamma_t^h w_t}{\gamma_{t+1}^h w_{t+1}}$. Then, banks' rate-of-return constraints can be rewritten as

$$r_t^f(\lambda) = \frac{\alpha_t(\lambda)}{\lambda}\gamma_t^f, \tag{30a}$$

$$r^h(\lambda) = \frac{\beta(\lambda)}{\pi}\frac{\gamma_t^h m_{t+1}^h}{m_t^h} = \frac{\alpha_t(\lambda)}{\lambda}\gamma_{t+1}^h\frac{w_{t+1}}{w_t}, \tag{30b}$$

$$r^n(\lambda) = \frac{1}{1-\pi-\lambda}\left[(1-\alpha(\lambda))\gamma_t^f + (1-\beta(\lambda))\gamma_{t+1}^h\frac{w_{t+1}}{w_t} + \left(1-\gamma_t^h-\gamma_t^f\right)\rho\right], \tag{30c}$$

Thus, banks maximize (28) subject to (30a) - (30c). After λ is realized, the first-order-condition for the choices of α and β are given by

$$(1-\pi-\lambda)E_t\left\{\frac{1}{(1-\alpha_t(\lambda))\gamma_t^f + (1-\beta_t(\lambda))\gamma_{t+1}^h\frac{w_{t+1}}{w_t} + \left(1-\gamma_t^h-\gamma_t^f\right)\rho}\right\} \le \frac{\lambda}{\alpha_t(\lambda)\gamma_t^f}, \quad \text{“ = ” if } \alpha_t(\lambda) < 1 \tag{31}$$

$$(1-\pi-\lambda)E_t\left\{\frac{\gamma_{t+1}^h\frac{w_{t+1}}{w_t}}{(1-\alpha_t(\lambda))\gamma_t^f + (1-\beta_t(\lambda))\gamma_{t+1}^h\frac{w_{t+1}}{w_t} + \left(1-\gamma_t^h-\gamma_t^f\right)\rho}\right\} \le \frac{\pi}{\beta_t(\lambda)}, \quad \text{“ = ” if } \beta_t(\lambda) < 1 \tag{32}$$

Banks' focs, (31) and (32), command that the expected value of a marginal peso (dollar) be equal across home (foreign) movers and non-movers, in terms of their marginal utilities of consumption. Equation (31) states that once λ is realized, the bank would like to equalize the expected marginal utilities of consumption across foreign movers and non-movers unless it has already exhausted all its dollar reserves. Similar conditions exist between movers and non-movers as expressed by (32). Notice that (31) and (32) can be more compactly written as

$$E_t\left\{\frac{c_t^f}{c_t^n}\right\} \le 1, \text{ “ = ” if } \alpha_t(\lambda) < 1;$$

$$E_t\left\{\frac{c_t^h}{c_t^n}\right\} \le 1, \text{ “ = ” if } \beta_t(\lambda) < 1;$$

Below we show that much depends on the realization of w_t. If w_t is sufficiently high, e_t is low and expected devaluation or I_t^h is high. Below, we show that the allocation rules then coincide with those under the deterministic case. On the other hand, if w_t is sufficiently low the allocation rules may be significantly different.

4.1.1 The case of $w_t > \hat{w}$ If w_t is sufficiently high the expected return on pesos is sufficiently low. We begin with the conjecture that then banks separate home-movers from the foreign-movers' insurance problem. As long as banks decide not to involve home-movers into the insurance problem, the fact that home-movers' rate of return is uncertain is of no consequence. In order to compute equilibrium allocations, we first look for cases where the *expected* rate of return on pesos is sufficiently low so that, as in the deterministic case, banks' γ^h equals π and $\beta(\lambda) = 1$ for all λ. Recall that the expected rate of return on pesos is $E_t\left\{\frac{p_t}{p_{t+1}}\right\} = E_t\left\{\frac{m_{T+1}}{m_t}\right\}$. Define $I_t^e \equiv \frac{\rho}{E_t\left\{\frac{m_{t+1}}{m_t}\right\}}$ as the opportunity cost of peso reserves relative to storage.[4] Then following Lemma 2 verifies that the above conjecture is indeed correct.

Lemma 2 *Let $I_t^e > I^p \equiv \frac{1}{1-\frac{\gamma^f}{1-\pi}\left(1-\frac{1}{\rho}\right)}$ where γ^f is computed as in Proposition 1. Then the equilibrium allocations coincide with those in Proposition 1 (a)*

$$\gamma_t^h = \pi;\ \beta_t(\lambda) = 1 \text{ for all } \lambda$$

(b)

$$\alpha_t(\lambda) = \begin{cases} \frac{\lambda}{1-\pi}\left[1 + \frac{1-\pi-\gamma_t^f}{\gamma_t^f}\rho\right] \equiv \frac{\lambda}{\hat{\lambda}_t}, & \text{if } \lambda \leq \hat{\lambda}_t, \\ 1, & \text{if } \lambda > \hat{\lambda}, \end{cases}$$

where

$$\hat{\lambda}_t \equiv \frac{(1-\pi)\gamma_t^f}{\gamma_t^f + \left(1-\pi-\gamma_t^f\right)\rho},$$

The result follows from the fact that with exchange rate uncertainty it is only the home-movers that face a real rate of return uncertainty in case all pesos are disbursed to them. Recall that the realization of λ does not affect the expected return of pesos. Since the expected rate of return on pesos $E_t\left\{\frac{e_t}{e_{t+1}}\right\} = E_t\left\{\frac{m_{t+1}}{m_t}\right\} = \frac{\rho}{I^e}$, it follows that as long as the expected rate of return on pesos is less than than the "pooled" return on dollars and storage, $\frac{\rho}{I^p}$, it is optimal for banks to choose $\gamma = \pi$ and $\beta(\lambda) = 1$ for all λ. Note that even the threshold value $\hat{\lambda}$ is now a function of w_t; hence, it is denoted with a time subscript.

Note as $\gamma_t = \pi$ for all $I_t^e > I^p$ it follows that $E_t\left(\frac{m_{t+1}}{m_t}\right) = \frac{E_t\left(\gamma_{t+1}w_{t+1}\right)}{\pi w_t} < \frac{I^p}{\rho}$. Since w is i.i.d. $E_t\left\{\gamma_{t+1}w_{t+1}\right\} = E\left\{\gamma w\right\}$. Furthermore, as I^p is independent of w_t, $I_t^e > I^p$ holds if and only

[4] In equilibrium $m_t = \gamma_t^h w_t$. As w is i.i.d. and banks' problem each period, after w_t is realized, is oththerwise identical, the portfolio allocation rules γ_t^h and γ_t^f are stationary functions of w_t. Thus $I_t^e = \frac{\rho\gamma_t w_t}{E\{\gamma w\}}$.

if $w_t > \hat{w} \equiv \frac{\rho E\{\gamma w\}}{\pi I^p}$. Intuitively, when the current endowment is high relative to the average, the current price level (exchange rate) is lower than the average. Then the peso is expected to depreciate.

Once again, recall that I^p is determined endogenously when ex-ante banks choose to separate. As expected return on pesos gets higher, such that $\gamma_t^h = \pi$ may lead to peso returns exceed the pooled return on dollars and storage, banks will find it ex-post efficient to not to disburse all pesos to homemovers if λ is sufficiently low. In this event, allocation rules will be substantially different as discussed in Section 2 and in Proposition 2. It is now obvious that this will happen if and only if $w_t < \hat{w}$. This is taken up next.

4.1.2 $w < \hat{w}$ Here $w_t < \hat{w}$. Then the peso is expected to appreciate. It is then not efficient to disburse all the pesos to home movers when λ is sufficiently low because pooled dollar and storage return falls below expected peso returns. Banks' focs, (31) and (32), command that the expected value of a marginal peso (dollar) be equal across home (foreign) movers and non-movers, in terms of their marginal utilities of consumption. Of course, if λ is sufficiently high, the banks will disburse all the dollars to foreign movers. Similarly, if λ is too high, a low number of non-movers obtain a high return from storage, and then all pesos will be disbursed to home-movers.

In particular, when $\lambda < \hat{\lambda}_t$, the pooled return from dollar and storage is below that expected on pesos. Then banks allocation rules α and β ensure that everyone gets the same return. This is done by letting α linearly increase with λ. At $\lambda = \hat{\lambda}_t$ all the dollars are given to foreign movers, yet some pesos are retained for the non-movers. As λ gets larger a higher amount of storage goes to non-movers. Now banks would like to keep less of pesos for non-movers. Thus, β begins to rise as λ rises beyond $\hat{\lambda}_t$. Finally, when $\lambda = \tilde{\lambda}_t$, $\beta = 1$. For all higher λ values, all pesos (dollars) are disbursed to home (foreign) movers, and non-movers simply consume out of storage.

Formally, for all $\lambda \in [0, \hat{\lambda}_t]$, it will be optimal for banks to equalize returns across all groups as some of the pesos can be carried over to provide goods to non-movers during the next period. Thus, as long as $\lambda < \hat{\lambda}_t$, the expected marginal utility across all three groups is equalized and both (31) and (32) hold with equality. Manipulating them, $\hat{\lambda}_t$ is now obtained as

$$\hat{\lambda}_t = (1-\pi)\frac{E_t\left\{\frac{\gamma_t^f}{\left(1-\beta_t\left(\hat{\lambda}_t\right)\right)\gamma_{t+1}^h\frac{w_{t+1}}{w_t}+\left(1-\gamma_t^h-\gamma_t^f\right)\rho}\right\}}{1+E_t\left\{\frac{\gamma_t^f}{\left(1-\beta_t\left(\hat{\lambda}_t\right)\right)\gamma_{t+1}^h\frac{w_{t+1}}{w_t}+\left(1-\gamma_t^h-\gamma_t^f\right)\rho}\right\}}, \tag{33}$$

where $\beta_t\left(\hat{\lambda}_t\right)$ is implicitly determined from

$$\beta_t\left(\hat{\lambda}_t\right) = \frac{\pi}{1-\pi}\frac{1+E_t\left\{\frac{1}{\left(1-\beta_t\left(\hat{\lambda}_t\right)\right)\gamma_{t+1}^h\frac{w_{t+1}}{w_t}+\left(1-\gamma_t^h-\gamma_t^f\right)\rho}\right\}}{E_t\left\{\frac{\gamma_{t+1}^h\frac{w_{t+1}}{w_t}}{\left(1-\beta_t\left(\hat{\lambda}_t\right)\right)\gamma_{t+1}^h\frac{w_{t+1}}{w_t}+\left(1-\gamma_t^h-\gamma_t^f\right)\rho}\right\}}$$

For all $\lambda \leq \hat{\lambda}_t$, $\alpha_t(\lambda)$ and $\beta_t(\lambda)$ are now implicitly determined from (31) and (32). For all λ between $\hat{\lambda}_t$ and $\tilde{\lambda}_t$, $\beta_t(\lambda)$ is determined from (32). Finally, $\beta_t\left(\tilde{\lambda}_t\right) = 1$. Hence, from (32), $\tilde{\lambda}_t$ is given by[5]

$$\tilde{\lambda}_t = (1-\pi) - \pi \frac{I_t^e}{\gamma_t^h}\left(1-\gamma_t^h-\gamma_t^f\right) \tag{34}$$

where, as before, $I_t^e = \frac{\rho\gamma_t w_t}{E\{\gamma w\}}$. Note, however, that unlike the deterministic case, peso returns are uncertain. Furthermore, ex-post these returns are different among various groups. Thus, given the above allocation rules, ex-post rate of returns are given by

$$\begin{aligned}
&r_t^f(\lambda) = \alpha_t(\lambda)\frac{\gamma_t^f}{\lambda},\ r_t^n(\lambda) = \frac{1}{1-\pi-\lambda}\left[(1-\alpha_t(\lambda))\gamma_t^f + (1-\beta_t(\lambda))\gamma_{t+1}^h\frac{w_{t+1}}{w_t} + \left(1-\gamma_t^h-\gamma_t^f\right)\rho\right],\\
&r_t^h(\lambda) = \frac{\beta_t(\lambda)}{\pi}\gamma_{t+1}^h\frac{w_{t+1}}{w_t},\ \text{ for } \lambda \leq \hat{\lambda};\\
&\quad r_t^f = \frac{\gamma_t^f}{\lambda},\ r_t^h = \frac{\beta_t(\lambda)}{\pi}\gamma_{t+1}^h\frac{w_{t+1}}{w_t},\ \text{and } r_t^n = \frac{1}{1-\pi-\lambda}\left[(1-\beta_t(\lambda))\gamma_{t+1}^h\frac{w_{t+1}}{w_t} + \left(1-\gamma_t^h-\gamma_t^f\right)\rho\right],\ \text{ for } \lambda \in \left[\hat{\lambda},\tilde{\lambda}\right];\\
&\quad r_t^f = \frac{\gamma_t^f}{\lambda},\ r = \frac{\gamma_{t+1}^h}{\pi}\frac{w_{t+1}}{w_t},\ r^t = \frac{1-\gamma_t^h-\gamma_t^f}{1-\pi-\lambda}\rho,\ \text{for } \lambda \in \left[\tilde{\lambda},\bar{\lambda}\right]
\end{aligned} \tag{35}$$

4.1.3 Welfare under flexible exchange rates Recall that the λ-contingent rules take γ_t^h and γ_t^f as given, which, however, have to be obtained from the welfare maximization problem. Having obtained the rules as in (33) - (35), we can write the indirect utility as function of γ_t^f and γ_t^h. At the point when banks choose γ_t^f and γ_t^h, only w_t is known. In addition, banks know the the aggregate w-contingent rules which helps them form expectations. Given the distribution, banks'choices of γ_t^f and γ_t^h and resulting λ-contingent rules are functions of w_t. In turn, these rules induce aggregate rules that banks take as given for the next period. It is in this sense that the portfolio allocations are a fixed point of a rational expectations equilibrium. Thus banks' maximization problem can be rewritten as

$$W(w_t) = \max_{\gamma_t^h,\gamma_t^f}\int_{w_{t+1}}\left\{\int_0^{\bar{\lambda}}\left[\pi\ln r_t^h(\lambda) + \lambda\ln r_t^f(\lambda) + (1-\pi-\lambda)\ln r_t^n(\lambda)\right]f(\lambda)\,d\lambda\right\}f(w_{t+1})\,dw_{t+1}$$

subject to (33) - (35). However, solving this problem analytically is not possible and we resort to numerical techniques. For comparing flexible exchange rate regime with the fixed, we evaluate ex-ante welfare given by

$$W^{flex} = \int_w W(w)\,f(w)\,dw$$

[5] Rewrite $\tilde{\lambda} = (1-\pi) - \pi\frac{\rho w_t}{E\{\gamma w\}}\left(1-\gamma_t^h-\gamma_t^f\right)$. Define $\tilde{w}$ implicitly as $\bar{\lambda} = (1-\pi) - \pi\frac{\rho\tilde{w}}{E\{\gamma w\}}\left(1-\gamma_t^h-\gamma_t^f\right)$determined from the above equation. Remember that γ_t^h and γ_t^f are also functions of w_t. Thus, for all $w < \tilde{w}$ it will imply that $\beta\left(\bar{\lambda}\right) < 1$. Furthermore, for all $w_t < \underline{w}$, where $\underline{w}$ solves $\gamma_t\left(I^e\left(\underline{w}\right)\right) = 1-\gamma_t^f\left(I^e\left(\underline{w}\right)\right)$, and where γ and γ^f as before maximize the expected utility, no storage will be held.

4.2 Welfare under fixed exchange rates

As before, we assume that the world rate of inflation is zero. Then, under fixed exchange rates

$$\frac{e_{t+1}}{e_t} = \frac{p_{t+1}}{p_t} = 1$$

Then $I^h = \rho = I^f$. Given that all rate of returns are now fixed, banks allocation rules are obtained as in the deterministic case. Note further that $I^h = I^f > I^p$, and the rules simply follow from Section 2.3. Clearly then $\gamma^h = \pi$ and $\beta(\lambda) = 1$ for all λ and t. Furthermore, γ^f and $\alpha(\lambda)$ will also be time-invariant and will follow Proposition 1.

Welfare However, under fixed exchange rates, peso reserves will fluctuate over time. Recall that households income net of transfers is given by $w_t + \tau_t$. Hence, $m_t = \pi(w_t + \tau_t)$. In order to maintain the peg, the government will conduct monetary injections and withdrawals through lump-sum transfers and taxes such that

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t - m_{t-1}$$

Then $m_t = \pi(w_t + \tau_t)$ can be rearranged to obtain

$$m_t = -\phi m_{t-1} + \phi w_t \tag{36}$$

where $\phi = \frac{\pi}{1-\pi}$. It is assumed that $\phi < 1$. Thus, (36) induces an AR(1) of peso balances. It is easy to show that

$$m^e = \frac{\phi}{1+\phi}\bar{w}$$
$$Var\{m\} = \frac{\phi^2}{1-\phi^2}\sigma_w^2$$

For comparing with flexible exchange rates, we evaluate ex-ante welfare by using the stationary distribution of m obtained from (36) to evaluate

$$W^{fixed} = \int_0^{\bar{\lambda}} \left[\pi \ln r^h(\lambda) + \lambda \ln r^f(\lambda) + (1-\pi-\lambda)\ln r^n(\lambda)\right] f(\lambda)\, d\lambda + E\left\{\ln \frac{m}{\pi}\right\} \tag{37}$$

where banks allocation rules are as given by Proposition 1. Note that the expectation of the second term is obtained using (36).

4.3 Numerical results

To compare the two regimes, we assume the following parameter values: $\pi = 0.2, \bar{\lambda} = 0.4, \mu = \mu^f = 1$, $\rho = 1.04, \underline{w} = 0.9$ and $\bar{w} = 1.1$. The following Table 1 presents the numerical value for utility and crisis probability under the two regimes:

Table 1

	Consumption equivalent of welfare	Crisis probability
Fixed exchange rates	0.9755	0.188305
Flexible exchange rates	0.9769	0.211089

Evidently, flexible exchange rate obtain a higher welfare relative to fixed exchange rates (0.15%). However, the crisis probability is higher under flexible exchange rates.

To get an intuitive albeit informal sense of this, we start by noting that under the flexible rate regime, the exchange rate adjustments and consequent interest rate changes induce banks to tilt their portfolios towards the highest paying assets. Suppose for instance that markets anticipate a low devaluation in any period. Banks will then tilt their portfolio towards domestic reserves, which in turn will induce price level to fall. As a result, the expected devaluation will now be lower. Thus, in equilibrium, exchange rate adjustments will be muted due to changes in portfolio demand. Thus, even though the flexible exchange rate regime suffers from the rate-of-return uncertainly, the price mechanism ensures that the range of fluctuations is diminished. Fixed exchange rate regime, on the other hand, eliminates rate-of-return uncertainty, but the inefficiency that stems from income redistribution makes it worse. Essentially, the exchange-rate adjustment mechanism along with portfolio choice implies that banks will be allocating their assets in the most efficient way. However, note that, despite these adjustments the domestic assets sometimes may become too attractive, and banks will put less weight on dollar reserves under such cases. As a result, the probability of crisis under flexible exchange rates is higher than under fixed exchange rates.

5 Conclusion

Banking liquidity crises are fairly frequent events, often accompanied by huge resolution costs as well as current and future output losses. These crises also affect both the future conduct and scope of domestic fiscal and monetary policy as well as the decision of countries to peg or float their currencies. This paper studies the potential for liquidity crises and their concommitant domestic and international repurcussions in a microfounded general equilibrium dynamic model in the tradition of Diamond and Dybvig (1983) and Chang and Velasco (2000). More specifically, we produce a small open economy pure exchange overlapping generations model with random relocation along the lines of Smith (2002). The combination of random relocation and the assumed role of currency in interlocation trade creates random location - and country - specific liquidity needs. Banks naturally arise to provide much-desired insurance against these liquidity shocks. In this setting, if withdrawal demand for the bank's deposits is high enough, the bank will exhaust all its cash reserves and a banking crisis will occur. We provide a complete characterization of optimal interest rate policies in this setting. In a deterministic environment, we find that nominal interest rates that are desirable from a welfare perspective may also decrease the probability of liquidity crisis.

We then study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes. To that end, we introduce time-varying random endowments into the above structure. This makes the banks' portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks' portfolio choice is deterministic and constant over time and this is achieved by the central bank injecting or withdrawing nominal balances. This in turn generates a novel inter-generational income redistribution. Under a flexible exchange rate regime, the money supply stays constant (hence no income redistribution) but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. We go on to study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes by introducing time-varying random endowments into the above structure. This makes the banks' portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks' portfolio choice is deterministic and constant over time and this is supported by the injection/removal of nominal balances by the central bank. Under a flexible exchange rate regime, the money supply stays constant but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. Our results show that a flexible exchange rate regime is superior in an ex-ante welfare sense relative to the fixed exchange rate regime; however, the ordering is reversed when it comes to the likelihood of liquidity and currency crises.

References

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6 Appendix

6.1 Banks' constraints

The λ-contingent returns to agents, who are relocated abroad, relocated domestically, or not relocated must satisfy the following:

$$\lambda r_t^f(\lambda)(w+\tau_t) \leq \alpha_t(\lambda) m_t^f \frac{p_t^f}{p_{t+1}^f},$$
$$\pi r_t^h(\lambda)(w+\tau_t) \leq \beta_t(\lambda) m_t^h \frac{p_t^h}{p_{t+1}^h},$$
$$(1-\pi-\lambda) r_t^n(\lambda)(w+\tau_t) \leq (1-\alpha_t(\lambda)) m_t^f \frac{p_t^f}{p_{t+1}^f} + (1-\beta_t(\lambda)) m_t^h \frac{p_t^h}{p_{t+1}^h} + s_t\rho$$

where $\alpha(\lambda)$ and $\beta(\lambda)$ denote the fraction of reserves returned to agents relocated abroad and domestically, respectively. Using (4) in the above equations yields (5a) - (5c) in the main text.

6.2 Proof of Proposition 1

The proofs assumes a uniform distribution for λ over support $[0,\bar{\lambda}]$. The proof proceeds in following steps:

Step I: derivation of cutoff $\hat{\lambda}$ We first begin with the conjecture that $\beta = 1$ for all λ. Then, substituting $\beta(\lambda) = 1$ in (9) yields

$$\frac{1}{1-\pi-\lambda}\left[\frac{(1-\alpha(\lambda))\gamma^f}{I^f} + \left(1-\gamma-\gamma^f\right)\right] \geq \frac{\alpha(\lambda)}{\lambda}\frac{\gamma^f}{I^f}, \quad \text{“} = \text{”} \text{ if } \alpha(\lambda) < 1$$

With some algebra, the above yields

$$\lambda\left[\gamma^f + \left(1-\gamma-\gamma^f\right) I^f\right] \geq \alpha(\lambda)\gamma^f(1-\pi), \quad \text{“} = \text{”} \text{ if } \alpha(\lambda) < 1$$

or

$$\alpha(\lambda) \leq \frac{\lambda}{1-\pi}\left[1+\frac{1-\gamma-\gamma^f}{\gamma^f}I^f\right] \equiv \frac{\lambda}{\hat{\lambda}}, \quad \text{“ = ” if } \alpha(\lambda) < 1,$$

where $\hat{\lambda} = \frac{1-\pi}{\gamma^f+(1-\gamma-\gamma^f)I^f}$. The above equation directly yields (13) in the main text. The state-contingent returns offered by the banks are easily computed using (13) in (7a) and (7c) which lead to (14) - (16) in the main text. Using our conjecture that $\beta = 1$, it is clear from (7b) that $r^h(\lambda) = \frac{\gamma^h}{\pi}\frac{\rho}{I^h}$, which does not depend on I^f or on the realization of λ.

Step II: deriving γ^h and γ^f Next, , using (14) - (16) in (6), along with some rearrangement yields

$$\begin{aligned} W = &\int_0^{\hat{\lambda}} (1-\pi)\ln\left(\frac{\gamma^f}{\hat{\lambda}}\frac{\rho}{I^*}\right) f(\lambda)\,d\lambda + \int_{\hat{\lambda}}^{\bar{\lambda}} \lambda \ln\left(\frac{\gamma^f}{\lambda}\frac{\rho}{I^*}\right) f(\lambda)\,d\lambda \\ &+ \int_{\hat{\lambda}}^{\bar{\lambda}} (1-\pi-\lambda)\ln\left(\frac{1-\gamma^h-\gamma^f}{1-\pi-\lambda}\rho\right) f(\lambda)\,d\lambda + \int_0^{\bar{\lambda}} \pi \ln\left(\frac{\gamma^h\rho}{I\pi}\right) f(\lambda)\,d\lambda \end{aligned}$$

The ex-ante choice of γ^f is derived by maximizing the expression for stationary welfare in (17). Some algebra yields

$$\frac{\hat{\lambda}(1-\pi)}{\frac{\gamma^f}{I^*}+1-\gamma^h-\gamma^f}\left(\frac{1}{I^*}-1\right)+\frac{\bar{\lambda}^2-\hat{\lambda}^2}{2\gamma^f}-\frac{1}{1-\gamma^h-\gamma^f}\left(\bar{\lambda}-\hat{\lambda}\right)\left(1-\pi-\frac{\bar{\lambda}+\hat{\lambda}}{2}\right)=0$$

which simplifies to

$$\frac{\bar{\lambda}^2+\hat{\lambda}^2}{2\gamma^f} = \frac{\hat{\lambda}^2}{\gamma^f}I^* + \frac{\bar{\lambda}-\hat{\lambda}}{(1-\gamma^h-\gamma^f)}\left[1-\pi-\frac{1}{2}\left(\bar{\lambda}+\hat{\lambda}\right)\right]. \tag{39}$$

Also, γ^h is similarly computed from

$$-\frac{\hat{\lambda}(1-\pi)}{\frac{\gamma^f}{I^*}+1-\gamma^h-\gamma^f}-\frac{1}{1-\gamma^h-\gamma^f}\left(\bar{\lambda}-\hat{\lambda}\right)\left(1-\pi-\frac{\bar{\lambda}+\hat{\lambda}}{2}\right)+\frac{\bar{\lambda}\pi}{\gamma^h}=0$$

which further simplifies to

$$\frac{\bar{\lambda}\pi}{\gamma^h} = \frac{\hat{\lambda}^2}{\gamma^f}I^* + \frac{\bar{\lambda}-\hat{\lambda}}{(1-\gamma^h-\gamma^f)}\left[1-\pi-\frac{1}{2}\left(\bar{\lambda}+\hat{\lambda}\right)\right] \tag{40}$$

By subsituting $\gamma^h_h = \pi$ in (39) and (40), both equations can be rewritten as $G_f\left(\gamma^f\right) = 0$ and $G_h\left(\gamma^f\right) = 0$ respectively. Then, after some algebra it can be shown that $\gamma^f G_f\left(\gamma^f\right) = (1-\pi)\,G_h\left(\gamma^f\right)$. Hence, $\gamma^h = \pi$ is clearly the solution. Finally, γ^f is computed by observing from (39) and (40) that

$$\gamma_f = \frac{\bar{\lambda}^2+\hat{\lambda}^2}{2\bar{\lambda}} = \frac{\hat{\lambda}}{2}\left(\frac{\hat{\lambda}}{\bar{\lambda}}+\frac{\bar{\lambda}}{\hat{\lambda}}\right) \geq \hat{\lambda} \tag{41}$$

which can be rewritten as a cubic equation

$$(1-\pi)^2\left(\gamma^f\right)^2-\left(2\bar{\lambda}\gamma^f-\bar{\lambda}^2\right)\left[\gamma_f+\left(1-\pi-\gamma^f\right)I^f\right]^2=0,$$

Note from the above that γ^f is a function of I^f. It can be shown that $\frac{d\gamma^f}{dI^f}<0$. Then, it is easy to show that $\frac{d\hat{\lambda}}{dI^f}<0$. The higher the foreign interest rate, the lower γ^f, and hence lower is the threshold $\hat{\lambda}$, at which the foreign reserves are exhausted. Thus the probability of a dollar liquidity crisis is thus increasing in I^f.

Step III: $\hat{\lambda}<\bar{\lambda}$ and $\gamma_f>\hat{\lambda}$. First note that $\hat{\lambda}>\bar{\lambda}$ would imply that $\alpha\left(\bar{\lambda}\right)<1$. As $I^f>1$, foreign cash reserve is dominated in rate or return relative to storage. Hence, $\hat{\lambda}>\bar{\lambda}$ is ruled out. Suppose $\hat{\lambda}=\bar{\lambda}$ instead. Then, from (41) $\gamma_f=\bar{\lambda}$. Since $\gamma^h=\pi$, equation (40) would imply that $I^f=1$, which is ruled out by assumption. Note that $\gamma_f=\hat{\lambda}=\bar{\lambda}$ if and only if $I^f=1$

Step IV: verifying $\beta(\lambda)=1$ for all $I^h>I^p$ Note from (9) and (11) that $\beta(\lambda)=1$ for all λ if and only if

$$\frac{\alpha(\lambda)}{\lambda}\frac{\gamma_f}{I^f}\geq\frac{1}{\pi}\frac{\gamma_h}{I^h}\text{ for all }\alpha(\lambda)<1$$

Further, using (13) and $\gamma^h=\pi$, the above equation can be rewritten as

$$\gamma_f\geq\hat{\lambda}\frac{I^f}{I^h}$$

which leads to

$$I^h\geq\hat{\lambda}\frac{I^f}{\gamma^f}=\hat{\lambda}\equiv\frac{(1-\pi)}{1+\frac{\gamma^f}{1-\pi}\left(\frac{1-I^f}{I^f}\right)}=I^p$$

which is what we assume.

6.3 Proof of Proposition 3

The ex-ante welfare can be written as

$$W=\int_0^1\left[\begin{array}{c}\pi\ln\left[\frac{\gamma}{\pi}\frac{1}{I^h}\right]+\lambda\ln\left[\frac{\gamma^f}{\lambda}\frac{1}{I^f}\right]\\+(1-\pi-\lambda)\ln\left[\frac{1}{1-\pi-\lambda}\left\{(1-\beta(\lambda))\gamma^h\frac{1}{I^h}+(1-\alpha(\lambda))\gamma^f\frac{1}{I^f}+\left(1-\gamma^h-\gamma^f\right)\right\}\right]\\+\ln\rho+\ln w-\ln\left[1-\left(1-\frac{1}{\mu}\right)\gamma^h\right]\end{array}\right]f(\lambda)\,d\lambda$$

where γ^h,γ^f,α, and β are optimally chosen at the bank's level. Thus, partial derivatives of W with respect to these rules will yield zero by the envelope condition. Recall that $I^h=\rho\mu$. Then, the first order condition can be written as

$$\frac{dW}{d\mu}=-\frac{\pi}{\mu}-\frac{(1-\pi-\lambda)(1-\beta(\lambda))\gamma\frac{1}{I^h}\frac{1}{\mu}}{(1-\beta(\lambda))\gamma\frac{1}{I^h}+(1-\alpha(\lambda))\gamma^f\frac{1}{I^f}+(1-\gamma-\gamma^f)}+\frac{\frac{\gamma}{\mu^2}}{1-\left(1-\frac{1}{\mu}\right)\gamma}=0.$$

We assume that the world rate of inflation equals zero, i.e., $p_t^f = p^f$ for all t. Then, $I^f = \rho$. We conjecture that $\mu = 1$ is a solution of the above first order condition. This implies that $I^h = I^f > I^p$. Then, $\gamma = \pi$ and $\beta(\lambda) = 1$. Then, the above first order condition implies

$$\tilde{\mu} = 1.$$

6.4 Proof of Lemma 1

First note that with $\mu^f = 1$, $I^f = \rho$. Further, for $I^h < I^p$, $\hat{\lambda}$ is a function $\hat{\lambda}\left(I^h, \rho\right)$ with $\hat{\lambda}_{I^h} > 0$ and $\hat{\lambda}_\rho < 0$. On the other hand, for all $I^h > I^p$, $\hat{\lambda} \equiv \hat{\lambda}(I^p, \rho)$, i.e. $\hat{\lambda}$ is independent of I^h. Thus, any $\mu = \frac{I^p}{\rho}$ obtains the minimum crisis probability. As $I^p < I^f = \rho$, $\mu = \frac{I^p}{\rho} < 1$ minimizes crisis probability. Trivially, then $\mu = 1$ minimizes crisis probability in addition to maximizing welfare.

6.5 Proof of Lemma 2

After substituting $\beta(\lambda) = 1$, it is easy to see from (31) that all the terms on both sides are known at t, and can be rewritten as

$$\frac{1}{1-\pi-\lambda}\left[(1-\alpha_t(\lambda))\gamma^f + \left(1-\gamma_t-\gamma_t^f\right)\rho\right] \geq \frac{\alpha_t(\lambda)\gamma^f}{\lambda}, \quad \text{“} = \text{”} \text{ if } \ \alpha_t(\lambda) < 1$$

Although (32) still has $t+1$ terms on the left hand side it can be rearranged as

$$\frac{1}{1-\pi-\lambda}\left[(1-\alpha_t(\lambda))\gamma^f + \left(1-\gamma_t-\gamma_t^f\right)\rho\right] > \frac{\gamma_t^h \rho}{\pi I_t^e}$$

Compare the above two focs with those in the deterministic case, equations (9) and (11). If $I^e > I^p$, all the equations that lead to Proposition 1 are satisfied.