# The Dynamics of Retail Oligopolies 

Arie Beresteanu* and Paul B. Ellickson ${ }^{\dagger}$<br>Duke University

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#### Abstract

This paper examines competition between retail firms using a dynamic model of strategic investment. Employing a panel dataset of store level observations spanning seven major retail industries, we propose and estimate a fully dynamic model of chain level competition. Since firm's investment, entry, and exit decisions are modeled at the level of the chain, the unique, store level dataset that underlies our empirical results is aggregated to the firm level using a variety of industry sources. Building on the methods proposed by Bajari, Benkard, and Levin (2002), we employ a two-step estimation procedure in which policy functions are first estimated from each firm's observed actions and outcomes are then matched to an equilibrium condition using forward simulation. The parameters of the structural model are then used to evaluate merger policy.


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## 1 Introduction

Retail firms account for a surprising fraction of economic activity. These firms employ over $20 \%$ of the private sector workforce and produce nearly $13 \%$ of US GDP. Firms like Wal-Mart and Target have played a prominent role in the development and diffusion of information technologies, forcing upstream producers to lower prices and make complementary investments in cost reducing innovations. The rise of the "big box" format and a continued emphasis on one stop shopping has both increased the variety of products and lowered their costs. At the same time, many retail industries have become highly concentrated. Most "category killers" compete locally with only one or two rivals. In some categories, like office supplies, there are only two or three chains nationwide. Viewed more broadly, these industries exhibit a highly skewed size distribution: a few giant chains compete with a large number of marginal players. While the explosion in variety and reduction in price is unambiguously beneficial to consumers, the increase in concentration may be cause for concern. In particular, it is unclear whether these industries are tending toward monopoly, or if there are competing forces that maintain some symmetry, at least among the largest firms. The goal of this paper is to develop a model of retail chain competition in which this and other questions can be evaluated.

Understanding whether markets will eventually become dominated by a single firm requires identifying the form of strategic investment (Athey \& Schmutzler (2001), Besanko \& Doraszelski (2004)). For example, in markets where investments exhibit forces of increasing dominance, it is well known that small asymmetries tend to be exacerbated over time, yielding outcomes which are highly skewed (Athey \& Schmutzler (2001)). In contrast, in markets that exhibit global catch-up forces, equilibria tend to remain relatively balanced, even when firms are subject to idiosyncratic shocks (Besanko \& Doraszelski (2004)). In the context of retail competition, theorists have produced models consistent with both possibilities, providing an obvious role for empirical analysis. In the current paper, we develop a model of retail competition which accommodates both possibilities. By confronting the model with data from several distinct industries, we hope to characterize how firms behave in practice and, in so doing, identify the forces that shape competition between retail chains.

The theoretical framework proposed in this paper is based on Besanko \& Doraszelski's
model of capacity accumulation, which extends the Markov perfect equilibrium (MPE) framework developed in Ericson \& Pakes (1995). In the context of retail competition, a firm's capacity corresponds to the number of outlets it chooses to operate in each period. ${ }^{1}$ Following Ryan (2004), we allow firms to fully adjust the size of their chain each period by either opening new stores or closing existing ones. After making these investments, firms compete in the product market, which is characterized by a reduced form profit function that depends on the current state of the industry and the level of population. Firms are also subject to idiosyncratic shocks, treated here as private information. One set of shocks impacts the investment process, allowing asymmetries to develop despite the assumption of initially symmetric firms. Firms are also allowed to enter and exit, subject to a second set of random shocks.

As Besanko \& Doraszelski demonstrate, industry dynamics depend on both the form of product market competition and the degree to which investments are reversible. While the same form of competition is likely to hold across different industries, the ease of recovering investments is not. For example, video stores may be much easier to sell off than office supply stores or movie theaters. This provides a potential explanation for heterogeneity in industry evolution.

Our empirical strategy is to estimate this model of competition using data from seven distinct retail oligopolies. Using a unique census of retail firms, we constructed separate panels for each industry that track the dominant chains over eleven consecutive periods. Our estimator is based on the two-step procedure proposed by Bajari, Benkard \& Levin (2002) (henceforth BBL) and implemented in a similar context to ours by Ryan (2004). In the first step, we recover the firm's policy functions governing entry, exit, and investment. These functions characterize firms beliefs regarding the evolution of their state variables and the actions of their competitors. In the second step, we use the structure of the MPE to recover the parameters that make those beliefs optimal. Following BBL, this is accomplished via forward simulation. Using these estimates, we can then simulate various futures, compare these to behavior observed in the data, and characterize the most likely evolution of each industry. Furthermore, since we have recovered the structural parameters of the underlying

[^1]model, we will then be able to perform policy experiments. In particular, we would like to evaluate the impact of various proposed mergers and, after obtaining additional data, perform some welfare analysis regarding the impact of overinvestment.

This paper builds on both the sizable literature on estimating static entry games as well as more recent work on dynamics. Until recently, the empirical entry literature has mainly employed static frameworks. As a consequence, the early papers were somewhat limited in scope, focusing primarily on characterizing the number of firms that could fit into markets of various size. In a series of seminal papers, Bresnahan \& Reiss examined the relative importance of strategic and technological factors in determining market structure (Bresnahan \& Reiss (1987, 1990, 1991)). By comparing the threshold market size at which only a single firm could survive to that which could sustain a second entrant, the authors were able to distinguish empirically between the impact of sunk costs and the role of price competition. Berry (1992) extended this analysis to include both heterogeneity across firms and the impact of firm characteristics. More recently, Mazzeo (2002) and Seim (2004) have extended the static approach to incorporate various aspects of product differentiation, documenting the empirical importance of both location and quality. In all of these studies, firms were assumed to provide only single products. Moreover, the static setting has clearly limited our ability to evaluate either merger policy or changes in the environment, as these are explicitly dynamic concepts. The emphasis on static (really two-period) frameworks was a direct result of the complexity associated with estimating a truly dynamic model of competition. Until recently, the burden was virtually insurmountable, as estimation required solving explicitly for an MPE via a nested fixed-point procedure. This computational burden placed severe restrictions on the ability to model complex interactions. However, the application of two-step estimation techniques has eased the burden substantially (Aguirregabiria \& Mira (2002), Bajari et al. (2002), Pakes, Ostrovsky \& Berry (2002), and Pesendorfer \& SchmidtDengler (2002)), opening the door to much more realistic modeling possibilities. ${ }^{2}$ Our goal is to use these methods to estimate a truly dynamic model of entry in which firms are able to constantly adjust their optimal size. Our paper is closest to the work of Ryan (2004), who estimates a fully dynamic model of entry and investment in the cement industry. Using a

[^2]panel of firms in geographically distinct markets, he is able to recover the full cost structure of the industry and evaluate the welfare impact of a change in environmental policy.

The paper is organized as follows. Section 2 describes the construction of the dataset. Section 3 describes the theoretical framework. The empirical framework is described in Section 4. The results of the first and second steps of the estimation are presented in Section 5 and the results of the policy experiments (TBD) are contained in Section 6. Section 7 concludes.

## 2 Data

This paper is aimed at characterizing competition between large retail oligopolies. For the most part, we have chosen to focus on the so called "category killers," retail firms that specialize in providing a wide array of choices in a relatively narrow class of products. This retail segment has grown dramatically over the past 20 years and has consistently attracted the attention of the anti-trust authorities. As a point of comparison, we will also examine the fast food industry, which is also characterized by large chains (although many of these stores are franchised). Store level data on the number of retail establishments (both firms and stores) were drawn from biannual versions of the American Business Disk (ABD). Our data span from the first half of 1998 to the first half of 2003, yielding a total of 11 periods. Although the ABD is primarily a library reference database, its publisher (InfoUSA) also markets several commercial versions of this product used in constructing a variety of mailing lists. The ABD contains information on the identity and locations of over 12 million retail firms based on Yellow and White Page listings. The entries are updated continuously. Since it is sold as a marketing and research tool to individuals and libraries, the accuracy of the listings are cross-checked by direct phone calls to local businesses and through comparisons with other independent resources. InfoUSA has been the leading firm in this industry for over 15 years and advertises a $95 \%$ level of accuracy on its website. An earlier version of this database was used by Bresnahan \& Reiss (1987), who actually travelled to several of the markets in their sample. They claimed to find very few errors. Although we have found some discrepancies in the total store counts with those published in alternative sources (i.e. time-lines from the firm's own websites), the differences are relatively small in magnitude.

We constructed our sample as follows. Starting from the primary (six digit) SIC codes associated with each retail category (e.g. 5812-08 for fast food, 5943-01 for office supplies), we extracted the full set of firms associated with that classification. The ABD records the top 20 SIC codes that each store associates itself with, so this process sometimes produced an odd collection of firms. We then eliminated any obvious mis-classifications (e.g. drug stores in the sporting goods category, non-profit firms) and merged the full set of years using store identification numbers unique to each observation. This created a store level panel with 11 periods of data. At this point, we found some cases where stores exited and re-entered the same location with the same phone number and manager. This occurred often enough to suggest that these were not remodels or temporary shut-downs but more likely coding errors in the ABD database. Based on this conclusion, we filled in the "missing observations" for all of these cases.

The most challenging part of constructing the dataset involved linking individual stores to their parent firms. Unfortunately, ABD does not record any firm identifier codes, so we had to construct these ourselves. For tractability, we decided to focus on only the largest chains. We used Hoover's Online database ${ }^{3}$ to identify the dominant firms in each industry. Consulting both Hoover's and the firm's own websites, we then constructed a list of all the operating names used by each firm at any point in time ${ }^{4}$. We then collapsed the data across all years by name and constructed firm identifiers based on our master list. The ID codes were then merged back into the store level dataset. We followed the same procedure for each industry. For each firm in every period, we know the physical location of each store (geocode), every SIC with which it is associated, the number of years it has been in operation, and a categorical variable indicating its level of yearly sales. However, only the store count is used in the subsequent analysis.

Table 1 contains summary statistics for each of our industries. The number of major chains varies substantially, from a low of 3 in office supplies, to a high of 23 in fast food. Surprisingly, across the six category killers, the share of total stores operated by the top chains is clustered around two points, $13 \%$ and $30 \%$. Store density varies from a low of under 1 store per 100,000 in pet supplies to a high of almost 30 in fast food. Among

[^3]the category killers, video chains operate the highest number of stores. Although there is substantial variation in both the number of stores operated and number of markets served across all of these industries, the number of firms that contest each market is relatively stable (again, with the exception of fast food). As has been noted elsewhere, the majority of these markets are dominated by only 2 or 3 major firms. These markets are also very concentrated. Over a quarter of the MSAs are served by only one firm and the Herfindahl indices correspond to one firm concentration ratios $\left(C_{1}\right)$ in the range of $50-70 \%$.

Finally, there is a fair amount of turnover between periods. Although it is relatively rare for a firm to exit a market completely, store closures are almost as common as openings. Also, when firms enter a market for the first time, they tend to open only a single store.

## 3 Model

Our model of competition between retail chains is based on Besanko \& Doraszelski (2004) as adapted to the empirical framework of Bajari et al. (2002). The game is in discrete time with an infinite horizon. We observe $N$ firms ( $i=1, . ., N$ ) in $M$ geographic markets $(m=1, . ., M)$, taken here to be Metropolitan Statistical Areas (MSAs). For each market/period combination, firms are classified as either incumbents or potential entrants, based on whether they currently operate any stores in the given market. In each period, each potential entrant privately observes an idiosyncratic shock determining their sunk cost of entry. Based on this private draw, these firms decide whether to enter. Next, both the incumbent firms and the potential entrants who have decided to enter observe an additional shock to the marginal cost of investment, again treated as private information. At this point, firms decide on the optimal level of investment (i.e. how many stores to open or close) based on this investment draw and their current state. Of course, incumbents may also choose to exit the market (entrants are only allowed to exit after competing in the product market in the subsequent period). This decision is subject to a third (privately observed) shock governing the scrap value associated with exiting the market. After observing all of these shocks and making their investments, incumbent firms then compete in the product market (new entrants compete only in the subsequent period). The (reduced form) profit function characterizing the static payoff from competing in the product market

Table 1: Summary Statistics

|  | Office <br> supplies | Book <br> stores | Record <br> stores | Video <br> rentals | Pet <br> stores | Sporting <br> goods | Fast food <br> restaurants |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total number of chains | 3 | 9 | 17 | 5 | 6 | 19 | 23 |
| Chain stores share of SIC | 0.28 | 0.13 | 0.3 | 0.28 | 0.14 | 0.13 | 0.18 |
| Stores per 100,000 | 1.25 | 1.23 | 1.33 | 3.23 | 0.71 | 1.42 | 29.7 |
| Chain size |  |  |  |  |  |  |  |
| Average | 848 | 242 | 154 | 1509 | 241 | 137 | 2447 |
| Minimum | 787 | 19 | 6 | 77 | 59 | 11 | 51 |
| $\quad$ Maximum | 902 | 656 | 698 | 4940 | 585 | 634 | 9921 |
| Number of MSA served |  |  |  |  |  |  |  |
| Average | 231 | 104 | 59 | 167 | 79 | 58 | 192 |
| Minimum | 215 | 8 | 3 | 37 | 16 | 6 | 16 |
| Maximum | 256 | 267 | 214 | 321 | 177 | 219 | 327 |
| Number of chains per MSA |  |  |  |  |  |  |  |
| Average | 2.12 | 2.93 | 3.13 | 2.55 | 1.8 | 3.5 | 13.5 |
| Minimum | 1 | 1 | 1 | 1 | 1 | 1 | 6 |
| $\quad$ Maximum | 3 | 8 | 10 | 5 | 5 | 12 | 19 |
| Number of entries | 308 | 222 | 270 | 278 | 260 | 321 | 448 |
| Stores built per entry | 1.17 | 1.06 | 1.06 | 1.25 | 1.27 | 1.06 | 3.3 |
| Number of exits | 57 | 226 | 386 | 132 | 50 | 295 | 400 |
| Stores closed per exit | 1.04 | 1.14 | 1.1 | 1.18 | 1.02 | 1.07 | 1.34 |
| Stores Built per period | 1.36 | 1.26 | 1.17 | 2.05 | 1.45 | 1.14 | 1.85 |
| Stores closed per period | 1.18 | 1.33 | 1.32 | 1.52 | 1.09 | 1.16 | 1.64 |
| Average Herfindahl Index |  |  |  |  |  |  |  |
| All periods | 6169 | 4578 | 4749 | 5941 | 7452 | 4556 | 1148 |
| First period | 7071 | 4831 | 4784 | 6967 | 7885 | 4481 | 1141 |
| Last period | 5802 | 4717 | 5211 | 5607 | 7184 | 4515 | 1171 |

is assumed to depend only on the number of stores operated by each firm in the current period and the level of population (i.e. there are no unobserved shocks in this stage). For notational convenience we will suppress the market subscript in what follows.

In period $t$, each market can therefore be described by an ( $N+1$ )-dimensional state vector $s_{t} \in S$. The first $N$ components of this vector describe the number of stores operated by each firm, so that $s_{i t}$ indicates the number of stores operated by firm $i$ in period $t$. The final component of $s_{t}$ describes the population at time $t$.

Given the state at time $t\left(s_{t}\right)$, firms choose their levels of investment $I_{t}$ (i.e. the number of stores to open or close) simultaneously. Recall that this decision is conditional on the realization of their privately observed cost shock $\nu_{i t}$. We assume that these shocks are iid draws from a commonly known distribution $G(\cdot)$. Firms therefore choose their level of investments to maximize their discounted sum of future profits as given by

$$
\begin{equation*}
E \sum_{t=0}^{\infty} \beta^{t} \pi_{i}\left(I_{t}, s_{t}, \nu_{i t}\right) \tag{1}
\end{equation*}
$$

where we assume that the common discount factor is a known constant. Denoting the period profits from the static stage game as $\pi_{i}^{s}\left(s_{t}\right)$, we can rewrite this as

$$
\begin{equation*}
E \sum_{t=0}^{\infty} \beta^{t}\left(\pi_{i}^{s}\left(s_{t}\right)-C\left(I_{t}, s_{t}\right)\right) \tag{2}
\end{equation*}
$$

where $C\left(I_{t}, s_{t}\right)$ is the cost of building new stores.
Finally, we assume that the transition between states can be characterized by the distribution $P\left(s_{t+1} \mid s_{t}, I_{t}\right)$, where the dependence on $I_{t}$ applies only to the evolution of store counts (i.e. we assume that population evolves exogenously). Following BBL, we focus only on pure strategy Markov Perfect Equilibrium (MPE) and assume that the equilibrium observed in the data is unique.

Given the Markov profile $\sigma$ mapping states into investments $\left(\sigma: S \times \mathbb{R}^{n} \rightarrow I\right)$, the value function of firm $i$ can be written in recursive form as:

$$
\begin{equation*}
V_{i}(s \mid \sigma)=E_{\nu}\left[\pi_{i}\left(\sigma(s, \nu), s_{t}, \epsilon_{i t}\right)+\beta V_{i}\left(s^{\prime} \mid \sigma(s, \nu)\right) d P\left(s^{\prime} \mid \sigma(s, \nu), s\right)\right] \tag{3}
\end{equation*}
$$

For a strategy profile $\sigma$ to be an equilibrium, we then require that there be no firm $i$ and alternative (Markov) strategy $\sigma^{\prime}$ such that firm $i$ will prefer the alternative strategy $\sigma^{\prime}$

Figure 1: Timing

| 1 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| t | $\varepsilon_{\text {pos }}$ and $\varepsilon_{\text {neg }}$ are realized | Investment decisions are made and building costs/ closing scrap values are born/collected. | Stores that existed at the beginning of the period produce profit (including those who will be closed next period). | t+1 |

to $\sigma$ given that all of its rivals use profile $\sigma$. Specifically, $\sigma$ is an MPE if

$$
\begin{equation*}
V_{i}(s \mid \sigma) \geq V_{i}\left(s \mid \sigma_{i}^{\prime}, \sigma_{-i}\right) \tag{4}
\end{equation*}
$$

for all $i, s$, and $\sigma_{i}^{\prime}$. It is this set of inequalities that forms the basis for the second step of the estimation. Intuitively, the first step involves recovering (as flexibly as possible) estimates of both $\sigma(s, \nu)$ and $P\left(s_{t+1} \mid s_{t}, I_{t}\right)$. With these estimates in hand, we then use the equilibrium condition (4) to recover the dynamic parameters.

## 4 Entry, exit and investment costs

A firm's decision regarding which action to take depends on the expected future value resulting from that decision minus the cost of taking that action. To estimate the Markov process governing the transition between states, we need to decompose the cost associated with moving from one state to another into several components. Denote the number of stores that chain $i$ owns in the market in the current period $s_{i}$ and the number of stores that the chain intends to operate next period $s_{i}^{\prime}$.

When a firm first enters a market it incurs a sunk cost related to establishing a presence in a new market regardless of how many stores it intends to operate in that market. This cost is assumed to be a constant denoted by $\overline{E N T R Y}$. If a firm is already present in the market it can decide to exit the market. In that case the chain recovers a scrap value
associated with leaving the market. This scrap value is assumed to be a constant denoted by $\overline{E X I T}$. A chain that is present in the market (either a new firm who decided to enter or an incumbent who decided not to exit) can either increase or decrease the number of stores it operates or do nothing at all. The fixed cost of building a store is denoted by $F C$ and is assumed to be a random variable. A firm that chooses to close stores will recover a scrap value equal to $S C R A P$. We assume the following on the random costs:

$$
\begin{aligned}
F C & =\overline{F C}+\varepsilon_{p o s} \\
S C R A P & =\overline{S C R A P}+\varepsilon_{n e g} .
\end{aligned}
$$

$\overline{E N T R Y}, \overline{E X I T}, \overline{F C}$ and $\overline{S C R A P}$ are unknown parameters and $\varepsilon_{\text {pos }} \sim N\left(0, \sigma_{\text {pos }}^{2}\right)$ and $\varepsilon_{i n} \sim N\left(0, \sigma_{i n}^{2}\right)$. The error term associated with the fixed cost of building a store and the error term associated with the scrap value a firm receives from closing a store can be correlated. The assumptions regarding the correlation between the two errors are explained in Section 5. The correlation between $\varepsilon_{\text {pos }}$ and $\varepsilon_{\text {neg }}$ comes from the fact that both costs are related to real-estate prices and it is reasonable to assume that the random element in both are correlated. Other costs like labor costs and local taxes may affect FC and SCRAP differently and therefore the correlation between $\varepsilon_{\text {pos }}$ and $\varepsilon_{n e g}$ is not perfect. The timing in which these shocks are realized is as follows. In the beginning of the period $\varepsilon_{\text {pos }}$ and $\varepsilon_{\text {neg }}$ are realized. It is important to note that these investment decisions include entry and exit decisions if those are relevant. A positive investment for stores that are not present in the market means entry and negative investment equal to the number of stores operated by achain means exit. The fixed cost of building stores is born or the scrap value from closing them is collected. Firms also bear entry cost or collect exit values if those are relevant. At the end of the period the revenue from the stores that were open at the beginning of the period is collected. Investment matures but will become productive only at $t+1$. This timing is described in Figure 1.

## 5 Estimation Strategy

In this section, we describe our estimation strategy, which builds on the methodology developed by Bajari et al. (2002). The model is estimated using a two-step procedure. In
the first step, we estimate the policy functions that govern the transition between states, as well as the exogenous process determining the evolution of population. In the second step, these estimators are used to recover the parameters of the profit function by simulating many possible future paths. These steps are described in detail below.

### 5.1 First stage

Our strategy for estimating the policy functions governing the transition between states requires several assumptions. First, the process governing the transitions is a first-order Markov: when firms decide which state they want to move to in period $t+1$, they condition only on the state of the market at time $t$. Second, the firms in each market are ex-ante symmetric. This implies that firm $i$ treats the case where firms $j$ and $k$ operate $a$ and $b$ stores respectively the same as it would treat the case where they operate $b$ and $a$ stores instead. Third, all markets are treated as random draws from the same Markov process. These assumptions allow us to pool together the observations from all periods, markets, and firms. Suppressing market and time subscripts, let $s_{i}$ denote the number of stores operated by firm $i$ and $s_{-i}$ the vector indicating the number of stores operated by firms other than $i$. The same variables for the following period are denoted $s_{i}^{\prime}$ and $s_{-i}^{\prime}$ respectively, while other demographics of the market are denoted by $x$.

Conditional on its own number of stores, the number of stores operated by its competitors, and other market demographics, the firm makes two decisions jointly: how many new stores to open and how many stores to close. The choice of how many stores to close is limited by the number of existing stores: a firm can't close more stores than it owns. Of course, the option to close stores is also unavailable to potential entrants. The joint decision to open and close stores is modeled using the following bivariate ordered probit model:

$$
\begin{aligned}
& \text { Pos }_{i}^{*}=\left(s_{i}, s_{-i}, x\right)^{\prime} \beta-\varepsilon_{\text {pos }} \\
& \text { Neg }_{i}^{*}=\left(s_{i}, s_{-i}, x\right)^{\prime} \gamma+\varepsilon_{\text {neg }}
\end{aligned}
$$

where

$$
\left[\begin{array}{l}
\varepsilon_{\text {pos }} \\
\varepsilon_{\text {neg }}
\end{array}\right] \sim N\left(\left[\begin{array}{l}
0 \\
0
\end{array}\right],\left[\begin{array}{cc}
1 & \rho \\
\rho & \sigma^{2}
\end{array}\right]\right)
$$

and $\beta$ and $\gamma$ are unknown vectors of coefficients of an appropriate dimension. We do not
observe $\left(\right.$ Pos $_{i}^{*}$, Neg $\left._{i}^{*}\right)$ but we observe $\left(\right.$ Pos $\left._{i}, N e g_{i}\right)$ such that

$$
\begin{aligned}
& \text { Pos }_{i}=p \text { if } \pi_{p} \leq \operatorname{Pos}_{i}^{*}<\pi_{p+1} \\
& \text { Neg }_{i}=g \text { if } \nu_{g} \leq N e g_{i}^{*}<\nu_{g+1}
\end{aligned}
$$

where $p=0 . . P$ and $g=0 . . G$ and $\pi_{0}=\nu_{0}=-\infty, \pi_{P+1}=\nu_{G+1}=\infty$. As mentioned earlier, $G$ can depend on the number of stores that the chain currently operates and can be equal to zero, yielding a univariate ordered probit model. We estimate the coefficients $\beta$ and $\gamma$, the thresholds $\pi_{1} \ldots \pi_{P}$, and $\nu_{1} \ldots \nu_{G}$ and the parameters $\rho$ and $\sigma^{2}$ using maximum-likelihood. The log-likelihood function is given by

$$
\begin{align*}
l & =\sum_{i=1}^{N} \sum_{p=0}^{P} \sum_{g=0}^{G} d_{i p g} \operatorname{Pr}\left(\pi_{p} \leq \text { Pos }^{*} \leq \pi_{p+1}, \nu_{g} \leq N e g^{*}<\nu_{g+1}\right)  \tag{5}\\
& =\sum_{i=1}^{N} \sum_{p=0}^{P} \sum_{g=0}^{G} d_{i p g}\left(-F_{p+1, g+1}+F_{p, g+1}+F_{p+1, g}-F_{p, g}\right)
\end{align*}
$$

where $d_{i p g}=1$ if for observation $i, \operatorname{Pos}_{i}=p$ and $N e g_{i}=g$ and

$$
F_{p, g}=\Phi\left(\left[\begin{array}{c}
X^{\prime} \beta-\pi_{p} \\
\nu_{g}-X^{\prime} \gamma
\end{array}\right],\left[\begin{array}{l}
0 \\
0
\end{array}\right],\left[\begin{array}{cc}
1 & \rho \\
\rho & \sigma^{2}
\end{array}\right]\right)
$$

is the cumulative bivariate normal distribution.
To facilitate estimation, we restrict $P$ and $N$ to a small set of possible values. The following tables describes the decisions made by incumbents and potential entrants (to be added in the near future). They show that choosing $P=2$ and $N=2$ covers most of the choices made by firms in our sample. Therefore, category " 2 j represents "two or more". For a firm that is not present in the market the entry decision is estimated using univariate ordered probit model. For a firms already present in the market the decision is estimated using a bivariate ordered probit model described by the likelihood function (5). Due to boundery condition the incumbents are separated into two groups: those who operate only one store and those who operate two or more stores. The model choice model can be estimated in several ways. First of all one should make a decision whether the likelihood functions describing the decision of the three sub groups of firms share any common parameters. One obvious set of prameters that they can share are $\rho$ and $\sigma^{2}$ which describe the shocks to (de)investment. If we assume that the three sub groups draw $\left(\varepsilon_{\text {pos }}, \varepsilon_{\text {neg }}\right)$ from potentially
different distributions, then the three likelihoods can be estimated separately. If they share a comon $\rho$ and $\sigma^{2}$ then two options are available. First, a joint likelihood function should be estimated with $\rho$ and $\sigma^{2}$ appearing in all components of that joint likelihood function. Second, first order conditions can be driven from (5) and then stucked together and the parameters will be estimated through a GMM method. In this version of the paper we choose to estimate the likelihood models separately for the three sub populations. The predictions from the estimates in this step were compared with the frequencies computed from the sample and were found to produce quite accurate predictions (see Section 5).

### 5.2 Second Stage

Given the estimates from the first stage, we are now ready to estimate the parameters of the payoff and cost functions. Following Bajari et al. (2002), we specify a profit function which is linear in its arguments. This reduces the computational burden substantially. The payoff function we use is

$$
\begin{equation*}
R_{t}=\left(\alpha+\beta_{1} \frac{O W N_{t}}{p o p_{t}}+\beta_{2} \frac{O T H E R_{t}}{p o p_{t}}+\beta_{3} O N E_{t}+\beta_{4} T W O_{t}\right) \cdot 1_{\left[O W N_{t}>0\right]} \tag{6}
\end{equation*}
$$

where $O W N$ is the number of stores the chain operates, $O T H E R$ is the total number of stores operated by competitors, $O N E$ is a dummy variable equal to one if the chain has only one competitor in the market and zero otherwise, and $T W O$ is a dummy variable equal to one if the chain has two competitors in the market. The costs related to investment or de-investment are

$$
\begin{align*}
C= & \overline{E N T E R} \cdot 1\left(O W N_{t}=0 \text { and } O W N_{t+1}>0\right)  \tag{7}\\
& +\left(\overline{F C}+\varepsilon_{\text {pos }}\right)\left(O W N_{t+1}-O W N_{t}\right)^{+}+\gamma_{1}\left[\left(O W N_{t+1}-O W N_{t}\right)^{+}\right]^{2}  \tag{8}\\
& -\overline{E X I T} \cdot 1\left(O W N_{t}>0 \text { and } O W N_{t+1}=0\right) \\
& +\varepsilon_{\text {out }}-(S C R A P+\varepsilon)\left(O W N_{t+1}-O W N_{t}\right)^{-}-\gamma_{2}\left[\left(O W N_{t+1}-O W N_{t}\right)^{-}\right]^{2} \tag{9}
\end{align*}
$$

where $(x)^{+}$equals x if $x>0$ and zero otherwise and $(x)^{-}$equals $|x|$ if $x<0$ and zero otherwise and $1(\cdot)$ is the indicator function. The profit is then $\pi_{t}=R_{t}-C_{t}$ and the present value of the profit is $\pi=\sum_{t=0}^{\infty} \beta^{t} \pi_{t}$.

The results from the first stage can be viewed as three vectors of coefficients fully describing the entry, exit and investment decisions. These vectors fully describe each firm's
strategy, denoted by $\sigma$. We denote an alternative strategy by $\tilde{\sigma}$, where alternative means changing the values of the parameters governing the entry, exit and investment decisions. As noted above, we consider only symmetric MPE. If $\sigma$ is the optimal strategy, then using an alternative strategy $\tilde{\sigma}$ while the competitors use the strategy $\sigma_{-i}$ should yield a lower present value of profits than if the firm were to use $\sigma$. This should hold for any market, regardless of initial conditions. Specifically, $\pi_{i}\left(\sigma, \sigma_{-i}, s_{0} ; \alpha\right) \geq \pi_{i}\left(\tilde{\sigma}, \sigma_{-i} s_{0} ; \alpha\right)$ where $s_{0}$ is the initial state of the market (i.e. a vector representing the number of stores that each chain operates as well as exogenous market characteristics) and $\alpha$ is the vector of structural parameters of the profit function. We then estimate $\alpha$ using minimum-distance criteria based on this inequality. The goal is to find a parameter $\alpha$ such that the squared differences between $\pi_{i}\left(\sigma, \sigma_{-i}, s_{0} ; \alpha\right)$ and $\pi_{i}\left(\tilde{\sigma}, \sigma_{-i} s_{0} ; \alpha\right)$ are minimized for the cases where $\pi_{i}\left(\sigma, \sigma_{-i}, s_{0} ; \alpha\right) \geq \pi_{i}\left(\tilde{\sigma}, \sigma_{-i} s_{0} ; \alpha\right)$ is violated. This can be estimated using the following integral

$$
\int 1_{\left[\pi_{i}\left(\sigma, \sigma_{-i}, s_{0} ; \alpha\right)-\pi_{i}\left(\tilde{\sigma}, \sigma_{-i}, s_{0} ; \alpha\right)<0\right]}\left[\pi_{i}\left(\sigma, \sigma_{-i}, s_{0} ; \alpha\right)-\pi_{i}\left(\tilde{\sigma}, \sigma_{-i}, s_{0} ; \alpha\right)\right]^{2} d F\left(\tilde{\sigma}, s_{0}\right)
$$

where $F\left(\tilde{\sigma}, s_{0}\right)$ is some distribution on the possible perturbations on the strategy $\sigma$ and starting state $s_{0}$. In practice we evaluate this integral by perturbing the vector $\sigma$ and by picking a starting point $s_{0}$ randomly from starting points observed in our sample. As a result we evaluate the above integral using all the markets that appear in our data.

## 6 Empirical results

In this section we report the estimation results from the first and second stages of estimation. The first stage, as we discussed above, estimates the transition probabilities regarding to moving from one state to another. For a firm that is not present in the market the entry decision is estimated using univariate ordered probit model. For a firms already present in the market the decision is estimated using a bivariate ordered probit model described in Section 5. Due to boundery condition the incumbents are separated into two groups: those who operate only one store and those who operate two or more stores. Therefore, over all we are estimating three ordered probit models: one univariate model and two bivariate models. The results are summarized in the Tables 2, 3 and 4. The variables used in estimating the Probit models are $S H A R E$ which represent the precent of stores that the firm operates
out of the total number of stores in the market, $\triangle P O P$ which is the percent change in population and $N$ which is the total number of competitors faced by the firm in the market.

The goal in step one is merely to get a good estimates of the equilibrium choice behavious of the firms. In order to check the fit of the models (beyond the value of the likelihood which is unimformative) we compared the predictions of the model to the observed frequencies in the data. The comparisson was done in the following way. We computed the frequencies of actions as predicted by the model when the explanatory variables are at their mean value. We also computed the (unconditional) frequencies of the various action pairs from the data. The comparison is described in tables 5, 6 and 7 .

The reader should interpret the numbers presented in Tables 2, 3 and 4 with caution. The parameters do not necessarily have the right sign or magnitude. The estimates merely discribe the equilibrium behavious and the goal is to achieve a good prediction for the equilibrium behavior.

We also present here initial results from estimating the second step. The results presented here are for the Office Supplies industry only. The results demonstrate the coefficient that can be estimated using the second step.

Table 2: Bivariate Probit model for incumbents with two or more stores

|  |  | Positive investment | Negative investment |
| :--- | :--- | :--- | :--- |
| Coeffishients | SHARE | 1.32 | -0.96 |
|  | $\Delta P O P$ | -6.81 | 3.00 |
|  | $N$ | 0.56 | -0.4 |
| Thresholds | lower | 2.12 | 1.89 |
|  | upper | 2.87 | 3.51 |
| other | $\rho$ | 0.23 |  |
| parameters | $\sigma$ | 3.79 |  |
| Likelihood |  | -3413.8 |  |

Table 3: Bivariate Probit model for incumbents with one store

| Coeffishients | SHARE | Positive investment | -0.25 |
| :--- | :--- | :--- | :--- |
|  | $\Delta P O P$ | -12.13 | -3.23 |
|  | Negative investment |  |  |
|  | $N$ | 0.33 | 2.55 |
| Thresholds | lower | 1.85 | 0.51 |
|  | upper | 2.60 | 2.92 |
|  | $\rho$ | 0.04 | - |
| other | $\rho$ | 2.11 |  |
| parameters | $\sigma$ | -1064.5 |  |
| Likelihood |  |  |  |

Table 4: Univariate Probit model for potential entrants

|  |  | Positive investment |
| :--- | :--- | :--- |
| Coeffishients | $\Delta P O P$ | -4.31 |
|  | $N$ | -0.17 |
| Thresholds | lower | 1.27 |
|  | upper | 2.22 |
| Likelihood | -1165.7 |  |

Table 5: Predictted and observed frequencies of actions - incumbents with two or more stores

| predicted by the model |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 0 | 1 | 2 | cumulative |
| 0 | 0.700 | 0.068 | 0.015 | 0.782 |
| 1 | 0.133 | 0.018 | 0.004 | 0.155 |
| 2 | 0.052 | 0.008 | 0.002 | 0.063 |
| cumulative | 0.885 | 0.094 | 0.021 | 1 |
| Observed in the data |  |  |  |  |
|  | 0 | 1 | 2 | cumulative |
| 0 | 0.694 | 0.065 | 0.011 | 0.770 |
| 1 | 0.1364 | 0.023 | 0.004 | 0.163 |
| 2 | 0.047 | 0.014 | 0.005 | 0.067 |
| cumulative | 0.878 | 0.102 | 0.020 | 1 |

Table 6: Predictted and observed frequencies of actions - incumbents with one store

|  | predicted by the model |  |  |
| :--- | :--- | :--- | :--- |
|  | 0 | 1 | cumulative |
| 0 | 0.940 | 0.009 | 0.948 |
| 1 | 0.043 | 0.0005 | 0.043 |
| 2 | 0.009 | 0.0001 | 0.009 |
| cumulative | 0.991 | 0.009 | 1 |
| Observed in the data |  |  |  |
|  | 0 | 1 | cumulative |
| 0 | 0.924 | 0.018 | 0.942 |
| 1 | 0.046 | 0.002 | 0.047 |
| 2 | 0.010 | 0.0007 | 0.011 |
| cumulative | 0.980 | 0.020 | 1 |

Table 7: Predictted and observed frequencies of actions - potential entrants

|  | predicted by the model |
| :--- | :--- |
|  | 0 |
| 0 | 0.932 |
| 1 | 0.060 |
| 2 | 0.007 |
| cumulative | 1 |
|  | Observed in the data |
|  | 0 |
| 0 | 0.917 |
| 1 | 0.074 |
| 2 | 0.009 |
| cumulative | 1 |

Table 8: Stage two estimates

|  | Office <br> Supplies |
| :--- | :--- |
| constant | 0.504 |
| OWN/pop | 0.197 |
| OTHER/pop | -0.459 |
| ONE | 0.505 |
| TWO | -0.256 |
| $\overline{E N T E R}$ | 0.320 |
| $\overline{F C}$ | 0.003 |
| $\gamma_{1}$ | 0.333 |
| $\overline{E X I T}$ | -1.05 |
| SCRAP | -1.38 |
| $\gamma_{2}$ | -0.122 |



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[^0]:    *Department of Economics, Duke University, Durham NC 27708. Email: arie@econ.duke.edu.
    ${ }^{\dagger}$ Department of Economics, Duke University, Durham NC 27708. Email: paul.ellickson@duke.edu.

[^1]:    ${ }^{1}$ Alternatively, one might think of stores as competing in store density by locating as close as possible to the full set of consumers. This setting would then correspond to the original Ericson \& Pakes (1995) quality ladder example.

[^2]:    ${ }^{2}$ See Benkard (2004) for an early application of these methods to learning and strategic pricing in the commercial aircraft industry.

[^3]:    ${ }^{3}$ http://www.hoovers.com
    ${ }^{4}$ It is not uncommon for retail firms to operate under several "flag" names, especially if the firm has gone through several mergers and consolidations.

