The Timing of Acquisitions*

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5 January 2005

Abstract

We analyse the timing of competitive bids. There are two buyers in our model, each of whom decide when and how much to bid for an object. These agents' valuations are commonly known, but are driven by a stochastic state variable that varies randomly over time. We assume that agent 1 (2) has the higher valuation when the state is high (low). We show first that there is *delay* in equilibrium: agent 1 bids only when the state is sufficiently high; and agent 2 bids only when the state is sufficiently low. This delay is not necessarily present when there is a single agent; it is present with two agents because competition makes payoffs convex. Secondly, the *extent of delay* increases with the degree of uncertainty (i.e., both agents wait for more extreme values of the state before bidding). Thirdly, we show that there is *too much delay* in equilibrium, relative to the efficient solution. Finally, we show that, in our model, the seller wishes to sell immediately, and to choose the lowest degree of uncertainty about the value of the object.

Keywords: Real options, Timing games, Auctions, Acquisitions.

JEL classification: D44, D81, G34.

*We are grateful to seminar participants in Cambridge, Essex and Southampton, and V. Bhaskar and Juuso Välimäki in particular, for helpful comments. Further comments are very welcome. The latest version of the paper can be found at `http://www.soton.ac.uk/~ram2`. Robin Mason gratefully acknowledges financial support from the ESRC Research Fellowship Award R000271265.

1 Introduction

Many papers have studied the economics of takeovers and acquisitions. These have tended to focus on mechanisms by which takeovers are enacted or hindered, free-rider problems amongst shareholders, and the efficiency (or otherwise) of the market for corporate control. Few, however, have examined the issue of timing of acquisitions. When a number of potential bidders have positive valuations at the prevailing market price, what determines when the auction for the target is held? What does this imply for the pattern of takeover activity? Is this efficient from the perspective of the target firm's shareholders, and for society as a whole?

In this paper, we analyse the timing of competitive bids in a dynamic model. There are two potential buyers in our model, each of whom decides when and how much to bid for an object. These agents' valuations are commonly known, but are driven by a stochastic state variable that varies randomly over time. We assume that agent 1 (2) has the higher valuation when the state is high (low). For all values of the state variable, both agents are credible buyers, as each one's valuation always exceeds the seller's reservation value (which is normalised at zero). When one buyer decides to bid, a bidding war results, and the object is awarded to the highest bidder.

There are a number of applications for our model. The most direct is a complete information auction of a single unit of a good, when bidders' valuations have a common value component so that they satisfy a single-crossing property.[1] A related application involves two firms bidding to take over a target firm. In this application, the state variable is the level of demand in the target firm's industry. The bidders have different comparative advantages in running the firm, depending on the level of demand. For example, firm 1 may have a lower marginal cost of production than the target; while firm 2 has a lower fixed cost of production. Hence the value of the target to firm 1 increases with the level of demand, while firm 2's advantage is independent of demand. A third application is a variant of the second, when the bidders are in different countries rather than industries.[2]

[1]For example, suppose that agent $i \in \{1, 2\}$ has a valuation of $\eta_i + \lambda_i\theta$, where $\eta_1 < \eta_2$ but $\lambda_1 > \lambda_2$, and θ is a (common) state variable.

[2]We are grateful to Pauli Murto for suggesting this example.

In this application, the state variable is the exchange rate between the countries. If firm 1 is in the same country as the target, it has the higher valuation when the exchange rate is strong; when the exchange rate is weak, the foreign firm 2 has the higher valuation.

We show the following results. First, there is *delay in equilibrium*: agent 1 bids only when the state is sufficiently high, while agent 2 bids only when the state is sufficiently low. Secondly, the *extent of delay* increases with the degree of uncertainty (i.e., both bidders wait for more extreme values of the state variable before bidding). In deciding when to bid, the bidders balance two factors. The first is the benefit from delay that arises due to real option values. The second is the cost from being pre-empted i.e., the state variable changing by a large amount so that the rival bidder can acquire the object. (This second factor is absent in a standard real option setting with a single agent.) Our result shows that the first factor dominates.

Thirdly, we show that there is *too much delay in equilibrium*, relative to the efficient solution. The efficient solution does involve delay: the object is allocated to a bidder only when its valuation is sufficiently greater than its rival's. But the extent of delay in the efficient solution is less than in equilibrium. Fourthly, if the seller can choose the time at which the object is sold, then it sells the object immediately. If it cannot directly control the timing of acquisition but can choose the degree of uncertainty (for example, the seller may be a target firm choosing the riskiness of its investment projects), then it chooses the lowest possible degree of uncertainty in order to reduce the extent of bidding delays. Finally, we use numerical analysis to contrast the seller's preferences to the efficient choice of the level of uncertainty.

The main results of inefficient delay in equilibrium arise because of two general features of our model. There is delay in equilibrium because *competition makes payoffs convex.* The agents face a combination of irreversibility (they do not get a second chance to acquire the object if they lose) and uncertainty (valuations are driven by a stochastic state variable). There are, therefore, real options involved in the bidding decision. But irreversibility and uncertainty on their own are not sufficient to cause delay in our model. We assume that the agents' valuations are not convex in the state. This assumption means that a single agent bidding for the object, not faced with a rival bidder, bids immediately: there is no (real) option value to the bidder and hence no incentive to delay. Competition is also required for delay to occur. When an agent is faced with a rival bidder, its payoff function is convex since if it loses, it receives a payoff of zero. The convexity induced by competition gives agents an incentive to delay their bids. This finding runs counter to most of the literature on competitive real options, where competition hastens, rather than delays, option exercise.

There is too much delay in equilibrium because *competition creates negative externalities between bidders*. These externalities affect the level and shape of the equilibrium payoff functions, relative to the efficient payoff. The equilibrium payoff to a bidder is lower than the efficient payoff (the difference being the other bidder's valuation). Also, the equilibrium payoff functions (determined by relative valuations) may be more or less convex than the efficient payoff function (determined by actual valuations). We concentrate on the levels effect by restricting the analysis of the efficient solution to the case where agent 1's valuation is a linear function of the state variable θ ($s_1(\theta) = \lambda\theta$ where $\lambda > 0$), while agent 2's valuation is a constant: $s_2(\theta) = \eta > 0$. The efficient payoff is then $\pi_E \equiv \max\{\eta, \lambda\theta\}$; the equilibrium payoff to agent 1 is $\pi_1 \equiv \max\{\lambda\theta - \eta, 0\} = \pi_E - s_2(\theta)$; to agent 2 is $\pi_2 \equiv \max\{\eta - \lambda\theta, 0\} = \pi_E - s_1(\theta)$. Hence the equilibrium payoff are lower than the efficient payoff, both when an agent wins and when it loses. The reduction in payoff on winning the object leads the agent to bid later (relative to the efficient solution); the reduction on losing leads to an earlier bid. We show that the former is more important when the agents choose their triggers optimally. This leads to inefficient delay in equilibrium—an effect that appears not to be specific to linear valuation functions.

The analysis has the following implications for takeover activity and the market for corporate control. First, it implies that even when there is competition between bidders, each of whom has a postive value of acquistion, the target may not be acquired immediately. Secondly, if the stochastic variable in each case is correlated with general economic variations, takeover activity will be highest during booms and busts, with lower activity in between. Finally, the market for corporate control may not be fully efficient as regards the timing of acquisition.

We are not aware of many papers that consider the timing of acquisitions. The field of empirical study of the pattern of firm takeovers is reasonably large (see, e.g., Weston, Chung, and Hoag (1990) for a review). There are fewer papers dealing with the issue theoretically. Lambrecht (2004) uses, like us, real option techniques to consider the timing of mergers. There are two crucial differences between his analysis and ours, however. First, he focuses on an agreed merger between two firms; thus, the key feature of competitive bidding for the target is missing from his framework. Secondly, the surplus from merger arises from economies of scale, and hence increases with industry demand; while we allow for more general payoff functions. It is this second assumption that drives Lambrecht's finding of pro-cyclical merger waves. Our paper also contributes to the small, but growing number of papers analysing strategic interaction in real options settings. See, for example, Smets (1991), Grenadier (1996), Hoppe (2000), Weeds (2002), Lambrecht and Perraudin (2003), and Mason and Weeds (2004).

In the rest of the paper, we first describe the model (section 2). We then characterise equilibrium in what we call *competitive cautious trigger strategies*; in this equilibrium, the *timing* of bidding is of primary interest. In section 4, we derive the efficient solution and contrast this to the equilibrium outcome. Section 5 summarises the main results, before section 6 considers the preferences of the seller.

2 The Model

Time is continuous and the time horizon is infinite with $t \in [0, +\infty)$. Two risk neutral agents each can bid to acquire an object. The decisions to bid and acquire can be delayed indefinitely. Once the object has been acquired by one bidder, no further actions are considered. This limits the analysis to one 'cycle' of acquisition, for simplicity. This implies no resale in the case of an auction; while in the context of a takeover, post-merger integration prevents subsequent acquisition of the original business unit. We do not assume a sunk cost of bidding (though this could be added).

We normalize the value of the object to the seller to zero. The valuations of agent 1 and 2, denoted $s_i(\theta) : \mathbb{R}_+ \to \mathbb{R}_+,\ i \in \{1, 2\}$, are functions of of an exogenous, stochastic variable $\theta \in \mathbb{R}_+$. The variable θ evolves according to a geometric Brownian motion (GBM) without drift:

$$d\theta_t = \sigma\theta_t dW_t \tag{1}$$

where $\sigma \in [0, +\infty)$ is the instantaneous standard deviation or volatility parameter, and dW_t is the increment of a standard Wiener process $\{W_t\}_{t\geq 0}$, so that $dW_t \sim N(0, dt)$. The continuous-time discount rate is $r > 0$. The parameters σ and r are common knowledge and constant over time. (The choice of continuous time and this representation of uncertainty is motivated by the analytical tractability of the value functions that result.)

The following assumptions are made about the bidders' valuations:

Assumption 1 *The functions $s_i(\theta) : \mathbb{R}_+ \to \mathbb{R}_+,\ i \in \{1, 2\}$ are continuous, twice-differentiable, positive and non-convex. Let $\delta(\theta) \equiv s_1(\theta) - s_2(\theta)$.*

(i) $\delta(\theta)$ is a (continuous, twice-differentiable) strictly increasing function of θ;

(ii) there is a unique $\theta^* > 0$ such that $\delta(\theta^*) = 0$, and $\delta(\theta) < (>)0$ when $\theta < (>)\theta^*$.

Continuous differentiability of valuations simplifies the analysis. Secondly, the fact that the bidders' valuations are non-negative means that we focus on the case in which it is efficient to sell the object; moreover, with two credible bidders at every value of θ, bidding occurs competitively in equilibrium. Thirdly, as we will make clear in section 2.1, assuming that valuations are non-convex ensures that there is a qualitative difference between the single-agent decision problem and the two-agent game. Finally, the single-crossing assumption on relative valuations implies that agent 1's valuation of the object is greater than agent 2's when θ is high (above θ^*); and that agent 2 has the higher valuation when θ is low (below θ^*).

We formulate the game form and the strategies of the agents to reflect the structure that we have imposed on the agents' valuations. Our main interest is in equilibria with the following two properties:

Property 1 (Competitive Cautious Bidding) *(i) if one agent bids, then so does the other; (ii) the agents epsilon-outbid each other; (iii) the losing agent bids cautiously.*[3]

Property 2 (Trigger Bids) *Agent* 1 *bids at the first instant that the state variable* θ *hits the interval* $[\theta_1, +\infty)$. *Agent* 2 *bids at the first instant that the state variable* θ *hits the interval* $[0, \theta_2]$.

Definition 1 (Competitive, Cautious, Trigger Equilibrium) *Any equilibrium that satisfies properties 1 and 2 is called a* CCT equilibrium.

(These properties will be restated later in terms of the strategies of the agents, which we are about to define.)

Restricting attention to competitive bidding means that we can focus on the *timing* (rather than the amount) of bids as our primary interest. We concentrate on cautious equilibria to rule out arbitrary outcomes in which the losing bidder effectively uses weakly dominated strategies. The same equilibrium notion has been used in the equilibrium analysis of e.g., Bergemann and Välimäki (1996) and Felli and Harris (1996). We only consider trigger strategies, for two reasons. First, they seem natural ones to consider, given the single-crossing property that we have assumed for the relative valuation function $\delta(\theta)$. The second reason is analytical tractability. There may be equilibria that involve non-trigger strategies i.e., where the 'stopping region' for an agent (e.g., the set $[\theta_1, +\infty)$ for player 1) is not an interval. Solving explicitly for such equilibria is not likely to be possible.

We therefore consider a game with two stages. The first stage is a timing game in which agents decide when to bid (i.e., at what level of the state variable). Once one or more of the agents decides to bid, the game then enters a second, bidding stage in which the agents both submit bids (if they wish). To formalize this story, at any time $t \geq 0$, let $x = 0$ if agent 1 has not made a bid for the target at any time $\tau \leq t$, and $x = 1$ otherwise; let $y \in \{0, 1\}$ indicate the same for agent 2. The state variable of the game is the triple (θ, x, y). At any time $t > 0$, past realizations of the process described by equation (1) and

[3] I.e., so as to be indifferent between winning and not winning at the equilibrium bids.

past decisions whether to bid constitute the history of the game. Agents are assumed to use stationary Markovian strategies: actions depend on only the current state and the strategy formulation itself does not vary with time.[4] A (pure) Markovian strategy for agent $i \in \{1,2\}$ (at time t) has two parts: (i) when $x_t = y_t = 0$, a measurable function[5] $m_i(\theta_t) : \mathbb{R}_+ \to \{0,1\}$ which takes the value 0 when the agent has not made a bid, and the value 1 when the agent makes a bid; (ii) when $x_t + y_t > 0$, a measurable function $b_i(\theta_t) : \mathbb{R}_+ \to \mathbb{R}$ describing the bid submitted by the agent.

In the bidding stage, agent i acquires the target if and only if its bid b_i exceeds its rival's b_{-i} and is greater than zero. If this is the case, then agent i receives a payoff $s_i(\theta)$ (which can be interpreted as the present discounted value of a flow payoff $\hat{s}_i(\theta)$ received in perpetuity after acquisition), while paying its bid to acquire the target. If agent i bids unsuccessfully, then it receives a (flow) payoff of zero. In the event of a non-zero tie ($b_1 = b_2 > 0$), the target is allocated randomly between the bidders. Hence the intertemporal payoff of agent i is

$$V_i(T, b_1, b_2; \theta_t) = \begin{cases} \mathbb{E}_t\left[(s_i(\theta_T) - b_i(\theta_T))e^{-r(T-t)}\right] & \text{if } b_i > \max[0, b_{-i}], \\ 0 & \text{if } b_i < \max[0, b_{-i}], \\ \frac{1}{2}\mathbb{E}_t\left[(s_i(\theta_T) - b_i(\theta_T))e^{-r(T-t)}\right] & \text{if } b_i = b_{-i} > 0, \\ 0 & \text{if } b_i = b_{-i} = 0, \end{cases} \tag{2}$$

where $T \equiv \min[T_1, T_2]$ and T_i is the (random) first time at which $m_i = 1$; the operator $\mathbb{E}_t$ denotes expectations conditional on information available at time t.

Definition 2 (Markov Perfect Equilibrium, MPE) *A pair of strategies* $(m_1^*, b_1^*), (m_2^*, b_2^*)$, *with* $T^* \equiv \min[T_1^*, T_2^*]$ *and* $T_i^* \equiv \inf\{t | m_i^*(\theta_t) = 1\}$, *is a Markov Perfect Equilibrium if*

[4] For further explanation, see Maskin and Tirole (1988) and Fudenberg and Tirole (1991). Non-Markovian equilibria may exist. Since we want to analyse how the resolution of uncertainty affects the take-over game, we concentrate on equilibria in Markovian strategies. This allows us to rule out collusive equilibria with continuation strategies that depend on information that is not payoff relevant.

[5] Measurability is with respect to the filtration $\mathcal{F}$ of the complete probability space $(\Omega, \mathcal{F}, P)$ on which the Wiener process $\{W_t\}_{t \geq 0}$ in equation (1) is defined.

and only if, for all θ_t and $i \in \{1,2\}$,

$$\begin{aligned} & V_i(T^*, b_i^*, b_{-i}^*; \theta_t) \geq V_1(T, b_i^*, b_{-i}^*; \theta_t), \forall\ m_i(\theta_t), \\ \text{where} \qquad & T \equiv \min[T_i, T_{-i}^*],\ T_i \equiv \inf\{t | m_i(\theta_t) = 1\}; \\ & V_i(T^*, b_i^*, b_{-i}^*; \theta_t) \geq V_i(T^*, b_i, b_{-i}^*; \theta_t), \forall\ b_i. \end{aligned}$$

Note that deviation strategies are not required to be Markovian.

2.1 The Single-Agent Decision Problem

In this model, it is relatively straightforward to determine when a single agent, not faced with a rival bidder, will bid. If the bidder's valuation function, $s(\theta)$, is not convex in the state θ (as we have assumed, in assumption 1), then it will bid immediately. In this case, there is no option value to the bidder and hence no incentive to delay bidding. Only if the valuation function is convex in θ will the bidder have an incentive to delay. As we shall see, this is in marked contrast to the competitive bidding situation: in the game between the two bidders, equilibrium always involves delay, even when the bidders' valuations are not convex in θ. This is because the prospect of a rival acquiring the target introduces a convexity into the bidder's payoff function; this convexity leads to delay in equilibrium.

3 Equilibrium

We are interested in equilibria that have cautious, competitive triggers bids. In terms of the agents' strategies and best responses, this implies the following properties:

Property 1 (Competitive Cautious Bidding) *For $i \in \{1,2\}$,*

$$b_i(\theta) \begin{cases} = b_{-i}(\theta) + \epsilon & \text{if } b_{-i} < s_i, \\ \leq s_i & \text{if } b_{-i} \geq s_i, \end{cases}$$

where $\epsilon > 0$ is arbitrarily small.

Property 2 (Trigger Bids)

$$m_1(\theta) = \begin{cases} 0 & \theta < \theta_1, \\ 1 & \theta \geq \theta_1; \end{cases} \qquad m_2(\theta) = \begin{cases} 0 & \theta > \theta_2, \\ 1 & \theta \leq \theta_2. \end{cases}$$

I.e., agent 1 (2) bids when θ rises (falls) to θ_1 (θ_2), where $\theta_1 > \theta_2$.

A CCT equilibrium is characterized, therefore, by the two trigger points θ_1 and θ_2. We first determine and analyze these trigger points; we then confirm that a CCT equilibrium exists.

The equilibrium analysis will be simplified by making the following assumptions on $\delta(\theta)$ (the difference in the agents' valuations):

Assumption 2 *1.*

(i) $-(\alpha+1) < \theta\delta''(\theta)/\delta'(\theta) < \alpha$ *for all* θ, *where* $\alpha > 0$ *is a constant determined below, and* $\delta'(\cdot)$ *and* $\delta''(\cdot)$ *denote the first and second derivatives of* $\delta(\cdot)$, *respectively;*

(ii) $\theta^2\delta(\theta)\delta''(\theta) - \alpha(\alpha+1)(\delta(\theta))^2 < 0$ *for all* θ *and* $\alpha > 0$.

Part (i) complements assumption 1 in ensuring that in equilibrium, agent 1 will be the successful bidder when θ is high, and agent 2 when θ is low (as the proof of proposition 1 makes clear). It requires that $\delta(\cdot)$ be neither too convex nor too concave. Part (ii) ensures that the agents' strategies are strategic complements; see the next section. Both parts of the assumption, as well as assumption 1, are satisfied by a linear $\delta(\cdot)$ function, for example.

3.1 Properties of a CCT Equilibrium

Detailed derivations of the agents' equilibrium value functions are contained in the appendix. There we show that the value function $V_1(\theta)$ of agent 1 has three components, holding over different ranges of θ:

$$V_1(\theta) = \begin{cases} 0 & \theta \leq \theta_2, \\ A_1\theta^{-\alpha} + B_1\theta^{\alpha+1} & \theta_2 < \theta < \theta_1, \\ s_1(\theta) - b_1(\theta_1) & \theta = \theta_1. \end{cases} \tag{3}$$

A_1 and B_1 are constants determined by boundary conditions, discussed below. α is the positive root of a 'characteristic' equation (see the appendix):

$$\alpha = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8r}{\sigma^2}}\right) \geq 0.$$

$B_1\theta^{\alpha+1}$ is an option term anticipating agent 1's bid for the target; $A_1\theta^{-\alpha}$ is an option-like term anticipating agent 2's bid.

By arbitrage, the critical value θ_1 for agent 1 must satisfy a value-matching condition; optimality requires a second, smooth-pasting condition to be satisfied. (See Dixit and Pindyck (1994) for an explanation.) This condition requires the components of the agent 1's value function to meet smoothly at θ_1. θ_2 is not chosen optimally by agent 1; hence the smooth-pasting optimality condition does not apply here for agent 1. Value functions are forward-looking, however, and so a value-matching condition applies at θ_2. Hence there are three relevant equations for agent 1—value-matching and smooth-pasting at θ_1; and value-matching at θ_2:

$$\begin{aligned} A_1\theta_1^{-\alpha} + B_1\theta_1^{\alpha+1} &= \delta(\theta_1), \\ -\alpha A_1\theta_1^{-\alpha-1} + (\alpha+1)B_1\theta_1^{\alpha} &= \delta'(\theta_1), \\ A_1\theta_2^{-\alpha} + B_1\theta_2^{\alpha+1} &= 0. \end{aligned}$$

These three equations can be combined to give

$$\begin{aligned} \frac{\theta_1^{2\alpha+1}}{\Theta(\theta_1)} &= \theta_2^{2\alpha+1}, \quad (4)\\ \text{where}\ \ \Theta(\theta) &\equiv \frac{\alpha\delta(\theta)+\theta\delta'(\theta)}{-(\alpha+1)\delta(\theta)+\theta\delta'(\theta)}. \end{aligned}$$

Equation (4) gives the reaction function of agent 1.[6] Note that the equation is well-defined iff $-(\alpha+1)\delta(\theta)+\theta\delta'(\theta)\neq 0$.

Definition 3 $\bar{\theta} > \theta^*$ *is such that* $-(\alpha+1)\delta(\bar{\theta})+\bar{\theta}\delta'(\bar{\theta}) = 0$.

Hence if $\theta < \bar{\theta}$, then equation (4) is well-defined, from assumption 2(i). From that assumption, if θ_2 is increased (decreased), agent 1's best response is lower (higher). Hence the agents' triggers are strategic complements—agent 1's best response to a more aggressive strategy from agent 2 (θ_2 closer to θ^*) is more aggressive (θ_1 is closer to θ^*). The following facts about $\bar{\theta}$ follow immediately from its definition (using assumptions 1 and 2) and so are stated without proof:

Result 1 *(i)* $\partial\bar{\theta}/\partial\sigma > 0$; *(ii)* $\lim_{\sigma\to 0}\bar{\theta} = \theta^*$.

A similar story holds for agent 2. Its value function is

$$V_2(\theta) = \begin{cases} s_2(\theta) - b_2(\theta_2) & \theta = \theta_2, \\ A_2\theta^{-\alpha} + B_2\theta^{\alpha+1} & \theta_2 < \theta < \theta_1, \\ 0 & \theta_1 \leq \theta. \end{cases} \quad (5)$$

Value-matching and smooth-pasting hold at θ_2, and value-matching at θ_1:

$$\begin{aligned} A_2\theta_2^{-\alpha} + B_2\theta_1^{\alpha+1} &= -\delta(\theta_2), \\ -\alpha A_2\theta_2^{-\alpha-1} + (\alpha+1)B_2\theta_2^{\alpha} &= -\delta'(\theta_2), \\ A_2\theta_1^{-\alpha} + B_2\theta_1^{\alpha+1} &= 0. \end{aligned}$$

[6] Equation (4) gives a reaction function rather than correspondence, since assumption 2 implies that the function on the left-hand side of equation (4) is strictly decreasing in θ.

These three equations can be combined to give

$$\frac{\theta_2^{2\alpha+1}}{\Theta(\theta_2)} \quad = \quad \theta_1^{2\alpha+1}. \tag{6}$$

Definition 4 *$\underline{\theta} < \theta^*$ is such that $\alpha\delta(\underline{\theta}) + \underline{\theta}\delta'(\underline{\theta}) = 0$.*

If $\theta > \underline{\theta}$, then equation (4) is well-defined, from assumption 2(i), and gives the reaction function of agent 2. Analogously to result 1:

Result 2 *(i) $\partial\underline{\theta}/\partial\sigma < 0$; (ii) $\lim_{\sigma\to 0}\underline{\theta} = \theta^*$.*

A necessary condition for a CCT equilibrium with $\theta_1 > \theta^* > \theta_2$ to exist is that there is a solution to the simultaneous equations (4) and (6) that satisfies this property. The next proposition establishes the existence of such a solution.

Proposition 1 *There exists a solution to equations (4) and (6) with $\bar{\theta} > \theta_1 > \theta^* > \theta_2 > \underline{\theta}$.*

Proof. The proof uses five facts about the reaction functions defined by equation (4) and (6). Note that the functions $\theta_1(\theta_2)$ and $\theta_2(\theta_1)$ defined by the two equations are continuously differentiable for $\theta \in (\underline{\theta}, \bar{\theta})$.

1. The reaction functions intersect at $\theta_1 = \theta_2 = \theta^*$. At these values, $\Theta(\theta_1) = \Theta(\theta_2) = 1$, since $\delta(\theta^*) = 0$; and $\theta_1/\theta_2 = 1$.

2. As $\theta_2 \to 0$, the solution to equation (4) must be such that $\Theta(\theta_1) \to \infty$ i.e., $\theta_1 \to \bar{\theta}$.

3. As $\theta_1 \to \infty$, the solution to equation (6) must be such that $\Theta(\theta_2) \to 0$ i.e., $\theta_2 \to \underline{\theta}$.

4. The derivative of agent 1's reaction function, when it exists, is obtained by total differentiation of equation (4):

$$\left((2\alpha+1)\frac{\theta_1^{2\alpha}}{\Theta(\theta_1)} - \frac{\theta_1^{2\alpha+1}}{(\Theta(\theta_1))^2}\Theta'(\theta_1)\right) d\theta_1 = (2\alpha+1)\theta_2^{2\alpha} d\theta_2.$$

By assumption 2(ii), the reaction function is downward-sloping. Straightforward calculations show that

$$\lim_{\theta_1 \downarrow \theta^*,\, \theta_2 \uparrow \theta^*} \frac{\partial \theta_1}{\partial \theta_2} = -\infty.$$

5. Similarly, the derivative of agent 2's reaction function, when it exists, is obtained by total differentiation of equation (6). Evaluated at $\theta_1 = \theta_2 = \theta^*$, this gives the derivative of agent 2's reaction function as

$$\lim_{\theta_1 \downarrow \theta^*,\, \theta_2 \uparrow \theta^*} \frac{\partial \theta_1}{\partial \theta_2} = 0.$$

Hence the solutions of equations (4) and (6) are equal at $\theta_1 = \theta_2 = \theta^*$ (fact 1); for any given value of θ_2 less than but close to θ^*, the solution of equation (4) is greater than the solution of equation (6) (facts 4 and 5); and the solution of equation (4) is less than the solution of equation (6) for θ_2 sufficiently close to $\underline{\theta}$ (facts 2 and 3). Therefore, by the intermediate value theorem, a solution to the equations exists that satisfies $\bar{\theta} > \theta_1 > \theta^* > \theta_2 > \underline{\theta}$. □

One possibility is shown in figure 1, which assumes that there is only one intersection point (labelled $(\hat{\theta}_1, \hat{\theta}_2)$ for clarity) of the reaction functions in the region $\theta_1 > \theta^* > \theta_2$. Note from the figure (see also fact 1 in the proof of the proposition) that the reaction functions also intersect at $\theta_1 = \theta_2 = \theta^*$. The following corollary, which follows immediately from the relative slopes of the reaction functions around the point $\theta_1 = \theta_2 = \theta^*$, means that this solution can be ignored in the remainder of the analysis.

Corollary 1 *The solution $\theta_1 = \theta_2 = \theta^*$ to equations* (4) *and* (6) *is not stable under the best-response dynamic (when strategies are restricted to be CCT).*

The two-agent game differs markedly from the single-agent decision problem. As we pointed out in section 2.1, with one agent, there is no delay: the object is acquired immediately. This is because we have assumed that individual agents' valuations are non-convex; and with non-convex valuations, there is no benefit from delaying a bid.[7]

[7] Note that we do not have a sunk cost of bidding in the model—any irreversibility arises because once the object is acquired, it cannot be resold and the bid recouped.

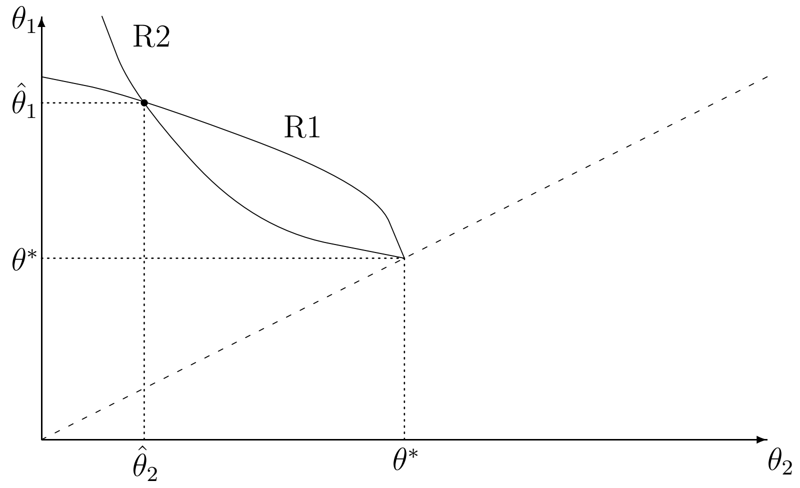

Figure 1: Reaction functions

When there is a rival bidder, however, each agent's payoff function becomes convex: it is bounded below by 0 when the agent loses, and is equal to the difference in the agents' valuations when the agent wins. This convexity induces an incentive to delay bidding: rather than bidding immediately, or at the first instant when it has the larger valuation, each bidder waits until the difference in valuations is sufficiently large. This feature—that competition makes payoffs convex—is quite general; we therefore expect to see bidder delay in more general environments than the one that we consider in this paper.

For the usual reasons, comparative statics are complicated if there are multiple equilibria. In order to ensure a unique solution to equations (4) and (6), the following assumption is made:

Assumption 3 (Uniqueness of CCT Equilibrium) $\theta\Theta(\theta)^{-1/(2\alpha+1)}$ *is a strictly concave function of* θ *for all* α.

There is, unfortunately, no intuitive interpretation of the technical condition required in the assumption. The assumption can, of course, be stated in terms of the function $\delta(\theta)$; it is satisfied by a linear $\delta(\cdot)$ function, for example.

An important comparative static involves the effect of an increase in uncertainty (i.e., the parameter σ) on the trigger points θ_1 and θ_2. A standard property of single-agent real option models is that irreversible actions are delayed (i.e., occurs at a higher level of the state variable for agent 1, and a lower level for agent 2) when uncertainty increases. The reason for this is that delay allows for the possibility that the random process (1) might change; if it goes in an adverse direction (down for agent firm 1, up for agent 2), then the agent need not act. The greater the variance of the process, the more valuable is the option created by this asymmetric situation, and so the more delay occurs.

The situation is complicated in the multiple agent case by the threat of pre-emption. If agent 1 delays, then it may lose its option altogether should agent 2 act in the meantime. This consideration can be seen in the value function in equation (3), for example. The term $B_1\theta^{\alpha+1} > 0$ is agent 1's valuation of the option to delay due to the single-agent effect; the option-like term $A_1\theta^{-\alpha} < 0$ is the decrease in the valuation due to the possibility that agent 2, not agent 1, acquires the target.

There are, therefore, two factors pulling in opposite directions when uncertainty increases; the comparative statics of θ_1 and θ_2 with respect to σ are determined by the balance between these two factors. Proposition 2 shows how these effects balance out.

Proposition 2 *$\partial\theta_1/\partial\sigma > 0$ and $\partial\theta_2/\partial\sigma < 0$ for all $\sigma \geq 0$.*

Proof. The proof of both parts concentrates on the reaction function of agent 1, equation (4); the symmetry in equations (4) and (6) means that the equivalent result for agent 2 follows immediately. Equation (4) can be re-written as

$$\frac{\theta_1}{(\Theta(\theta_1))^{\frac{1}{2\alpha+1}}} = \theta_2.$$

Consider the denominator of the left-hand side of this reaction function. Taking logs and differentiating with respect to σ gives

$$\frac{\partial}{\partial\sigma}\ln\left((\Theta(\theta))^{\frac{1}{2\alpha+1}}\right) = \frac{2\alpha_\sigma}{(2\alpha+1)^2}\left(-\ln\Theta(\theta) + \frac{1}{2}\left(\frac{\Theta(\theta)-1}{\Theta(\theta)}\right)(1+\Theta(\theta))\right)$$

where $\alpha_\sigma \equiv \partial\alpha/\partial\sigma$. Since $\alpha_\sigma < 0$, the sign of this expression is determined by the sign of

$$-\ln\Theta(\theta) + \frac{1}{2}\left(\frac{\Theta(\theta)-1}{\Theta(\theta)}\right)(1+\Theta(\theta)). \tag{7}$$

Consider the function $\phi(x) \equiv -\ln x + \frac{1}{2}\left(\frac{x-1}{x}\right)(1+(x-1))$. $\phi(1) = 0$; and for $x > 1$, the first derivative is

$$\phi'(x) = \frac{1}{2}\left(\frac{x-1}{x}\right)^2 > 0.$$

Hence $\phi(x) > 0$ for $x > 1$. Hence the expression in equation (7) is positive for all values of σ below some critical value σ^*. Therefore the denominator of the left-hand side of the reaction function of agent 1 is decreasing in σ; and consequently, the reaction function of agent 1 shifts upwards when σ increases. With the symmetric argument for agent 2's reaction function, the proposition follows. □

We show in the proposition that the standard option effect (which acts e.g., to increase the trigger θ_1 as σ increases) outweighs the pre-emption effect (which acts in the opposite direction). This need not be the case. When the random process in equation (1) has a positive drift, μ,[8] there are cases when the pre-emption effect dominates. We have shown analytically (in a proof available on request) that when $\mu > 0$, in the limit as $\sigma \to +\infty$, $\partial\theta_1/\partial\sigma < 0$ and $\partial\theta_2/\partial\sigma > 0$. Numerical analysis suggests that this outcome occurs when the drift parameter is large (close to, but below the interest rate r) and there is a substantial degree of uncertainty. When μ is large, the opportunity cost of holding the option to bid is small, and so the option value is large. When σ is large, the option value is large. Hence both conditions (μ and σ large) ensure that the agents' options to bid are very valuable. But when σ is large, each agent assesses a high probability that the state variable hits the other agent's trigger point. The optimal response of each agent to an increase in σ is then to decrease delay (so that θ_1 falls and θ_2 rises), to limit the probability of pre-emption and so preserve its large option value. (See Mason and Weeds (2004) for a more general analysis of when the standard comparative static is reversed

[8] I.e., $d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t$ where $\mu \in [0, r)$ is the drift parameter. The restriction that $\mu < r$ ensures that there is a positive opportunity cost to holding the option to bid to acquire the good. This means that each agent bids at some finite value of the state variable, rather than holding the option in perpetuity.

because of pre-emption.)

In a sense, therefore, the result in proposition 2 is not general: the comparative static with respect to σ is unambiguous only when $\mu = 0$. But the numerical analysis suggests that, even with a large drift, the extent of delay increases with σ for almost all values of the volatility parameter. The unusual outcome, in which delay decreases with uncertainty, occurs only for extreme values of σ.

3.2 Equilibrium Existence

In this section, we show that the best response to a CCT strategy is itself a CCT strategy i.e., that a CCT equilibrium exists. Consider first the bidding stage when at least one of the agents has made a bid. By the standard 'Bertrand' argument in common knowledge bidding games, when one agent bids competitively and cautiously, then the best response is a competitive, cautious bid. Given this behaviour in the bidding stage, now consider the prior stage when agents must decide when to bid; and suppose that agent 2 bids at the first time that the state variable hits the interval $[0, \theta_2]$. Agent 1's value function V_0 when it is not bidding is

$$V_0 = \begin{cases} 0 & \theta \leq \theta_2, \\ A_1\theta^{-\alpha} + B_1\theta^{\alpha+1} & \theta > \theta_2; \end{cases}$$

see the appendix. Its value function when it makes a bid is given by

$$\begin{matrix} \delta(\theta) - \epsilon & s_1(\theta) > s_2(\theta) \\ 0 & \text{otherwise.} \end{matrix}$$

As is argued in the appendix, the continuation region (i.e., in which no bid is made) for agent 1 is the half-open interval $[0, \hat{\theta})$, while the stopping region (in which a bid is made) is the interval $[\hat{\theta}, \infty)$, for some $\hat{\theta}$. (In fact, the analysis above determines the optimal point as θ_1.)

An analogous argument holds for agent 2's best response when agent 1 uses a CCT strategy. In summary: the best response to a CCT strategy is itself a CCT strategy. Hence

Proposition 3 *A CCT equilibrium exists.*

Of course, non-CCT equilibria exist—there are Markovian equilibria that are non-CCT (for example, the continuum of equilibria with non-cautious bidding in the bidding stage of the game); and there are non-Markovian equilibria. As we have explained, our focus on Markovian CCT equilibrium is motivated by our interest in the timing of noncollusive bids.

4 The Efficient Solution

Efficiency requires that the object be acquired by the agent with the higher valuation. The identity of that agent is stochastic—when $\theta < \theta^*$, it is agent 2, when $\theta > \theta^*$, it is agent 1. The timing of acquisition must also be efficient; we denote the efficient trigger points by θ_L for the lower threshold and θ_H for the upper one. Hence the efficient allocation rule takes the form "award the object to agent 2 immediately if $\theta \in [0, \theta_L]$, to agent 1 immediately if $\theta \in [\theta_H, \infty)$; otherwise wait". Familiar derivations give the efficient value function as

$$W = \begin{cases} s_2(\theta) & \theta \leq \theta_L, \\ A_E\theta^{-\alpha} + B_E\theta^{\alpha+1} & \theta \in (\theta_L, \theta_H), \\ s_1(\theta) & \theta \geq \theta_H. \end{cases} \tag{8}$$

Value-matching and smooth-pasting conditions apply at the efficient triggers θ_L and θ_H. These four equations yield, after elimination of the value function coefficients,

$$\left(\frac{\theta_H}{\theta_L}\right)^{\alpha} = \frac{-(\alpha+1)s_2(\theta_L) + \theta_L s_2'(\theta_L)}{-(\alpha+1)s_1(\theta_H) + \theta_H s_1'(\theta_H)}, \tag{9}$$

$$\left(\frac{\theta_H}{\theta_L}\right)^{\alpha+1} = \frac{\alpha s_1(\theta_H) + \theta_H s_1'(\theta_H)}{\alpha s_2(\theta_L) + \theta_L s_2'(\theta_L)}. \tag{10}$$

Little analytical progress can be made without strong assumptions on the agent's valuation functions $s_1(\cdot)$ and $s_2(\cdot)$. We assume the following simple linear form.

Assumption 4

$$s_1(\theta) = \lambda\theta, \qquad \lambda > 0, \tag{11}$$

$$s_2(\theta) = \eta, \qquad \eta > 0. \tag{12}$$

Hence $\delta(\theta) = -\eta+\lambda\theta$, $\theta^* = \eta/\lambda$, $\bar{\theta} = (\alpha+1)\theta^*/\alpha$ and $\underline{\theta} = \alpha\theta^*/(\alpha+1)$. These functional forms are chosen for their analytical convenience. One informal story to support them is that agent 1 is a firm that operates in the same industry as the target and has a lower unit cost of production than the target. When the state θ is the level of demand in the industry, agent 1 can generate an extra profit, of the form $\lambda\theta$ (where λ is related to the cost difference). In contrast, agent 2 has a fixed cost saving, relative to the target, of η.

With assumption 4, equations (4) and (6), determining the equilibrium triggers, and equations (9) and (10), determining the efficient triggers, become

$$\left(\frac{\theta_1}{\theta_2}\right)^{2\alpha+1} = \frac{(\alpha+1)\theta_1 - \alpha\theta^*}{-\alpha\theta_1 + (\alpha+1)\theta^*}, \tag{13}$$

$$\left(\frac{\theta_1}{\theta_2}\right)^{2\alpha+1} = \frac{-\alpha\theta_2 + (\alpha+1)\theta^*}{(\alpha+1)\theta_2 - \alpha\theta^*}, \tag{14}$$

$$\theta_H = \left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{2\alpha+1}} \theta^*, \tag{15}$$

$$\theta_L = \left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{2\alpha+1}} \theta^*. \tag{16}$$

The symmetry of the equations means that $\theta_1 = 1/\theta_2$ and $\theta_H = 1/\theta_L$.

Since pre-emption is not an issue in the efficient solution, the triggers θ_H and θ_L should have the the usual property of delay for situations of irreversible action under uncertainty; that is, θ_H should be increasing and θ_L decreasing in σ. The next proposition confirms this conjecture.

Proposition 4 *If assumption 4 holds, then*

$$
\begin{array}{ll}
\text{(i)} & \lim_{\sigma\to 0} \theta_H/\theta_L = 1; \\
\text{(ii)} & \lim_{\sigma\to\infty} \theta_H/\theta_L = \infty; \\
\text{(iii)} & \partial\theta_H/\partial\sigma > 0, \quad \partial\theta_L/\partial\sigma < 0.
\end{array}
$$

Proof. Equations (15) and (16) imply that

$$\frac{\theta_H}{\theta_L} = \left(\frac{\alpha+1}{\alpha}\right)^{\frac{2}{2\alpha+1}}.$$

Parts (i) and (ii) of the proposition follow immediately from observing that as $\sigma \to 0$ (∞), $\alpha \to \infty$ (0), respectively. Part (iii) of the proposition will be proved for θ_H; an equivalent argument holds for θ_L. Since α is a decreasing function of σ,

$$\frac{\partial}{\partial\sigma}\left(\frac{\alpha+1}{\alpha}\right) > 0 \text{ and } \frac{\partial}{\partial\sigma}\left(\frac{1}{2\alpha+1}\right) > 0.$$

This implies immediately that θ_H is increasing in σ. □

Assumption 4 also allows us to compare the equilibrium and efficient triggers:

Proposition 5 *If assumption 4 holds, then* $\theta_1 \geq \theta_H$ *and* $\theta_2 \leq \theta_L$.

Proof. The proof works by showing (through contradiction) that $\theta_1/\theta_2 \geq \theta_H/\theta_L$; since $\theta_1 = 1/\theta_2$ and $\theta_H = 1/\theta_L$, this gives the result immediately. To simplify expressions, let $\tilde{\theta}_k \equiv \theta_k/\theta^*$ for $k \in \{1, 2, H, L\}$. From equations (15) and (16),

$$\frac{\tilde{\theta}_H}{\tilde{\theta}_L} = \left(\frac{\alpha+1}{\alpha}\right)^{\frac{2}{2\alpha+1}};$$

from equation (13),

$$\frac{\tilde{\theta}_1}{\tilde{\theta}_2} = \left(\frac{(\alpha+1)\tilde{\theta}_1 - \alpha}{-\alpha\tilde{\theta}_1 + \alpha + 1}\right)^{\frac{1}{2\alpha+1}}.$$

Suppose that $\tilde{\theta}_H/\tilde{\theta}_L \geq \tilde{\theta}_1/\tilde{\theta}_2$. Manipulation of the above expressions shows that this would mean that

$$\tilde{\theta}_1 \leq \frac{\alpha^3 + (\alpha+1)^3}{\alpha(\alpha+1)(2\alpha+1)}.$$

Similarly, from equation (14),

$$\frac{\tilde{\theta}_1}{\tilde{\theta}_2} = \left(\frac{-\alpha\tilde{\theta}_2 + \alpha + 1}{(\alpha+1)\tilde{\theta}_2 - \alpha} \right)^{\frac{1}{2\alpha+1}}.$$

$\tilde{\theta}_H/\tilde{\theta}_L \geq \tilde{\theta}_1/\tilde{\theta}_2$ then implies that

$$\tilde{\theta}_2 \geq \frac{\alpha(\alpha+1)(2\alpha+1)}{\alpha^3 + (\alpha+1)^3}.$$

Hence $\tilde{\theta}_H/\tilde{\theta}_L \geq \tilde{\theta}_1/\tilde{\theta}_2$ implies that $\tilde{\theta}_1/\tilde{\theta}_2 \leq 1$—a contradiction (since $\theta_1 \geq \theta_2$). Therefore $\tilde{\theta}_H/\tilde{\theta}_L \leq \tilde{\theta}_1/\tilde{\theta}_2$. The proposition follows. $\square$

Proposition 5 tells us that there is too much delay in equilibrium. The efficient payoff is $\pi_E \equiv \max\{s_1, s_2\}$; the equilibrium payoff of bidder 1 is $\pi_1 \equiv \max\{s_1 - s_2, 0\}$, and of bidder 2 is $\pi_2 \equiv \max\{s_2 - s_1, 0\}$. Equilibrium payoffs are lower, therefore, because *competition creates negative externalities*: $\pi_i = \pi_E - s_{-i} < \pi_E$ for $i \in \{1, 2\}$. Notice that the equilibrium payoff of an agent is lower than the efficient payoff, both when it wins the object and when it loses to the agent. The reduction in payoff on winning the object leads the agent to bid later (relative to the efficient trigger); the reduction on losing leads to an earlier bid. When an agent chooses its trigger point optimally, the former effect dominates, so that negative externalities lead to more delay in equilibrium.

To see why, consider first the choice of trigger point by agent 1. When assumption 4 holds, the equilibrium payoff to agent 1 is $\max\{0, \lambda\theta - \eta\}$; the efficient payoff is $\max\{\eta, \lambda\theta\}$ i.e., an upward shift of the equilibrium payoff. The equilibrium payoff of agent 1 is therefore lower than the efficient payoff by an amount η (i.e., the valuation of agent 2) both when it wins the object and when it loses to agent 2. Agent 1 will be the first to bid only when $\theta \geq \theta^*$ i.e., when it has the higher valuation. Hence, the payoff reduction on winning is more important than the reduction on losing because the former occurs immediately, while the latter occurs in the future (when θ falls below θ^*) and so is discounted. Since the undiscounted payoff reductions are equal, the reduction on winning has greater weight. So, for a given lower trigger point (θ_L, say), the equilibrium upper trigger is greater than the efficient upper trigger. The shape of the reaction functions (established in proposition 1) then ensures that $\theta_1 \geq \theta_H \geq \theta_L \geq \theta_2$.

This argument for agent 1 relied on agent 2's valuation being a constant, η. A more general intuitive argument can be made. Agent 1's value function in equation (3) has two components. The first $OL_1(\theta) \equiv A_1\theta^{-\alpha} < 0$ is an option-like term anticipating agent 2's successful bid; the second $O_1(\theta) \equiv B_1\theta^{\alpha+1} > 0$ is an option term relating to its own successful bid. At agent 1's optimally-chosen trigger point θ_1, value-matching and smooth-pasting imply that $O_1(\theta_1) > -OL_1(\theta_1) > 0$ and $O_1'(\theta_1) > -OL_1'(\theta_1) > 0$ (where the prime denotes the derivative with respect to θ). Hence the option term is greater in terms of both level and slope. In short, the value from winning is more important than the value from losing.[9] Hence agent 1's trigger point lies above the efficient level. This argument suggests that the result in proposition 5 should generalize beyond linear valuation functions, although we have been unable to show this analytically.[10]

In general, a second effect may arise: the negative externalities may not only shift equilibrium payoffs, but also change the degree of convexity, relative to the efficient payoff. If the externalities create more (less) convexity, then they will tend to lead to more (less) delay in equilibrium, other things equal. Agent 1's equilibrium payoff function is (weakly) more convex than the efficient payoff function iff

$$\frac{s_1''(\theta)}{s_1'(\theta)} \geq \frac{s_2''(\theta)}{s_2'(\theta)} \quad \forall\ \theta,$$

assuming non-zero first derivatives. This condition is satisfied with equality when assumption 4 is satisfied (since $s_1'' = s_2'' = 0$);[11] hence this effect does not arise in our

[9] Equivalently, for agent 2: its value function in equation (5) has two components: the option term $O_2(\theta) \equiv A_2\theta^{-\alpha} > 0$ relating to its own successful bid; and the option-like term $OL_2(\theta) \equiv B_2\theta^{\alpha+1}$ anticipating agent 1's successful bid. Value-matching and smooth-pasting at agent 2's trigger point θ_2 imply that $O_2(\theta_2) > -OL_2(\theta_2) > 0$ and $-O_2'(\theta_2) > -OL_2'(\theta_2) > 0$. Again, the option term is greater in level and slope.

[10] Numerical investigation supports this optimism.

[11] Or when both valuation functions are (strictly increasing) linear functions—in this case, the equilib-

analysis.

5 Summary of Results

In figure 2, the various triggers are shown against different values of σ. We use assumption 4 for the agent's valuation functions, and set the interest rate r to 5%, agent 2's value η to 10 and the slope of agent 1's value λ to 1. With these values, $\theta^* = 10$.

Summarising, our analytical results are that

1. $\bar{\theta}$ is increasing and $\underline{\theta}$ is decreasing (results 1 and 2);
2. θ_1 is greater than θ^*, less than $\bar{\theta}$ and increasing; θ_2 is less than θ^*, greater than $\underline{\theta}$ and decreasing (propositions 1 and 2);
3. θ_H is increasing and θ_L decreasing (proposition 4);
4. $\theta_1 \geq \theta_H$ and $\theta_2 \leq \theta_L$ (proposition 5).

These results are all confirmed by the numerical analysis shown in figure 2.

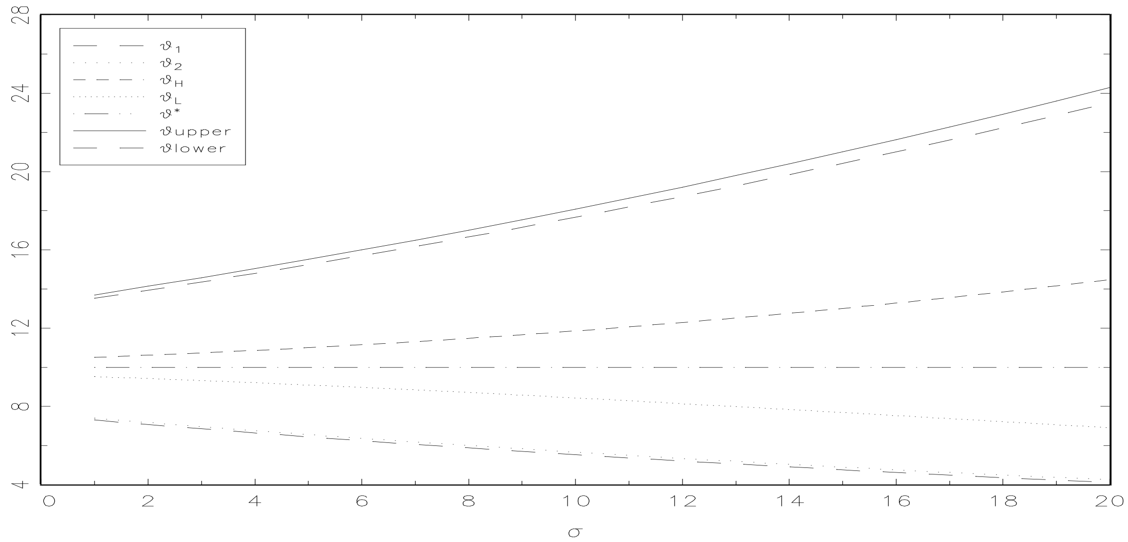

Figure 2: Triggers against the degree of uncertainty σ

rium payoff of e.g., agent 1 is shifted vertically and rotated, relative to the efficient payoff; the degree of convexity is unchanged.

6 Incentives of the Seller

We now investigate the question of the seller's preferences with respect to the timing of acquisition, and towards the volatility parameter σ of the stochastic process followed by θ. One motivation for this analysis is the case of the seller as a target firm in a take-over situation. If the target can influence when bids are made, at what time would its shareholders prefer this to be? A firm can affect its value by choosing projects with different risk profiles: when volatility affects the timing of takeover bids, how might this be manipulated?

When the bidders' payoffs satisfy assumption 4, there is an alternative interpretation of volatility. Note that the stochastic process θ drives the gap between the valuations of the target firm and bidder 1, with this difference being scaled by λ. Thus, the volatility σ may be interpreted as the degree of correlation between the value of these assets when controlled by the target and under the alternative ownership of bidder 1—a higher value of σ is a reduction in the correlation between these valuations. Under this interpretation, the target's choice of a lower value of σ corresponds to a higher degree of correlation between itself and the bidder with lower marginal cost. (The degree of correlation with bidder 2, with a fixed cost reduction, is not affected.)

The seller receives the payoff of the losing bidder when acquisition occurs i.e., its payoff is

$$S(\theta) = \begin{cases} s_1(\theta) & \theta \leq \theta_2, \\ s_2(\theta) & \theta \geq \theta_1. \end{cases}$$

Using familiar calculations, the seller's value function can then be shown to be

$$U = \begin{cases} s_1(\theta) & \theta \leq \theta_2, \\ A_S\theta^{-\alpha} + B_S\theta^{\alpha+1} & \theta_2 < \theta < \theta_1, \\ s_2(\theta) & \theta \geq \theta_1. \end{cases}$$

The coefficients A_S and B_S are determined by value-matching conditions at the triggers

θ_1 and θ_2; there are no corresponding smooth-pasting conditions as the trigger points involve no optimality on the part of the seller.

The first observation is immediate: the seller's payoff function $S(\theta)$ is concave. Thus greater delay (i.e., a widening of the trigger points θ_1 and θ_2) is harmful to the seller. Note that this finding does not require a functional form assumption, such as assumption 4, for $s_1(\cdot)$ and $s_2(\cdot)$. It depends only on the assumption that the difference in the valuations is (weakly) increasing; bidding is competitive, so that the seller receives the lower of the valuations.[12]

From proposition 2, we know that the bidders' triggers widen when the degree of uncertainty σ increases. An increase in σ reduces the seller's pre-bid value: for any two parameters such that $\sigma_1 < \sigma_2$, the associated value functions U_1 and U_2 are such that $U_1 > U_2$. Hence the seller always prefers a lower value of σ. This is illustrated in figure 3, in which an increase in σ from 0.2 to 0.4 shifts θ_2 to the left and θ_1 to the right, and results in a downward shift in the seller's value function.

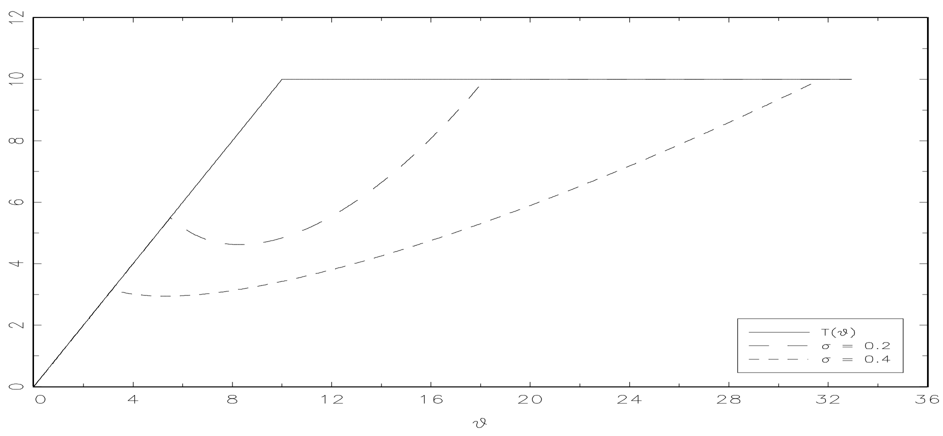

Figure 3: The target's value function against σ

These arguments are summarised in the next proposition.

Proposition 6 *If the seller can choose the time at which the object is sold, then it will*

[12]Recall that assumption 1 ensures that the valuation functions $s_i(\cdot)$ are not convex.

sell the object immediately. If, instead, the seller can choose the volatility parameter σ of the state variable process (1)*, then it will choose the lowest possible value of σ.*

To complete this section, we consider the efficient choice of the parameter σ, *given* the behaviour of the bidders. The efficient value function is of the same form as in equation (8), but the value function coefficients A_E and B_E are now determined by value-matching conditions at θ_1 and θ_2. (In section 4, smooth-pasting conditions apply at θ_H and θ_L , since these triggers are fully efficient.) We are unable to obtain analytical results in this case. Numerical analysis shows that the efficient choice of σ depends on the initial level θ_0 of the process 1. If θ_0 is close to θ^*, the efficient choice of σ is high, as illustrated in figure 4. But when the initial value is further away from θ^*, the efficient choice of σ is low. This result is due to the tension between two factors. Since the efficient payoff function is convex, greater uncertainty increases the (option) value of take-over. This effect is particularly strong close to the kink at θ^*. However, in non-cooperative equilibrium, the bidders tend to delay too much; greater uncertainty worsens this inefficiency. At values of θ further away from θ^*, this inefficiency dominates and the efficient choice of σ is low, despite the convexity of the efficient payoff, in order to avoid excessive delay.

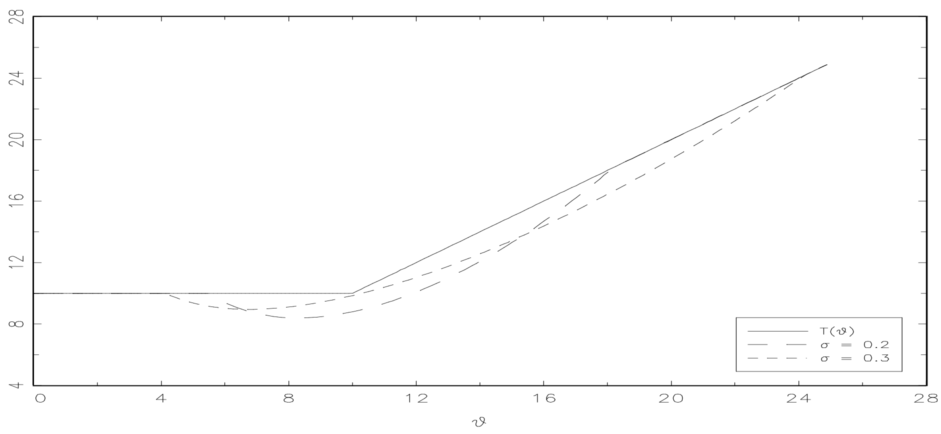

Figure 4: The efficient value function against σ

7 Conclusions

We have examine the timing of competitive bids. Our main results demonstrate that there is inefficient delay in equilibrium. Delay occurs because competition between bidders makes their payoffs convex. Too much delay occurs in equilibrium because competition creates negative externalities. While we had to specialize the analysis to linear valuation functions when demonstrating the inefficiency of equilibrium, we believe that the result is more general.

In future work, we plan to examine in more detail the seller's decision of when to offer the object for sale. When the seller has incomplete information about the buyers' valuations, the choice of the selling mechanism (e.g., auction format) is likely to influence the timing of sale. The separation of ownership and control implies that managerial, rather than shareholder, incentives determine the behaviour of the target; in this context an analysis of managers' payoffs becomes relevant.

Appendix

In this appendix, we derive the value function of agent 1 when agent 2 uses a CCT strategy. The derivation of agent 2's value function is very similar and so is omitted.

Agent 1's value function $V_1(\theta_t)$ given a level θ_t of the state variable is given by equation (2). In the 'continuation' region before either agent has bid, in any short time interval dt starting at time t agent 1 receives a flow payoff of 0 and experiences a capital gain or loss dV_1. The Bellman equation for the value of the entry opportunity is therefore

$$V_1 = \exp(-rdt)\mathbb{E}_t\left[V_1 + dV_1\right].$$

Itô's lemma and the GBM equation (1) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 V_1''(\theta) - rV_1(\theta) = 0.$$

The general solution of this homogeneous ordinary differential equation is $V_1 = A_1\theta^\alpha +$

$B_1\theta^{\alpha+1}$, where A_1 and B_1 are constants, and $\alpha > 0$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z+1) - r$.

If agent 1 bids and wins, it receives a flow payoff of $s_1(\theta)$ while paying its bid b_1. With CCT bidding, $b_1 = b_2 + \epsilon$ if $b_2 < s_1$, or $b_1 \leq s_1$ otherwise. Hence agent 1's payoff at the point $\hat{\theta}$ at which it bids and wins is $\delta(\theta)$ (minus an arbitrarily small constant ϵ). By assumption 1, $\delta(\cdot)$ is a strictly increasing function. Hence by a standard argument (see e.g., Dixit and Pindyck (1994)), the 'stopping' region for agent 1 i.e., the region of the state space in which it bids, is the interval $[\hat{\theta}, +\infty)$.

References

Bergemann, D., and J. Välimäki (1996): "Learning and Strategic Pricing," *Econometrica*, 64(5), 1125–49.

Dixit, A. K., and R. S. Pindyck (1994): *Investment under Uncertainty.* Princeton University Press, Princeton.

Felli, L., and C. Harris (1996): "Job Matching, Learning and Firm-Specific Human Capital," *Journal of Political Economy*, 104, 838–868.

Fudenberg, D., and J. Tirole (1991): *Game Theory.* MIT Press, Cambridge.

Grenadier, S. R. (1996): "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets," *Journal of Finance*, 51(5), 1653–1679.

Hoppe, H. (2000): "Second-mover Advantages in the Strategic Adoption of New Technology under Uncertainty," *International Journal of Industrial Organization*, 18, 315–338.

Lambrecht, B. M. (2004): "The Timing and Terms of Mergers Motivated by Economies of Scale," *Journal of Financial Economics*, 72(1), 41–62.

Lambrecht, B. M., and W. Perraudin (2003): "Real Options and Pre-emption under Incomplete Information," *Journal of Economic Dynamics and Control*, 27(4), 619–643.

Maskin, E., and J. Tirole (1988): "A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs," *Econometrica*, 56(3), 549–569.

Mason, R. A., and H. F. Weeds (2004): "Can Greater Uncertainty Hasten Investment?," Mimeo.

Smets, F. (1991): "Exporting versus Foreign Direct Investment: The Effect of Uncertainty, Irreversibilities and Strategic Interactions," Mimeo.

Weeds, H. F. (2002): "Strategic delay in a real options model of R&D competition," *Review of Economic Studies*, 69(3), 729–747.

Weston, J. F., K. S. Chung, and S. E. Hoag (1990): *Mergers, Restructuring, and Corporate Control*. Prentice Hall International.