# The Welfare Gains of Improving Risk Sharing in Social Security

Conny Olovsson*

March 29, 2004

## Abstract

This paper shows that improved intergenerational risk sharing in social security may imply very large welfare gains, amounting to up to 15 percent of the per-period consumption relative to the current U.S. consumption. Improved risk sharing raises welfare through a direct effect, i.e., by correcting an initially inefficient allocation of risk, and through a general equilibrium (GE) effect. The GE effect is due to the fact that the allocation of risk in the pay-as-you-go system influences the demand for capital. As a result, with an efficient risk sharing arrangement, the crowding out effect associated with an unfunded system can actually be completely eliminated. Efficient risk sharing in social security implies highly volatile and pro-cyclical benefits, i.e., that retirees' exposure to productivity risk is increased. Consequently, a policy involving completely safe benefits will unambiguously be welfare reducing.

*JEL* Classification E21 H21 H55

*Keywords*: Social Security, risk sharing

# 1 Introduction

This paper deals with the problem of how aggregate productivity risk should be allocated between taxpayers and retirees in a pay-as-you-go (PAYGO) social security system. The question is motivated by the fact that overlapping generations (OLG) models are known to be inefficient in an *ex ante* sense, stemming from the inability of the unborn to insure themselves.[1] With standard CRRA preferences and Cobb-Douglas technology, the laissez-faire allocation of risk is inefficient by imposing too little productivity risk on retirees and too much on future generations.[2]

[1] See Peled (1982) and Wright (1987).
[2] This is a very general result that will be true when the intertemporal elasticity of substitution for consumption is less than one, and/or the depreciation rate less than 100 percent. See Bohn (1998) and Bohn (2003).

1

A PAYGO system has the potential of correcting these inefficiencies, due to the immanent intergenerational link found by current pension payments being immediately transferred to retirees. More precisely, productivity risk can be transferred between taxpayers and retirees through benefits being allowed to respond to macroeconomic shocks. When the economy is hit by a productivity shock, the government can keep the social security budget in balance, by adjusting benefits or contributions (or, naturally, by a combination thereof). In the former case, the exposure to productivity risk is amplified for retirees and, in the latter case, for taxpayers. Using the PAYGO system to transfer productivity risk between these two groups is thus straightforward.[3] In the current U.S. social security system, an element of intergenerational risk sharing can be found in the mechanism of wage-indexed benefits, implying that benefits respond to younger generations' income.[4]

To analyze the importance of improved intergenerational risk sharing in social security, I set up a three-period overlapping generations model with endogenous production, aggregate uncertainty and a PAYGO system with simple (linear) schemes that can be used to allocate risk between taxpayers and retirees. Specifically, wage- and capital-indexed benefits are considered.[5] The three-period model is motivated on the grounds that three is the minimal number of periods that captures the heterogeneity of consumers across age groups that I wish to emphasize: the *uninsured* young, the *saving* middle-aged and the *dissaving* old. Moreover, the three-period model is needed to evaluate the effects of shortselling constraints.[6] The utility function developed by Epstein and Zin (1989, 1991) and Weil (1989) is used to make the model consistent with empirically revealed attitudes towards risk.

The experiment I carry out is to find the coefficients of the schemes maximizing the expected lifetime utility of a newborn agent, and compare the outcomes, both to the current U.S. and the social optimum. The analysis thus quantifies the importance of obtaining a better intergenerational risk sharing allocation, and it sheds lights on the way benefits and taxes should respond to macroeconomic shocks. I also model

---

[3] However, the suboptimality of the risk allocation in the existing U.S. social security system has been discussed in several recent papers, including Shiller (1998), Ball and Mankiw (2001).

[4] An individual's earnings are indexed to the average wage level 2 years prior to the year of eligibility, i.e., when the agent reaches the age of 62.

[5] The production function is assumed to be of the Cobb-Douglas form, implying that the arrangement with wage-indexed benefits is identical to one where benefits are indexed to aggregate income.

[6] More specifically, the two-period model imposes implicit short-selling constraints. For instance, in that setting, the young can never take a short position in capital unless some institution (for example the government) takes a long position in capital. The case is similar for bondholdings since, in equilibrium, the young must hold the whole amount of government debt, thereby also implying a long position in bonds.

unexpected transitions from the current U.S. economy to the efficient equilibria, and compute the implied welfare effects.

The findings are that improved risk sharing in social security may imply very large welfare gains. The welfare gain of being born into an economy with efficient wage-indexation is between 12 and almost 15 percent of per-period consumption in the current U.S., depending on whether agents are subject to shortselling constraints. In contrast, the welfare loss of being born into the inefficient economy with completely safe benefits is roughly 10 percent of the per-period consumption. Hence, welfare effects are, by all means, very large. Efficient risk sharing in social security implies highly volatile and pro-cyclical benefits and tax rates. The pro-cyclical arrangement eases the risk exposure of the young, since they are subject to lower taxes in bad states and vice versa. In this way, social security provides some insurance to the uninsured young. Highly volatile benefits are motivated by the fact that the allocation of risk in the PAYGO system has a major influence on demand for capital. More specifically, highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. In fact, in the absence of shortselling constraints, the crowding out effect generally associated with an unfunded system can be completely eliminated by the use of efficient risk sharing arrangements. The welfare effects of improved risk sharing in social security can thus be decomposed into a *direct* welfare effect and a *general equilibrium* (GE) effect. The direct welfare effect raises welfare by correcting an initially inefficient allocation of risk, whereas the welfare effects stemming from the GE effect are due to the higher average level of the capital stock.[7] The welfare gains associated with the risk allocations analyzed in this paper are highly correlated with the capital stock, indicating that the major part of the welfare gain associated with improved risk sharing actually comes from the GE effect.

I also find a considerable difference between the social optimum and all other economies, including the laissez-faire economy. Despite the large welfare gains associated with improved risk sharing, no economy actually comes close to the social optimum. The capital stock in the social optimum is roughly 3.5 times the capital stock in the laissez-faire economy, indicating that the social planner builds up a very large

---

[7] The fact that social security may be used to change national saving and investment is also discussed in Abel (2003).

buffer to be able to smooth the consumption of future generations.

An intuition for the findings in this paper is that the welfare cost of exposing the old to aggregate risk is relatively small, as long as the risk is predictable and known in advance. The reason is that the old have had a lifetime for setting up their hedging portfolios. In contrast, the earlier in life agents are exposed to risk, the smaller are their possibilities to hedge.

The benchmark model is calibrated to be consistent with the risk free rate, the Sharpe ratio and the capital output ratio in the U.S. However, since the coefficient of relative risk aversion (RRA) is controversial, a sensitivity experiment is also carried out to evaluate the importance of this parameter. In this exercise, CRRA utility is used and the RRA coefficient is set to 2, while the rest of the calibration is kept constant. In every important aspect, the results are very similar to those found with Epstein-Zin utility. The welfare effects are still substantial, indicating that even if agents are only moderately risk averse, they still care a great deal about the allocation of risk, mainly because of the large GE effects at stake.

The results in this paper are closely related to earlier findings on intergenerational risk sharing. First, they are in line with the findings of Storesletten et al. (1998) and Krueger and Kubler (2002), who both find the major part of the (negative) welfare effects associated with social security to come from general equilibrium effects (i.e., it crowds out capital). Second, the finding of pro-cyclical benefits and taxes resembles the findings in Smetters (2002), where it is shown that the government can use negative capital income taxes (implying pro-cyclical wage taxes) to correct the "biological trading constraint" preventing living generations from negotiating contingent contracts with the unborn. These are also in line with Bohn (1998, 2003) who find the young in OLG models to be generally to exposed to aggregate risk.

This paper abstracts from population growth, demographic uncertainty, labor supply decisions, debt policy and preferences motivating a PAYGO system. Admittedly, these are all potentially important issues. For instance, it would be more realistic to consider an endogenous labor supply, since that would give the young one more degree of freedom. However, as shown by Olovsson (2004a), the response in labor supply to a tax increase will, to a large extent, depend on whether agents also work in home production. Introducing home production significantly complicates the analysis. Debt policy is, of course, a natural

candidate for handling issues of risk sharing. The question is then how to construct the debt policy response function in an efficient way. When hit by a bad productivity shock, the government can either increase the debt and provide safe benefits to the old, or keep it constant and reduce benefits. However, due to the general similarities between unfunded social security and debt, the use of debt as an instrument to handle risk sharing can in every relevant aspect be expected to resemble one using the PAYGO system. Most important might be the fact that it is difficult to provide a rational for the PAYGO system using standard preferences. However, Gonzalez-Eiras and Niepelt (2003) show that in a model with standard preferences, a PAYGO system of realistic size may be introduced and sustained in a political equilibrium.

# 2 The Economic Model

## 2.1 The Consumers

The basic model is a three-period overlapping generations model. Each generation is modeled as a representative consumer. There is one consumption good in each period and it perishes at the end of the period. The index $i = 0, 1$ and $2$ is used to denote the young, the middle-aged and the old, respectively. An agent inelastically supplies labor for two periods and retires in the third period when old. During their working years, agents are endowed with a level of productivity $e_i$ and they receive wage $W$. When retired, they collect the social security benefit, $\widetilde{\varphi}_t$.

There are two types of securities in the economy, bonds and capital denoted by $b$ and $k$, respectively. Bonds are assumed to be in zero net supply, while the supply of capital is endogenous. A consumer born in period $t$ has zero endowment of assets. This consumer makes a portfolio decision $a_{t+1,0} = (b_{t+1,0}, k_{t+1,0})$ in period $t$, when young; adjusts this decision to $a_{t+2,1} = (b_{t+2,1}, k_{t+2,1})$ in period $t+1$, when middle-aged; and sells the portfolio in period $t+2$, when old. As usual, a negative position in bonds or stocks denotes a short position in that asset. The price of the bond and the gross rate of return on capital are denoted by $p$ and $R$, respectively, and the tax rate is denoted by $\tau_t$. The budget constraints in period $t$ are then given by

$$c_{t,0} \leq (1 - \tau_t) e_0 W_t - k_{t+1,0} - p_t b_{t+1,0} \tag{1}$$

$$c_{t,1} \leq (1 - \tau_t) e_1 W_t + R_t k_{t,0} + b_{t,0} - k_{t+1,1} - p_t b_{t+1,1} \tag{2}$$

$$c_{t,2} \leq \widetilde{\varphi}_t + R_t k_{t,1} + b_{t,1}, \tag{3}$$

where (1), (2) and (3) are the budget constraints faced by the period $t$ young, middle-aged and old, respectively.

Since this paper is concerned with the welfare effects of different risk allocations, it is important to use preferences that are, at least in principle, consistent with empirically revealed attitudes towards risk. It is well known that there is no way of fitting both the level of the risk-free rate and the risk premium with standard preferences (i.e., with power utility).[8] The more flexible utility function developed by Epstein and Zin (1989,1991) and Weil (1989) will therefore be used instead. If we denote the subjective discount factor by $\beta$, the Epstein-Zin-Weil objective function can be recursively defined by

$$U_{t,i} = \frac{1}{1 - \gamma} \left[ (1 - \beta) c_{t,i}^{\frac{1-\gamma}{\theta}} + \beta \left[ (1 - \gamma) E_t U_{t+1,i+1} \right]^{\frac{1}{\theta}} \right]^{\theta}, \tag{4}$$

where $\theta$ is defined by $\theta = (1 - \gamma) / [1 - (1/\sigma)]$, $\gamma$ is the coefficient of relative risk aversion and $\sigma$ is the elasticity of intertemporal substitution. Note that when $\gamma = 1/\sigma$, i.e., when $\theta = 1$, (4) collapses to the standard time-separable power utility function with relative risk aversion, $\gamma$.

Finally, the following assumption is made:

$$U_{t,i} \equiv 0 \quad \text{for } i \geq 3,$$

which implies that the old do not buy any assets (and that altruistic bequests are ruled out). The

---

[8] By construction, the basic power utility model makes the elasticity of intertemporal substitution the reciprocal of the coefficient of relative risk aversion. This is restrictive, since no theoretical or empirical evidence supports such a tight link between these two concepts. The strength of the Epstein-Zin-Weil model is that it allows us to break this link.

maximization problem of an agent aged $i \in \{0, 1\}$ is then

$$U_{t,i} = \max_{c_{t,i}, a_{t+1,i}} \frac{1}{1 - \gamma} \left[ (1 - \beta) c_{t,i}^{\frac{1-\gamma}{\theta}} + \beta \left[ (1 - \gamma) E_t U_{t+1,i+1} \right]^{\frac{1}{\theta}} \right]^{\theta}. \tag{5}$$

## 2.2 Firms

There is a representative firm, which in each period uses labor and capital to produce the consumption good, according to a constant returns to scale production function. Since firms make their decision on how much capital and labor to hire after the realization of shock $Z_t$, they face no uncertainty and simply maximize their current-period profits. The aggregate production function takes the form

$$Y_t = Z_t K_t^{\alpha} L^{1-\alpha}, \tag{6}$$

where $Z_t$ is a stochastic productivity shock, $K_t$ the aggregate capital stock and $L$ the aggregate labor supply.

In order to allow the total return on capital to vary somewhat independently from the wage rate, I assume a stochastic depreciation rate. Given aggregate consumption $C$, and the rate of depreciation on aggregate capital $\delta$, the law of motion for aggregate capital is given by

$$K_{t+1} = Y_t - C_t + (1 - \delta_t) K_t. \tag{7}$$

## 2.3 The Government Sector

The government administrates a PAYGO social security system, i.e. it collects taxes from workers and pays out social security benefits to the retired. In the current U.S. social security system, there are basically three important factors determining the social security benefits received by an agent: his/hers average income, the replacement ratio and the evolvement of average wages. The first is important because the level of benefits when reaching the retirement age is based on lifetime earnings. However, for computational ease, I will not base benefits on agents' income histories, but instead on the average

labor income in the economy.[9] This average is denoted by $\overline{W} = \frac{(e_0 + e_1)}{2} E[W]$, where $E[W]$ is the unconditional expected long-run wage rate in the stationary economy (and $e_0$ and $e_1$ are once more age-specific productivities).

The second determinant of the social security benefits is the replacement ratio $\eta$, i.e. the rate at which social security replaces past earnings. In a world without uncertainty, the social security benefit would thus be given by

$$\varphi = \eta \overline{W}. \tag{8}$$

However, benefits are also related to the aggregate wage-index, i.e., the evolvement of aggregate labor income. More specifically, an individual's earnings are indexed to the average wage level at the time of retirement.[10] The variability of the aggregate wage rate thus influences the variability of benefits. The period $t$ benefit can then finally be specified by the following two equations

$$\widetilde{\varphi}_t = a_0 \varphi + a_w \frac{W_t}{E[W]} \varphi , \tag{9}$$

and

$$a_0 + a_w = 1. \tag{10}$$

The wage-indexation scheme specified by the two equations (9) and (10) constitutes an easy way of transferring aggregate (wage) risk between agents participating in the social security system.[11] Equation (10) implies that unconditional expected benefits always equal $\eta \overline{W}$.[12] The importance of this equation is to ensure that only risk is transferred between taxpayers and retirees. In this paper, I am concerned with the welfare gains of improving risk sharing in social security, and violating (10) would shift the focus away from the issue of risk sharing to that of the optimal size of social security. This is obviously an

---

[9] This assumption can be viewed as a rough first approximation to the highly redistributive nature of the actual link between actual earnings and benefits. The assumption significantly eases the computational burden as individuals' income histories are not needed as state variables. Finally, Olovsson (2004b) shows that these histories are not really quantitatively important.

[10] Actually, an individual's earnings are indexed to the average wage level 2 years prior to the year of eligibility, i.e., when the agent reaches the age of 62.

[11] The notation in (9) is chosen to illustrate that the benefit $\varphi$ consists of a safe part and a risky part.

[12] However, no further restrictions are placed on $a_0$ and $a_w$.

important issue, but beyond the scope of this paper.

When $a_w = 0$, benefits are completely safe, in the sense that all productivity risk in social security is solely borne by the tax payers. This risk can then be shifted to the retirees, and arbitrarily increased by increasing $a_w$. Following Bohn (1998), the current U.S. social security system is modeled by setting $a_w = 1$. Benefits thus respond to aggregate wages with a factor of 1.

It may also be interesting to consider the alternative with capital-indexed benefits. In a world where aggregate uncertainty hits the economy according to (6), and the returns to labor and capital are perfectly correlated, the scheme given by (9) would in every aspect be equivalent to one where benefits are indexed to capital income or aggregate income. In the long run, labor and capital returns are indeed highly positively correlated in the U.S. and other OECD countries.[13] However, a stochastic depreciation rate was introduced in the previous section to make it possible for the total return on capital to vary somewhat independently from the wage rate. To analyze the implications of a PAYGO scheme where benefits are instead indexed to the total return on capital, I here introduce the following alternative function

$$\widetilde{\varphi}_t = a_0\varphi + a_k \frac{R_t K_t}{E\left[R_t K_t\right]}\varphi \ , \tag{11}$$

which must once more be combined with (10).

Finally, the government is required to balance its budget in each period. Since the government bond is in zero net supply, the government budget constraint is given by

$$\widetilde{\varphi}_t = \tau_t L W_t. \tag{12}$$

## 2.4 Shortselling Constraints

Shortselling constraints may be a potentially important issue, since they restrict agents to smooth consumption across states. More specifically, shortselling constraints prevent agents from borrowing against their future income, when facing a bad shock. Agents may therefore find the allocation of risk in social security more important when subject to shortselling constraints restricting them in their financial

---

[13] Baxter and Jermann (1997).

behavioral. Most likely, these constraints are most severe for the uninsured young. Both cases with and without shortselling constraints will therefore be considered. A shortselling constraint can purely be motivated on realistic grounds: it is a well known fact that human capital alone does not collateralize major loans in modern economies (for reasons of moral hazard and adverse selection).

The shortselling constraints are given by

$$b_{t+1,i} \geq 0, k_{t+1,i} \geq 0 \quad i = 0, 1, 2.$$

# 3 Equilibrium

## 3.1 The Decentralized Competitive Equilibrium

I am now searching for a stationary equilibrium where decisions made in a given period are determined by the aggregate shock $s_j = (Z_j, \delta_j)$, $s_j \in S$, the aggregate capital stock $K$ and the current wealth of the middle-aged $a_{-1} = (b_{-1}, k_{-1})$.[14] An equilibrium can be defined to consist of market clearing prices $R$, $W$ and $p$ and a set of age-specific functions $k_0' = k_0(s, K, a_{-1})$; $b_0' = b_0(s, K, a_{-1})$ and $k_1' = k_1(s, K, a_{-1})$; $b_1' = b_1(s, K, a_{-1})$ such that

**(1)** *The firm's profit maximization problem is satisfied*

$$R = \alpha \frac{Y}{K} + 1 - \delta \tag{13}$$

$$W = (1 - \alpha) \frac{Y}{L}. \tag{14}$$

**(2)** *Individual optimization problems are satisfied, i.e.* $\{U_i, c_i, b_i', k_i'\}_{i=0}^2$ *satisfies equations (5)*

**(3)** *Markets clear and aggregate quantities result from individual decisions*

$$K' = \sum_{i=0}^{1} k_i(s, K, a_{-1}) \tag{15}$$

---

[14] The young are born with zero assets.

$$B' = \sum_{i=0}^{1} b_i \left( s, K, a_{-1} \right).$$ (16)

**(4)** *The government budget constraint (12) is satisfied with equality.*

**(5)** *The resource constraint (7) holds with equality.*

All the above conditions should be considered as standard.

## 3.2 The Command Optimum

To properly evaluate different equilibria, the social optimum needs to be solved for. This optimum gives us the best possible outcome and is therefore a natural benchmark case. The social planner's problem is to maximize the welfare function at some initial date $t = 0$:

$$W_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t U_t \right],$$

subject to the resource constraint (7) and where the initial capital stock $K_0$ and the past consumption of the middle-aged and old $c_{t-1,0}$; $c_{t-1,1}$; $c_{t-2,0}$ are given. The planner is assumed to use a constant discount factor equal to the agents' subjective discount factor. Because utilities are evaluated in expectation, the allocation will be efficient *ex ante*, contingent on the initial conditions. There is some controversy in the literature about *ex ante* versus interim efficiency.[15] Under an interim perspective, agents born in different states of nature are considered to be distinct. A Pareto improvement will then require that no birth-contingent agent in any birth state is made worse off. For the purpose of this paper, i.e. policy analysis, interim efficiency is rather uninteresting, since policies almost always shift resources across states of nature and are therefore not comparable by interim standards.

The planner's first-order conditions provide the necessary conditions for ex ante efficiency

$$\frac{\partial E_t U_t}{\partial c_{0,t}} = \frac{\partial E_t U_{t-1}}{\partial c_{1,t}} = \frac{\partial U_{t-2}}{\partial c_{2,t}}$$ (17)

---

[15] See, for example, Wright (1987).

$$\frac{\partial E_t U_t}{\partial c_{0,t}} = \beta E_t \left[ \frac{\partial U_t}{\partial c_{1,t+1}} R_{t+1}^K \right]. \tag{18}$$

The first equation is, in the words of Bohn (1998), "the distributional optimality conditions" linking the consumption of the agents alive in period $t$. The second equation is "the intertemporal optimality condition" which reveals how the planner allocates resources over time. Equation (18) is actually identical to the individual optimality condition for savings (i.e., the Euler equation). In contrast, however, the efficiency condition (17) is generally not satisfied by the market allocation.[16] For time-separable utility, for example, (17) calls for a deterministic link between the contemporaneous consumption of all agents alive in a given period.

# 4   Computation of the Equilibrium

I use a spline collocation algorithm to numerically approximate the equilibrium. The strength of the three-period model is twofold: (i) it is sufficiently rich to allow agents to intergenerationally share risk and (ii) it allows me to rely on standard techniques when solving for the equilibrium. To solve their optimization problems, agents need to keep track of the aggregate capital stock, the (constant) net supply of bonds and the wealth of the middle aged (alternatively, the wealth of the old)

$$\underbrace{k_{0,t-1}\left(1+r_K\right)+b_{0,t-1}}_{\text{Wealth of the mid-aged}} + \underbrace{k_{1,t-1}\left(1+r_K\right)+b_{1,t-1}}_{\text{Wealth of the old}} = K\left(1+r_K\right)+B. \tag{19}$$

The approximation of the equilibrium is therefore straightforward.

A more serious challenge is the numerical computation of the social optimum. Generally, the social optimum is easier to compute than the decentralized equilibrium. With Epstein-Zin preferences, however, one also needs to keep track of consumption histories. I approximate this equilibrium by using the aggregate capital stock, the period $t-1$ consumption of the currently mid-aged and a variable summarizing

---

[16] See Bohn (1998) for a more profound discussion.

the consumption history of the currently old as state variables. The result is a very large state space, consisting of three continuous endogenous state variables. In addition, the system of equations shows a very high degree of curvature ($\theta$ is equal to -51 with the chosen calibration, thereby implying that some equations are raised to the power of -52).[17]

# 5  Calibration

## 5.1  Income Profiles and Social Security

**The average share of wage income going to the young** $\frac{e_0}{e_1}$. Recall that the productivity of the young and the old, respectively, is given by $e_0$ and $e_1$. Labor endowments are deterministic and set so that $\frac{e_0}{e_1} = 0.70$. This is basically consistent with the estimates from PSID data in Storesletten et al. (2003).

 **The expected replacement rate** $\eta$. In the U.S., the current payroll tax is 12.4 percent, and benefits replace 43.7 percent of the average pre-taxed wage.[18] However, up to 85 percent of the received benefits may also be subject to income tax. Since I do not want to put any restrictions on the possible allocations of risk between taxpayers and retirees, I will not assume benefits to be taxable.[19] Instead, I set $\eta = 0.4$, in the sense of benefits replacing 40 percent of the (unconditional expected) average life-time after-tax wage in the economy. This corresponds to a replacement ratio of 43.7 percent of the average pre-taxed wage, where roughly 50 percent of the received benefits are subject to income tax.

## 5.2  Aggregate uncertainty

Aggregate productivity is assumed to be driven by a four-state Markov process with state space $S = [s_1, s_2, s_3, s_4]$ and the transition probability matrix $\pi = (\pi_{ij})$.[20] An aggregate state is characterized by a TFP shock, and a depreciation rate $s_j = (Z_j, \delta_j)$, $s_j \in S$. The four states are in order

---

[17] Due to the high degree of curvature, a tensor product approach to the three-dimensional approximation is called for. The total number of unknowns then becomes: the No. of gridpoints times the No. of exogenous states times the No. of endogenous variables. Even with a simple 10x10x10 grid and 4 states, the total number of unknowns becomes 12000.

[18] See, for example, Mchale (1999).

[19] Obviously, if benefits are taxable, both workers and retirees will be equally exposed to tax risk.

[20] This assumption implies that markets are somewhat incomplete. The setting is needed to account for a high, but not perfect, correlation between return to capital and return to labor.

$$
S = \begin{cases}
(1-v), (\overline{\delta} + \zeta) \\
\left(1 - \frac{v}{3.5}\right), (\overline{\delta} - \zeta) \\
\left(1 + \frac{v}{3.5}\right), \left(\overline{\delta} + \frac{\zeta}{3.5}\right) \\
(1+v), \left(\overline{\delta} - \frac{\zeta}{3.5}\right)
\end{cases} ,
$$

where $\overline{\delta}$ is the average depreciation rate.[21] I set $\overline{\delta} = 0.626$, implying the average annual depreciation rate to be between 4 and 5 percent.[22] The stochastic process is assumed to be *i.i.d.* over time. Although it is well established that aggregate productivity shocks are highly autocorrelated at annual and quarterly frequencies, there does not seem to exist any conclusive evidence indicating such positive serial correlation at generational frequencies (i.e. 20-30 year periods).[23] As a benchmark, aggregate shocks are therefore assumed to be uncorrelated across time.

With the above representation, the aggregate state $s_1$ is characterized by a bad TFP shock and a bad depreciation shock whereas $s_4$ is given by a good TFP shock, and a good depreciation shock. In aggregate states $s_2$ and $s_3$, the TFP shock and the depreciation shocks move in opposite directions. It is also assumed that $\pi_1 = \pi_4$ and $\pi_2 = \pi_3$.

In order to pin down $v, \zeta$ and $\pi_1$, I set out to match the following statistics.[24]

**The coefficient of variation of the 20-year aggregate income, $\frac{\sigma(y)}{E(y)}$.** It is is rather problematic to calibrate this statistic, due to the fact that even a century-long time series only provides five non-overlapping observations, resulting in large standard errors of the point estimates. I follow Constantinides et al. (1998), and set the coefficient of variation of the 20-year aggregate income to 0.2.

**The coefficient of variation of the 20-year aggregate capital, $\frac{\sigma(K)}{E(K)}$.** Capital in this model is not just a claim to corporate dividends, but to all risky capital in the economy. According to Baxter and Jermann (1997), the return to labor is less volatile than the return to capital. In the U.S., the volatility of the return to labor is estimated to be 85 percent of the volatility of the return to capital. Lacking a

---

[21] The four aggregate states are somewhat asymmetric since this setup makes the numerical computation of the social optimum somewhat simpler. However, the results are not sensitive to the asymmetric setup.

[22] As in Cooley and Prescott (1995).

[23] This assumption is also in line with several other papers dealing with OLG-models with two or three periods. See, for instance, Bohn (1999), Kreuger and Kubler (2002) and Smetters (2002).

[24] Very roughly, $\nu$ determines the variation of aggregate income, $\varsigma$ determines the variation of aggregate capital and $\pi_1$ determines the correlation between returns to labor and capital. The calibration procedure involved a simple grid search algorithm.

better estimate, I set $\frac{\sigma^2(K)}{E(K)} = 1.25 \frac{\sigma^2(Y)}{E(y)}$, which makes the model consistent with that estimate.[25]

**The cross-correlation of aggregate labor income and aggregate capital income** $corr\,(w_t, R_t)$. According to Baxter and Jermann (1997) and Bohn (1999), the return to capital, and the return to labor are highly correlated in the long run. With these findings in mind, I set $corr\,(w_t, R_t) = 0.9$.

## 5.3 Preferences

Since this paper is concerned with the welfare effects of different allocations of risk, it is important to assign values to the preference parameters that actually reflect households' attitudes to risk. The preference parameters in the model are the RRA coefficient $\gamma$, the coefficient of intertemporal substitution $\sigma$, and the subjective discount factor $\beta$. To calibrate these preference parameters, I set out to make the model consistent with the following empirical findings:

**The annualized capital output ratio** $\frac{K}{Y}$. This statistic is calibrated to be 3.3, which is the number reported in Cooley and Prescott (1995).

**The average annual real risk free rate** $r_f$. The risk free rate varies over time so it is not really obvious what is a reasonable level for the safe real rate. When Mehra and Prescott announced the existence of a equity premium puzzle in 1985, they found the average riskless real interest rate to be 0.8 percent for the period 1947-1976.[26] Since the mid-eighties, however, the average real risk free rate has been somewhat higher than in the period studied by Mehra and Prescott. According to Campbell (1999), the average real risk free rate was 1.955 percent for the period 1891-1995.[27]

**The Sharpe ratio**, $\frac{E[r_k]-r_f}{std(r_k)}$. Since in the model, capital is not just a claim to corporate dividends but to all risky capital in the economy, I do not try to match the observed equity premium, the reason, of course, being that equity returns are much more volatile than capital return. Instead, I set out to match the Sharpe ratio, which in this case is the risk premium households demand for holding risky capital, divided by the standard deviation of the return to capital. Constantinides et al. (2002) report data on 20-year holding period real returns and standard deviations for bonds and equity. The implied Sharpe

---

[25] Due to the Cobb-Douglas technology, aggregate income and labor income are equally volatile.

[26] Mehra and Prescott (1985).

[27] The annual return in the model is defined as $100\left[R_f^{1/20} - 1\right]$, since a period is assumed to be 20 years.

ratio can then be calculated to 1.65.

I set $\gamma = 18$, $\sigma = 0.75$ and $\beta = 0.68$, which generates an annualized capital ratio of 3.3, an interest rate of 1.85 and a Sharpe ratio of 1.61 in the model without borrowing constraints and 1.67 in the model with borrowing constraints.[28] A $\beta$ equal to 0.68 corresponds to an annual discount factor of 0.98. To sum up, technology and preference parameters are set to

Table 1: PRODUCTION AND PREFERENCE PARAMETERS

| $v$ | $\zeta$ | $\pi_1$ | $\beta$ | $\gamma$ | $\sigma$ |
|-------|-------|-------|-------|-------|-------|
| 0.295 | 0.274 | 0.175 | 0.68 | 18 | 0.75 |

# 6  Results

## 6.1  Long-Run Equilibria

Results are presented in this section. All results presented in table 2 are expressed in relation to the current U.S. economy and, consequently, the capital stock and the welfare gain of being born into this economy are both normalized to 1.[29] Welfare gains are expressed from the perspective of an unborn agent. More precisely, a welfare gain of x percent of being born into a specific economy implies that the per-period consumption in the present U.S. must be increased by x percent, for the agent to be indifferent between being born into these two economies. Wage-Index* and Capital-Index* refer to economies with optimized indexation with respect to wages and capital, respectively. The laissez-faire economy is characterized by the absence of a state, taxes and thus, social security.

I summarize the results as follows.

- **General equilibrium effects are substantial**. First, the capital stock in the laissez-faire econ-omy is roughly 64 percent higher than in the current U.S., implying the crowding out effect to be 39 percent. Interestingly, this is very close to the empirical estimate of 38 percent in Feld-

---

[28] Once more, very roughly, $\beta$ determines the capital output ratio, $\sigma$ determines the risk-free rate and $\gamma$ determines the Sharpe ratio.

[29] All economies are simulated for 10 000 periods.

Table 2: CAPITAL STOCK AND WELFARE GAINS

|  | SHORTSELLING | | NO SHORTSELLING | |
|---|---|---|---|---|
|  | $E[K]$ | WELFARE GAIN | $E[K]$ | WELFARE GAIN |
| U.S. | 1 | 1 | 1 | 1 |
| SAFE BENEFITS | 0.9 | -10.09 | 0.95 | -9.29 |
| WAGE-INDEX* | 1.69 | 14.86 | 1.46 | 12.09 |
| CAPITAL-INDEX* | 1.71 | 13.83 | 1.40 | 9.41 |
| LAISSEZ-FAIRE | 1.64 | 19.89 | 1.63 | 19.59 |
| SOCIAL OPTIMUM | 5.8 | 50.88 | 6.3 | 52.19 |

stein (1974).[30] Second, the allocation of risk in the PAYGO system heavily influences the demand for capital. In the absence of shortselling constraints, the capital stock is actually higher under optimized wage and capital-indexation than in the laissez-faire illustrating that the crowding out effect can be completely eliminated. However, welfare is still higher in the laissez-faire economy, at least partially as a result of the fact that agents in the economies with social security are forced to service the debt associated with providing an unfunded transfer to the initial generations (those who were retired when the system was introduced), whereas this is not the case for agents in the laissez-faire.[31] Finally, wage-indexation seems to be slightly preferable to capital-indexation. Recall that in an OLG settings, it is the young that need to be insured and the risk they are facing is wage-risk.

- **Welfare effects are very large**. By all standards, the welfare gains associated with the different economies are huge. Welfare gains are also obviously very highly correlated with the size of the capital stock, indicating that the major part of the welfare gain associated with improved risk sharing actually comes from the GE effect. These results are thus in line with the findings of

---

[30] More specifically, Feldstein uses U.S. data for the period 1929 through 1971 to assess how the introduction and growth of social security have affected aggregate personal savings and the national accumulation of capital. See also Feldstein (1996).

[31] The welfare gains of the laissez-faire economy should therefore be corrected for these transfer effects for it to be the relevant object of comparison (as in Storesletten et al. (1999)).

Storesletten et. al (1998) and Krueger and Kubler (2002), who both find the major part of the (negative) welfare effects associated with social security to come from general equilibrium effects (i.e., it crowds out capital). Finally, welfare effects are of similar magnitude, irrespective of whether agents are subject to shortselling constraints. Thus, there seems to be no support for the view that the allocation of risk in social security is without importance, when agents can trade in several assets. On the contrary, welfare effects are actually somewhat higher in the absence of shortselling constraints.

- **Safe benefits are significantly worse than the status quo**. The welfare loss of being born into an economy with safe benefits is rather large: roughly 10 percent of per-period consumption. This welfare loss is due to the fact that the regime with safe benefits implies that the direct welfare effect and the GE welfare effects are both negative. The direct welfare effect is obviously negative, since safe benefits require contra-cyclical taxes, which inevitably increases the exposure of the young to aggregate risk.[32] The GE effect is also negative, since the capital stock is significantly lower under the regime of safe benefits.

- **A considerable difference between the social optimum and other economies, including the laissez-faire**. The capital stock in the social optimum is roughly 3.5 times the capital stock in the laissez-faire economy, indicating that the social planner builds up a very large buffer to be able to smooth the consumption of future generations. Consequently, the welfare gain of being born into the social optimum is also considerably larger than the gain of being born into the laissez-faire economy. These results illustrate the quantitative importance of *ex ante* inefficiency, and they show that agents actually find the risk of being born into the wrong state a serious issue.

## 6.2 The Allocation of Risk

Table 3 gives the optimal coefficients of the respective schemes (i.e., equations (9) and (11)). The coefficients are all positive, thereby implying that benefits, as well as taxes, should be pro-cyclical. This

---

[32] As stated in the introduction, the young are already too exposed to aggregate risk in the decentralized equilibrium when the intertemporal elasticity of substitution for consumption is less than 1 (see Bohn (1998)).

Table 3: Coefficients for efficient risk sharing

|  | NO SHORTSELLING | SHORTSELLING |
|---|---|---|
| WAGE-INDEX* | $a_w = 4.39$ | $a_w = 4.28$ |
| CAPITAL-INDEX* | $a_k = 2.72$ | $a_k = 3.0$ |

arrangement eases the risk exposure of the young, since they are subject to lower taxes in bad states, and vice versa. Social security thus provides some insurance to the uninsured young. As a contrast, the regime with safe benefits calls for contra-cyclical taxes which increase the risk exposure of the young, since they are forced to pay higher taxes in bad states. These results resemble the findings of Smetters (2002), who shows that the government can use negative capital income taxes (implying pro-cyclical wage taxes), to correct the "biological trading constraint" preventing living generations from negotiating contingent contracts with the unborn.[33] These are also in line with Bohn (1998, 2003) who find the young in OLG models to be generally exposed to aggregate risk.[34]

In addition, the coefficients are all much larger than 1. Highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. This is illustrated in Fig. 1, where capital demand is plotted as a function of $a_w$ when agents are subject to shortselling constraints. Capital demand by the young is basically constant, whereas capital demand by the middle-aged is strictly increasing in $a_w$. Maybe somewhat surprising, the magnitude of the respective coefficients in table 3 is very much the same, irrespective of whether agents are subject to shortselling constraints.

Since the value of the RRA coefficient is controversial, I carry out a sensitivity experiment in the Appendix with respect to this parameter. Instead of Epstein-Zin preferences, power utility (CRRA) is used, i.e., the instantaneous utility for household $i$ in period $t$ is specified by

$$U\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma}, \tag{20}$$

---

[33] A negative capital tax is valid when production takes the Cobb-Douglas form, the depreciation rate is less than 100 percent, and the intertemporal substitution elasticity is equal to one.

[34] Moreover, Bohn finds wage-indexed social security (i.e. $a_w = 1$) to be a neutral policy, implying that $a_w$ must be larger than 1 to correct the inefficient outcome.
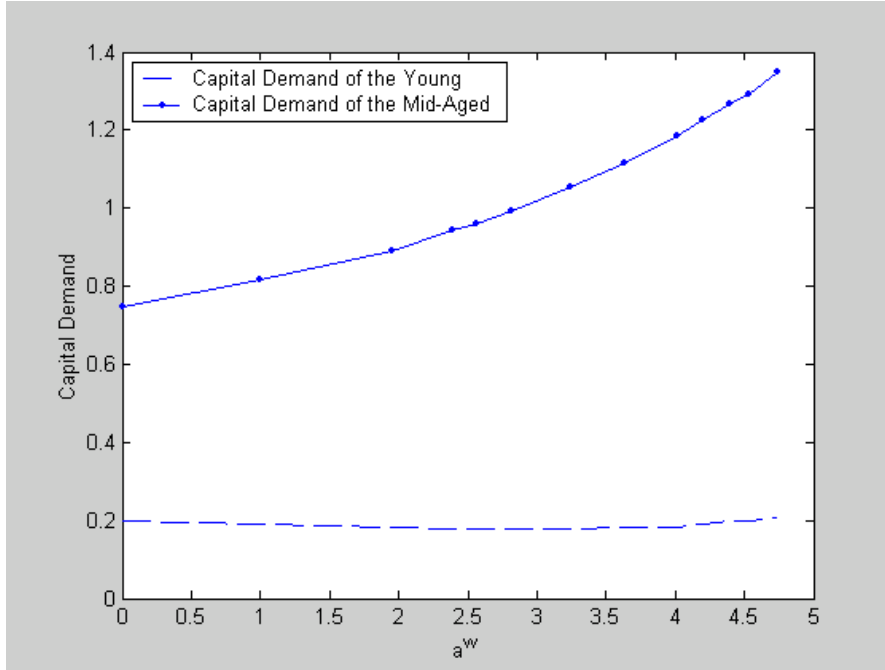
Figure 1: ASSET DEMAND WHEN SHORT-SELLING IS NOT ALLOWED

where $\gamma$ is set equal to 2. The rest of the parameters are kept constant, however (only the case with short selling constraints is considered). Comparing table 2 and table 5 in the Appendix, we see that the results from this exercise with a more moderate RRA coefficient are very similar to those presented above. The actual numbers are somewhat changed but basically, all the main findings above still go through. GE effects are very large, welfare effects are substantial, safe benefits generate the worst outcome and there is still a substantial difference between the social optimum and all other economies, including the laissez-faire. This result indicates that even if agents are only moderately risk averse, they still care a great deal about the allocation of risk, mainly because of the large GE effects at stake.

Finally, if agents are less risk averse, they respond less to changes in the allocation of risk. As a result, to generate GE effects, benefits must be made even more volatile than with Epstein-Zin utility (compare table 3 with table 6 in the Appendix).

20

## 6.3 Transitions

In this section, I carry out unexpected transitions from the current U.S. to the different economies. The results are presented in table 4.[35]

Table 4: UNEXPECTED TRANSITIONS

| | WELFARE GAIN | |
| --- | --- | --- |
| | NO SHORTSELLING | SHORTSELLING |
| U.S. | 1 | 1 |
| SAFE BENEFITS | -4.29 | -5.16 |
| WAGE-INDEX* | -1.62 | -0.51 |
| CAPITAL-INDEX* | 0.6 | -0.13 |
| LAISSEZ FAIR | 3.17 | 2.14 |
| SOCIAL OPTIMUM | 8.59 | 9.38 |

Almost all welfare gains are turned into losses when the transition is considered. The main explanation is that since these transitions are unexpected, the agents alive at the introduction of the new regimes are taken from a relatively safe environment and placed in a much more risky one, without a chance of hedging themselves against this new risk. The long-run gains found in the previous section rest upon the notion that productivity risk should be predictable and placed upon those agents who actually have the possibility of hedging this risk (i.e., the old). In the unexpected transitions, these hedging possibilities are simply not just there, obviously resulting in welfare losses.

However, since the welfare gains found in the previous section are so large, there should be some way of extracting these gains, by choosing a more sophisticated transition policy. There are several possibilities from which to choose. The most straightforward way is probably just to announce the transition several periods in advance. Another possibility could be that the government hedges the first generations in the transition to some degree, either by going into debt, or decumulating a buffer they have built up a before the transition. I leave this for future research.

---

[35]Results are averaged over 100 randomly chosen initial allocations in the benchmark economy called the U.S.

# 7   Conclusions

The importance of improved risk sharing in social security has been analyzed. The findings are that improved risk sharing in social security may imply very large welfare gains. The welfare gain of being born into an economy with efficient wage-indexation is between 12 and almost 15 percent of per-period consumption in the current U.S., depending on whether agents are subject to shortselling constraints. In contrast, the welfare loss of being born into the inefficient economy with completely safe benefits is roughly 10 percent of the per-period consumption. Efficient risk sharing in social security implies highly volatile and pro-cyclical benefits and tax rates. The pro-cyclical arrangement eases the risk exposure of the young, since they they are subject to lower taxes in bad states and vice versa. In this way, social security provides some insurance to the uninsured young. Highly volatile benefits are motivated by the fact that the allocation of risk in the PAYGO system has a major influence on demand for capital. More specifically, highly volatile benefits strengthen the precautionary savings motive of the middle-aged agents. As a result, they save more and hold more capital to hedge their coming volatile benefits. In fact, in the absence of shortselling constraints, the crowding out effect generally associated with an unfunded system can be completely eliminated by the use of efficient risk sharing arrangements. The welfare gains associated with the risk allocations analyzed in this paper are highly correlated with the capital stock, indicating that the major part of the welfare gain associated with improved risk sharing actually comes from the GE effect.

I also find a considerable difference between the social optimum and all other economies, including the laissez-faire economy. Despite the large welfare gains associated with improved risk sharing, no economy actually comes close to the social optimum. The capital stock in the social optimum is roughly 3.5 times the capital stock in the laissez-faire economy, indicating that the social planner builds up a very large buffer to be able to smooth the consumption of future generations.

An intuition for the findings in this paper is that the welfare cost of exposing the old to aggregate risk is relatively small, as long as the risk is predictable and known in advance. The reason is that the old have had a lifetime for setting up their hedging portfolios. In contrast, the earlier in life agents are exposed to risk, the smaller are their possibilities to hedge.

# References

[1] Abel, Andrew B. "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security." *Econometrica* 72 (2), March 2003, pp. 551-578.

[2] Ball, Laurence and Mankiw, Gregory N. "Intergenerational Risk Sharing in the Spirit of Arrow, Debreu, and Rawls, with Applications to Social Security Design." NBER working paper 8270, 2001.

[3] Baxter, Marianne and Jermann, Urban J. "The International Diversification Puzzle is Worse than You Think." *The American Economic Review* 87 (1), 1997, pp. 170-180.

[4] Bohn, Henning. "Risk Sharing in a Stochastic Overlapping Generations Economy". Mimeo, University of California, 1998.

[5] Bohn, Henning. "Should the Social Security Trust Fund Hold Equities? An Intergenerational Welfare Analysis". *Review of Economic Dynamics* 2, July 1999, pp. 666-697.

[6] Bohn, Henning. "Intergenerational Risk Sharing and Fiscal Policy". Mimeo, University of California, 2003.

[7] Campbell, John Y., Cocco, João F., Gomes, Francisco J., and Maenhout, Pascal J. "Investing Retirement Wealth: A Life Cycle Model." NBER working paper 7029, 1999.

[8] Constantinides, George M., Donaldson, John B. and Mehra, Rajnish. "Junior Can't Borrow: a New Perspective on the Equity Premium Puzzle." *Quarterly Journal of Economics* 117 (1), 2002, pp. 269-2976.

[9] Cooley, Thomas F and Prescott, Edward C. "Economic Growth and Business Cycles", in Cooley, Thomas F (ed.), *Frontiers of Business Cycle Research*. Princeton: Princeton University Press, 1995, pp.1-38.

[10] Epstein, Larry G., and Zin, Stanley E. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57 (4), 1989, pp. 937-969.

[11] Epstein, Larry G., and Zin, Stanley E. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis" *Journal of Political Economy* 99 (2), 1991, pp. 263-286.

[12] Feldstein, Martin. "Social Security, Induced Retirement, and Aggregate Capital Accumulation." *Journal of Political Economy* 82 (5)1974, pp. 905-926.

[13] Feldstein, Martin. "Social Security and Saving: New Time Evidence." *National Tax Journal* 49 (2) June 1996, pp. 151-164.

[14] Gonzalez-Eiras, Martin and Niepelt, Dirk. "Sustaining Social Security." Mimeo, Institute for International Economic Studies, Stockholm University 2003.

[15] Kreuger, Dirk and Kubler, Felix., "Pareto Improving Social Security Reform when Financial Markets are Incomplete!?" NBER working paper 9410, 2002.

[16] McHale, John "The Risk of Social Security Benefit Rule Changes: Some International Evidence." NBER working paper 7031, 1999.

[17] Mehra, Rajnish and Prescott, Edward C. "The Equity Premium: A Puzzle."*Journal of Monetary Economics* 15 (2), 1985, pp. 145-161.

[18] Olovsson, Conny "Why do Europeans Work so Little?" Seminar Paper 727. Institute for International Economic Studies, Stockholm University 2004a.

[19] ——."Why Social Security Increases the Equity Premium". Ph.D. Mimeo, Institute for International Economic Studies, Stockholm University 2004b.

[20] Peled, Dan "Informational Diversity and Optimality" *Journal of Economic Theory*. 28.(2), 1982, pp. 255-274.

[21] Shiller, Robert J. "Social Security and Institutions for Intergenerational, Intragenerational, and International Risk Sharing." NBER working paper 6641, 1998.

[22] Smetters, Kent A., "Trading with the Unborn: A New Perspective on Capital Income Taxation" NBER working paper 9412, 2002.

[23] Storesletten, Kjetil; Telmer, Chris I and Yaron Amir "The Risk Sharing Implications of Alternative Social Security Arrangements." Carnegie-Rochester Conference Series on Public Policy, 1999, vol. 50, pp. 213-259.

[24] ——."Consumption and Risk Sharing over the Life Cycle." 2003. Forthcoming in *Journal of Monetary Economics*.

[25] Weil, Philippe. "The Equity Premium Puzzle and the Risk-Free Rate Puzzle." *Journal of Monetary Economics* 24 (3), 1989, pp. 401-421.

[26] Wright, Randall D. "Market Structure and Competitive Equilibrium in Dynamic Models." *Journal of Economic Theory* 41 (1), 1987, pp. 189-201.

# A    Appendix

## A.1    A Sensitivity Experiment

### A.1.1    The Case of CRRA Utility

In this section, the RRA coefficient is set to 2, in order to evaluate the importance of this parameter. Except for this change, the rest of the calibration is kept constant. Only the case with short selling constraints is considered. The results are presented in tables 5 and 6.

Table 5: CAPITAL STOCK AND WELFARE GAINS, RRA=2

| | NO SHORTSELLING | |
| --- | --- | --- |
| | $E\left[K\right]$ | WELFARE GAIN |
| U.S. | 1 | 1 |
| SAFE BENEFITS | 1.02 | -1.74 |
| WAGE-INDEX* | 2.00 | 9.20 |
| CAPITAL-INDEX* | 2.03 | 7.87 |
| LAISSEZ-FAIRE | 1.95 | 28.09 |
| SOCIAL OPTIMUM | 3.03 | 43.90 |

Table 6: Coefficients for efficient risk sharing, RRA=2

| | NO SHORTSELLING |
| --- | --- |
| WAGE-INDEX* | $a_w = 6.43$ |
| CAPITAL-INDEX* | $a_R = 4.71$ |