A Theory of the Matching Function*

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Abstract

This paper develops a mechanical model of mismatch. Workers and jobs are randomly assigned to labor markets. Each labor market clears but some labor markets have more workers than jobs, hence unemployment, and some have more jobs than workers, hence vacancies. This simple model is quantitatively consistent with the U.S. Beveridge curve. I then augment the model to introduce a flow of workers and jobs across labor markets. This yields a matching function—the transition rate from unemployment to employment as a function of the vacancy-unemployment ratio—that is indistinguishable from Cobb-Douglas, a finding that is consistent with many empirical studies. I discuss other predictions that can be tested using microeconomic data.

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1 Introduction

The goal of this paper is to develop a simple theory of the matching function. Pissarides (2000) writes in his textbook,

> Trade in the labor market is a nontrivial economic activity because of the existence of heterogeneities, frictions, and information imperfections. If all workers were identical to each other and if all jobs were also identical to each other, and if there was perfect information about their location, trade would be trivial.... The matching function ... is a modeling device that captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit. (pp. 3–4)

This paper develops a model of heterogeneities, frictions, and information imperfections and hence a theory of the matching function.

There are fixed set of labor markets, workers, and jobs. Each labor market represents a different occupation, geographic location, etc.. At each instant, workers and jobs are randomly assigned to a labor market and each labor market clears. Variation in the average number of jobs per labor market, holding fixed the average number of workers per labor market, induce a downward sloping Beveridge curve (unemployment-vacancy locus). I find that a model with only a single free parameter, the average number of workers per labor market, can replicate both the location and slope of the U.S. Beveridge curve during the time period when reliable nationwide vacancy data are available, December 2000 to April 2005 (Figure 2).

I then extend the model to introduce flows of workers and jobs between labor markets (quits and layoffs).[1] I assume that once matched, a worker and job remain matched until either the worker quits (the worker leaves the labor market) or the firm lays the worker off (the job leaves the labor market). Both events are exogenous. I examine the relationship between the rate at which unemployed workers find jobs and the vacancy-unemployment ratio, analogous to a reduced-form matching function. I find that this is indistinguishable from a Cobb-Douglas matching function (Figure 3); however the theoretical elasticity of the matching function, 0.21, is somewhat lower than recent estimates using U.S. data, e.g. 0.28 in Shimer (2005a). Although the job finding rate is proportional

[1] The model is truly mechanical. A worker's "decision" to leave a labor market does not depend on how many workers and jobs are in the labor market, and similarly for a job.

to the total separation rate—the sum of the quit and layoff rates—I find that it is insensitive to the breakdown of separations between quits and layoffs. This is reassuring since in practice is often difficult to distinguish between these events.

I perform two other exercises with this model. First, I show that the theory also predicts that the job-to-job transition rate should be procyclical, consistent with recent evidence (Shimer 2005b). Second, I show that jobs are significantly more likely to hire a worker if they lose one during the month than if they start the month vacant. Moreover, the ratio of these probabilities depends systematically on the vacancy-unemployment ratio. These predictions can be tested using microeconomic data such as the Job Openings and Labor Turnover Survey (JOLTS) dataset.[2] Similarly, a worker is significantly more likely to find a job if she loses one during the month than if she starts the month unemployed, a prediction that can be tested using the Current Population Survey (CPS) microeconomic data.

The model that I develop in this paper is a variant on the "stock-flow" matching model (Taylor 1995, Coles and Muthoo 1998), which puts mismatch in a central position. I treat the allocation of workers and jobs to labor markets more explicitly than those earlier papers, but the most important difference is one of emphasis. These earlier papers stressed the differences between the stock-flow matching model and the matching function approach, whereas I find that a model of mismatch can explain why the matching function does such a good job of describing the data. For example, Lagos (2000, p. 854) emphasizes that "the matching function is an equilibrium object and is sensitive to policy." Coles and Petrongolo (2003) pitch their paper as a horse race between the stock-flow matching model and the matching function approach. Burdett, Shi, and Wright (2001, p. 1062) stress that "the standard matching function used in the literature is misspecified.... It should also depend on whether there are many firms each with a few vacancies or a few firms with many vacancies." None of these papers observe that the stock-flow matching model can explain the exact position and slope of the Beveridge curve and the robust finding that the matching function is Cobb-Douglas.[3]

Section 2 discusses data on the Beveridge curve in the U.S.. Section 3 describes and solves the model. Section 4 briefly concludes by mentioning my plans for future versions of this paper.

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[2] Unfortunately these data are not publicly available.

[3] But Lagos (2000) and Coles and Petrongolo (2003) emphasize that the stock-flow matching model predicts a constant returns to scale matching function.

2 The U.S. Beveridge Curve

Since December 2000, the Bureau of Labor Statistics (BLS) has measured vacant jobs using the JOLTS. According to the Bureau of Labor Statistics news release, July 30, 2002, available at `http://www.bls.gov/jlt/jlt_nr1.pdf`, "A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the Internet, posting help-wanted signs, accepting applications, or using similar methods."

By comparing the behavior of this measure of vacant jobs with the unemployment rate during the 2001 recession and subsequent slow recovery, it is possible to trace a Beveridge curve in the United States from December 2000 to April 2005. I construct the unemployment rate as the ratio of unemployed workers to the sum of employed and unemployed workers, where employment and unemployment are measured by the BLS from the CPS. The vacancy rate is the ratio of job openings to the sum of employed workers plus job openings.

Figure 1 plots the U.S. Beveridge curve. Two facts stand out. First, there is a strong negative correlation between the two variables, -0.963 in the raw monthly data. A one percent increase in the unemployment rate (e.g. from 5.00 to 5.05 percent) is associated with a 1.12 percent decrease in the vacancy rate (e.g. from 2.50 to 2.47 percent).[4] Second, there are many fewer vacant jobs than unemployed workers; in an average month during this time period, the ratio of the vacancy rate to the unemployment rate (v-u ratio) was 0.431 and it ranged between 0.310 and 0.816. While the labor market had a lot of slack during part of this period, by historical standards it was very tight in December 2000. Thus the empirically relevant range for the v-u ratio includes values that are significantly smaller than 1.

An important goal of this paper is to see whether a model of mismatch can reproduce Figure 1.

[4] The data also suggests the possibility of "counterclockwise loops", with both vacancies and unemployment higher during a recovery than during a downturn. This typically occurs if vacancies lead unemployment, but in this data set the two variables are contemporaneous.

3 A Model of Mismatch

In this section I develop a simple, essentially static, model of the coexistence of unemployed workers and vacant jobs due to mismatch. I then introduce dynamic considerations to study workers' "job finding rate" and firms' "job filling rate"—the rates at which unemployed workers find jobs and vacant jobs find workers, respectively.

3.1 A Theoretical Beveridge Curve

At each instant in time, there is a measure 1 of labor markets, a measure M of workers and a measure N of jobs. Each worker and each job is assigned to exactly one labor market. I assume that the allocation of workers to labor markets is independent across workers, with each one equally likely to be in each labor market. This implies that the number of workers in any labor market is a Poisson random variable with expected value M.[5] Similarly, I assume that the number of jobs in any labor market is a Poisson random variable with expected value N, independent of the number of workers in that labor market. Putting this together, the measure of labor markets with i workers and j jobs is

$$p(i,j) = \frac{e^{-(M+N)} M^i N^j}{i!j!}. \tag{1}$$

Each labor market clears at each instant, so unemployed workers and vacant jobs never coexist in a particular market. Instead, there are unemployed workers in a particular labor market at a particular moment in time if there are more workers than jobs in that market and there are vacant jobs if there are more jobs than workers. The unemployment rate u and vacancy rate v are given by

$$u = \frac{1}{M} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} (i-j) p(i,j) \quad \text{and} \quad v = \frac{1}{N} \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} (j-i) p(i,j). \tag{2}$$

This model provides a static theory of the coexistence of unemployment and vacancies. Before developing a theory of the Beveridge curve, consider some

[5] With a finite number of labor markets L' and a finite number of workers M', the fraction of labor markets with i workers would be

$$\frac{M'!}{i!(M'-i)!} \left(\frac{1}{L'}\right)^i \left(1 - \frac{1}{L'}\right)^{M'-i}.$$

Taking the limit as L' and M' go to infinity but holding $M = M'/L'$ fixed gives the Poisson approximation to a Binomial random variable, $e^{-M} M^i / i!$.

simple comparative statics. A proportional increase in both M and N tends to reduce both u and v by reducing the frictions in the labor market. An increase in the firm-worker ratio N/M tends to raise the v-u ratio $\theta = v/u$, consistent with a downward sloping Beveridge curve.

To make this into a theory of the Beveridge curve, I assume that the number of workers per labor market M is fixed over time but the number of jobs per labor market N varies. At each instant the fraction of labor markets with i workers and j firms is given by (1). And since the labor market clears at each instant, unemployment and vacancies are given by (2). Although I cannot say much about these expressions analytically, it is simple to compute them numerically.

The main finding is that the model-generated Beveridge curve is not only downward sloping but it is quantitatively reasonable (Figure 2). For example, if $M = N = 250$, both the unemployment rate and vacancy rate are 3.6%. Reducing N to 242 raises the unemployment rate to 5.4% and reduces the v-u ratio to 0.42, about the same as the mean value experienced in the U.S. between December 2000 and April 2005. According to the model, a 1 percent increase in unemployment caused by a decline in the number of jobs is associated with about a 1.2 percent decline in vacant jobs. The comparable elasticity in U.S. data is 1.1.

It is worth emphasizing that this model has only one free parameter, the number of workers per labor market M. I choose this parameter to get the level of unemployment and vacancies correct. The only other parameter, the number of firms per labor market N, varies so as to trace out a Beveridge curve. One could instead fix N and allow M to vary cyclically. The Beveridge curve would look similar.

3.2 A Theoretical Matching Function

The model of the Beveridge curve is essentially static since the unemployment and vacancy rates only depend on the number of workers and firms at each instant. But the matching function, which gives the rate at which unemployed workers find jobs as a function of the v-u ratio is inherently dynamic. I therefore must turn this model into one of worker and job flows.

Assume that each worker is forced to leave her labor market—quit—at rate q. Similarly, assume that each job leaves its labor market—laying off its worker—at rate l. These shocks are independent across workers, jobs, and labor markets. At the same time, new workers and jobs enter each labor market. Let m denote the flow of workers entering a labor market and n denote the flow of jobs entering a labor market. These workers and jobs may either be movers or new entrants. Movers and new entrants are independently assigned to labor markets. This is consistent with the earlier assumption that the joint distribution of workers and jobs is Poisson at each moment.

I assume that the number of workers is always in steady state, $m = qM$, but allow for deviations between the inflow and outflow of jobs:

$$\dot{N}(t) = n(t) - l(t)N(t),$$

where the argument t indicates time. In what follows I assume that l is constant but allow for time variation in $n(t)$ and hence $N(t)$. Still, I focus on situations where $n(t)$ has been constant for some time, so $N(t) = l/n(t)$ has adjusted to its stochastic steady state value.

Finally, I assume that if a worker and job are matched, they remain matched until one exits the labor market, which occurs at rate $q + l$. This means new matches are formed in some labor market only when a matched worker or job exits the labor market or a new worker or job enters.

To be precise, unemployed workers form matches in one of three ways. First, an employed worker may quit when there are unemployed workers in a labor market. This occurs at rate q for each of the j employed workers in a labor market with unemployed workers, $j < i$:

$$\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} qjp(i, j).$$

Second, a job may enter a labor market with excess workers. This occurs at rate $n = lN$ among labor markets with $i > j$:

$$\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} lNp(i, j).$$

Finally, an unemployed worker may move to a labor market in which there are excess jobs. Each of the uM unemployed workers leaves her labor market at rate q and then may find a new labor market with available jobs, $j > i$:

$$quM \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} p(i, j)$$

Summing these and dividing by unemployment uM gives the job finding rate for unemployed workers:[6]

$$F = \frac{\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} (qj + lN) p(i,j)}{\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} (i-j) p(i,j)} + q \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} p(i,j). \tag{3}$$

Note that F is linearly homogeneous in the quit rate q and the layoff rate l. All three objects are measured as an arrival rate per unit of time. It what follows, I think of the time unit as a month.

With $M = N = 250$ and $q = l = 0.02$, unemployed workers find jobs according to a Poisson process with arrival rate $F = 0.550$. If N falls to 240 with no change in the layoff rate l, the job finding rate falls to 0.436. More generally, Figure 3 shows that the model yields a reduced form matching function that is indistinguishable from a Cobb-Douglas, $f(\theta) = 0.551\theta^{0.214}$.

Starting with Pissarides (1986) and Blanchard and Diamond (1989), a number of authors have estimated matching functions; see the recent survey by Petrongolo and Pissarides (2001). For example, in Shimer (2005a) I compare a measure of the transition rate from unemployment to employment with the v-u ratio for the United States from 1951 to 2003. The correlation between the detrended time series is about 0.95 and least squares regressions suggest that $f(\theta)$ is well-described by $\mu\theta^{0.28}$, a somewhat higher elasticity than the one predicted by the model.

As noted before, doubling the quit and layoff rates doubles the job finding rate, effectively by cutting the period in half. For example, raising q and l to 0.06 turns the model period into a quarter. What about a change in the composition of separations between quits q and layoffs l? Empirically it is difficult to tell quits from separations but fortunately the model indicates that this is not very important (Figure 4). The job finding rate is virtually unaffected by the breakdown between quits and layoffs. This would also be true if there were cyclical changes in the composition of separations between quits and layoffs.

Finally, this model also yields a theory of job-to-job transitions. These can occur in one of two ways. First, an employed worker may be laid off in a market with available jobs, $j > i$:

$$\sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} lip(i,j);$$

[6]One can also compute the job-to-job transition rate. A job-to-job transition occurs when an employed worker switches to a labor market with excess jobs or when an employed worker's job exists but there are excess jobs in the labor market.

and second an employed worker may quit but move to a labor market with available jobs:

$$q(1-u)M\sum_{i=0}^{\infty}\sum_{j=i+1}^{\infty}p(i,j).$$

Summing these and dividing by $(1-u)M$ gives the job-to-job transition rate:

$$J=\frac{\sum_{i=0}^{\infty}\sum_{j=i+1}^{\infty}lip(i,j)}{M-\sum_{i=1}^{\infty}\sum_{j=0}^{i-1}(i-j)p(i,j)}+q\sum_{i=0}^{\infty}\sum_{j=i+1}^{\infty}p(i,j). \tag{4}$$

Figure 5 shows that the job-to-job transition rate moves with the v-u ratio, consistent with the evidence discussed in Shimer (2005b). Again, this finding is insensitive to the breakdown of separations between quits and layoffs.

3.3 Other Empirical Predictions

The mismatch model has a number of other empirical predictions.[7] First, a worker who has been unemployed for a long time is relatively unlikely to get a job, since she is probably in a labor market with few jobs and many workers. Of course, there are other explanations for duration dependence in the hazard of exiting unemployment, so this is a weak test of the model. Second, an unemployed worker's job finding rate depends not only on the stock of unemployed workers and vacant jobs u and v but also on the inflow rates, e.g. n and m. But this test is weak because Figure 3 suggests that the v-u ratio may be a good predictor of the job finding rate even in the mismatch model.

To develop additional tests, I consider which firms are most likely to hire workers. I compare two jobs. One is vacant at the start of the month while the other is filled but experiences a separation at some point during the month. A vacancy indicates a firm's inability to hire, and so one might expect a job whose worker quits to be more likely to hire during the month.

To be more precise, a vacancy is filled according to a Poisson process with arrival rate

$$G=\frac{\sum_{j=1}^{\infty}\sum_{i=0}^{j-1}(li+qM)p(i,j)}{\sum_{j=1}^{\infty}\sum_{i=0}^{j-1}(j-i)p(i,j)}+l\sum_{j=0}^{\infty}\sum_{i=j+1}^{\infty}p(i,j), \tag{5}$$

analogous to equation (3). The probability that a vacancy is not filled during a full month of searching for a worker is $\exp(-G)$, the probability that it is not

[7] See Coles and Petrongolo (2003) for a test of these and similar predictions.

hit by the Poisson shock.

In contrast, when a worker leaves her job, the firm can fill it instantaneously if there are more workers than jobs at her labor market, with probability

$$\sum_{i=1}^{\infty}\sum_{j=0}^{i-1} p(i,j).$$

Even if the firm fails to fill the job instantaneously, there is still a chance it manages to hire a worker during the remaining fraction of the month. If the worker is equally likely to leave at any time during the month, the probability the firm does not fill the job during the month is

$$\int_0^1 \exp(-Gt)dt = \frac{1-\exp(-G)}{G}.$$

I use these expressions to compute the probability that a job which loses its worker is unfilled by the end of a month.

Figure 6 compares these two probabilities. When the v-u ratio is high, the probability that a vacancy is not filled within the month is about 1.5 times the probability that a firm which loses a worker fails to hire a new one within the month. At lower v-u ratios, the ratio exceeds to 2 since the job is almost always immediately refilled. Again, this result seems to be quantitatively insensitive to the composition of separations between quits q and layoffs l.

There is one important caveat: some separations occur because of layoffs rather than quits; and following some layoffs a firm may not create a job in another labor market. In this event, the firm will obviously not hire another worker. This suggests a comparison between two types of firms. Firm x has a vacancy at the beginning of the month while firm y has a filled job. The mismatch model predicts that, conditional on firm y experiencing a separation during the month and still having a (vacant or filled) job at the end of the month, firm y is more likely to have hired a worker than is firm x. This is true even if one conditions on the event that firm x still has a (vacant or filled) job at the end of the month. This prediction can be explored using the microdata behind JOLTS.

One can similarly compare which worker is more likely to find a job, one who starts the month unemployed or one who loses her job during the month. The model predicts that, contingent on remaining in the labor market, the later worker is more likely to find a job (Figure 7). Moreover, the probability of that a quitter or job loser finds a new job is significantly more sensitive to the vacancy-unemployment ratio. The next iteration of this paper will explore these predictions using the CPS microdata.

4 Conclusions

A mechanical model of mismatch can quantitatively explain the U.S. Beveridge curve. It can also explain the apparent existence of a Cobb-Douglas matching function, although the theoretical elasticity is 0.21 rather than 0.28 in the data. The model has other predictions that can be tested using microeconomic data.

The next steps in this research program are (i) to test the microeconomic predictions and (ii) to relax some of the model's stronger mechanical assumptions, e.g. allow workers and jobs to move to another labor market when doing so is optimal.

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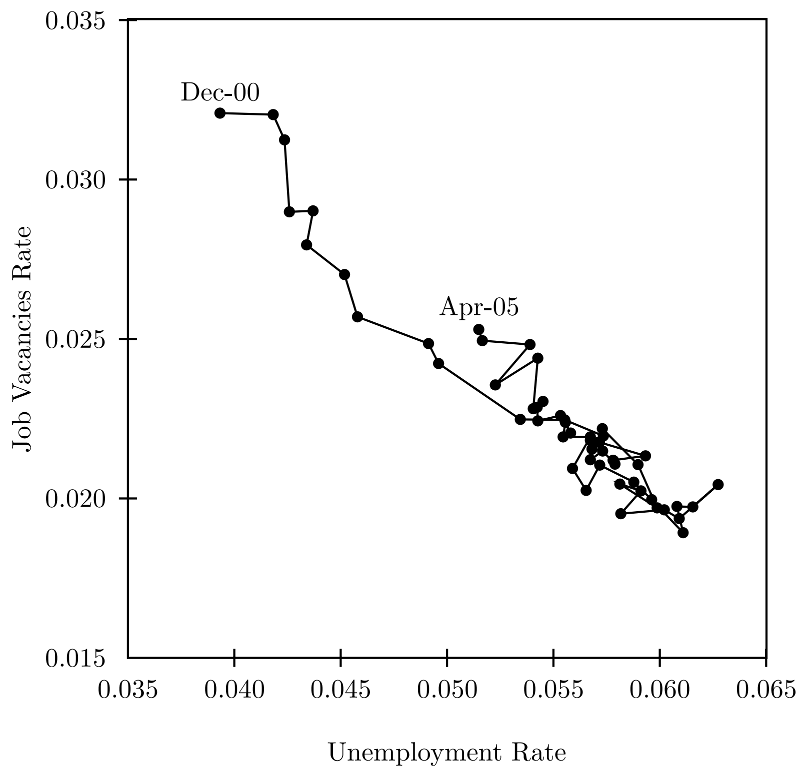

Figure 1: U.S. Beveridge curve, December 2000 to April 2005. Unemployment and Employment are measured by the BLS from the CPS. Job vacancies are measured by the BLS from the JOLTS.

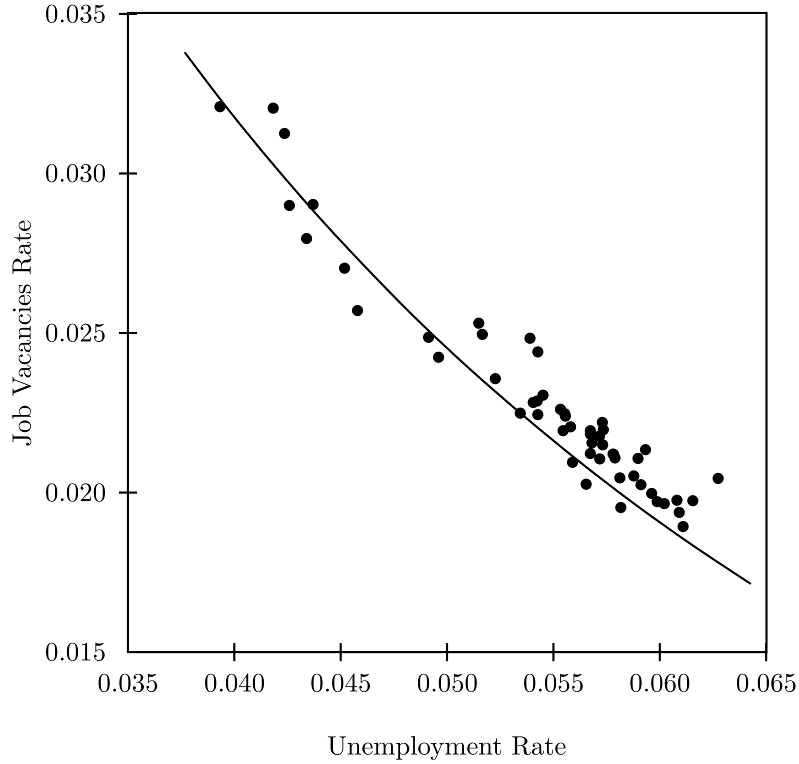

Figure 2: Beveridge Curve in Mismatch Model. The number of workers per labor market is fixed at $M = 250$ and the number of jobs per labor market N varies from 238 to 249. The U.S. Beveridge curve from Figure 1 is repeated for comparison.

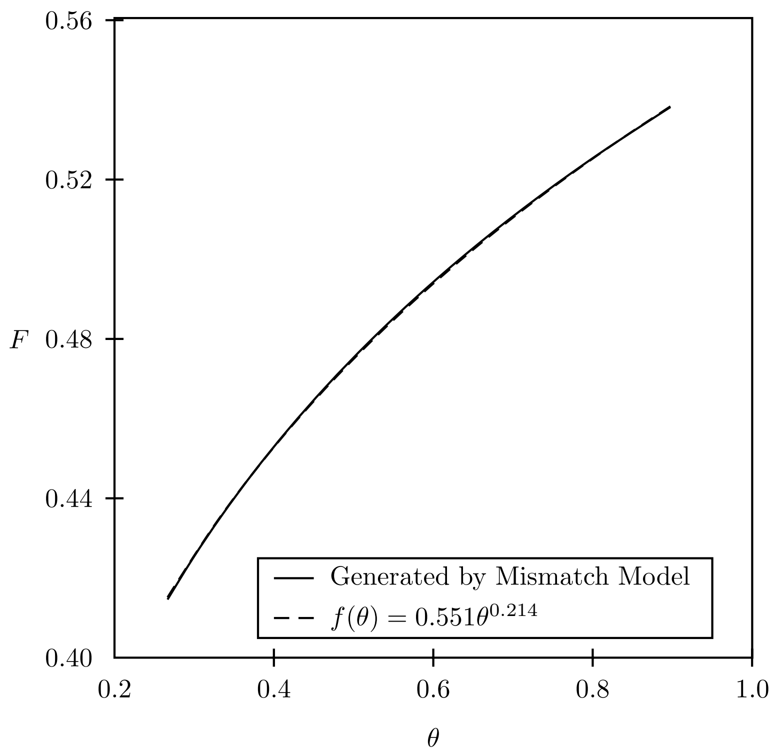

Figure 3: Job Finding Rate in the Mismatch Model. The solid line shows the relationship between the job finding rate F and the $v-u$ ratio θ when number of workers per labor market is fixed at $M = 250$, the quit rate is fixed at $q = 0.02$, the layoff rate is fixed at $l = 0.02$, and the number of jobs per labor market N is allowed to vary from 238 to 249. The dashed line shows $f(\theta) = 0.551\theta^{0.214}$.

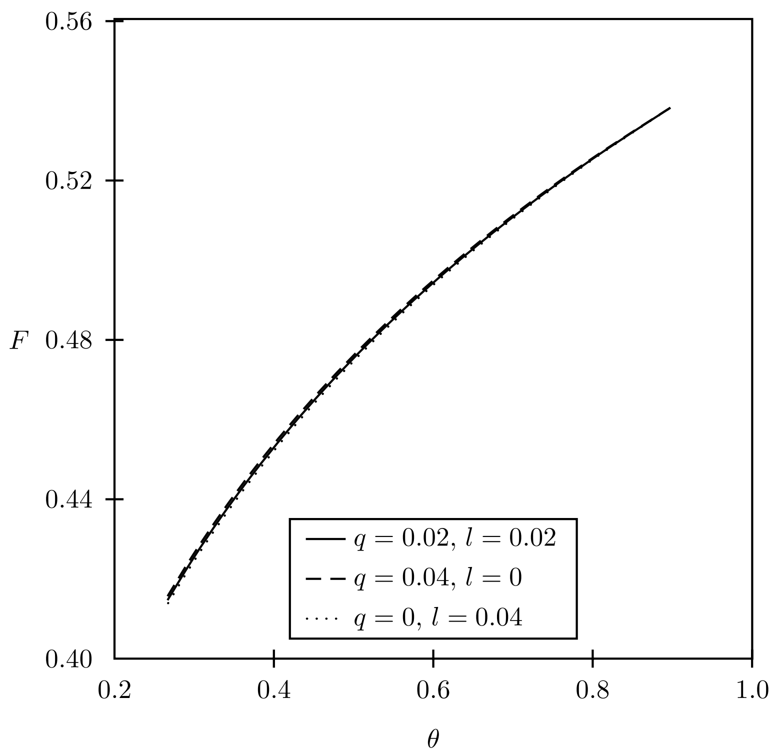

Figure 4: Variation in the composition of separations. The solid line shows the relationship between the job finding rate F and the $v-u$ ratio θ when number of workers per labor market is fixed at $M = 250$ and the number of jobs per labor market N is allowed to vary from 238 to 249 for three different values of q and l with $q + l = 0.04$.

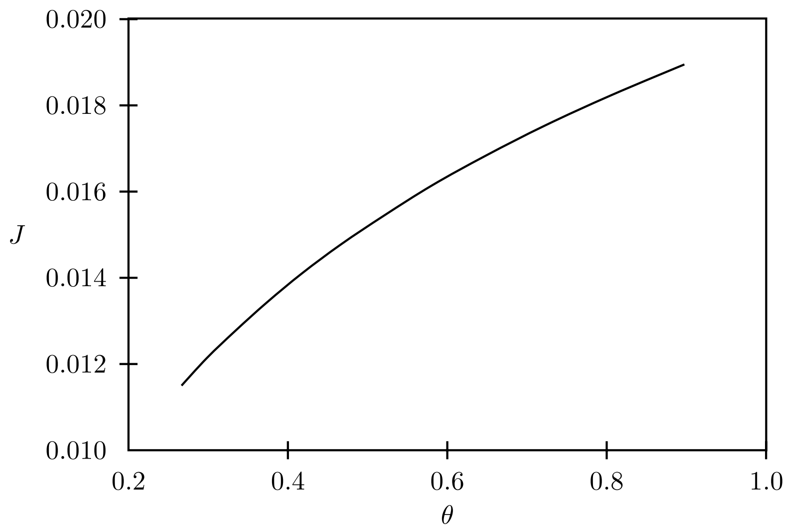

Figure 5: Job-to-Job Transition Rate. The number of workers per labor market is fixed at $M = 250$, the quit and layoff rates are fixed at $q = l = 0.02$, and the number of jobs per labor market N is allowed to vary from 238 to 249.

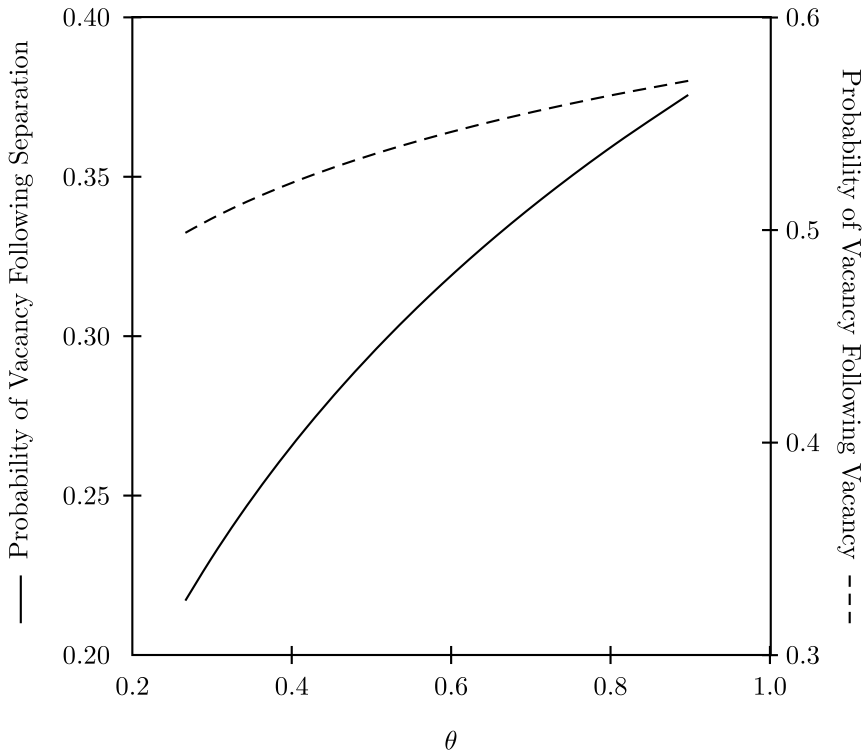

Figure 6: Probability that a Job Opening is Not Filled in the Mismatch Model. The solid line (left axis) shows the probability that a job is not filled by the end of the month after a worker separation. The dashed line (right axis) shows the probability that a vacancy is not filled during a full month of search. The number of workers per labor market is fixed at $M = 250$, the quit rate is fixed at $q = 0.02$, the layoff rate is fixed at $l = 0.02$, and the number of jobs per labor market N is allowed to vary from 238 to 249.

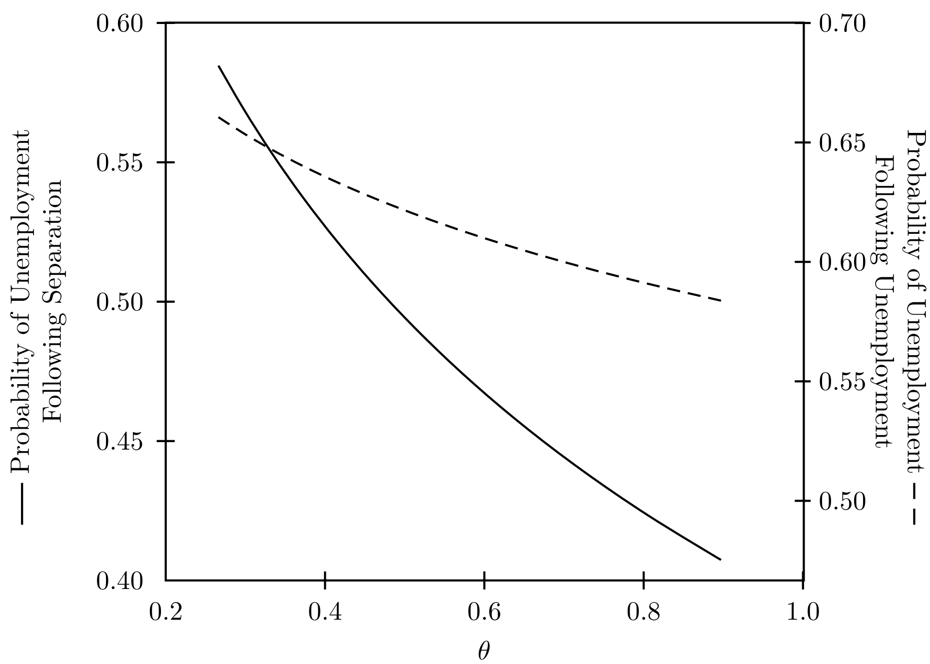

Figure 7: Probability of Not Finding a Job in the Mismatch Model. The solid line (left axis) shows the probability that a worker does not find a job by the end of the month after a separation conditional on remaining in the labor market. The dashed line (right axis) shows the probability that an unemployed worker does not find a job during a full month of search. The number of workers per labor market is fixed at $M = 250$, the quit rate is fixed at $q = 0.02$, the layoff rate is fixed at $l = 0.02$, and the number of jobs per labor market N is allowed to vary from 238 to 249.