# Sovereign Default and Debt Renegotiation

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## Abstract

In this paper, we develop a small open economy model to study sovereign default and debt renegotiation within a dynamic borrowing framework. The model features both endogenous default risk and endogenous debt recovery rates. A country's future borrowing and default decisions affect the determination of debt recovery rates in a Nash bargaining game; whereas the endogenous debt recovery rates, in turn, influence the country's ex ante incentive to default. Sovereign bonds are priced to compensate creditors for the risks of default and debt restructuring in equilibrium. We find that both equilibrium debt recovery rates and sovereign bond prices decrease with the level of debt. In a quantitative analysis, the model successfully accounts for the volatile and countercyclical bond spreads, countercyclical current account and other empirical regularities of the Argentine economy. The model also replicates the dynamics of bond spreads during the recent debt crisis in Argentina. Furthermore, we show that introducing an endogenous debt recovery schedule leads to a higher default probability and greater interest rate volatility.

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# 1 Introduction

The markets of sovereign debt for emerging economies have developed rapidly over the past few decades.[1] Associated with the enormous growth of sovereign debt markets are the recurrent large-scale sovereign debt crises.[2] To resolve the debt crises in the absence of an international bankruptcy law, the defaulting countries and lenders usually renegotiate over the reduction of defaulted debt.[3] Despite the importance of post-default debt renegotiation to sovereign borrowing and default, the existing literature does not contain a model that adequately captures the strategic considerations at play in the international capital markets. It remains a challenge to incorporate both sovereign default risk and debt renegotiation into a dynamic equilibrium model and to account for the dynamics of sovereign bond spreads in emerging economies.

In this paper, we develop a small open economy model to investigate the connection between sovereign default, debt renegotiation, and interest rates in a dynamic borrowing framework. The model features both endogenous default risk and endogenous debt recovery rates. With this model, we theoretically and quantitatively study the determination of debt recovery rates and how debt renegotiation interacts with a country's default decision. Moreover, we analyze the valuation of sovereign bonds and map the model to the Argentine data in a quantitative exercise.

In the model, a risk-averse country and risk-neutral competitive financial intermediaries trade one-period discount bonds in incomplete capital markets. The country faces stochastic endowments and has an option to default. Default may result in the loss of future access to the capital markets, or incur direct sanctions imposed by the lenders. However, through renegotiation over debt reduction, the inefficient sanctions can be lifted and the defaulting country can restore its reputation, regaining access to capital markets once the renegotiated debt is repaid in full. In the meantime, the lenders can recover at least a part of the defaulted bonds. Debt recovery rates, which are endogenously determined in a Nash bargaining game, affect a country's ex ante incentive to default. In equilibrium, sovereign bonds are priced to compensate the lenders for both the default risk and the risk of debt restructuring.

We first establish the existence of a recursive equilibrium in the model economy. In equilibrium bond prices and debt recovery rates are jointly determined, and vary with the domestic endowment and the level of debt. We analytically characterize the equilibrium bond prices and equilibrium debt recovery schedule. The debt recovery rates decrease with the indebtedness,

[1] The total amount of debt outstanding in developing countries has increased from \$553 billion to \$2,433 billion from 1980 to 2003. In 2003 the total international debt issued by emerging economies reached \$167.2 billion.

[2] There are 84 events of sovereign default from 1975 to 2002 according to Standard and Poor's (2002). One recent example is the Argentina's default on the international bonds of over \$82 billion in 2001.

[3] The most recent renegotiation is the Argentine sovereign debt restructuring closed in 2005. See Chuhan and Sturzenegger (2003) for the cases of sovereign debt renegotiation in 1980 to 2000.

but for low debt levels, there is no debt reduction. We also show that default may arise in equilibrium, and a country is more likely to default if it has a higher level of debt. Finally, interest rates increase with the level of debt due to the higher default probability and lower debt recovery rate.

We use the model to analyze quantitatively the sovereign debt of Argentina, calibrating the model to match the default probability, debt recovery rate, and debt-to-output ratio. The model successfully accounts for the volatility of the short-term Argentine bond spreads. Moreover, it generates the countercyclicality of bond spreads, which is found to have an important effect on the business cycles of emerging economies.[4] In the model, when a country gets a bad shock, default risk is higher, and expected debt recovery rate is smaller. Therefore, the sovereign bond spreads are higher in recessions. Furthermore, the model generates volatile consumption and countercyclical current account, which are in line with the data. We also show that the model can replicate the time series of Argentine bond spreads from 1992 to 2002.

In addition, we quantitatively examine the role of debt renegotiation. Compared to the models with endogenous default but no renegotiation,[5] our model can generate higher default probabilities and more volatile bond spreads. The introduction of debt renegotiation induces more frequent defaults because the international financial intermediaries are willing to lend more and the default option is more appealing to the borrowing country. At the same time, the endogenous debt recovery rates contribute to the higher bond price volatility because the movement in the debt recovery rates amplifies default risk. We also demonstrate that the changes in bargaining power have a great impact on debt recovery rates as well as on the sovereign bond spreads, shedding light on the policy implications of sovereign debt restructuring procedure. When the sovereign borrower has a higher bargaining power, the equilibrium debt recovery rates decrease, which drives up the sovereign bond spreads. However, the average sovereign bond spreads may not increase because the country could borrow less in equilibrium and is less likely to default. As a result, the sovereign bond spreads do not increase monotonically with the bargaining power.

The remainder of the paper is organized as follows. The next subsection discusses the related literature. In Section 2, we describe the model environment. Section 3 presents the sovereign borrower and lenders' problems and defines a recursive equilibrium. We then demonstrate the existence of a recursive equilibrium and characterize the equilibrium bond prices and debt recovery rates. Section 4 presents the model calibration and results of the quantitative analysis. We conduct sensitivity analysis and policy experiments in Section 5. Finally, Section 6 offers

[4] Neumeyer and Perri (2004) and Uribe and Yue (2005) document the countercyclical country interest rates for emerging markets. They show that countercyclicality of sovereign bond spreads exacerbates the business cycle fluctuations in these countries.

[5] Arellano (2004) and Aguiar and Gopinath (2004b) assume these default penalties in their quantitative study of sovereign default.

concluding remarks. The proofs and computational procedure are in the Appendix.

### 1.1 Related Literature

Our work builds on several strands of literature. A significant part of the sovereign debt literature focuses on the question of why countries repay their debt. Reputation and sanctions have been identified as two main enforcement mechanisms of debt repayment. In their pioneering work on reputation models, Eaton and Gersovitz (1981) argue that a country's incentive to make repayments is to preserve its reputation as a good borrower. In their model, the country uses international capital markets to share its income risk, but default causes a permanent exclusion from the markets. Grossman and Van Huyck (1987), Atkeson (1991), Cole and Kehoe (1998), Kletzer and Wright (2000), and Wright (2002), explore other aspects of reputation mechanism in enforcing debt repayment in models with dynamic borrowing. In the second approach, the country's debt repayment motive comes from the creditors' threat of direct sanctions. In particular, Bulow and Rogoff (1989a) argue that direct default punishment is necessary to support international debt when cash-in-advance contract is available. Sachs and Cohen (1982) and Cole and Kehoe (2000) also assume direct default cost in analyzing the sovereign debt crises. However, all these papers assume that a country either fully repays its debt or defaults completely, incurring the default penalties. The manner in which a debt crisis is resolved is not taken into account and thus plays no role in the country's default decision in these papers.

The literature on sovereign debt renegotiation analyzes the impact of debt renegotiation on international lending and borrowing. Bulow and Rogoff (1989b) present a model of sovereign debt renegotiation in which direct sanctions are lifted through a continuous bargaining. Fernandez and Rosenthal (1990) analyze debt renegotiation by assuming that the borrowing country gains improved future access to capital markets when the renegotiated debt is repaid in full. Recent studies (see Eichengreen, Kletzer and Mody (2003), Weinschelbaum and Wynne (2003) and Bolton and Jeanne (2004)) focus on the implication of several proposals about the reforms of sovereign debt renegotiation process, including Collective Action Clauses (CACs) and Sovereign Debt Restructuring Mechanism.[6] However, the dynamic bargaining games analyzed in this literature are embedded in a static one-shot borrowing model. Therefore, a country's reputation for future borrowing plays no role in the renegotiation. The distinguishing feature of our paper relative to the two strands of literature above is that we incorporate both default and debt renegotiation into a dynamic borrowing model. Thus the defaulting country's reputation is important in the bargaining and the valuation of sovereign bonds depends on both default risk and debt recovery rates. Moreover, the framework in our paper helps to evaluate the policy

[6] Collective Action Clauses (CACs) in sovereign bond contracts are majority renegotiation clauses, under which the changes endorsed by a specified majority of bond holders are binding on all bondholders. Sovereign Debt Restructuring Mechanism is a statutory code proposed by IMF. See Roubini and Setser (2004) for more details.

reform of debt renegotiation process.

There are relatively few papers that study sovereign debt pricing. Gibson and Sundaresan (2001) present a model of sovereign debt valuation with endogenous default risk and restructuring risk in a static borrowing framework. Arellano (2004), and Aguiar and Gopinath (2004b) explore the connection between endogenous default, interest rates and income fluctuations in a dynamic model. Their models can generate countercyclical bond spreads. However, their models are less successful in explaining the high default frequency and large interest rate volatility in Argentina. Moreover, they rule out debt renegotiation by assuming a zero debt recovery rate post default. Our equilibrium model not only endogenizes debt recovery rates, but also quantitatively accounts for the volatile and countercyclical sovereign bond spreads as well as the high default frequency.

This paper is also related to the literature of optimal contract with endogenous risk of default, such as Kehoe and Levine (1993), Kocherlakota(1996), and Alvarez and Jermann (2000). In this literature, perfect risk sharing is not achieved due to the commitment problem even though the asset markets are complete. Default does not arise in equilibrium, and agents with high endowment have higher default incentive, which is counterfactual. Several papers in this literature, such as Phelan (1995), Krueger and Uhlig (2004) and Cooley, Marimon and Quadrini (2004), endogenize an agent's outside options. They assume that the defaulted agents can start a new credit relationship with a competing financial intermediary. This paper is related to these works in endogenizing the value of default. The difference is that our model does generate default in equilibrium, and the default risk is higher at bad times.

Several papers on consumer default are also relevant. Athreya (2002) develops a dynamic equilibrium model of personal bankruptcy with stochastic punishment spells to investigate the welfare implication of the bankruptcy reform. Chatterjee, Corbae, Nakajima and Rios-Rull (2002) construct an equilibrium model of unsecured consumer debt and default to evaluate bankruptcy law. The theoretical framework in their paper explains the endogenous borrowing limit, and our model is closely related to theirs. Li and Sarte (2003) and Athreya (2004) assess the role of production and asset exemption in consumer bankruptcy, respectively. Livshits, MacGee and Tertilt (2003) compare the consumer bankruptcy rules in the U.S. and Germany. In contrast to the case of consumer default studied in these papers, bankruptcy rules are absent in sovereign default. Therefore, the post-default debt renegotiation plays an important role in resolving sovereign debt crises, which is the focus of our paper.

## 2 The Model Environment

We study sovereign default and debt renegotiation in a dynamic model of a small open economy. We consider a risk-averse sovereign country that can not affect the world interest rate. The country's preference is given by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

where $0 < \beta < 1$ is the discount factor of the sovereign, $c_t$ denotes the consumption in period $t$ and $u : \mathbb{R}_+ \to \mathbb{R}$ is the period utility function, which is assumed to be continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. The discount factor reflects both the sovereign's pure time preference and the probability that the current sovereignty will sustain in the next period.[7]

The model analyzes an endowment economy. In each period the country receives an exogenous endowment of the single non-storable consumption good $y_t \in Y$. The endowment $y_t$ is stochastic, reflecting the variation in a country's output due to changes in trade environment, production technology, or other factors. We assume that $y_t$ has a compact support $Y$ and is drawn from a atomless probability space where $\mu_y(y_t|y_{t-1})$ is the probability distribution function of a shock $y_t$ conditional on the previous realization $y_{t-1}$.

International financial intermediaries are risk-neutral and have perfect information on the country's endowment and asset position. We also assume that they behave competitively on the international capital markets and can borrow or lend as much as needed at a constant world risk-free interest rate $r$. Each period, one financial intermediary is randomly selected to trade with the sovereign government.[8]

The capital markets are incomplete. The sovereign government and financial intermediaries can borrow or lend via one-period zero-coupon bonds, but no state-contingent contracts are available. The face value of a discount bond is denoted as $b'$, specifying the amount to be repaid next period. When the sovereign government purchases bonds, $b' > 0$, and when it issues new bonds, $b' < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} < -\frac{\overline{y}}{r}$, which is the largest debt level that could be repaid by the country. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate. Such an upper bound exists when the interest rates on a country's saving are sufficiently small compared to the time discount factor. Specifically, let $q(b', y)$ be the price of a bond with face value $b'$ issued by the sovereign with an endowment shock $y$. We assume that for $b > 0$, bond price $q(b, y) > \beta$. The bond price function will be determined in equilibrium.

[7] Grossman and Van Huyck (1988) construct a model of sovereign borrowing where the time discount factor is a product of time preference coefficient and the government's survival probability of staying in power next period.

[8] An alternative assumption is that there exists a coordinating mechanism for creditors in the debt renegotiation. That is, we rule out the strategic "holdouts" behavior of creditors in the post-default debt renegotiation. Our assumption corresponds to the sovereign bonds issued with CAC clauses, or the sovereign loans restructured through the London Club or the Paris Club.

## 2.1 Default Option and Renegotiation on Debt Reduction

As in most sovereign debt studies, we assume that sovereign debt contracts have one-sided commitment.[9] The financial intermediaries always keep their promise. But the sovereign government is free to decide whether to repay or default on its debt. Default can result in two types of punishment. The first type of punishment is the reputation cost of losing its access to the capital markets, as in Eaton and Gersovitz (1981). The defaulting country cannot borrow because the debt contract signed first has to be honored first before new debt can be issued.[10] We also assume that the country cannot save after default. This assumption can be rationalized if the assets that the country attempts to accumulate in the default periods will be seized by the creditors. Alternatively, the financial intermediaries can collude and apply the "cheat the cheater" strategy (see Kletzer and Wright (2000) and Wright (2002)). In either case, countries with default on record are unable to save, and the saving opportunity analyzed in Bulow and Rogoff (1989a) does not apply. We also assume that there is a loss equal to a fraction $\lambda_d$ of endowment for a country with a default record, which captures the direct cost associated with a bad reputation.[11]

The second type of punishment is the direct sanctions that creditors may impose on the defaulting country. Creditors may litigate on foreign courts or apply trade sanctions, forcing the country to conduct its trade in roundabout ways to avoid seizure, as discussed in Bulow and Rogoff (1989b) and Rose (2002). Due to these retaliatory actions, the country's endowment decreases by a fraction $\lambda_s$.

However, when debt renegotiation is allowed, sovereign default does not necessarily lead to an imposition of full default punishment and complete disruption of the debt relationships. Direct sanctions, $\lambda_s y$, can be avoided if the creditors can be persuaded to waive sanctions in debt renegotiations. The financial exclusion can also be alleviated to a temporary exclusion from the markets. Thus, both types of default penalty can be reduced through debt renegotiation. Such a renegotiation is beneficial to both parties: the sovereign country avoids severe default punishment and the creditors get partial payments in debt renegotiation.

We assume that the lender bargains with the sovereign government over debt reduction. A debt reduction renegotiation involves the following concessions:

1) Upon agreement, the government's next-period debt arrear is reduced to some fraction of the unpaid debt $b$.

2) The intermediaries do not impose the direct sanctions $\lambda_s y$ on the country.

[9] Kletzer and Wright (2000) present a model of sovereign default with two-sided commitment problem.

[10] We can assume that the debt contract contains a "negative pledge" clause to ensure the seniority of debt contracts.

[11] For example, the defaulting country may not obtain technology, direct investment, and other aids from the foreign countries, which reduces its output. Cole and Kehoe (1998) analyze a model in which sovereign default affects a government's reputation in other economic relationship.

The debt recovery rate is given by the function $\alpha(b, y)$, which determines the fraction of debt to be repaid if the sovereign defaults on debt $b$ with endowment shock $y$. Should the renegotiation fail, the international investors would lose their investment, and the country stays in autarky forever from that period on.

We denote the country's credit score by a discrete variable $h \in \{0, 1\}$. Let $h = 0$ denote no debt crises on a country's record, that is, a good credit score; whereas $h = 1$ indicates a record of sovereign default in the country's credit history, or a bad credit score. A country's credit score deteriorates after a default event. We assume that the government with a bad credit score after default cannot save or borrow. The defaulting country can regain its access to the capital markets once the renegotiated debt is repaid in full. Therefore, with renegotiation, the country can restore its reputation after default.[12]

### 2.2 Timeline

Figure 1 displays the timing of decisions within one period in the model economy. At the beginning of the period, an endowment shock $y$ is realized. The sovereign government has assets $b$. When the government has a good credit score ($h = 0$), it decides to repay its debt or default. If the government decides not to default, then it chooses $b'$ and its credit score remains good for the next period ($h' = 0$). If the government chooses to default, its debt is reduced to $\alpha(b, y)\, b$ through debt renegotiation. But its credit score deteriorates ($h' = 1$). When the government starts with a bad credit score ($h = 1$) with unpaid debt arrear $b$, it determines how much debt to repay. If some debt arrear is not repaid ($b' > 0$), then the country's credit score remains bad ($h' = 1$). Once all the debt arrear is repaid ($b' = 0$), the government regains its reputation ($h' = 0$).

## 3 Recursive Equilibrium

In this section, we define and characterize a dynamic recursive equilibrium. In Subsection 3.1 we analyze the sovereign government's problem when the bond price schedule and the debt recovery plan are taken as given. In the subsequent two subsections, 3.2 and 3.3, we study the renegotiation problem and the lenders' problem to determine the debt recovery schedule and bond price schedule respectively. Subsection 3.4 defines a dynamic recursive equilibrium, in which both bond prices and debt recovery rates are endogenized. Finally, we demonstrate the existence of the equilibrium and characterize the equilibrium in Subsection 3.5.

[12] Fernandez and Rosenthal (1990) study a sovereign debt renegotiation game in which the defaulting country benefits from accessing capital markets when it fully repays the debt arrear. Cole, Dow and English (1995) develop a signaling model of sovereign debt in which a defaulting country regain access to capital markets by making partial debt repayment.

### 3.1 Sovereign Government's Problem

The sovereign government's objective is to maximize the expected lifetime utility of a domestic representative agent. The government makes its default decision and decides the amount of assets for next period, given the current asset position $b$ and the endowment shock $y$. Only the country with a good credit history can have debt. Hence the set $\mathcal{L} = B \times \{0\} \times Y \cup B_{--} \times \{1\} \times Y$ lists all possible $(b, h, y)$ triplets of state variables, where $B_{--} = B \cap R_{--} = [b_{\min}, 0)$. Let $v(b, h, y) : \mathcal{L} \to \mathcal{R}$ be the life-time value function for the country that starts the current period with the credit score $h$, asset position $b$, and endowment shock $y$. Let $V$ be the set of all continuous functions $v : \mathcal{L} \to \mathcal{R}$. We restrict the space of bond price functions to be $Q = \left\{q | q(b, y) : B \times Y \to \left[0, \frac{1}{1+r}\right]\right\}$, and the space of debt recovery schedules to be $A = \{\alpha | \alpha(b, y) : B_{--} \times Y \to [0, 1]\}$. Given any bond price function $q \in Q$ and debt recovery schedule $\alpha \in A$, the sovereign government solves its optimization problem. The government's problem can be written in a recursive way as follows.

For $b \geq 0$ and $h = 0$, the country has a good credit score and savings from last period. There is no debt to default in this case. The government receives payments from the financial intermediaries and determines its next-period asset position $b'$ to maximize utility. The country's credit score remains good.

$$v(b, 0, y) = \max_{b' \in B} u\left(y + b - q\left(b', y\right) b'\right) + \beta \int_Y v\left(b', 0, y'\right) d\mu\left(y' | y\right) \tag{2}$$

For $b < 0$ and $h = 0$, the country has a good credit score and outstanding debt. The government determines to default or not optimally. If the government chooses to honor its debt obligation, it is a standard consumption-saving problem as in the previous case. If the government defaults, it can not borrow or save in the current period. Moreover, the country's credit score deteriorates to $h' = 1$. But the government gets debt reduction $(1 - \alpha(l, y))$.

$$v(b, 0, y) = \max \left\{ \begin{array}{c} \max\limits_{b' \in B} u\left(y + b - q(b', y) b'\right) + \beta \int_Y v(b', 0, y') d\mu(y' | y) \\ u(y) + \beta \int_Y v\left(\left[\alpha(b, y)(1 + r) b\right], 1, y'\right) d\mu(y' | y) \end{array} \right\} \tag{3}$$

For $h = 1$, the country has a bad credit score and unpaid debt arrear $b < 0$. The country cannot save or borrow in the current period because of the bad reputation. In the current period, the endowment suffers a proportional loss of $\lambda_d y$. We assume that the creditors can enforce the interest payment.[13] The government chooses to optimally pay back the debt arrear. If the government repays partially, the debt level in the beginning of next period is the agent's choice

[13]This assumption is reasonable because it is common to pledge a renegotiated debt contract with risk-free bonds. For instance, according to the Brady plan, bonds which are issued to exchange for defaulted bank loans are backed with U.S. Treasury zero coupon bonds. See Chuhan and Sturzenegger (2003).

$b'$ and its credit score remains bad. The debt arrear rolls over at the interest rate $r$. When all the debt arrear is paid up, the country regains its full access to the markets.

$$v(b,1,y) = \max_{b' \in [b,0]} u\left((1-\lambda_d)\, y + b - \frac{b'}{1+r}\right) + \beta \int_Y v\left(b',1,y'\right) d\mu\left(y'|y\right) \tag{4}$$

where

$$v\left(0,1,y'\right) = v\left(0,0,y'\right) \tag{5}$$

In the model economy, the country may have an incentive to default because the default option and debt renegotiation introduce contingencies into normally non-contingent sovereign debt contracts, and thereby facilitate interstate consumption smoothing. But as we will show in the subsequent analysis, default comes at the cost of a higher equilibrium interest rate and more limited access to the financial markets, both of which hurt the country's intertemporal consumption smoothing. The sovereign government's default policy can be characterized by default sets. Let $D(b) \subset Y$ be the set of endowment shock $y$'s for which default is optimal given the debt position $b$. That is

$$D(b) = \left\{ y \in Y : v^r(b,0,y) \leq v^d(b,0,y) \right\}$$

where

$$\begin{aligned} v^r(b,0,y) &= \max_{c,b' : c + q(b',y)b' = y + b} u(c) + \beta \int v\left(b',0,y'\right) \mu\left(y'|y\right) \\ v^d(b,0,y) &= u(y) + \beta \int v\left(\alpha(b,y)\, b\,(1+r), 1, y'\right) \mu\left(y'|y\right) \end{aligned}$$

## 3.2 Debt Renegotiation Problem

If a country defaults, the sovereign government and financial intermediaries can renegotiate over the debt reduction. The debt reduction plan consists of reducing the net present value of payments due. With a debt reduction agreement, the government pays back the reduced debt to the creditors. The creditors, in turn, do not impose the direct sanctions on the country. Let $(1-\alpha(b,y))$ denote the debt write-off in the debt reduction plan. Alternatively, the ratio $\alpha(b,y)$ is the realized debt recovery rate. Once the reduced debt is fully repaid, the country regains its access to the capital markets.

We model the debt renegotiation as a generalized Nash bargaining game.[14] The threat point

[14] Nash bargaining is efficient and the debt reduction agreement is reached without any delay. This static bargaining model in our paper provides a succinct framework to analyze the interaction between sovereign default and debt renegotiation. The model can be extended to incorporate the strategic bargaining in a stochastic environment.

of the bargaining game is that the country stays in autarky and the creditors get nothing. The expected value of autarky to the country, $v^{aut}(y)$, is given in a recursive form:

$$v^{aut}(y) = u\left(\left(1-\left(\lambda_d+\lambda_s\right)\right)y\right) + \beta \int_Y v^{aut}\left(y'\right) d\mu\left(y'|y\right) \tag{6}$$

Note that permanent autarky implies that a country has no access to capital markets and faces direction sanctions and faces output loss by the fraction of $\lambda_d + \lambda_s$.

We now specify the Nash bargaining problem. In a debt renegotiation, for any debt recovery plan $a$, the surplus accruing to the sovereign government is the difference between the value of accepting the deal and the value of rejecting it. We denote the country's surplus by $\Delta^B(a; b, y)$, a function of recovery rate $a$ given amount of defaulted debt $b$ and current endowment $y$.

$$\Delta^B(a; b, y) = \left[u(y) + \beta \int_Y v\left(a(1+r)b, 1, y'\right) d\mu\left(y'|y\right)\right] - v^{aut}(y) \tag{7}$$

The term in the bracket is the expected life-time utility of defaulting when the debt recovery rate is $a$. The surplus to the country comes from two sources. First, the direct output loss is smaller with the renegotiation agreement because no direct sanctions are imposed. Second, although the country's credit score becomes bad in the next period, the expected length of financial exclusion is finite. Thus, the defaulting country gains from losing the access to capital markets for a temporary periods rather than permanently.

The surplus to the risk-neutral financial intermediaries is the present value of recovered debt.

$$\Delta^L(a; b, y) = -ab \tag{8}$$

If the lenders can make take-it-or-leave-it offers, then they could extract debt repayments up to the full amount of a country's cost of default. If, on the other hand, the borrowing country has all the bargaining power, then it gets a complete debt reduction in the bargaining. However, a realistic debt renegotiation is not a take-it-or-leave-it offer from either party. We assume that the borrower has a bargaining power $\theta$ and the lenders have a bargaining power $(1-\theta)$. The parameter of bargaining power $\theta$ summarizes the institutional arrangement of debt renegotiation.

To ensure that the bargaining problem is well defined, restrictions on the bargaining power parameter $\theta$ are required. We define the bargaining power set $\Theta \subset [0, 1]$ such that for $\theta \in \Theta$ the renegotiation surplus has a unique optimum for any debt position $b$ and endowment shock $y$. Given the debt level $b$ and endowment $y$, the debt recovery rate, $\alpha(b, y)$ which is a function of $b$ and $y$, solves the following bargaining problem:

$$\alpha(b,y) = \arg\max_{a\in[0,1]} \left[\Delta^B(a;b,y)^\theta \left(\Delta^L(a;b,y)\right)^{1-\theta}\right] \tag{9}$$
$$\text{s.t. } \Delta^B(a;b,y) \geq 0$$
$$\Delta^L(a;b,y) \geq 0$$

The debt renegotiation determines the debt recovery schedule $\alpha(b,y) \in A$. Because the value of defaulting to the country depends on the bond price function and debt recovery rates, the renegotiation is endogenous. The recovery schedule will be solved in the equilibrium.

One remark is that we assume the renegotiation takes place only once for one default event and is costless. The empirical evidence shows that extended repeated renegotiations are seldom observed. Our assumption keeps the model tractable to analyze the interaction between default and renegotiation. An alternative way in modelling renegotiation is to allow for continuous costly renegotiations. In that case, the number of rounds of bargaining is a state variable and the pricing problem becomes quite complicated. The model in the paper is a special case where most of the economic intuitions still hold. Allowing for continuous costless renegotiation will generate either risk-free debt or no international lending.

### 3.3 International Financial Intermediaries' Problem

Taking the bond price function as given, the financial intermediaries choose the amount of debt $b'$ to maximize their expected profit $\pi$. The expected profit is given by

$$\pi(b',y) = \begin{cases} q(b',y)\,b' - \frac{1}{1+r}b' & \text{if } b' \geq 0 \\ \frac{[1-p(b',y)+p(b',y)\cdot\gamma(b',y)]}{1+r}(-b') - q(b',y)(-b') & \text{if } b' < 0 \end{cases} \tag{10}$$

where $p(b',y)$ is the expected probability of default for a country with an endowment $y$ and indebtedness $b'$, and $\gamma(b',y)$ is the expected recovery rate of such a debt contract.

Because we assume that the sovereign debt market is completely competitive, the financial intermediaries' expected profit is zero in equilibrium. Using the zero expected profit condition, we get

$$q(b',y) = \begin{cases} \frac{1}{1+r} & \text{if } b' \geq 0 \\ \frac{[1-p(b',y)]}{1+r} + \frac{p(b',y)\cdot\gamma(b',y)}{(1+r)} & \text{if } b' < 0 \end{cases} \tag{11}$$

When the country lends to the intermediaries, $b' \geq 0$, default probability is zero. Thus the sovereign bond price is equal to $\frac{1}{1+r}$, which is the price of a risk-free bond. In contrast when the country borrows from the intermediaries, $b' < 0$, there exist the risks of default and debt restructuring. As a result, the sovereign bond is priced to compensate the financial intermediaries for bearing these risks. The default probability and expected debt recovery rate are endogenous, depending on the country's incentive to default and renegotiation outcome.

Since $0 \leq p(b', y) \leq 1$ and $0 \leq \gamma(b', y) \leq 1$, the bond price $q(b', y)$ lies in $\left[0, \frac{1}{1+r}\right]$. The interest rate on sovereign bonds, $r^s(b', y) = \frac{1}{q(b', y)} - 1$, is bounded below by the risk-free rate. The difference between the country interest rate and the risk free rate is the country's credit spread , $s(b', y) = r^s(b', y) - r$. We will focus on the dynamics of the sovereign credit spreads in the subsequent quantitative analysis.

### 3.4 Recursive Equilibrium

We can now define a stationary recursive equilibrium in the model economy.

**Definition 1** *A recursive equilibrium is a set of functions for (i) the sovereign government's value function $v^*(b, h, y)$, asset holdings $b'^*(b, h, y)$, default set $D^*(b)$, consumption $c^*(b, h, y)$ (ii) recovery rate $\alpha^*(b, y)$ and (iii) pricing function $q^*(b, y)$ such that*

*1. Given the bond price function $q^*(b, y)$ and debt recovery rate $\alpha^*(b, y)$, the country's asset holdings $b'^*(b, h, y)$, consumption $c^*(b, h, y)$ and default set $D^*(b)$ satisfy the sovereign government's optimization problem (2), (3), and (4).*

*2. Given the bond price function $q^*(b, y)$, the recovery rate $\alpha^*(b, y)$ solves the debt renegotiation problem (9).*

*3. Given the recovery rate $\alpha^*(b, y')$, the bond price function $q^*(b, y)$ satisfies the zero expected profit condition for intermediaries (11), where the default probability $p^*(b', y)$ and expected recovery rate $\gamma^*(b, y)$ are consistent with the sovereign's default policy and renegotiation agreement.*

In equilibrium, the default probability is related to the sovereign government's default policy in the following way:

$$p^*\left(b', y\right) = \int_{D^*(b')} d\mu\left(y'|y\right) \tag{12}$$

When $D^*(b') = \emptyset$, the country with debt $b'$ does not default for any realization of the endowment shock. Hence, the equilibrium default probability is zero and the bond price is $\frac{1}{1+r}$. When $D^*(b') = Y$, the sovereign government always chooses to default regardless of the endowment shock realization. Then the equilibrium default probability is equal to 1 and bond price is simply the expected recovery rate. In the models with endogenous default but a full debt discharge,[15] bond prices are zero when default probability is one. In our model, however, the recovery rate is endogenized. Therefore, bond prices may not be zero even when the sovereign is expected to default with probability one.

[15] Examples are Chatterjee, Corbae, Nakajima and Rios-Rull (2002) on consumer default, and Arellano (2004) and Aguiar and Gopinath (2004b) on sovereign default.

The expected recovery rate is the expected proportion of defaulted debt that the financial intermediaries can recover, conditional on default. In equilibrium, it is determined by

$$\begin{aligned}\gamma^*\left(b',y\right) &= \frac{\int_{D^*(b')}\alpha^*\left(b',y'\right)d\mu\left(y'|y\right)}{\int_{D^*(b')}d\mu\left(y'|y\right)} \\ &= \frac{\int_{D^*(b')}\alpha\left(b',y'\right)d\mu\left(y'|y\right)}{p^*\left(b',y\right)}\end{aligned} \tag{13}$$

The numerator is the expected proportion of debt that the investors can recover, and the denominator is the default probability.

Before we establish the existence of a recursive equilibrium in the next subsection, two more comments are in order. First, the debt recovery rate $\alpha\ (b,y)$ solves the Nash bargaining problem in equilibrium. Therefore, the debt recovery rate $\alpha\,(b,y)$ is endogenous in equilibrium. If the debt recovery rate function $\alpha\ (b,y)$ were given exogenously, we could define a partial equilibrium in which conditions 1 and 3 are the only equilibrium conditions. As a special case, when $\alpha$ $(b,y)=0$, the equilibrium is similar to the ones in Chatterjee, Corbae, Nakajima and Rios-Rull (2002), Arellano (2004) and Aguiar and Gopinath (2004b). Second, both the government and intermediaries take the debt recovery function as given and behave optimally. At the same time, the debt recovery rate solves the ex post debt renegotiation after default. Therefore, the equilibrium is subgame perfect.

## 3.5 Existence and Characterization of a Recursive Equilibrium

In the model, the sovereign government decides to default or to repay its debt. At the same time the post-default debt reduction is determined endogenously through debt renegotiation. Due to the introduction of discrete default decision and an endogenous bargaining problem, the existence of a dynamic recursive equilibrium does not follow from a standard argument. In this subsection, we first establish the existence of a recursive equilibrium and then demonstrate some of its properties.

### 3.5.1 Existence of Equilibrium

**Theorem 1** *Given any bargaining power $\theta\in\Theta$, a recursive equilibrium exists.*

**Proof.** We give a sketch of the proof and defer the details of the proof to the Appendix. The proof consists of three steps. First, given any debt recovery rate schedule $\alpha\,(b,y)\in A$, we show that there exists a bond price function $q\,(b',y)\in Q$ that is consistent with the government's default policy and the financial intermediaries' zero expected profit condition.[16] Second, given

[16] When debt recovery rate is taken as exogenous, the partial equilibrium for the model is similar to the one in Chatterjee et al (2002). They show that for atomless endowment distribution and no tie-breaking rule for

any bond price schedule $q(b',y) \in Q$, we show that there exists a debt recovery schedule $\alpha(b,y) \in A$ that solves the Nash bargaining problem. Lastly, we demonstrate that an equilibrium bond price function and an equilibrium debt recovery schedule exist.

(1) Taking any debt recovery schedule $\alpha(b,y) \in A$ as given, we define a price correspondence $\varphi(q)$ that takes points in $Q$.

$$\varphi(q)\left(b',y;\alpha\right) = \begin{cases} \begin{array}{c} \left(1 - p(q)(b',y;\alpha)\right)/(1+r) \\ +p(q)(b',y;\alpha)\cdot\gamma(q)(b',y;\alpha)/(1+r) \end{array} & \text{if } b' \geq 0 \\ \\ 1/(1+r) & \text{if } b' \leq 0 \end{cases} \tag{14}$$

where $p(q)(b',y;\alpha)$ and $\gamma(q)(b',y;\alpha)$ satisfy (12) and (13). Thus, $\varphi(q)(b',y;\alpha)$ is the set of prices for a debt contract of type $(b',y)$ that are consistent with zero profits given the price function $q$. We can show that the bond price correspondence $\varphi(q)$ has a closed graph and is convex-valued. Then via Kakutani's fixed point theorem, the correspondence has a fixed point $q = \varphi(q)(b',y;\alpha)$, which is an equilibrium bond price function, given the debt recovery schedule $\alpha$.

(2) Taking any bond price function $q(b,y) \in Q$ as given, we first define a debt recovery schedule correspondence $\psi(\alpha)$ that takes point in $A$.

$$\begin{aligned} \psi(\alpha)(b,y;q) \quad = \quad & \arg\max_{a\in[0,1]} \left[\left(\Delta^B(a;b,y,q,\alpha)\right)^{\theta}\left(\Delta^L(a;b,y,q,\alpha)\right)^{1-\theta}\right] \\ & \text{s.t. } \Delta^B(a;b,y,q,\alpha) \geq 0 \\ & \quad \Delta^L(a;b,y,q,\alpha) \geq 0 \end{aligned} \tag{15}$$

$\psi(\alpha)(b,y;q)$ is the set of deb recovery rates for debt contract of type $(b,y)$ that are consistent with Nash bargaining game, given debt recovery schedule $\alpha$. We can show that the debt recovery function correspondence is upper hemicontinuous. Since $\theta \in \Theta$ , the solution $\psi(\alpha)(b,y;q)$ is unique, thus is a continuous function. Then via Kakutani's fixed point theorem, the correspondence $\psi(\alpha)$ has a fixed point $\alpha = \psi(\alpha)(b,y;q)$, which is an equilibrium debt recovery schedule, given the bond price function $q$.

(3) We construct a functional mapping operator $T : Q \times A \rightarrow Q \times A$ such that

$$T(q,\alpha)(b,y) = \begin{bmatrix} \varphi(q)(b,y;q,\alpha) \\ \psi(\alpha)(b,y;q,\alpha) \end{bmatrix}$$

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defaulting and not defaulting, equilibrium bond price schedule exists.

We can show that the correspondence $T(q, \alpha)$ has a closed graph and is convex valued. Therefore, we can apply Kakutani's fixed point theorem and show the existence of a fixed point.

$$T(q^*, \alpha)(b, y) = (q^*, \alpha^*)$$

Thus, a recursive equilibrium exists. ■

### 3.5.2 Properties of the Equilibrium

We can proceed to establish some properties of a recursive equilibrium. First, we can characterize the sovereign government's value function $v(b, h, y)$ under the following condition.

**Condition 1** *Debt recovery schedule $\alpha(b, y) \in A$ satisfies*

$$\alpha(b_1, y) b_1 \leq \alpha(b_2, y) b_2 \tag{16}$$

*for all $y \in Y$ and $b_{\min} \leq b_1 \leq b_2 < 0$.*

This condition implies that the absolute value of reduced debt is increasing in the amount of defaulted debt. This condition is intuitive and, as we will show below, is satisfied by the equilibrium debt recovery rate. The set of debt recovery rate $\alpha(b, y)$ that satisfies condition 1 is denoted by $\mathcal{A}$; that is,

$$\mathcal{A} = \{\alpha | \alpha \in A \text{ such that } \alpha(b_1, y) b_1 \geq \alpha(b_2, y) b_2 \text{ for } b_1 \leq b_2 < 0\}$$

**Theorem 2** *Given any debt recovery schedule $\alpha \in \mathcal{A}$, the sovereign government's value function $v(b, h, y)$ is increasing in $b$.*

**Proof.** See Appendix A. ■

As a result of the debt renegotiation, the debt recovery rate $\alpha(b, y)$ optimally divides the bargaining surplus. The surplus to the financial intermediaries, $\Delta^L(a; b, y)$, is a linearly decreasing function of the country's debt $b$. By Theorem 2, the sovereign borrower's surplus $\Delta^B(\alpha(b, y); b, y)$ is an increasing function of $b$, given the debt recovery rate $\alpha(b, y) \in \mathcal{A}$. There are two special cases in the Nash bargaining game. If the sovereign borrower's bargaining power $\theta$ is large enough, the total surplus is an increasing function of $\alpha(b, y) b$. In this case, a full debt discharge is optimal so that $\alpha(b, y) = 0$. If $\theta$ is sufficiently small, the total surplus is a decreasing function of $\alpha(b, y) b$. In this case, the debt recovery rate is $\alpha(b, y) = 1$; the creditors do not agree on any debt reduction. The analysis in the following theorem generalizes the solution to the Nash bargaining problem for bargaining power $\theta$ in the set $\Theta$.

**Theorem 3** *For a bargaining power $\theta \in \Theta$, there exists a threshold $\overline{b}(y) \leq 0$ such that the equilibrium debt recovery function $\alpha$ satisfies*

$$\alpha^*(b, y) = \begin{cases} \frac{\overline{b}(y)}{b} \text{ if } b \leq \overline{b}(y) \\ 1 \text{ if } b \geq \overline{b}(y) \end{cases}$$

**Proof.** See Appendix A. ■

The intuition for Theorem 3 is the following: The benefit to the defaulting country of paying off their debt depends on the expected duration of financial exclusion, which is determined by the total amount of reduced debt. On the other side of the renegotiation, the financial intermediaries are also solely concerned with the total recovery on defaulted debt. Therefore, the bargaining problem is equivalent to the one with the reduced debt as the renegotiation subject. We find that there is an optimal value of reduced debt as a threshold, and that there is no debt reduction for debt levels smaller than the threshold. A similar result is found in Cole, Dow and English (1995), which studies the role of debt renegotiation settlement in signalling a government's type. An immediate result following the above theorem is that an equilibrium debt recovery function is in the set $\mathcal{A}$.

**Corollary 1** *An equilibrium debt recovery schedule $\alpha^*(b, y)$ satisfies Condition* (1)*.*

Now we can analyze the sovereign government's debt policy in equilibrium. The government's incentive to default depends on the *ex post* renegotiation agreement on debt reduction. Given the equilibrium debt recovery schedule $\alpha(b, y)$, characterized by Theorem 3, and the endowment shock $y$, the value function of a defaulting country is independent of the level of debt if it is larger than $\overline{b}(y)$. Therefore, we can show that the maximal default set increases with the country's indebtedness and the equilibrium default probability increases with the level of debt.

**Theorem 4** *Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, for $b^0 < b^1 \leq \overline{b}(y)$, if default is optimal for $b^1$, then default is also optimal for $b^0$. That is $\overline{D}^*(b^1) \subseteq \overline{D}^*(b^0)$.*

**Proof.** See Appendix A. ■

**Theorem 5** *Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, the sovereign government's probability of default in equilibrium satisfies $p^*(b^0, y) \geq p^*(b^1, y)$, for $b^0 < b^1 \leq \overline{b}(y) \leq 0$.*

**Proof.** See Appendix A. ■

Given the endogenous debt recovery rates, our model predicts that default probability increases with the level of debt. Eaton and Gersovitz (1981), Chatterjee, Cobae, Nakajima and

Vios-Rull (2002) and Arellano (2004) all obtain similar results, although they assume a zero debt recovery rate and rule out the possibility of debt renegotiation.

We can also characterize the equilibrium bond price schedule.

**Theorem 6** *Given an endowment $y \in Y$, for $b^0 < b^1 \leq \overline{b}(y) \leq 0$, an equilibrium bond price $q^*(b^0, y) \leq q^*(b^1, y)$.*

**Proof.** See Appendix A. ∎

In equilibrium, the sovereign bond prices depend on both the risk of default and the expected debt recovery rates. By the theorems above, for a high level of debt, the default probability is higher, but the expected debt recovery rate is lower. Therefore, equilibrium bond prices decrease with indebtedness. This result is consistent with the empirical evidence, for example, $b$ (1984).

The next theorem characterizes the debt arrear repayment policy for a defaulting country.

**Theorem 7** *Given an endowment $y \in Y$, if there exists a level of debt $\widetilde{b} < 0$ that satisfies*

$$\begin{aligned} & \sup_{b'<0}\left((1-\lambda_d)\,y+\widetilde{b}-\frac{b'}{1+r+\delta}\right)+\beta y^{1-\sigma}\int_G v\left(b',1,y'\right)d\mu\left(y'|y\right) \qquad (17) \\ = \; & u\left((1-\lambda_d)\,y+\widetilde{b}\right)+\beta y^{1-\sigma}\int_G v\left(0,0,y'\right)d\mu\left(y'|y\right) \end{aligned}$$

*then for all $b \in B_{--}$ and $b > \widetilde{b}$, it is strictly optimal for the defaulting country to repay its debt arrear in full, and for all $b \in B_{--}$ and $b < \widetilde{b}$, a partial repayment is strictly optimal.*

**Proof.** See Appendix A. ∎

This theorem implies that for all $b \in B_{--}$, if the sovereign government fully repays the debt arrear $b$ and regains access to financial markets for next period, then it also chooses to do so with a lower level of debt arrear. If the government decides not to repay the debt in full, it will do the same for higher debt arrear. In this case, it will take the country at least more than one period to re-enter the financial market after default. With the above theorem, the expected duration of financial exclusion increases with the amount of debt arrear.

A last remark about the theoretical model concerns the number of equilibria in the model economy. We cannot show the uniqueness of equilibrium in the model. There may exist many types of equilibria: one equilibrium implies high default risk, low debt recovery rates, and corresponding low bond price schedule; another equilibrium may support low default risk, high debt recovery rates, and high bond price schedule. However, we cannot formally establish the existence of multiple equilibria. The above equilibrium properties apply to all the equilibria. In the subsequent quantitative analysis, we compute an equilibrium as a fixed point starting with high debt recovery rates and high bond prices.

# 4 Quantitative Analysis

In the quantitative analysis, we examine whether the model can account for the debt crises and the dynamics of sovereign bond interest rates for emerging economies. We calibrate the model to analyze quantitatively the sovereign debt of Argentina.

## 4.1 Calibration

We define one period as a quarter. We assume the period utility function for the sovereign government is a constant relative risk-aversion utility function (CRRA), so that

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{18}$$

where $\sigma$ is the coefficient of risk aversion. We set the risk aversion coefficient to 2, which is standard in the macroeconomics literature.

We calibrate the endowment process to the Argentine output. The Argentine output, plotted in Figure 2, is characterized by a stochastic trend. Therefore, we model the output growth rate as an AR(1) process:

$$\log g_t = \left(1-\rho_g\right)\log \mu_g + \rho_g \log g_{t-1} + \varepsilon_t^g \tag{19}$$

where growth rate $g_t = \frac{y_t}{y_{t-1}}$, growth shock $\varepsilon_t^g \overset{iid}{\sim} N\left(0, \sigma_g^2\right)$ and $\log \mu_g$ is the expected log growth rate of the country's endowment. The quarterly output data are seasonally adjusted real GDP for the period of 1980Q1 to 2003Q4 from the Ministry of Finance (MECON). We estimate the endowment process to match the average growth rate, as well as the standard deviation and autocorrelation of HP detrended output.[17] We conduct Cochrane's (1988) variance ratio test using the Argentine output data and the model simulation. The result (Figure 3) shows that the estimated endowment process captures the stochastic trend in the data.

Because a realization of the growth shock permanently affects endowment, the model economy is nonstationary. In the quantitative analysis, we detrend the model by the lagged endowment level $y_{t-1}$ and denote the detrended counterpart of any variable $x_t$ by $\widehat{x}_t$, with

$$\widehat{x}_t = \frac{x_t}{y_{t-1}}$$

The normalization by $y_{t-1}$ ensures that if $x_t$ is in the agent's information set at $t$, so is the detrended variable $\widehat{x}_t$.

The risk-free interest rate $r$ is the cost of capital for intermediaries to engage in the interna-

[17] Aguiar and Gopinath (2004a) find that emerging economies are subject to substantial volatility in the trend growth rate. Aguiar and Gopinath (2004b) use an endowment process with stochastic trend to analyze sovereign default and interest rates.

tional financial markets. In the benchmark economy, the risk-free interest rate $r$ is a constant. We set $r$ to 1%, the average quarterly yield on 3 month US treasury bills. When a country defaults, its output drops by a fraction $\lambda_d$. Sturzenegger (2002) estimates the percentage of output contraction after default using a panel of 100 countries, and finds that the output loss is equal to 2%. Accordingly we set $\lambda_d = 2\%$.

In the last part of the calibration, we pick the time discount factor $\beta$, sovereign government's bargaining power $\theta$, and direct sanctions $\lambda_s$ to match the average default frequency, average debt recovery rate, and debt-to-output ratio of Argentina. Because the country could default in equilibrium, default and debt renegotiation are the equilibrium outcome, which is consistent with the data. Reinhart, Rogoff and Savastano (2003) report four episodes of sovereign defaults in Argentina's external debt from 1824 to 1999. In 2001, Argentina defaulted a fifth time on its external debts, making its average default frequency 2.78% annually or 0.694% quarterly.[18] The average debt recovery rate is estimated as the first available bid price for sovereign bonds 30 days after default. According to the Moody's (2003) report on average defaulted debt recovery rates for sovereign issuers, Argentina's average recovery rate is 28%.[19] We use the foreign debt service-to-output ratio as one of the target statistics. The debt service-to-output ratio includes both short term debt and debt service on long term debt, which is calculated using data from the World Bank. For the period 1980-2003, the ratio of Argentina's debt service to its gross national income is 9.538%.

We use the simulated method of moments (SMM) to estimate the time discount factor $\beta$, the bargaining power $\theta$, and the direct sanctions $\lambda_s$. These three parameters are estimated jointly because the target statistics are endogenous in equilibrium. Table 1 presents the statistics for Argentina that we use as the calibration target. Table 2 summarizes the calibration results.

The time discount factor $\beta$ is 0.740. This high degree of impatience helps to generate the high default probability.[20] It also reflects the high political instability in Argentina over the past years where there have been 14 presidents from 1981 to 2004.[21] Therefore, the government's quarterly survival probability is 84.7%, which makes the value of pure time discount factor 0.873.[22] The output loss due to trade sanctions, 1.22%, shows that the creditors have some power to impose direct sanctions on the country. Notice, however, that the effect of direct sanctions on output is less than the output drop due to a bad reputation. Finally, the estimated Argentina's bargaining

[18] Reinhart (2001) estimates an unconditional default probability for a sample of 56 emerging economies from 1970 to 1999. Her result implies that the average annual default probability is 0.12%.

[19] In the recent proposal of bond exchanges for the Argentine defaulted debt, the recovery value is 25%. The country average recovery rate is higher at 41% in Moody's (2003).

[20] Chatterjee, Corbae, Nakajima and Rios-Rull (2002) study a similar model to the case of U.S. consumer default and find the time discount factor to be 0.819 in their calibration. Their default rate statistic is 0.5%. Argentina's annual default probability is much higher at 2.78%, so the low discount rate of 0.740 is needed.

[21] Argentina has 72 presidents/governments in power from 1833 to 2004

[22] See Grossman and Van Huyck (1988)

power is 0.83, which shows that it has a more favorable position in debt renegotiation than the international investors.

### 4.2 Comparison of Model to Data

After calibrating the parameters to match the Argentine economy, we simulate the model and compare the results with the Argentine data. Table 3 compares the data with the model statistics. The statistics are calculated from 2000 simulations of 50 observations each. In each simulation, we simulate the model for 1000 periods and extract the last 50 observations to explore the behavior of the stationary distribution of the model economy.

Overall, the model matches the Argentine interest rate volatility, consumption volatility, and the correlations between interest rates, output, consumption and current account. We also show that the model can replicate the recent default crises and the time series of the Argentine interest rate over the past 10 years.

The bond spreads are quarterly spreads for Argentine foreign bonds with the maturity of three year from 1994Q1 to 2001Q3, taken from Broner, Lorenzoni and Schmukler (2004).[23] They compute the spread curve with maturities of 3, 6, 9, and 12 years using a modified term structure model developed by Nelson and Siegel (1987) and Diebold and Li (2004). As discussed in Broner, Lorenzoni and Schmukler (2004), the sovereign bond spreads have a significant term structure variation. The average bond spreads and volatility for Argentine foreign bonds decrease with the maturity for the period of 1994Q1 to 2001Q3.[24] Because the calibrated model generates interest rates for 3-month bonds, we compare the model result to the bond spreads data with short maturity.[25]

The model simulation closely matches the volatility of the Argentine interest rates in the data, which is hard to explain in the literature. The model can account for about 78% of volatility in the 3-year bond spreads in the data. This improves the result in the previous models on sovereign bonds pricing without endogenous debt renegotiation.[26] The bond spreads in our model are jointly determined by the default probabilities and debt recovery rates. Therefore, allowing for debt renegotiation breaks the one-to-one matching from default probabilities to bond spreads even though lenders are risk neutral. Because the debt recovery rates are endogenous and are correlated with default probability, our model can generate more volatile bond spreads.

[23] We are grateful to Broner, Lorenzoni and Schmukler for kindly providing their dataset on sovereign bond spread curve.

[24] According to the estimated bond spread curve in Broner, Lorezoni and Schmukler (2004), the average quarterly bond spreads increase from 1.02 to 1.56 from 3-year bonds to 12-year bonds. The volatilities increase from 0.42% to 0.78% from 3-year bonds to 12-year bonds.

[25] The commonly used J.P. Morgan's Emerging Markets Bond Indices is calculated using bonds with a long maturity of 7-10 years on average.

[26] Arellano (2004) and Aguiar and Gopinath (2004b) find that their equilibrium models cannot generate high interest rate volatility.

In particular, default probability is higher when a larger fraction of debt reduction is expected in the post-default renegotiation. Therefore, the endogenous debt renegotiation amplifies the default risk and thus the volatility of bond spreads. We will explore the effect of endogenous debt renegotiation in generating high volatility of bond spreads in the next section.

The average quarterly bond spreads is 0.46% in the simulation, which is about 50% of the mean spreads for 3-year bonds in the data. Although this shows that our model falls short in this regard, the simulated average bond spreads are conceivably higher than the results in Arellano (2004) and Aguiar and Gopinath (2004b). In our model, the mean spread is equal to the product of average default probability and average debt reduction rate in the model. Since the default frequency in the data is 0.695% and the average debt reduction rate is 72%, the mean of bond spreads due to default risk and restructuring risk is about 0.5% for 3-month bonds. Data on 3-month bond spreads is desirable to assess the ability of such models in explaining sovereign bond spreads. On corporate bonds, Huang and Huang (2003) find that credit risk account only for 19% of the yield observed in investment grade corporate bonds.

A third result is that the model accounts for the relation between bond spreads, outputs and current account. Current account data is from the Ministry of Finance (MECON) for 1980Q1 to 2003Q4 and is calculated as a ratio of output. Bond spreads are negatively correlated with output and positively correlated with current account. Moreover, current account is also countercyclical in the model, although the magnitude of correlation in the model is lower than in the data. These results imply that the country borrows more in good times at a lower bond spreads. Because the growth shock is persistent in the model, when the country gets a good shock today, its permanent income increases even more. Therefore, the country has an incentive to borrow more. The equilibrium bond spreads increase with the level of debt. However, bond spreads also decrease with output because the bond price schedule shifts up with lower default risk and higher expected debt recovery rates. The equilibrium outcome indicates that the shift in bond price schedule dominates. Therefore, bond spreads are countercyclical and positively correlated with current account. Aguiar and Gopinath (2004b) explore this effect in a sovereign default model with shocks to endowment growth.

The model also generates volatile consumption and current account at the business cycle frequency. The volatility of consumption using seasonally adjusted consumption data from the Ministry of Finance (MECON) for 1980Q1 to 2003Q4. Consumption volatility is higher than output volatility in the data, which is a common feature of emerging economies as documented by Neumeyer and Perri (2004). Our model can account for such consumption volatility. The reason why consumption volatility is higher endowment volatility in our model is that the endowment process has a stochastic trend. Because a good endowment shock increases permanent income more than proportionally, the country borrows to smooth consumption. Therefore, consumption is more volatile than endowment. This is also explained in Aguiar and Gopinath (2004b).

Regarding the current account volatility, the model fails to generate volatile current account as in the data. Because the model replicates a low level of international borrowing, the magnitude of current account is small. Therefore, the volatility of current account is smaller than the data.

To better understand the bond pricing mechanism in the model, we plot the bond price, default probability, and debt recovery rate schedule in equilibrium. Figure 4 shows the bond price function faced by a country with the highest and the lowest endowment shock in the current period. The bond price is increasing with the level of assets-to-output ratio (decreasing in the level of debt-to-output ratio), which reflects our theoretical result and is consistent with the data. Regarding the endowment shock, when the country has a good economic shock, it faces a larger bond price.

Figure 5 plots the default probabilities. For a country with a very low debt-to-output ratio, there is virtually no default, regardless of the value of the economic shock. Consistent with the theory, as the indebtedness increases, the default probability increases. We also find that the default probability is higher for a country that experiences a bad growth shock. When a country in debt gets a bad shock, default is more likely. This relationship helps to generate the countercyclicality of interest rates.

Figure 6 plots the debt recovery schedule. For the defaulting country with a good shock, the debt recovery rate is higher, and vice versa. This relation between recovery rate with output is intuitive: a country which defaults with a good economic shock is asked to pay more. Such a result contributes to the countercyclicality of interest rates because the ex ante incentive to default depends on *ex post* debt reduction. When the country gets a bad shock, the benefit from defaulting is higher partly with a higher debt reduction. Therefore, the default risk is even higher. A higher default probability and a lower debt recovery rate generate a higher sovereign bond spread, thus a negative correlation between spreads and endowment.

Figure 6 also displays the model prediction regarding the relation between debt recovery rates and the level of debt. If a country defaults with a small amount of debt, there is no debt reduction . As the amount of defaulted debt increases, the debt recovery rate decreases. Due to the limited availability of data on sovereign debt renegotiation, there is little empirical evidence on the property of debt recovery rate. However, some observations from the recent sovereign bond exchanges are largely consistent with the model prediction. For example, the bond exchanges by Ukraine, Uruguay, and Pakistan achieved no debt reduction because the eligible debt is very small, less than \$5.4 billion. The cases of Ecuador \$6.7 billion bond exchange and Russian \$31.8 billion bond exchange resulted in 40% and 36.5% debt reduction respectively. The current renegotiation for Argentina \$82.2 billion is expected to result in a debt reduction of 75%. Therefore, the model prediction is roughly consistent with the empirical observation.

The model successfully predicts the recent Argentine debt crisis and replicates the time series of the Argentine bond spreads. We feed the Argentine GDP growth rate into the model and compare the time series of bond spreads. Figure 7 plots the H-P detrended output, the 3 year Argentine bond spreads, and the simulated bond spreads over the period of 1994Q2 and 2001Q4. The figure demonstrates that the model can explain the recent Argentinian default episode. Before a default occurs, the country faces countercyclical interest rates. When the country gets a really bad shock, it defaults on its sovereign debt.

Regarding the endogenous length of punishment, the country receives one period financial exclusion after default in the model. Because the country has a high bargaining power and gets a high debt reduction, the benefit of paying back a small debt arrear to gain access to capital markets outweighs the cost of reducing current consumption. Therefore, the model predicts that the country returns to the market in one period. Gelos et al (2003) find that it takes less than one year for defaulted countries to regain access to international financial markets in the 1990s. Since the renegotiation agreement in the model is reached right after default, the length of financial exclusion in the model is shorter than what they find.[27]

# 5 Additional Model Implications

In this section, we explore the role of endogenous recovery rates and examine how the equilibrium changes with the bargaining power. Results of sensitivity analysis are also presented.

## 5.1 Role of Endogenous Debt Renegotiation

We study the role of endogenous debt renegotiation by comparing our model to the one without renegotiation. In the comparison model, default leads to a full debt discharge and the defaulting country regains access to the capital markets with an exogenous return probability.[28] Complete debt discharge corresponds to an exogenous zero debt recovery rate $\alpha = 0$ in our model. An exogenous return probability $\delta$ determines the expected length of exclusion from financial markets.

We calibrate the time discount factor and the return probability in the model without renegotiation to match the target statistics of average default frequency and debt-to-output ratio. The estimated time discount factor is 0.768, and the return probability is 0.274. The other parameters take the same values as in benchmark model. Table 4 summarizes the results from our model and the comparison model without renegotiation.

The calibration of the comparison model does not match the target statistics. In particular, the model without renegotiation generates significantly lower default frequency than what we see

[27] We get longer periods of financial exclusion in the sensitivity analysis when the country has lower bargaining power.

[28] Chatterjee, Corbae, Nakajima and Rios-Rull (2002), Arellano (2004), Aguiar and Gopinath (2004b) apply the comparison model to study consumer default and sovereign default, respectively.

in the data. It implicates that endogenous renegotiation is needed to account for the observed high default frequency for Argentina. The comparison model also generates a lower level and volatility of Argentine bond spreads. Although the financial intermediaries do not recover any fraction of the defaulted debt, the sovereign bonds bear lower interest rates due to a smaller default risk in this case. In equilibrium, the sovereign government is more willing to accumulate debt with a less strict borrowing condition, and the country's consumption is less volatile. Regarding the volatility of bond spreads, because default risk is the sole determinant of the bond price, and in equilibrium default is a rare event, the simulated bond spreads are not volatile enough. In contrast, our model with endogenous renegotiation generates higher default probability and exacerbates interest rate volatility.[29]

## 5.2 Bargaining Power and Collective Action Clauses

In this subsection, we compute the model economy for different bargaining powers. The results are summarized in Table 5. The bargaining power parameter has a direct impact on debt recovery rate. It is intuitive that higher bargaining power for the country implies less debt recovery for financial intermediaries. Keeping other things fixed, the lower recovery rate increases the average bond interest rates. On the other hand, for a higher bargaining power for the country, the corresponding decreased debt recovery rate makes the bond price schedule shift down. As a result, borrowing is discouraged and thus the equilibrium debt-to-output ratio is smaller. Also, with more restricted borrowing terms, the default probability goes down and the bond interest rates decreases, ceteris paribus. Therefore, the increasing bargaining power for the country has two opposite effects on the bond interest rates. How the equilibrium interest rates change depends on which effect dominates. Table 5 shows that the average interest rates do not change monotonically with the bargaining powers.

For a country with full bargaining power, the debt renegotiation always results in a zero debt recovery rates. In this case, the bond price schedule shifts so that borrowing cost increases dramatically. The debt-to-output ratio is 2% and default probability is 0.02%. When the country has little bargaining power, for example $\theta = 0.35$, the debt recovery rate is very high. In this case, the bond price, as a function of indebtedness, becomes much flatter, given the endowment shock. Figure 8 plots the bond price schedule. Therefore, the model with debt renegotiation can potentially generate a wider range of debt-to-output ratio, as shown for $\theta = 0.35$.

The results in Table 5 with different bargaining powers can be viewed as outcomes of policy experiments. To evaluate the impact of different policy on the sovereign country's welfare, we also calculate the country's ex ante utilitarian welfare in the stationary distribution. The

[29] Cooley, Marimon and Quadrini (2004) endogenize the value of repudiation for entrepreneurs with optimal financial contracts. They find that economies with low enforceability of contracts have greater macroeconomic volatility.

country gets higher utility when it has a higher bargaining power. We also calculate the change in consumption that makes the country indifferent between the model economy with a given bargaining power and the benchmark economy. As discussed in Athreya (2002), we denote the change in consumption $\phi$. Let $\Lambda^0$ denote benchmark welfare, and $\Lambda^p$ denote welfare in the model economy with a given bargaining power. $\phi$ satisfies the following:

$$\phi = \left( \frac{\Lambda^p + 1/\left(1-\sigma\right)\left(1-\beta\right)}{\Lambda^0 + 1/\left(1-\sigma\right)\left(1-\beta\right)} \right)^{1/(1-\alpha)} - 1$$

If $\phi > 0$, the country is better off with the new bargaining power than in the benchmark case. The converse also holds.

Our results shed light on the impact of reform in sovereign bond restructuring on the international financial market. Recently, voluntary and market-friendly debt restructuring clauses in bond contracts are viewed as an improvement of the current debt restructuring process. One example is the use of Collective Action Clauses (CACs), the contractual clauses that allow the terms of contract to change if there is consent from a predetermined supermajority of bond holders. Because sovereign bond holders are diverse, and one investor may only hold a small fraction of debt, in the event of default one investor would always find it incentive compatible to hold the debt rather than cooperate in the renegotiation process. This "hold out", or free riding problem results in a very costly renegotiation and reduces the country's bargaining power. However, CACs can align bondholders' incentives to ease the renegotiation process by specifying a majority rule that binds all bondholders to eliminate the hold out problem. Several empirical studies analyze the market impact of CACs inclusion. Eichengreen and Mody (2000) show that bonds with CACs have higher yields. Becker, Richards and Thaicharoen (2003) argue that bond prices are not affected much by the CACs. However, these exercises suffer from the Lucas critique. Through the experiments on our model, we find that when the sovereign borrower has higher bargaining power, the country's borrowing cost does not necessarily increase. The amount of sovereign debt issued on the market is also affected and the extent of risk sharing differs with the bargaining power. Although our model does not have a structural bargaining game, it points out the priority to analyze default and renegotiation in a dynamic framework to evaluate the impact of renegotiation reforms.

### 5.3 Sensitivity Analysis

We study the sensitivity of our model to some key structural parameter values. First we study the effect of risk free interest rates on the model. The first panel in Table 6 reports the results. For a higher risk-free interest rate, the average recovery rate is smaller because the intermediaries' cost of capital is higher. The renegotiation results in a higher default loss for impatient lenders.

For the country, a higher risk free rate implies higher borrowing costs as well as a better debt renegotiation deal. In general, the country borrows less and the default probability decreases. The combined effect of the default risk and a smaller recovery rate is that the bond interest rate increases with the risk free rate, but less proportionally. This is consistent with what Eichengreen and Mody (1998) find in their empirical study of sovereign bond spreads.

The results in the benchmark model are sensitive to the choice of time discount factor, as shown in the second panel in Table 6. On the debt renegotiation, a more patient country gets less debt reduction. Because the patient sovereign government cares more about the reentry to capital markets in the future, the value of renegotiation agreement is relatively higher than the cost of repaying more reduced debt. Therefore, the bargaining results in a higher recovery rate. Default probability also decreases when the country is more patient because the intertemporal consumption smoothing is highly valued. Accordingly, the average sovereign spreads decrease with discount factor.

Finally, we examine the sensitivity of the model to changes in default punishment. First, We compare the benchmark model with the case when the creditors do not have any sanction technology beyond the ability to cease lending to the defaulted country. That is, $\lambda_s = 0$. Because the sovereign government implicitly has a higher value at its threat point. Therefore, the debt renegotiation results in a smaller recovery rate and the bond price schedule shifts down. In this case, the country's debt-to-output ratio is lower and defaults less frequently. Table 6 shows that the average interest rate is lower in this experiment. Regarding the output loss due to default, we find that an increase in this output loss decreases default probability. But the debt recovery rate decrease because the creditors' direction sanctions $\lambda_s$ becomes relatively less important. As a result, the debt-to-output ratio increases. At the same time, the interest rate volatility decreases.

# 6 Conclusion

It is well observed that sovereign debt crises have a great impact on the borrowing countries and international capital markets. Therefore, it is crucial to understand the sovereign default risk and the role of debt crises resolution in the sovereign debt markets. This paper studies sovereign default and debt renegotiation in a small open economy model. This model allows us to investigate the interaction between default and debt renegotiation within a dynamic borrowing framework. We find that debt recovery rates decrease with indebtedness and, in turn, affect the country's *ex ante* incentive to default. In equilibrium, sovereign bonds are priced to compensate creditors for the risks of default and restructuring. Consistent with the empirical evidence, the model predicts that interest rates increase with the level of debt.

We use the model to analyze quantitatively the sovereign debt of Argentina. The model successfully accounts for the high bond spreads, countercyclical country interest rates, and other key features of the Argentine economy. The model also replicates the dynamics of bond interest rates during the recent Argentine debt crisis. Furthermore, we show that the introduction of an endogenous debt recovery schedule leads to a higher default probability and greater interest rate volatility. We also demonstrate that the changes in bargaining power have a great impact on debt recovery rates as well as on the sovereign bond spreads, shedding light on the policy implications of sovereign debt restructuring procedure. In particular, we find that the sovereign bond spreads do not increase monotonically with the bargaining power.

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| Statistics | Source | Data | Model |
|---|---|---|---|
| Panel A | | | |
| World risk-free interest rate | US Treasury-bill quarterly interest rates | 1% | 1% |
| Output loss in debt crises | Sturzenegger (2002) | 2% | 2% |
| Panel B | | | |
| Average output growth rate | MECON | 0.420% | 0.420% |
| Output standard deviation | MECON | 4.346% | 4.346% |
| Output autocorrelation | MECON | 0.824 | 0.824 |
| Panel C | | | |
| Average debt service/output | the World Bank | 9.538% | 9.695% |
| Default frequency | Reinhart, Rogoff and Savastano (2003) | 0.695% | 0.540% |
| Average recovery rate | Moody's (2003) | 28% | 28% |

Table 1: Target Statistics for Argentina (1980.1-2003.4)

| Parameter | Symbol | Value |
|---|---|---|
| Panel A | | |
| Coefficient of Risk Aversion | $\sigma$ | 2 |
| Risk Free Interest Rate | $r$ | 1% |
| Output Loss in Default | $\lambda_d$ | 2% |
| Panel B | | |
| Average Endowment Growth | $\mu_g$ | 1.004 |
| Std Dev. to Endowment Growth Shock | $\sigma_g$ | 0.025 |
| Endowment Growth AR(1) coefficient | $\rho_g$ | 0.406 |
| Panel C | | |
| Time Discount Factor | $\beta$ | 0.740 |
| Sanction Threat | $\lambda_s$ | 1.221% |
| Bargaining Power | $\theta$ | 0.832 |

Table 2: Model Parameter Values in the Model

| Non-target Statistics | Data | Model |
|---|---|---|
| Average Bond Spreads (quarterly) | 1.02% | 0.46% |
| Bond Spreads Std. Dev.(quarterly) | 0.42% | 0.33% |
| Correlation between Bond Spreads and Output | -0.12 | -0.18 |
| Correlation between Bond Spreads and Current Account | 0.49 | 0.54 |
| Correlation between Current Account and Output | -0.88 | -0.14 |
| Consumption Std. Dev./Output Std. Dev. | 1.15 | 1.03 |
| Current Account Std. Dev. | 1.35 | 0.58 |

Table 3: Model Statistics for Argentina

Note: Bond spreads series are quarterly spreads for Argentine foreign bonds with maturity of three year from 1994Q1 to 2001Q3. Output and consumption are log HP filtered. Current account is the ratio of trade balance to output and is HP detrended.

| Target Statistics | Data | Model | Comparison Model |
|---|---|---|---|
| Default Probability | 0.69% | 0.54% | 0.191% |
| Average Debt Service/Output | 9.54% | 9.69% | 10.24% |
| Other Statistics | | | |
| Average Recovery Rate | 28% | 28% | 0 |
| Average Bond Spreads | 1.02% | 0.46% | 0.10% |
| Bond Spreads Std. | 0.42% | 0.33% | 0.13% |
| Consumption Std./Output Std. | 1.15 | 1.03 | 1.01 |

Table 4: Role of Endogenous Renegotiation

Note: The default frequency is the quarterly average from 1824 to 2003. Bond spreads series are quarterly spreads for Argentine foreign bonds with maturity of three year from 1994Q1 to 2001Q3. Output and consumption are log HP filtered.

| bargaining power | recovery rate | $\frac{\text{debt}}{\text{output}}$ | default prob. | mean $r^s$ | utilitarian welfare | $\phi$ change in consumption |
|---|---|---|---|---|---|---|
| $\theta$=0.35 | 89.01% | 93.14% | 1.339% | 0.315% | -0.208 | -0.356% |
| $\theta$=0.7 | 33.68% | 13.46% | 0.33% | 1.24% | -0.008 | -0.034% |
| **$\boldsymbol{\theta}$=0.83** | **27.93%** | **9.69%** | **0.54%** | **1.46%** | **-0.007** | – |
| $\theta$=0.9 | 19.64% | 7.83% | 0.33% | 1.31% | -0.006 | 0.025% |
| $\theta$=1 | 0 | 2.25% | 0.02% | 1.03% | -0.005 | 0.062% |

Table 5: Statistics for Different Bargaining Powers
Note: The default frequency is the quarterly average from 1824 to 2003. Bond spreads series are quarterly spreads for Argentine foreign bonds with maturity of three year from 1994Q1 to 2001Q3. Output and consumption are log HP filtered.

| | Default prob. | Recovery rate | debt/output | Mean $s^s$ | Std.$(r^s)$ |
|---|---|---|---|---|---|
| Risk free rate | | | | | |
| $\mathbf{r = 0.01}$ | **0.540%** | **27.930%** | **9.689%** | **0.460%** | **0.325%** |
| $r = 0.02$ | 0.193% | 27.366% | 8.985% | 0.148% | 0.094% |
| $r = 0.03$ | 0.105% | 27.739% | 8.742% | 0.079% | 0.077% |
| Time discount factor | | | | | |
| $\boldsymbol{\beta} = \mathbf{0.74}$ | **0.540%** | **27.930%** | **9.689%** | **0.460%** | **0.325%** |
| $\beta = 0.8$ | 0.333% | 34.153% | 9.823% | 0.234% | 0.115% |
| $\beta = 0.9$ | 0.099% | 47.947% | 11.770% | 0.051% | 0.029% |
| Endowment loss | | | | | |
| $\boldsymbol{\lambda_s} = \mathbf{0.012}$ | **0.540%** | **27.930%** | **9.689%** | **0.460%** | **0.325%** |
| $\lambda_s = 0$ | 0.175% | 26.299% | 9.130% | 0.164% | 0.178% |
| $\boldsymbol{\lambda_d} = \mathbf{0.02}$ | **0.540%** | **27.930%** | **9.689%** | **0.460%** | **0.325%** |
| $\lambda_d = 0.03$ | 0.306% | 23.541% | 14.239% | 0.271% | 0.146% |
| $\lambda_d = 0.04$ | 0.173% | 21.488% | 18.963% | 0.157% | 0.128% |

Table 6: Sensitivity Analysis for Benchmark Model
Note: Output and consumption are log HP filtered.

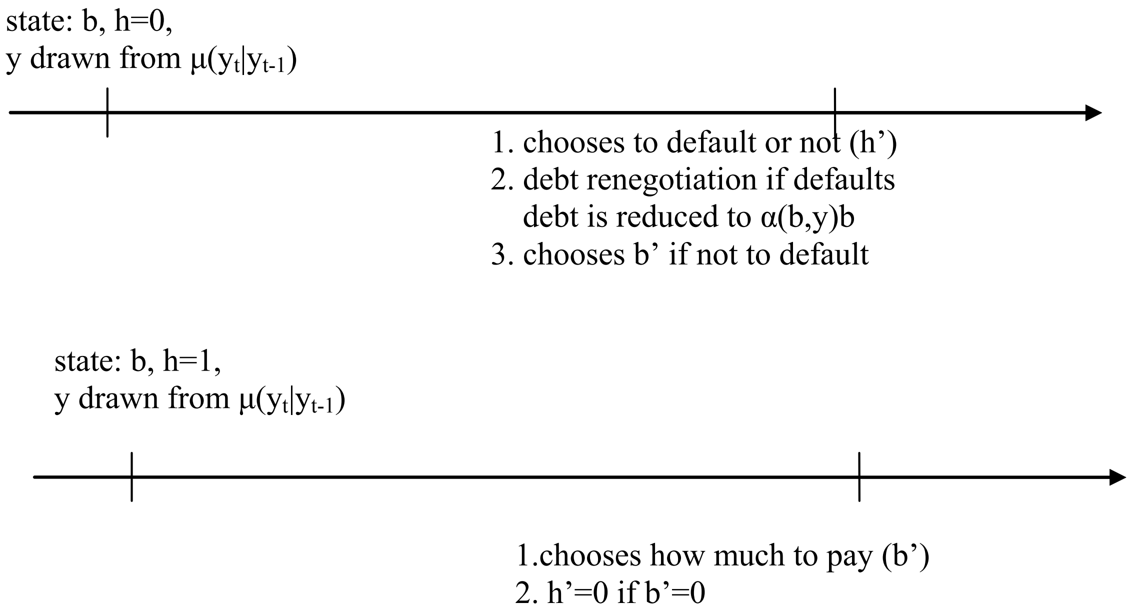


Figure 1: Timeline of the Model

Note: The upper panel plots the timing of events within one period for a country with a good credit score ($h = 0$) and asset position $b$. The lower panel plots the timing of events within one period for a country with a bad credit score ($h = 1$) and debt arrear $b$.

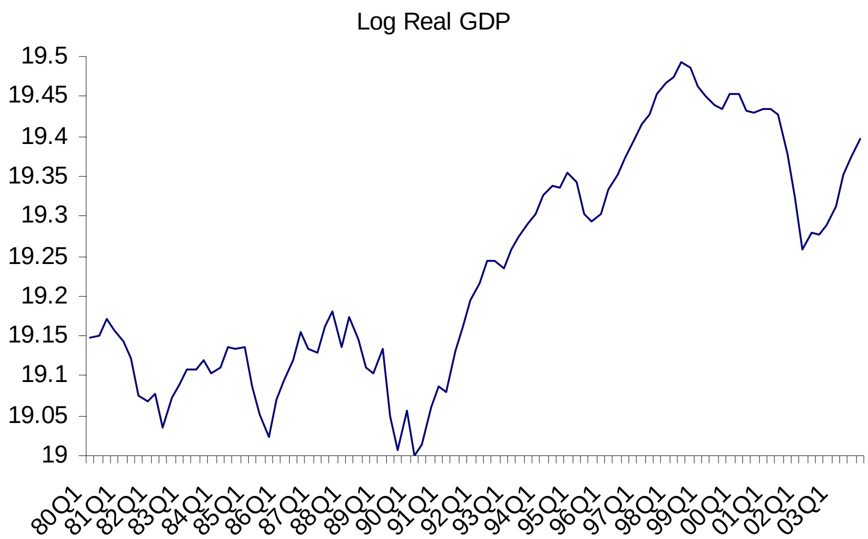


Figure 2: Log Real GDP of Argentina, 1980.1-2003.4

The figure plots the log real GDP for Argentina from 1980Q1 to 2003Q4. The output data is taken from MECON and is seasonally adjusted.

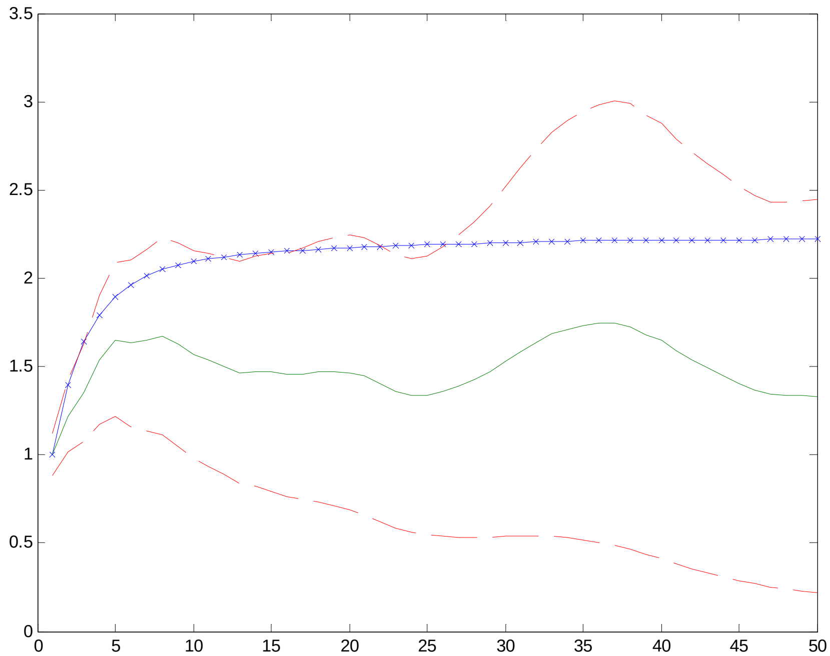


Figure 3: Variance Ratio for Argentine GDP, 1980.1-2003.4

The figure plots the variance ratio of output in the data and in the model simulation. Variance ratio is calculated as1/k times the variance of k-differences of output divided by variance of first difference, as defined in Cochrane (1988). The solid line shows the variance ratio from the data, and the dashed lines show the one standard error band. The line with crosses shows the variance ratio calculated with model simulation.

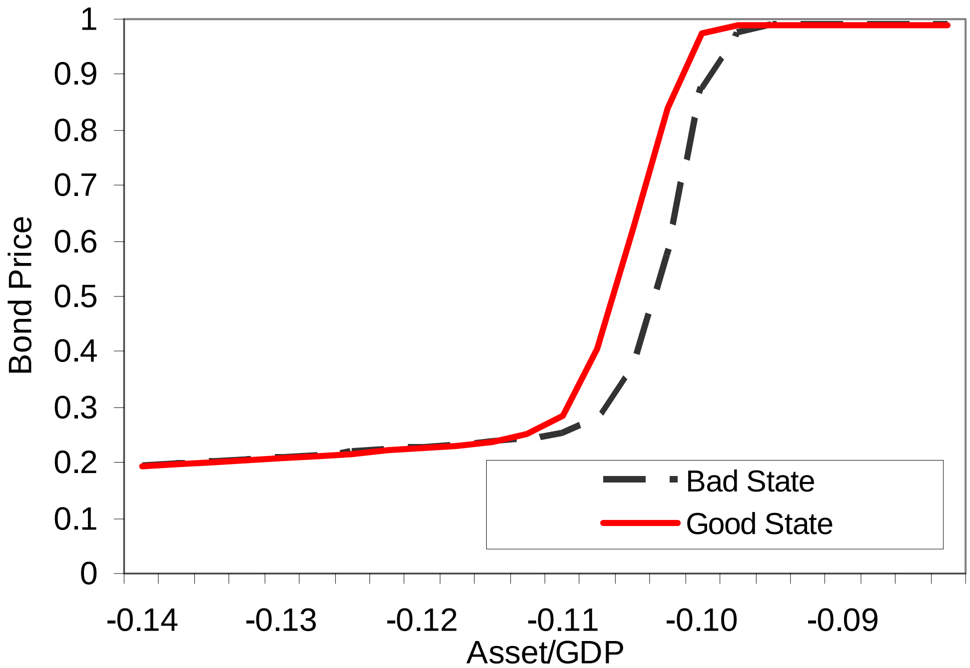


Figure 4: Bond Price in Benchmark Model

Note: The figure shows the bond price as a function of assets for the highest and lowest value of endowment shock. The solid line corresponds to bond price function with the low endowment shock (bad state), and the dashed line corresponds to the bond price function with the high endowment shock (good state)

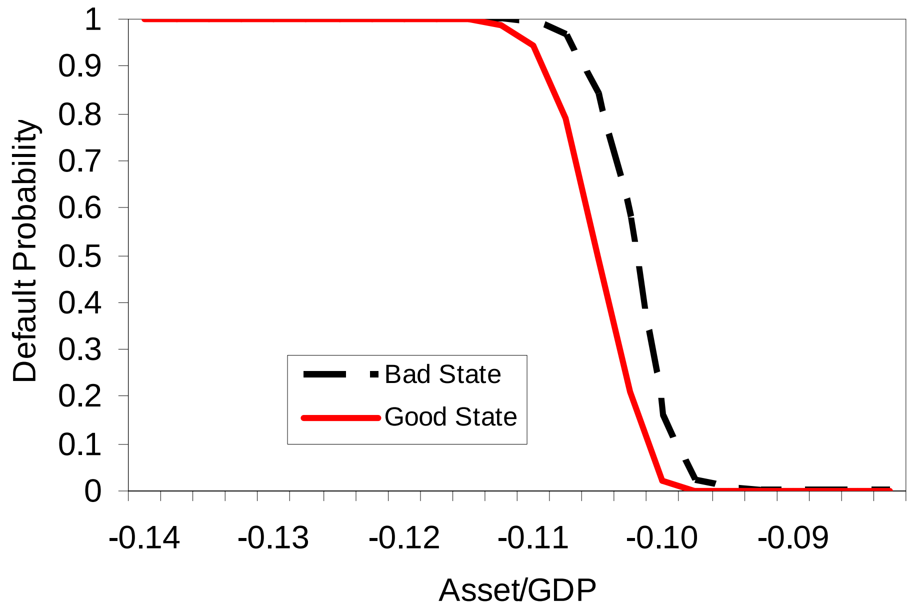


Figure 5: Default Probability in Benchmark Model

The figure plots the default probability as a function of assets for the highest and lowest value of endowment shock. The solid line corresponds to default probability function with the low endowment shock (bad state), and the dashed line corresponds to the default probability function with the high endowment shock (good state)

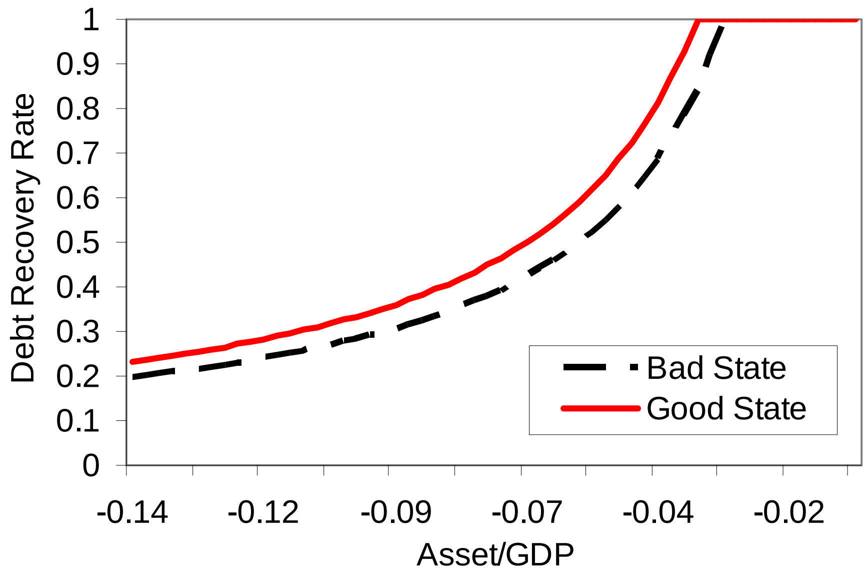


Figure 6: Recovery Rate in Benchmark Model

The figure plots the debt recovery rates as a function of assets for the highest and lowest value of endowment shock. The solid line shows the debt recovery rates function with the low endowment shock (bad state), and the dashed line corresponds to the debt recovery rates function with the high endowment shock (good state)

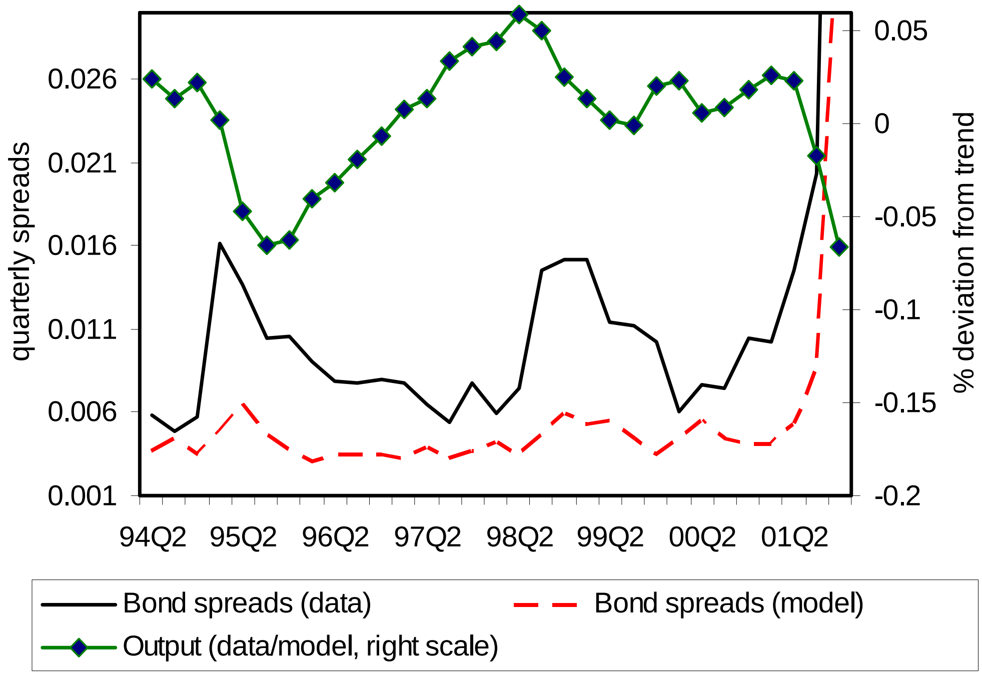


Figure 7: Output and Bond Spreads in the Data and in the Model (1994.2-2001.4)

The figure plots the bond spreads and output in the data and in the model when the output data is fed in the simulation. The solid line is the bond spreads data (3-year bond spreads). The dashed line is the bond spreads generated by the model. The diamond line is the output data for Argentina, which is log HP detrended.

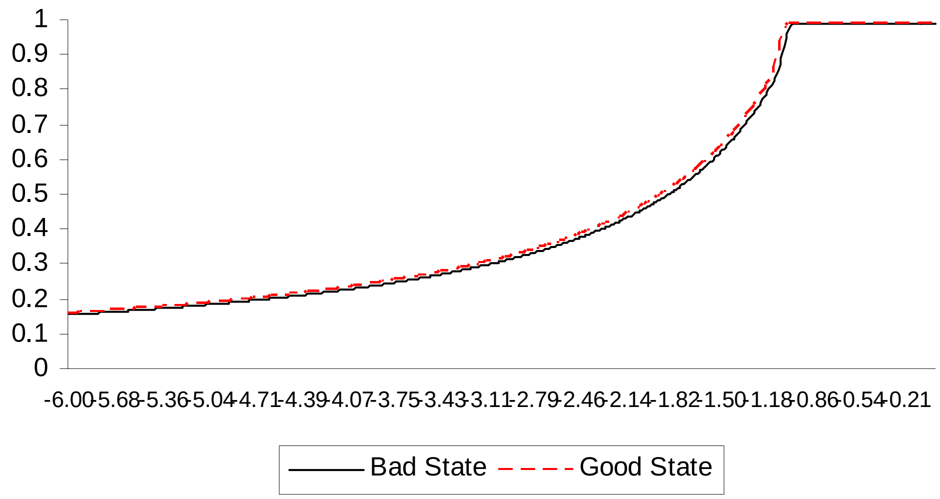


Figure 8: Bond Price for $\theta = 0.35$

The figure plots the bond price as a function of assets for the highest and lowest value of endowment shock when the country's bargaining power is 0.35. The solid line corresponds to bond price function with the low endowment shock (bad state), and the dashed line corresponds to the bond price function with the high endowment shock (good state)

# Appendix

## A. Proofs

**Construction of Bond Price Correspondence.** Given $\alpha(b,y)$ and $q(b,y)$, we first define $M^*(b',y)(q)$ as the set of default probabilities consistent with the sovereign's default choice for debt level $b'$ and type $y$, given the price schedule $q$. If $m_y \in M^*(b',y)(q)$ then there is a $d^*(b',y',q)$ such that $\int_Y d^*(b',y',q)\,d\mu_y(y') = m_y$. We then define the equilibrium bond price correspondence given debt recovery rate schedule $\alpha(b,y)$

$$\varphi(q)\left(b',y;\alpha\right) = \begin{cases} \{x : x = \overline{q}\,[m\cdot\alpha(b',y) + (1-m)]\} \text{ for some } m \in M^*(b',y)(q) & \text{if } b' \geq 0 \\ \overline{q} & \text{if } b' \leq 0 \end{cases}$$

Then, $q^*$ is an equilibrium price vector if $q^*(b',y;\alpha) \in \varphi(q^*)(b',y;\alpha)$ for all $b',y \in B\times Y$, given $\alpha \in C^0$. ■

**Lemma 1** *Given $\alpha$, for each $q$, $\varphi(q)(b',y;\alpha)$ is a closed interval in $R$ and the correspondence $\varphi(q)(b',y;\alpha)$ has a closed graph.*

**Proof.** We can show that $\varphi(q)(b',y;\alpha)$ is a closed interval in $R$ using a similar proof for Lemma App 5 in Chatterjee, Corbae, Nakajima and Rios-Rull (2002). We also follow the proof for Lemma 8 in Chatterjee, Corbae, Nakajima and Rios-Rull (2002) to show that the correspondence $\varphi(q)(b',y;\alpha)$ has a closed graph. ■

**Proof of Step 1 in Theorem 1.** Given $\alpha$, for each $b',y$, $\varphi(q)(b',y;\alpha)$ is closed-valued with compact Hausdorff range space, therefore $\varphi(q)(b',y;\alpha)$ is an upper hemicontinuous correspondence (see Aliprantis and Border (1999) Thm 16.12 pp. 529). For any $q \in Q$, let $\varphi(q) \subset Q$ be the product correspondence $\Pi_{b',y\in B\times Y}\varphi(q)(b',y;\alpha)$. Since $\varphi(q)(b',y;\alpha)$ is convex-valued for each $b'$,$y$, $\varphi(q)$ is convex-valued as well. Furthermore, since $\varphi(q)(b',y;\alpha)$ is upper hemicontinuous with compact values for each $b',y$, the product correspondence $\varphi(q)$ is upper hemicontinuous with compact values as well. (see Aliprantis and Border (1999), Thm 16.28, pp. 537). Therefore, $\varphi(q;\alpha)$ is a closed convex-valued correspondence that takes elements of the compact, convex set $Q$ and returns sets in $Q$. By Kakutani-Fan-Glicksberg FPT (Corollary of Halpern-Bergman FPT, see Aliprantis and Border (1999), Thm 16.51, pp. 550) there is $q^* \in Q$ such that $q^* \in \varphi(q^*)$. In other words, given $\alpha$, there exists $q^*$ such that $q^*(b',y)(\alpha) \in \varphi(q^*)(b',y;\alpha)$ for all $b',y \in B\times Y$. Hence, given $\alpha$, a competitive equilibrium exists. ■

**Construction of Debt Recovery Correspondence.** Given $q(b,y)$, we first define the correspondence for the domain of debt recovery rate in renegotiation $\omega(\alpha)(b,y;q) : A \to [0,1]$

$$\omega(\alpha)(b,y;q) = \left\{ a \in [0,1] : \Delta^B(a;b,y,\alpha,q) \geq 0, \Delta^L(a;b,y,\alpha,q) \geq 0 \right\}$$

We then define the debt recovery correspondence by

$$\psi(\alpha)(b,y;q) = \arg\max_{a \in \omega(\alpha)(b,y;q)} \left[ \left(\Delta^B(a;b,y,\alpha,q)\right)^{\theta} \left(\Delta^L(a;b,y,\alpha,q)\right)^{1-\theta} \right]$$

■

**Lemma 2** *The correspondence $\omega(\alpha)(b,y;q)$ is a continuous correspondence with nonempty compact value.*

**Proof.** Step 1: We establish that $\omega(\alpha)(b,y;q)$ is an upper hemicontinuous correspondence. First, $\omega(\alpha)(b,y;q)$ is closed-valued. If $a^n \in \omega(\alpha)(b,y;q)$ and $a^n \to a^*$, then $\lim_{n\to\infty}$ $\Delta^B(a^n;b,y,\alpha,q) = \Delta^B(a^*;b,y,\alpha,q) \geq 0$ and $\lim_{n\to\infty} \Delta^L(a^n;b,y,\alpha,q) = \Delta^B(a^*;b,y,\alpha,q) \geq 0$ because $\Delta^B(a,\alpha)(b,y;q)$ and $\Delta^L(a,\alpha)(b,y;q)$ are continuous in $a$. Second, $\omega(\alpha)(b,y;q)$ has a closed graph. For $\alpha^n \to \alpha, a^n \to a$, Let $\{(\alpha^n, a^n)\}$ be any sequence in $A \times [0,1]$ such that $a^n \in \omega(\alpha^n)(b,y;q)$ and $\{(\alpha^n, a^n)\}$ converges to $(\alpha, a)$, $\alpha \in Q$. We need to establish that $a \in \omega(\alpha;q)(b,y)$. We need to show that $\Delta^B(a;b,y,\alpha,q) \geq 0, \Delta^L(a;b,y,\alpha,q) \geq 0$. This is true because $\Delta^B(a;b,y,\alpha,q)$ and $\Delta^L(a;b,y,\alpha,q)$ are continuous in $a$ and $\alpha$, $\lim_{n\to\infty} \Delta^B(a^n;b,y,\alpha^n,q)$ $= \Delta^B(a;b,y,\alpha,q) \geq 0$ and $\lim_{n\to\infty} \Delta^L(a^n;b,y,\alpha^n,q) = \Delta^L(a;b,y,\alpha,q) \geq 0$. Last, the correspondence $\omega(\alpha;q)(b,y)$ has compact Hausdorff range space. Hence by closed graph theorem ( see Aliprantis and Border (1999), Thm 16.12, pp. 529), $\omega(\alpha;q)(b,y)$ is upper hemicontinuous.

Step 2: We establish that $\omega(\alpha)(b,y;q)$ is a lower hemicontinuous correspondence. For any closed subset $F$ of $[0,1]$, we want to show that the upper inverse of $F$, $\omega^u(F)(b,y)$ is also closed. The upper inverse of $F$ is $\omega^u(F)(b,y) = \{\alpha \in A : \omega(\alpha)(b,y) \subset F\}$. Let $\alpha_n \in \omega^u(F)(b,y)$ and $\alpha_n \to \alpha$. Because $\omega(\alpha_n)(b,y) \subset F$, and $\Delta^B(:;b,y,\alpha_n)$ and $\Delta^L(:;b,y,\alpha_n)$ are continuous, $\omega(\alpha_n)(b,y) \to \omega(\alpha)(b,y)$. Since $F$ is closed set, $\omega(\alpha)(b,y) \in F$. Therefore,

$\alpha \in \omega^u(F)(b,y)$. The upper inverse of any closed set is also closed. Hence $\omega(\alpha)(b,y;q)$ is a lower hemicontinuous correspondence.

Step 3: From step 1 and 2, $\omega(\alpha;q)(b,y)$ is a continuous correspondence. Because

$\omega(\alpha)(b,y;q)$ is closed valued and bounded, $\omega(\alpha)(b,y;q)$ also has compact value.

$\omega(\alpha)(b,y;q)$ is nonempty because $0 \in \omega(\alpha)(b,y;q)$. ■

**Proof of Step 2 in Theorem 1.** Given $q$, for each $b', y$, because $[0,1]$ is Hausdorff and the correspondence $\omega(\alpha)(b, y; q)$ is a continuous correspondence with nonempty compact value, $\psi(\alpha)(b', y; q)$ is an upper hemicontinuous correspondence with nonempty compact values from Berge's Maximum Theorem (see Aliprantis and Border (1999) Thm 16.31 pp. 539). For any $\alpha \in A$, let $\psi(\alpha; a) \subset A$ be the product correspondence $\Pi_{b',y \in L \times Y} \psi(\alpha)(b', y; q)$. Since $\psi(q)(b', y; \alpha)$ is upper hemicontinuous with compact values for each $b', y$, the product correspondence $\psi(q; \alpha)$ is upper hemicontinuous with compact values as well. (see Aliprantis and Border (1999), Thm 16.28, pp. 537). For bargaining power $\theta \in \Theta$, $\psi(\alpha)(b', y; q)$ is single-valued, so is the product correspondence $\psi(q; \alpha)$. Therefore, $\psi(q; \alpha)$ is a closed convex-valued correspondence that takes elements of the compact, convex set $A$ and returns sets in $A$. By Kakutani-Fan-Glicksberg FPT (Corollary of Halpern-Bergman FPT, see Aliprantis and Border (1999), Thm 16.51, pp. 550) there is $\alpha^* \in A$ such that $\alpha^* \in \omega(\alpha^*; q)$. In other words, given $q$, there exists $\alpha^*$ such that $\alpha^*(b', y)(\alpha) \in \psi(\alpha^*)(b', y; q)$ for all $b', y \in L \times Y$. ■

**Proof of Step 3 in Theorem 1.** We construct a functional mapping correspondence $T : Q \times \mathcal{A} \rightarrow Q \times \mathcal{A}$ such that

$$T(q, \alpha)(b, y) = \begin{bmatrix} \varphi(q)(b', y; q, \alpha) \\ \psi(\alpha)(b, y; q, \alpha) \end{bmatrix}$$

Because $\varphi(q)(b', y; q, \alpha)$ and $\psi(\alpha)(b, y; q, \alpha)$ are upper hemicontinuous, $T(q, \alpha)$ is upper hemicontinous. (see Aliprantis and Border (1999) Thm 16.23 pp. 539). Therefore, the correspondence $T(q, \alpha)$ has a closed graph. We can also show that $T(q, \alpha)$ is convex valued. Suppose $(q_1, \alpha_1) \in T(q, \alpha)$ and $(q_2, \alpha_2) \in T(q, \alpha)$. Because $\varphi(q)(b', y; q, \alpha)$ is convex valued, $\gamma q_1 + (1-\gamma) q_2 \in \varphi(q; \alpha)$. Because $\psi(\alpha)(b, y; q, \alpha)$ is single valued, $\alpha_1 = \alpha_2 = \gamma\alpha_1 + (1-\gamma)\alpha_2 \in \psi(\alpha; q)$. Therefore, $(\gamma q_1 + (1-\gamma) q_2, \gamma\alpha_1 + (1-\gamma)\alpha_2)$. Hence, we can apply Kakutani's fixed point theorem and show the existence of a fixed point.

$$T(q^*, \alpha)(b, y) = (q^*, \alpha^*)$$

A recursive equilibrium exists. ■

**Proof of Theorem 2.** Define the functional mapping $M$ for the sovereign's value function. For $b \geq 0,\ h = 0$,

$$M\left(v\right)\left(b,0,y\right) = \max_{b' \in B} u\left(y + b - q\left(b',y\right)b'\right) + \beta \int_Y v\left(b',0,y'\right) d\mu\left(y'|y\right)$$

For $b < 0,\ h = 0$,

$$M\left(v\right)\left(b,0,y\right) = \max \left\{ \begin{array}{l} \max_{b' \in B} u\left(y + b - q\left(b',y\right)b'\right) + \beta \int_Y v\left(b',0,y'\right) d\mu\left(y'|y\right) \\ u\left(\left(1-\lambda_d\right)y\right) + \beta \int_Y v\left(\left[\alpha\left(b,y\right)\left(1+r\right)b\right],1,y'\right) d\mu\left(y'|y\right) \end{array} \right\}$$

For $h = 1$,

$$M\left(v\right)\left(b,1,y\right) = \max_{b' \in [b,0]} u\left(\left(1-\lambda_d\right)y + b - \frac{b'}{1+r}\right) + \beta \int_Y v\left(b',1,y'\right) d\mu\left(y'|y\right)$$

where

$$v\left(0,1,y'\right) = v\left(0,0,y'\right)$$

It is easy to show that $M$ is a contraction mapping and its fixed point is the sovereign's value function $v^*$. Therefore, to prove the theorem is equivalent to show that $M\left(v\right)$ maps an increasing function to an increasing function.

Assume $v\left(b,h,y\right)$ is an increasing function in $b$. Given state variables $\left(b,h,y\right)$, denote the budget set by $B\left(b,h,y,d\right)$ for the sovereign which makes its default decision $d$. Let $b^0 \leq b^1$ and $b^0, b^1 \in L$. For $h = 0$, if $0 > b^0 \geq b^1$, $B\left(b^0,0,y,0\right) \supseteq B\left(b^1,0,y,0\right)$ when $d = 0$, so $M\left(v\right)\left(b,0,y\right)$ increases in $b$. When $d = 1$, $c\left(b^0,0,y,1\right) = c\left(b^1,0,y,1\right) = \left(1-\lambda_d\right)y$ and $\alpha\left(b^0,y\right)b^0 \geq \alpha\left(b^1,y\right)b^1$ by assumption on $\alpha \in A$, so $M\left(v\right)\left(b,1,y\right)$ increases in $b$. If $b^0 \geq 0 \geq b^1$, then $B\left(b^0,0,y,0\right) \supseteq B\left(b^1,0,y,d\right)$; and if $b^0 \geq b^1 > 0$, then $B\left(b^0,0,y,0\right) \supseteq B\left(b^1,0,y,0\right)$. For both cases, $M\left(v\right)\left(b,1,y\right)$ increases in $b$. For $h = 1$, if $0 > b^0 \geq b^1$, then $c^0 \in \left[\left(1-\lambda_d\right)y - b^0, \left(1-\lambda_d\right)y - \frac{rl^0}{1+r}\right]$ and $c^1 \in \left[\left(1-\lambda_d\right)y - b^1, \left(1-\lambda_d\right)y - \frac{rl^1}{1+r}\right]$. Because $\left(1-\lambda_d\right)y + b^0 > \left(1-\lambda_d\right)y + b^1$ and $\left(1-\lambda_d\right)y + \frac{rl^0}{1+r} > \left(1-\lambda_d\right)y + \frac{rl^1}{1+r}$, if the optimal $c^1 \geq \left(1-\lambda_d\right)y - b^0$, the valid $B\left(b^0,1,y,d\right) \supseteq B\left(b^1,1,y,d\right)$, $v\left(b^0,1,y\right) \geq v\left(b^1,1,y\right)$. If the optimal $c^1 < \left(1-\lambda_d\right)y + b^0$, $u\left(c^1\right) < u\left(c^0\right)$, and $b' \leq 0$. So, $M\left(v\right)\left(b,1,y\right)$ increases in $b$.

We have shown that $M\left(v\right)\left(b,h,y,1\right)$ increases in $b$. Therefore, the fixed point $v^*\left(b,h,y\right)$ of functional mapping $M$ is also increasing in $b$. ■

**Proof of Theorem 3.** Because $\Delta^B\left(a;b,y\right)$ and $\Delta^L\left(a;b,y\right)$ are both function of $al$, define $\Delta^B\left(a;b,y\right) = \widetilde{\Delta}^B\left(ab;y\right)$, and $\Delta^L\left(a;b,y\right) = \widetilde{\Delta}^L\left(ab;y\right)$. The bargaining problem is equivalent to the following

$$\max_{al} \left[ \left( \widetilde{\Delta}^B (ab; y) \right)^\theta \left( \widetilde{\Delta}^L (ab; y) \right)^{1-\theta} \right]$$
$$\text{s.t. } \widetilde{\Delta}^B (al; y) \geq 0$$
$$\widetilde{\Delta}^L (al; y) \geq 0$$

where the functional form of $\widetilde{\Delta}^B (al; y)$ and $\widetilde{\Delta}^L (al; y)$ are simple transformation of $\Delta^B (a; b, y)$ and $\Delta^L (a; b, y)$. For bargaining power $\theta \in \Theta$, given $(b, y)$, the renegotiation surplus has a unique optimum. In the transformed problem, the optimal solution is solely a function of endowment $y$ and we denote it as $b_y \leq 0$. The bargaining over debt reduction has constraint $a \in [0, 1]$ . When $b \leq b_y$, the constraint $a \in [0, 1]$ is not binding, so $a = \frac{b_y}{b}$. If $b \geq b_y$, the constraint $a \in [0, 1]$ is binding, so $a = 1$.

Therefore,

$$\psi (\alpha; q) (b, y) = \begin{cases} \frac{b_y}{b} \text{ if } b \leq b_y \\ 1 \text{ if } b \geq b_y \end{cases}$$

Because an equilibrium debt recovery rate function is a fixed point of the correspondence $\psi (\alpha; q) (b, y)$, the debt recovery rate also satisfies

$$\alpha (b, y) = \begin{cases} \frac{b_y}{b} \text{ if } b \leq b_y \\ 1 \text{ if } b \geq b_y \end{cases}$$

■

**Proof of Theorem 4.** >From Theorem 2, $v (b, 0, y)$ is increasing in $b$. Since the equilibrium debt recovery schedule satisfies Theorem 3, given endowment $y$, the debt arrear after defaulting is independent of $b$. Thus, the utility from defaulting is independent of $b$. Therefore, if $v \left( b^1, 0, y \right) = u ((1 - \lambda_d) y) + \beta v (b_y, 1, y)$, then it must be the case that $v \left( b^0, 0, y \right) = u ((1 - \lambda_d) y) + \beta v (b_y, 1, y)$. Hence, any $y$ that belongs in $\overline{D} \left( b^1 \right)$ must also belong in $\overline{D} \left( b^0 \right)$. ■

**Proof of Theorem 5.** Let $d^* (b, 0, y')$ be the equilibrium default functions. Equilibrium default probability is then given by

$$p \left( b', y \right) = \int_G d^* \left( b', 0, y' \right) d\mu \left( y' | y \right)$$

>From Theorem 4, if $d^*\left(b^1, 0, y'\right) = 1$, then $d^*\left(b^0, 0, y'\right) = 1$. Therefore,

$$p\left(b^0, y\right) \geq p\left(b^1, y\right)$$

■

**Theorem 8** *Given $y$, for $b^0 < b^1 \leq b_y \leq 0$, equilibrium bond price $q\left(b^0, y\right) \leq q\left(b^1, y\right)$.*

**Proof of Theorem 6.** Let $p^*(b, y)$ be the equilibrium default probability function and $\alpha^*(b, y)$ be the equilibrium debt recovery schedule. The expected debt recovery rate is then given by

$$\gamma\left(b', y\right) = \frac{\int_Y d\left(b', 0, y'\right) \alpha\left(b', y'\right) d\mu\left(y'|y\right)}{\int_Y d\left(b', 0, y'\right) d\mu\left(y'|y\right)}$$

>From Theorem 3, given $y$, for $b^0 < b^1 \leq b_y \leq 0$, $\alpha^*\left(b^0, y\right) < \alpha^*\left(b^1, y\right) \leq 1$. Therefore, the equilibrium expected debt recovery rate $\gamma^*\left(b^0, y\right) < \gamma^*\left(b^1, y\right) \leq 1$. And from Theorem 5, $p^*\left(b^0, y\right) \geq p^*\left(b^1, y\right)$. For the country's indebtedness, the equilibrium bond price is given by

$$\begin{aligned} q\left(b', y\right) &= \frac{1 - p\left(b', y\right)}{1 + r} + \frac{p\left(b', y\right) \cdot \gamma\left(b', y\right)}{1 + r} \\ &= \frac{1 - p\left(b', y\right)\left(1 - \gamma\left(b', y\right)\right)}{1 + r} \end{aligned}$$

Hence, we obtain that

$$q\left(b^0, y\right) \leq q\left(b^1, y\right)$$

■

**Proof of Theorem 7.** Because $u(.)$ is concave function, given $b$, for all $b' \leq 0$,

$$\frac{d}{dl'}\left[u\left(\left(1 - \lambda_d\right) y + b\right) - u\left(\left(1 - \lambda_d\right) y + b - b'/\left(1 + r\right)\right)\right] \geq 0$$

If $b \in L_{--}$ and $b \geq b$, for all $b' < 0$,

$$\begin{aligned} & u\left(\left(1 - \lambda_d\right) y + b\right) - u\left(\left(1 - \lambda_d\right) y + b - b'/\left(1 + r\right)\right) \\ \geq\ & u\left(\left(1 - \lambda_d\right) y + b\right) + u\left(\left(1 - \lambda_d\right) y + b - b'/\left(1 + r\right)\right) \\ \geq\ & \beta y^{1-\sigma} \int_Y v\left(0, 0, y'\right) d\mu\left(y'|y\right) + \beta \int_Y v\left(b', 1, y'\right) d\mu\left(y'|y\right) \end{aligned}$$

Thus,

$$\begin{aligned} & u\left(\left(1-\lambda_d\right) y+b\right)+\beta \int_Y v\left(0,0, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \\ \geq\ & \sup_{b^{\prime}<0} u\left(\left(1-\lambda_d\right) y+b-b^{\prime} /(1+r)\right)+\beta \int_Y v\left(b^{\prime}, 1, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \end{aligned}$$

which implies

$$v(b, 1, y)=u\left(\left(1-\lambda_d\right) y+b\right)+\beta \int_Y v\left(0,0, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right)$$

If $b \in L_{--}$ and $b \geq b$, for all $b' < 0$, suppose

$$\begin{aligned} & u\left(\left(1-\lambda_d\right) y+b\right)+\beta \int_Y v\left(0,0, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \\ >\ & \sup_{b^{\prime}<0} u\left(\left(1-\lambda_d\right) y+b-b^{\prime} /(1+r)\right)+\beta \int_Y v\left(b^{\prime}, 1, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \end{aligned}$$

then, according the above analysis,

$$\begin{aligned} & u\left(\left(1-\lambda_d\right) y+b\right)+\beta \int_Y v\left(0,0, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \\ >\ & \sup_{b^{\prime}<0} u\left(\left(1-\lambda_d\right) y+b-b^{\prime} /(1+r)\right)+\beta \int_Y v\left(b^{\prime}, 1, y^{\prime}\right) d \mu\left(y^{\prime} \mid y\right) \end{aligned}$$

contradiction. ■

## B. Computational Procedure

This appendix outlines the procedure to compute the equilibrium of the model economy.

First we set grids on the spaces of asset holdings and endowment. The asset space is discretized into 600 grids, and we use 51 equally spaced grids to discretize the space for endowment shocks. The limits of the asset space are set to ensure that the limits do not bind in equilibrium. The limits of endowment space are large to include big deviations from the average value of shocks. We approximate the distribution of endowment shock using a discrete Markov transition matrix. Then, we use the following procedure to compute an equilibrium.

1. Guess an initial debt recovery schedule $\alpha_0$. Our initial guess is 1.

2. Given a debt recovery schedule $\alpha_0$, we solve for equilibrium bond price $q_0$

(a) Guess an initial price of discounted loans $q_{00}$. Our initial guess is the risk free bond price 1/(1+r*).

(b) Given a price for loans, $q_{00}$, we solve the country's optimization problem. This procedure includes finding the value function as well as the default decisions. We first guess value function $v^0$ and iterate it using the Bellman equation to find the fixed out $v^*$, given bond price and debt recovery rates. For the problem of a country with debt and a good credit score, we also obtain the optimal default choice, which requires comparison between the implications of defaulting and not defaulting. This comparison also enables us to calculate the corresponding default set.

(c) Using the default set derived in step (b) and the zero profit condition for international investors, we compute the new price of discounted bonds $q_{01}$. If $q_{01}$ is sufficiently close to $q_{00}$, stop iterating on $q$ and go on to the step 3, else go back to step (b).

3. Solve the bargaining problem given converged bond price $q_0^*$ and compute the new debt recovery schedule for every $(b, y)$. If the new debt recovery schedule $\alpha_{01}$ is sufficiently close to $\alpha_{00}$, stop iterating on $\alpha$, else, go back to step 2.